OPTIMAL CEO COMPENSATION AND STOCK OPTIONS*

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RESUMEN

Este artículo estudia el problema de incentivos que surge entre los dueños de una empresa y el ejecutivo que la dirige, fruto de la imposibilidad de observar directamente las acciones del directivo. El modelo difiere del modelo estándar en la literatura en dos puntos clave. En primer lugar, tiene en cuenta que las acciones que toma el directivo tienen un efecto persistente en el tiempo; esta persistencia no la consideran los modelos estándar de riesgo moral repetido. En segundo lugar, el efecto del esfuerzo del directivo en el precio de las acciones de la empresa se deriva de los primitivos del modelo: el esfuerzo determina la distribución de probabilidad de los beneficios de la empresa, y no directamente la distribución de precios. Los compradores en el mercado de valores determinan el precio de las acciones basándose en la información disponible sobre los beneficios pasados. El artículo presenta, como marco de referencia, una caracterización del contrato óptimo asumiendo responsabilidad limitada por parte del directivo. Para el caso en que se pueden emitir múltiples paquetes de opciones, se presentan condiciones suficientes para la implementación del contrato óptimo. Para un caso simplificado en el que la compensación se realiza con un solo paquete de opciones, se analizan las características del mismo. Los resultados del análisis indican que la fecha de ejercicio óptima se determina balanceando los beneficios y los costes de esperar un periodo más: por un lado, aumenta la calidad de información; por el otro, aumenta el coste de proveer incentivos, por tener que estar estos concentrados en un horizonte temporal menor. El número de opciones en el paquete, el salario, y especialmente el precio de ejercicio se usan para explotar la correlación entre los cambios en precios y los cocientes de probabilidad relativa correspondientes a las historias de beneficios que generan esos precios. Por ejemplo, cuando los precios bajos están débilmente correlacionados con los correspondientes cocientes, el paquete óptimo de opciones tiene un precio de ejercicio positivo, que permite explotar la correlación existente en el rango de precios alto mejor que un paquete que incluyera simplemente acciones (i.e, acciones de venta restringida). Estos resultados sugieren cautela a la hora de aprobar regulación que pueda distorsionar la elección de los precios de ejercicio de las opciones en los paquetes de compensación de directivos de empresa.

Palabras clave: Riesgo Moral, Contratos Óptimos, Persistencia, Compensación de Directivos, Opciones

ABSTRACT

We study the incentive problem between the owners of a firm and its CEO's due to the unobservability of the manager's actions. Our model departs from the literature in two ways. First, we acknowledge that, in contrast with standard repeated moral hazard models, actions taken by CEO's have a persistent effect in time. Second, we derive the effect of effort on stock prices from primitives; i.e., effort affects directly the conditional distribution of profits, and not the distribution of prices. The stock market determines the price of the stock of the firm using information about past profits. A complete characterization of the Second Best contract assuming limited liability is given as a benchmark. Allowing for an arbitrary number of option grants to be awarded, sufficient conditions are given for the implementation of the Second Best contract by an Options Scheme. For a stylized scheme with a unique option grant, the characteristics of the solution are analyzed. We find that the optimal time of exercise balances the increase in quality of information of waiting one extra period with the cost of the poorer smoothing of incentives of doing so. The number of options in the grant, the constant wage, and especially the exercise price are used to best exploit the correlation between the changes in prices and in the likelihood ratios of the histories of profits generating them. As an example, whenever low prices are poorly correlated with the likelihood ratios, the optimal option scheme implies a positive exercise price, which allows for a better use of a higher correlation over the high stock price range than a simple restricted stock scheme. Our results suggest caution regarding regulations that influence the setting of exercise prices.

Keywords: Moral Hazard, Optimal Contracts, Persistence, CEO Compensation, Stock Options.

JEL classification: D30, D31, D80, D82.

1 Introduction

Compensation of chief executive officers (CEOs) with stock options is a widely spread practice nowadays. The purpose of this paper is to study the implications that the persistent nature of actions taken by a CEO has for the main characteristics of compensation packages that include stock options.

It is generally accepted that delegating the management of the firm to a CEO creates an agency problem, since, absent the right incentives, CEOs could base their decisions on short term or personal objectives, which imply immediate reward, rather than trying to serve the interests of the stock holders by increasing the long term value of the firm. Granting options and restricted stock is considered a simple and convenient way of aligning the objective of CEOs with that of the stock holders. Indeed, in recent years options have become a very important portion of the total pay of the CEO, along with restricted stock grants.[1] Tax advantages associated with performance related pay and especial accounting treatment for options granted at the money (with exercise price equal to the market price of the stock at the time of granting) make at the money options particularly attractive for firms. This paper presents a new model of the agency problem between the CEO and the owners of the firm which takes into account the persistent nature of actions taken by the CEO. It explicitly models the effect of the manager's actions on long term profits, providing a formal framework for determining his long term compensation. In this context, we explore the implied form of compensation packages that approximate the optimal contract, given that they can include stock options. The analysis is a first step in studying the potential effects of tax incentives and recent changes in disclosure and accounting rules.

The model in this paper differs from the standard moral hazard models used in the literature in two ways. First, it acknowledges that the actions that CEOs are asked to exert are persistent: they affect the profit of the firm for many periods after they are implemented. Examples of the kinds of actions that we are modeling include decisions over mergers, investment in alternative technologies, the downsizing or expansion of the firm, and hiring of new personnel, etc. The effect of any such action taken by the manager of the firm is persistent, since the decisions are difficult to reverse immediately, and will determine the company's profits for several periods afterwards. Following

[1] As documented in Murphy (1999), in the mid 90's stock options represented 17 to 36% of total executive compensation. Bonuses represented 19 to 26% of compensation, while the base salary proportion ranged from 21 to 40%. Restricting the sample to companies with sales above the median, options amounted to 39% of the total compensation, while salary was 24%. In Jensen and Murphy (2004) numbers are provided up to 2002: the proportion of total pay represented by options valued at the time of granting reached a pick of 54% in 2000, decreasing to a sizeable 47% in 2002. Both data sets include all companies in the S&P 500, based on ExecuComp data.

Hopenhayn and Jarque (2007), persistence is modelled in this paper as follows: the CEO takes a single action at the beginning of the contractual relationship, which determines the conditional distribution of the firm's profit in each of the following periods, until the end of the contract.

The second departure from the literature is that the effect of the manager's action on the probability distribution over the firm's stock prices is not assumed directly, but is derived from a more fundamental structure: effort affects the firm's profits, and the stock market rationally determines prices based on the history of profits.

For our analysis, we proceed in two steps. First, assuming an unlimited set of compensation instruments, we study how the persistence of the actions taken by the CEO changes the properties of the optimal contract, i.e. we characterize the sequence of wages contingent on observed profit that induces high effort from the CEO at the minimum cost to the firm. The characterization of the optimal contract highlights the role of information gathering over time: variation in wages is postponed until later periods of the contract, when more information is available.

Second, using our model of stock prices we study how the relationship between profit announcements and prices determines the characteristics of an option-like (limited instruments) optimal compensation package. Our simple model of the stock market aims at illustrating explicitly the effect of the manager's action on the stock price of the firm. The probability distribution over profits is affected by both the effort of the manager and the quality of the firm's technology. This quality is an unknown parameter to the board, the manager and the stock market. The assumption that the manager does not know nor can he control the quality of the firm implies that the optimal contract would provide insurance against low profits if it were known that they are due to a bad quality realization. Buyers in the stock market price a share of the firm at the expected stream of future profits. In order to calculate this expectation, they incorporate all past public information. More precisely, they use the history of profit realizations to update their priors over the technology parameter. They also understand that the firm provides the right incentives to the manager, and hence they correctly condition their beliefs on the equilibrium action taken by the CEO. Given this, any variation in the price of the stock comes from learning about the quality of the firm's technology, and not from learning about the action taken by the manager. However, in the presence of learning, an off the equilibrium path change in the effort of the manager (shirking) will affect the market's posterior about the firm's type, and thus it will decrease its market value. Payment schemes that include options exploit the indirect effect that low effort has on the price of the firm: lower effort increases the probability of poor outcomes; the market interprets these as a sign of a bad quality firm and this lowers the stock price, decreasing the manager's profits from selling the stock options.

In this context and with the optimal contract in mind we try to derive implications for the structure of compensation packages that include option grants. Allowing for an arbitrary set of option grants, we provide sufficient conditions for the compensation package to implement exactly the optimal contract. In doing so, we stress the asymmetry between the stock prices movements and the desired sensibility of pay for proper incentives. Since the Board is relying on exogenous variation of prices to set the wage of the CEO, the richness in the compensation instruments is key for providing the right mix of incentives and insurance. Real life compensation packages use a fairly reduced set of instruments. Firms provide simple schemes with few at the money option grants, only very rarely using indexed exercise prices or price contingent vesting times. Tax and accountancy advantages presumably stand behind this uniformity in compensation practices. Are these advantages distorting the provision of incentives? When using stock options to provide incentives, is the exercise price a redundant instrument? We try to address this question by analyzing a one option grant model. We show how the vesting time, the number of options in the grant and especially the exercise price are used to best exploit the correlation between stock prices and likelihood ratios of profits. The learning process about the quality of the firm determines the stock price for each history of profit realizations: it implies a certain sensitivity of stock prices to profits. Granting at the money options, which set the exercise price equal to the stock price at the time of granting, implies arbitrarily setting the sensitivity of compensation to prices. Using the full range of possible exercise prices, instead, allows to transform the market–given sensitivity of prices into the sensitivity of compensation in the optimal contract. In other words, the exercise price is used to best exploit the correlation between the changes in prices and in the likelihood ratios of the histories of profits generating them. As an example, whenever low prices are poorly correlated with the likelihood ratios, the optimal option scheme implies a positive exercise price; this allows for a better use of a higher correlation over the high stock price range than a simple restricted stock scheme. Hence, we find a big range of variation in optimal exercise prices, from simple restricted stock to in the money options, depending on the parameter values. We complete the analysis by characterizing, in a two period model, the optimal compensation package including wages and an unrestricted number of option grants. We provide examples in which the firms' characteristics influence strongly the structure of exercise prices. Our results suggest caution regarding regulations that influence the setting of exercise prices.

1.1 Related Literature

In addition to the empirical literature on CEO compensation (see Yermak (1995), Kole (1997), Jensen and Murphy (1990) and finally Murphy (1999) for a nice review), some papers have lately looked at the problem from a contract theory perspective. Clementi, Cooley and Wang (2006) propose a justification of the use of stock options as a commitment device that increases efficiency, in a context of repeated moral hazard. Wang (1997) calibrates a repeated moral hazard model to gives some quantitative results on sensibility of CEO pay, using standard contingent consumption contracts that do not include options.

Some studies have explicitly focused on the form that real life compensation packages should take as part of an optimal contract. Aseff and Santos (2005) study CEO compensation through options schemes, in a one period setup – thus, with no persistence of effort. Kadan and Swinkles (2008) compare options with restricted stocks in a dynamic model in which the CEO can choose to hedge against the risk in his compensation. They show that stocks dominate options for forms with higher risk of bankruptcy, and they provide some empirical evidence that supports their result.

All the above mentioned papers assume a certain distribution of prices of the firm contingent on the effort choice, as opposed to assuming an induced distribution over output and then letting the market price the stock, as it is modeled in the present paper. This reduced form allows them to study the spanning possibilities of an exogenously given distribution of prices, but not to explore the endogenous determination of those possibilities, as we do in this paper. Holmström and Tirole (1993) do present a model in which prices are endogenously set. They assume the existence of stock traders with different levels of information and liquidity needs. They characterize the optimal contract including options in a three period model. The manager decides at the initial period how much effort he devotes to both short and long term activities, but these two efforts are set separately, unlike in our framework. Despite this, the effect of effort combined with the signal extraction implies the optimality of granting shares and stock appreciation rights in their linear compensation contract. They provide results on how the concentration of ownership and the liquidity of the market influence the monitoring value of stock prices. Recently, Bolton, Sheinkman and Xiong (2006) build on this model to argue that the possibility of speculative movements in prices may rend optimal to construct option packages that emphasize short term stock performance.

Hall and Murphy (2000) study the setting of optimal exercise prices, also motivated by the seemingly puzzling uniformity in real life option grants. Using a certainty equivalent approach, they provide a computational model to evaluate options from the point of view of risk averse managers, as opposed to the standard Black and Scholes valuation that assumes risk neutrality. For an arbitrary set of grant values, their numerical exercise finds that the market price at the time of the grant is usually included in the range of exercise prices optimal for the provision of incentives. They conclude that granting options at the money is not generally harming for firms. Their analysis, however, is done ignoring the effects that the effort of the manager would have, on and off the equilibrium, on the value of the firm, and hence on the value of the option both for the firm and for the manager.

The paper is organized as follows: the model is introduced in the first section. The optimal contract with unrestricted compensation instruments is analyzed in section 3. Section 4 introduces the payment scheme including options; a One Option Grant scheme is analyzed in detail and some numerical examples are provided. A two period example of an exact implementation of the Second Best through stock options is derived in the last subsection. Section 5 concludes.

2 The Model

We model the moral hazard problem between the owners of a firm and the CEO of the company. The stockholders of the firm, acting as a unique risk neutral principal, delegate the design of the contract to the board of directors.[2] The manager is assumed to be risk averse, with a strictly concave utility function $u(\cdot)$ with $u'(0)=\infty$. He has an outside opportunity with an expected utility of $\underline{U}$ at period zero. The contract lasts for an exogenously determined number of periods $\overline{T}$, after which the effort of the CEO does not affect the profit of the firm. In any period, profits can take two values, $y_L=0$, and $y_H=1$. Both the manager's effort level e, and the quality of the technology of the firm θ, affect the probability of the outcome. Effort can take two values, e_L and e_H, (with $e_L<e_H$) which also stand for the manager's disutility of effort. We assume that the parameters are such that the firm owners always want to implement e_H. The firm type θ can be "good" (G), or "bad" (B). Both the owners and the CEO have the same prior q_0 of $\theta=G$. New observations are used to update this prior according to Bayes' rule. The posteriors are denoted $q\left(y^t\right)$ if the action is e_H and $\widehat{q}\left(y^t\right)$ if the action is e_L.

For a given quality of the firm and a given effort of the CEO, the probabilities of high profits are:

	G	B
e_L	$\hat{\pi}_G$	$\hat{\pi}_B$
e_H	π_G	π_B

We assume that $0<\pi_\theta<1$ and $0<\hat{\pi}_\theta<1$ for both $\theta=G,B$. Fig. **??** illustrates the timing of the game and the actual probabilities driving output realizations. First, nature (N) decides the type of the firm. Second, the manager (M), without knowing the realization of the type, chooses between the high and the low levels of effort. This determines the probability for each history of outputs to follow. The CEO, the board and the stock market all calculate the ex ante probabilities over output histories under imperfect information about θ.

[2] The board of directors of the firm may, in turn, may delegate the design of the compesation package to a compensation committee. This potentially gives rise to additional incentive problems. Here simplify and assume that the board decides compensation and its incentives are completely aligned with those of the shareholders.

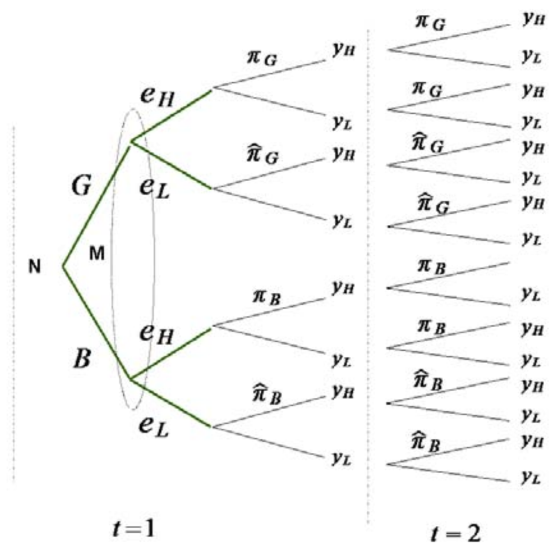

Figure 1: Timing of the game and probabilities.

The probability of a high level of profits at period t, when the CEO chooses e_H, is determined as follows:

$$\Pr\left(y_H | y^{t-1}\right) = q\left(y^{t-1}\right)\pi_G + \left(1 - q\left(y^{t-1}\right)\right)\pi_B, \tag{1}$$

where π_θ is the probability of y_H induced by the high effort when the firm is of type $\theta = G, B$. Let $q\left(y^t\right)$ denote the Bayesian posterior on the probability of $\theta = G$ conditional on history y^t. The Bayesian updating is done in the usual way; after observing a high output at time t, the prior over $\theta = G$ for period $t+1$ will be:

$$q\left(y^{t+1}\right) = \frac{q\left(y^t\right)\pi_G}{q\left(y^t\right)\pi_G + \left(1 - q\left(y^t\right)\right)\pi_B}. \tag{2}$$

If the manager chooses e_L instead, the corresponding probability to that in (1) will be:

$$\widehat{\Pr}\left(y_H | y^{t-1}\right) = \widehat{q}\left(y^t\right)\widehat{\pi}_G + \left(1 - \widehat{q}\left(y^t\right)\right)\widehat{\pi}_B, \tag{3}$$

with $\pi_G > \widehat{\pi}_G$ and $\pi_B > \widehat{\pi}_B$, and where $\widehat{q}\left(y^t\right)$ is the posterior under the assumption of low effort, calculated in the same way as in (2). The probability of each individual period's outcomes under imperfect information about θ depends on the history through the Bayesian updating of the probability of the good technology. Given our assumptions about the stochastic structure, we can construct the probability of a given history y^t as:

$$\Pr\left(y^t\right) = \prod_{\tau=1}^{t} \Pr\left(y_\tau | y^{\tau-1}\right).$$

3 Benchmark: The Optimal Contingent Consumption Scheme

In this section we assume an unlimited set of compensation instruments are available to the board. This implies that in the optimal contract wages can vary arbitrarily with the history of profit realizations. We call this a contingent wage contract, and following the moral hazard literature we refer to the optimal one as the Second Best (as opposed to the First Best, when effort is observable).

The optimal contract defines an unrestricted sequence of contingent consumption levels that minimizes the cost of implementing the high effort level. It is noteworthy that any Options Scheme naturally puts a lower bound on the available punishments to the agent (he can always choose not to exercise the options and he incurs no loss). Correspondingly, we assume that there exists a minimum consumption level b that will be guaranteed to the agent under any contingent consumption scheme.[3]

Let $c_t\left(y^t\right)$ be the agent's consumption levels, contingent on the realizations of the firm's profits. The cost of the principal is:

$$K\left(\underline{U}, \left\{c_t\left(y^t\right)\right\}_{t=0}^{\overline{T}}, \omega\left(y^{\overline{T}}\right)\right) = \sum_{t=0}^{\overline{T}} \sum_{y^t} \beta^t \left\{c_t\left(y^t\right)\right\} \Pr\left(y^t\right)$$

The Participation Constraint of the CEO at period 0 is:

$$\underline{U} \leq \sum_{t=0}^{\overline{T}} \sum_{y^t} \beta^t u\left(c_t\left(y^t\right)\right) \Pr\left(y^t\right) - e_H, \tag{PC}$$

The Incentive Constraint needs to be satisfied:

$$\sum_{t=0}^{\overline{T}} \sum_{y^t} \beta^t u\left(c_t\left(y^t\right)\right) \Pr\left(y^t\right) - e_H \geq \sum_{t=0}^{\overline{T}} \sum_{y^t} \beta^t u\left(c_t\left(y^t\right)\right) \widehat{\Pr}\left(y^t\right) - e_L. \tag{IC}$$

We also impose the limited liability constraint:

$$c_t\left(y_t\right) \geq b \quad \forall y^t. \tag{LL}$$

The model presented here is a variation of the one in Hopenhayn and Jarque (2007). They provide a characterization of the Second Best contract for a class of stochastic processes that includes the

[3]Note that $u\left(b\right)$ may be interpreted as the CEO's per period consumption if he were to work elsewhere. Although we are not explictely allowing for firing or quitting decisions, in our model those will never occur in equilibrium provided $c\left(y^t\right) > b$ is satisfied for every y^t.

one in this paper, but do not consider an explicit lower bound on utility. Our next proposition is rephrasing their main characterization result under a limited liability constraint.[4]

Let the Likelihood Ratio of a given history be defined as follows:

$$LR\left(y^t\right) \equiv \frac{\widehat{\Pr}\left(y^t\right)}{\Pr\left(y^t\right)}.$$

Proposition 1 *(Characterization of the Second Best) There exists a LR^* such that:*

(i) any history y^t with $LR\left(y^t\right) \geq LR^*$ is assigned the minimum consumption level: $c\left(y^t\right) = b$;

(ii) for histories with $LR\left(y^t\right) < LR^*$ consumption levels are ranked by likelihood ratios:

$$c\left(y^t\right) > c\left(\widetilde{y}^t\right) \Leftrightarrow LR\left(y^t\right) < LR\left(\widetilde{y}^t\right).$$

Proof. See Appendix. ■

Longer histories contain more information, so the dispersion of likelihood ratios and compensation increases over time. In other words, postponing incentives until later periods, when better information is available, reduces the cost of implementing high effort: it reduces the need to spread consumption in earlier periods, bringing down the average variance of compensation in the contract.

It follows from the above characterization that only when the likelihood ratio values are monotone in the outcome of the firm, is it optimal to have compensation increasing in the performance of the firm. When learning about the firm's type is possible, this will not always be the case, as indicated by the following proposition.

Proposition 2 *Optimal consumption is not necessarily monotonic in profit.*

Proof. See Appendix for a generalization of the proof in Miller(99). ■

The existence of non-monotonicities comes from combining learning with the provision of incentives. Since the quality of the firm's technology is not controllable by the CEO, an ideal contract would insure him against this risk. However, under such a contract, the manager would shirk and blame poor performance on a bad technology. The optimal contract demands exposing the agent to some technology-related risk, which can imply non monotonicities in consumption. The owners of the firm evaluate the relative likelihood of effort and learn about the quality of the technology at the same time. The likelihood ratio of a history consists of the ratio of the probability of the history if effort is low, divided by the probability if effort is high. A high outcome following low ones

[4] See Kadan and Swinkels (2006) for a study of the effect of bounds on utility on the optimal contract under moral hazard.

may increase the likelihood that previous bad draws were the result of low effort and is thus "bad news" for the agent. In other words, the weights given to each type's probability distribution (i.e., the posteriors,) are different for high and low efforts, so the ordering of each type's probabilities is not preserved in the probability unconditional on the type. To clarify, we can think of the following example:

	G	B
e_L	$\hat{\pi}_G$	0
e_H	π_G	0

with $\pi_G > \hat{\pi}_G$. In words, output can never be high if the firm is of the bad type, regardless of the effort of the agent, but it happens with positive probability if the firm is good. If the first couple of outcomes are low, the agent is given some compensation taking into account that the firm might be of the bad type, so the observed output is quite likely under the high effort level. In the third period, output can be low or high. If the third outcome is high, then the owners of the firm know for sure that they are facing a good type. The whole history is judged now under the new beliefs. If the third realization is low, the bad type has still positive probability (in fact, it has a higher posterior). This may mean that the agent's wage in the second period is lower if we observe a high outcome than if we observe a low one, since the two first low outputs have a much higher likelihood when knowing that the firm is good. This sort of learning dynamic is what gives rise to non monotonic compensation.[5]

4 Compensating CEO's with Stock Options

We now introduce a stylized compensation scheme that tries to capture the main features of typical executive compensation. The compensation instruments that are available to the designers of the contract are a constant wage c_0 to be paid at any period before any option is exercised, and a set of H different option packages or option grants, each defined by the following elements:

$$\{T_j, n_j, z_j\}_{j=1}^{H},$$

[5] In an independent study, Celentani and Loveira (2206) apply this same logic to explain the Relative Performance Evaluation Puzzle (the documented lack of a negative relationship between CEO compensation and comparative performance measures, such as industry or market performance), as well as the tendency to insulate a CEO's rewards from bad luck, but not from good luck

In our model, if we were to allow for economy–wide shocks (i.e., orthogonal to both the quality of the firm and the effort of the CEO), the same logic behind Proposition 2 would, for the right parameter values, imply this type of "puzzling" insurance for the CEO.

where T_j is the time at which options in the grant j are exercised, n_j is the number of stock options in the grant and z_j is the price at which the stock can be bought if the options are exercised.

The first thing to notice about this compensation scheme is that, in order for options to be used as an incentive mechanism, the firm's stock price sequence $\{x_t\}$ must be sensitive to profit realizations. In our model, this dependence comes from learning. Investors, who are the third player of the game, want to learn about the quality of the technology of the firm, so they can accurately price the expected future stream of profits. Given the stochastic structure defined in the previous section, and for a given interest rate r, we can calculate the price of the stock to be:

$$x_t\left(y^t\right) = E_t\left[\sum_{\tau=t}^{\infty} y_\tau \left(\frac{1}{1+r}\right)^{\tau-t}\right]. \tag{4}$$

The assumptions under this pricing rule are that there is competitive (actuarially fair) pricing in the Stock Market, and that the market understands that the incentive problem is solved inside the firm, so the equilibrium level of effort is the relevant one for calculating the expectations.

Remark 3 *Stock prices are always increasing in output (See Appendix).*

The timing of the game is as follows: First, the firm Owners (O) decide on the Options Contract. Second, nature (N) decides whether the firm is good or bad, and the manager (M) decides his level of effort without knowledge of the firm type. Finally, the firm profit level is realized conditional on the choice of effort, and the stockmarket participants (S) price the stock based on their Bayesian updating of the firm quality (see Fig. 1).

Note that, if there were no extra noise in the firm's outcome, a payment with options would not be feasible in a pure strategy equilibrium, since the price of the stock would be constant and independent of the effort level of the CEO. This is in the spirit of Holmström's influential paper on dynamic incentives for managers [8]. Although in Holmström's setup the manager is not required to exert a certain level of effort, he is paid in the future based on the beliefs of the market about his productivity. The manager does not know his own productivity (as the manager in our model does not know the type of the firm), but he exerts positive effort in order to increase the probability of good outcomes, which influence upwards the beliefs of the market about his productivity, and hence his future wage. In our model, the pricing rule of the market is independent of the actual effort exerted, since the market assumes the equilibrium effort is chosen. The manager understands, however, that lowering his effort would increase the probability of low output, which would drive downwards the beliefs about the firm and thus prices. Lower prices would mean, in turn, lower profits from exercising his options.

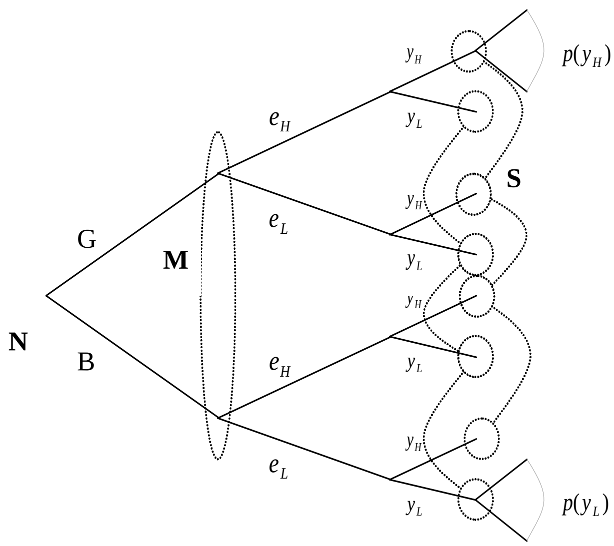

Figure 1: Game tree for a given options contract.

In our set-up with learning, we explicitly make the distribution over stock prices depend on the recommended level of effort: when the CEO is considering a deviation, he understands that prices are determined under the equilibrium beliefs of the market. As a result, the reduced form probability vector over prices will depend on both the actual effort chosen (e) and the recommended effort (e_H):

$$\Pr\left(x\left(y^t\right)|e;e_H\right).$$

The problem of the owners is to choose the Options Scheme that minimizes the cost of implementing the high level of effort.

Definition A Perfect Bayesian Equilibrium of this game is an Options Contract $\left\langle c_0^*, \left\{T_j^*, n_j^*, z_j^*\right\}_{j=1}^{H^*}\right\rangle$, a level of effort of the agent equal to e_H, and a pricing function $x^*\left(y^T\right)$, such that

a) $\left\langle c_0^*, \left\{T_j^*, n_j^*, z_j^*\right\}_{j=1}^{H^*}\right\rangle \in \arg\min_{\{\cdot\}} K\left(e_H\right)$

b) The utility of the agent choosing e_H is higher than if choosing e_L, and is as large as his outside utility $\underline{U}$

c) the pricing function $x\left(y^T\right)$ and the beliefs of the stockmarket participants are consistent with the agent choosing e_H

d) beliefs are updated according to Bayes' rule

Since the probability of observing any history is positive under the equilibrium level of effort, Bayesian updating provides consistent beliefs, and no refinement is necessary.

If we allow for a big enough number of available grants H, at different exercise prices and times, and we also let n take negative values, the executive compensation package could reproduce any contract that uses arbitrary consumption transfers.

Proposition 4 *Whenever $LR\left(y^t\right) \neq LR\left(y^{t\prime}\right)$ implies $x\left(y^t\right) \neq x\left(y^{t\prime}\right)$ for all y^t, the Second Best contingent wage contract can be implemented through a multiple stock Options Scheme.*

Proof. See Appendix. ■

To understand this result, recall that the Second Best scheme ranks consumption levels according to likelihood ratios. If market prices discriminate between any two histories with different likelihood ratios, then there exists enough variation in prices to span the vector of consumption levels implied by the Second Best scheme.

Real-life options schemes, however, do not include as much variety of options as the above proposition suggests. We do observe changes in salaries and sequential award of different options with several vesting times, but payment schemes including options are fairly simple. As documented by Murphy (1999) in his analysis of stock options granting practices for 1,000 firms in 1992, there seems to be little cross–sectional variation in granting practices. In 83% of the firms in his sample, all options expired in 10 years, and in 95% of the firms they were granted with exercise prices equal to the "fair market value" on date of grant. A second data source cited by Murphy, a survey conducted by Towers Perrin in 1997, when asked about the rules used to determine option grants, 40% of the firms answered they used a fixed value of the options as a target (adjusting the number of options according to the market price at the date of the grant), and another 40% answered they used a fixed number of shares rule. Moreover, the above result may imply negative exercise prices, i.e., payments from the CEO to the firm, a practice that is not usually observed in real life.

These empirical regularities make it hard to argue for a level of sophistication as the one suggested in the previous proposition. Although we do not explicitly model the trade–off that is behind the simplification of real life option grant awards, in the next sections we restrict to more realistic cases by restricting the Options Scheme to take a very simple form.

4.1 Analysis of a One Option Grant

In order to gain some analytical insight on the role and determination of the individual instruments of the scheme, we analyze a payment scheme including only one option grant. A One – Option Scheme is a vector:

$$\{c_0, T, c_1, n, z\},$$

where c_0 is the constant consumption for the CEO for every period before the time at which options are exercised (his base salary); T is the time at which options are exercised, n is the number of stock options granted (which, for the rest of the paper, is assumed to take only positive values); z is the price at which the stock can be bought if the options are exercised; and c_1 is his consumption at T without the potential profit from the exercise of the options (his base salary at the time of exercise).

This scheme is a major simplification in several ways. First, all the stock options share the same exercise time, while in reality we observe that some proportions of the same stock options award become available for exercise at different times. Second, the base wage is constant, whereas usually stock grants are complemented with bonus plans based on accounting measures. As pointed out by Holmström (79), any informative signal should be included in an optimal compensation scheme. It is reasonable to think that the accounting measures in which bonus payments are made contingent on contain information not included in stock prices, and vice versa. Hence, the coexistence of the two variable compensation instruments can, in principle, be part of an optimal compensation scheme in the presence of agency problems. Here, we concentrate on the information contained in stock prices and the implications it has for the characteristics of option grants. We are assuming for our analysis that incentives are provided exclusively through option granting. This simple framework allows us to learn about the trade–off present in the optimal use of stock options as compensation instruments.

Some properties of a one–option grant can be pointed out right away.

Remark 5 *The optimal scheme will always have a positive number of stock options and the exercise price will be such that the options will be exercised after at least one of the possible histories of profits.*

This property is necessary to satisfy the Incentive Constraint when using only a one option grant.[6] Note as well that our option grant could have a zero exercise price, so that in fact it would be in the form of restricted stock.

Remark 6 *The consumption of the manager is bounded below, since he always has the right to not exercise the option if the stock price is below the exercise price.*

[6]If accounting measures are not a sufficient statistic for stock prices, however, this would be true even in a more general compensation scheme including, for example, a bonus program.

This is, in fact, an important limitation of the design of the contract. It is this fact that motivated us to include an assumption of limited liability of the CEO in the previous section.

Remark 7 *In an Options Scheme, the utility of the CEO is always weakly monotonic in the profit of the firm; i.e., after a given history y^t, whenever the observation at time $t+1$ is a high outcome, the manager is given higher utility than when the realization is low.*

In the payment with options, the designer of the contract is limited to payments that are spannable by changes in the stock price, after information on the firm's output is released. Traders in the market are only trying to learn about the type of the firm, and the effort level is taken to be the equilibrium one. Hence, as stated in a previous remark, prices are weakly increasing in output. This is a key departure from the Second Best contract, where under some parameters the new evidence about the effort level chosen and the firm type can give rise to non-monotonicities, as established in Prop. 2. Since the non–monotonic cases are rare, however, for the rest of the analysis in the paper we rule out the parameter specifications for which consumption in the optimal contract is non monotonic in output, unless specifically noted.

We can split the problem of finding the optimal One–Option Scheme into two stages. First, we find the values of $c_0(T)$, $c_1(T)$, $n(T)$ and $z(T)$ that implement the high effort at the minimum cost, for a given T. Given this partial solution, we can then calculate the cost associated with each T, which we denote by $K_1(T)$. The optimal exercise time is the one with the smaller cost. In order to keep the notation as simple as possible, denote the possible histories at T by $\{y_i\}_{i=1}^{2^T}$, and their probability by $\{p_i\}_{i=1}^{2^T}$ for the high effort, and $\{\widehat{p}_i\}_{i=1}^{2^T}$ for the low effort. Similarly, denote $\{x_i\}_{i=1}^{2^T}$ all the possible stock prices at T. For a given T, let $\gamma_0 = \frac{1-\beta^{T-1}}{1-\beta}$ be the weight of the payments that occur up to the time T, when the options are exercised. Similarly, $\gamma_1 = \frac{\beta^{T-1}}{1-\beta}$ is the weight of payments at T.

$$K_1(T) \quad \min_{c_0,c_1,n,z} \gamma_0 c_0 + \gamma_1 \left\{ c_1 + \sum_{i\in\Gamma(z)} p_i n (x_i - z) \right\} \tag{5}$$

$$s.t.$$

$$\underline{U} \geq \gamma_0 u(c_0) + \gamma_1 u(c_1) F(z) + \gamma_1 \sum_{i\in\Gamma(z)} p_i u(c_1 + n(x_i - z)) - e_H$$

$$\begin{aligned} & \gamma_1 u(c_1) F(z) + \gamma_1 \sum_{i\in\Gamma(z)} p_i u(c_1 + n(x_i - z)) - e_H \\ \geq \; & \gamma_1 u(c_1) \widehat{F}(z) + \gamma_1 \sum_{i\in\Gamma(z)} \widehat{p}_i u(c_1 + n(x_i - z)) - e_L \end{aligned}$$

where

$$\Gamma = \left\{ y^T : x(y^T) > z \right\}.$$

In the second stage of the problem, the optimal exercise time is determined as follows:

$$T^* = \arg\min_T K_1(T),$$

and correspondingly, $c_0^* = c_0(T^*)$, $c_1^* = c_1(T^*)$, $n^* = n(T^*)$ and $z^* = z(T^*)$.

The limitations of the one-option contract with respect to the Second Best are two. On one hand, consumption can only be made contingent on realizations after time T, and it is constant before that.[7] On the other hand, at time T, the payments are constrained to be history-dependent in a very specific way: they are an affine function of the difference between the market price assigned to the given history and the exercise price. In the Second Best, all the information contained in the history of realizations is used at each point in time, and consumption depends on the histories in an arbitrary way.

Both limitations of the Options Scheme would be present, although to a lesser extent, in a more general compensation scheme including more instruments. We observe a limited number of different vesting times in real life schemes; if these limitations are partially exogenous to the incentive problem (for example, simplicity needed for transparency concerns, or tax deduction considerations), the tensions presented here should be present in real life compensation schemes.

In order to analyze the roles of both limitations separately, it is useful to look at a problem which is constrained in information in the same way as the Options Scheme, but which can determine contingent consumption at T in an unrestricted way. We refer to this problem as the period–T optimal contract.

The period–T compensation scheme consists of a constant consumption $\widetilde{c}_0$ to be delivered to the agent until time $\widetilde{T}$ and a set of contingent consumption levels at time $\widetilde{T}$, $\widetilde{c}_i$. We can split the problem into the same two stages as before; taking $\widetilde{T}$ as given, we first find $\widetilde{c}_0\left(\widetilde{T}\right)$, and $\left\{\widetilde{c}_i\left(\widetilde{T}\right)\right\}$ that solve the following cost minimization problem:

$$\widetilde{K}\left(\widetilde{T}\right) = \min_{\widetilde{c}_0, \{\widetilde{c}_i\}} \gamma_0 \widetilde{c}_0 + \gamma_1 \sum_i \widetilde{c}_i p_i \tag{6}$$

$s.t.$

$$\underline{U} = \gamma_0 u(\widetilde{c}_0) + \gamma_1 \sum_i u(\widetilde{c}_i) p_i - e_H$$

[7] The problem could potentially be generalized for a higher number of exercise times. Since we are not modelling here the trade–off that would endogenize this variable, we choose to present the simplest case for the analysis.

$$e_H - e_L = \gamma_1 \sum_i u\left(\widetilde{c}_i\right)\left(p_i - \widehat{p}_i\right)$$

It is possible to characterize the solution to this problem, which constitutes a useful benchmark for analyzing the limitations imposed by the affine structure of the options when trying to optimally allocate consumption over the different histories at T. The optimal consumption in the period–T problem satisfies the following set of first order conditions:

$$\frac{1}{u'\left(c_0\right)} = \lambda$$

$$\frac{1}{u'\left(c_i\right)} = \lambda + \mu\left(1 - \frac{\widehat{p}_i}{p_i}\right) \qquad \forall c_i.$$

The intuition for this solution parallels that of the Second Best scheme characterized in Prop. 1: histories with lower likelihood ratios receive higher payments than histories with high likelihood ratios, except that here we only look at histories of length T. The spread of consumption levels at time T and the concavity of the CEO's utility function determine the constant wage c_0, since from the FOC's we have

$$\frac{1}{u'\left(c_0\right)} = \sum_i p_i \frac{1}{u'\left(c_i\right)}. \tag{7}$$

This condition was derived by Rogerson (1985) in a two period repeated moral hazard problem. Here, it implies that the expected wage of the CEO at the time of exercise will be higher than his base salary, c_0, if the inverse of his marginal utility is concave. It also implies that c_0 is always higher than the lowest consumption established by the contract at T.

The main lesson that we take from this benchmark model is that, ideally, the Options Scheme would deliver big profits from exercising the options when a history associated with a low likelihood ratio is realized, while keeping the benefits low (maybe setting a high enough exercise price so the option cannot be exercised at all) when the corresponding likelihood ratio is high. In the following subsections, we propose very simple and stylized examples that illustrate the roles of the exercise time and price, the number of options and the constant consumption, in tailoring incentives to the likelihood ratios ordering.

4.1.1 Determining the Exercise Time

The main trade–off that the Options Scheme presents when determining the optimal time of exercise are mostly all present in the period–T Second Best scheme. Like in the unconstrained case, waiting longer increases information quality. Higher exercise time means longer histories to base the compensation on, so more extreme values for the likelihood ratios. At the same time, delaying incentives is expensive. Delaying means that the same spread in utilities has to be delivered in less periods, so more variation overall is needed, increasing the risk premium. This requires also a higher constant wage up to the period when incentives are given, following the intuition in eq.7 – the inverse of the marginal utility evaluated at c_0 equals the expected inverse of the marginal utility of contingent consumption at T. The optimal time of exercise is one for which the marginal benefit from the increase in information when waiting one more period does not overcome the increase in cost associated with the higher variability in the continuation utility.

This intuition can be formalized for the period– T scheme. Assume the CEO has utility function $u(c) = 2\sqrt{c}$. Using this functional form, we can find explicit solutions for the multipliers of the PC (λ) and the IC (μ) in problem 6:

$$\lambda = \frac{\underline{U} + e_H}{2}(1-\beta)$$

$$\mu = \frac{e_H - e_L}{2\gamma_1}\frac{1}{v_T},$$

where v_T is the variance of the likelihood ratio distribution at time T.[8] Consumption in the optimal period–T scheme will be as follows:

$$c_0^* = \left(\frac{\underline{U} + e_H}{2}\right)^2 (1-\beta)^2$$

$$c_i^* = \left[\frac{\underline{U} + e_H}{2}(1-\beta) + \frac{e_H - e_L}{2\gamma_1}\frac{1}{v_T}\left(1 - \frac{\widehat{p}_i}{p_i}\right)\right]^2.$$

After some algebra, we get the following expression for the cost of the optimal contract:

$$\begin{aligned} \widetilde{K}^*\left(\widetilde{T}\right) &= \gamma_0 \widetilde{c}_0^* + \gamma_1 \sum_i \widetilde{c}_i^* p_i && (8) \\ &= \left(\frac{\underline{U} + e_H}{2}\right)^2 (1-\beta) + \left(\frac{e_H - e_L}{2\gamma_1}\right)^2 \frac{1}{v_T}. && (9) \end{aligned}$$

The last step is to select $\widetilde{T}^*$, which is the $\widetilde{T}$ that has the smallest $\widetilde{K}^*\left(\widetilde{T}\right)$. Note that there are two elements in 8 that depend on T : the weight of payments at the time of exercise, γ_1, and v_T. Since $\gamma_1 = \frac{\beta^T}{1-\beta}$, we know $\gamma_1 \in [0,1]$ and that it is decreasing in T. This force makes cost increase with T, and agrees with the intuition that the longer we wait to provide incentives the more variation in consumption will be necessary to implement high effort; this translates into a higher marginal cost of incentives, μ, and ultimately in higher cost. The second element that varies with the time of exercise, v_T, could be increasing or decreasing in T. In an i.i.d. output case, this variance can be shown to

[8] Each possible history at T, y_i, has a likelihood ratio associated, $LR(y_i)$ .The distribution function over the likelihood ratios is constructed by attaching to each value $LR(y_i)$ a probability equal to that of the histories that have it associated.

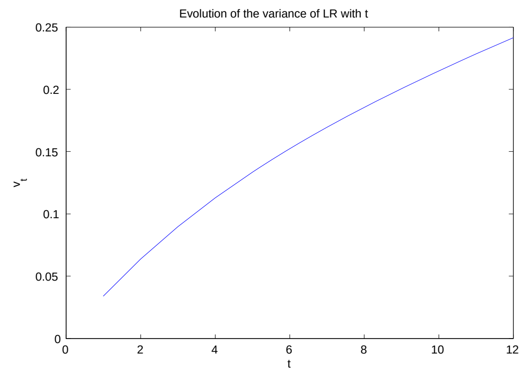

Figure 2: Example: $\pi_B = .6$, $\widehat{\pi}_B = .55, \pi_G = .85$, $\widehat{\pi}_G = .75$, $q_0 = .6$

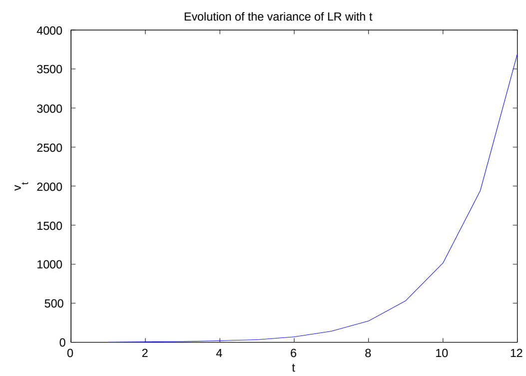

Figure 3: Example: $\pi_B = .8$, $\widehat{\pi}_B = .41, \pi_G = .85$, $\widehat{\pi}_G = .45$, $q_0 = .6$

be increasing (see Hopenhayn and Jarque (2006) for details). In the presence of learning, however, the result does not follow through. Numerical simulations suggest v_T will generally be increasing, although it could be a concave or convex function of time (Fig. 2 and 3 present two examples of these situations). Assuming v_T is always increasing with T, this force goes against the one coming from γ_1. The optimal period $-T$ contract determines $\widetilde{T}^*$ by balancing the benefits of waiting for richer information with the cost of delaying incentives.

Another issue associated with the Options Scheme is the fit of the price behavior of the period in question with respect to the likelihood ratios of the histories. As we argue in the next subsection, this will determine the solution for a given exercise time, and thus, will imply different costs.

4.1.2 Providing Incentives at T

As mentioned in the above discussion of the period–T model, the choice of the option grant parameters is aimed to match low-likelihood-ratios-histories with high profits, and high-likelihood-ratios-histories with low profits. Recall that we assume that the parameters are such that likelihood ratios are monotonic in outcomes. As established in Prop. 7, the compensation of the CEO under the Options Scheme is also monotonic in the outcome of the firm. Market prices increase with the number of high realizations because high profits increase the posterior of the good type. As for likelihood ratios, observing an extra good outcome lowers the likelihood ratio because the probability of a better history is higher under the high level of effort than under the low effort. The increase of the price for an extra high realization is not necessarily commensurate to the corresponding decrease in the likelihood ratio. The problem that the design of the option grant must address is precisely that of transforming the change in market prices into a change in compensation that matches, as much as possible, the increase in consumption that the likelihood ratios would recommend.

In our model, prices are determined endogenously; in order to get a clear picture of the role of each instrument in the mapping from prices to consumption, we construct the simplest possible examples that capture the effect of the independent behavior of prices and likelihood ratios. In particular, we take prices to be exogenously and arbitrarily assigned to histories, and we pose two different specifications for the likelihood ratio structure. In section 4.1.3 we provide some numerical examples for an optimal one–option grant in which prices are endogenously determined from primitives, and we corroborate the intuition described in this section.

The following examples show how the use of options with a positive exercise price can sometimes be more efficient than a simple linear compensation, such as restricted stock grants. For a given price structure, the number of options granted and the constant consumption c_1 affect the change in consumption for a given increase in market prices: a high n reinforces the increase determined by the market, while a low one combined with higher c_1 forces consumption to be more even across histories with different market prices. The exercise price, however, adds flexibility to the linear compensation: if likelihood ratios of histories with different low market prices are similar, an exercise price above those prices allows the Options Scheme to set consumption to be equal in those states, matching closely the optimal incentive scheme. In a linear compensation scheme, this effect could be achieved only by lowering n, but this would imply a flat profile for all histories. Using the exercise price, consumption can be flat for histories with low market price without compromising steep compensation for high prices, which can be achieved by choosing a high n.

Two Outcome Example Let the possible prices at a given exogenous T be x_L and x_H, with $x_L < x_H$, and the corresponding probabilities be p and $1-p$. For notational convenience, define

$$w_i = c_1 + n \max\{0, (x_i - z)\}, \quad \text{for } i = L, H,$$

i.e., w_i is the implied consumption under the Options Scheme, for a given outcome realization i, at the time of exercise. The choice of the exercise price influences the compensation in two ways. First, it determines the benefit per option exercised, through the difference $(x_i - z)$. Second, it determines the set of states for which the option is exercised: when $z < x_L$ the option is always exercised, while for $x_L < z < x$, it is only exercised when the market price is high. As it turns out, the principal is indifferent between any $z \in [0, x_H)$.

Proposition 8 *For a two-outcome case, the cost of the Options Scheme does not change with the exercise price, and it is equal to the cost of the period–T optimal scheme.*

Proof. In a two outcome scheme, with the three instruments available in the Options Scheme, the principal can replicate exactly the two consumption levels implied by the period–T optimal scheme. Any triple $\{c_1, n, z\}$ that solves the system

$$\begin{aligned} c_L &= c_1 + n(x_L - z) \\ c_H &= c_1 + n(x_H - z) \end{aligned}$$

implements exactly the optimal consumption. Since there is one extra instrument, for any $z \in [0, x_H)$ we can find c_1 and n that give c_L and c_H. ■

Three Outcome Example Now let there be three possible prices at a given exogenous T : x_L, x_M and x_H, with $x_L < x_M < x_H$, with corresponding probabilities p_L, p_M and p_H summing up to one (e.g., period 2 in the binomial output case analyzed). Choosing z, we determine the set of states for which options are exercised. There are three possible cases: $z < x_L$, so options are always exercised, $x_L < z < x_M$, or $x_M < z < x_H$.

Claim In analyzing the three outcome case, we can restrict attention to prices $z \in [0, x_M]$.

Proof. We can rule out $z \in (x_M, x_H]$ from the analysis since the cost of the contract for any z in this interval is the same as the cost for $z = x_M$. To see this, note that when z is in this range the option is only exercised in the high state, so

$$\begin{aligned} w_L &= w_M = c_1 \\ w_H &= c_1 + n(x_H - z). \end{aligned}$$

As in the two outcome case analyzed in the previous subsection, the three instruments available let us span any pair of consumption levels. Since in this rage of possible exercise prices we are constrained to pool together the low and the medium states, the pair of consumption levels that minimizes the cost of implementing the high level of effort is the solution to the period–T optimal scheme for a two outcome case in which the combined state of low and medium happens with probability $p_L + p_M$. The cost minimizing solution for n and c_1, for any given $z \in [x_M, x_H]$, is such that it implements the same $w_L = w_M$ and w_H; thus, the cost of the contract is constant in this range. ■

We now show, with two examples, that the solution for the optimal z is not trivial. Depending on the way that likelihood ratios change across outcomes, having a positive exercise price may constitute a cheaper compensation scheme than having the wage be just a linear function of the firm's value.

Example 9 *Let $p_L, p_M, \widehat{p}_L$ and $\widehat{p}_M$ be such that the likelihood ratios of the low and the medium price take the same value:*

$$\frac{\widehat{p}_L}{p_L} = \frac{\widehat{p}_M}{p_M} > \frac{\widehat{p}_H}{p_H}.$$

Looking at the FOC's of the three outcome period–T optimal contract, we see that equal likelihood ratios imply that consumption levels at x_L and x_M should be equal at the optimum. Take $z = x_M$; this exercise price equates consumption at the low and medium realizations: $w_L = w_M$. In turn, this reduces the problem to a two outcome problem, and as we saw before, c_1 and n are chosen to get $w_L = w_M = c_L = c_M$ and $w_H = c_H$. No other Options Scheme can do better than the proposed one.

In this example, any options contract with $z < x_M$ would have a higher cost of implementing high effort. As we see in the next example, however, there exist cases in which the optimal exercise price is lower.

Example 10 *Let $p_M, p_H, \widehat{p}_M$ and $\widehat{p}_H$ be such that the likelihood ratio of the medium and the high price take the same value:*

$$\frac{\widehat{p}_L}{p_L} > \frac{\widehat{p}_M}{p_M} = \frac{\widehat{p}_H}{p_H}.$$

Assume, by means of contradiction, that $z = x_M$. The IC of the problem can be written as:

$$e_H - e_L \leq \left(1 - \frac{\widehat{p}_H}{p_H}\right)\left[p_H u\left(c_1 + n\left(x_H - z\right)\right) + p_M u\left(c_1\right)\right] + \left(1 - \frac{\widehat{p}_L}{p_L}\right) p_L u\left(c_1\right).$$

As it is evident from this rearrangement of the IC, incentives are given in this case by differentiating the expected consumption of the agent in the states in which he exercises the option (whenever the high or the medium price occur) and consumption in the low realization case, when he only consumes the constant wage. Define:

$$\overline{u} \equiv p_H u\left(c_1 + n\left(x_H - z\right)\right) + p_M u\left(c_1\right).$$

Consider the following deviation: set a lower price of exercise $z = x_L$ and decrease the number of options granted as much as needed so that the expected utility delivered to the agent whenever he exercises the option stays equal to $\overline{u}$. Call this new number of options n':

$$p_H u\left(c_1 + n'\left(x_H - x_L\right)\right) + p_M u\left(c_1 + n'\left(x_M - x_L\right)\right) = \overline{u}.$$

For a given c_1 this change in z and n does not affect the PC and the IC:

$$\underline{U} = \overline{u} + p_L u\left(c_1\right) - e_H$$

$$e_H - e_L \leq \left(1 - \frac{\widehat{p}_H}{p_H}\right)\overline{u} + \left(1 - \frac{\widehat{p}_L}{p_L}\right) p_L u\left(c_1\right).$$

As for the cost of this alternative payment scheme, we can show that it is lower than the one with $z = x_M$. By lowering the exercise price the scheme is shifting consumption from the high realization to the medium. As it is obvious form the definition of $\overline{u}$ and by the concavity of the utility function of the agent, marginal utility of consumption is higher in the medium state than in the high one. This implies that, under the more even distribution of utility implied by the pair $z = x_L$ and n', the cost in terms of consumption of achieving $\overline{u}$ is lower. This implies that $z = x_M$ could never be optimal.[9]

In light of the second example, the intuition for the high exercise price in the first example follows the same logic. In that case, the likelihood ratios of the first two observations are equal, so they should be grouped in the same interval, either in the exercise set or in the no exercise set. Given that they are the two lowest values, the optimal contract makes them both part of the non-exercising set. In this case, any $z < x_M$ would introduce a difference in consumption between the two states, making more expensive the delivery of the necessary average utility, as established by the incentive problem. This is what drives the exercise price up. More generally, these examples suggest that when likelihood ratios are a decreasing but concave function of the stock price, the exercise price should be higher; when they are convex, the exercise price should be lower. We will return to this point as we examine numerical results below.

[9] In fact, for a non empty set of parameter values the optimal exercise price can be zero. In order to satisfy the IC and the PC with $z = 0$ the new number of options n'' has to satisfy both

$$\pi_H u\left(c_1 + n'' p_H\right) + \pi_M u\left(c_1 + n'' p_M\right) = \overline{u}$$

and $c_1 - n'' p_L \geq 0$ since there is a nonnegativity (more generally, a limited liability) constraint on consumption. Small p_L relative to p_M, very concave utility functions, small $\frac{\widehat{\pi}_L}{\pi_L}$ or $\pi_M >> \pi_H$ all favor exercise prices close to zero, since they make $c_1 - n'' p_L \geq 0$ slacker.

4.1.3 Numerical Examples

The analysis of the previous subsections looked at changes in the choice for T, or for z, for given values of the other variables, prices and likelihood ratios. With the results of that partial analysis in mind we go back to the one–option grant and use numerical examples to illustrate that the intuition provided by our previous analysis prevails when deriving $\{c_0, T, c_1, n, z\}$ jointly as the outcome of the cost minimization problem in 5. To corroborate the intuition derived from those examples, we consider cases in which the graph of likelihood ratios as a function of stock prices has a concave or a convex shape. The parameters used in these examples are given in Table 4.1.3.

$\overline{T}$	β	$\underline{U}$	e_H	e_L	π_G	π_B	$\widehat{\pi}_G$	$\widehat{\pi}_B$
9	.96	.85	3	2.5	.8	.4	.6	.3

Parameters used in the numerical examples.

In our first example we set the prior $q_0 = .9$.[10] The specification of the parameters satisfies the MLRP, so the Second Best scheme is monotonic in the number of high outcomes of the history. The optimal time of exercise is $T = 8$. In Figure 4 we graph the likelihood ratio of the nine possible histories at period 8 with respect to their corresponding market prices. Prices are monotonic in output, so the lowest corresponds to a history of eight low outcomes and the highest to one of eight high outcomes. The exercise price is 14.1, in between the fifth and fourth highest market prices. It is clearly at the high end of possible stock prices. Notice that, as in example 9, likelihood ratios are concave almost everywhere with respect to stock prices.

Now consider the same numerical example with a different prior for the good state, $q_0 = .5$ instead of $q_0 = 0.9$. The optimal time of exercise is period 6. When we graph the likelihood ratio of each of the possible histories at $T = 6$ against the market prices, in figure 5, we observe that the curve is now convex for the most part. The optimal exercise price ($z = 8.5$) is very close to the second lowest market price, so the options are exercised almost in every state. This confirms our previous intuition: the exercise price is higher in the concave case and lower in the convex one.

It is interesting to examine why this change in priors affects in such a way the shape of the likelihood ratios function. The change in prior induces a change in both the likelihood ratios associated with each outcome and in market prices. When the prior changes the relative importance of the learning effect and the effort effect (how the likelihood ratio changes with observations) changes. In an extreme case with no learning about the type of the firm, the likelihood ratio would vary and the market price would take only one value: the graph would be a vertical line. If, instead, the

[10] Solution: $c_0 = 1.2227$, $n = 2.5436$, $z =$14.0776, $T = 8$.

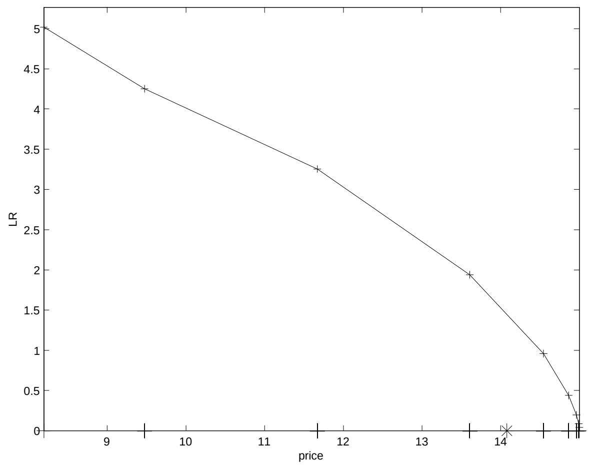

Figure 4: Likelihood ratios plotted against stock prices and the optimal exercise price ($q_0 = 0.9$).

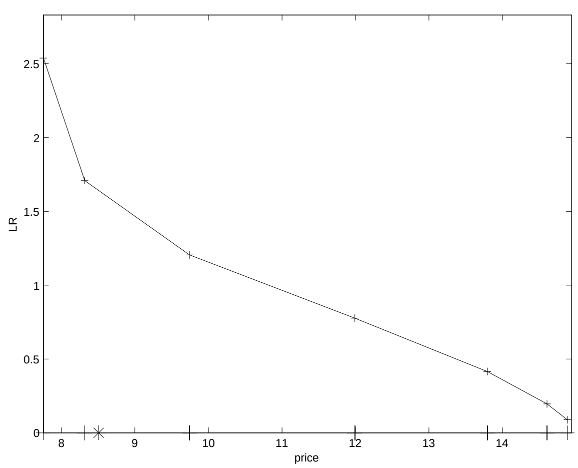

Figure 5: Likelihood ratios plotted against stock prices and the optimal exercise price ($q_0 = 0.5$).

probabilities were equal across efforts in both states, there would be no learning about the effort and the graph would be a flat line. In between cases determines the final shape of the graph of likelihood ratios against prices.

In our example, the relative importance of the two effects is determined by the value of the prior. For example, take a low price market of the firm; if the history was to have an extra high realization, for the case of $q_0 = 0.9$ that would imply a strong learning effect that would drive prices up. Note that lowering the posterior equivalent to giving more weight in the likelihood ratio to the probabilities of the bad type of the firm (π_G =0.4, $\widehat{\pi}_G = 0.2$); for these probabilities, the informational content about effort of the CEO is weaker than for the bad type (π_B =0.7, $\widehat{\pi}_B = 0.6$), since the likelihood ratio of the good outcome is the same in both states but it is much higher for the bad outcome in the good state.

Recall that optimal consumption in the period–T optimal contract is ordered according to likelihood ratios. For simplicity consider the case of logarithmic utility, for which consumption in the period–T scheme is linear on the likelihood ratios:

$$c\left(y^T\right) = \lambda + \mu\left(1 - LR\left(y^T\right)\right).$$

The final consumption of the agent under the Options Scheme, although it tries to replicate the optimal scheme defined by the ordering and values of the likelihood ratio, is constrained to be spanned through the market prices. In particular, it is linear on prices:

$$w\left(y^T\right) = c_1 + n\left(x\left(y^T\right) - z\right).$$

the Options Scheme can prevent that the increase in prices translates into higher consumption of the CEO by setting a high enough exercise price. This implies that, in our numerical example, when the prior decreases the optimal options contract should, first, reward lower prices (lower exercise price), and second, lower the difference in consumption among the high states (lower n). The first point is due to the fact that now low outcomes are not associated with such large likelihood ratios, and at the same time the ratio of the increase in market price due to an extra high outcome for a poor history with respect to a high history is lower under the low prior; the two effects reinforce each other. As for the second point, prices under the lower prior are more different for histories that are similar in terms of their likelihood ratio (good histories), so the sensitivity of consumption to changes in prices (n) should be lower.

4.2 Optimal Option Scheme: a Two Period Example

The analysis of compensation schemes containing options in the previous sections was limited to schemes including only one option grant; this provides with a clear understanding of the role of each of the elements in an option grant. Ultimately, we would like to derive the best possible compensation scheme using as many options as needed, and then evaluate its performance against the Second Best. Although finding the optimal option scheme this is a complicated problem in general, in this section we study a two period version of our problem and we provide an example in which the Second Best contract can be implemented though option grants. Moreover, we examples in which a negative exercise price is necessary to provide incentives in an efficient manner.

Assume the CEO has $u(c) = 2\sqrt{c}$ and $\overline{T} = 2$. Denote by p_1 the probability of the high outcome in the first period:

$$\begin{aligned} p_1 &= \Pr{}_1 (y_H | e_H) \\ \widehat{p}_1 &= \Pr{}_1 (y_H | e_L) . \end{aligned}$$

Denote the prior over the good type as q_0, and the posterior following realization y_i as q_{1i}. Denote the second period probability by

$$\begin{aligned} p_{2i} &= \Pr{}_2 (y_H | y_i, e_H) \\ \widehat{p}_{2i} &= \Pr{}_2 (y_H | y_i, e_L) . \end{aligned}$$

First, we find the Second Best scheme, to then show the implementation through options. For simplicity, assume no discounting and also that the limited liability constraint does not bind. The solution to the two period problem of the principal is characterized in Prop. 1. From the First Order conditions of the problem, we get an explicit solution for the multipliers:

$$\lambda = \frac{(\underline{U} + e)}{4}.$$

$$\mu = \frac{e}{2} \frac{1}{v_1 + v_2},$$

where v_1 and v_2 are the variances of the likelihood ratios in period 1 and 2, correspondingly. Using these expressions we can get the optimal consumption assigned by the Second Best contract to each history. We can see the solution graphically for a numerical example.[11] In Fig. 6 the pairs (p_L, c_L) and (p_H, c_H) are plotted, corresponding to the first period.

It is easy to see that a simple restricted stock grant, combined with a base salary, implement exactly the Second Best consumption in the first period. Let the base salary in the first period be denoted by c_1, the number of option grants by n_1 and the corresponding exercise price by z_1. Let

$$c_1 = c_L$$

[11] Parameters: $\underline{U} = 100$, $e_H = 10$, $e_L = 0$, $\widehat{\pi}_B = .1$, $\pi_B = .65$, $\widehat{\pi}_G = .75$, $\pi_G = .85$, $q_0 = .6$. Solution: $c_L = 2,6851$, $c_H = 3,1304$, $c_{LL} = 1,8060$, $c_{HL} = c_{LH} = 3,0577$, $c_{HH} = 3,1508$. Stock prices: $x_L = 0.7283$, $x_H = 0.7825$, $x_{LL} = 0.6932$, $x_{HL} = x_{LH} = 0.7413$, $x_{HH} = 0.7939$.

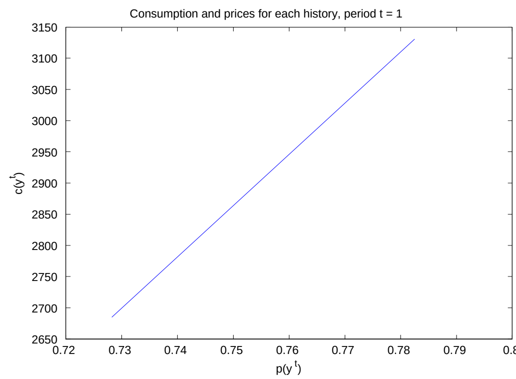

Figure 6: Example: (p_L, c_L) and (p_H, c_H)

$$n_1 = \frac{c_H - c_L}{x_H - x_L}$$
$$z_1 = 0.$$

In Fig. 7 the pairs (p_{LL}, c_{LL}), (p_{LH}, c_{LH}) and (p_H, c_H) are plotted, corresponding to the second period. The plot highlights the concavity of the compensation scheme, which is difficult to reproduce by granting restricted stock alone, which would imply a constant sensitivity of compensation to output. Using a combination of stock options, however, we can implement the Second Best consumption. Let the salary in the second period be $c_2 = c_{LL}$. Let one option grant consist of n_{21} options, at an exercise price of $z_{21} = x_{LL} + \varepsilon$, where ε is a small positive number and:

$$n_{21} = \frac{c_{LH} - c_{LL}}{x_{LH} - x_{LL}}.$$

Let a second option grant consist of n_{22} options, at an exercise price of $z_{22} = x_{HL} + \varepsilon$, with:

$$n_{21} = \frac{c_{HH} - c_{LH}}{x_{HH} - x_{HL}}.$$

Other patterns for consumption and prices correlations may arise in which implementation of the Second Best is only possible using negative exercise prices, as indicated in Prop. 4. In Fig. 8 we present an example of a concave graph; the optimal options scheme will involve a negative exercise price for options exercisable if pHH is realized.

In Fig. 9 we plot an example in which the optimal consumption is not monotonic in stock prices; again, an exact implementation of the optimal scheme violates the limited liability of CEO's. In this case, it is clear that an imperfect implementation through and option scheme that equates consumption in the LL and the LH states, and then provides exactly c_{HH} in the HH state, will

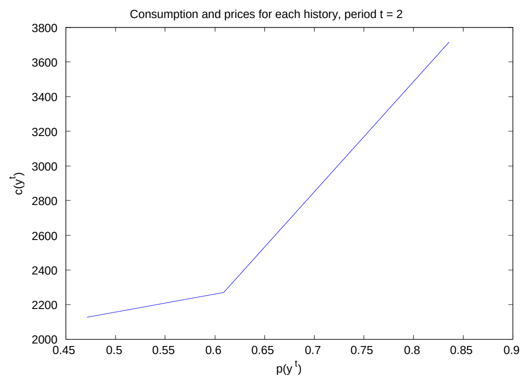

Figure 7: Example: convex graph of consumption and prices.

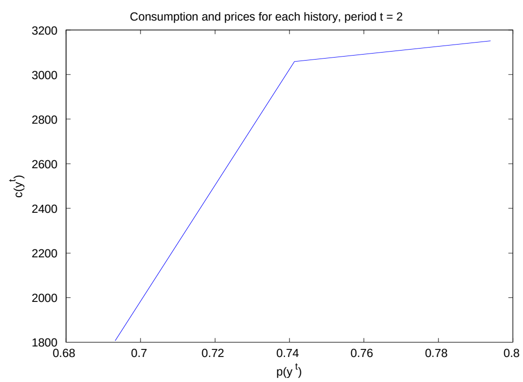

Figure 8: Example: (p_{LL}, c_{LL}), (p_{LH}, c_{LH}) and (p_H, c_H)

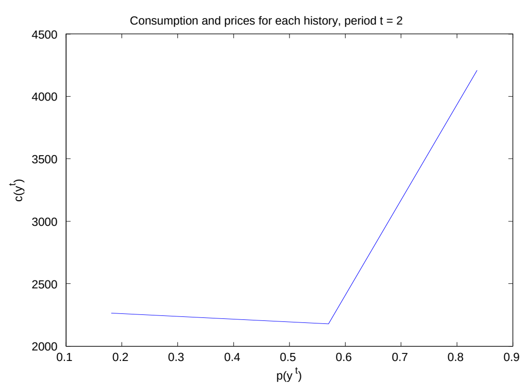

Figure 9: Example: non monotonic graph of consumption and prices.

implement high effort at a lower cost than a restricted stock scheme that is constraint to strictly increasing compensations. From our analysis of a two period problem we conclude that the intuition derived in the one-option example is still useful in the more general options scheme.

5 Conclusion

This paper studies the form of contracts used to solve the incentive problem between CEO's and the owners of a firm due to the unobservability of the manager's actions. We provide an innovative framework to evaluate the decentralization of CEO compensation through stock; we model the long lasting effects of CEO's actions and we derive the effect of effort on stock prices from primitives. Effort affects directly the conditional distribution of profits, and not the distribution of prices.

In this setup we are able to analyze the extent to which Options Schemes can be a good approximation to an optimal contract with unrestricted contingent consumption. In order for options to be used as an incentive mechanism, the price sequence for the stock of the firm $\{x_t\}$ must be sensitive to profit realizations, and thus to the effort of the CEO. In our model, we assume that buyers in the stock market understand that the owners of the firm design the compensation of the CEO in order to provide him with incentives to choose the right effort; under this assumption, the dependence of prices on profits comes only from learning about the quality of the technology of the firm through time. This points out the main characteristic of compensation schemes that include stock options: the link between effort and payoff is indirect and can make the use of options a more expensive way of providing incentives.

From the analysis of the optimal Options Scheme we conclude that there exist situations in which a compensation scheme including options with a positive exercise price performs better in terms of cost that a simple linear contract, i.e. a payment in the form of restricted stock. One example of these situations is when, for a low price market of the firm, the arrival of good news has a strong learning effect that drives prices up; if the informational content about effort of the CEO is not as strong, the Options Scheme can prevent this increase in prices to translate into higher consumption of the CEO by setting a high enough exercise price. On the other hand, the main drawback of using options is the concentration of incentives in time; setting a high exercise time constitutes a simple way of making the compensation of the manager contingent on better information, but it can raise the cost of the payment scheme due to the higher volatility of compensation needed to implement the high cost action. Big rewards are a consequence of the use of simple Options Schemes instead of more complicated wage contracts that incorporate all information available at each point in time.

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6 Appendix

Proof of Proposition 1 Using the Kuhn-Tucker theorem, the first order conditions with respect to $c_t\left(y^t\right)$ will be:

$$\frac{1}{u'\left(c_t\left(y^t\right)\right)} = \lambda + \mu\left[1 - \frac{\widehat{\Pr}\left(y^t\right)}{\Pr\left(y^t\right)}\right] + \gamma\left(y^t\right)\frac{1}{\Pr\left(y^t\right)} \qquad \forall y^t.$$

Whenever $\gamma\left(y^t\right) > 0$, we know that the limited liability constraint holds, so $c_t\left(y^t\right) = b$. For histories such that $\gamma\left(y^t\right) = 0$,

$$\frac{1}{u'\left(c_t\left(y^t\right)\right)} = \lambda + \mu\left[1 - \frac{\widehat{\Pr}\left(y^t\right)}{\Pr\left(y^t\right)}\right]$$

holds and by strict concavity of the utility function a higher value for the likelihood ratio $\frac{\widehat{\Pr}\left(y^t\right)}{\Pr\left(y^t\right)}$ implies lower consumption.

Proof of Proposition 2 (Generalization of Miller 1999) The first order conditions of the problem say:

$$c\left(y_i\right) = \lambda + \mu\left(1 - LR\left(y_i\right)\right) \quad i = L, H$$

Proof. We have that

$$LR\left(y_L\right) = \frac{q_0\left(1-\widehat{\pi}_G\right) + \left(1-q_0\right)\left(1-\widehat{\pi}_B\right)}{q_0\left(1-\pi_G\right) + \left(1-q_0\right)\left(1-\pi_B\right)}$$

$$LR\left(y_H\right) = \frac{q_0\widehat{\pi}_G + \left(1-q_0\right)\widehat{\pi}_B}{q_0\pi_G + \left(1-q_0\right)\pi_B}$$

Since

$$\begin{aligned} \pi_G &> \widehat{\pi}_G \\ \pi_B &> \widehat{\pi}_B, \end{aligned}$$

we find that, as in the standard moral hazard problem, consumption in the first period is monotonic in the outcome:

$$LR\left(y_L\right) > LR\left(y_H\right) \Rightarrow c\left(y_L\right) < c\left(y_H\right).$$

However, in the second period non–monotonicities may arise. The difference $c\left(y^t, y_H\right) - c\left(y^t, y_L\right)$ will have the same sign as $LR\left(y^t, y_L\right) - LR\left(y^t, y_H\right)$. To simplify the notation, let p and $\widehat{p}$ be the probability of the high outcome after history y^t under the high and low effort respectively. Then :

$$\begin{aligned} LR\left(y^t, y_L\right) - LR\left(y^t, y_H\right) &= \frac{\widehat{p}}{p} - \frac{1-\widehat{p}}{1-p} \\ &= \frac{p-\widehat{p}}{p\left(1-p\right)}. \end{aligned}$$

So,

$$sign\left(c\left(y^t,y_H\right)-c\left(y^t,y_L\right)\right)=sign\left(p-\widehat{p}\right).$$

$$\begin{aligned}
p-\widehat{p} &= \left[q\left(y^t\right)\left(\alpha^t\pi_G+(1-\alpha)\right)\overline{\pi}_G\right] \\
&\quad +\left(1-q\left(y^t\right)\right)\left[\alpha^t\pi_B+\left(1-\alpha^t\right)\overline{\pi}_B\right] \\
&\quad -\widehat{q}\left(y^t\right)\left[\alpha^t\widehat{p}_G+(1-\alpha)\overline{\pi}_G\right] \\
&\quad +\left(1-\widehat{q}\left(y^t\right)\right)\left[\alpha^t\widehat{p}_B+\left(1-\alpha^t\right)\overline{\pi}_B\right]
\end{aligned}$$

or, rearranging terms,

$$\begin{aligned}
p-\widehat{p} &= \alpha^t\left[q\left(y^t\right)\pi_G+\left(1-q\left(y^t\right)\right)\pi_B\right] \\
&\quad +\left(1-\alpha^t\right)\left[q\left(y^t\right)\overline{\pi}_G+\left(1-q\left(y^t\right)\right)\overline{\pi}_B\right] \\
&\quad -\alpha^t\left[\widehat{q}\left(y^t\right)\widehat{p}_G+\left(1-q\left(y^t\right)\right)\widehat{p}_B\right] \\
&\quad +\left(1-\alpha^t\right)\left[\widehat{q}\left(y^t\right)\overline{\pi}_G+\left(1-q\left(y^t\right)\right)\overline{\pi}_B\right]
\end{aligned}$$

We can add and subtract the term $\alpha^t\left[q\left(y^t\right)\widehat{p}_G+\left(1-q\left(y^t\right)\right)\widehat{p}_B\right]$:

$$\begin{aligned}
p-\widehat{p} &= \alpha^t\left[q\left(y^t\right)\pi_G+\left(1-q\left(y^t\right)\right)\pi_B\right] \\
&\quad +\left(1-\alpha^t\right)\left[q\left(y^t\right)\overline{\pi}_G+\left(1-q\left(y^t\right)\right)\overline{\pi}_B\right] \\
&\quad -\alpha^t\left[\widehat{q}\left(y^t\right)\widehat{p}_G+\left(1-q\left(y^t\right)\right)\widehat{p}_B\right] \\
&\quad +\left(1-\alpha^t\right)\left[\widehat{q}\left(y^t\right)\overline{\pi}_G+\left(1-q\left(y^t\right)\right)\overline{\pi}_B\right] \\
&\quad +\alpha^t\left[q\left(y^t\right)\widehat{p}_G+\left(1-q\left(y^t\right)\right)\widehat{p}_B\right] \\
&\quad -\alpha^t\left[q\left(y^t\right)\widehat{p}_G+\left(1-q\left(y^t\right)\right)\widehat{p}_B\right]
\end{aligned}$$

and rearrange:

$$\begin{aligned}
p-\widehat{p} &= \alpha^t\left[q\left(y^t\right)\left(\pi_G-\widehat{\pi}_G\right)+\left(1-q\left(y^t\right)\right)\left(\pi_B-\widehat{\pi}_B\right)\right. \\
&\quad \left.+\left(\widehat{\pi}_G-\widehat{\pi}_B\right)\left(q\left(y^t\right)-\widehat{q}\left(y^t\right)\right)\right] \\
&\quad +\left(1-\alpha^t\right)\left(q\left(y^t\right)-\widehat{q}\left(y^t\right)\right)\left(\overline{\pi}_G-\overline{\pi}_B\right)
\end{aligned}$$

If $q\left(y^t\right)>\widehat{q}\left(y^t\right)$, compensation will be higher after a good observation. If $q\left(y^t\right)<\widehat{q}\left(y^t\right)$, then we have to check the conditions under which the effect of learning will overcome the standard MLRP effect. ■

Remark: *Stock prices are always increasing in output.*

Proof. Given the assumptions about the conditional distribution of profit, a good outcome is always stronger evidence of a good technology, i.e., $q\left(y^t, y_H\right) > q\left(y^t, y_L\right)$. From eq. 4,

$$\begin{aligned} x\left(y^t, y_H\right) - x\left(y^t, y_L\right) &= \frac{1+r}{1+r+\alpha}\left[q_t\left(y^t, y_H\right) - q_t\left(y^t, y_L\right)\right] \\ &\times \left[\left(\pi_G - \overline{\pi}_G\right) - \left(\pi_B - \overline{\pi}_B\right)\right] \end{aligned}$$

$$+\frac{1+r}{r}\left[q_t\left(y^t, y_H\right) - q_t\left(y^t, y_L\right)\right]\left(\overline{\pi}_G - \overline{\pi}_B\right) > 0.$$

This implies that market prices will always be monotonic in profit: $x\left(y^t, y_H\right) > x\left(y^t, y_L\right)$. From the PC of the problem, the utility of the CEO is an increasing function of market prices.

Proof of Proposition 3 The Second Best contingent consumption scheme can be thought of as a set of vectors, one for each period:

$$\overrightarrow{c}_1 = \begin{bmatrix} c_1(0) \\ c_1(1) \end{bmatrix}, \quad \overrightarrow{c}_2 = \begin{bmatrix} c_2(0,0) \\ c_2(0,1) \\ c_2(1,0) \\ c_2(1,1) \end{bmatrix}, \quad \ldots, \quad \overrightarrow{c}_t = \begin{bmatrix} c_t(0,\ldots,0) \\ \vdots \\ c_t\left(y^t\right) \\ \vdots \\ c_t(1,\ldots,1) \end{bmatrix}$$

The dimension of the consumption vector at each t equals 2^t; let this dimension be denoted by $d(t)$. We need the Options Scheme to span any point in $\mathbb{R}^{d(t)}$. Denote as $w_t(\cdot)$ the consumption of the CEO under the Options Scheme at time t and following a history with a given y^t. It will be given by:

$$w_t\left(y^t\right) = \sum_{i=1}^{d(t)} \max\left\{n_{t,i}\left[x\left(y^t\right) - z_{t,i}\right], 0\right\}.$$

The set of vectors of consumption spannable with the multiple Options Scheme will be:

$$\begin{bmatrix} w_{t,1} & \ldots & w_{t,d(t)} \end{bmatrix} = \begin{bmatrix} n_{t,1} & \ldots & n_{t,d(t)} \end{bmatrix} \cdot$$

$$\begin{bmatrix} \max\left\{x\left(y_1^t\right) - z_{t,1}, 0\right\} & \ldots & \max\left\{x\left(y_{d(t)}^t\right) - z_{t,1}, 0\right\} \\ \max\left\{x\left(y_1^t\right) - z_{t,2}, 0\right\} & & \max\left\{x\left(y_{d(t)}^t\right) - z_{t,2}, 0\right\} \\ \vdots & \ddots & \vdots \\ \max\left\{x\left(y_1^t\right) - z_{t,d(t)}, 0\right\} & & \max\left\{x\left(y_{d(t)}^t\right) - z_{t,d(t)}, 0\right\} \end{bmatrix}.$$

For the result to hold, we need the rank of the matrix of payoffs equal to $d(t)$. For each t, order the market prices from lowest to biggest, where $x_i < x_i$ for $i = 1, \ldots, d(t)$. Let $z_{t,1} = 0$, and $z_{t,i} = \frac{x_i + x_{i-1}}{2}$. The resulting matrix will be

$$\begin{bmatrix} \max\{x_1 - 0, 0\} & \cdots & \max\{x_{d(t)} - 0, 0\} \\ \max\{x_1 - \frac{x_2+x_1}{2}, 0\} & & \max\{x_{d(t)} - \frac{x_2+x_1}{2}, 0\} \\ \vdots & \ddots & \vdots \\ \max\left\{x_1 - \frac{x_{d(t)}+x_{d(t)-1}}{2}, 0\right\} & & \max\left\{x_{d(t)} - \frac{x_{d(t)}+x_{d(t)-1}}{2}, 0\right\} \end{bmatrix}.$$

Given the construction of the prices, the matrix becomes:

$$\begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_d \\ 0 & \frac{x_2-x_1}{2} & x_3 - \frac{x_1+x_2}{2} & \cdots & x_d - \frac{x_1+x_2}{2} \\ 0 & 0 & \frac{x_3-x_2}{2} & \cdots & x_d - \frac{x_3+x_2}{2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{x_{d(t)}-x_{d(t)-1}}{2} \end{bmatrix},$$

which is diagonal and thus of full rank. We can always find $(n_{t,1}, \ldots, n_{t,t+1})_{t=1}^{T}$ so that $\overrightarrow{w}_t = \overrightarrow{c}_t$ $\forall t$.