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Wellbeing Opportunity

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Wellbeing opportunity

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Abstract

This paper presents an evaluation protocol for assessing the relative wellbeing of different population subgroups using subjective wellbeing (SWB) as primary data. The key element of our approach is the comparison of wellbeing distributions in terms of the probability of achieving better outcomes, which we interpret as a measure of wellbeing opportunity. We adopt an ordinal approach by treating SWB levels as ordered categories rather than magnitudes.

Keywords: Subjective wellbeing; domination probability; ordinality; wellop function; wellbeing opportunity comparisons.

JEL classification: I30, I31

Resumen

Este artículo presenta una forma de evaluar el bienestar relativo de diferentes subgrupos poblacionales, utilizando el bienestar subjetivo (BS) como dato primario. El elemento clave de nuestro enfoque es la comparación de las distribuciones de bienestar en términos de la probabilidad de lograr mejores resultados, que interpretamos como una medida de la oportunidad de bienestar. Adoptamos un enfoque ordinal al tratar los niveles de BS como categorías ordenadas en lugar de magnitudes.

Palabras clave: bienestar subjetivo; probabilidad de dominación; ordinalidad; función *wellop* (de bienestar); comparaciones de oportunidades de bienestar.

Clasificación JEL: I30, I31

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"The ideas which are here expressed so laboriously are extremely simple and should be obvious. The difficulty lies, not in the new ideas, but in escaping from the old ones, which ramify, for those brought up as most of us have been, into every corner of our minds".

J. M. KEYNES (1935)

1.

Introducción y motivación

1.1 *Subjective wellbeing measures*

Subjective wellbeing (SWB) is a concept intended to capture people's happiness or life satisfaction. The OECD (2013) defines it as "good mental states, including all of the various evaluations, positive and negative, that people make of their lives, and the affective reactions of people to their experiences". SWB is usually measured by a numerical scale (e.g., the $[0{,}10]$ Likert scale), in response to questions such as "Overall, how satisfied are you with your life nowadays?". Besides overall evaluations of happiness or life satisfaction, specific SWB measures assess particular aspects, such as job satisfaction, mental health, or social engagement.

Measures of subjective wellbeing are systematically elaborated by Gallup World Poll since 2003 and are the basis for the UN World Happiness Report, which is an interesting complement of the Human Development Reports, which rely on objective data. They are also part of the OECD's welfare assessment on country members, and appear in the European Union official statistics (EU-SILC).[2] Subjective wellbeing has become a common tool for the evaluation of policy measures in several countries, including Austria, Belgium, Ecuador, Finland, Italy, Israel and the United Kingdom (Global Happiness Council, 2018, ch. 8). The UK, in particular, has been very active in the application of this approach (Dolan, 2011, HM Treasury, 2021), taking the Wellby as the key measure (Frijters et al, 2024). Academic interest in this topic has also steadily grown, dealing with a wide range of related issues (Layard, 2020).

There is consensus that SWB measures offer advantages in evaluating the impact of policy interventions (Frijters & Krekel, 2021; McGuire, Dupret & Plant, 2022; Mahoney, 2023). SWB provides a comprehensive metric that captures the overall benefit that individuals derive from policy interventions, eliminating the need to model the interplay between the variables that may affect individual wellbeing, such as health, income, or freedom (McGuire, Dupret & Plant, 2022; Coleman, 2022; Plant, 2022). SWB measures are easy to collect, widely applicable, and have help uncovering

[2] Eurostat included in 2021 a life satisfaction question into the EU Survey on Income and Living Conditions (EU-SILC) household survey, after a couple of specific surveys run in 2013 and in 2018.

the relevance of some latent variables, often underestimated, such as mental health or underemployment (Dolan & Metcalfe, 2012; Clark et al., 2018). Moreover, those measures satisfy the standard four psychometric properties that are usually required: reliability (stability of scores over time and contexts), validity (providing sensible evaluations), sensitivity (scores are responsive to changes in conditions), and discriminant validity (proper correlations with other concepts like optimism and self-esteem). See Kahneman & Krueger (2006), Clark et al. (2018), Dolan et al. (2008), Frijters & Krekel (2021).

There are some sources of concern, though, regarding the use of SWB measures obtained from numerical scales to assess collective wellbeing (see Samuelsson et al, 2023, for a discussion). First, a problem always present in this type of empirical studies, the sensitivity of the responses to the framework, including the precise wording of the questions, the truthfulness of the answers, or their dependence on the environment (Krueger & Schkade, 2008; Benjamin et al, 2023).

Second, the interpretation of those scores. The endpoints and the "neutral point" of the numerical scale may vary in meaning across individuals, as they are linked to personal and social factors, such as health status, past experiences, expectations, family and socioeconomic background, life stage, or cultural patterns, among others. Additionally, individuals may assign different implicit weights to wellbeing components, considering different time frames, or take different social environments as reference when reporting SWB scores (Benjamin et al., 2023). These differences are expected to be smaller within more homogenous population subgroups, where individuals share similar cultural and socioeconomic traits, allowing for more consistent interpretation of the scale.

A third source of concern refers to the implicit cardinality assumption, which is extremely handy from an operational viewpoint, as it permits treating SWB scores as magnitudes and apply conventional mathematical and statistical tools to analyse societal wellbeing, its relationship with explanatory variables, welfare patterns, and changes induced by policy interventions. This assumption implies that, for each individual, the distance between any two consecutive steps in the wellbeing scale is the same. That is, in a [0, 10] Likert scale an 8/10 score represents a level of wellbeing twice of 4/10. This is a strong assumption that is not clearly justified, because people's sensitivity may well change along the different parts of the scale, akin to conventional utility functions (see Ferrer-i-Carbonell & Frijters, 2004). When combined with the interpersonal comparability requirement (i.e., individuals with the same wellbeing score are equally happy), this assumption implies that an individual with an 8/10 wellbeing score is twice as happy as another individual with a 4/10 score. This is a much stronger assumption indeed but follows inevitably maintaining both the linearity assumption and the interpersonal comparability.[3]

[3] A personal experience, for what it's worth: I coordinated the evaluation of research projects in the Spanish national research program for several years, specifically in the fields of Economics and Law. Each project was reviewed by two referees, who provided both a qualitative assessment and a numerical score ranging from 0 to 10. Economists and lawyers had noticeably different perceptions of what constituted a "neutral point" (essentially, a passing score). For economists, scores of 7–8 indicated outstanding projects. For lawyers, the same scores signified poor projects that did not merit funding. Economists applied scores in a linear fashion, while lawyers did not. One might assume that lawyers were simply more demanding in their funding criteria. However, their qualitative evaluations suggested the opposite. They seemed reluctant to assign scores below 6 or 7, as if doing so were inappropriate. As a result, discussions only took place for projects scoring above 8.5 in Law, whereas in Economics, projects with scores as low as 6 were open for debate. Ultimately, we decided to eliminate numerical scores in the panel that assessed both types of projects and relied solely on qualitative evaluations.

We propose in this paper an evaluation protocol for collective wellbeing that allows for an ordinal treatment of individual wellbeing, following the standard treatment of utility functions.[4] The evaluation of societal wellbeing is obtained by comparing the wellbeing distributions of social groups, based on the probability of achieving higher levels of wellbeing. This approach yields a complete, transitive and cardinal assessment of the wellbeing of those social groups being compared, which we interpret as a measure of their relative wellbeing opportunity. We call this protocol the wellop function, as a shorthand for "wellbeing opportunity function".

1.2 *The probability of achieving higher levels of wellbeing*

The key element of our proposal is the comparison of social groups in terms of their probabilities of achieving higher levels of wellbeing. This strategy permits to obtain a cardinal and well-defined measure of societal wellbeing, without having to assume that subjective wellbeing is linear. Moreover, those probabilities can be interpreted in terms wellbeing opportunity, representing what the different social groups offer to a newcomer from the veil of ignorance viewpoint (and idea borrowed from Herrero & Villar, 2021). The backbone of our approach is the assumption that a social group offers better opportunities of wellbeing than another, if a randomly selected member of the first group is more likely to achieve a higher wellbeing level than a randomly selected member of the second group. Since these probabilities depend on the number of individuals above each level of the wellbeing scale, this comparison aligns with the utilitarian principle of "more happiness for more people" (Bentham, 1789). Such an elementary statement already points out that we should care about welfare distributions rather than average values, as both the total level of happiness and the number of happy people are to be considered.

The idea of comparing pairs of outcome distributions in terms of the probability of obtaining better outcomes appears in the literature dealing with the analysis of discrimination. Gastwirth (1975) introduced this principle to study wage discrimination between men and women, by calculating the "probability that a randomly chosen woman earns at least as much as a randomly selected man" (Cf. p. 32). The deviation of this probability from 0.5 is a measure of discrimination. In a similar vein, Lieberson (1976) uses this notion to measure segregation between two social groups, A and B. He builds an indicator comparing the probability that an agent from group A achieves higher outcomes than one from group B and the complementary event. Lieberson measures discrimination by the absolute value of the difference of those probabilities.

This powerful idea presents the inconvenient that it cannot be directly applied to more than two groups, because this criterion is not transitive. That is, for more than two groups we may find cycles in the outcomes of pairwise comparisons. There are several ways to circumvent this problem. One is by recurring to an indirect evaluation, comparing each group with a distinguished reference group (LeBreton et al. 2012). A different solution is that proposed by Herrero & Villar (2018), who apply an extended principle of proportionality, by which the evaluation of two groups

[4] In terms of social choice parlance, we would be substituting full cardinal unit comparability by ordinal level comparability, which is much weaker an assumption. Several arguments can be found in favour of cardinal unit comparability (Frijters & Krekel, 2021), including logical reasoning, joint use of language, and empirical tests showing consistent recall of scores.

is proportional to their corresponding probabilities of obtaining better outcomes. Such an extension, named balanced worth, can be regarded as the result of a consistency requirement and corresponds to the stable solution of a Markov chain.

We propose here another way of handling this transitivity problem, which consists of making pairwise comparisons between each group and the result of merging all others into a single one. The formula we obtain is intuitive, transparent, computationally simple, and can be easily characterised in terms of elementary properties.

2.

Wellbeing opportunity

2.1 *The framework*

We now describe a way of measuring wellbeing opportunity, that compares subjective wellbeing distributions in terms of their probabilities of achieving better results. Assume that our informational inputs are the data obtained from a numerical scale made of an interval of integer numbers, $[a, b] \subset \mathbb{Z}$ (e.g., a conventional [0, 10] Likert scale), in the understanding wellbeing levels are categories, rather than magnitudes (i.e., we adopt an ordinal approach). Consider a society N, with n individuals, which is partitioned into m population subgroups, with n_i individuals each, $i = 1,2, \dots, m$. We can think of society N as a country and the population subgroups as different types of individuals –according to demographic or socioeconomic profiles–, different regions, or the same population in different time periods or in different situations (e.g., before and after a policy intervention).[5]

Let $[a, b]$ stand for the numerical scale that describes SWB categories, ordered from worst to best. We denote by n_{ih} the number of individuals in group i with wellbeing level h, and by $s_{ih} = \frac{n_{ih}}{n_i} \geq 0$ the corresponding share of individuals in i at this level. Then, we let $s(i) = (s_{ia}, \dots, s_{ib})$ be the distribution vector of wellbeing levels in group i. The cumulative share of individuals in subgroup j with wellbeing below level k is given by:

$$r_{jk} = \sum_{h=a}^{k-1} s_{jh}$$

Let p_{ij} stand for the probability that an individual from group i exhibits a level of wellbeing higher than or equal to that of an individual from group j. We refer to p_{ij} as the *domination probability* of i over j, which is defined as follows:

$$p_{ij} = \sum_{h=a}^{b} s_{ih} r_{jh} + \frac{1}{2} \sum_{h=a}^{b} s_{ih} s_{jh} \qquad [1]$$

The first term on the r.h.s. of equation [1] corresponds to the probability that a member chosen at random from i exhibits a higher level of wellbeing than a member chosen at random from j. The second term is half the probability that a member chosen at random from i exhibits the same level of wellbeing that a member chosen at random from j. That is, we split evenly the probability of a tie between the two groups, so that $p_{ij} + p_{ji} = 1$.

We can interpret p_{ij} as the relative wellbeing opportunity of group i with respect to group j. To see that, think of a newcomer to society i whose probabilities of landing in the different levels of wellbeing are given by the distribution $s(i) = (s_{ia}, \dots, s_{ib})$, and a newcomer to society j whose

[5] We can also consider the case in which the society is a federation of some sort (e.g., the EU or the OECD) and the population subgroups as different country members. Note though that, as this type of evaluation provides a relative measure of collective wellbeing (i.e., how a group fares relative to the rest), it is important that group comparison be meaningful. That is, the more related the groups, the more informative the evaluation.

wellbeing probabilities are given by $s(j) = (s_{ja}, \dots, s_{jb})$. Then we compare how likely it is that the newcomer to i will obtain a higher level of wellbeing than a newcomer to j. The term p_{ij} tells us, therefore, the wellbeing opportunity that i offers relative to j. As $p_{ji} = 1 - p_{ij}$, the value 0.5 defines the boundary of having better wellbeing opportunity between these two groups.

Remark: *We use an integer numerical scale for the sake of simplicity in exposition. Yet, we can also consider an interval of the real line (a continuum), in which case the way of computing the probabilities must be adjusted in an obvious way.*

2.2 *The wellop function*

Let $P = \{\{p_{ij}\}_{j \neq i}\}_{i=1}^{m}$ stand for the collection of all those domination probabilities, p_{ij}, in a given evaluation problem. We assume that this is the required information to evaluate wellbeing for a society N made of m population subgroups, so that we identify the evaluation problem with the associated set P. Let Ω^m stand for the set of all possible evaluation problems involving m population subgroups. An evaluation function is a mapping $v: \Omega^m \to \mathbb{R}^m$, that, to each problem $P \in \Omega^m$, associates a vector $v(P) = (v_1(P), v_2(P), \dots, v_m(P))$, in the understanding that $v_i(P) > v_j(P)$ means that subgroup i does better than j, regarding the wellbeing opportunities of their members.

We now define the wellbeing opportunity function, or the *wellop function*, for short, as follows:

$$v_i(P) = \frac{1}{m-1} \sum_{j \neq i} p_{ij},\ i = 1,2, \dots, m \qquad [2]$$

The wellop function associates to each population subgroup the average domination probability with respect to all other groups. That is, the value $v_i(P) \in [0,1]$ tells us the probability that an individual in group i achieves a level of wellbeing higher than or equal to an individual from some other group. We interpret this probability as the wellbeing opportunity that this group offers to its members, relative to the other groups. Here again 0.5 is the critical value that defines when a group has better wellbeing opportunity than the average of the others.

The wellop function is a well-defined vector-valued mapping that provides a cardinal evaluation of the relative wellbeing opportunities of the population subgroups. A formal discussion in the Appendix establishes that the wellop function is the only evaluation function, $v: \Omega^m \to \mathbb{R}^m$, that satisfies the following three independent and intuitive properties:

(1) *Separability:* The evaluation of group i depends only on its domination probabilities relative to other groups, and not on the domination probabilities between groups that do not involve i.

(2) *Substitutability:* A change in the domination probabilities of group i regarding just two different groups, j, k with opposite signs but equal magnitude, does not affect the evaluation of i.

(3) *Scale:* When all domination probabilities are equal, we can take this value as the corresponding evaluation.

Besides these properties that characterise the wellop function, this evaluation protocol also satisfies other standard requirements. In particular: anonymity (the evaluation only depends on the individuals' wellbeing levels and not on other aspects such as labels or names), symmetry (two groups with identical distributions obtain the same evaluations), monotonicity (if the members of group j improve their wellbeing whereas all other groups do not change, then the evaluation of group j will increase),[6] and boundary condition (if a group is strictly dominated by all others, its evaluation is zero; if it dominates all others, its evaluation is one).

The wellop function can be interpreted as applying a binary criterion that compares each group and the fictitious one obtained by averaging the wellbeing distributions of all others (a form of consistency). Besides, this evaluation protocol satisfies a strong cardinality requirement, as it always distributes the constant amount $0.5m$ among the groups being evaluated. That is, for all $P \in \Omega^m$, we have: $\sum_{i=1}^{m} v_i(P) = 0.5m$ (see Proposition 1 in the Appendix). This feature can be viewed as follows: We start from the egalitarian distribution of opportunities, by assigning to each group an initial score $v_i(P) = 0.5$, which corresponds to the wellbeing opportunity that matches that of all other groups combined. The aggregate score so obtained, $0.5m$, is then redistributed according the actual distribution of opportunities.

Observe that, if we think of the geometric mean of those $v_i(P)$ values as a social welfare indicator,[7] then the egalitarian distribution is the one that maximises social welfare. Hence, we can define an elementary index of inequality of wellbeing opportunity as follows:

$$I_{op}(P) = 1 - 2\mu_G(P) \in [0,1]$$

Where $\mu_G(P)$ is the geometric mean of the $v_i(P)$ values. This index tells us how unequal wellbeing opportunities are between population subgroups and can be used to compare the degree of fairness of different societies.

2.3 *Changes in wellbeing opportunities*

The wellop function provides a measure of the likelihood of achieving a higher level of wellbeing in each group relative to others. However, it does not directly measure absolute levels for each group taken in isolation. This might appear as a limitation, as welfare measures are commonly used to assess the impact of policy interventions, track societal welfare trends, and estimate the effects of economic shocks. A natural way to address this issue is by applying a standard index number strategy, as outlined below.

Consider a society composed of m population subgroups in two different states, labeled as 0 (initial) and 1 (new). Suppose these states represent conditions before and after a policy intervention. How can the wellop function be used to assess the impact of such an intervention?

To determine whether a specific population subgroup has actually improved, we compare its wellbeing distributions before and after the intervention. Let P_0, P_1 denote the corresponding sets of

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[6] This, in turn, implies stochastic dominance: If the distribution of one group stochastically (first order) dominates the distribution of another, then it will exhibit a larger v_i value.

[7] We can take $W = \mu(P)(1 - A_1(P))$, where $A_1(P)$ is Atkinson's index of inequality for $\epsilon = 1$ and $\mu(P)$ the average of the $v_i(P)$ values.

domination probabilities. A change in the wellop function, $v_i(P_1) > v_i(P_0)$, does not imply that group i has improved in absolute terms; rather, it indicates that its wellbeing opportunities relative to other groups have improved. This is compatible with an overall reduction in welfare. To determine whether a specific population subgroup has actually improved, we compare its wellbeing distributions before and after the intervention.

Take group i as reference. The probability that an individual chosen at random after the intervention exhibits a higher level of wellbeing than an individual chosen before the intervention is denoted as $p_{1,0}(i)$, with $p_{0,1}(i)$ as its complement. A value $p_{1,0}(i) > 0.5$ indicates an improvement in the wellbeing opportunity for group i. The distance from 0.5 serves as a measure of the magnitude of this improvement, in line with Gastwirth (1975). A straightforward measure of this improvement is:

$$\beta(i) = \frac{p_{1,0}(i) - p_{0,1}(i)}{p_{0,1}(i)} = \frac{p_{1,0}(i) - 0.5}{0.5 - \frac{1}{2}p_{1,0}(i)} \qquad [3]$$

An elementary assessment of the impact of an intervention on subgroup i that takes into account both the change of level and the change in the relative position with respect to the others, is the following:

$$\Delta w_i(P_0, P_1) = c_i\beta(i) - (1 - c_i)\frac{v_i(P_1) - v_i(P_0)}{v_i(P_0)} \qquad [4]$$

Where $c_i \in [0,1]$ is a coefficient that ponders the relevance of the change in the level of wellbeing in group i with respect to itself, and the change in the relative situation of this group, relative to the others.

A similar approach can be used for the overall society by examining aggregate distributions. Define $s_h(t) = \frac{n_h(t)}{n(t)}$ as the share of individuals at each wellbeing level in the whole society in state $t = 0{,}1$, and let $s(t) = (s_a(t),, \dots, s_b(t))$. Letting $p_{1,0}$ represent the domination probability of society in state 1 relative to state 0, we can measure the overall impact of the policy intervention as:

$$\beta(S) = \frac{p_{1,0} - 0.5}{0.5 - \frac{1}{2}p_{1,0}}$$

Thus, while the wellop function only provides a relative measure of societal wellbeing, it allows us to measure changes in wellbeing over time for each individual group and for society. This makes it possible to apply standard optimisation techniques to policy selection, such as choosing interventions that maximise improvements in wellbeing opportunities for a target group or for society as a whole, subject to budget constraints.

2.4 *Marginal impacts*

A related question is to measure the impact on the wellop function of rising the wellbeing status of one individual, from one level to the next, say from h to $(h + 1)$, in group i (a comparison in the spirit of the Wellby; see Frijters et al, 2021). Given the nature of domination probabilities, that impact will change depending not only on the group but also on the level at which the change occurs. This suggests that it is also interesting to consider the marginal impact on the wellop function from moving upwards an individual at each level of wellbeing (except at level 10, obviously), to obtain an overall impact measure.[8]

A bit of elementary algebra shows that the marginal impact of improving the wellbeing of an individual from step h to $(h+1)$ in group i, $\Delta v_h(i)$, is directly proportional to the aggregate share of individuals within those steps, in all other groups. And also, that the marginal impact of a change of an individual from each level to the next, $\Delta v(i)$, is directly proportional to the number of people below the top level, b, in all other groups. In both cases the coefficient of proportionality corresponds to averaging those aggregate values. That is,

$$\Delta v_h(i) = \frac{1}{2n_i(m-1)} \sum_{j \neq i} (s_{jh} + s_{j(h+1)}) \qquad [5]$$

$$\Delta v(i) = \frac{1}{2n_i} \sum_{j \neq i} r_{jb} \qquad [6]$$

It is easy to see that the range of variation of those changes is given by:

$$\Delta v_h(i) \in [0, \frac{1}{2n_i}], \qquad \Delta v(i) \in [0, \frac{m-1}{2n_i}]$$

[8] Note that one of the implications of linearity is that the marginal impact of moving one step upwards an individual is independent on the level at which it happens.

3.
A numerical example

To illustrate the wellop function in practice, consider three population subgroups with the following wellbeing distributions: Distribution A, which is uniformly spread across all wellbeing levels; Distribution B, concentrated around the mean; and Distribution C, which is fully polarised, with most individuals either at the lowest or highest wellbeing levels. Table 1 describes the distribution of the eleven members of each group along a conventional [0, 10] wellbeing scale.

How should we rank those distributions? How does the wellop function evaluate them?

Table 1. Wellbeing distributions in three population subgroups

Likert scale	0	1	2	3	4	5	6	7	8	9	10
A	1	1	1	1	1	1	1	1	1	1	1
B	1	0	0	0	0	5	5	0	0	0	0
C	5	0	0	0	0	1	0	0	0	0	5

Although all three distributions have the same mean and median (5), their structures are extremely diverse. Distribution A allows for all possible wellbeing levels, which could be seen as beneficial but also introduces uncertainty. In distribution B ten of its eleven members score above or at the mean, with only one scoring below. This is appealing, even though one might argue that this reflects a mediocre distribution of wellbeing opportunity. The chances of achieving top levels are much higher in distribution C, but are just the same as those of ending up at the bottom level. Ranking the overall wellbeing of these three population subgroups, therefore, is not obvious and may depend on risk aversion attitudes.[9]

How does the wellop function evaluate this case?

The wellop function sets distribution B at the top of the ranking, with a value $v(B) = 0.5475$, then distribution A, with $v(A) = 0.5$, followed by distribution C, with $v(C) = 0.4525$. The uniform distribution thus provides the same wellbeing opportunities that the distribution resulting from merging that concentrated on the mean and that concentrated over the tails. As a consequence,

[9] To gain informal insights into the evaluation of these distributions, I conducted a brief WhatsApp survey among some fifty university professors from my personal contacts. While this is a highly biased and non-representative sample, it provides valuable perspectives from individuals with diverse academic backgrounds, including economics, law, history, and medicine. The results indicate a strong preference for distribution B, with 75% of respondents selecting it as the best option. Meanwhile, 14% favoured distribution C, and 11% considered distribution A the most desirable. These findings, though anecdotal, suggest an inclination toward a distribution concentrated around the mean, potentially reflecting an implicit preference for stability over polarisation or uniformity.

the opportunity distance between A and B, and A and C is the same in absolute terms, that is, $|v(A) - v(B)| = |v(A) - v(C)| = 0.0475$.

Next, we consider how the evaluation changes when we alter distribution C by spreading uniformly the population at the bottom level between levels 0 and 4 (Distribution C′), and when the population in the top-level spreads uniformly between the levels 10 and 6 (Distribution C′′). Table 2 describes those changes. Neither of those changes alter the mean or the median of distribution C.

Table 2. Spreading the tails of distribution C

Likert scale	0	1	2	3	4	5	6	7	8	9	10
C′	1	1	1	1	1	1	0	0	0	0	5
C′′	5	0	0	0	0	1	1	1	1	1	1

We are interested in knowing how those alternative distributions compare with A and B, regarding two aspects. First, if changes in the evaluation of C are of different sign. Second, whether both spreads have a symmetric effect on the evaluation. The answer is yes to the first question and no to the second. Table 3 presents the evaluations obtained with the wellop function, comparing the three situations, distributions {A, B, C}, {A, B, C′} and {A, B, C′′}.

Common to the three situations is that distribution B is valued most. The value of distribution A moves below 0.5 when we spread the bottom part of C and above 0.5 when we spread its upper part instead (by the same absolute amount, 0.0413). The value of distribution C′ is above 0.5 whereas that of distribution C′′ is further below 0.5 than it was originally.

Table 3. The wellop function for three different situations

Distributions {A, B, C}	Distributions {A, B, C′}	Distributions {A. B. C′′}
v(A) = 0.5	v′(A) = 0.4587	v′′(A) = 0.5413
v(B) = 0.5475	v′(B) = 0.5393	v′′(B) = 0.5868
v(C) = 0.4525	v′(C′) = 0.5021	v′′(C′′) = 0.3719

Assume the change from C to C′ is the result of a policy measure intended to improve the wellbeing of those individuals at the bottom of distribution C. We can measure the size of the improvement by comparing distributions C and C′. Elementary calculations yield $p_{1,0}(C') = 0.5826$, so that we observe a 40% wellbeing improvement in this group. Similarly, if we think of C′′ as the result of adverse conditions on people's wellbeing at the upper side of C, we get $p_{1,0}(C'') = 0.4174$, which represents a 28% reduction of wellbeing.

4.
Final comments

The core of our proposal is the comparison of population subgroups using the likelihood of achieving higher levels of wellbeing as the key variable. This approach allows for an ordinal interpretation of subjective wellbeing (SWB) and, more importantly, introduces a new way of comparing societal wellbeing by considering wellbeing distributions rather than relying solely on summary values.[10] This provides a more comprehensive use of primary data, as illustrated by the numerical example above: although the three distributions share the same mean and median, they depict extremely different situations.

The wellop function is a transparent evaluation protocol that quantifies the wellbeing opportunities available to different social groups. The score assigned to each population subgroup reflects the probability of achieving better outcomes compared to the rest. A value of 0.5 serves as a natural neutral point, indicating whether a social group has greater wellbeing opportunities than others.

In addition to evaluating wellbeing disparities, the wellop function is useful for policy analysis. By tracking changes in domination probabilities over time or under different circumstances, it provides an effective tool for assessing the impact of policy interventions on social groups. This methodology can help guide policy decisions aimed at reducing wellbeing disparities and improving overall societal welfare. Moreover, it can be used to validate conventional wellbeing evaluations —particularly to test whether cardinality assumptions influence assessments of societal wellbeing— and to compare results obtained through this methodology, not only in terms of rankings but also in score distributions (especially regarding the discriminatory power of different approaches).

Interestingly, this approach applies whenever ordered categories are used to record subjective perceptions across various domains. For example, it can be employed in medical treatments (such as pain scales for evaluating alternative treatments) and in other ordinal assessments, including judgments of quality, performance, or consumers' satisfaction.

Overall, the wellop function offers a user-friendly, robust, and policy-relevant evaluation framework that complements existing tools for wellbeing analysis. It is easy to compute and provides a clear, intuitive interpretation.

[10] There is some discussion on the empirical relevance of the cardinality assumption (e.g., Ferrer-i-Carbonell & Frijters, 2004), which may suggest that the ordinal approach is just a case of theoretical parsimonia. Yet the use of all the information in the wellbeing distributions goes well beyond this methodological element.

Appendix

We first present four properties that characterise the wellop function:

Separability: Let $P, P' \in \Omega^m$ be two evaluation problems such that $p_{ij} = p'_{ij}, \forall j \neq i$. Then, $v_i(P) = v_i(P')$.

Substitutability: Let $P, P' \in \Omega^m$ be such that $p'_{it} = p_{it}, \forall t \neq j, k,\ p'_{ij} - p_{ij} = -(p'_{ik} - p_{ik})$. Then, $v_i(P) = v_i(P')$.

Scale: Let $p_{ij} = \beta, \forall j \neq i, \forall i$. Then, $v_i(P) = \beta$.

The following result is obtained:

Proposition 1: An evaluation function $v: \Omega^m \to \mathbb{R}^m$ satisfies separability, substitutability, and scale if and only if, for each problem $P \in \Omega^m$, it adopts the form:

$$v_i(P) = \frac{1}{m-1} \sum_{j \neq i} p_{ij}\ , \qquad i = 1,2, \dots, m \qquad [2]$$

With $\sum_{i=1}^{m} v_i(P) = 0.5m$. Moreover, those properties are independent.

Proof

The average values of the domination probabilities clearly satisfy those three properties.

To prove the reciprocal, note that the property of separability implies that we can write, without loss of generality,

$$v_i(P) = f_i(p_{i1}, p_{i2}, \dots, p_{im})\ , \quad j \neq i$$

Substitutability implies that $\frac{\Delta v_i}{\Delta p_{ij}} = \frac{\Delta v_i}{\Delta p_{ik}}$ whenever $\Delta p_{ij} = \Delta p_{ik}$. Therefore, v_i must be a linear function that adopts the form:

$$v_i(P) = k_i \sum_{j \neq i} p_{ij} + b_i$$

The property of scale ensures that $k_i = \frac{1}{m-1}, b_i = 0, \forall i$.

To show that $\sum_{i=1}^{m} v_i(P) = 0.5m$, note that the result is trivial when $p_{ij} = 0.5$, $\forall \{i,j\}, j \neq i$. If this is not the case, suppose first there is a single pair, $p_{ij} \neq 0.5 \neq p_{ji}$. We would have: $v_t(P) = 0.5, \forall t \neq i, j$, with:

$$v_i(P) = \frac{1}{m-1}((m-2)0.5 + p_{ij})\ , \quad v_j(P) = \frac{1}{m-1}((m-2)0.5 + (1 - p_{ij}))$$

From this it follows that $v_i(P) + v_j(P) = 1$, so that:

$$\sum_{k=1}^{m} v_k(P) = (m-2)0.5 + 1 = 0.5m$$

By repeating this argument as many times as required, we obtain the desired result.

To check for independence, consider the following examples:

(i) Function $v_i(P) = \frac{1}{m(m-1)} \sum_{i=1}^{m} \sum_{j \neq i} p_{ij}$, satisfies all properties but separability.

(ii) Function $v_i(P) = p_{ij}$, for a given j, satisfies all properties but substitutability.

(iii) Function $v_i(P) = \sum_{j \neq i} p_{ij}$ satisfies all properties but scale.

Q.e.d.

Proposition 2: (i) The marginal impact of moving an individual from wellbeing level h to level $(h+1)$, in group i, is given by:

$$\Delta v_h(i) = \frac{1}{2n_i(m-1)} \sum_{j \neq i} (s_{jh} + s_{j(h+1)}) \quad [6]$$

(ii) The marginal impact of moving an individual from each level of wellbeing to the next in group i, is given by:

$$\Delta v(i) = \frac{1}{2n_i} \sum_{j \neq i} r_{jb} \quad [7]$$

<u>Proof.-</u>

(i) From equation [1] it follows that a change of an individual from level h to level $(h+1)$ in group i, entails a change in the domination probability given by:

$$\Delta p_{ij}(h) = (s'_{ih} - s_{ih})r_{jh} + (s'_{i(h+1)} - s_{i(h+1)})r_{j(h+1)} + \frac{1}{2}((s'_{ih} - s_{ih})s_{jh} + (s'_{i(h+1)} - s_{i(h+1)})s_{j(h+1)})$$

Now observe that $s'_{ih} - s_{ih} = \frac{-1}{n_i}, s'_{i(h+1)} - s_{i(h+1)} = \frac{1}{n_i}$, so that we can rewrite the expression above as:

$$\Delta p_{ij}(h) = \frac{1}{n_i}(r_{j(h+1)} - r_{jh}) + \frac{1}{2n_i}(s_{j(h+1)} - s_{jh}) = \frac{1}{n_i}\frac{s_{jh} + s_{j(h+1)}}{2}$$

Hence,

$$\Delta v_h(i) = \frac{1}{2n_i(m-1)} \sum_{j \neq i} (s_{jh} + s_{j(h+1)})$$

(ii) Note that,

$$\sum_{h=a}^{b-1}(s_{jh}+s_{j(h+1)}) = 2r_{jb}+s_{jb} = 1+r_{jb}$$

So that, adding up over the wellbeing levels we obtain:

$$\Delta v(i) = \frac{1}{2(m-1)n_i}\sum_{j\neq i}(1+r_{jb}) = \frac{1}{2n_i}\sum_{j\neq i} r_{jb}$$

Q.e.d.

References

BENJAMIN, D.J., GUZMAN, J.D., FLEURBAEY, M., HEFFET, O. & KIMBALL, M. (2023), What do happiness data mean? Theory and survey evidence, Journal of the European Economic Association, 21: 2377–2412.

BENTHAM, J. (1789), An Introduction to the Principles of Morals and Legislation. Batoche Books, 14.

CLARK, A.E., FLÈCHE, S., LAYARD, R., POWDTHAVEE, N. & WARD, G. ET AL. (2018), The Origins of Happiness: The Science of Well-Being over the Life Course, Princeton University Press.

COLEMAN, M. (2022), Affective forecasting, Happier Lives Institute.

DOLAN, P., LAYARD, R., METCALFE, R. (2011), Subjective Well-being for Public Policy, Office of National Statistics, UK.

DOLAN, P. & METCALFE, R. (2012), Valuing Health: A Brief Report on Subjective Well-Being versus Preferences, Medical Decision Making, 32 : 578-582.

DOLAN, P., PEASGOOD, T. & WHITE, M. (2008), Do we really know what makes us happy? A review of the economic literature on the factors associated with subjective well-being, Journal of Economic Psychology, 29 : 94-122.

FERRER-I-CARBONELL, A. & FRIJTERS, P. (2004), How important is methodology for the estimates of the determinants of happiness, The Economic Journal, 114 : 641-659.

FRIJTERS, P. & KREKEL, C. (2021), A Handbook for Wellbeing Policy-Making. History, Theory, Measurement, Implementation, and Examples, Oxford University Press.

FRIJTERS, P., KERNEL, C., SANCHIS, R. & SANTINI, Z.I., (2024), The WELLBY: a new measure of social value and progress, Humanities and Social Sciences Communications, 1:736 | https://doi.org/10.1057/s41599-024-03229-5.

GASTWIRTH, J.L. (1975), Measures of earnings differentials, American Statistic Association, 29 : 32–35.

GLOBAL HAPPINESS COUNCIL (2018), Global Happiness Policy Report.

HERRERO, C. & VILLAR, A. (2018), The Balanced Worth: A procedure to evaluate performance in terms of ordered attributes, Social Indicators Research, 140: 1279-1300.

HERRERO, C. & VILLAR, A. (2021), Opportunity advantage between income distributions, Journal of Economic Inequality, 19 : 785-799.

HM Treasury (2021), Wellbeing Guidance for Appraisal: Supplementary Green Book Guidance, https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/1005388/Wellbeing_guidance_for_appraisal_-_supplementary_Green_Book_guidance.pdf .

Kahneman, D. & A.B. Krueger (2006), Developments in the Measurement of Subjective Well-Being, Journal of Economic Perspectives, 20 : 3–24.

Krueger, A. B. & Schkade, D. A. (2008), The reliability of subjective well-being measures, Journal of public economics, 92 : 1833-1845.

Layard, R. (2020), Can We Be Happier? Evidence and Ethics, A Pelikan Book.

Le Breton, M., Michelangeli, A. & Peluso, E. (2012), A stochastic dominance approach to the measurement of discrimination, Journal of Economic Theory, 147 : 1342–1350.

Lieberson, S. (1976), Rank-sum comparisons between groups, Sociological Methodology, 7 : 276–291.

Mahoney, J. (2023), Subjective wellbeing measurement: Current practice and new frontiers, OECD papers o Wellbeing and Inequalities, working paper nº 17.

McGuire, J., Dupret, S. & M. Plant (2022), To WELLBY or not to WELLBY? Measuring non-health, non-pecuniary benefits using subjective wellbeing, Open Philanthropy, Worldwide Investigations.

OECD (2013), OECD Guidelines on Measuring Subjective Well-being, OECD Publishing. http://dx.doi.org/10.1787/9789264191655-enDurand (2018).

Plant, M. (2021), A happy possibility about happiness (and other subjective) scales: an investigation and tentative defence of the Cardinality Thesis, Happier Lives Institute, https://www.happierlivesinstitute.org/report/the-comparability-of-subjective-scales/.

Samuelsson, C., Dupree, S., Plant, M. & Kaiser, C. (2023), Can we trust surveys? A pilot study of comparability, linearity and neutrality, Happier Lives Institute. https://www.happierlivesinstitute.org/wp-content/uploads/2023/03/Comparability-linearity-and-neutrality-Pilot-report.pdf.


Event
Event Budget for Event : EXPENSES
Actual Cost Breakdown
Estimated vs. Actual
Estimated
Actual
$500.00
$100.00
$200.00
$300.00
$1,100.00
$1,000.00
$2,300.00
$2,500.00
$1,500.00
$2,000.00
Site
Publicity
Refreshments
Prizes
Program
Decorations
Miscellaneous
9%
7%
10%
19%
$2,500.00
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