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A measure of insufficient performance

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Abstract

This paper introduces and characterises a measure of insufficient performance that is decreasing and convex with respect to deviations from a minimal threshold. The measure is defined as the logarithm of the geometric mean of these deviations, weighted by the relative frequency of faulty outcomes. Our measure exhibits an elementary mathematical structure, is easy to interpret, and satisfies some desirable properties, such as scale independence, replication invariance, and decomposability by observational subgroups. We provide two illustrations on the applicability of this evaluation protocol, regarding the labour market and life satisfaction.

Keywords: insufficient performance, logarithmic evaluation, geometric mean, decomposability.

JEL classification: G3, G32, H32.

Resumen

Este artículo presenta y caracteriza una medida de rendimiento insuficiente que es decreciente y convexa en las desviaciones con respecto a un umbral mínimo. La medida se define como el logaritmo de la media geométrica de estas desviaciones, ponderada por la frecuencia relativa de resultados defectuosos. Este indicador presenta una estructura matemática sencilla, es fácil de interpretar y cumple algunas propiedades deseables, como independencia de la escala, repica de poblaciones y descomponibilidad por subgrupos observacionales. Presentamos dos ejemplos de la aplicabilidad de este protocolo de evaluación en relación con el mercado laboral y la satisfacción con la vida.

Palabras clave: rendimiento insuficiente, evaluación logarítmica, media geométrica, descomponibilidad.

Clasificación JEL: C02, C60.

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1.

Introducción

We address the problem of evaluating a system or organisation's performance regarding some outcome, with a focus on insufficient performance. This evaluation could apply to individuals, teams, firms, institutions, educational systems, consumers' satisfaction, or adherence to a medical treatment of a group of patients, among other examples. The common feature is that the system produces one or several outcomes and that we are concerned about insufficient performance.

The problem is of interest because insufficient performance can have wide-ranging negative implications, such as reputation damage, loss of market value, decreased productivity, strategic disadvantages, or operational inefficiencies. These issues can hinder the smooth functioning of the system and even threaten its sustainability. Measuring properly insufficient performance is key for understanding the extent and nature of performance gaps, enabling continuous monitoring, preventing resource waste, and providing guidelines for improvement. See Basseville & Nikiforov (1993), Ogata (2010) or Montgomery (2017) for a comprehensive discussion of this question.

To evaluate insufficient performance, we collect data from a series of trials, $t = 1,2, \ldots, T$, with $n(t)$ observations each. These trials may involve repeated measurements of a single outcome, different outcomes (systems with multiple outputs), or several parameters. We assume that those observations consist of positive values of a quantitative variable. Each trial t has defined a performance threshold, $z(t)$. An observation $y_j(t)$ exhibits insufficient performance if it falls below the threshold, that is, if $y_j(t) < z(t)$.

We propose a straightforward approach to evaluating insufficient performance, inspired in the literature on poverty measurement (see Chakravarty 2009 or Villar 2017 for a discussion, and Watts 1968 for the closest reference). Insufficient performance is calculated as a function of the ratios between the performance threshold and the values of faulty observations. The evaluation protocol implements a basic principle: larger deviations from the threshold receive progressively higher values, i.e., the function that evaluates insufficient performance is decreasing and convex in the relative deviations, $z(t)/y_j(t)$.

Our measure has an elementary mathematical structure, is easy to interpret, and satisfies desirable properties such as scale independence, replication invariance, and decomposability by observational subgroups, among others. Some of these properties fail in the usual indicators of faulty performance, as the fault rate (that measures incidence), the mean of faulty observations (that measures severity), the weighted faulty performance index (that combines incidence and intensity), or the root mean square error (that measures the distance of faulty observations to the threshold).

The paper is organised as follows. Section 2 presents the model and characterises the basic evaluation formula in terms of three properties: differentiability, inverse proportionality (changes are inversely proportional to the levels), and scale (the measure goes to zero as faulty observations vanish). The resulting formula corresponds to the logarithm of the geometric mean of the relative deviations, weighted by the relative frequency of faulty outcomes. Section 3 illustrates the application of this measure with two examples. A few comments in Section 4 close the work.

2.

Measuring insufficient performance

We call ***trial*** t to a collection of $n(t)$ observations on the performance of a system, measured by a quantitative variable. The term $y_i(t)$ describes the value of the i th observation at trial t while $y(t) \in \mathbb{R}_{++}^{q(t)}$ is the corresponding outcome distribution vector. Let $z(t) > 0$ denote a performance threshold and $Q(t)$ stand for the set of observations such that $y_i(t) < z(t)$, with cardinal $q(t)$. We implicitly assume that $n(t)$ is large enough to be considered as a representative sample of the system's output. Performance thresholds $z(t)$ are treated as external parameters, even though they can also be defined as a function of the outcome distribution (e.g., values below a given percentile).

An ***experiment*** is a finite collection of trials, $t = 1,2, \ldots, T$, which may be realised in different periods or places, under different circumstances, for several parameters of an output, or for different components of the system. As different trials might be of difference importance, we consider a vector $\boldsymbol{\delta} = (\delta(1), \ldots, \delta(T))$ of parameters, whose elements weigh the relevance of those trials.

We aim at assessing the overall performance of a system from the data of an experiment, focusing on insufficient performance. Let $\boldsymbol{P} = (\boldsymbol{y}, \boldsymbol{z}, \boldsymbol{\delta})$ denote an evaluation problem. Insufficient performance in trial t, with $n(t)$ observations, is measured by a function ρ_t: $\mathbb{R}_{++}^{q(t)} \to \mathbb{R}$, where $q(t)$ is the number of observations with $y_i(t) < z(t)$. We say that $\frac{z(t)}{y_i(t)}$ is the relative deviation of the ith observation.

We now impose three properties on this evaluation function. The first property, differentiability, is a smoothness requirement on the behaviour of the function. The second property, inverse proportionality, describes the behaviour of the evaluation function and requires ρ_t to be decreasing and convex in $y_i(t)$. This implies that the impact of insufficient performance on the evaluation is higher the smaller the achievement. More precisely, we assume that changes in the evaluation are inversely proportional to the performance level. The third property, scale, says that the value of the function is zero when there are no faulty observations.[2]

Formally:

Differentiability: ρ_t is a differentiable function.

Inverse proportionality: $\frac{\Delta \rho_t}{\Delta y_i(t)} = \frac{-k_t}{y_i(t)}, k_t > 0.$

Scale: $\rho_t(z(t), z(t), \ldots, z(t)) = 0.$

Those three properties determine the form of the evaluation function, as the next result shows:

[2] To be precise one should say that $\rho_t \to 0$ as $y_i(t) \to z(t)$, $\forall i$.

Proposition: *An evaluation function* ρ_t*:* $\mathbb{R}_{++}^{q(t)} \rightarrow \mathbb{R}$ *that satisfies differentiability, inverse proportionality, and scale, takes the form:*

$$\rho_t\big(y(t)\big) = \frac{q(t)}{n(t)} \times \; ln\tilde{\mu}(y(t)) \qquad [1]$$

Where $\tilde{\mu}(y(t))$ is the geometric mean of the relative deviations, $z(t)/y_i(t)$.

Proof.-

First observe that differentiability and inverse proportionality imply that the evaluation function is additive, so that we can write:

$$\rho_t\big(y(t)\big) = \sum\nolimits_{i\in Q(t)} f(y_i(t))$$

Hence, $\frac{\partial \rho_t}{\partial y_i(t)} = -\frac{k_t}{y_i(t)} = \frac{df}{dy_i(t)}$. Integrating the last equality we obtain:

$$f\big(y_i(t)\big) = -k_t ln y_i(t) + C$$

It follows form scale that $0 = -\, k_t lnz(t) + C$, so that $C = k_t lnz(t)$.

Consequently,

$$f\big(y_i(t)\big) = k_t\big(lnz - lny_i(t)\big) = \; k_t ln\frac{z(t)}{y_i(t)}$$

Then,

$$\rho_t\big(y(t)\big) = k_t \sum\nolimits_{i\in Q(t)} ln\frac{z(t)}{y_i(t)}$$

Without loss of generality, we let $k_t = \frac{1}{n(t)}$, so that the value of the indicator is pondered by the number of observations. Now, multiplying and dividing by *q(t)*, we can rewrite the equation above as:

$$\rho_t\big(y(t)\big) = \frac{q(t)}{n(t)} \; x \; ln \prod\nolimits_{i\in Q(t)} \left(\frac{z(t)}{y_i(t)}\right)^{\frac{1}{q(t)}}$$

As $\tilde{\mu}\big(y(t)\big) = \prod_{h\in Q(t)} \left(\frac{z(t)}{y_i(t)}\right)^{\frac{1}{q(t)}}$, the result follows.

Q.e.d.

Equation [1] says that assuming the properties of differentiability, inverse proportionality, and scale, amounts to evaluating insufficient performance in trial *t*as the product of the *incidence* of the faulty observations, $\frac{q(t)}{n(t)}$, and the log of the geometric mean of the relative deviations, $ln\tilde{\mu}(y(t))$, which measures the severity of the failure.[3]

Given an evaluation problem $P = (y, z, \delta)$, we define the **performance handicap** of the experiment, $\rho(P)$, as the weighted average of insufficient performance across trials. That is,

$$\rho(P) = \frac{1}{T}\sum_t \delta\,(t)\rho_t(y(t)) \qquad [2]$$

Where $\delta(t)$ are the parameters that weigh the relevance of the trials.

Equation [2] can be rewritten as

$$\rho(P) = ln\prod_t \left([\tilde{\mu}(y(t))]^{\frac{\delta(t)q(t)}{n(t)}}\right)^{\frac{1}{T}}.$$

Now observe that $\prod_t \left([\tilde{\mu}(y(t))]^{\frac{\delta(t)q(t)}{n(t)}}\right)^{\frac{1}{T}}$ is the geometric mean of the weighted geometric means of the deviations in the different trials, where the weights are given by the associated incidence of faulty observations, pondered by the relevance of the trials. By letting $\prod_t \left([\tilde{\mu}(y(t))]^{\frac{q\delta(t)(t)}{n(t)}}\right)^{\frac{1}{T}} = \tilde{\mu}_w(P)$, we have:

$$\rho(P) = ln\tilde{\mu}_w(P) \qquad [2']$$

The performance handicap thus corresponds to the log of the weighted geometric mean of the losses across trials. It is always positive, decreasing and convex in its variables, and tends to zero when all faulty observations tend to their thresholds.

This evaluation function satisfies several desirable properties for an index of this type. Besides *continuity*, it is *homogeneous of degree zero* (i.e. changes in the units of measurement do not affect the function's value), monotonically decreasing and convex in its variables, its value goes to zero as faulty observations vanish, it is independent of the values of non-faulty observations (a property known as *focus*), and satisfies *replication invariance*, meaning that an experiment consisting of two identical trials will yield the same evaluation as any of the individual trials, which is essential to compare experiments or trials with different numbers of observations.

Perhaps most importantly, the evaluation function satisfies *decomposability*. That is, it can be decomposed additively by trials and *observational subgroups*. For instance, if each trial consists of observations from different groups (e.g., different demographic subgroups), the evaluation can be expressed as the sum of the results from each subgroup. To see this, suppose that the observations can be divided into *G* different

[3] This measure corresponds to a slight variant of Watts (1968) poverty index. See also Zheng (1993) for an alternative characterisation.

observational subgroups, g = 1,2, ..., G. The set of observations below the threshold at trial t can be describes as $Q(t) = \cup_{g=1}^{G} Q_g(t)$, where $Q_g(t)$ is the set of faulty observations in subgroup g, with cardinal $q_g(t)$. In this context, it is easy to check that we can express equation [1] as follows:

$$\rho_t\big(y(t)\big) = \frac{q(t)}{n(t)} \sum_{g=1}^{G} \underbrace{\frac{q_g(t)}{q(t)}\ ln \prod_{i \in Q_g(t)} \left(\frac{z(t)}{y_i(t)}\right)^{\frac{1}{q_g(t)}}}_{\rho_t^g(y(t))}$$

That is,

$$\rho_t\big(y(t)\big) = \tfrac{q(t)}{n(t)} \textstyle\sum_{g=1}^{G} \rho_t^g\big(y(t)\big) \qquad [3]$$

Hence,

$$\rho(P) = \tfrac{1}{T} \textstyle\sum_t \delta\,(t) \tfrac{q(t)}{n(t)} \sum_{g=1}^{G} \rho_t^g\big(y(t)\big) \qquad [4]$$

Also observe that we can define:

$$\rho^g(P) = \tfrac{1}{T} \textstyle\sum_t \delta\,(t) \rho_t^g\big(y(t)\big) \qquad [5]$$

Which is a measure of the contribution of observational subgroup g to the handicap of the experiment.

3.
Two illustrations

We now present two specific evaluation problems in which this measure of insufficient performance proves useful. The first is an index of unemployment that accounts for both the unemployment rate and the duration of unemployment (see also Gorjón et al, 2020). The second is a measure of dissatisfaction, based on life satisfaction as the reference variable (see also Frijters et al, 2024). There are two features of this evaluation protocol that are especially relevant in these illustrations. First, the decomposability of the evaluation by population subgroups. Second, the incentive to apply policies to the worst cases as the way of reducing more quickly the performance handicap.[4]

3.1 An index of unemployment

Consider a society with $n(t)$ workers in period t. The labour force consists of those who are working and those who are unemployed.[5] Most Labour Force Surveys provide quarterly information on the number of unemployed with different unemployment spells, usually measured in months, besides information on some demographic characteristics (gender, age, education, region of residence, etc.). The available data are, therefore, binary in nature: a worker is unemployed or not in the current period (usually a quarter) and, if it is unemployed, we have information about how many months has been in that situation. With those features in mind, assume that we consider a time span consists of H months (we can think of 60 months, to fix ideas) and that we have a binary information on whether a worker is unemployed or not, and on how long has been unemployed. This can be treated as follows.

Let $m_i(t)$ the number of months that this agent has been unemployed at and $z(t)=H$ stand for the number of months that this worker should have been working at time t. Define[6] $y_i(t) = H - m_i(t)$, $\forall i \in U(t)$, where $U(t)$ is the set of the unemployed at t, with cardinal $u(t)$. Therefore, by applying our evaluation formula we obtain:

$$\rho_t\big(y(t)\big) = \frac{u(t)}{n(t)} \times \ln \prod_{i \in U(t)} \left(\frac{H}{H - m_i(t)}\right)^{\frac{1}{u(t)}}.$$

That is, the unemployment index is the product of its incidence, the unemployment rate, and its severity, measured by the log of the geometric mean of the unemployment relative deviations. This function is increasing and convex in $m_i(t)$, so that longer unemployment spells have proportionally more impact on the index than shorter ones.

[4] See also Villar (2024) for a related application regarding poverty measurement.

[5] This is a convention which seems to be changing, to include at least those discouraged (ready to work but not looking actively for a job) and those that unwillingly work part-time. We shall keep the standard definition for the sake of simplicity in the exposition.

[6] We implicitly assume that $H - m_i(t) > 0$, to make the formulation coherent (e.g., $y_i(t) = 60 - \min\{m_i(t), 48\}$ in our reference example).

Note that if a person unemployed "yesterday" for $m_i(t-1)$ months finds an employment today, all that burden of unemployment disappears. So, it is sensible to evaluate unemployment by considering the average of a number *T* of periods, to avoid losing all the information about former values. In this context, we can let $\delta(t)$ = 1, for all *t*. The corresponding evaluation will be given by:

$$\rho(y) = \frac{1}{T}\sum\nolimits_t \rho_t(y(t)$$

Once again, this is an expression easy to understand and interpret, and computationally friendly from the usual data sets on the labour market.

The property of decomposability appears in this context as especially useful, to understand the nature and implications of unemployment, and to help design policies to fight unemployment in the population subgroups that suffer more.

3.2 A measure of dissatisfaction

Another interesting application refers to the use of the conventional Likert scales to evaluate aspects such as pain relief, self-perceived levels of health or happiness, or customers' satisfaction. Likert scales provide information on the intensity of people's feelings regarding a given subject, usually transforming qualitative assessments into quantitative evaluations.

Take the case of *life satisfaction* (see Mahoney 2023, OECD 2020, 2024). Consider a society with members, $N = \{1,2, \ldots, n\}$ at a given point in time, that we call *today*, labelled by 0. We consider a time span of $T + 1$ periods, $t = 0,1, 2, \ldots, T$, whereindicates the time distance from today (that is, $t = 1$ stands for "yesterday", $t = 2$ for the "previous day", and so forth up to $t = T$). For each agent $h \in N$ there is a vector $y_h=(y_h(0), y_h(1), \ldots, y_h(T))$ describing the level of satisfaction in each of the considered periods. That information can be summarised by a matrix $Y=(y_1,\ldots,y_n) \in \mathbb{R}_{++}^{n(T+1)}$. We denote by $\boldsymbol{z} = (z(0), z(1), \ldots, z(T))$ the vector of satisfaction thresholds, and by *Q* the set of the unsatisfied,[7] with cardinal *q*.

We can think of dissatisfaction as a social welfare loss, by aggregating individual utility losses of those below the satisfaction threshold (e.g., those below 4 in the 0 – 10 subjective life satisfaction scale). Now observe that, by letting agent *h*'s utility function be given by $u_h(x) = \ln x$, a standard increasing and concave utility function, the term $\ln z(t) - \ln y_h(t)$ describes the agent's utility loss at t, as the difference between the minimum utility admissible and the actual one. But this is just our basic evaluation criterion, $\ln \frac{z(t)}{y_h(t)}$.

To compute the agents' utility losses during the period, we set $\delta(t) = \delta^t$, where $\delta \in [0,1]$ is a discount factor that gives us today's value of $\rho_t(y(t))$. We can think of this parameter as a function of the "interest rate", e.g., $\delta = 1/(1 + r)$. In this way we give progressively less relevance to dissatisfaction in older periods.

Let $\boldsymbol{P} = (\boldsymbol{Y}, \boldsymbol{z}, \delta)$ denote a satisfaction evaluation problem. The average utility loss of individual $h \in Q$ along the $(T + 1)$ periods, with a discount factor δ, is given by:

$$d_h(P) = \frac{1}{T+1}\sum_{t=0}^{T} max\{0,\ \delta^t(lnz(t) - lny_h(t))\}.$$

[7] We can define the set of the unsatisfied as those with $y_h(0) < z(0)$ (i.e., those who are unsatisfied today), or as those with $y_h(t) < z(t)$ for some *t*.

Let $\tilde{\mu}(y_h)$ stand for the geometric mean of the agent's time-adjusted deviations, $\left(\frac{z(t)}{y_h(t)}\right)^{\delta^t}$, during the entire time span. That is,

$$\tilde{\mu}(y_h) = \prod_{t=0}^{T} max\left\{1, \left[\left(\frac{z(t)}{y_h(t)}\right)^{\delta^t}\right]^{\frac{1}{T+1}}\right\}$$

Then, we can rewrite the expression above as:

$$d_h(P) = ln\tilde{\mu}(y_h)$$

That is, the utility loss of agent *h* corresponds to the log of the geometric mean of the time-adjusted deviations.

We define the **index of dissatisfaction** as the average value of the aggregate utility loss due to insufficient satisfaction. The welfare cost relative to a problem ***P*** = (***Y***, ***z***, δ), denoted by *C*(***P***), is thus given by:

$$C(P) = \frac{1}{n}\sum_{h\in Q} ln\tilde{\mu}\,(y_h) = \frac{q}{n}\times\; ln\prod_{h\in Q}[\tilde{\mu}(y_h)]^{\frac{1}{q}}$$

That is, the cost of dissatisfaction is the product of its incidence, $\frac{q}{n}$ and the geometric mean across agents of the geometric means of their time adjusted deviations. Here again decomposability turns out very handy, to better understand the nature of social unrest.

4. Final comments

The evaluation function presented in this paper provides a comprehensive measure of insufficient performance, defined as the log of the weighted geometric mean of the losses across trials. Insufficient performance in each trial is in turn approached by the product of the incidence and the severity of the failure. This approach, therefore, offers a simple and intuitive method for performance assessment.

By using the geometric mean, the evaluation penalises systematic insufficient performance more than scattered deviations, reflecting the informational value of consistent failure. Indeed, this approach shares similarities with entropy in information theory (Khinchin 1957, Kullback 1959, Theil, 1967), providing a novel way of estimating the "entropy" of an experiment.[8] The decomposability of the measure further enhances its utility, allowing for in-depth analysis and policy recommendations for specific subgroups or trials.

The flexibility of this measure makes it applicable to a wide range of problems, providing valuable insights for improving system performance across various domains.

[8] Let us recall that, given a series of possible events, 1, 2,..., n, with probabilities $\pi_1, \ldots, \pi_n,$ the information function is given by $ln(1/\pi_i)$ and the corresponding entropy measure is the expected value of the situation. Thus, by letting $\pi_i = y_i/z$, the information function above yields $ln(z/y_i)$, which is precisely our basic evaluation tool. The formula for the evaluation of a trial is also a weighted sum of those values, even though the weights differ, as our values are not probabilities and we contemplate the evaluation of repeated trials. Yet the measurement approach is quite similar in spirit and mathematical structure, so that our formula can be interpreted as a different way of estimating the entropy of an experiment.

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Event
Event Budget for Event : EXPENSES
Actual Cost Breakdown
Estimated vs. Actual
Site
Decorations
Publicity
Miscellaneous
Refreshments
Program
Prizes
$0.00
$10.00
$20.00
$30.00
$40.00
$50.00
$60.00
$70.00

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