SKEWNESS IN INDIVIDUAL STOCKS AT DIFFERENT FREQUENCIES*

Amado Peiró

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Correspondence to: Amado Peiró, Universitat de València, Facultad de Economía, Departamento de Análisis Económico, Campus dels Tarongers, s/n, 46022 Valencia, e-mail: Amado.Peiro@uv.es.

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ABSTRACT

This paper examines the (a)symmetry of twenty-four individual stock returns at different frequencies: daily, weekly and monthly. While some asymmetries are observed in daily returns, they disappear almost completely at lower frequencies. The explanation for this fact lies in the convergence to normality that takes place when frequency decreases. These features allow one to question several financial models; in particular, they question the preference for positive skewness as a factor for investments in stock markets.

Key words: Diversification, skewness, symmetry.
JEL classification: E32

RESUMEN

Este artículo examina la (a)simetría de las rentabilidades de veinticuatro valores individuales para diferentes frecuencias: diaria, semanal y mensual. Aunque se observan algunas asimetrías en las rentabilidades diarias, éstas desaparecen casi completamente en frecuencias menores. La explicación a este fenómeno reside en la convergencia a la normalidad que se produce al disminuir la frecuencia. Estos hechos cuestionan varios modelos financieros; en concreto cuestionan la preferencia por la asimetría positiva como un factor de inversión en los mercados de acciones.

Palabras clave: Diversificación, simetría.
Clasificación JEL: E32

I. Introduction

Traditionally, many financial models have been based on a risk-return framework, and these two properties have usually been measured by using the first two moments of the distribution of returns. However, this framework may be somewhat limited. When restricting the analysis to the first two moments, one is neglecting the importance of higher order moments, which would be reasonable only in two cases: i) when investors' utility functions are quadratic, or ii) when the distribution of returns is normal. But there is ample evidence that makes these assumptions questionable.

In an attempt to generalize many financial models and to go beyond the mean-variance framework, several researchers have considered higher order moments. In particular, the third order moment (skewness) has frequently been taken into account. Underlying many of the contributions that consider the skewness of returns is the presumption that many investors may have preference for positive skewness. This preference would also explain why many rational people take unfair gambles (Garrett and Sobel, 1999). Brennan (1979) and He and Leland (1993) have shown that if the market's portfolio rate of return has constant mean and volatility, the average investor has a power utility function. As this function has a positive third derivative, it implies skewness preference that is positively valued by investors.

Several researchers have realized the potential importance of skewness, and, consequently, have incorporated it in different financial models. In this way, Arditti and Levy (1975) build a three-parameter multi-period model, and Kraus and Litzenberger (1976) extend the capital asset pricing model to include the effect of skewness on valuation, and present empirical evidence consistent with their extension. Simkowitz and Beedles (1978) and Conine and Tamarkin (1981) explain the low diversification of many investors' portfolios by the preference for positive skewness, and Lai (1991) and Chunhachinda et al. (1997) have analysed the problem of portfolio selection taking into account the skewness of returns. Chunhachinda et al. (1997) find that the incorporation of skewness into the investor's portfolio decision substantially alters the construction of the optimal portfolio, and that investors trade expected return for skewness.

The issue of skewness in financial returns is also important for option pricing theories. The widely used Black-Scholes option pricing model frequently misprices deep-in-the-money and deep-out-the-money options. Hull (1993) has explained this anomaly, known as volatility skew, as a consequence of non-normality, and Corrado and Su (1996 and 1997) attribute this fact to the skewness and kurtosis of the returns' distribution. They find significant non-normal skewness and kurtosis implied by option prices, and show that when skewness- and kurtosis-adjustment terms are added to the Black-Scholes formula, improved accuracy is obtained for pricing options.

Finally, interest in the skewness of returns has recently increased due to several contributions that point to skewness (as an alternative to risk) as a goal for many individuals. Golec and Tamarkin (1998), in the context of horse races, and Garrett and Sobel (1999), in the context of lottery games, show that bettors pursue skewness instead of risk. Both studies explain the behavior of risk-averse individuals through their preference for positive skewness.

Parallel with the importance of skewness, lies the issue of its measurement. To measure the (a)symmetry of the returns, most researchers have used sample skewness, central third-order moment divided by the cube of standard deviation,

$$\hat{\alpha} = \frac{\sum_{t=1}^{T}\left(R_t - \bar{R}\right)^3}{T\hat{\sigma}^3}, \tag{1}$$

where R_t denotes the return in date *t*, $\bar{R}$ is the sample mean, *T* is the sample size, and $\hat{\sigma}$ is the sample standard deviation. Its asymptotic distribution, *under normality*, is given by:

$$\hat{\alpha} \rightarrow N(0,\ 6/T). \tag{2}$$

Though (1) may be a proper measure of (a)symmetry, serious problems arise when using (2) to test for symmetry. Many researchers have used the asymptotic distribution shown in (2) to test for the symmetry of financial returns, but these are really tests of normality and not tests of symmetry. As, obviously, the distribution may be symmetric though not normal, the tests may incorrectly conclude the asymmetry of returns when the parent distribution is perfectly symmetric but not normal, and there is enormous evidence since Mandelbrot (1963) of the non-normality of returns at high frequencies. As the distribution of $\hat{\alpha}$ may be very different under alternative distributions (see Peiró, 1999), tests of symmetry with (1) and (2) are greatly misleading.

With this (problematic) measure of skewness, several researchers have examined interesting topics, like the evolution of skewness over time (persistence), its magnitude at different frequencies, and its relationship with diversification. This last point is especially important, as it could help to explain a long-standing puzzle. An intriguing feature in financial economics is the low diversification of portfolios held by investors. However, this behavior could be perfectly rational if investors have preference for positive skewness, and lower-diversified portfolios present more positive skewness than higher-diversified ones. Indeed, empirical evidence suggests that this is the case. Simkowitz and Beedles (1978) observed that skewness in monthly stock returns decreases and becomes negative with diversification, and Alles and Kling (1994) find that smaller capitalized stock indices are more negatively skewed than larger stock indices. If investors crave for positive skewness, and skewness decreases with diversification, then they would rationally hold low-diversified portfolios, in sharp contrast with common financial orthodoxy.

The purpose of this paper is to examine skewness in individual stocks. To avoid the problems that involve (1) and (2), distribution-free methods proposed by Peiró (1999) will be used. These methods have shown that returns on well-diversified portfolios (stock indexes) are mostly symmetric, or, at least, do not present strong evidence of asymmetry. However, no evidence is provided on individual stocks. If these robust methods conclude that individual stocks do present asymmetry, but that it decreases or disappears with diversification, a possible explanation of the low-diversification puzzle will lie in the preference for positive skewness. To cast some light on these issues, the rest of the paper is organized as follows. Section II presents the data used, twenty-four stocks listed in the New York Stock Exchange. Section III analyses the (a)symmetry of their returns at different frequencies with several tests. Finally, Section IV summarizes the main results and conclusions.

II. Returns Series

In what follows, twenty-four Dow-Jones stocks listed in the New York Stock Exchange have been considered. These companies are shown in Appendix 1. The series of daily closing prices cover the period from 12/26/1995 to 5/25/2000 and, after excluding non-trading days, are composed of 1116 observations. Daily returns were obtained by logarithmic differences; that is by $R_{d,t} = \log\left(P_t / P_{t-1}\right)$, where $R_{d,t}$ is the return for day t and P_t is the closing price for the same day. Lower frequency returns were also considered. Five-day (weekly) returns were obtained by $R_{w,t} = \log\left(P_t / P_{t-5}\right)$, and twenty-day (monthly) returns were obtained by $R_{m,t} = \log\left(P_t / P_{t-20}\right)$. One must notice that weekly (monthly) returns are equal to the aggregation of five (twenty) daily returns. The series of daily, weekly and monthly returns are formed by 1115, 223 and 55 observations, respectively.

Table 1 shows some basic statistics on the returns of these series. According to (2), standard deviation of skewness is equal to 0.073, 0.164 and 0.330, for daily, weekly and monthly returns, respectively. Therefore, many statistics are highly significant. As said above, this fact must be understood as a rejection of normality, not as a rejection of symmetry. It is interesting to observe that the number of rejections decrease with frequency; at the 5% significance level, there are thirteen rejections with daily returns, twelve with weekly and only five with monthly. This fact suggests convergence to normality. We shall come back to this point later.

Table 1. Basic statistics

	Daily Returns			**Weekly Returns**			**Monthly Returns**		
	Mean	**Std. Dev.**	**Skewness**	**Mean**	**Std. Dev.**	**Skewness**	**Mean**	**Std. Dev.**	**Skewness**
AA	0.076%	0.021	0.459	0.381%	0.047	0.195	1.673%	0.089	0.106
AX	0.114%	0.023	0.049	0.568%	0.051	-0.279	2.282%	0.094	-1.202
BA	-0.001%	0.022	-0.480	-0.006%	0.045	-0.421	-0.032%	0.086	-0.903
BS	-0.114%	0.030	0.334	-0.569%	0.066	0.664	-1.805%	0.131	-0.301
CA	0.022%	0.023	-0.132	0.111%	0.048	-0.025	0.446%	0.092	0.011
DD	0.031%	0.021	-0.024	0.156%	0.046	-0.196	0.643%	0.090	-0.164
DI	0.061%	0.022	0.023	0.306%	0.047	0.401	1.239%	0.082	0.233
EK	-0.013%	0.019	-0.695	-0.066%	0.045	-0.024	-0.387%	0.074	-0.042
GE	0.129%	0.018	0.054	0.647%	0.041	-0.117	2.637%	0.068	-0.231
GM	0.052%	0.020	-0.001	0.261%	0.043	-0.343	1.389%	0.075	-0.323
GT	-0.053%	0.020	0.165	-0.264%	0.046	-0.243	-0.830%	0.095	-0.606
IB	0.138%	0.024	-0.354	0.688%	0.052	0.431	2.808%	0.093	-0.071
IP	-0.007%	0.022	0.181	-0.034%	0.046	0.044	0.006%	0.089	-0.396
JP	0.043%	0.021	0.018	0.215%	0.043	-0.490	0.770%	0.087	-0.583
KO	0.031%	0.020	0.032	0.154%	0.044	-0.389	0.477%	0.080	-0.736
MC	0.046%	0.019	0.094	0.228%	0.041	0.508	0.847%	0.078	0.145
MO	-0.006%	0.023	-0.186	-0.029%	0.053	-1.132	-0.448%	0.090	-0.753
MR	0.072%	0.020	-0.014	0.361%	0.044	-0.310	1.272%	0.078	-0.677
PG	0.041%	0.023	-4.265	0.205%	0.049	-3.318	0.667%	0.097	-2.858
SE	-0.003%	0.024	0.279	-0.017%	0.051	0.342	-0.085%	0.103	0.237
TX	0.031%	0.018	0.294	0.153%	0.036	-0.260	0.463%	0.054	-0.348
UK	0.030%	0.023	0.697	0.148%	0.047	0.456	0.843%	0.107	0.422
UT	0.082%	0.019	0.017	0.411%	0.041	-0.273	1.740%	0.085	-0.311
XO	0.062%	0.017	0.291	0.310%	0.034	0.023	1.215%	0.049	-0.164

III. Analysis of symmetry

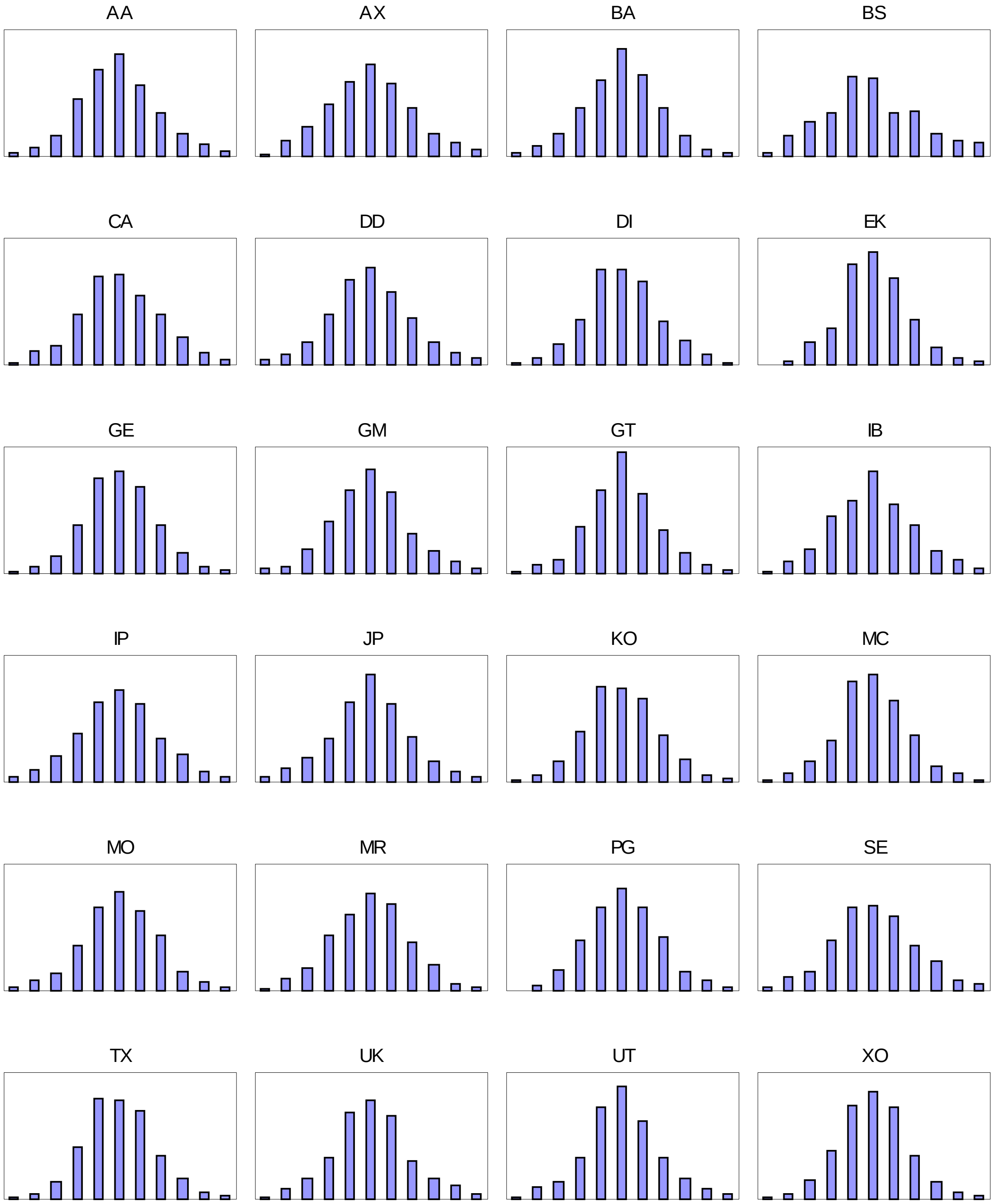
As the skewness coefficient is of little use in judging the symmetry or asymmetry of returns, one must follow alternative approaches. First, a graphic approximation will be considered. If the distribution of returns is symmetric, then the median must necessarily be the axis of symmetry, and coincides with the mean, if it exists. Then, the symmetry of returns will be reflected in the symmetry of the histogram about its mean. As there are twenty-four series, and each of them has its own mean, it is easier to subtract their own mean from the returns of each series, thus shifting the axis of symmetry to zero for all the series of excess returns. In this case, symmetry of returns would be reflected in the symmetry of the histograms of these excess returns about zero.

Figure 1 shows the histograms of daily excess returns. While some histograms seem symmetric, others present clear asymmetries. Thus, for example, AX and BA are fairly symmetric, but BS present strong asymmetries. One could judge the (a)symmetry of returns by examining these histograms, but, though these histograms provide intuitive insight, the information contained inside each interval (rectangle) is wasted away, and, most importantly, they are not at all statistical tests of symmetry. Therefore, to test for the symmetry of returns, the distribution of negative excess returns taken in absolute values will be compared with the distribution of positive excess returns taken in absolute values. If returns are symmetric, then both distributions must be equal. These comparisons may be carried out with conventional tests or with distribution-free tests.

To test for the equality of distributions, the mean and the variance of negative excess returns (henceforth, always in absolute values) will be compared with the mean and the variance of positive excess returns through the usual *t*- and *F*-tests. Table 2 shows the results of these tests. The equality of means cannot be rejected for any company at the 1% significance level, and the statistics are significant in only four cases at the 5% level.[1] Nor can the equality of variances be rejected in most companies; however, the results of the *F*-tests for eight companies (one third) suggest a different dispersion between negative and positive excess return. These are the only signs of asymmetry that can be observed with these tests.

[1] As, by construction, the sum of negative excess returns (in absolute values) is equal to the sum of positive excess returns, the test for the equality of means can also be regarded as a test for the equal number of negative and positive excess returns.

Figure 1. Histograms of daily excess returns.



In all histograms the intervals are the following: (-∞, -0.055), (-0.055, -0.045), (-0.045, -0.035), ..., (-0.005, +0.005), ..., (0.035, 0.045), (0.045, 0.055), (0.055, ∞).

Table 2. Tests of symmetry with daily returns

	t	*P*-value	*F*	*P*-value	*KS*	*P*-value	*W**	*P*-value	*ST**	*P*-value
AA	2.165*	0.031	1.538**	0.000	0.083*	0.044	1.051	0.293	2.761**	0.006
AX	0.574	0.566	1.096	0.281	0.040	0.764	0.230	0.818	0.758	0.448
BA	1.142	0.254	1.109	0.223	0.083*	0.042	2.291*	0.022	2.516*	0.012
BS	2.251*	0.025	1.537**	0.000	0.304**	0.000	5.140**	0.000	8.237**	0.000
CA	2.229*	0.026	1.019	0.826	0.098**	0.009	2.856**	0.004	0.040	0.968
DD	1.951	0.051	1.035	0.683	0.063	0.215	2.261*	0.024	-0.300	0.764
DI	0.937	0.349	1.054	0.536	0.071	0.119	1.046	0.296	0.854	0.393
EK	0.088	0.930	1.258**	0.007	0.074	0.095	0.094	0.925	2.422*	0.015
GE	0.251	0.802	1.065	0.456	0.047	0.567	0.077	0.939	1.478	0.139
GM	1.621	0.105	1.097	0.275	0.059	0.296	1.382	0.167	0.608	0.543
GT	0.031	0.975	1.118	0.189	0.079	0.062	0.523	0.601	2.228*	0.026
IB	0.697	0.486	1.003	0.970	0.050	0.488	0.296	0.767	1.108	0.268
IP	1.238	0.216	1.211*	0.024	0.097*	0.011	2.491*	0.013	2.947**	0.003
JP	0.958	0.338	1.070	0.425	0.053	0.403	0.918	0.359	-0.429	0.668
KO	1.194	0.233	1.406	0.160	0.070	0.135	1.047	0.592	-0.131	0.896
MC	2.291*	0.022	1.163	0.077	0.099**	0.009	2.532*	0.011	-0.687	0.492
MO	0.452	0.651	1.118	0.190	0.064	0.208	0.200	0.842	0.942	0.346
MR	0.463	0.644	1.022	0.801	0.051	0.452	0.450	0.653	-0.867	0.386
PG	0.132	0.895	2.628**	0.000	0.042	0.702	1.011	0.312	-0.235	0.814
SE	0.346	0.729	1.084	0.341	0.092*	0.019	0.685	0.494	2.893**	0.004
TX	1.386	0.166	1.329**	0.001	0.049	0.528	0.716	0.474	0.680	0.497
UK	1.930	0.054	1.383**	0.000	0.115**	0.001	2.182*	0.029	-1.640	0.101
UT	1.167	0.244	1.037	0.667	0.057	0.322	1.301	0.193	0.388	0.698
XO	0.255	0.799	1.294**	0.002	0.044	0.648	0.528	0.598	0.907	0.364

t is the usual test statistic for equality of means. *F* is the usual test statistic for equality of variances. *KS*, *W* and ST** are respectively the Kolmogorov-Smirnov, the standardized Wilcoxon and the standardized Siegel-Tukey two-sample test statistics for equality of distributions. * (**) denotes statistics significant at the 5% (1%) significance level. In all cases, the first sample is formed by negative excess returns, and the second sample is formed by positive excess returns.

Although *t*-tests for the equality of means are rather robust to distributional assumptions, *F*-tests are rather sensitive to these assumptions (see, for example, Stuart and Ord, 1987). Therefore, it would be desirable to corroborate all these results, especially the different dispersion observed in eight companies, with distribution-free methods. The distribution-free methods are especially suitable in these cases because the distribution of the test statistic does not depend on the specific distribution function of the population; these methods only require minimal assumptions about the underlying distribution, and, besides, do not depend to such an extent on extreme returns.

Three distribution-free methods will be used: the Kolmogorov-Smirnov two sample test, the Wilcoxon rank-sum test and the Siegel-Tukey test. These are two-sample tests, which will allow the comparison of the distributions of negative and positive excess returns. In all of them the null hypothesis establishes the equality of the populations underlying the two samples. But, while the Kolmogorov-Smirnov test is sensitive to any difference in the distribution of the two samples, the Wilcoxon rank-sum test is especially appropriate for detecting differences in location, and the Siegel-Tukey test is especially appropriate for detecting differences in dispersion (see Gibbons and Chakraborti, 1992).

In the Kolmogorov-Smirnov two-sample test, the test statistic, *KS*, is obtained by computing the maximum absolute difference between the empirical distributions,

$$KS = \max_{0<x<\infty} \left| F^{-}(x) - F^{+}(x) \right| , \tag{3}$$

where F^{-} and F^{+} are the empirical distribution functions of negative and positive excess returns, respectively. The critical values of the asymptotic distribution of *KS* under the hypothesis of equal distributions are tabulated in Gibbons and Chakraborti (1992).

In the Wilcoxon rank-sum test, the absolute values of negative and positive excess returns are combined. The test statistic, *W*, is given by the sum of the ranks of the absolute values of the negative excess returns in the ordered combined sample,

$$W = \sum_{t=1}^{T} I_t \, r\left(\overline{R} - R_t\right), \tag{4}$$

where

$$I_t = \begin{cases} 1 & \text{if } R_t < \overline{R} \\ 0 & \text{if } R_t > \overline{R}, \end{cases} \tag{5}$$

and $r(\cdot)$ is the rank operator. Under the null hypothesis of equal distributions, the asymptotic distribution of *W* is given by

$$W \to N\left(\frac{T_1 (T+1)}{2}, \ \frac{T_1 \, T_2 (T+1)}{12} \right), \tag{6}$$

where T_1 is the number of negative excess returns (first sample), T_2 is the number of positive excess returns (second sample) and $T_1 + T_2 = T$.

In the Siegel-Tukey test the absolute values of negative and positive excess returns are also combined and ordered. The test statistic is

$$ST = \sum_{t=1}^{T} I_t w_t \,, \tag{7}$$

where I_t is defined as in (5); that is, I_t has value 1 if the place t in the ordered combined sample is occupied by an observation coming from the first sample (negative excess returns), and 0 otherwise. If T is even, the weights are

$$W_t = \begin{cases} 2t & \text{if} \quad t \leq \dfrac{T}{2}, \quad t \text{ even} \\ 2t\text{-}1 & \text{if} \quad t \leq \dfrac{T}{2}, \quad t \text{ odd} \\ 2(T-t)+2 & \text{if} \quad t > \dfrac{T}{2}, \quad t \text{ even} \\ 2(T-t)+1 & \text{if} \quad t > \dfrac{T}{2}, \quad t \text{ odd} \end{cases} \tag{8}$$

If T is odd, then the middle observation is dropped, and these weights are applied to the resulting number of observations. Thus, the lower weights are assigned to the extremes, and the higher weights to the middle of the ordered combined sample. Under the null hypothesis of equal distributions, the asymptotic distribution of ST is the same as that of W,

$$ST \rightarrow N\left(\frac{T_1(T+1)}{2}, \ \frac{T_1\,T_2(T+1)}{12}\right). \tag{9}$$

The results of these distribution-free tests with daily returns are shown in Table 2. While the Kolmogorov-Smirnov tests lead to the rejection of the hypothesis of equal distributions in eight cases (four of them at the 1% significance level), the Wilcoxon- and the Siegel-Tukey tests lead to the rejection in seven cases each. There is a sound relation between these different tests. In the eight cases where symmetry is rejected with the KS tests, at least one of the W and ST tests also leads to the rejection of symmetry. On the other hand, the four rejections in the t-tests are also rejected in the KS test, and three of them are also rejections in the Wilcoxon tests (especially sensitive in detecting differences in location). The general conclusions that follow from this table is that daily returns present clear asymmetries in some cases, but asymmetry does not seem to be a ubiquitous and common feature of daily returns.

With weekly returns, the results are rather different. Table 3 shows the results obtained with returns generated over a period of five trading days (weekly returns). Neither of the *t*-statistics, and only one of the seventy-two distribution-free tests, allows the rejection of symmetry of weekly returns (the *P*-value in the Siegel-Tukey test for EK is equal to 2.8%). The *F*-tests yield several rejections of symmetry, but, as said above, these tests seem to be of little value, given their sensibility to normality. The results are similar with monthly returns (generated over a period of twenty trading days). The *t*-tests never allow the rejection of symmetry of returns, and the distribution-free tests allow the rejection only in four out of seventy-two cases (in three out of four only at the 5% significance level). Two clear messages arise from these tables: i) though there are some relatively slight asymmetries in daily returns, asymmetry does not seem to be a pervasive characteristic of daily returns, and ii) the (relatively weak) asymmetries observed in daily returns disappear at lower frequencies (weekly or monthly returns).

The symmetry observed at low frequencies is not very surprising. As these returns are continuously compounded, weekly and monthly returns may be generated by aggregation of 5 or 20 daily returns, respectively. Then, one may invoke a central limit theorem, which under relatively general conditions implies convergence to normality. Only in the case that one accepts stable Paretian distributions (see MandelBrot, 1963 or Fama, 1963 and 1965) with a characteristic exponent less than 2 for daily returns, would convergence to normality not take place. To cast some light on these issues, different tests of normality were run on all the series of returns. Table 5 shows the number of rejections of normality at the 5% significance level and, in parenthesis, at the 1% significance level. For every test, the number of rejections decreases with the frequency. Thus, the main conclusion of this table is that returns converge to normality as they cover broader horizons, and, therefore, that they necessarily become symmetric whether they were or not at higher frequencies. In addition, one may observe in Table 5 that the main source of non-normality is kurtosis, not asymmetry. The higher number of rejections occur in those tests that rely on the kurtosis of the distribution: kurtosis, of course, and also Jarque-Bera and Shapiro-Wilk tests. Conversely, tests with skewness clearly present fewer rejections. The same occurs with the Kolmogorov-Smirnov normality tests, as these tests have low power against distributions with high kurtosis.

Table 3. Tests of symmetry with weekly returns

	t	*P*-value	*F*	*P*-value	*KS*	*P*-value	*W*	*P*-value	*ST*	*P*-value
AA	1.461	0.145	1.161	0.440	0.141	0.220	1.880	0.060	-1.135	0.256
AX	1.391	0.166	1.377	0.097	0.099	0.737	1.179	0.238	-0.375	0.708
BA	0.514	0.607	1.217	0.305	0.140	0.226	0.857	0.392	0.789	0.430
BS	0.771	0.441	1.866**	0.001	0.083	0.616	0.242	0.809	0.804	0.421
CA	0.228	0.820	1.128	0.526	0.140	0.222	0.823	0.410	-1.873	0.061
DD	0.721	0.472	1.236	0.270	0.075	0.561	0.564	0.573	-0.145	0.884
DI	0.558	0.577	1.495*	0.035	0.158	0.124	1.582	0.114	0.168	0.866
EK	0.843	0.400	1.020	0.913	0.164	0.100	1.348	0.178	2.201*	0.028
GE	1.513	0.132	1.203	0.339	0.133	0.279	1.457	0.145	-0.979	0.327
GM	0.231	0.818	1.261	0.225	0.067	0.965	0.001	0.999	-0.394	0.693
GT	1.735	0.084	1.394	0.087	0.129	0.316	1.568	0.117	-0.609	0.542
IB	0.074	0.941	1.347	0.119	0.074	0.921	0.445	0.656	-0.509	0.611
IP	0.974	0.331	1.042	0.828	0.131	0.300	1.507	0.132	0.640	0.522
JP	0.078	0.938	1.509*	0.032	0.105	0.576	0.505	0.613	0.338	0.735
KO	0.590	0.556	1.399	0.080	0.094	0.713	0.335	0.737	1.097	0.273
MC	0.687	0.493	1.578*	0.018	0.071	0.942	0.279	0.780	0.089	0.929
MO	1.042	0.300	2.298**	0.000	0.076	0.907	0.072	0.943	-1.176	0.240
MR	1.470	0.143	1.374	0.100	0.170	0.082	1.408	0.159	0.239	0.811
PG	1.249	0.213	5.104**	0.000	0.106	0.567	0.151	0.880	-1.914	0.056
SE	0.672	0.502	1.372	0.100	0.107	0.553	0.425	0.671	1.136	0.256
TX	0.751	0.453	1.352	0.116	0.063	0.978	0.356	0.722	-0.380	0.704
UK	0.074	0.941	1.552*	0.022	0.118	0.420	0.790	0.430	1.351	0.177
UT	0.767	0.444	1.328	0.139	0.084	0.825	0.554	0.580	0.089	0.929
XO	0.605	0.545	1.098	0.625	0.096	0.683	0.605	0.545	0.999	0.318

t is the usual test statistic for equality of means. *F* is the usual test statistic for equality of variances. *KS*, *W* and ST** are respectively the Kolmogorov-Smirnov, the standardized Wilcoxon and the standardized Siegel-Tukey two-sample test statistics for equality of distributions. * (**) denotes statistics significant at the 5% (1%) significance level. In all cases, the first sample is formed by negative excess returns, and the second sample is formed by positive excess returns.

Table 4. Tests of symmetry with monthly returns

	t	*P*-value	*F*	*P*-value	*KS*	*P*-value	*W*	*P*-value	*ST*	*P*-value
AA	0.438	0.663	1.181	0.679	0.212	0.568	0.497	0.619	0.405	0.686
AX	1.725	0.090	4.190**	0.001	0.212	0.593	0.971	0.332	-1.306	0.192
BA	1.147	0.257	2.877*	0.011	0.202	0.641	0.314	0.754	0.882	0.378
BS	0.155	0.877	1.354	0.442	0.148	0.924	0.126	0.900	-0.202	0.840
CA	0.418	0.678	1.018	0.970	0.183	0.748	0.531	0.595	0.506	0.613
DD	0.508	0.614	1.296	0.516	0.139	0.953	0.329	0.742	-0.590	0.555
DI	1.144	0.258	1.139	0.757	0.368	0.051	1.502	0.133	-2.223*	0.026
EK	0.170	0.866	1.099	0.812	0.164	0.853	0.093	0.926	-0.673	0.501
GE	0.555	0.581	1.751	0.152	0.298	0.174	0.885	0.376	-1.602	0.109
GM	0.493	0.624	1.552	0.270	0.117	0.992	0.194	0.846	-0.843	0.399
GT	0.480	0.633	2.678*	0.015	0.393*	0.029	0.649	0.516	-3.355**	0.001
IB	0.615	0.541	1.213	0.618	0.174	0.802	0.750	0.453	0.152	0.879
IP	0.136	0.893	1.535	0.274	0.247	0.370	0.631	0.528	-1.280	0.201
JP	0.704	0.485	1.534	0.288	0.160	0.876	0.685	0.494	0.152	0.879
KO	0.741	0.462	1.999	0.076	0.247	0.378	1.513	0.130	-0.473	0.636
MC	0.901	0.371	1.192	0.666	0.193	0.688	0.955	0.340	0.051	0.960
MO	1.630	0.109	3.102**	0.008	0.242	0.414	1.032	0.302	-1.587	0.113
MR	0.154	0.878	1.600	0.232	0.152	0.908	0.429	0.668	0.589	0.556
PG	1.302	0.199	9.231**	0.000	0.106	0.998	0.266	0.790	-0.052	0.959
SE	0.161	0.873	1.151	0.721	0.209	0.586	0.177	0.860	-0.640	0.522
TX	0.972	0.336	1.806	0.148	0.265	0.299	0.382	0.703	-2.410*	0.016
UK	0.178	0.860	1.393	0.401	0.200	0.643	0.210	0.833	-1.010	0.312
UT	0.448	0.656	1.460	0.342	0.194	0.683	0.261	0.794	-1.585	0.113
XO	0.893	0.376	1.436	0.369	0.253	0.346	0.668	0.504	-2.011	0.044

t is the usual test statistic for equality of means. *F* is the usual test statistic for equality of variances. *KS*, *W* and ST** are respectively the Kolmogorov-Smirnov, the standardized Wilcoxon and the standardized Siegel-Tukey two-sample test statistics for equality of distributions. * (**) denotes statistics significant at the 5% (1%) significance level. In all cases, the first sample is formed by negative excess returns, and the second sample is formed by positive excess returns.

Table 5. Normality tests

	Daily	Weekly	Monthly
Skewness	13 (10)	12 (7)	6 (3)
Kurtosis	24 (24)	19 (18)	7 (6)
Stud. Range	24 (24)	16 (11)	7 (3)
JB	24 (24)	19 (17)	8 (7)
KS	17 (13)	2 (0)	0 (0)
SW	24 (24)	18 (16)	9 (4)

Stud. Range denotes the Studentized Range normality test, *JB* denotes the Jarque-Bera normality test, *KS* denotes the Kolmogorov-Smirnov normality test and *SW* denotes the Shapiro-Wilk normality test. The figures indicate the number of rejections of the null of normality at the 5% significance level and, in parenthesis, at the 1% significance level.

As lower-frequency returns do not seem to be asymmetric, and as asymmetry in daily returns is not a strong and pervasive feature, these results question several authors' explanations of the low diversification of many investors' portfolios due to the preference for positive skewness. On the other hand, if stock returns are not skewed, the phenomenon known as "volatility skew" in option pricing cannot be a consequence of the skewness in the distribution of returns. Finally, two limitations of this study must be noticed. Firstly, the values that have been analyzed are blue chips, with a very high capitalization. It is possible that lower-capitalized values could present asymmetries. Secondly, as investors may be specially concerned about extreme movements, an interesting asymmetry would be the different sizes and shapes of the tails of the distributions. The tests carried out in this study compare the whole distributions, and, therefore, though they may detect different tails, their power against asymmetries in extreme movements is limited. In these circumstances, specific tests should be conducted that take into account the low sample sizes in each tail.

IV. Conclusions

The (a)symmetry of returns may be very important for different financial issues: asset pricing, diversification, portfolio selection, option pricing, and, in general, investment analysis. In spite of this potential importance, in sharp contrast with other characteristics of stocks as risk, serious problems arise in the measurement of (a)symmetry.

This study examines the (a)symmetry of returns following a new approach. This approach encompasses both conventional and distribution-free tests, which may be specially appropriate in these circumstances. The results obtained with daily returns on twenty-four values detect some asymmetries, although asymmetry does not seem to be a stylized fact characteristic of daily returns. These asymmetries disappear almost completely in weekly or monthly returns, as the distributions converge to normality. These results call several hypotheses into question, like the explanation of low diversification as a result of preference for positive skewness, or the importance of skewness in option pricing.

Appendix 1

AA	Alcoa
AX	American Express
BA	Boeing
BS	Bethlehem Steel
CA	Caterpillar
DD	Du Pont
DI	Disney
EK	Eastman Kodak
GE	General Electric
GM	General Motors
GT	Goodyear
IB	International Business Machines
IP	International Paper
JP	JP Morgan
KO	Coca-Cola
MC	McDonalds
MO	Philips Morris
MR	Merk
PG	Procter & Gamble
SE	Sears Roebuck
TX	Texaco
UK	Union Carbide
UT	United Technologies
XO	Exxon

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