serie ad

WP-AD 2014-01

Information and Consumer Fraud in a Signalling Model

Silvia Martínez-Gorricho

Ivie

Working papers Working papers

Versión: enero 2014 / Version: January 2014

Edita / Published by:
Instituto Valenciano de Investigaciones Económicas, S.A.
C/ Guardia Civil, 22 esc. 2 1º - 46020 Valencia (Spain)

WP-AD 2014-01

Information and Consumer Fraud in a Signalling Model

Silvia Martínez-Gorricho*

Abstract

This article considers a two-sided private information model. We assume that two exogenously given qualities are offered in a monopolistic market. Prices are fixed. A low quality seller chooses to be either honest (by charging the lower market price) or dishonest (by charging the higher price). We discuss the signaling role of the consumer's private information on the equilibrium level of dishonesty, incidence of fraud and trade. We demonstrate that the equilibrium incidence of fraud is nonmonotonic in the buyer's private information when the prior belief favors the low-quality seller strongly enough. This result holds as long as information is noisy and regardless of its private or public nature. Welfare consequences are ambiguous.

Keywords: Consumer Fraud; Asymmetric Information; Price Signalling.

JEL classification numbers: D42; D82; G14; L15; L51.

* Corresponding address: University of Alicante, Department of Economics, Crta. San Vicente s/n, 03690, San Vicente del Raspeig (Alicante). E-mail: silviamg@ua.es. Phone: +34 965903400 (ext. 3218). Fax: +34 965903898.

I thank Ettore Damiano, Martin J. Osborne, Aloysius Siow, Sumon Majumdar and Miguel A. Sánchez Villalba for helpful comments and suggestions. An earlier version of this article appeared as a chapter in my University of Toronto Ph. D dissertation. Financial support from the Ministerio de Ciencia e Innovación and FEDER funds under project SEJ2007-62656 and from the Spanish Ministry of Economics and Competitiveness (ECO2012-34928) is gratefully acknowledge.

1 Introduction

The market is the place set apart where men may deceive one another.

Anacharsis, 600 B.C.

Consumer fraud is a common international occurrence. As an illustration, the international seafood industry suffers from an activity called seafood fraud: lower-quality and less expensive fish are often mislabeled as desirable species for financial gain. For instance, a piece of sushi sold as the luxury treat white tuna usually turns out to be Mozambique tilapia or even escolar. Recent studies[1] using DNA bar coding techniques have found that seafood may be mislabeled at restaurants and stores as often as 25% to 70% of the time for fish like red snapper, wild salmon, Atlantic cod, tuna and grouper among others. Generally, consumer fraud is a well documented phenomenon in the United States. The National Institute of Justice sponsored a November 1991 survey [2] finding that 15% of participants had been the victim of a successful personal fraud[3] in the 12 months prior to the interview and 13% of them reported suffering direct monetary injury. Similarly, the American Association of

[1]Refer to reports about investigations carried out by Globe (Abelson and Daley (2011) and Daley and Abelson (2011)); by Consumer Reports Magazine in 2011; by Oceana (Warner (2011)); and by the U.S. Government Accountability Office (GAO) in 2009.

[2]The results of this survey have been published in Titus, Heinzelmann and Boyle (1995).

[3]The survey asked about "local" fraud problems, such as problems with automobile or appliance repairs and with home repairs and improvements.

Retired Persons (AARP) conducted a survey in December 1998. Three quarters of those in the study reported that they had at least one bad experience when buying a product or service in the year preceding the interview. Further, 17% answered that they were the subject of major consumer fraud. The Federal Trade Commission (FTC) commissioned a 2003 survey of 2,500 randomly chosen adults about their consumer experiences during the previous year by targeting specific types of fraud. The results suggest that 25 million adult Americans (around 11.2% of adult population) were victims of one or more of the fraud actions covered by the survey.

Information provision is considered a market response to the buyers' incomplete information. Clients rationally take into account imperfect signals on quality when taking decisions. Some examples are information provided by independent third parties such as consumer report magazines, consumer opinion websites and institutional warnings. If the information privately collected by the consumer is informative but noisy, then two-sided information asymmetries exist in the market. Both suppliers and consumers are unable to identify the type of the market participant they are confronting.

Our theoretical model is motivated by this evidence. The main purpose of this article is to investigate the following relevant policy questions: is the relationship between information asymmetries and the equilibrium incidence of fraud monotonic? What is the role played by the accuracy of consumers' private information on the equilibrium levels of trade and welfare?

The information gap between consumers and traders is lowered as the consumers' private information, which is imperfectly correlated with the true quality of the good, gets more precise. Perfect information about quality helps consumers make informed choices but at the same time, it deprives consumers of any informational rent. This fact prevents consumers from appropriating any gains from trade if the seller is a monopolist price setter and prices are flexible. Martínez-Gorricho (2012) shows that under the assumption of flexible prices, a decrease in the consumers' informational rent may increase the seller's market power resulting in higher equilibrium prices for high quality products. A positive relationship between the equilibrium prices and the precision of the imperfect private information owned by consumers means that a priori costless exogenous information provision becomes costly to consumers. The benefit to consumers of making a more informed choice is outweighed by the price distortion for some parameter values. In this setting, an increase in the precision of the consumer's private information may be harmful for consumers. These results imply that the empowerment of consumers by a policy favoring accurate information may go against the goal of enhancing consumers' welfare if prices are flexible.

In regulated markets, prices are fixed, and so is, in essence, the seller's market power. This allows consumers to capture some gains from trade in a monopoly regime under full information if the market prices are set below the consumers' valuations for the goods. Given that the previous reasoning does not longer apply, we are interested in answering the following question: may better private information harm consumers in regulated markets? Furthermore, regulated markets are interesting for two additional reasons: (i) the fixed-prices could serve as tools with which governments can tackle the problem of fraud; and (ii) the monopolist cannot perfectly price discriminate consumers even if consumers' private information became public. If consumers' types were perfectly identified by the seller, then there would be one-sided information asymmetries instead of two-sided information asymmetries in the market. Ceteris paribus, for a given information precision level, this would lower the information gap between consumers and the monopolist. However, the reduction in the information gap is not symmetrically distributed among market participants because the seller is the only one who obtains additional information. As a result, it is not a priori obvious how publicly revealing the buyer's private information affects welfare. This raises the following interesting questions: can consumers benefit from their identification, that is, from public revelation of their private information? What is the effect of publicly revealing the buyer's private information on the equilibrium incidence of fraud and welfare? And so, should governments promote and subsidize public signals?

To address these policy questions, we introduce a simple price-quality signaling model of an experience good with private information on both sides of the market. The supplier is a monopolist who sells a good of exogenously determined quality, either high or low.[4] The seller knows the quality of his[5] good and he demands one of the two exogenously fixed prices: p_L and $p_H > p_L$ for his good. A consumer observes the price demanded and a costless noisy binary signal of quality and chooses whether to purchase the good or not. We assume that the consumer is not willing to pay the high price for a low quality product. The informational asymmetries between the buyer and the seller create incentives for attempting fraud in this context: the seller with a low quality good could try to manipulate the consumer's beliefs by charging the high price. If the buyer is fooled into purchasing a low-quality product at the high price, then we say that the buyer becomes a victim of fraud.

We prove the existence of equilibria involving fraud for all parameter values and perform comparative statics with respect the parameters of interest. If prices are fixed, it seems intuitive to conjecture that an increase in the accuracy of the buyer's private information reduces her probability of being deceived. After all, if the buyer becomes better informed about the quality of the object for sale, the seller will be less successful in misrepresenting it and he will be able to deceive the buyer less frequently. This lower probability of trade should restrain the seller from overcharging and induce him to be more honest. However, this argument is incomplete. The equilibrium level of dishonesty may increase in the precision of the buyer's private

[4] This description typically fits a market for an experience good whose quality is subject to some stochastic process, being examples, products which are generated by R&D, crops, or even a fish.

[5] For ease of exposition, we refer to a seller as 'he' and to a consumer as 'she'.

information. This result has an intuitive explanation. As the signal's precision increases, a buyer who receives a favorable private signal becomes more convinced of the product's good quality whereas the opposite is true when the buyer receives an unfavorable signal. If the prior belief is not very biased toward the high quality good and the private information is not too accurate, only a buyer with a favorable signal is willing to pay the high price in equilibrium. An increase in the quality of the buyer's information increases the willingness to pay of such buyer, and ultimately induces the low quality seller to attempt a fraud more often. The equilibrium incidence of fraud is therefore nonmonotonic in the precision of the buyer's private information. In this particular case, better information is beneficial from the seller's point of view but it hurts consumers. In general, better private information has ambiguous welfare consequences but both the consumer and the seller always benefit in expected terms from better buyer's information if this information is precise enough. Ex-ante, the seller may be harmed by more precise customer's information for an intermediate range of signal precision levels because trade is nonmonotonic in information.

Next, we show that a key requirement for the existing nonmonotonic relationship between the incidence of fraud and information is the noisiness of information. We prove it by examining other (more extreme) information structures often analyzed in the literature. These information structures assume that a fraction of consumers have access to perfect information about the quality of the good on sale whereas the rest of consumers remain uninformed. Under this assumption, the equilibrium incidence of fraud is a nonincreasing function of the fraction of informed consumers in the market. This implies that the relationship between information asymmetries and the equilibrium incidence of fraud is (weakly) monotonic when a perfect knowledge of quality is in hands of a few consumers and the supplier is unable to detect which consumers are informed. This implies that the relationship between these two variables depends on how information is sorted among consumers.

Finally, we prove that our findings are robust to the perfect identification of buyer's types. The relationship between the equilibrium incidence of fraud and information precision remains nonmonotonic when the consumer's private information becomes public. Publicly revealing the buyer's private information can increase or decrease the incidence of fraud as well as the ex-ante expected payoffs of the buyer and the seller. However, (i) it maximizes total surplus because the potential gains of trade are fully realized for all parameter values; and (ii) it is a Pareto improvement relative to the case of private information provision under some conditions.

The article is organized as follows. Section 2 provides a literature review. The model is formalized in section 3 and some preliminaries are shown in section 4. Section 5 carries out the equilibrium analysis and presents the central results of the article. Section 6 concludes. All proofs are contained in the Appendix.

2 Related Literature

The existing industrial organization literature on price as a signal of quality can be embodied into two main areas of research: a first area is concerned with the moral hazard aspects of the choice of quality, whereas the second focuses on the classical adverse selection problem, pioneered by Akerlof (1970). The latter line of research is the one pursued in this article. In these models, quality is not treated as a choice variable but, instead, is exogenously given. The vast majority of the articles that belong to this body of literature assume extreme information structures: the potential customers are either perfectly informed or completely uninformed about the quality of the good on sale. Several articles (Wolinsky (1983), Judd and Riordan (1994), Voorneveld and Weibull (2004)) relax this assumption. Our modeling approach is in the spirit of Voorneveld and Weibull (work): we specify that each consumer observes a private and imperfect binary signal without incurring any cost. As a result, the model is categorized as a two-sided asymmetric information model. The central question addressed by the price-quality signaling models consists on determining whether in such settings, prices alone are capable of conveying information on quality in equilibrium. Therefore, they restrict attention to pure strategy sequential equilibria and investigate the existence of fully separating equilibrium outcomes that survive selection criteria. Thus, in such equilibrium outcomes, honest reporting prevails in the market. In contrast, we focus on pooling and mixed strategies, which may exhibit various degrees of fraud. Furthermore, we perform comparative statics with respect to different parameters of interest in order to determine whether more or less information is revealed in equilibrium, and calculate its impact on the level of dishonesty and incidence of fraud in the market.

A related literature that focuses on the analysis of fraud includes work on credence goods.[6] Dulleck and Kerschbamer (2006) provide a unifying framework that can reproduce the majority of results of the literature. Most of the existing literature also assumes extreme information structures. An exception is Hyndman and Ozerturk (forthcoming) who introduce non-identifiable heterogeneously informed consumers. The authors show that when a positive fraction of consumers observe a noisy but informative signal about the seriousness of the problem and the expert cannot distinguish between informed and uninformed consumers, there is a unique equilibrium outcome in which the expert is always truthful to all types of consumers. Instead, we prove the existence of equilibria involving fraud for all parameter values.

Our article also contributes to the literature on the value of information. Several related articles that belong to this literature also belong to the literature on quality uncertainty. In all these models, sellers are uncertain about the objective quality or the buyers' tastes for their products and as a result, there is no scope for signalling. Instead, the monopolist is perfectly informed about the quality of his product in our

[6] With credence goods (Darby and Karni (1973)), consumers cannot judge actual quality either before or after purchase.

model and the fixed prices could be used as signals of quality to induce purchases by the potential imperfectly informed buyers.

Using a one-sided asymmetric information framework with private information on the demand side, Lewis and Sappington (1994) discuss the value of buyers' private information for a monopolist as we do assuming a balanced prior belief. Moscarini and Ottaviani (2001) extend the model introducing price competition in a duopolistic market. Both articles find a result which is opposite to ours: while a poorly informed buyer likes better information, a well-informed one dislikes it. This is because suppliers optimally sells to a broad market when the quality of information is sufficiently low while they target only the customers with favorable information for its own good otherwise. They also find that the supplier's expected profit increases (declines) as buyers' information about the product improves in the range where the signals are (not) very accurate. In contrast, the seller can benefit from better private information when the quality of this information is low in a signalling framework with two-sided information asymmetries.

Levin (2001) finds that the relationship between information asymmetries and trade (measured by the maximum probability with which a good can be traded) is nonmonotonic in a competitive market with private information on the supply side.

Ottaviani and Prat (2001) analyze a non-linear pricing model categorized as a two-sided asymmetric information model. The monopolist can choose to commit to publicly reveal his private information. The authors show that the monopolist is always better off by committing to reveal directly the information inferred by the buyer in equilibrium and to destroy the remaining information.[7] Finally, Schlee (1996) analyzes a model in which quality information is public and hence, there are no information asymmetries. The author identifies two properties of the cost functions[8] that lead to a negative value of information for consumers: increasing returns to scale and "sufficiently" convex marginal costs. We demonstrate that the result that consumers may sometimes prefer less public information about product quality also extends to a signalling model with information asymmetries.[9]

[7]In addition, under affiliation and supermodularity, the monopolist achieves higher expected profits by committing to reveal his private information in full.

[8]It is assumed that the cost of production is independent of quality and it is strictly increasing in quantity.

[9]The author also provides an example where private information about quality may hurt consumers facing a monopolist price setter. However, his example is constructed under the assumption that trade is not desirable in the low-quality state under full information. We obtain a similar finding in a signalling setting under the assumption that trade is always desirable under full information.

3 The Model

Consider a market for an indivisible experience good[10] (commodity or service) whose quality is the only characteristic relevant to consumers. We assume that there is only one potential buyer and one seller. The seller produces one unit of the good for sale and the customer is willing to purchase at most one unit. Both agents are risk neutral. The seller maximizes expected profits and the buyer maximizes the expected valuation net of the price paid.

We simplify our analysis by assuming that two exogenously given qualities are offered in the market. A good can be of either low quality or high quality: $\Theta = \{L, H\}$. The seller and consumer differ from each other in their valuation for different quality items. The consumer values quality at $0 < v_L < v_H$ respectively. The seller's valuation or reservation price for both types of items is normalized to zero. As a result, potential gains from trade are positive in both scenarios.

The seller's good is of high quality ($\theta = H$) with probability $\pi \in (0, 1)$ and of low quality with probability $1 - \pi$. This probability distribution is common knowledge. The unit production costs are also common knowledge and normalized to zero. The seller knows his actual, realized quality, but his potential customer does not and

[10]For experience goods, quality must be verifiable at least after consumption. Thus, beliefs are given by a probability distribution over quality and are known ex ante or at least, can be deduced from own experience after consumption. (Nelson (1970), Nelson (1974)).

there is no credible direct way by which the seller can provide this information before the customer makes her purchasing decision.

Nevertheless, the seller cannot completely disguise the true quality of his product. Assume that prior to purchase, the consumer obtains without cost a private binary signal, which conveys a certain amount of information about product quality. Let the signal structure be common knowledge and be given by: $Pr(s = \theta|\theta) = \delta \ \forall\theta \in \Theta$ and $Pr(s = \theta'|\theta) = 1 - \delta \ \forall\theta \neq \theta'$ and $\theta, \theta' \in \Theta$ where $(1/2) < \delta \leq 1$. The number δ is the probability of observing the correct signal realization and it is interpreted as the precision of the consumer's signal. In the limit, when it is equal to one, quality can be deduced with certainty by pure inspection before consumption. This is the case of symmetric information. In the other extreme case, if δ were equal to one half, the signal would become totally uninformative as it would be uncorrelated with the quality of the good, corresponding to the case of one-sided asymmetric information. In the intermediate cases in which the signal is imperfectly correlated with the true quality of the product, this model is categorized as a two-sided asymmetric information model.

We study the case in which price is the only signaling variable available to the seller. The seller sets a take-it-or-leave-it price $p \in \{p_L, p_H\}$ for his unit, knowing its quality. Prices are exogenously fixed[11] and satisfy $0 < p_L < v_L < p_H < v_H$, so that

[11] If prices are endogenized, then a plethora of equilibria emerge in a one period monopoly setting and among the refinements suggested in the literature, only criteria D1, which is equivalent

under perfect information, the buyer is not willing to purchase a low quality item at a high price. Otherwise, trade is desirable under perfect information. From now on, selling a low quality object at a high price will be referred to as "fraud".[12] Note also that the buyer and the seller share the gains from trade if trade occurs. That is, the seller is unable to extract the entire buyer's surplus under full information due to the price rigidity.[13]

The consumer is Bayesian rational; she has beliefs over the seller's types and she uses all the available information to update her beliefs according to Bayes rule.

The buyer's strategy is simply whether to accept or reject the offer proposed by the seller. If the offer proposed by the seller is accepted by the buyer, the buyer consumes the good and she enjoys its true valuation. The seller's and buyer's respective (ex-post) utilities if the offer at price p is accepted are given by $u_s = p$ and $u_b = v - p$, where $v \in \{v_L, v_H\}$. If the offer is rejected, the buyer does not

Universal Divinity and Never a Weak Best Response in this setting, has any power in pruning the set of outcomes, and its power is very limited. Refer to Martínez-Gorricho (2012a).

[12] In the English dictionary by Oxford University Press, fraud is defined as the action or an instance of deceiving somebody in order to make money. Alternatively, it is defined as a thing that is not what is claimed to be.

[13] The assumption $p_L < v_L$ can be justified using the following argument: if there were some heterogeneity in the valuation for the low-quality good, the consumer with higher valuation for the low-quality good will enjoy a strictly positive surplus. As this extended version of the model does not provide any significant additional insights, this assumption is useful in letting the model be as simple as possible.

consume the good and both agents' utilities are normalized to zero.

3.1 Equilibrium

We confine attention to (weak) Perfect Bayesian Equilibrium (PBE). Let ϕ_H^* and ϕ_L^* denote the probabilities that each type of seller charges the high price in equilibrium. Let $b_H^*(p)$ and $b_L^*(p)$ denote the probabilities that the buyer who observes the high signal realization and low signal realization respectively accepts the offer p proposed by the seller in equilibrium.

Definition 3.1. A Perfect Bayesian Equilibrium (PBE) consists of beliefs and strategies (μ^*, ϕ^*, b^*) satisfying the following requirements:

(A) Given the players' beliefs, their strategies are sequentially rational. At each information set, the action taken by the player with the move must be optimal given the player's belief at the information set and the other players' subsequent strategies:

(1) The seller's strategy ϕ^* is a best reply to the buyer's strategy:

$$\forall \theta, \quad \phi_\theta^* \in \arg\max_{\phi} Eu(\bigcup_{p,s} b_s^*(p), \phi|\theta)$$

where $Eu(\bigcup_{p,s} b_s^*(p), \phi|\theta) \equiv \phi[\delta b_\theta^*(p_H) + (1-\delta) b_{\theta'}^*(p_H)]p_H + (1-\phi)[\delta b_\theta^*(p_L) + (1-\delta) b_{\theta'}^*(p_L)]p_L$ denotes the seller's expected utility.

(2) The buyer's strategy b^* is optimal given p, s and her associated beliefs:

$$\forall p, s, \quad b_s^*(p) \in \arg\max_b Eu(p, s, \mu_b(\theta|p,s), b)$$

where $Eu(p, s, \mu_b(\theta|p,s), b) \equiv \sum_{\theta\in\Theta} \mu_b(\theta|p,s) b(v_\theta - p)$ denotes the buyer's expected utility.

(B) At each information set, the player with the move has a belief about which node in the information set has been reached by the play of the game. At information sets on the equilibrium path, beliefs are determined by Bayes' rule and the players' equilibrium strategies:

(3) At each seller's information set $\{H, L\}$, the system of seller's beliefs satisfies:

$$\mu_s^*(s|\theta) = \begin{cases} \delta & \text{if } s = \theta \\ 1-\delta & \text{if } s \neq \theta \end{cases}$$

(4) At each buyer's information set $\{(H, p_H), (H, p_L), (L, p_H), (L, p_L)\}$, the system of consumer beliefs is Bayes-consistent, that is,

$$\mu_b^*(\theta|p_H, s) = \begin{cases} \frac{Pr(\theta)Pr(s|\theta)\phi_\theta^*}{\sum_{\theta'\in\Theta} Pr(\theta')Pr(s|\theta')\phi_{\theta'}^*} & \text{if } \sum_{\theta'\in\Theta} Pr(\theta')Pr(s|\theta')\phi_{\theta'}^* > 0 \\ Arbitrary & \text{if } \sum_{\theta'\in\Theta} Pr(\theta')Pr(s|\theta')\phi_{\theta'}^* = 0 \end{cases}$$

$$\mu_b^*(\theta|p_L, s) = \begin{cases} \frac{Pr(\theta)Pr(s|\theta)(1-\phi_\theta^*)}{\sum_{\theta'\in\Theta} Pr(\theta')Pr(s|\theta')(1-\phi_{\theta'}^*)} & \text{if } \sum_{\theta'\in\Theta} Pr(\theta')Pr(s|\theta')(1-\phi_{\theta'}^*) > 0 \\ Arbitrary & \text{if } \sum_{\theta'\in\Theta} Pr(\theta')Pr(s|\theta')(1-\phi_{\theta'}^*) = 0 \end{cases}$$

4 Preliminaries

4.1 Symmetric Information

The classical case of symmetric information, that is, when the signal is perfectly informative, corresponds to the boundary case $\delta = 1$ in the present model. Due to the absence of information asymmetries, a unique separating equilibrium exists. The buyer's optimal strategy is to accept trade at a high price offer if and only if she observes a high signal realization and to accept trade at a low price for every possible signal realization. As a result, the seller's unique best reply to the buyer's strategy is to be honest by setting the low price if he has a low quality good and a high price otherwise. Trade occurs with certainty in equilibrium so that the expected potential gains from trade are fully realized:

$$\bar{TS} \equiv \pi v_H + (1 - \pi) v_L \tag{1}$$

4.2 Asymmetric Information: The Buyer's Decision Problem

Let $\bar{v}$ denote the ex-ante buyer's expected valuation for a given object: $\bar{v} = \pi v_H + (1 - \pi) v_L$. Assume that the buyer's private information is imperfectly correlated with the true quality of the good, ie. assume $\delta \in (1/2, 1)$. Once the potential customer observes the price-signal pair (p, s), she updates her beliefs as to which type of seller she faces. The potential customer's optimal decision is to purchase the good if and only if the price posted by the seller does not exceed the ex-post expected valuation of the good, which is given by $v_L + \mu_b(H|p,s)(v_H - v_L)$. As a result, $\forall s \in \{L,H\}$, the buyer's optimal trading decision rule takes the form:

$$b_s^*(p_H) = \begin{cases} 1 & \text{if } \mu_b(H|p_H,s) > \frac{p_H - v_L}{v_H - v_L} \\ [0,1] & \text{if } \mu_b(H|p_H,s) = \frac{p_H - v_L}{v_H - v_L} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

and

$$b_s^*(p_L) = 1, \tag{3}$$

where $(p_H - v_L)/(v_H - v_L) \in (0,1)$ could be interpreted as a proxy for the extent of fraud (in relative terms) existing in the market, provided that the sellers engage in fraudulent actions. The closer the value of $(p_H - v_L)/(v_H - v_L)$ is to 0, the less severe is the fraud committed by sellers. This is because the price charged by the seller with the low quality good is only slightly above the consumer's true valuation for this good. Likewise, the closer the value of $(p_H - v_L)/(v_H - v_L)$ is to 1, the more severe is the seller's fraudulent behavior.

Upon having received a high price offer, the posterior beliefs of the customer who observes the high signal realization are at least as high as the posterior beliefs of the customer who observes the low signal realization. This implies $b_H^*(p_H) \geq b_L^*(p_H)$ and therefore, $\delta b_H^*(p_H) + (1-\delta)b_L^*(p_H) \geq (1-\delta)b_H^*(p_H) + \delta b_L^*(p_H)$. Although in this model there does not exist an explicit cost of signalling high quality by charging the high price, there exists an implicit opportunity cost in terms of the probability of trade. As in the standard signalling games, this opportunity cost is (weakly) higher for the low quality seller than for the high quality seller. As a result, it must be the case that the high quality seller charges the high price in equilibrium with a probability at least as high as the probability at which the low quality seller charges this price: $\phi_H^* \geq \phi_L^*$.

Note that if no customer type accepts the high price offer, then the sellers' optimal response is to pool on the low price. If instead all customer types always accept the high price offer, then the sellers' optimal response is to pool on the high price. Otherwise, the probability of trading the object at the high price is strictly greater for the high quality seller than for the low quality seller according to (2).

These simple observations allow us to state the following lemma:

Lemma 4.1. *Suppose* $\frac{1}{2} < \delta < 1$. *An equilibrium is of one of the following types:*

(i) $\phi_H^* = \phi_L^* = 0$.

(ii) $\phi_H^* = 1$ *and* $\phi_L^* \in (0, 1]$.

The main focus of this article is analyzing the effect of information on the level of dishonesty and incidence of fraud in equilibrium. We consider the *candidate* pooling equilibria in which both types of sellers charge the low price not interesting because there is no fraud attempted in such *potential* equilibria.[14] Consequently,

[14] These potential equilibria can be shown to fail an extension of criterion D1 to a setting with two types of receivers and the reasonable assumption about out-of-equilibrium beliefs $\mu_b(H|p_H, H) \geq \mu_b(H|p_H, L)$.

we restrict our analysis to study only the remaining candidate equilibria. In those potential equilibria, the high quality seller always offers his item for sale at the high price whereas the low quality seller may pool on the high price or may randomize between both prices. Completely fraudulent behavior by the low quality seller is encompassed in pooling equilibria whereas partial dishonest behavior by the low quality seller is revealed in hybrid equilibria.

Definition 4.1. A Fraudulent Pooling Equilibrium is an equilibrium in which both sellers pool on the high price, that is, $\phi_H^* = \phi_L^* = 1$, and therefore, the low quality seller displays a completely fraudulent behavior.

Definition 4.2. A Fraudulent Hybrid Equilibrium is an equilibrium in which the high quality seller chooses p_H and the low quality seller randomizes between p_H and p_L, that is, $\phi_H^* = 1$ and $\phi_L^* \in (0, 1)$, and therefore, the low quality seller displays a partial dishonest behavior.

5 Equilibrium Analysis

Given the sellers' strategies in equilibrium, the buyer's posterior beliefs after receiving signal s and a high price offer are by Bayes consistency:

$$\mu_b^*(H|p_H, s) = \begin{cases} \frac{\pi\delta}{\pi\delta+(1-\pi)(1-\delta)\phi_L^*} \equiv \hat{\mu}_H(\phi_L^*) & \text{if } s = H \\ \frac{\pi(1-\delta)}{\pi(1-\delta)+(1-\pi)\delta\phi_L^*} \equiv \hat{\mu}_L(\phi_L^*) & \text{if } s = L \end{cases} \tag{4}$$

$$\mu_b(L|p_H, s) = 1 - \mu_b(H|p_H, s) \tag{5}$$

These posterior beliefs satisfy the monotone likelihood ratio property and, therefore, first-order stochastic dominance.

Figure 1 displays the buyer's posterior beliefs as a function of the accuracy of the buyer's private information after a high price quotation. Due to the informativeness of the private signal, the buyer's posterior beliefs upon receiving a high (low) signal realization are increasing (decreasing) in the signal precision level. Both posterior beliefs are decreasing in the level of dishonesty exerted by the low quality seller in equilibrium, ϕ_L^*. Ceteris paribus, the lower is ϕ_L^*, the more certain are both types of customer that the quality of the good conditional on having been offered the item at the high price is high and the higher is their expected valuation of the good. Note that ϕ_L^* denotes the precision of the high price as a signal of quality. The lower is ϕ_L^*, the more informative is the price set strategically by the seller. In addition, both posterior beliefs increase with the prior belief π. Note also that after being offered the item at the high price and having observed a favorable signal, the buyer's posterior beliefs about facing a high-quality seller are necessarily higher than the prior. Instead, the posterior beliefs of the buyer with unfavorable signal and a high price offer could be higher or lower than the prior depending on the value of ϕ_L^* (the lower-bound is zero).

Let δ_H (δ_L) denote the signal accuracy levels at which a customer who observes a high (low) signal realization is indifferent between accepting or rejecting a high price offer under completely fraudulent behavior by the low quality seller. Thus, if $p_H > \bar{v}$,

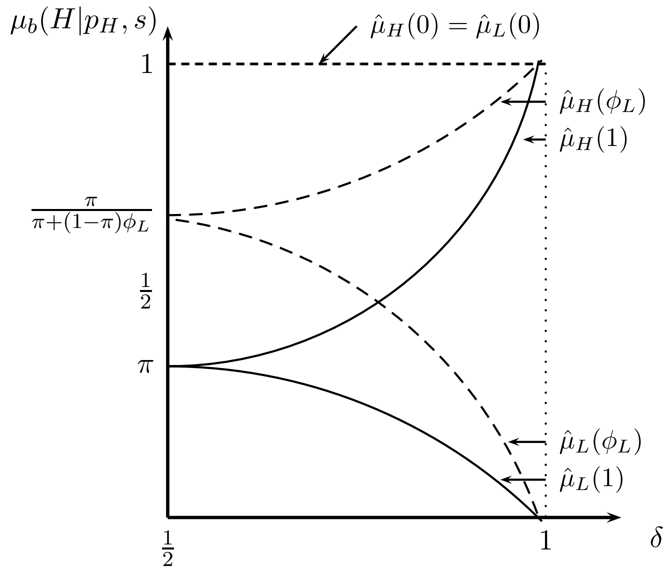

Figure 1: Buyer's posterior beliefs of confronting a high quality seller as a function of the signal precision if p_H is offered.

then $\delta_H = [1 + (\pi/(1-\pi))((v_H - p_H)/(p_H - v_L))]^{-1}$, whereas the low-signal customer cannot be made indifferent because then $\hat{\mu}_L(1) < \pi < (p_H - v_L)/(v_H - v_L)$. Alternatively, if $p_H < \bar{v}$, then $\delta_L = 1 - [1 + (\pi/(1-\pi))((v_H - p_H)/(p_H - v_L))]^{-1}$ whereas the high-signal customer cannot be made indifferent because then $(p_H - v_L)/(v_H - v_L) < \pi < \hat{\mu}_H(1)$. Let define $\bar{\pi} \equiv [1 + (p_L/(p_H - p_L))((v_H - p_H)/(p_H - v_L))]^{-1}$ and $\underline{\pi} \equiv [1 + ((p_H - p_L)/p_L)((v_H - p_H)/(p_H - v_L))]^{-1}$. Now, we proceed to establish conditions for the existence of pooling and hybrid equilibria involving consumer fraud.

Proposition 5.1. *Assume* $1/2 < \delta < 1$ *and* $\pi \geq \frac{p_H - v_L}{v_H - v_L}$.

(i) Fraudulent Pooling Equilibria exist if and only if $\delta \leq \max\{\delta_L, 1 - (p_L/p_H)\}$.

(ii) Fraudulent Hybrid Equilibria exist if and only if $\delta \geq \max\{\delta_L, 1 - (p_L/p_H)\}$ *and* $\delta \neq \delta_L$.

Proposition 5.2. *Assume* $1/2 < \delta < 1$ *and* $\pi < \frac{p_H - v_L}{v_H - v_L}$.

(i) If $\pi \in \left[\underline{\pi}, \frac{p_H - v_L}{v_H - v_L}\right)$,

(a) Fraudulent Pooling Equilibria exist if and only if $\delta \in [\delta_H, 1 - (p_L/p_H)]$.

(b) Fraudulent Hybrid Equilibria exist if and only if $\delta \in \left(\frac{1}{2}, \delta_H\right) \cup [1 - (p_L/p_H), 1)$.

(ii) If $\pi < \underline{\pi}$, *no fraudulent pooling equilibrium exists whereas fraudulent hybrid equilibria exist* $\forall \delta \in (1/2, 1)$.

Corollary 5.3. *Equilibria involving fraud exist for all* $\delta \in (1/2, 1)$.

Uniqueness[15] of fraudulent pooling and hybrid equilibrium outcomes is guaranteed for almost all values of δ. For the non-generic cases $\delta = \delta_H$ and $\delta = \delta_L$, there exists a continuum of fraudulent pooling equilibrium outcomes parameterized by the randomization strategy of the customer who observes the high and low-signal realization respectively. For the non-generic case $\delta = 1 - (p_L/p_H)$, there exists a continuum of fraudulent hybrid equilibrium outcomes in accordance with the seller's randomization strategy, which is not pinned down but bounded from above and below. The complete characterization of the fraudulent equilibria is provided in the appendix. However, the following partial characterizations will be proved useful for following up the discussion:

Definition 5.1. Assume $\delta \in \left(\frac{1}{2}, 1\right)$. Four different types of fraudulent equilibrium can be distinguished for generic values of δ:

(i) Fraudulent Pooling HL equilibrium, which is characterized by $\phi_H^* = \phi_L^* = 1$ and $b_H^*(p_H) = b_L^*(p_H) = 1$.

(ii) Fraudulent Pooling H equilibrium, which is characterized by $\phi_H^* = \phi_L^* = 1$, $b_H^*(p_H) = 1$ and $b_L^*(p_H) = 0$.

(iii) Fraudulent Hybrid $H\bar{L}$ equilibrium, which is characterized by $\phi_H^* = 1$, $\phi_L^* = \underline{\phi}(\delta)$, $b_H^*(p_H) = 1$ and $b_L^*(p_H) = 1 - \frac{1}{\delta}\left(1 - \frac{p_L}{p_H}\right)$.

[15] Recall that our analysis ignores the possible existence of pooling equilibria in which both types of seller charge the low price as discussed before.

(iv) Fraudulent Hybrid $\bar{H}$ equilibrium, which is characterized by $\phi_H^* = 1$, $\phi_L^* = \bar{\phi}(\delta)$, $b_H^*(p_H) = \left(\frac{1}{1-\delta}\right)\frac{p_L}{p_H}$[16] and $b_L^*(p_H) = 0$.

where $\underline{\phi}(\delta) \equiv \left(\frac{\pi}{1-\pi}\right)\left(\frac{1-\delta}{\delta}\right)\left(\frac{v_H-p_H}{p_H-v_L}\right)$ and $\bar{\phi}(\delta) \equiv \left(\frac{\pi}{1-\pi}\right)\left(\frac{\delta}{1-\delta}\right)\left(\frac{v_H-p_H}{p_H-v_L}\right)$

As illustrated in Figure 2, when $p_H > 2p_L$, the parameter space can be partitioned into four regions, each corresponding to a different specification of the equilibrium strategies. We focus our discussion on the case $p_H > \max\{\bar{v}, 2p_L\}$ as this range yields the most interesting results.

If information is sufficiently noisy, $\delta < \min\{\delta_H, 1-(p_L/p_H)\}$, then observing a favorable signal does not necessarily imply so good news in terms of quality. If the low-quality seller always attempts fraud, the posterior beliefs of the customer who observes a favorable signal are updated slightly upwards and the expected valuation, although higher than its ex-ante expected value ($\bar{v}$), is still lower than the requested high price. As a result, this buyer is not willing to purchase the item on sale at the high price. Hence, no pooling equilibrium exists for this range of information precision. By contrast, if there existed some level of honesty in the market, this customer's posterior beliefs could be updated upwards strongly enough that her expected valuation could beat the price. This implies that with private information of bounded precision, there is room for the existence of a fraudulent hybrid equilibrium

[16] This function is increasing and convex in δ. As $\delta \to 1/2$, $b_H^*(p_H) \to 2(p_L/p_H)$; as $\delta \to 1-(p_L/p_H)$, $b_H^*(p_H) \to 1$.

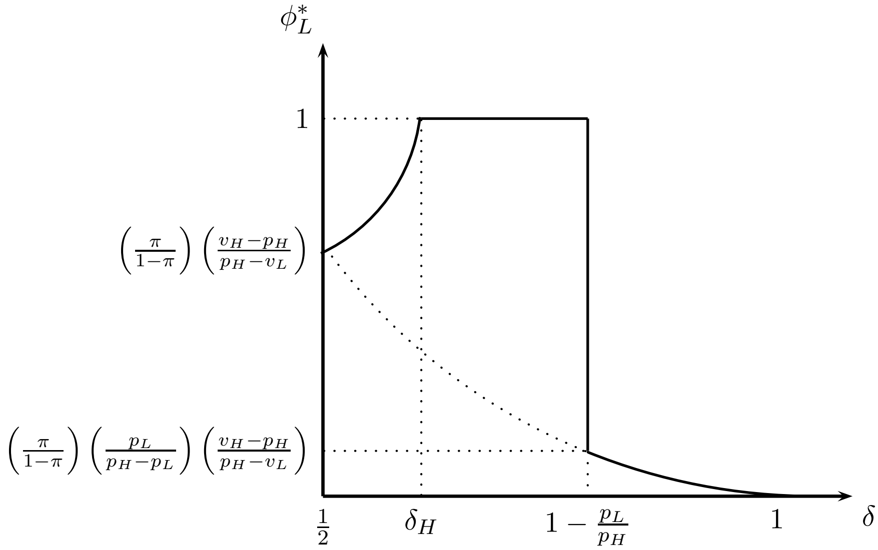

Figure 2: The equilibrium level of dishonesty as a function of the signal precision if $p_H > \max\{\bar{v}, 2p_L\}$ & $\delta_H < 1 - \frac{p_L}{p_H}$.

(*Fraudulent Hybrid* $\bar{H}$) in which the low quality seller targets only the customer with favorable information for his good, fully extracting her rent.[17] When the signal is fairly accurate, $\delta > \min\{\delta_H, 1-(p_L/p_H)\}$, observing a favorable signal does convey a positive concomitant meaning in terms of quality. The posterior beliefs of this buyer are updated upwards strongly enough that it is rational for this buyer to purchase the item on sale at the high price even in the most pessimistic scenario of completely fraudulent behavior. On the other hand, observing an unfavorable signal is bad news in terms of quality and the posterior beliefs of the low-signal customer are updated downwards so that this buyer is not willing to accept the high price offer if completely fraudulent behavior prevails in the market place. The essential trade-off the low-quality supplier faces in choosing ϕ_L^* is whether to sell his product to a segmented market or a broad market. If $\delta_H < \delta \leq 1-(p_L/p_H)$, it is profitable for the low-quality seller to always attempt fraud targeting only the customer with favorable information (*Fraudulent Pooling* H). This is because the unit profit is high despite a lower volume of sale. In contrast to previously, this buyer type is now able to enjoy rents. Otherwise, if $\delta > 1-(p_L/p_H)$, then it is

[17] This hybrid equilibrium outcome is robust with respect to the introduction of noisy quality signals (the same hybrid outcome can be supported in equilibrium if $\delta = 1/2$ as if $\delta \to 1/2$). Technically however, and in comparison with the case in which the buyer does not receive any signal at all, the high-signal buyer must accept now the high price offer twice as many times to compensate for the outright rejection of the offer by the low-signal customer.

not rational for the low-quality seller to be always completely dishonest because this seller does not confront a customer who observes a favorable signal frequently enough. Hence, no fraudulent pooling equilibrium can be supported for this range of signal precision values. Instead, a unique hybrid outcome *(Fraudulent Hybrid $H\bar{L}$)* can be supported in equilibrium in which the low-quality seller commits a level of fraud that guarantees that the buyer buys also when receiving an unfavorable signal but it leaves no rents to this buyer type. Note that as $\delta \to 1$, $\underline{\phi}(\delta) \to 0$ and $b_L^*(p_H) \to p_L/p_H$. Therefore, as the signal is made arbitrarily precise, this hybrid equilibrium converges to a separating equilibrium in which the customer who observes the low signal realization agrees to trade at the high price with a positive probability which is strictly lower than one.[18]

5.1 Comparative Statics

5.1.1 Private Information

The purpose of this subsection is to analyze the effect of the buyer's private information on the equilibrium level of dishonesty and incidence of fraud as well as on trade and welfare. Total surplus is measured by the expected gains of trade that are realized in equilibrium. The equilibrium incidence of fraud is defined as the

[18] Consequently, we have a discontinuity. This separating equilibrium outcome is absent under perfect information and therefore, it is not robust. The slightest decrease in signal precision causes this hybrid equilibrium outcome to emerge.

customer's expected probability of becoming a victim of fraud:

$$\Phi^* \equiv (1-\pi)\phi_L^*[(1-\delta)b_H^*(p_H) + \delta b_L^*(p_H)]. \tag{6}$$

Similarly, the equilibrium expected loss of the consumer due to fraud is defined as the product of the equilibrium incidence of fraud and the extent of fraud, $Eloss = \Phi^*((p_H - v_L)/(v_H - v_L))$.

Except for trade, the equilibrium values of these variables of interest are continuous in the private information. A main insight of this subsection is that more precise customer's private information may lead to higher levels of fraud committed in equilibrium, benefiting the sellers and harming consumers. Additionally, trade may be nonmonotonic in information even if the previous result does not hold, implying that total surplus may decrease as the private information gets more precise despite lower levels of fraud.

Proposition 5.4. *Assume $p_H \leq \max\{\bar{v}, 2p_L\}$. The equilibrium level of dishonesty ϕ_L^* is nonincreasing in the quality of the buyer's information δ.*

Proposition 5.5. *Assume $p_H > \max\{\bar{v}, 2p_L\}$. The equilibrium level of dishonesty ϕ_L^* is nonmonotonic in the quality of the buyer's information δ. For relatively imprecise signals ($\delta < \min\{\delta_H, 1-(p_L/p_H)\}$), more precision leads to more dishonesty in equilibrium whereas for relatively precise signals ($\delta > 1-(p_L/p_H)$), more precision leads to less dishonesty in equilibrium.*

As previously, we focus our discussion on the case $p_H > \max\{\bar{v}, 2p_L\}$ which corresponds to Proposition 5.5. Figure 3 illustrates the equilibrium level of dishonesty under conditions $p_H > 2p_L$ and $\pi \in [\underline{\pi}, (p_H - v_L)/(v_H - v_L))$.

The nonmonotonic effect of an increase in information on the level of market dishonesty if $p_H > \max\{\bar{v}, 2p_L\}$ is due to a change in the identity of the marginal customer (the customer who is made indifferent between purchasing or not the product ex-post). In the Fraudulent Hybrid $\bar{H}$ equilibrium outcome, the marginal customer is the one who observes a favorable signal. The fact that this customer becomes more certain that the quality of the good is high as the signal becomes more informative intensifies the seller's dishonest behavior. This implies that the public information revealed by the seller behaves as a substitute of the buyer's private information since the more precise is the customer's private information, the less precise is the high price as a public signal of quality. In the Fraudulent Hybrid $H\bar{L}$ equilibrium outcome, the marginal customer is the one who observes an unfavorable signal. The fact that this customer is more certain that the quality of the good is low as the signal gets more precise moderates the seller's unethical behavior. Consequently, a better buyer's private information improves the quality of the public information supplied strategically by the seller. This reinforcement stresses the complementary nature shared by the private and public signals in this region of the parameter space.

If $\delta < \min\{\delta_H, 1 - (p_L/p_H)\}$, better private information leads the customer who observes a favorable signal to accept trade at the high price more often. Consequently, the equilibrium trade probability for high quality-items ($t_H^* = \delta b_H^*(p_H) = (\delta/(1-\delta))\,(p_L/p_H)$) is increasing in δ while the equilibrium trade probability for low items offered at the high price is invariant and equal to $t_L^*(p_H) = (1-\delta)b_H^*(p_H) = (p_L/p_H)$. As a result, better private information leads to more fraud attempted and committed in the market ($\Phi^* = (1-\pi)(p_L/p_H)\bar{\phi}(\delta)$). The expected equilibrium trade probability for low-quality units ($t_L^* = 1 - \phi_L^*(p_H)(1 - t_L^*(p_H)) = 1 - \bar{\phi}(\delta)\,(1 - (p_L/p_H))$) is decreasing in δ. All low-quality items which are offered at the low price are traded in the market with certainty. However, as the quality of private information increases, the less often are low-quality goods offered at the low price. This negative effect on trade of low quality products is strong enough to reverse the positive effect on trade of high quality products so that the expected level of trade in equilibrium ($t^* = 1 - \pi\,[1 + (\delta/(1-\delta))\,(1 - 2(p_L/p_H))]$) is decreasing in the quality of information. Because trade of high-quality items is increasing in the quality of information, so is the expected utility of the high-quality supplier. By definition of hybrid equilibrium, the low-quality seller is indifferent between charging either price in equilibrium. His utility is constant thereby and equal to the low price. Therefore, the ex-ante monopolist benefits from a stronger signal of the buyer:

$$Eu_s^* = \left[\pi\left(\frac{\delta}{1-\delta}\right) + 1 - \pi\right] p_L \tag{7}$$

The buyer who observes a favorable signal is made indifferent between accepting and rejecting the high price offer. Thus, her expected utility conditional on being offered the item at the high price is zero. The customer with an unfavorable signal rejects all high price offers so that her utility conditional on receiving a high price offer is also zero. Any customer who is offered the item at the low price obtains a strictly positive surplus which is independent of the private information precision level. However, the increase in the dishonesty level worses the equilibrium expected payoff of the buyer:

$$Eu_b^* = (1-\pi)(1-\bar{\phi}(\delta))(v_L - p_L) \tag{8}$$

A higher low price increases the seller's equilibrium payoff while it reduces the buyer's equilibrium payoff. Therefore, the total surplus achieved in equilibrium is increasing and convex in δ if and only if the low price is high enough ($p_L > ((v_H - p_H)/(v_H - v_L))\, v_L$). Otherwise, it is a decreasing and concave function of the signal precision:

$$TS^* = \pi \left(\frac{\delta}{1-\delta}\right) \frac{p_L}{p_H} v_H + (1-\pi)\left[1 - \bar{\phi}(\delta)\left(1 - \frac{p_L}{p_H}\right)\right] v_L < \bar{TS} \tag{9}$$

If $\delta_H < \delta \leq 1 - (p_L/p_H)$, the low-quality seller always attempts fraud so that the high price becomes an uninformative signal of quality and the customer only relies on her private information to deduce the quality of the object on sale. The more precise is the customer's private information, the lower is the probability that a low-quality seller confronts a customer with a favorable signal, and the lower are the equilibrium incidence of fraud ($\Phi^* = (1-\pi)(1-\delta)$) and the expected loss of fraud. The equilibrium trade probabilities for high-quality ($t_H^* = \delta$) and low-quality products ($t_L^* = t_L^*(p_H) = 1 - \delta$) are respectively increasing and decreasing in information. Therefore, the seller's equilibrium expected profits are increasing in δ if and only if $\pi > 1/2$:

$$Eu_s^* = [\pi\delta + (1-\pi)(1-\delta)]p_H \tag{10}$$

The customer whose signal realization is low is not willing to trade at the high price. This implies that the buyer's expected utility is increasing in the precision of her information as the expected valuation of the customer who observes a favorable signal is increasing in δ:

$$Eu_b^* = \pi\delta(v_H - p_H) - (1-\pi)(1-\delta)(p_H - v_L) \tag{11}$$

As a result, a more precise signal leads to an increase in the total surplus only if the prior probability for a high-quality product is large enough ($\pi > v_L/(v_H + v_L)$).

$$TS^* = \pi\delta v_H + (1-\pi)(1-\delta)v_L < \bar{TS} \tag{12}$$

If $\delta > 1 - (p_L/p_H)$, as the quality of information improves, the low-quality seller confronts a customer who observes an unfavorable signal more often but this customer type accepts trade at the high price more frequently. As a result, the equilibrium trade probability for low-quality products which are offered at the high price ($t_L^*(p_H) = p_L/p_H$) remains invariant leading to a lower incidence of fraud

($\Phi^* = (1-\pi)(p_L/p_H)\ \underline{\phi}(\delta)$) and a lower expected loss due to fraud in equilibrium. This implies that the equilibrium trade probability for low-quality units ($t_L^* = 1 - \underline{\phi}(\delta)\,(1-(p_L/p_H))$) is increasing in δ as a stronger signal leads the low-quality seller to quote the low price more often and this offer is always accepted by the buyer in equilibrium. The equilibrium trade probability for high quality products ($t_H^*(p_H) = 1 - ((1-\delta)/\delta)\,(1-(p_L/p_H))$) and the expected level of trade in equilibrium ($t^* = 1 - \pi\,((1-\delta)/\delta)\,(1-(p_L/p_H))\,((v_H - v_L)/(p_H - v_L))$) are also increasing in δ. Consequently, the more precise is the private information, the higher is the ex-ante seller's expected profits in equilibrium:

$$Eu_s^* = \pi\left[1 - \left(\frac{1-\delta}{\delta}\right)\left(1 - \frac{p_L}{p_H}\right)\right]p_H + (1-\pi)p_L \tag{13}$$

A more precise private information makes the ex-ante buyer better off since it leads to less fraud *attempted* in the market place:

$$Eu_b^* = \pi\left[1 - \left(\frac{1-\delta}{\delta}\right)\left(1 - \frac{p_L}{p_H}\right)\right](v_H - p_H) + (1-\pi)\left[v_L - p_L - \underline{\phi}(\delta)\left(1 - \frac{p_L}{p_H}\right)v_L\right] \tag{14}$$

The total surplus converge to the potential gains from trade in the limit, as the signal is made arbitrarily precise. Overall, the total surplus is an increasing and concave function of the signal precision.

$$TS^* = \pi\left[1 - \left(\frac{1-\delta}{\delta}\right)\left(1 - \frac{p_L}{p_H}\right)\right]v_H + (1-\pi)\left[1 - \underline{\phi}(\delta)\left(1 - \frac{p_L}{p_H}\right)\right]v_L < \bar{TS} \tag{15}$$

Finally, it is worth mentioning that the Fraudulent Pooling HL equilibrium outcome can be supported if the prior is sufficiently biased toward the high quality product and the buyer's private signal is not too informative. In this region of the parameter space, trade always takes place between the buyer and the seller in equilibrium independently of the signal realization observed. As a result, the potential gains from trade are fully realized so that this equilibrium is efficient. However, the incidence of fraud remains constant at its highest possible value, $1-\pi$.

All these results lead to the following proposition.

Proposition 5.6. *Assume $\delta \in (1/2, 1)$. The effect of the consumer's private information δ on the incidence of fraud (Φ^*), the expected consumer's loss due to fraud ($ELoss^*$), the expected level of trade (t^*), the buyer's expected utility (Eu_b^*), the ex-ante seller's expected profits (Eu_s^*) and the expected total surplus (TS^*) in equilibrium is as follows:*

(i) The Incidence of Fraud Φ^* and the Consumer's Expected Loss due to Fraud $ELoss^*$ are nonincreasing in the quality of private information δ if $p_H \leq \max\{\bar{v}, 2p_L\}$. Otherwise, they increase with the quality of the buyer's information δ for $\delta < \min\{\delta_H, 1-(p_L/p_H)\}$ and decrease with δ for $\delta > \min\{\delta_H, 1-(p_L/p_H)\}$.

(ii) The expected level of trade t^* is strictly increasing in the quality of the buyer's information δ if $\bar{v} < p_H < 2p_L$. Otherwise, it is nonmonotonic and noncon-

tinuous in δ.

(iii) The ex-ante seller's equilibrium profits Eu_s^* are increasing in the quality of the buyer's information δ if $\pi \leq \underline{\pi}$ or if $\max\{1/2, \underline{\pi}\} < \pi \leq (p_H - v_L)/(v_H - v_L)$ or if $\underline{\pi} < \pi < \min\{1/2, (p_H - v_L)/(v_H - v_L)\}$ and $p_H \leq 2p_L$. Otherwise, they are nonmonotonic in δ.

(iv) The buyer's equilibrium payoff Eu_b^* is nondecreasing in the quality of private information δ if $p_H \leq \max\{\bar{v}, 2p_L\}$. Otherwise, the buyer's equilibrium payoff Eu_b^* decreases with the quality of the buyer's information δ for $\delta < \min\{\delta_H, 1 - (p_L/p_H)\}$ and increases for $\delta > \min\{\delta_H, 1 - (p_L/p_H)\}$.

(v) The equilibrium total surplus TS^* is increasing in the quality of private information δ if $\bar{v} < p_H < 2p_L$ or if $\pi \leq \underline{\pi}$ and $p_L > ((v_H - p_H)/(v_H - v_L))\, v_L$ or if $\max\{v_L/(v_L + v_H), \underline{\pi}\} < \pi < (p_H - v_L)/(v_H - v_L)$ and $p_L > ((v_H - p_H)/(v_H - v_L))\, v_L$. Otherwise, it is nonmonotonic in δ.

(v) The ex-ante seller's equilibrium profits Eu_s^*, the buyer's equilibrium payoff Eu_b^* and the equilibrium total surplus TS^* are not convex in the quality of the buyer's private information δ.

When the private signal is sufficiently precise and the prior belief is not very biased in favor of the high quality seller (the Hybrid $H\bar{L}$ region of the parameter space), the complementary nature shared by both the buyer's private information and the endogenous information revealed by the seller implies that policies that favor accurate private information provision lead to a Pareto improvement and less fraud committed in the market place. This result also holds when the private signal is not sufficiently precise and the prior is biased but not too biased toward the high quality seller. When the precision of the private signal is low and the prior takes intermediate values favoring the low quality seller, accurate private information provision policies are beneficial for the buyer but detrimental for the seller. Instead, when both the prior and precision of the private signal are low (the Hybrid $\bar{H}$ region), the consumer is harmed while the seller is benefitted due to the sustitutability nature shared by the exogenous and endogenous information. Consequently, governments concerned about consumers' welfare and about protecting them from unfair commercial practices should not favor more accurate private information provision policies in this region of the parameter space.

5.1.2 Prior Belief

A lower prior belief π that the good on sale is of high-quality disciplines the low-quality seller in hybrid equilibria favoring the trade of low quality products and resulting in a lower incidence of fraud and expected loss due to fraud. The seller's expected profits decrease in π whereas the impact on the buyer's equilibrium expected payoff is positive in Fraudulent Hybrid $\bar{H}$ equilibrium but ambiguous in Fraudulent Hybrid $H\bar{L}$ equilibrium. In the latter case, the buyer is better off for weakly enough signals. Let define $\rho \equiv [(v_H - v_L)/(p_H - p_L) - 1]\,[(p_H - v_L)/(v_H - p_H)]$.

Proposition 5.7. *The seller's equilibrium payoff Eu_s^* is nondecreasing and globally not convex in the prior π.*

Proposition 5.8. *CHANGE For high enough quality of private information ($\delta > \max\{(1+\rho)^{-1}, 1-(p_L/p_H)\}$), the buyer's equilibrium payoff Eu_b^* is increasing in the prior π. Otherwise, the buyer's equilibrium payoff Eu_b^* is nonmonotonic in the prior π. For all quality levels of private information, the buyer's equilibrium payoff Eu_b^* is globally convex in the prior π.*

5.1.3 Regulation and Welfare Analysis

Suppose that a regulator could manipulate the values of the prices of the products exogenously fixed in our market. The value of the low price does not have an effect on the level of dishonesty[19] but it does so on the incidence of fraud and the expected loss due to fraud through the acceptance probability in fraudulent hybrid equilibria. On the one hand, a higher low-price p_L favors trade in equilibrium and therefore, it increases total surplus but at a cost of a higher incidence of fraud and expected loss due to fraud. This has a negative (positive) impact on the buyer's (seller's) expected payoff.

[19]However, note that in the limiting case in which the low price is set arbitrarily small, the equilibrium level of dishonesty is nondecreasing in the signal accuracy level for all signal precision levels and high price values.

A higher high price p_H disciplines the low-quality seller and discourages trade in fraudulent hybrid equilibria. Hence, the incidence of fraud is lowered at the cost of aggravating the extent of fraud. The first effect dominates the second effect reducing the consumer's expected loss due to fraud in hybrid equilibria. The first effect does not exist in pooling equilibria so that the second effect increases the expected loss of the consumer due to fraud. Hence, a higher high-price decreases the expected loss of fraud for sufficiently low values of π ($\pi < \pi_H$ if $\delta < 1 - (p_L/p_H)$ and $\pi < \pi_L$ if $\delta > 1 - (p_L/p_H)$) while it increases the expected loss of fraud otherwise. The impact on the agents' expected utilities is non-negative in Fraudulent Hybrid $\bar{H}$ equilibrium whereas it is ambiguous for the buyer in Fraudulent Hybrid $H\bar{L}$ equilibrium: it is non-negative for sufficiently low signal precision levels ($\delta \leq (1 + \lambda)/(2 + \lambda)$ where $\lambda \equiv (v_H - v_L)(v_L - p_L)/(p_H - v_L)^2$).[20]

In addition, the high and low prices have an effect on the support for the different types of fraudulent pooling and hybrid equilibria. a higher low-price $p_L$favors the existence of Fraudulent Hybrid $H\bar{L}$ equilibrium, widening its region in the parameter space. This change in the equilibrium regime decreases the equilibrium incidence of fraud. A higher high price p_H increases δ_H and $1 - (p_L/p_H)$ whereas it decreases δ_L. That is, a higher high price p_H favors the existence of fraudulent hybrid equilibria and Fraudulent Pooling H equilibrium versus Fraudulent Pooling HL equilibrium,

[20]The expected total surplus is increasing in p_H for all signal precisions in hybrid equilibria.

inducing the low-quality seller to be more honest for a wider-range of signal precision values.

The regulator could make the consumer's incidence of fraud and expected loss of fraud arbitrarily small by setting the high price arbitrarily close to the buyer's valuation of a high-quality product v_H. In such case, and for all prior values, only Hybrid $\bar{H}$ equilibrium can be supported for sufficiently low signal precision levels ($\delta < 1 - (p_L/v_H)$) while only Hybrid $H\bar{L}$ equilibrium can be supported otherwise. Its effect on the trade of high-quality products and the consumer's surplus under Hybrid HL would be negative.

5.2 Other Information Structures

This section briefly sketches how the main results of the model change when the information structure is modified. We now assume, as the vast majority of the articles in the literature do, that some consumers can ascertain the quality of the product perfectly by inspection whereas the remaining consumers are completely uninformed about the quality of the good and they ex ante believe that quality is high with probability π. Let α denote the probability that the consumer is perfectly informed. This case is closely related to the case in our model in which consumers are heterogeneous in the degree of signal precision: a fraction α of them observe a perfectly informative signal ($\delta_1 = 1$) whereas the rest observe an uninformative signal ($\delta_2 = 1/2$). Note that the seller with the low quality product who charges the high price can potentially capture only a totally uninformed consumer. Informed consumers will accept the high price offer only if the seller's good is of high quality. The results differ from those in our model as follows:

(*i*) A sufficiently large fraction of informed consumers ($\alpha > 1 - (p_L/p_H)$) enables the high price to signal high quality, leading to the emergence of a separating equilibrium. By definition, there is no fraud committed in any separating equilibrium and the potential gains from trade are fully realized.

(*ii*) If the high price is higher than the ex-ante expected valuation of the good ($p_H > \bar{v}$) and the fraction of informed consumers is not sufficiently large ($\alpha < 1 - (p_L/p_H)$), a unique hybrid equilibrium outcome exists: the high quality seller always charges the high price, the low quality seller randomizes strictly between setting both prices and charges the high price with a probability $\phi_u \equiv (\pi/(1-\pi))\,((v_H - p_H)/(p_H - v_L))$. The uninformed customer accepts the high price offer with probability $b_u^* = (1/(1-\alpha))\,(p_L/p_H)$. Both types of customer always accept the low price offer. Note that the level of fraud does not depend on the fraction of informed consumers because the uninformed consumer's posterior beliefs do not depend on the value of α. On the other hand, the acceptance probability by the uninformed customer is increasing in α. Ceteris paribus, the higher is α, the higher is the probability that a low quality seller confronts an informed customer and therefore, the higher is the probability that his offer may end up being rejected. In order for this seller to remain indifferent and still be willing to randomize, the uninformed customer must accept the high price offer more frequently so that the probability of trade for the low quality seller remains invariant. The incidence of fraud is not a function of the fraction of informed consumers and it is equal to $\Phi^* = \pi\left((v_H - p_H)/(p_H - v_L)\right)(p_L/p_H)$. The equilibrium trade probability for low-quality items is given by: $t_L^* = \phi_u(1-\alpha)b_u^* + (1-\phi_u) = 1 - \phi_u\left(1 - (p_L/p_H)\right)$. It is independent of α because the level of dishonesty does not depend on it. The equilibrium probability with which high-quality items are traded is increasing α because informed consumers always accept trade at the high price: $t_H^*(p_H) = \alpha + (1-\alpha)b_u^* = \alpha + (p_L/p_H)$. As a result, a higher fraction of informed buyers in the market is beneficial for the ex-ante seller's economic interests: $Eu_s^* = p_L + \alpha\pi p_H$ and for the realization of the potential gains from trade: $W^* = \pi v_H\left(\alpha + (p_L/p_H)\right) + v_L\left[(1-\pi) - \pi\left((v_H - p_H)/(p_H - v_L)\right)\left(1 - (p_L/p_H)\right)\right]$. Finally, consumers always benefit from the presence of more informed consumers in ex-ante terms because these consumers never become victims of fraud: $Eu_b^* = \pi(v_H - p_H)\left(\alpha + (p_L/p_H)\right) + (1-\pi)(v_L - p_L) - \pi v_L\left((v_H - p_H)/(p_H - v_L)\right)\left(1 - (p_L/p_H)\right)$.

(iii) If the high price is lower than the ex-ante expected valuation of the good ($p_H < \bar{v}$) and the fraction of informed consumers is not sufficiently large

($\alpha < 1 - (p_L/p_H)$), then a unique pooling equilibrium outcome exists: both sellers always charge the high price and the uninformed customers always accept any offer. Trade is not monotonic in information because informed customers are never fooled into purchasing low-quality products at the high price. As a result, the incidence of fraud is decreasing in the fraction of informed consumers whereas the ex-ante buyer's expected utility is increasing in it. By the same reasoning, the seller's ex-ante expected utility and total surplus are decreasing in α.

It is immediate that the equilibrium incidence of fraud is a non-increasing function of the fraction of informed consumers in the market. Furthermore, consumers always benefit from more information in ex-ante expected terms. Under this more extreme information structure, more information is always beneficial for all agents in expected terms if $p_H > \bar{v}$. Therefore, the equilibria can be Pareto ranked according to the value of the fraction of informed consumers in the market if $p_H > \bar{v}$. This result contradicts our previous result of a possible negative value of private information for some agents if $p_H > \bar{v}$.

In addition, if $\pi > [1 + (p_L/(p_H - p_L))((p_H - v_L)/(v_H - p_H))]^{-1}$ then $p_H < \bar{v}$ and $\delta_L > 1 - (p_L/p_H)$. Under these assumptions, no agent is made worse off in ex-ante terms by an increase in the precision of the buyer's private imperfect information in our original model. However, under this more extreme information structure, the effect of an increase in the fraction of informed consumers on the monopolist's ex-ante expected profits and total surplus is negative if $\alpha \geq 1-(p_L/p_H)$.

5.3 Public Revelation of the Buyer's Private Information

This subsection briefly sketches how the main results of the model change when the consumer's private information about the good's quality becomes public. Then, the model is categorized as a one-sided asymmetric information model on the supply side. Because market prices are fixed, they cannot be made contingent on the realization of the public signal. As in Fong (2005), the seller replaces price discrimination with cheating his identifiable customers selectively.

The main difference with respect to our previous analysis is that the ex-post opportunity cost of signalling high quality by charging the high price (in terms of the trade probability) is the same for both types of seller. As a result, for each value of the signal precision, there exists a pooling[21] equilibrium in which the prices offered are not signals of quality. Let ϕ^s denote the probability with which both types of seller offer the high price to the potential customer in equilibrium once

[21] There also exists a continuum of hybrid equilibria outcomes in accordance with the sellers' randomization strategies which are themselves not pinned down but their ratio is pinned down. For example, any strategies $\bigcup_{\theta \in \Theta, s \in S}\{\phi_\theta^{*s}\}$ and $\bigcup_{s \in S}\{b_s^*(p_H), b_s^*(p_L)\}$ such that $\phi_L^{*H}/\phi_H^{*H} = \bar{\phi}(\delta)$, $\phi_L^{*L}/\phi_H^{*L} = \underline{\phi}(\delta)$, $b_H^*(p_H) = b_L^*(p_H) = p_L/p_H$ and $b_H^*(p_L) = b_L^*(p_L) = 1$ can be supported in equilibrium. Unfortunately, given that the equilibrium dishonesty level is not pinned down, comparative statics cannot be performed.

signal s is publicly observed. Three main cases can be distinguished for generic values of δ:

(*i*) If $p_H > \bar{v}$ and $\delta < \delta_H$, the sellers' optimal strategy is to always pool on the low price so that there is no fraud committed in equilibrium. The equilibrium is efficient as the potential gains from trade are fully realized.

(*ii*) If $p_H > \bar{v}$ and $\delta > \delta_H$ or if $p_H < \bar{v}$ and $\delta > \delta_L$, both types of seller always offer the good at the high price in equilibrium if a high signal realization is publicly observed whereas they offer the item at the low price after the observation of a low signal realization: $\phi^H = 1 > \phi^L = 0$. Trade is always accepted in equilibrium. As a result, the equilibrium is efficient: $W^* = \pi v_H + (1-\pi)v_L$. As the public information becomes more precise, the observation of a positive signal becomes more rare if the good on sale is of low-quality. Consequently, the incidence of fraud ($\Phi^* = (1-\pi)(1-\delta)$) is lower and in the limit, it approaches a zero value as the signal becomes arbitrarily precise. The seller's and buyer's ex-ante expected utilities are respectively:

$$Eu_s^* = [\pi\delta + (1-\pi)(1-\delta)]p_H + [\pi(1-\delta) + (1-\pi)\delta]p_L \tag{16}$$

$$Eu_b^* = \pi\delta(v_H - p_H) + (1-\pi)(1-\delta)(v_L - p_H) + \pi(1-\delta)(v_H - p_L) + (1-\pi)\delta(v_L - p_L) \tag{17}$$

The value of public information is negative (positive) to the buyer (seller) if and only if $\pi > 1/2$.

(*iii*) If $p_H < \bar{v}$ and $\delta < \delta_L$, the sellers' optimal strategy is to always pool on the high price and the buyer always agrees to trade at this price independently of the signal realization observed. The incidence of fraud is at its highest possible value in equilibrium. The potential gains from trade are fully realized so the equilibrium is efficient. The seller's and buyer's ex-ante expected utilities do not depend on the value of δ.

Publicly reveling the consumer's private information favors the existence of a truthful equilibrium if public information is sufficiently noisy and $p_H > \bar{v}$. This is because if the public signal precision were fairly low, the inability of the seller with a high-quality product to signal its quality forces him to offer the item at the low-price in order to make positive profits. Instead, the level of dishonesty is strictly positive if public information is sufficiently precise and $p_H > \bar{v}$. As a consequence, the previous result that the incidence of fraud is non-monotonic in information if $p_H > \bar{v}$ continues to be robustly present. However, the incidence of fraud can now be eradicated for sufficiently noisy public signals whereas it was strictly positive for all possible values of private information precision.

Under the assumptions $p_H > \max\{\bar{v}, 2p_L\}$ or $p_H < \bar{v}$ and $\delta_L > 1 - (p_L/p_H)$, we found that the value of private information can be negative for some agents only if information is not accurate enough. On the contrary, we find now that the value of public information can be negative for some agents only if information is sufficiently precise.

Do agents benefit from public revelation of the consumer's private information? What is the welfare impact of such public revelation? It is immediate that total surplus is unambiguously higher when consumers' private information becomes public because the potential gains from trade are fully realized for all parameter values. Its impact on the rest of variables of interest is ambiguous.

Under the assumption $p_H > \max\{\bar{v}, 2p_L\}$, the incidence of fraud is (weakly) lower when the signal is revealed publicly than when it remains private. However, under the assumption $p_H < \bar{v}$, the incidence of fraud is higher under the perfect identification of the consumer's type only if the information noise is small: $\delta > \left[1 + [(\pi/(1-\pi))\,((v_H - p_H)/(p_H - v_L))\,(p_L/p_H)]^{-1}\right]^{-1}$.

The impact of public revelation on the ex-ante buyer's expected utility is positive[22] if $\pi \geq 1/2$ but ambiguous if $\pi < 1/2$. Suppose that the probability with which a high-quality product is supplied is less than one half. Then, the buyer achieves a higher expected utility in ex-ante terms when her information is revealed publicly if and only if the signal is not sufficiently precise.[23]

[22] Suppose the information noise is small. Then, the higher incidence of fraud due to the public revelation of the buyer's private information could be more than compensated by the possibility of acquiring a high-quality item at the low price, a possibility non-existent when information is made private.

[23] The requirement is $\delta < (\pi/(1-2\pi))\,((v_H - p_H)/(p_H - v_L))$ under the assumption $p_H > \max\{\bar{v}, 2p_L\}$ and $\delta < (\pi/(1-2\pi))\,((v_H - p_H)/(p_H - v_L))\,((v_L - p_L)/(p_H - p_L))$ under the as-

Under the assumptions $p_H < \bar{v}$ and $\pi < 1/2$, the seller always benefits from public revelation of the buyer's private information. Otherwise, the monopolist benefits in ex-ante terms from this revelation only if the signal is sufficiently precise.[24]

This analysis implies that both market participants could simultaneously benefit from public revelation of the buyer's private information under some conditions, leading to a Pareto improvement.

6 Concluding Remarks

This article has explored the role played by the accuracy of consumers' private information on the equilibrium levels of dishonesty, incidence of fraud and welfare in regulated markets with monopoly power and two-sided information asymmetries. In brief, our main findings are three-fold. First, equilibria involving fraud exist for all parameter values. Second, the level of fraud and incidence of fraud are not monotonic in information if the buyer's information is imperfect. This result is robust to public revelation of the buyer's information. Third, a more precise information can harm the buyer and/or the seller for some parameter values.

---

sumption $p_H < \bar{v}$.

[24]Under the assumption $p_H > \max\{\bar{v}, 2p_L\}$, the requirement is $\delta > \delta_H$. Under the assumptions $p_H < \bar{v}$ and $\pi > \frac{1}{2}$, the requirement is $\delta < \max\{\max\{\delta_L, 1 - (p_L/p_H)\}, \pi/(2\pi - 1)\}$.

References

[1] Abelson, J., and Daley, B., (2011), "On the Menu, But Not On Your Plate", Boston Globe, October 23 2011.

[2] Akerlof, G., (1970), "The Market for Lemons: Quality Uncertainty and the Market Mechanism", Quarterly Journal of Economics 84, 488-500.

[3] Bester, H., and Ritzberger, K., (2001), " Strategic Pricing, Signalling, and Costly Information Acquisition", International Journal of Industrial Organization 19, 1347-1361.

[4] Daley, B., and Abelson, J., (2011), "From Sea to Sushi Bar, a System Open to Abuse", Boston Globe, October 24 2011.

[5] Darby, M., and Karni, E., (1973), "Free Competition and the Optimal Amount of Fraud", Journal of Law and Economics 16, 67-88.

[6] Dulleck, U., and Kerschbamer, R., (2006), "On Doctors, Mechanics and Computer Specialists: The Economics of Credence Goods", Journal of Economic literature 44, 5-42.

[7] Fong, Y.F., (2005), "When Do Experts Cheat and Whom Do They Target?", RAND Journal of Economics 36, 113-130.

[8] Judd, K.L., and Riordan, M.H., (1994), "Price and Quality in a New Product Monopoly", Review of Economics Studies 61, 773-789.

[9] Hyndman, K. and Ozerturk, S., Forthcoming, "Consumer Information in a Market for Expert Services", Journal of Economic Behavior and Organization.

[10] Levin, J., (2001), "Information and the Market for Lemons", RAND Journal of Economics 32, 657-666.

[11] Lewis, T.R., and Sappington, D.E.M., (1994), "Supplying Information to Facilitate Price Discrimination", International Economic Review, 309-327.

[12] Martinez-Gorricho, S., (2012) "Beneficial Consumer Fraud", IVIE Working Paper WP-AD-2012-13.

[13] Nelson, P., (1970), "Information and Consumer Behavior", Journal of Political Economy 78, 311-329.

[14] Oceana, (2011), "Bait and Switch: How Seafood Fraud Hurts Our Oceans, Our Wallets and Our Health", Oceana's Report, May 25 2011.

[15] Ottaviani M., and Prat, A. (2001) "The Value of Public Information in Monopoly", Econometrica 69, 1673-1683.

[16] Schlee, E., (1996), "The Value of Information about Product Quality", RAND Journal of Economics 27, 803-815.

[17] Titus, R.M., Heinzelmann, F., and Boyle, J.M., (1995), “Victimization of Persons by Fraud”, Crime and Deliquency 41, 54-72.

[18] U.S. Government Accountability Office, (2009), “Seafood Fraud: FDA Program Changes and Better Collaboration among Key Federal Agencies Could Improve Detection and Prevention”, GAO-09-258, February 2009.

[19] Voorneveld, M., and Weibull, J.W., (2004), “Prices and Quality Signals”, Working Paper 551, Department of Economics, Stockholm School of Economics.

[20] Warner, K., (2011), “Seafood Fraud Found in Boston-Area Supermarkets”, Oceana’s Report, October 24 2011.

[21] Wolinsky, A., (1983), “Prices as Signals of Product Quality”, Review of Economic Studies 50:4, 647-658.

Appendix A: Proofs

Proof of Lemma 4.1. We must prove that no separating equilibrium in which the high type seller sets the high price and the low type seller sets the low price nor hybrid equilibria in which the high type seller strictly randomizes exist for $\delta \in (1/2, 1)$. The proof is by contradiction.

(i) Suppose that a separating equilibrium, in which the high type seller sets the high price and the low type seller sets the low price, exists. Consider an information set $(p_H, s) \forall\ s \in S$ of the buyer. By consistency of beliefs along the equilibrium path, she believes that the item is of high quality with probability one. Because $v_H > p_H$, her optimal decision is to purchase the product for all possible signal realizations. But then, this implies that the low quality seller could increase his expected payoff by charging the high price. A contradiction.

(ii) Suppose that a separating equilibrium, in which the high type seller sets the low price and the low type seller sets the high price, exists. Consider an information set $(p_H, s) \forall\ s \in S$ of the buyer. By consistency of beliefs along the equilibrium path, she believes that the item is of low quality with probability one. Because $v_L < p_H$, her optimal decision is not to purchase the product for all possible signal realizations. But then, this implies that the low quality seller could increase his expected payoff by charging the low price. A contradiction.

(iii) Suppose that an hybrid equilibrium, in which both seller types strictly randomize between charging both prices, exists. Then, the low quality seller must be indifferent between charging either price. But the high quality seller could increase his expected payoff by charging only the high price given his strictly higher probability of trade at this price relative to that of the low quality seller. A contradiction.

(iv) Suppose that an hybrid equilibrium in which the high type seller strictly randomizes between charging both prices and the low quality seller charges only the low price, exists. This results in a contradiction by the same argument as in (i).

(v) Suppose that an hybrid equilibrium in which the high type seller strictly randomizes between charging both prices and the low quality seller charges only the high price, exists. Then, the high quality seller must be indifferent between charging either price. But then, the low quality seller could increase his expected earnings by not charging the high price but the low price. A contradiction.

(vi) Suppose that an hybrid equilibrium in which the high type seller sets the low price and the low quality seller strictly randomizes between charging both prices, exists. This results in a contradiction by the same argument as in (iii).

Proof of Proposition 5.1. We prove this proposition through a series of lemmas.

Lemma A.1: *Assume $p_H \leq \bar{v}$. Fraudulent Pooling equilibria exist if and only if $\delta \leq \max\left\{\delta_L, 1 - \frac{p_L}{p_H}\right\}$.*

Proof. For necessity, suppose not, so that if $p_H \leq \bar{v}$, fraudulent pooling equilibria exist if $\delta > \max\{\delta_L, 1 - (p_L/p_H)\}$. By Bayes consistency, the posterior beliefs $\mu_b^*(H|p_H, s)$ must be given by equation (4). Because $\delta > \delta_L$, then $\mu_b^*(H|p_H, H) > (p_H - v_L)/(v_H - v_L) > \mu_b^*(H|p_H, L)$ so that the buyer's optimal strategies conditional on the signal received are $b_H^*(p_H) = 1$ and $b_L^*(p_H) = 0$. The low quality seller's expected profit if he charges the high price p_H is given by $(1-\delta)p_H < p_L$ due to the fact that $\delta > 1 - (p_L/p_H)$. The low quality seller would find it more profitable to deviate and charge exclusively the low price. A contradiction. We prove sufficiency through claims A.1.1-A.1.3:

Claim A.1.1: *Assume $p_H < \bar{v}$. There exists a pooling equilibrium (Fraudulent Pooling HL) characterized by the following strategies $\phi_H^* = \phi_L^* = 1$, $b_H^*(p_H) = b_L^*(p_H) = 1$ and the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (4) if $\delta < \delta_L$. This equilibrium is unique.*

Proof. Consider the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (4). To verify that (μ^*, ϕ^*, b^*) is a PBE if $\delta < \delta_L$, note that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta < \delta_L$, $\mu_b^*(H|p_H, s) > (p_H - v_L)/(v_H - v_L)\ \forall s \in S$. By (2), the buyer's strategy is rational with respect to them. Now consider any seller. Because the buyer agrees to trade at both prices always, and $p_H > p_L$, charging the high price is the seller's best reply. Uniqueness follows from the necessity proof of Lemma A.2.

Claim A.1.2: *Assume $p_H < \bar{v}$. There exists a continuum of Fraudulent Pooling equilibria characterized by $\phi_H^* = \phi_L^* = 1$, $b_H^*(p_H) = 1$, $b_L^*(p_H) \in [1 - (1/\delta)(1 - (p_L/p_H)), 1]$ and the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (4) if $\delta = \delta_L$.*

Proof. Consider the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (4). To verify that (μ^*, ϕ^*, b^*) is a PBE if $\delta = \delta_L$, note that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta = \delta_L$, then $\mu_b^*(H|p_H, H) > (p_H - v_L)/(v_H - v_L)$ and $\mu_b^*(H|p_H, L) = (p_H - v_L)/(v_H - v_L)$. By (2), the buyer's strategy is rational with respect to them. Now consider the low quality seller. By charging the high price, his expected profit is given by $[(1 - \delta) + \delta b_L^*(p_H)]p_H \geq p_L$. Thus, charging the high price is the low quality seller's best reply and so is it for the high quality seller.

Claim A.1.3: *Assume $p_H \leq \bar{v}$. There exists a pooling equilibrium (Fraudulent Pooling H) characterized by $\phi_H^* = \phi_L^* = 1$, $b_H^*(p_H) = 1$, $b_L^*(p_H) = 0$ and the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (4) if $\delta \in (\delta_L, 1 - (p_L/p_H)]$. This equilibrium is unique if $\delta \in (\delta_L, 1 - (p_L/p_H))$.*

Proof. Consider the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (4). To verify that (μ^*, ϕ^*, b^*) is a PBE if $\delta \in (\delta_L, 1 - (p_L/p_H)]$, note that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta > \delta_L$, then $\mu_b^*(H|p_H, H) > (p_H - v_L)/(v_H - v_L) > \mu_b^*(H|p_H, L)$. By (2), the buyer's strategy is rational with respect to them. Now consider the low quality seller. Because the buyer agrees to trade at the high price only if she observes the high signal realization, his expected profit if he charges the high price p_H is given by $(1 - \delta)p_H \geq p_L$. Hence, charging the high price is this seller's best reply and so is it for the high quality seller. Uniqueness if $\delta \in (\delta_L, 1-(p_L/p_H))$ follows from an argument similar to that in the necessity proof of Lemma A.2.

This completes the proof of Lemma A.1.

Lemma A.2: *Assume $p_H \leq \bar{v}$. Fraudulent Hybrid Equilibria exist if and only if $\delta \geq \max\{\delta_L, 1-(p_L/p_H)\}$ and $\delta \neq \delta_L$.*

Proof. For necessity, suppose not, so that hybrid equilibria exist if $\delta < \max\{\delta_L, 1-(p_L/p_H)\}$ and $p_H \leq \bar{v}$. By Bayes consistency, the posterior beliefs $\mu_b^*(H|p_H, s)$ must be given by equation (4). Given $p_H \leq \bar{v}$, then $\mu_b^*(H|p_H, H) > (p_H - v_L)/(v_H - v_L)$ $\forall \phi_L \in (0,1)$ and $\delta > 1/2$ so that the buyer's optimal strategy conditional on observing a high signal realization and being offered the high price is $b_H^*(p_H) = 1$. As a result, the low quality seller's expected profits if he charges only the high price are at least $(1-\delta)p_H > p_L$ given that $\delta < 1-(p_L/p_H)$. Consequently, the low quality seller would find it more profitable to deviate and charge the high price rather than randomize. Suppose now that hybrid equilibria exist if $\delta = \delta_L$ and $p_H \leq \bar{v}$. As a result, any buyer's optimal strategy is to purchase the item if $\phi_L^* < 1$. But then, the low quality should deviate and charge only the high price, obtaining higher profits. We prove sufficiency through claim A.2.

Claim A.2: *Assume $p_H \leq \bar{v}$. There exists a hybrid equilibrium (Fraudulent Hybrid $H\bar{L}$) characterized by $\phi_H^* = 1$, $\phi_L^* = \underline{\phi}(\delta)$, $b_H^*(p_H) = 1$, $b_L^*(p_H) = 1 - (1/\delta)(1-(p_L/p_H))$ and the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (4)if $\delta \geq \max\{\delta_L, 1-(p_L/p_H)\}$ and $\delta \neq \delta_L$. This equilibrium is unique if $\delta >$*

$\max\{\delta_L, 1-(p_L/p_H)\}$.

To verify that (μ^*, ϕ^*, b^*) is a PBE if $\delta \geq \max\{\delta_L, 1-(p_L/p_H)\}$ and $\delta \neq \delta_L$, note that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta > \delta_L$, then $\mu_b^*(H|p_H, H) > (p_H - v_L)/(v_H - v_L) = \mu_b^*(H|p_H, L)$. By (2), the buyer's strategy is rational. Now consider the low quality seller. Because the buyer accepts to trade at the high price with probability one if she observes the high signal realization and with probability strictly less than one if she observes the low signal, the low quality seller's expected profit if he charges the high price is given by $[(1-\delta) + \delta b_L^*(p_H)]p_H = p_L$. The low quality seller is indifferent between charging either price and therefore, a randomization strategy is a best reply for this seller. Charging the high price is the high quality seller's optimal response given the buyer's strategy. Uniqueness follows from the necessity proof of Lemma A.1.

This completes the proof of Lemma A.2 and Proposition 5.1.

Proof of Proposition 5.2. We prove this proposition through a series of lemmas.

Lemma A.3: *If $p_H > \bar{v}$ and $\delta_H \leq 1-(p_L/p_H)$, Fraudulent Pooling equilibria exist if and only if $\delta \in [\delta_H, 1-(p_L/p_H)]$.*

Proof. For necessity, suppose not, so that if $p_H > \bar{v}$ and $\delta_H \leq 1-(p_L/p_H)$, fraudulent pooling equilibria exist if $\delta \in \left(\frac{1}{2}, \delta_H\right) \cup (1-(p_L/p_H), 1)$. By Bayes consistency, the posterior beliefs $\mu_b^*(H|p_H, s)$ must be given by equation (4). If $\delta < \delta_H$, then $(p_H - v_L)/(v_H - v_L) > \mu_b^*(H|p_H, H) > \mu_b^*(H|p_H, L)$ so that the buyer's optimal strategies are $b_H^*(p_H) = b_L^*(p_H) = 0$. Any seller who charged the high price would see his offer rejected and therefore, a low quality seller would be strictly better off if he charges the low price. If $\delta \in (1 - (p_L/p_H), 1)$, $\mu_b^*(H|p_H, H) > (p_H - v_L)/(v_H - v_L) > \mu_b^*(H|p_H, L)$, so that the buyer's optimal strategies are $b_H^*(p_H) = 1$ and $b_L^*(p_H) = 0$. The low quality seller's expected profit of charging the high price is given by $(1-\delta)p_H < p_L$ due to the fact that $\delta > 1-(p_L/p_H)$. The low quality seller would find it more profitable to deviate and charge the low price. A contradiction. We prove sufficiency through claims A.3.1.-A.3.2.:

Claim A.3.1: *There exists a continuum of pooling equilibria characterized by* $\phi_H^* = \phi_L^* = 1$, $b_H^*(p_H) \in [(1/(1-\delta))\,(p_L/p_H), 1]$, $b_L^*(p_H) = 0$ *and the buyer's posterior beliefs* $\mu_b^*(H|p_H, s)$ *given by equation (4)if* $\delta = \delta_H$.

Proof. Consider the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (4). To verify that (μ^*, ϕ^*, b^*) is a PBE if $\delta = \delta_H$, note that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta = \delta_H$, then $\mu_b^*(H|p_H, H) = (p_H - v_L)/(v_H - v_L) > \mu_b^*(H|p_H, L)$. By (2), the buyer's strategy is rational with respect to them. The low quality seller's expected profit conditional on charging the high price is given by $(1-\delta)b_H^*(p_H)p_H \geq p_L$. Thus, charging the high price is a best reply for the low quality seller and so is it for the high quality seller.

Claim A.3.2: *There exists a pooling equilibrium (Fraudulent Pooling H) characterized by* $\phi_H^* = \phi_L^* = 1$, $b_H^*(p_H) = 1$, $b_L^*(p_H) = 0$ *and the buyer's posterior beliefs* $\mu_b^*(H|p_H, s)$ *given by equation (4) if* $\delta \in (\delta_H, 1 - (p_L/p_H)]$. *This equilibrium is unique if* $\delta < 1 - (p_L/p_H)$.

Proof. Consider the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (4). To verify that (μ^*, ϕ^*, b^*) is a PBE if $\delta \in (\delta_H, 1 - (p_L/p_H)]$, note that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta > \delta_H$, then $\mu_b^*(H|p_H, H) > (p_H - v_L)/(v_H - v_L) > \mu_b^*(H|p_H, L)$. By (2), the buyer's strategy is rational with respect to them. Now consider the low quality seller. Because the buyer agrees to trade at the high price only if she observes the high signal realization, his expected profit if he charges the high price is given by $(1-\delta)p_H \geq p_L$. Charging the high price is a best reply for this seller and so is it for the high quality seller too. Uniqueness follows from the necessity proof of Lemma A.4.

This completes the proof of Lemma A.3.

Lemma A.4: *Assume* $p_H > \bar{v}$ *and* $\delta_H < 1-(p_L/p_H)$. *Then, Fraudulent Hybrid Equilibria exist if and only if* $\delta \in (1/2, \delta_H) \cup [1 - (p_L/p_H), 1)$.

Proof. For necessity, suppose not, so that if $p_H > \bar{v}$ and $\delta_H \leq 1 - (p_L/p_H)$, hybrid equilibria exist if $\delta \in [\delta_H, 1 - (p_L/p_H))$. By Bayes consistency, the posterior beliefs $\mu_b^*(H|p_H, s)$ must be given by equation (4). Then $\mu_b^*(H|p_H, H) > (p_H -$

$v_L)/(v_H - v_L)$ $\forall \phi_L \in (0,1)$ so that $b^*_H(p_H) = 1$. The low quality seller's expected profit if he charges the high price is given by at least $(1-\delta)p_H > p_L$. Hence, the low quality seller would find it more profitable to deviate and charge the high price instead of randomizing. A contradiction. We prove sufficiency through claims A.4.1-A.4.2:

Claim A.4.1: *Assume $p_H > \bar{v}$. Then, a hybrid equilibrium (Fraudulent Hybrid H) characterized by $\phi^*_H = 1$, $\phi^*_L = \bar{\phi}(\delta)$, $b^*_H(p_H) = (1/(1-\delta))\,(p_L/p_H)$, $b^*_L(p_H) = 0$ and the buyer's posterior beliefs $\mu^*_b(H|p_H, s)$ given by equation (4) exists if $\delta <$* $\min\{\delta_H, 1-(p_L/p_H)\}$. *This equilibrium is unique.*

Proof. To verify that (μ^*, ϕ^*, b^*) is a PBE if $\delta < \min\{\delta_H, 1-(p_L/p_H)\}$, note that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. In addition, $\mu^*_b(H|p_H, H) = (p_H - v_L)/(v_H - v_L) > \mu^*_b(H|p_H, L)$. By (2), the buyer's strategy is rational. Now consider the low quality seller. Given the buyer's strategy, the low quality seller's expected profit if he charges the high price is given by $(1-\delta)b^*_H(p_H)p_H = p_L$. The low quality seller is then indifferent between charging p_H or p_L so that a randomization strategy is an optimal reply. The high quality seller's optimal response given the buyer's strategy is to charge the high price. Uniqueness follows from the necessity proof of Lemma A.3.

Claim A.4.2: *Assume $p_H > \bar{v}$. Then, a hybrid equilibrium (Fraudulent Hybrid H$\underline{L}$) characterized by $\phi^*_H = 1$, $\phi^*_L = \underline{\phi}(\delta)$, $b^*_H(p_H) = 1$, $b^*_L(p_H) = 1 -$*

$(1/\delta)\,(1-(p_L/p_H))$ *and the buyer's posterior beliefs* $\mu_b^*(H|p_H,s)$ *given by equation (4)exists if* $\delta \geq 1-(p_L/p_H)$.

Proof. To verify that (μ^*, ϕ^*, b^*) is a PBE if $\delta \geq 1-(p_L/p_H)$, note that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. In addition, $\mu_b^*(H|p_H,H) > (p_H - v_L)/(v_H - v_L) = \mu_b^*(H|p_H,L)$. By (2), the buyer's strategy is rational. Now consider the low quality seller. Given the buyer's strategy, the low quality seller's expected profit if he charges the high price is given by $[(1-\delta)+\delta b_L^*(p_H)]p_H = p_L$. The low quality seller is indifferent between charging either price and therefore, a randomization strategy is a best reply for this seller. Charging the high price is the high quality seller's optimal response given the buyer's strategy. When $\delta > 1-(p_L/p_H)$, uniqueness follows from an argument similar to that in the necessity proof of Lemma A.3.

This completes the proof of Lemma A.4.

Lemma A.5: *Assume* $p_H > \bar{v}$ *and* $\delta_H > 1-(p_L/p_H)$. *Then, no Fraudulent Pooling equilibrium exists.*

Proof. Suppose not, so that if $p_H > \bar{v}$ and $\delta_H > 1-(p_L/p_H)$, a fraudulent pooling equilibrium exists. By Bayes consistency, the posterior beliefs $\mu_b^*(H|p_H,s)$ must be given by equation (4). If $\delta < \delta_H$, then $(p_H - v_L)/(v_H - v_L) > \mu_b^*(H|p_H,H) > \mu_b^*(H|p_H,L)$ so that the buyer's optimal strategies are $b_H^*(p_H) = b_L^*(p_H) = 0$. Any seller who charged the high price would see his price offer rejected. Thus, a low quality seller would be strictly better off if he charged the low price. If $\delta \geq \delta_H$, $\mu_b^*(H|p_H, H) \geq (p_H - v_L)/(v_H - v_L) > \mu_b^*(H|p_H, L)$, so that the buyer's optimal strategies are $b_H^*(p_H) \leq 1$ and $b_L^*(p_H) = 0$. The low quality seller's expected profit if he charges the high price is at most $(1-\delta)p_H < p_L$ because $\delta > 1-(p_L/p_H)$. The low quality seller would find it more profitable to deviate and charge the low price. A contradiction.

This completes the proof of Lemma A.5.

Lemma A.6: *Assume $p_H > \bar{v}$ and $\delta_H \geq 1-(p_L/p_H)$. Then, Fraudulent Hybrid equilibria exist* $\forall \delta \in (1/2, 1)$.

Proof. The proof follows from claims A.4.1 and A.4.2.

The proof of proposition 5.2 is completed.

Proof of Proposition 5.4. If $p_H \leq 2p_L$ then $1-(p_L/p_H) \leq 1/2$ and the result is immediate as only either Fraudulent Pooling or Fraudulent Hybrid $H\bar{L}$ can be supported in equilibrium for any $\delta \in (1/2, 1)$ by the proofs of Lemmas A.1-A.2 and Claim A.4.2. If $2p_L < p_H \leq \bar{v}$ then only Fraudulent Pooling equilibria can be supported if $\delta < 1-(p_L/p_H)$ by Lemma A.1 and only Fraudulent Hybrid $H\bar{L}$ can be supported in equilibrium for $\delta > 1-(p_L/p_H)$ by Lemma A.2. A continuum of equilibria characterized only by $\phi_H^* = 1$, $\phi_L^* \in [(\pi/(1-\pi))\,(p_L/(p_H - p_L))\,((v_H - p_H)/(p_H - v_L))\,, 1]$, $b_H^*(p_H) = 1$, $b_L^*(p_H) = 0$ and the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (4) can be supported for $\delta = 1-(p_L/p_H)$. The result is then immediate.

Proof of Proposition 5.5. Under these conditions, only Fraudulent Hybrid $\bar{H}$ can be supported in equilibrium for any $\delta < \min\{\delta_H, 1-(p_L/p_H)\}$ by the proof of Claim A.4.1 whereas only Fraudulent Hybrid $H\bar{L}$ can be supported in equilibrium for any $\delta > 1-(p_L/p_H)$ by the proof of Claim A.4.2. The result is then immediate.

Ivie

Guardia Civil, 22 - Esc. 2, 1º
46020 Valencia - Spain
Phone: +34 963 190 050
Fax: +34 963 190 055

Website: www.ivie.es
E-mail: publicaciones@ivie.es