KUZNETS CURVE AND TRANSBOUNDARY POLLUTION*

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A B S T R A C T

This work presents a clearer way of solving the optimisation problem addressed by Selden and Song (1995) in order to derive the J curve for abatement. The proposed framework is also extended to a two-country model. Results are consistent, also for the two country case, with empirical evidence that shows a positive relationship between environmental quality and economic growth for high income levels. A static comparative analysis confirms that the smaller the rate of discount and/or the less polluting a technology is, the higher the steady state stock of capital will be. Moreover, the lack of cooperation is proved to result in smaller efforts to abate emissions.

Keywords: environmental quality, growth, pollution abatement.

R E S U M E N

Este trabajo presenta una forma más clara de resolver el problema de optimización planteado por Selden y Song (1995) con el fin de obtener la curva en J para las actividades de control de la contaminación. El esquema propuesto se extiende también a un modelo de dos países. Los resultados son consistentes, también para el caso de dos países, con la evidencia empírica que muestra una relación positiva entre calidad medioambiental y crecimiento económico para niveles altos de renta. Un análisis de estática comparativa confirma que cuanto menor es la tasa de descuento y/o menos contaminante la tecnología utilizada, mayor será el stock de capital de estado estacionario. Además, se comprueba que la ausencia de cooperación se traduce en menores esfuerzos por controlar las emisiones.

Palabras clave: calidad medioambiental, crecimiento económico, control de la contaminación.

JEL Classification: D62, O41, Q20.

1 Introduction

There is a growing empirical literature on how environmental quality changes at different income levels. Hettige, Lucas and Wheeler (1992) find inverted-U relationships with economic development for an index of toxic pollutants. Likewise, Selden and Song (1994) show that per capita emissions of some air pollutants exhibit a 'Kuznets' relationship with per capita GDP. The World Bank Development Report (1992), Shafik (1994), Grossman and Krueger (1995), Holtz-Eakin and Selden (1995) are other works that confirm this pattern. Therefore, empirical evidence seems to support the fact that environmental quality declines in the first stages of economic growth and it improves once economies reach a certain degree of development.

Previous studies on growth and the natural environment consider pollution either as an input (see Bovenberg and Smulders (1996) and Stokey (1998)) or as a by-product of the economic process (see Forster (1973), Gruver (1976), Ploeg and Withagen (1991), Thavonen and Kuuluvainen (1993), John and Pecchenino (1994), Selden and Song (1995), Withagen (1995), Smulders and Gradus (1996) and Hettich (1998)). Thus, improving environmental quality requires substitution between pollution and other inputs, in the former case, and spending money on cleaning up activities to abate pollution, in the later situation. However, only Gruver (1976), John and Pecchenino (1992), Selden and Song (1995) and Stokey (1998), explicitly analyse the transition paths for pollution, abatement efforts and development.

Selden and Song (1995) use the simplest and earliest framework, Forster's model, to derive the J curve for abatement and the inverted U curve for pollution. An important result is that if pollution is taken into account the marginal product of capital net of depreciation no longer equals the social rate of discount, the economy tends to a lower stock of capital and consumes at a lower level. In order to account for corner solutions, Selden and Song (1995) relax the assumption that marginal efficiency of abatement tends to infinite when expenditures on pollution control are equal to zero and find a J curve for abatement. However, the procedure used to obtain the optimal saving rate is not clear as one of the first order conditions of the optimisation problem is derived by incorporating the assumption that they previously relax.

Stokey (1998) presents three models to generate a U shape relationship between per capita income and environmental quality. Restrictions on the technology and preferences consistent with the observed hump-shaped relationship between income and pollution levels are defined by analysing a static model. Kuznets relationships are also reproduced by an exogenous growth model, however, the proposed endogenous growth model requires that the economy stops growing in order to maintain a constant level of environmental quality.

Finally, Andreoni and Levinson (1998) analyse a static model with a commodity that people like but whose consumption generates a bad, pollution, confronting consumers with a trade-off. They show that the pollution-income relationship behind the environmental Kuznets curve depends on the technological link between consumption of a desired good and abatement of its undesirable by-product. These authors conclude that, for a utility function quasiconcave in consumption and 'lack' of pollution, an abatement technology with increasing returns to scale can explain the inverse U-shape without introducing growth, released constraints or even externalities.

The present work focuses on Selden and Song's paper and assumes decreasing returns to scale for cleaning activities. The main objective consists of proposing, following Ulph (1998), an alternative framework to sort out Selden and Song's analysis and extend it to a multi-country case in order to verify whether the relationship between pollution and national income changes when pollution is a transboundary problem. Moreover, the analysis of the multi-country case provides an appropriate setting to study the sources of asymmetries that could explain the observed differences among abatement efforts across countries.

Thus, the model involved will be a Ramsey-Cass-Koopmans model with pollution as a regional reciprocal externality.[1] The main difference with respect to previous literature on Kuznets curve is that the optimisation problem is decomposed into two stages, the abatement and the accumulation problem. This framework clarifies Selden and Song's (1995) analysis. Moreover, it allows to identify the required properties for preferences and technology to reproduce the observed relationship between pollution and income, in a dynamic context, as Stokey (1998) does through the static model. Additionally, the analysis is extended to a multi-country case.

The simplest way of analysing interactions is by considering two countries. In this context, the action of each government affects welfare of the other and optimal behaviours can be modelled as a differential game where players may cooperate or, on the contrary, they may act independently.

Results are consistent, also for the two-country case, with empirical studies that show a J curve for abatement. Furthermore, it is shown that the smaller the rate of discount and/or the cleaner a technology is, the higher the steady state stock of capital will be. It is also proved that each govern-

[1] See Barro and Sala-i-Martín (1995) for more details.

ment has incentives to free ride on the abatement efforts of the other and that externalities due to the lack of cooperation result in smaller efforts to abate emissions. Regarding asymmetries, it is shown that differences in the rate of discount and the kind of technology used to produce final goods and capital explain different efforts to abate emissions across countries.

The following section states the model. Section 3 sets an alternative framework to derive the J curve for abatement with respect to Selden and Song's (1995) work. Section 4 deals with a two-country model and section 5 concludes the paper.

2 The Model.

The economic process produces goods and services that provide utility to consumers and wastes that generate certain disutility. Thus, social welfare depends positively on consumption and negatively on the damage caused by pollution.

Assume that the flow of gross benefits provided by consumption is an increasing and concave function, $B(c_t)$, and that marginal benefits tend to infinity as consumption approaches zero. Suppose also that pollution, P_t, is more harmful as it increases. Thus the damage function, $D(P_t)$, will have the following properties:

$D'(P_t) > 0, D''(P_t) > 0, \forall P_t \geq 0$, with $D(0) = 0$.

Additionally, consider a production function that depends exclusively on capital and labour. Assuming, for simplicity, a time invariant production function and a stationary population, the production function can be defined in terms of capital per capita. Suppose then that production is a concave function of capital per capita, $F(k_t)$, with $F(0) = 0$.

Since pollution is regarded as a negative externality of production, gross emissions are positively related to the stock of capital, $g_t = G(k_t)$. In their paper Selden and Song (1995) assume that $G''(k_t) \geq 0$. However, if emissions are associated with output and output is a concave function of capital it is not possible that emissions are a convex function of capital and concavity of $G(.)$, rather than convexity, should be assumed. For simplicity, we will make the stronger assumption that emissions per unit of output are constant, $g_t = G(k_t) = \theta F(k_t)$, with $\theta > 0$.[2]

Moreover, suppose that cleaning up activities, defined by the function $R(a_t)$, become less efficient as abatement expenditures increase, so that $R'(a_t) > 0$, $R''(a_t) < 0, \forall a_t \geq 0$ with $R(0) = 0$. Additionally, following Selden and Song (1995), assume that $\lim_{a \to 0} R'(a) = \gamma < +\infty$. This assumption incorporates corner solutions to Forster's (1973) model.

Assume also that net emissions, P_t, are additively separable into gross emissions, g_t, minus the reduction in pollution that depends on the amount of abatement expenditures, a_t, and ignore (without loss of generality) the fact that the natural environment absorbs certain amount of acid. That is, $P(k_t, a_t) = g_t - R(a_t)$.

Additionally, consider that welfare is additively separable into the flow of gross benefits provided by consumption minus the damage generated by net emissions of some toxic pollutants.[3] Consumption and expenditures on abatement have to be optimally chosen over time. Thus, considering that

[2] This assumption is the unique change regarding Selden and Song's (1995) model and it does not affect the results concerning the J curve for abatement.

[3] Following Selden and Song (1995), we are assuming that pollution does not affect the marginal utility of consumption. Other authors, such as Stokey (1998), make the same simplifying assumption.

the representative consumer discounts the future at a rate ρ and he/she is altruist in the sense that cares about future generations, the social planner´s problem can be formulated as follows:

$$\begin{aligned} & \max_{c_t, a_t} \int_0^{\infty} e^{-\rho t} \left\{ B\left(c_t\right) - D\left(P_t\right) \right\} dt, \qquad (1) \\ s.t.\ \dot{k}_t &= F\left(k_t\right) - c_t - a_t - \delta k_t,\ k\left(0\right) = k_0 \geq 0, \\ P_t &= \theta F\left(k_t\right) - R\left(a_t\right),\ a_t \geq 0, \end{aligned}$$

where δ is the rate of depreciation of capital. The optimal solution will solve the following Hamiltonian:

$$\begin{aligned} H\left(c_t, a_t, k_t, \mu_t, \lambda_t\right) &= B\left(c_t\right) - D\left[\theta F\left(k_t\right) - R\left(a_t\right)\right] \\ & \quad + \mu_t \left[F\left(k_t\right) - \delta k_t - c_t - a_t\right] + \lambda_t a_t. \end{aligned}$$

Thus, first order conditions for the state and control variables are given by the following equations:[4]

$$B^{'}\left(c\right) = \mu, \qquad (2)$$

$$R^{'}\left(a\right) D^{'}\left[\theta F\left(k\right) - R\left(a\right)\right] + \lambda = \mu, \lambda \geq 0, a \geq 0, \lambda a = 0, \qquad (3)$$

$$\dot{\mu} = \left[\rho + \delta - F^{'}\left(k\right)\right] \mu + \theta F^{'}\left(k\right) D^{'}\left[\theta F\left(k\right) - R\left(a\right)\right], \qquad (4)$$

$$\dot{k} = F\left(k\right) - c - a - \delta k. \qquad (5)$$

Hence, from (3) we can rewrite (4) as:

[4] Subscripts referring time will be omitted to simplify notation.

$$\dot{\mu} = \left[\rho + \delta + \frac{\theta F^{'}(k)}{R^{'}(a)} - F^{'}(k)\right]\mu - \frac{\theta F^{'}(k)}{R^{'}(a)}\lambda.$$

The last term of the above equation does not appear in Forster's (1973) paper because he assumes that efficiency of cleaning up activities tends to infinite as abatement expenditures approaches zero. On the contrary, as Selden and Song (1995) relax this assumption in order to account for corner solutions, this term should disappears only for interior solutions, as long as $a > 0$ implies that $\lambda = 0$, but it affects the dynamic of the costate variable μ when $a = 0$ and $\lambda > 0$. As a result, the phase diagram represented by them is not well defined under $\widetilde{E}(c, k)$, that is, for corner solutions, where $\widetilde{E}(c, k)$ is defined making (2) equal to (3) for $\lambda = a = 0$.

Thus, if $a > 0$, then, $\lambda = 0$, and we have a particular case of Forster´s (1973) model for which the same phase diagram applies. Nevertheless, for corner solutions, $a = 0$, taking the time derivative of equation (2) and substituting (4) we get:

$$\dot{c} = \frac{B^{'}(c)}{B^{''}(c)}\left\{\rho + \delta - F^{'}(k)\left[1 - \frac{\theta D^{'}[\theta F(k_t)]}{B^{'}(c)}\right]\right\}. \tag{6}$$

Remember that $R(0) = 0$. Following Forster (1973) define:

$$\begin{aligned} M(c,k) &= \frac{B^{''}(c)}{B^{'}(c)}\dot{c} = \rho + \delta + F^{'}(k)\left\{\frac{\theta D^{'}[\theta F(k)]}{B^{'}(c)} - 1\right\}, \\ N(c,k) &= \dot{k} = F(k) - c - \delta k. \end{aligned}$$

The function $N(c, k)$ is identical to that obtained by Forster (1973), however, $M(c, k)$ is different. Differentiating with respect to c and k we have:

$$\left.\frac{dc}{dk}\right|_{\dot{C}=0} = -\frac{M_k}{M_c}, \tag{7}$$

where,

$$\begin{aligned} M_k &= -F^{''}(k)\left[1-\frac{\theta D^{'}[\theta F(k)]}{B^{'}(c)}\right] \\ &\quad -\theta F^{'}(k)\frac{\theta D^{''}[\theta F(k)]F^{'}(k)B^{'}(c)-D^{'}[\theta F(k)]B^{''}(c)}{[B^{'}(c)]^2}, \\ M_c &= -F^{'}(k)\frac{\theta D^{'}[\theta F(k)]B^{''}(c)}{[B^{'}(c)]^2} > 0. \end{aligned}$$

Hence, given the assumptions of the model, the sign of M_k depends on the sign of $1-\left\{\theta D^{'}[\theta F(k)]/B^{'}(c)\right\}$. Therefore, the sign of (7) is ambiguous and the phase diagram could not be the same as that got by Forster (1973) for values of consumption and capital under $\widetilde{E}(c,k)$. However, for small values of the parameter θ, that is, for clean technologies, or for a small marginal willingness to pay for a reduction in pollution, defined by the ratio $D^{'}/B^{'}$, M_k could be positive and we would get Selden and Song´s (1995) phase diagram. In any case, a resolution of the optimisation problem in two steps allows to avoid the ambiguity. This alternative method would consist in reducing the control variables to one, consumption plus abatement expenditures, and represent the phase diagram in terms of capital and current expenditure rather than consumption. Then, the optimisation problem would be solved in two stages: accumulation and pollution control decisions. The former is a dynamic problem that determines the optimal level of current expenditure and the later is a static problem that establishes the optimal allocation rule of current expenditure to consumption or abatement.

3 The J-curve for abatement.

This section sets a clearer way of solving the optimisation problem addressed by Selden and Song in order to derive the J curve for abatement. Firstly, the abatement problem consists in optimally allocating current expenditure to consumption or abatement, such that $e = c + a$. Thus, at each period, we look for the level of expenditures on abating pollution that maximises net welfare for a given amount of current expenditure.

$$\max_{e \geq a \geq 0} u(a) = B(e-a) - D[g - R(a)]. \tag{8}$$

Under our assumptions, the maximand in (8) is strictly concave in a. Thus, the necessary and sufficient condition for an optimum is:

$$B'(e-a) \geq D'[g - R(a)] R'(a), a \geq 0. \tag{9}$$

This condition implicitly defines a function for abatement that depends on current expenditure and gross emissions.[5] Notice that for small values of consumption the marginal damage generated by emissions might be smaller than the marginal benefits provided by consumption. In that case, if current expenditure is small enough it would be optimal to spend no resources on abating pollution.

Define $e = E(g)$ by $B'[e] = D'[g] R'(0)$. This condition specifies the boundary of a range of values of gross emissions and current expenditure for which the solution to the above optimisation problem, defined

[5] The upper bound $a = e$ will never be reached as long as the marginal benefits of consumption tend to infinite as consumption approaches zero.

as $a^* = \alpha(e, g)$, is zero. As far as marginal damage is an increasing function in gross emissions, whereas marginal benefits decrease as consumption increases, an increase in gross emissions requires a decrease in current expenditure in order to satisfy the above equality. Therefore, $E(g)$ is a strictly decreasing function in gross emissions or, equivalently, in capital. Abatement expenditures are equal to zero in the region below and on this function and, consequently, consumption and current expenditure are the same. Thus, if $e \leq E(g)$, then $\alpha(e, g) = 0$, otherwise, $\alpha(e, g) > 0$.

For $e > E(g)$, the optimal amount for abatement expenditures is implicitly defined by $B'(e - a) = D'[g - R(a)] R'(a)$.

Hence, by applying the theorem of the implicit function we have $\alpha_e = \frac{-B''(c)}{\Delta} \in [0, 1]$ and $\alpha_g = \frac{R'(a)D''(P)}{\Delta} > 0$,where $\Delta = \left[R'(a)\right]^2 D''(P) - B''(c) - R''(a) D'(P) > 0$. That is, optimal abatement is an increasing function of current expenditure and gross emissions and we can state the following.

Proposition 1 *If current expenditure is small enough it would be optimal to spend no resources on abating pollution. However, if marginal damage is an increasing function of gross emissions and marginal benefits decrease as consumption increases, there is a critical value for current expenditure, $e = E(g)$, from which welfare increases by allocating resources to abate emissions. Then, abatement expenditures can be defined as an increasing function in current expenditure and gross emissions.*

Furthermore, the indirect utility function, $u(a^*) = v(e, g)$, has the following properties:

$$
\begin{aligned}
v_e &= B^{'}\left[e-\alpha\left(e,g\right)\right]>0,\ v_g=-D^{'}\left[g-R\left(\alpha\left(e,g\right)\right)\right]<0, \\
v_{ee} &= B^{''}\left[e-\alpha\left(e,g\right)\right]\left(1-\alpha_e\right)<0,\ v_{eg}=-B^{''}\left[e-\alpha\left(e,g\right)\right]\alpha_g>0, \\
v_{gg} &= -D^{''}\left[g-R\left(\alpha\left(e,g\right)\right)\right]\left[1-R^{'}\left(\alpha\left(e,g\right)\right)\alpha_g\right]<0.
\end{aligned}
$$

Let $m\left(e,g\right)$ be the marginal willingness to pay for a reduction in pollution defined as the marginal damage of gross emissions in terms of the marginal utility of current expenditure:

$$
m\left(e,g\right)\equiv-\frac{v_g}{v_e}=\frac{1}{R^{'}\left[\alpha\left(e,g\right)\right]}>0. \tag{10}
$$

Given our assumptions about the technology for abating emissions, the marginal willingness to pay for a reduction in damage is an increasing function of current expenditure and gross emissions, $m_e=\left[\frac{D^{'}(P)}{v_e}\right]^2\left[-R^{''}\left(a\right)\right]\alpha_e>0$ and $m_g=\left[\frac{D^{'}(P)}{v_e}\right]^2\left[-R^{''}\left(a\right)\right]\alpha_g>0$.

Consider now the optimal accumulation problem. This is the classical problem of allocating resources to consumption that provides utility by itself and investment that affords future consumption. In this model the trade-off involves investment and current expenditure, instead of consumption. Therefore, the optimal accumulation problem consists of choosing time paths for capital and current expenditure in order to maximise the present value of welfare for an infinite time horizon.

$$
\begin{aligned}
& \max_{e>0}\int_0^{\infty}e^{-\rho t}v\left[e,\theta F\left(k\right)\right]dt, \qquad (11) \\
s.t.\ \dot{k} &= F\left(k\right)-\delta k-e,\ k\left(0\right)=k_0\geq 0.
\end{aligned}
$$

The Maximum Principle defines two first-order differential equations, one in the state, k, and another in the costate variable, μ, and the requirement that the Hamiltonian is maximised with respect to the control variable at every point in time.

$$
\begin{aligned}
v_e &= \mu, && (12)\\
F'(k)\left[1-\theta m\left(e,\theta F(k)\right)\right] &= (\rho+\delta)-\frac{\dot{\mu}}{\mu}, && (13)\\
\dot{k} &= F(k)-\delta k-e. && (14)
\end{aligned}
$$

It can be inferred from (12) that it is optimal to accumulate capital to the point in which its shadow price equals the increment of welfare provided by the last unit of current expenditure. Condition (13) is the familiar Ramsey equation that states that the marginal revenue product of capital must be equal to the interest rate (pure rate of discount, plus rate of depreciation of capital, plus rate of decline of the marginal utility of current expenditure). Notice, however, that the price of capital is corrected by a Pigouvian tax to reflect the marginal damage being generated by output. This tax should be equal to the amount of emissions per unit of output, θ, multiplied by the marginal willingness to reduce damage, m. Finally, (14) is the equation of motion for the state variable.

As we are dealing with an infinite horizon 'autonomous' problem, the transversality condition needed to provide a boundary condition can be replaced by the assumption that the optimal solution approaches a steady state.[6] Thus, the steady state levels of current expenditure and capital,

[6] As Kamien and Schwartz (1991) point out, this is a quite reasonable assumption since one might expect that, in the long run, the optimal solution would tend to 'settle down' as the environment is stationary by hypothesis.

(e^*, k^*), will be given by the intersection between a concave and a decreasing function of capital.

$$
\begin{aligned}
e^* &= F(k^*) - \delta k^*, &(15)\\
F'(k^*)\{1 - \theta m[e^*, \theta F(k^*)]\} &= \delta + \rho. &(16)
\end{aligned}
$$

A necessary and sufficient condition that guarantees the existence of a unique steady state is that $1 - \theta m > 0$. That is, according to (10), equation (16) provides a unique solution if and only if the marginal product of abatement is smaller than θ. This means that if there is no technology for abating pollution and emissions are controlled by reducing production, that is, $R'(a) = \theta$, the steady state is not well defined. Thus, we will assume that $R'(a) > \theta$. Then, $0 < 1 - \theta m < 1$ and condition (16) implies that the stock of capital at the steady state is smaller than the one of the Ramsey-Cass-Koopmans model.

Regarding the transition to the steady state, (13) and (14) define the evolution of the costate and state variables, respectively. We can also obtain the equation of motion for the control variable by differentiating completely (12) with respect to time. Taking into account (13), the locus of points for which current expenditure is constant over time will be defined by the following equation:

$$
F'(k^*)\left\{\theta \frac{v_{eg}}{v_e}[F(k^*) - \delta k^* - e^*] + 1 - \theta m[e^*, \theta F(k^*)]\right\} = \rho + \delta. \quad (17)
$$

Notice that (15), (16) and (17) intersect at the point (e^*, k^*) and that the curve (17) is flatter than (16), as long as the additional term in (17) is positive (negative) under (above) the locus $\dot{k} = 0$.

Therefore, as Selden and Song (1995) prove, pollution control results in a smaller stock of capital at the steady state. We also get a phase diagram defined by both a concave and a negative sloped function. Nevertheless, in this case, the negative sloped line refers to the locus of points at which current expenditure, instead of consumption, is constant.

Define the system of motion for capital and current expenditure as:

$$\begin{aligned}
\dot{k} &= f(k,e) = F(k) - \delta k - e, \\
\dot{e} &= g(k,e) = -\frac{v_e}{v_{ee}} \left\{ F'(k)\,\theta \frac{v_{eg}}{v_e} \left[F(k) - \delta k - e\right] \right. \\
&\quad \left. +1 - \theta m(e, \theta F(k)) - (\rho + \delta) \right\}.
\end{aligned}$$

The behaviour of this non-linear system can be analysed by the study of the approximating linear differential equation system in the neighbourhood of the steady state (see appendix) and it is possible to conclude that:

Proposition 2 *Current expenditure and capital increase along their time path to the steady state. Moreover, pollution control results in a smaller stock of capital at the steady state.*

In conclusion, small levels of income or, alternatively, capital are related with no abatement control. However, once countries reach a given level of development they are better of by caring about environmental quality. Moreover, the marginal willingness to pay for reducing the damage generated by pollution increases with current expenditure and capital. Thus, as current expenditure and capital increase along the time path to the steady state, abatement efforts also increase. Therefore, previous propositions define a J curve for abatement and capital.

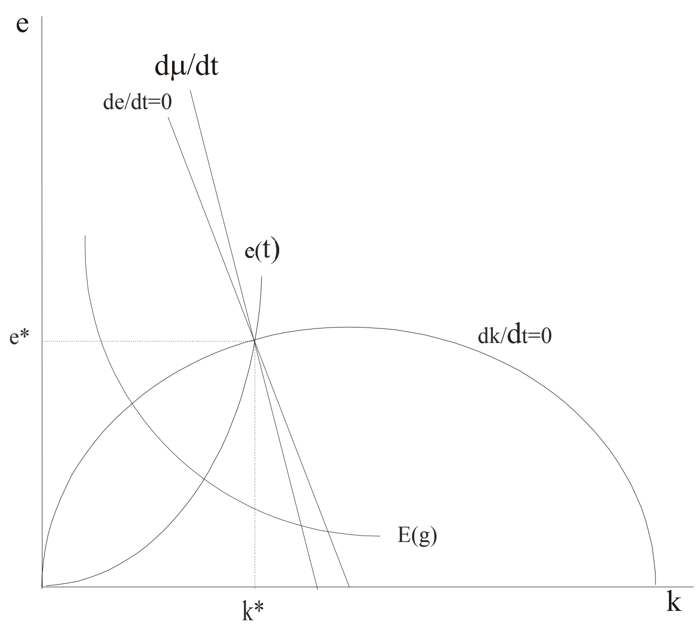

Figure 1: Steady state and stable path for current expenditure and capital.

Finally, it could be interesting to see how changes in the parameters ρ and θ affect the steady state. By differentiating equation (16) we have: $\frac{dk}{d\rho} = \frac{1}{\Lambda}, \frac{dk}{d\theta} = \frac{F^{'}(.)m(.)}{\Lambda}$, where $\Lambda = F^{''}(.)\left[1-\theta m(.)\right] - \theta F^{'}(.)\left[m_e\frac{de}{dk} + m_g\frac{dg}{dk}\right]$. Differentiating (15) we check that $de^*/dk^* = F^{'}(.) - \delta > 0$[7] and $dg^*/dk^* = \theta F^{'}(.) > 0$. Hence, the steady state stock of capital depends negatively on both parameters.

Moreover, as we stated in the first proposition, expenditure on abatement is an increasing function in current expenditure and gross emissions. Thus, since the steady state stock of capital is negatively related to ρ and θ and current expenditure and gross emissions increase with the stock of capital, abatement expenditures increase as these parameters decrease. How-

[7] From (16) we have that $F^{'}(k^*) - \delta = \theta m\left[e^*, \theta F(k^*)\right] F^{'}(k^*) + \rho > 0$.

ever, it is not possible to infer the effects of these changes on consumption without defining particular functions. Notice that net pollution could either increase or decrease as far as gross emissions and abatement expenditures have the same behaviour regarding changes in ρ and θ, $dP = \theta F^{'}(k)\left[1 - R^{'}(a)\,\alpha_g\right]dk - R^{'}(a)\,\alpha_e de$.[8] Therefore, the optimal allocation rule of current expenditure defined by the first order conditions (2) and (3) are not enough to infer the effects of changes in ρ and θ on consumption. When current expenditure and abatement decrease, consumption can either increase, if the decrease in current expenditure is smaller than the decrease in abatement expenditure, or, otherwise, decrease. Nevertheless, we could expect that a smaller stock of capital due to a higher rate of discount were the result of devoting more resources to consumption. On the contrary, the decrease in the stock of capital resulting from an increase in θ could be the consequence of higher expenditures on pollution control. These results are summarized in the next proposition.

Proposition 3 *The smaller the rate of discount and/or the less polluting the technology used to produce final goods and capital is, the higher the steady state levels of capital, current expenditure and abatement will be.*

Graphically, the locus (15) does not depend on parameters ρ and θ, however, (16) and (17) move on the right as either ρ or θ decreases.

4 A two-country model.

Pollution is usually a global or transboundary problem involving interactions between countries. This section considers two countries to check whether

[8] Notice that $1 - R^{'}(a)\,\alpha_g = -\left[R^{''}(a)\,D^{'}(P) + B^{''}(e-a)\right]/\Delta > 0$.

these interactions affect the relationship between air pollution and income. In the acid rain case, for instance, winds transport a percentage, σ_i, of pollutants emitted by each country, $P_i = g_i - R_i(a_i)$, to its neighbour. Thus, the flow of emissions in country i will be $\psi_i[g_i - R_i(a_i)] + \sigma_i[g_j - R_j(a_j)]$, where $\psi_i = 1 - \sigma_i$.[9]

In this context, as Barret (1990) points out, the marginal abatement cost for each country depends on its own abatement level, while each country´s marginal abatement benefit depends on world-wide abatement. However, as the externalities are reciprocal, every country has some incentive to take unilateral actions even in the absence of agreement. The difference between the global net benefits for the cooperative and non-cooperative outcomes defines the potential gains to cooperation.

Suppose that preferences, technologies and natural environments are exactly the same in both countries.

4.1 Cooperative equilibrium.

Each country chooses the optimal values for the control variables taking into account the damage due to its emissions of pollutants on both its own welfare and that of its neighbour. Thus, the optimisation problem can be formulated as if a social planner maximised the sum of welfare of both countries. Once again, the optimisation problem will be decomposed into two stages: environmental and investment decisions.

The optimal amount of abatement expenditure is implicitly defined by the following condition:[10]

---

[9] Once again, we are ignoring the amount of acid absorbed by the natural environment, otherwise, we would have that $\psi_i + \sigma_i \leq 1$.

[10] Notice that for $\sigma = 0$, that is, if pollution is a local problem, this equation is exactly

$$
\begin{aligned}
B_i^{'}\left(e_i-a_i\right) \quad \geq \quad & \Big\{\psi_i D_i^{'}\left[\psi_i\left(g_i-R_i\left(a_i\right)\right)+\sigma_i\left(g_j-R_j\left(a_j\right)\right)\right] \qquad (18)\\
& +\sigma_j D_j^{'}\left[\psi_j\left(g_j-R_j\left(a_j\right)\right)+\sigma_j\left(g_i-R_i\left(a_i\right)\right)\right]\Big\} R_i^{'}\left(a_i\right), a_i \geq 0.
\end{aligned}
$$

Since $v_{e_i}^c = B_i^{'}(.) > 0$ and $v_{g_i}^c = -\psi_i D_i^{'}(.) - \sigma_j D_j^{'}(.) < 0, \forall i, j = 1, 2, i \neq j$, the willingness to pay for abating pollution is:

$$
m_i^c\left(e_i, g_i, g_j, a_j\right) \equiv -\frac{v_{g_i}^c}{v_{e_i}^c} = \frac{1}{R_i^{'}(.)} > 0. \qquad (19)
$$

The optimal solution to the accumulation problem must satisfy the following conditions:

$$
\begin{aligned}
v_{ie_i} &= \mu_i, && (20)\\
F_i^{'}\left(k_i\right)\left[1-\theta_i m_i^c\right] &= \rho_i+\delta_i-\frac{\dot{\mu}_i}{\mu_i}, && (21)\\
\dot{k}_i &= F_i\left(k_i\right)-\delta_i k_i-e_i, && (22)
\end{aligned}
$$

$\forall\ i, j = 1, 2$ such that $i \neq j$. Hence, the steady state levels of current expenditure and capital are given by equations:

$$
\begin{aligned}
e_i^c &= F_i\left(k_i^c\right)-\delta_i k_i^c, && (23)\\
F_i^{'}\left(k_i^c\right)\left[1-\theta_i m_i^c\left(e_i, g_i, e_j, g_j\right)\right] &= \rho_i+\delta_i. && (24)
\end{aligned}
$$

4.2 Noncooperative equilibrium.

Consider now that countries do not cooperate and each government chooses time paths for abatement and capital in order to maximise the present dis-

the same as (9).

counted value of profits for an infinite time horizon, taken the time path of the other country net emissions, $P_j = g_j - R(a_j)$, as given.

The necessary and sufficient condition that the amount of abatement expenditures must satisfy to maximise welfare is:

$$B_i^{'}(e_i - a_i) \geq \psi_i D_i^{'}\left[\psi_i\left(g_i - R_i(a_i)\right) + \sigma_i P_j\right] R_i^{'}(a_i), a_i \geq 0. \qquad (25)$$

An interior solution, $a_i = \alpha_i^{ol}(e_i, g_i, P_j)$, to the optimal abatement problem of country $i = 1, 2$ has the following properties: $\alpha_{ie_i}^{ol} = \frac{-B_i^{''}(.)}{\Delta}$, $\alpha_{ig_i}^{ol} = \frac{\psi_i^2 R_i^{'}(.) D_i^{''}(.)}{\Delta} > 0$, $\alpha_{iP_j}^{ol} = \frac{\sigma_i \psi_i R_i^{'}(.) D_i^{''}(.)}{\Delta} > 0$, $da_i/da_j = -R_j^{'}(.)\, da_i/dP_j < 0$, where $\Delta = \psi_i \left\{ \psi_i \left[R_i^{'}(.)\right]^2 D_i^{''}(.) - R_i^{''}(.) D_i^{'}(.) \right\} - B_i^{''}(.) > 0$. That is, the optimal level of abatement expenditures increases with current expenditure and pollution and countries have incentives to free ride on the abatement efforts of their neighbour. Moreover, as $v_{ie_i} = B_i^{'}(.) > 0$ and $v_{ig_i} = -\psi_i D_i^{'}(.) < 0$,the marginal willingness to pay for abating pollution can be defined as:

$$m_i^{ol}(e_i, g_i) \equiv -\frac{v_{g_i}^{ol}}{v_{e_i}^{ol}} = \frac{1}{R_i^{'}\left[\alpha_i^{ol}(e_i, g_i, P_j)\right]} > 0. \qquad (26)$$

By comparing the cooperative and noncooperative solutions we see that condition (18) has an additional term regarding condition (25). As long as this additional term is positive, and given the properties of the functions, for a given level of current expenditure and net emissions from the other country, the amount of abatement expenditures that satisfies (18) must be higher than the one that satisfies (25).

These results can be summarised as follows:

Proposition 4 *When pollution is a global problem it is also optimal to spend no resources on abating emissions for small levels of current expenditure. However, there is a critical value, $e_i = E_i(g_i, P_j)$, from which welfare improves by allocating resources to abatement. This critical value is smaller for the cooperative solution. Furthermore, the optimal level of expenditures on abatement can be defined as an increasing function of pollution and current expenditure. Moreover, for each level of current expenditure and pollution, optimal abatement expenditures are larger in the cooperative equilibrium.*

Regarding the solution to the accumulation problem, it must satisfy the following conditions:

$$
\begin{aligned}
v_{ie_i} &= B_i' \left[e_i - \alpha_i^{ol}(e_i, g_i, P_j) \right] = \mu_i, && (27)\\
F_i'(k_i) \left[1 - \theta_i m_i^{ol}(e_i, g_i, P_j) \right] &= \rho + \delta - \frac{\dot{\mu}_i}{\mu_i}, && (28)\\
\dot{k}_i &= F_i(k_i) - \delta_i k_i - e_i. && (29)
\end{aligned}
$$

Thus, the steady state levels of current expenditure and capital are given by these expressions:

$$
\begin{aligned}
e_i^{ol} &= F_i\left(k_i^{ol}\right) - \delta_i k_i^{ol}, && (30)\\
F_i'\left(k_i^{ol}\right) \left[1 - \theta_i m_i^{ol}(e_i, g_i, P_j) \right] &= \rho_i + \delta_i. && (31)
\end{aligned}
$$

Notice that as $m_i^c(.) > m_i^{ol}(.)$, for the same values of ρ_i and δ_i, $F_i'\left(k_i^{ol}\right)$ in (31) must be smaller than $F_i'(k_i^c)$ in (24) and, consequently, the locus

$\dot{e} = 0$ defined by (24) for the cooperative solution is on the left of the locus defined by (31) for the noncooperative equilibrium. Then we can conclude that:

Proposition 5 *Cooperation results in smaller stationary levels of capital and current expenditure.*

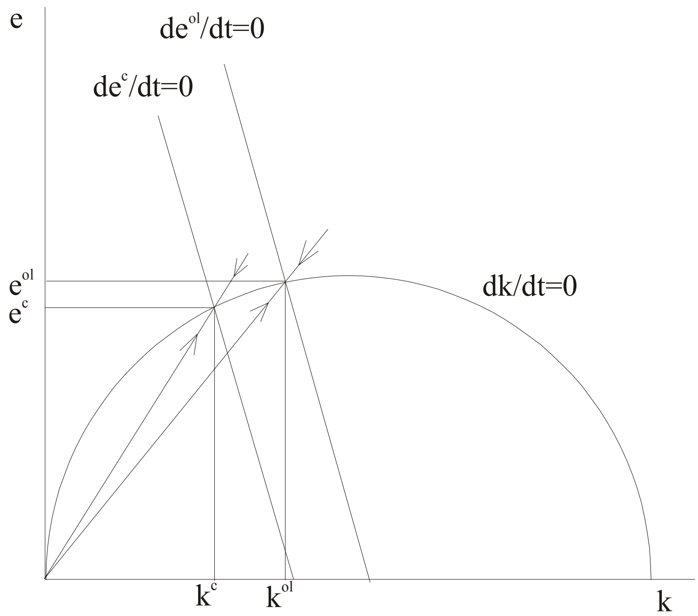

Figure 2: Steady state. Cooperative equilibrium versus non-cooperative equilibrium

Furthermore, changes in the rate of discount, ρ_i, and/or the technology used to produce final goods and capital, defined by the parameter θ_i, will affect the steady state levels of capital and current expenditure.

By differentiating (31) we get $\frac{dk_i}{d\rho_i} = \frac{1}{\Pi}$ and $\frac{dk_i}{d\theta_i} = \frac{F_i^{'}(.)m_i(.)}{\Pi}$, where $\Pi = F_i^{''}(.)\left[1-\theta_i m_i(.)\right] - \theta_i F_i^{'}(.)\left[m_{ie_i}\frac{de_i}{dk_i} + m_{ig_i}\frac{dg_i}{dk_i} + m_{iP_j}\frac{dP_j}{dk_i}\right]$, $m_{ie_i} = -\frac{R_i^{''}(.)\alpha_{ie_i}}{\left[R_i^{'}(.)\right]^2} > 0$, $m_{ig_i} = -\frac{R_i^{''}(.)\alpha_{ig_i}}{\left[R_i^{'}(.)\right]^2} > 0$, $m_{iP_j} = -\frac{(\sigma_i/\psi_i)\alpha_{ig_i}R_i^{''}(.)}{\left[R_i^{'}(.)\right]^2} > 0$.

Moreover, $de_i/dk_i = F_i^{'}(.) - \delta_i > 0$, $dg_i/dk_i = \theta_i F_i^{'}(.) > 0$ and $dP_j/dk_i = -\frac{\sigma_j\alpha_{jg_j}R_j^{'}(.)}{\psi_j}\left[1-\alpha_{ig_i}R_i^{'}(.)\right]dg_i/dk_i < 0$.[11] Accordingly, an increase in the stock of capital increases the marginal willingness to pay for abating pollution by increasing current expenditure and gross emissions. Nevertheless, as the reaction functions for abatement expenditures have a negative slope, a higher steady state stock of capital for country i will increase the level of abatement expenditures of country j and, as a result, net pollution P_j will decrease. Notice, however, that $m_{iP_j} = (\sigma_i/\psi_i)\, m_{ig_i} < m_{ig_i}$[12] and $|dP_j/dk_i| < |dg_i/dk_i|$ because $\sigma_j/\psi_j < 1$, $\alpha_{jg_j}R_j^{'}(.) < 1$ and $0 < 1 - \alpha_{ig_i}R_i^{'}(.) < 1$. Thus, differences in the rate of discount and/or the technology used to produce final goods and capital could explain asymmetries in the efforts to abate emissions and we can state the following.

Proposition 6 *The country with the smallest temporal rate of discount and/or the less polluting technology will be characterized by the highest steady state levels of capital, current expenditure and abatement.*

5 Conclusions.

This paper has proposed an alternative framework to Selden and Song's (1995) work. It has focused on the effects of emissions of some pollutants on the steady state levels and time paths for current expenditure, consumption

[11] Notice that $0 < 1 - R_i^{'}(.)\,\alpha_{ig_i} < 1$ for $i = 1, 2$.

[12] If we make the reasonable assumption that $\psi_i > \sigma_i$.

plus abatement expenditures, and capital. Pollution has been regarded as a negative externality that arises as a non-planned output of the economic process.

It has been shown that for low levels of current expenditure and capital it is optimal to care exclusively about economic growth and postpone cleaning activities. However, once a certain level of income is reached, welfare increases by reallocating resources to pollution control. Expenditures on abatement have been proved to be an increasing function of gross emissions and current expenditure. Moreover, current expenditure and capital increase along the time path to the steady state and, consequently, abatement expenditures also increase with income. Therefore, a J curve for abatement has been replicated by the proposed model. A static comparative analysis has confirmed that the smaller the rate of discount and/or the less polluting a technology is, the higher the steady state stock of capital will be; in the first case probably because of a higher level of consumption and in the second one as a consequence of greater efforts to abate emissions.

Additionally, pollution has been dealt as a transboundary problem by considering a simple model involving two countries. The lack of cooperation has been proved to result in smaller efforts to abate emissions along the time path to the steady state, characterized by larger levels of current expenditure and capital. Finally, it has been shown that differences in the rate of discount and the technology used to produce final goods and capital could explain different efforts to abate pollution across countries.

Nevertheless, an important interaction between countries like trade has not been addressed. Moreover, the model presented is an exogenous growth model and a more appropriate analysis should also include the computation of feedback strategies.

A Stability Analysis.

The local stability of the steady state can be inferred from the Jacobian matrix of the non-linear system evaluated at the steady state.

$$|J^*| = \begin{vmatrix} f_k^* & f_e^* \\ g_k^* & g_e^* \end{vmatrix}, \tag{32}$$

where, $f_k^* = F'(k^*) - \delta > 0$, $f_e^* = -1$, $g_k^* = -\frac{v_{eg}}{v_{ee}}\theta F'(k^*)\left[F'(k^*) - \delta\right] - \frac{v_e}{v_{ee}}\left\{F''(k^*)(1-\theta m) - \left[\theta F'(k^*)\right]^2 m_g\right\}$, $g_e^* = \left(\frac{v_{eg}}{v_{ee}} + \frac{v_e}{v_{ee}}m_e\right)\theta F'(k^*) < 0$.

Let x_1 and x_2 be the solutions to the characteristic equation. A well known property is that $x_1 x_2 = |J|$, where:

$$|J| = \theta F'(.)\left[F'(.) - \delta\right]\frac{v_e}{v_{ee}}m_e - \frac{v_e}{v_{ee}}\left\{F''(.)(1-\theta m) - \left[\theta F'(.)\right]^2 m_g\right\} < 0.$$

Hence, one root will be positive and the other negative and, consequently, the steady state will be saddle point stable. Equation (14) states that above $\dot{k} = 0$ the stock of capital decreases whereas it increases under that line. Likewise, equation (13) establishes that on the right-hand side of $\dot{\mu} = 0$, the shadow price of capital, μ, increases, whereas it decreases on the left-hand side.

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