INDIVIDUAL RIGHTS AND COLLECTIVE RESPONSIBILITY: THE RIGHTS-EGALITARIAN SOLUTION*

Carmen Herrero, Michael Maschler & Antonio Villar**

* Financial support from the Dirección General de Investigación Científica y Técnica, under project PB92-0342 and grant SAB95-0050 is gratefully acknowledged.

** Carmen Herrero & Antonio Villar: University of Alicante and Instituto Valenciano de Investigaciones Económicas

Michael Maschler: The Center for Rationality, The Hebrew University of Jerusalem.

INDIVIDUAL RIGHTS AND COLLECTIVE RESPONSIBILITY: THE RIGHTS-EGALITARIAN SOLUTION

Carmen Herrero, Michael Machler & Antonio Villar

A B S T R A C T

The problem of distributing a given amount of a divisible good among a set of agents which may have individual entitlements is considered here. A solution to this problem, called the *Rights-Egalitarian Solution*, is proposed and characterized. This allocation rule divides equally among the agents the difference between the aggregate entitlements and the amount of the good available. A relevant feature of the analysis developed is that no sign restriction is established on the parameters of the model (that is, the aggregate entitlements may exceed or fall short of the amount of the good, agents' rights may be positive or negative, the allocation may involve a redistribution on agents' holding, etc.)

KEYWORDS: Distributive Problems; Allocation Rules; Solution Functions.

1 INTRODUCTION

This paper refers to a distributive problem involving the allocation of a given amount of "money" among a number of "agents", each of which is characterized by a monetary *entitlement*. The money being distributed will be called the *budget*. It represents the worth jointly owned by the agents. Its origin can stem from one of many circumstances (e.g., administrative decisions, an enterprise to be liquidated, inheritance, etc.). The vector of entitlements represents the agents' individual rights (e.g., needs, claims, benefits or private loans, shares, inheritance will and others). By "agents" we mean people or more general instances, such as expenditure categories, departments or institutions. "Money" may well refer to actual money, or to any unit of account of rights and worth pertinent to the problem under consideration (e.g., square miles, calories, gallons, "utils", etc.). A distribution problem will thus be described by a triple $[N, E, \mathbf{c}]$, where N represents the number of agents, $E \in \mathbb{R}$ the budget, and $\mathbf{c} \in \mathbb{R}^n$ the vector of entitlements.

Examples of this type of situation are: (i) Dissolving a partnership with perfectly divisible value (the liquidation of an enterprise, the division of an estate, or a divorce settlement); (ii) Distributing the proceeds of a joint-venture (a collective investment, the yields of a research program); (iii) The allocation of funds by a central authority (distributing the budget of a University among the Departments, distributing the available supply of water in a refugees' camp, or allocating funds for the less-favoured regions). (iv) Paying for a collective utility (say, allocating the cost of an indivisible public good).

Related problems have been widely analyzed in the literature from different viewpoints. Let us mention here in particular the cost/surplus-sharing approach (see for instance Moulin [1988 Chs. 4-6]), and the analysis of bankruptcy-like situations (e.g., Aumann & Maschler [1988], Curiel, Maschler & Tijs [1988], Dagan & Volij [1993], Dagan [1995]).

An allocation of the budget which respects agents' entitlements will be such that it gives to each agent no less than her right, if the budget exceeds the aggregate entitlement, and no more than her claim, if the budget falls short of the collective rights. An *allocation rule* to this type of problem will thus be defined as a function F such that for any given problem $[N, E, c]$ associates a unique point $\mathbf{x} = F([N, E, c]) \in \mathbb{R}^n$ such that:

a) $\sum_{i \in N} x_i = E$,

b) Either $\mathbf{x} \geq \mathbf{c}$, if $E \geq \sum_{i \in N} c_i$, or $\mathbf{x} \leq \mathbf{c}$ (otherwise).

A distinctive feature of our approach his that the domain of problems considered does not impose any restriction on the values that the parameters of the model take. In particular, E as well as $(E - \sum_{i \in N} c_i)$ can be either positive or negative, and vector $\mathbf{c}$ may well have negative components. Similarly, the solution function $F([N, E, c])$ may also take both positive and negative components. A negative budget simply means a cost to be shared. A negative value of $E - \sum_{i \in N} c_i$ corresponds to the case of a budget which falls short of the claims. A value $c_i < 0$ can be interpreted as a debt of agent i (a negative entitlement). Finally, negative values in vector $\mathbf{x}$ with a positive budget may represent a *redistribution* among agents.

The main topic of this paper is the analysis of a particular allocation rule, that will be called the *rights-egalitarian solution*. This is a simple rule which splits equally the net common worth $(E - \sum_{i \in N} c_i)$ among the agents. It can be thought of as solving the distributive problem into two steps: First, it gives each agent her entitlement, and then it distributes equally the remaining worth (be it positive or negative). Note that the rights-egalitarian solution is always defined and meaningful for all possible values of $[N, E, \mathbf{c}]$. This is not the case for other solution concepts, such as the proportional solution (think of the case of $E > 0$ and $\mathbf{c} = 0$, or the case $E = 0$ and $\mathbf{c}$ containing positive and negative components).

In this paper we assume that *the budget is absolute*, in the sense that it must be fully distributed and *the rights are absolute too*, in the sense that they must be satisfied and the group is fully responsible for that. For example, if the budget exceeds the sum of the rights then in many situations the surplus is not distributed. It stays in the hands of an extraneous institution that owns the budget to begin with. In other words, it is lost to the claimants. This is *not* the case that we consider here: In this paper the whole budget belongs to the agents and they are assumed to share all of it. For example, this is the case when an inheritance is shared by several people.

As to the rights: Perhaps the sum of the rights, or even the rights of one agent exceed the budget. In some circumstances one can say: "Too bad. We cannot distribute what is not available, so we shall truncate the rights and share the budget the best we can." Again, this is *not* the case that we consider in this paper. In the cases that we consider it is the responsibility of the agents to satisfy all the rights even if they have to use their private resources to achieve that. The following example will illustrate the difference between these two situations:

Consider a bankruptcy problem with $N = \{1, 2\}$, $E = 100$ and $\mathbf{c} = (30, 120)$. Under one approach one decides that 120 is not a realistic entitlement: It should be truncated to 100. Even the 150 is not realistic so one deducts 15 from each claim and the final share is then (15, 85). The reader is referred to Aumann & Maschler (1988), where this case is pursued. But there is another way of viewing the rights, under which each agent is entitled to the full amount of her rights (for example, she will sue if she is not compensated) and since a debt of 50 results, it must be shared equally among the agents who are responsible to the debt and the final balance is $(5, 95)$. This paper studies such situations. Notice that with different parameters in the above example, an agent may end up with a negative balance.

The analysis is carried out following two standard approaches, which are the subject of sections 2 and 3, respectively. Section 2 contains the formal description of the problem , and provides us with alternative characterizations of the rights-egalitarian solution, in terms of conventional axioms that an allocation rule may be asked to satisfy. We provide several characterizations because each additional one extends the scope of cases to which our solution can be applied.

Section 3 analyzes the properties of the cooperative game (in characteristic form) that can naturally be associated with the distribution problem. It is shown first that allocation rules yield core outcomes, and that the rights-egalitarian solution coincides both with the Shapley value and the nucleolus of the associated game. It is also shown that this game has the *reduced game property* (à la Davis & Maschler) and that the diagram relating allocation problems, the associated games, reduced allocation problems and reduced games, commutes.

Let us conclude this section by presenting a simple example which gives us a hint on how this rule behaves, in a context whereby it is not usually applied:

Example.- Two friends agree to buy a bond of \$ 10,000, which yields a profitability 10 %, twice as much as they can get in their private saving accounts. Mary contributes \$ 6,000 and Bob \$ 4,000. Here $E =$ \$ 11,000 and vector $\mathbf{c}$ is given by (6,300, 4,200) (their opportunity costs). The net worth of this joint venture is 11,000 - (6,300 + 4,200) = 500. The rights-egalitarian solution gives Mary 6,550 [= 6,300 + (1/2)500] and Bob 4,450 [= 4,300 + (1/2)500]. Suppose now that things go the wrong way, the firm issuing the bond gets bankrupt and the liquidation worth of the bond is \$ 5,000. The net worth is now -5,500, so that the rights-egalitarian solution will give Mary 3,550 [= 6,300 - (1/2)5,500] and Bob 1,450 [=4,200 - (1/2)5,500].

In the worst of the situations, when the whole investment is lost, we have $E = 0$, and a net worth of -10,500. Our solution establishes in this case that Mary should receive \$ 1,050 from Bob (as a compensation for her higher risk).

2 ALLOCATION RULES

Let us start by presenting formally the key notions of the analysis:

Definition 1 : *An* ***allocation problem*** *is a triple* $[N, E, \mathbf{c}]$, *such that* N *is a set of agents,* $|N| = n$, $\mathbf{c} \in \mathbb{R}^n$ *describes a vector of entitlements (or rights) and* $E \in \mathbb{R}$ *a given budget.*

Let us call Ω the family of all allocation problems, and Ω^N the set of allocation problems where the set of agents is N. For any $\omega = [N, E, \mathbf{c}] \in \Omega$, call $C(\omega) = \sum_{i \in N} c_i$, and let $H(\omega)$ stand for the hyperplane $H(\omega) = \{z \in \mathbb{R}^n \mid z_1 + ... + z_n = E\}$.

Definition 2 : *An* ***allocation rule*** *is a function* $F : \Omega \to \bigcup_{n=1}^{\infty} \mathbb{R}^n$, *such that for any* $\omega = [N, E, \mathbf{c}] \in \Omega$,

(i) $F(\omega) \in H(\omega)$.

(ii) $\begin{cases} F(\omega) \geq \ \mathbf{c}, & \text{if } C(\omega) \leq E \\ F(\omega) \leq \ \mathbf{c}, & \text{if } C(\omega) \geq E \end{cases}$

Thus, an allocation rule is a mechanism such that: (a) It always provides us with a unique solution for any problem in Ω; (b) It exhausts the budget; and (c) Either no agent gets more than she claims for or no agent gets less than she has right to.

Let $\omega = [N, E, \ \mathbf{c}]$ be an allocation problem, and F an allocation rule. For any agent i, call

$$r_i(\omega) = E - \sum_{j \neq i} c_j = E - C(\omega) + c_i$$

The number $r_i(\omega)$ tells us the difference between the budget and the aggregate entitlement of all agents other than i.

Let now $\mathbf{r}(\omega)$ denote the n-vector whose components are $r_i(\omega)$, $i \in N$, and let $R(\omega) = \sum_{i \in N} r_i(\omega)$. Note that for any $\omega \in \Omega$, exactly one of the following alternatives occurs:

(1) $c_i < r_i(\omega)$ for all i;

(2) $c_i = r_i(\omega)$ for all i;

(3) $c_i > r_i(\omega)$ for all i.

That is, either $H(\omega)$ strictly separates $\mathbf{c}$ and $\mathbf{r}(\omega)$, or they coincide and lie on $H(\omega)$. Consequently, and by Definition 2,

$$(i)\ C(\omega) = E \Longrightarrow F(\omega) = \mathbf{c}$$
$$(ii)\ \min\{c_i, r_i(\omega)\} \leq F_i(\omega) \leq \max\{c_i, r_i(\omega)\}, \quad \textit{for all } i \in N.$$

Property (i) says that if the entitlements vector $\mathbf{c}$ is feasible, then $\mathbf{c}$ is precisely the solution provided by the allocation rule. Property (ii) establishes that the *ith* agent cannot get less than the minimum and more than the maximum between her claim and the difference between the available budget, and the amount which results from satisfying all others' rights.[1] Thus, the point $\mathbf{r}(\omega)$ can be thought of as an endogenous pseudo status quo (resp. an endogenous pseudo ideal point), which establishes a lower bound (resp. an upper bound) for the values that an allocation rule can take on, depending upon the relative situation of $\mathbf{c}$ and $H(\omega)$. We refer to $\mathbf{r}(\omega)$ as the reference vector.

We are now in a position to introduce the allocation rule which is the subject of this paper:

Definition 3 : *The rights-egalitarian allocation rule* F^{RE} *associates to each problem* $\omega = [N, E, \mathbf{c}]$, *a point* $\boldsymbol{x}^* = F^{RE}(\omega)$, *where* $x_i^* = c_i + \lambda$, *and* $\lambda \in \mathbb{R}$ *solves* $\sum_{i \in N}(c_i + \lambda) = E$, *that is,* $C(\omega) + n\lambda = E$.

This rule divides equally the net worth $E - C(\omega)$ among the n agents. When $E > C(\omega)$, the resulting allocation coincides with the equal-gains solution from the rights point c. If $E < C(\omega)$, then our solution corresponds to the equal-loss (or claims-egalitarian) solution from the claims point $\mathbf{c}$. Because the reference vector $\mathbf{r}(\omega)$ establishes a natural bound on the admissible values of allocation rules, we can think of this rule as solving the distribution problem as a two-step problem: In the first step consider the problem $[N, R(\omega), \mathbf{r}(\omega)]$, (whose solution, by definition is $\mathbf{r}(\omega)$). In the second step consider the 'remaining problem': $[N, E - R(\omega), \mathbf{c} - r(\omega)]$ which is symmetric and has the natural solution of given everybody the same.

[1]This relation will be used in Section 3 in order to define the cooperative game in characteristic form, associated with the allocation problem.

The rights-egalitarian rule can actually be viewed as a combination of the equal-award/equal-loss principles. Both principles are common in the literature dealing with the division of a surplus, Moulin [1987], the bankruptcy problem, Young [1985], Dagan [1995], the axiomatic bargaining theory, Kalai [1977], Chun [1988a], Chun & Peters [1991], Herrero & Marco [1993], and the bargaining with claims problem, Bossert [1993], Herrero [1994]. Nevertheless our solution function differs from the standard treatment of these problems in two respects:

(i) It applies over a bigger domain of problems; and

(ii) It uses a different reference vector as the proper upper (or lower) bound for admissible solutions.

Several properties will now be considered. These will serve the purpose of characterizing the rights-egalitarian rule.

Axiom 1 *(SYMMETRY).- For any* $\omega = [E, N, \ \mathbf{c}]$, *if* $c_i = c_j$ *for all* $i, j \in N$, *then* $F_i(\omega) = E/n$, *for all* $i \in N$.

Symmetry is a very mild condition which says that if all agents have identical entitlements, then the rule should divide the budget equally among them.

The next property is related to the possibility of solving distribution problems sequentially. Let $\omega = [N, E, \ \mathbf{c}]$ and let E_1, E_2 be such that $E_1 + E_2 = E$. We consider now the possiblity of solving the distribution problem ω into two steps. First we solve the problem $\omega_1 = [N, E_1, \ \mathbf{c}]$ in which we take E_1 instead of E. Let $\mathbf{x}_1 = F(\omega_1)$. Then we consider the complementary problem given by $\omega_2 = [N, E_2, \ \mathbf{c} - x_1]$, whose solution is $\mathbf{x}_2$. We require the outcome to be independent of such a sequential process, that is $F(\omega) = F(\omega_1) + F(\omega_2)$. This is a property which prevents the manipulation of the outcome by framing conveniently the sequential process. Slightly abusing the terminology, we call this property path independence. Formally:

Axiom 2 *(PATH INDEPENDENCE).- A distribution problem* $\omega = [N, E, \ \mathbf{c}] \in \Omega$ *satisfies path independence if for any* $E_1, E_2 \in \mathbb{R}$ *such that* $E_1 + E_2 = E$, *it follows that* $F(\omega) = \ \boldsymbol{x}_1 + \ \boldsymbol{x}_2$, *where* $\boldsymbol{x}_1 = F\left([N, E_1, \ \mathbf{c}]\right), \ \boldsymbol{x}_2 = F\left([N, E_2, \ \mathbf{c} - x_1]\right)$.

These properties bring us to the first characterization of F^{RE}:

Proposition 1 : F^{RE} *is the unique allocation rule in* Ω *satisfying symmetry and path independence.*

Proof. Obviously, F^{RE} satisfies symmetry. To see that it also satisfies path independence, take an arbitrary problem $\omega = [N, E,\ \mathbf{c}] \in \Omega$, and let $\omega_1 = [N, E_1,\ \mathbf{c}]$, $\omega_2 = [N, E_2,\ \mathbf{c} - F^{RE}(\omega_1)]$, with $E_1 + E_2 = E$. By definition, $F_i^{RE}(\omega) = c_i + \frac{E - C(\omega)}{n}$. Now observe that $F_i^{RE}(\omega_1) = c_i + \frac{E_1 - C(\omega)}{n} = x_{1i}$ and $F_i^{RE}(\omega_2) = c_i - x_{1i} + \frac{E_2 - (C(\omega) - E_1)}{n} = -\frac{E_1 - C(\omega)}{n} + \frac{E - C(\omega)}{n} = \frac{E_2}{n}$. Hence, $F_i^{RE}(\omega_1) + F_i^{RE}(\omega_2) = F_i^{RE}(\omega)$ for all i.

Let us consider now an allocation rule F that satisfies both requirements, and let $\omega = [N, E,\ \mathbf{c}] \in \Omega$. If we take $\omega_1 = [N, C(\omega),\ \mathbf{c}]$, it follows that $F(\omega_1) = \mathbf{c}$. Let now $\omega_2 = [N, E - C(\omega),\ \mathbf{0}]$. As ω_2 is a symmetric problem, it follows that $F_i(\omega_2) = [E - C(\omega)]/n$ for all $i \in N$. Path independence implies that, for every i, $F_i(\omega) = c_i + \frac{E - C(\omega)}{n} = F_i^{RE}(\omega)$. ■

The next two axioms introduce additional desirable properties that an allocation rule may be asked to satisfy:

Axiom 3 *(CONCAVITY).- Let* $\omega = [N, E,\ \mathbf{c}]$, $\omega' = [N, E,\ \mathbf{c}'] \in \Omega$, *and* $\lambda \in [0, 1]$. *Then,*

$$F([N, E, \lambda\mathbf{c} + (1 - \lambda)\mathbf{c}']) \geq \lambda F(\omega) + (1 - \lambda) F(\omega').$$

Concavity may be seen as dealing with the case in which the agents are uncertain about the rights point (it may be either $\mathbf{c}$ or $\mathbf{c}'$). The rule makes it appealing to sign a contingent contract before the uncertainty is resolved. Related properties appear in Chun & Thomson [1992] and Herrero [1995] in the bargaining with claims case. Notice that the requirement $F(\omega) \in H(\omega)$ combined with concavity implies that agents are indifferent between both situations. This is the condition of linearity:

$$F([N, E, \lambda\mathbf{c} + (1 - \lambda)\mathbf{c}']) = \lambda F(\omega) + (1 - \lambda) F(\omega').$$

Axiom 4 *(DUALITY).- Let* $\omega = [N, E,\ \mathbf{c}]$, *and let* $\omega' = [N, E,\ \mathbf{r}(\omega)]$. *Then,* $F(\omega) = F(\omega')$.

Duality refers to two different problems having identical set of agents and the same budget, and such that the rights point in one problem is the reference vector of the other. Duality requires that both problems have identical solutions. Notice that since $\mathbf{c}$ and $\mathbf{r}(\omega)$ are separated by $H(\omega) = H(\omega')$, ω and ω' are problems of different type, namely in one of them we have to allocate losses (with respect to the claims point), and in the other we have to allocate gains.

The intuitive idea behind the principle of duality is: Suppose $[N, E,\ \mathbf{c}]$ is a distribution problem in which, say, $\sum_{i=1}^{n} c_i > E$. The agents can view it in two ways:

(i) Since the rights exceed the budget, each agent expects and considers her loss $x_i - c_i$, where x_i is whar she gets in the outcome.

(ii) Each agent realizes that she will certainly get at least $r_i(\omega)$, because the worst thing that can happen to her is that all other agents get their rights and because E will be distributed. So, when evaluating the outcome, she will expect and consider her gains $x_i - r_i(\omega)$.

The principle of duality requires that it does not matter whether the agents' rights are the original claims or the rights are the 'undisputed amounts', the solution function will yield each of them the same outcome. (See Aumann & Maschler [1988], where the duality principle is discussed in a different setup).

This bring us to two further characterizations of F^{ER} :

Proposition 2 : *F^{RE} is the unique allocation rule in Ω satisfying symmetry and concavity.*

Proof. Obviously, F^{RE} satisfies these properties. Consider now a rule F satisfying Symmetry and Concavity. To prove the converse part, observe that if $E = C$, then every allocation rule coincides with F^{RE}. We shall therefore assume that $E \neq C$.

For any two points $\mathbf{x}, y \in \mathbb{R}^n$, denote by $L[\ \mathbf{x}, y]$ the straight line containing $\mathbf{x}$ and $\mathbf{y}$. Let $\omega = [N, E,\ \mathbf{c}]$ and take a point $\mathbf{d} \in \mathbb{R}^n$ such that $d_i = d_j$ for all $i, j \in N$, and d_i is strictly in between E/n and $C(\omega)/n$. Let now $\mathbf{b} \in \mathbb{R}^n$ be a point given by $\mathbf{b} = L\ [c, d] \cap H(\omega)$. By construction, there exists $\lambda \in (0, 1)$ such that $\mathbf{d} = \lambda\ \mathbf{b} + (1 - \lambda)\ \mathbf{c}$. Symmetry implies that $F_i([N, E,\ \mathbf{d}]) = E/n$, for any $i \in N$. Now consider the problem $\omega' = [N, E,\ \mathbf{b}]$. Since $\mathbf{b} \in H(\omega')$, the definition of an allocation rule implies that $F(\omega') =\ \mathbf{b}$. Now, applying concavity (hence, linearity), it follows that $1 \cdot E/n = \lambda\ \mathbf{b} + (1 - \lambda) F(\omega)$, so that $L[\ \mathbf{c}, F(\omega)]$ and $L[\ \mathbf{d}, 0]$ are parallel lines. Therefore, $F(\omega) = F^{RE}(\omega)$. ■

Proposition 3 : *An allocation rule F satisfies concavity and duality if and only if $F = F^{RE}$.*

Proof. F^{RE} satisfies concavity. It is easy to check that it also satisfies duality: Take $\omega = [N, E,\ \mathbf{c}]$, and $\omega' = [N, E,\ \mathbf{r}(\omega)]$. Then,
$F_i^{RE}(\omega) = c_i + \frac{E - C(\omega)}{n} = c_i + E - C(\omega) + \frac{E - nE + nC(\omega)}{n} = r_i(\omega) + \frac{E - R(\omega)}{n} =$

$F_i^{RE}(\omega')$.
On the other hand, take the point $\mathbf{e} \in \mathbb{R}^n$, defined by $e_i = c_i + \frac{E-C(\omega)}{n}$. It is immediate to observe that $\mathbf{e} = \lambda\ \mathbf{c} + (1-\lambda)\ \mathbf{r}(\omega)$, where $\lambda = (n-1)/n$. Thus, consider also the problem $\omega'' = [N, E,\ \mathbf{e}]$. Since $C(\omega'') = E$, we know that $F(\omega'') = \ \mathbf{e}$. Furthermore, by duality, $F(\omega) = F(\omega')$. Now, by concavity, $\mathbf{e} = \lambda F(\omega) + (1-\lambda) F(\omega') = F(\omega)$, and therefore, $F(\omega) = \ \mathbf{e} = F^{RE}(\omega)$. ■

The following properties are weaker versions of duality and concavity, respectively:

Axiom 5 *(BI-CONCAVITY).- Let* $\omega = [\{i,j\}, E,\ \mathbf{c}]$, $\omega' = [\{i,j\}, E,\ \mathbf{c}']$ *and* $0 \le \lambda \le 1$. *Then,*
$F[\{i,j\}, E, \lambda\ \mathbf{c} + (1-\lambda)\ \mathbf{c}'] \ge \lambda F(\omega) + (1-\lambda) F(\omega')$.

Axiom 6 *(BI-DUALITY).- Let* $\omega = [\{i,j\}, E,\ \mathbf{c}]$, *and let* $\omega' = [\{i,j\}, E,\ \mathbf{r}(\omega)]$. *Then,* $F(\omega) = F(\omega')$.

Bi-concavity and bi-duality ask for concavity and duality, respectively, only for two-person problems. Note that the proofs of Propositions 2 and 3 are valid for each particular N. Thus, these propositions are valid if the class of problems is restricted to Ω^2. For the application of Proposition 4 below, we state these facts for the class of two-person distribution problems:

Corollary 1 : *An allocation rule* F*, defined for the class of two-person distribution problems, that satisfies symmetry and bi-duality must be* F^{RE}.

Corollary 2 : *An allocation rule* F*, defined for the class of two-person distribution problems, that satisfies bi-concavity and bi-duality must be* F^{RE}.

The next property refers to the effect of a change in the number of agents. The following notation will be used: If $\omega = [N, E,\ \mathbf{c}]$, and $S \subset N$, $S \neq N$, we shall call $\mathbf{c}_S = (c_i)_{i \in S}$. We shall call $\omega_S = \left[S, \sum_{i \in S} F_i(\omega),\ \mathbf{c}_S\right]$ the reduced problem for the subset of agents S.

Axiom 7 *(CONSISTENCY).- Let* F *be an allocation rule.* F *is consistent if for any* $\omega = [N, E,\ \mathbf{c}] \in \Omega$, *and any subset of agents* $S \subset N$, $S \neq N$, *for any* $i \in S$, $F_i(\omega_S) = F_i(\omega)$.

Consistency has to do with the possibility of renegotiation among a group of agents, whenever they face the total amount assigned to them by the solution. When F is consistent, if a subset of agents leave bringing with them their alloted shares, the remaining agents cannot change their outcomes by using again the rule over the reduced problem. Consistency is considered an important stability feature of solution concepts (cf. Young [1977], Thomson & Lensberg [1989]). Trivially, F^{RE} satisfies consistency.

A weaker requirement is that of requiring this property to hold only for two-person problems. The interest of this particular case stems from the fact that these properties are easier to interpret in such circumstances. Formally:

Axiom 8 *(BI-CONSISTENCY).- For any $\omega = [N, E,\ \mathbf{c}] \in \Omega$, and for any pair $\{i, j\}$ of individuals in N, $F_k\left(\omega_{\{i,j\}}\right) = F_k(\omega)$, $k = i, j$.*

The following result provides us with a different characterization of the rights-egalitarian rule:

Proposition 4 *: An allocation rule satisfies bi-concavity, bi-duality and bi-consistency if and only if $F = F^{RE}$.*

Proof. Consistency of F^{RE} follows directly from the definition. Let us now consider an allocation rule F that satisfies the conditions of the proposition. It is F^{RE} if $|N| = 2$ (and $|N| = 1$). Consider now a problem $\omega = [N, E,\ \mathbf{c}] \in \Omega$, with $|N| > 2$. Denote $\mathbf{x} \equiv F(\omega)$. Now, for two agents $\{i, j\} \subset N$, consider the problem $\omega_{ij} \equiv [\{i, j\}, x_i + x_j; (c_i, c_j)]$. By bi-consistency, $F_i(\omega_{ij}) = x_i$, $F_j(\omega_{ij}) = x_j$.

Moreover, since F coincides with F^{RE} for two person problems, there exist some $\lambda \in \mathbb{R}$, such that $x_i = c_i + \lambda$, $x_j = c_j + \lambda$. Take now a third agent, k, and the problem $\omega_{jk} \equiv [\{j, k\}, x_j + x_k; (c_j, c_k)]$. Again, by bi-consistency, $F_j(\omega_{jk}) = x_j, F_k(\omega_{jk}) = x_k$. Since F coincides with F^{RE} for two person problems, for some $\mu \in \mathbb{R}$, $x_j = c_j + \mu$, $x_k = c_k + \mu$. In consequence, $\lambda = \mu$. Since previous construction can be done for any $i, j, k \in N$, it follows that $F(\omega) = F^{RE}(\omega)$. ■

Remark 1 *One can also consider the sensitivity of the solution to changes in the population in a different way. Let $\omega = [N, E,\ \mathbf{c}]$ be a distributive problem in Ω, and let $\boldsymbol{x} = F(\omega)$ be the solution provided by an allocation rule F. Suppose that new agents enter the picture, while the budget remains unaltered. How should the solution be affected?. It can be seen that F^{RE} is characterized by the following property (that may be called uniform population monotonicity): For any two problems $\omega = [N, E,\ \mathbf{c}]$, and $\omega' = [N \cup M, E, (\ \mathbf{c}, c')] \in \Omega$,*

$F_i(\omega) - F_i(\omega') = F_j(\omega) - F_j(\omega')$, *for all* $i, j \in N$. *According to this property, all agents initially present are equally affected by the incorporation of new claimants. (Cf. with the usual population monotonicity property in Thomson & Lensberg [1989] or Chun & Thomson [1992]).*[2]

To conclude this section, let us look at these problems in connection with a particular decentralization concept. Let ω be an allocation problem for two agents, $\omega = [\{i,j\}, E, \mathbf{c}]$. An allocation $x = (x_i, x_j)$ is a feasible distribution of the estate among the players. The set of admissible allocations $A(\omega)$, is given by:

$\{ \mathbf{x} = (x_i, x_j) \mid x_i + x_j = E, \min[r_k(\omega), c_k] \leq x_k \leq \max[r_k(\omega) c_k], k = i, j\}$

Consider, for any $\omega = [\{i,j\}, E, \mathbf{c}]$, a non cooperative game such that both agents have $A(\omega)$ as their strategy set, and the payoff functions, π_i, π_j, are given by $\pi_k(s_i, s_j) = \frac{1}{2}(s_{ik} + s_{jk})$, $k = i, j$, where ($\mathbf{s}_i$, $\mathbf{s}_j$) stands for any strategy combination [$\mathbf{s}_i = (s_{ii}, s_{ij})$, $\mathbf{s}_j = (s_{ji}, s_{jj})$, $\mathbf{s}_i, \mathbf{s}_j \in A(\omega)$]. That is, each player's payoff is the average of the amounts proposed by herself and the other player as her share of the state.

Consider now the following property:

Axiom 9 *(TWO-PERSON AVERAGE DECENTRALIZATION).- For any two person problem* $\omega = [\{i,j\}, E, \mathbf{c}]$, $F(\omega) = \pi_\omega(\mathbf{s}^*)$, *where* $\mathbf{s}^*$ *is a dominant strategies equilibrium point of the two-person game defined above.*

This property says that, for the two-person case, the allocation rule may be regarded as the outcome of a non-cooperative game that gives each player the average of the shares proposed by both players. Two-person average decentralization involves then a fairness feature of the allocation mechanism which is an expression of anonymity.

The following lemma shows that this property characterizes the solution for the class of two-person distribution problems[3]:

Lemma 1 *: An allocation rule F, defined on the class of of two-person distribution problems, satisfies two-person average decentralization if and only if* $F = F^{RE}$.

[2]As F^{RE} is characterized by this single property, one can take it as an alternative definition of the rule. The proof is trivial and thus omitted.

[3]This construction is closely related to that in Corchón & Herrero (1995)

Proof. Consider a two-person problem $\omega = [\{i,j\}, E, \mathbf{c}]$. Since π_i (π_j) is increasing in s_{ii} (s_{jj}), then $\mathbf{s}_i^* = (z_i, E - z_i)$, $\mathbf{s}_j^* = (E - z_j, z_j)$, where $z_k = \max\{c_k, r_k(\omega)\}$, $k = i, j$. Thus:

$$\pi\left(\mathbf{s}_i^*, \mathbf{s}_j^*\right) = \frac{1}{2}\left(E + z_i - z_j, E - z_i + z_j\right).$$

Notice that either $z_i = c_i$ and $z_j = c_j$ or $z_i = E - c_j$ and $z_j = E - c_i$. In both cases, $\pi\left(\mathbf{s}_i^*, s_j^*\right) = \frac{1}{2}\left(E + z_i - z_j, E - z_i + z_j\right) = \frac{1}{2}\left(E + c_i - c_j, E - c_i + c_j\right)$ $= (c_i + \lambda, c_j + \lambda)$, where $\lambda = [E - C(\omega)]/2$. That is, $F(\omega) = F^{RE}(\omega)$. ■

Proposition 5 : *An allocation rule F is bi-consistent and satisfies two-person average decentralization if and only if $F = F^{RE}$.*

Proof. It follows from lemma 1 in a similar way as in proposition 4. ■

3 ALLOCATION RULES AND GAMES

One can look at these allocation problems from a cooperative game theoretic standpoint. To start with, note that for each problem $\omega = [N, E, c]$ in Ω, we may define a cooperative game in characteristic form $\langle N, v_\omega \rangle$ (where N is the set of "players" and v_ω is the characteristic function), in the following way [4]:

$$v_\omega(T) = \begin{cases} 0 & if\ \ T = \emptyset \\ \min\{\sum\limits_{i \in T} c_i, E - \sum\limits_{j \notin T} c_j\} & if\ \ \emptyset \neq T \neq N \\ E & if\ \ T = N \end{cases}$$

$T \subset N$ is a "coalition" which can guarantee the amount of money which will never be disputed by the complementary group.

The following result characterizes those allocations which are in the core of the induced game.

Proposition 6 : *Let $\omega = [N, E, \mathbf{c}] \in \Omega$. Then, the core of the associated game $\langle N, v_\omega \rangle$ is nonempty. Furthermore, an imputation x is in the core if and only if $x \in H(\omega)$ and $Min\{\mathbf{r}(\omega), c\} \leq x \leq Max\{\mathbf{r}(\omega), c\}$.*

[4]This definition differs from the usual one associated to a bankruptcy problem, namely, $v(T) = \left(E - \sum\limits_{j \notin T} c_j\right)^+$ or the associated to a surplus problem, namely, $v(T) = \sum\limits_{j \in T} c_j$ [cf. Curiel, Maschler & Tijs (1988) or Moulin (1987)].

Proof. (i) Let us show first that this game is convex, that is, for any pair of coalitions $K, L \subset N$, such that $K \cap L = \emptyset$ we have:

$$v_\omega (K \cup L) \geq v_\omega (K) + v_\omega (L)$$

Notice first that if $C(\omega) < E$, then $v_\omega (K) = \sum_{i \in K} c_i$, for any $\emptyset \neq K \neq N$, whereas, if $C(\omega) > E$, then $v_\omega (K) = E - \sum_{j \notin K} c_j$, for any $\emptyset \neq K \neq N$.

Then, if $C(\omega) < E$, $v_\omega (K) = \sum_{i \notin K} c_i$, $v_\omega (L) = \sum_{i \in L} c_i$ and $v_\omega (K \cup L) = \sum_{i \in K \cup L} c_i = v_\omega (K) + v_\omega (L)$.

If $C(\omega) > E$, $v_\omega (K) = E - \sum_{j \notin K} c_j$, $v_\omega (L) = E - \sum_{j \notin L} c_j$, and therefore,

$v_\omega (K \cup L) = E - \sum_{j \notin K \cup L} c_j = v_\omega (K) + v_\omega (L) - E + \sum_{j \notin N} c_j \geq v_\omega (K) + v_\omega (L)$

(ii) By definition, $x \in Core(v_\omega)$ iff $\sum_{i \in N} x_i = E$, and if $C(\omega) < E$, $\sum_{i \in T} x_i \geq \sum_{j \in T} c_j$, for all $T \subset N$.
or, if $C(\omega) > E$, $\sum_{i \in T} x_i \geq E - \sum_{j \notin T} c_j$, for all $T \subset N$.
Therefore, if $C(\omega) < E$, $c_i \leq x_i = E - \sum_{j \neq i} x_j \leq E - \sum_{j \neq i} c_j = r_i(\omega)$. Moreover, if $c_i \leq x_i \leq r_i(\omega)$ then, $E = \sum_{i \in T} x_i + \sum_{j \notin T} x_j$, and therefore

$$\sum_{i \in T} x_i = E - \sum_{j \notin T} x_j \leq E - \sum_{j \notin T} c_j.$$

If $C(\omega) > E, E = x_i + \sum_{j \neq i} x_j \geq x_i + E - c_i$, and thus $r_i \leq x_i \leq c_i$
Moreover, if $r_i \leq x_i \leq c_i$, then $E = \sum_{i \in T} x_i + \sum_{j \notin T} x_j$, and therefore,
$\sum_{i \in T} x_i = E - \sum_{j \notin T} x_j \geq E - \sum_{j \notin T} c_j$. ■

Proposition 6 tells us that the bounds required by core allocations are exactly those associated with the notion of an allocation rule (recall that an allocation rule satisfies $\min\{ \mathbf{r}(\omega), c\} \leq F(\omega) \leq \max\{ \mathbf{r}(\omega), c\}$, $\forall \omega \in \Omega$. This indicates that the core property is not a selection criterion within this context.

The next result provides us with a way of selecting core allocations which is consistent with the rights-egalitarian rule.

Proposition 7 : *Let $\omega = [N, E, \mathbf{c}] \in \Omega$, and consider the associated cooperative game in characteristic form $\langle N, v_\omega \rangle$. Let λ be any game solution con-*

cept satisfying symmetry, covariance[5] *and optimality. Then,* $\lambda(\langle N, v_\omega\rangle) = F^{RE}(\omega)$.

Proof. (i) Take the case $C(w) < E$, and $\beta_i = -c_i, i = 1,...,n$. Then $\beta = (\beta_i)_{i=1}^n$. By covariance, $\lambda(\langle N, v_\omega + \beta\rangle) = \lambda(\langle N, v_\omega\rangle) + \beta$. Now, for any $S \subseteq N, (v_\omega + \beta)(S) = 0$ for all $S \neq N$, and $(v_\omega + \beta)(N) = E - C(\omega)$. Then, by symmetry and optimality, $\lambda(\langle N, v_\omega\rangle) = \frac{E-C(\omega)}{n}1$.
In consequence, $\lambda(\langle N, v_\omega\rangle) = \frac{E-C(\omega)}{n}1 - \beta$, i.e.,
$\lambda_i(\langle N, v_\omega\rangle) = \frac{E-C(\omega)}{n} + c_i = F_i^{RE}(\omega)$.
(ii) If $C(\omega) > E$, take $\beta_i = \sum_{j\neq 1} c_j - E$, $1,...,n$, and $\beta = (\beta_i)_{i=1}^n$. Thus, by covariance, $\lambda(\langle N, v_\omega + \beta\rangle) = \lambda(\langle N, v_\omega\rangle) + \beta$. Now, for any $S \subseteq N$,
$(v_\omega + \beta)(S) = E - \sum_{k\notin S} c_k + \sum_{i\in S}\left(\sum_{k\neq i} c_k - E\right) = (|s| - 1)[C(\omega) - E]$.
Therefore, by symmetry and optimality, $\lambda(\langle N, v_\omega + \beta\rangle) = \left(\frac{n-1}{n}[C(\omega) - E]\right)1$.
In consequence, $\lambda(\langle N, v_\omega\rangle) = \left(\frac{n-1}{n}[C(\omega) - E]\right)1 - \beta$, i.e.,
$\lambda_i(\langle N, v_\omega\rangle) = E - \sum_{j\neq i} c_j + \left(\frac{n-1}{n}[C(\omega) - E]\right) = c_i + \frac{E-C(\omega)}{n} = F_i^{RE}(\omega)$. ■

Corollary 3 : *Let* $\omega = [N, E, \mathbf{c}] \in \Omega$, *and consider the associated cooperative game in characteristic form* $\langle N, v_\omega\rangle$. *Let* $\sigma(\omega)$, $\nu(\omega)$ *denote the imputations associated with the Shapley value and the nucleolus, respectively. Then:* $\sigma(\omega) = \nu(\omega) = F^{RL}(\omega)$.

Definition 4 *:[Davis & Masschler (1965)].- Let* $\langle N, v\rangle$ *be a TU game, and let* ϕ *be a single-valued solution concept, such that* $\phi(\langle N, v\rangle) = x$. *Let* S *be a subset of the set of players,* $N, S, \subset N$. *Then, the reduced game* $\langle S, v_s^x\rangle$ *is defined in the following way:*

$$v_s^x(T) = \begin{cases} 0 & if\ T = \emptyset \\ \max\limits_{Q\subseteq N\setminus S} [v(T \cup Q) - x(Q)] & if\ T \subset S,\ T \neq S \\ v(N) - x(N\setminus S) & if\ T = S \end{cases}$$

In the following result we obtain a commutative diagram between allocation problems, games, reduced allocation problems and reduced games.

[5] A solution concept λ over the class G^N of games in characteristic function form with set of players N satisfies covariance if $\lambda(\alpha v + \beta) = \alpha\lambda(v) + \beta, \forall v \in G_N, \forall \beta \in \Re^n$.

Proposition 8 *Let $\omega = [N, E, c]$ be a problem in Ω, and let $\langle N, v_\omega \rangle$ be the TU game associated to ω. Let $x = F^{RE}(\omega)$. Then, for any coalition $S \subset N$, $S \neq N$, the game associated to the reduced problem $\left[S, \sum_{i \in S} x_i, c_s\right]$ coincides with the reduced game $(v_\omega)_s^x$.*

Proof. (i) if $C(\omega) > E$, then $v_\omega(S) = E - \sum_{j \in S} c_j$, $v_\omega(N) = E$.
Now, for any $Q \subset N$, if $q = |Q|$, $x(Q) = \sum_{i \in Q} c_i - q\frac{C(\omega)-E}{n}$, and $v_\omega(T \cup Q) = E - \sum_{j \notin T \cup Q} c_j$. In consequence,
$v_\omega(T \cup Q) - x(Q) = E - \sum_{i \notin T} c_i + q\frac{C(\omega)-E}{n} = v_\omega(T) + q\frac{C(\omega)-E}{n}$. Thus,
$(v_\omega)_s^x(T) = \max_{Q \subset N \setminus S} [v_\omega(T \cup Q) - x(Q)] = v_\omega[T \cup (N \setminus S)] - x(N \setminus S) = v_\omega(T) + (n-s)\frac{C(\omega)-E}{n}$; $(v_\omega)_s^x(S) = x(S)$.
Take now the reduced problem $\omega_s = \left[S, \sum_{i \in S} x_i, c_s\right]$. Then, the game associated to this problem, $v_{\omega_s}(T) = \sum_{i \in S} x_i - \sum_{j \in S \setminus T} c_j$ if $T \neq S$, $v_{\omega_s}(S) = \sum_{i \in S} x_i$.
$\sum_{i \in S} x_i = E - \sum_{j \in N \setminus S} x_j = E - \sum_{j \in N \setminus S} c_j + (n-s)\frac{C(\omega)-E}{n}$, and therefore,
$v_{\omega_S}(T) = E - \sum_{j \in N \setminus T} c_j + (n-s)\frac{C(\omega)-E}{n} = v_\omega(T) + (n-s)\frac{C(\omega)-E}{n} = (v_\omega)_s^x(T)$.
Furthermore, $(v_\omega)_s^x(S) = v_\omega(N) - \sum_{i \notin S} x_i = \sum_{i \in S} x_i = v_{\omega_S}(S)$.
(ii) Let us now consider $C(\omega) < E$. Thus, $v_\omega(T) = \sum_{i \in T} c_i$, if $T \neq S$, $v_\omega(N) = E$.
$(v_\omega)_s^x(T) = \max_{Q \subset N \setminus S} [v_\omega(T \cup Q) - x(Q)] = \max_{Q \subset N \setminus S} \left[\sum_{i \in T \cup Q} c_i - \sum_{i \in Q} c_i - q\frac{E-C(\omega)}{n}\right] = \max_{Q \subset N \setminus S} \left[\sum_{i \in T} c_i - q\frac{E-C(\omega)}{n}\right] = \sum_{i \in T} c_i$
$(v_\omega)_s^x(S) = E - x(N \setminus S) = x(S)$
Take now the reduced problem $\omega_s = \left[S, \sum_{i \in S} x_i, c_s\right]$. Then, the game associated to this problem, $v_{\omega_S}(T) = \sum_{i \in T} c_i$ if $T \neq S$, $v_{\omega_S}(S) = \sum_{i \in S} x_i$. ■

An allocation problem $\omega \in \Omega^n$ can also be regarded as a bargaining problem, by letting $a(\omega) = Min\ \{c, \mathbf{r}(\omega)\}$ be the "disagreement point", and thinking of the opportunity set as follows:

$$S(\omega) \equiv \{v \in \mathbb{R}^n / \sum_{i=1}^{n} v_i \leq E\}$$

The associated bargaining problem is $[S(\omega), a(\omega)]$, which turns out to be symmetric for any $\omega \in \Omega^n$. In consequence, all symmetric and weakly Pareto Optimal bargaining solutions coincide for $[S(\omega), a(\omega)]$, and the final outcome associated to any of these bargaining solutions is again the distribution suggested by the rights egalitarian rule (e.g., Nash, Kalai-Smorodinski, etc.). Thus, we have the following immediate result:

Proposition 9 : *Let $\omega = [N, E, c] \in \Omega$, and consider the associated bargaining problem $[S(\omega), a(\omega)]$. Let B denote any symmetric and weakly Pareto optimal bargaining solution. Then, $B[S(\omega), a(\omega)] = F^{RL}(\omega)$.*

4 FINAL REMARKS

In this paper, a solution for general problems of division of a estate by taking entitlements into account has been proposed and characterized. It is interesting to observe that according to legal regulations, the dissolution of a partnership follows in some cases the rights-egalitarian rule. This is the case, for example, for the legal Spanish system applied to a divorce or to unlimited liability societies.

The characterization results provided for the two-person case are significant not only because of their applicability in the characterization results for the general case, but also by themselves. It is commonly argued that (a) consistency is a desirable property for a solution concept, and (b) that in order to infer the type of justice values a group of agents may share, it is easier to explore the simpler two-person case than any other situation. Thus, if agents agree in the way of solving two-person problems, the general solution may be inferred by using consistency.

Several characterization results for the rights-egalitarian solution have been provided. In order to properly separate the axioms used in the aforementioned characterization results, consider the following allocation rules:

PROPORTIONAL RULE, P: If $\omega = [N, E, c]$, then
$P\omega = \lambda c$, with $\lambda C(\omega) = E$, if either $C(\omega) \neq 0$ or $E \neq 0$.
$P(\omega) = c$, if $C(\omega) = 0$ and $E = 0$.

DICTATORIAL RULE, D_j : If $\omega = [N, E, c]$, $j \in N$, then
$D_j(\omega) = c_j$
$D_k(\omega) = Min\{c_k, r_k\} + A/(n-1)$, for all $k \neq j$, where $A = E - c_j - \sum_{k \neq j} Min\{c_k, r_k(\omega)\}$

ALTERNATING DICTATORIAL RULE, A_{ij} : If $\omega = [\{i, j\}, E, c]$, then
$A_{ij}(\omega) = D_i(\omega)$ if $C(\omega) > E$
$A_{ij}(\omega) = D_j(\omega)$ if $C(\omega) < E$
$A_{ij}(\omega) = c$, if $C(\omega) = E$.

ADJUSTED DICTATORIAL RULE, AD_j : If $\omega = [N, E, c]$, $j \in N$, then
$AD_j(\omega) = \mathbf{r}(\omega) + D_j(\omega')$, where $\omega' = [N, E - C(\omega), c - \mathbf{r}(\omega)]$.

Now, in Proposition 1, P satisfies symmetry but does not satisfy congruence, and AD_j satisfies congruence but does not satisfy symmetry. In Proposition 3, P satisfies symmetry and does not satisfy concavity, whereas D_j *satisfies* concavity but does not satisfy symmetry. In Lemma 1, D_j satisfies bi-concavity, but does not satisfy bi-duality, and A_{ij} satisfies bi-duality but does not satisfy bi-concavity. Appropiate extensions also go in Proposition 4. In Proposition 5, D_j is bi-consistent and bi-concave but is not bi-dual; A is bi-consistent and bi-dual but it is not bi-concave; finally, if we define $F(\omega) = F^{RE}(\omega)$ if $|N| = 2$, $F(\omega) = P(\omega)$ if $|N| > 2$, then F is bi-dual and bi-concave but is not bi-consistent. In Proposition 6, if we define $F(\omega) = F^{RE}(\omega)$ if $|N| = 2$, $F(\omega) = P(\omega)$ if $|N| > 2$, then F is two-person average decentralizable but is not bi-consistent, and P is consistent but is not two-person average decentralizable.

Letting aside the appealingness of the properties used in the characterization results, the game-theoretical approach to our solution provides with additional support. First, it is interesting to observe that by means of the associated characteristic function v_ω to a problem ω, we tried to capture the traditional idea of "what a coalition can guarantee for its members". With such a spirit, we used a characteristic function form different from those traditionally used in bankruptcy problems and problems of sharing a surplus. These differences refer to two points: firstly, the idea of taking zero as a natural status-quo is meaningless in our context; secondly, we choose the minimum between the two traditional formulations, in order to capture the most natural expression of the characteristic function, depending upon the type of problem at hand.

It is interesting to stress that previous formulation reveals to be really natural in our context. Actually, Proposition 7 indicates that by means of such a characteristic function, the bounds for the Core are exactly the natural bounds in our problem. Proposition 8 indicates the appropriateness of our formulation and the strength of the rights-egalitarian solution: by means of its game-theoretic formulation any "well-behaved" single-valued solution concept will coincide with our proposal.

Finally, Proposition 9 indicates that our formulation adapts to the Davis-Maschler reduced game form, since the diagram

$$\Omega^N \xrightarrow{f} G^N$$

$$g \qquad\qquad\qquad h$$

$$\Omega^S \xrightarrow[f]{} G^S$$

commutes, where $f(\omega) = \langle N, v_\omega \rangle$, $g(\omega) = \omega_s$, and $h(\langle N, v_\omega \rangle) = \langle S, (v_\omega)_s^x \rangle$. Additionally, it again provides with an alternative way of obtaining the consistency property for our solution concept.

Concerning the decentralizability property, is in interesting to stress that the rights-egalitarian solution satisfies a similar property for any set of agents. Let $\omega = [N, E, c]$. Then, if any agent chooses the following allocation: $s_i = (c_i, e_{-i}(\omega) + \lambda)$, where $e_j(\omega) = Min\{c_i, r_i(\omega)\}$, for all $j \neq i$, and λ satisfies $(n-1)\lambda = E - \sum_{j \neq i} r_j(\omega)$, by means of the average mechanism, $\pi_k = \frac{1}{n} \sum_{i \in N} s_{ik}$, we may implement the rights-egalitarian solution.

Finally, we may mention that even though the claims-egalitarian solution coincides with the Shapley value of the associated game, the reduced game property à la Hart-Mas Collel [see Hart & Mas-Collel (1989)] does not work for our characteristic function.

REFERENCES

AUMANN, R. & MASCHLER, M., (1988), Game Theoretic Analysis of a Bankrupt cy Problem from the Talmud, Journal of Economic Theory, 36:195-213.

BOSSERT, W., (1993), An Alternative Solution for Bargaining Problems with Claims, Mathematical Social Sciences, 25: 205-220.

CURIEL, I.J., MASCHLER, M. & TIJS, S.H., (1988), Bankruptcy Games, Zeitsch rift fur Operations Research, 31: A143-A159.

CHUN, Y. (1988a), The equal-loss pricnciple for bargaining problems, Economics Letters, 26, 103-106.

CHUN, Y. (1988b), The proportional solution for rights problems, Mathematical Social Sciences, 15: 231-246.

CHUN, Y. & PETERS, H. (1991), The lexicographic equal-loss solution, Mathematical Social Sciences,22: 151-161.

CHUN, Y. & THOMSON, W., (1992), Bargaining Problems with Claims, Mathematical Social Sciences, 24, 19-13.

CORCHON, L. & HERRERO, C. (1995), Decent Proposals, mimeo, University of Alicante.

DAGAN, N. (1995), New Characterizations of Old Bankruptcy Rules, Social Choice and Welfare, 12.

DAGAN, N. & VOLIJ, O., (1993), The Bankruptcy Problem: A Cooperative Bargaining Approach, Mathematical Social Sciences, 26: 287-297.

DAVIS, J. & MASCHLER, M. (1965), The Kernel of a Cooperative Game, Naval Research Logistics Quarterly, 12, 223-259.

GUPTA, S. & LIVNE, Z.A., (1988), Resolving a conflict situation with a referance outcome: An Axiomatic Model, Management Science, 34, No. 11, 1303-1314.

HART, S. & MAS-COLLEL, A. (1989), Potential, Value and Consistency, Econometrica, 57, 589-614.

HERRERO, C. (1995), Endogenous Reference Points and the Adjusted Proportional Solution for Bargaining Problems with Claims, mimeo.

HERRERO, C. (1994), Bargaining with reference points-bargaining with claims_ egalitarian solutions reexamined, A Discusion, WP AD 94- 15.

HERRERO, C. & MARCO, M.C. (1993), Rational Equal-Loss Solutions for Bargaining Problems, Mathematical Social Sciences, 26: 273-286.

KALAI, E. (1977), Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons, Econometrica 45: 1623-1630.

MOULIN, H. (1987), Equal or proportional division of a surplus, and other methods, International Journal of Game Theory, 16(3): 161-186.

MOULIN, H. (1988), Axioms of Cooperative Decision Making, Cambridge University Press.

O'NEILL, B. (1982), A problem of rights arbitration in the Talmud, Mathematical Social Sciences, 2, 345-371.

THOMSON, W. & LENSBERG, T. (1989) Axiomatic Theory of Bargaining with a Variable Number of Agents, Cambridge, Cambridge University Press.

YOUNG, H.P. (1987), On dividing an amount according with individual claims or liabilities, Mathematics of Operations Research, 12: 397-414.