STRATEGIC CONSUMER LOCATION IN SPATIAL COMPETITION MODELS

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WP-EC 95-17

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* Financial support by the *Instituto Valenciano de Investigaciones Económicas* and the *Fundació Caixa Castelló* (proyecto P1A94-10) is gratefully acknowledged. This paper was presented at the *1995 EARIE Conference* (France) and the *XI Jornadas de Economía Industrial* (Spain). The paper benefitted substantially from comments by the participants in both events and, especially, A. Gil, P.Kujal and M. Peitz, as well as from comments by an anonymous referee of this working paper series. All remaining errors are the authors' responsibility alone.

**Universitat Jaume I

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ABSTRACT

We consider a number of individual, discrete consumers, deciding their location on Hotelling's line in a non-cooperative way. Agglomeration emerges as a non-cooperative equilibrium, implying high transportation costs. No restriction is required concerning the functional form of transport costs except that they increase with the distance. In a two-dimensional market and a two-complementary-products-four-firm version of the model, the importance of transportation costs for consumer location is recovered, with consumer location determined by the interaction of two factors: minimisation of transportation costs and the *intensity-of-competition effect* of consumer agglomeration.

KEYWORDS: *agglomeration, consumer location, intensity-of-competition effect, spatial competition.*

RESUMEN

Se considera un número de consumidores que deciden su localización sobre la línea de Hotelling en una manera no cooperativa. El resultado del equilibrio no cooperativo es la aglomeración que, además, implica altos costes de transporte. No se impone ninguna restricción sobre la función de costes de transporte, excepto la de ser creciente respecto a la distancia. La importancia de los costes de transporte en la ubicación de los consumidores se recupera en una versión bi-dimensional del modelo, con dos bienes complementarios y cuatro empresas. La localización del consumidor se obtiene de la interacción entre dos factores: la minimización de los costes de transporte y el *efecto de estímulo de la competencia* de la aglomeración de los consumidores.

PALABRAS CLAVE: *aglomeración, localización de consumidores, efecto de estímulo de la competencia, competencia espacial.*

1. INTRODUCTION

Static models of spatial competition[1] assume that consumer distribution is a continuous, exogenously given function, defined for all points of the space under consideration. Considering various types of consumer distributions and their implications for firm location and transportation costs (and thus on social welfare) has been the main objective of a large number of studies[2]. However, this set-up does not allow for any type of endogenisation of consumer location based on individual consumer -or group of consumers- choice given that, in the case of a continuous consumer distribution, individual consumers do not exist.

In a dynamic set-up, Krugman[3] considers a number of forces that favor agglomeration of labor and, therefore, of consumers around the locations of less mobile production factors (like land). Wage differentials and population mobility are combined with increasing returns to scale in order for the location of the agglomeration of consumers and producers to be endogenously determined. An alternative and, probably, complementary explanation for the phenomenon of population agglomeration is offered by Berliant and Konishi (1994). Consumers act in a cooperative way to set up marketplaces and gain from mass transportation of traded goods.

In García *et al.* (1995a), we consider a family of linear consumer distributions and show that agglomeration is preferred by the whole population of consumers, the majority of them, as well as by those located far from the two sellers. Later, in an extension of the analysis with consumers distributed along the line according to a *Beta-function*[4], we obtain agglomeration of consumers away from the locations of firms.

In this paper, which is inspired by the results of our earlier work with specific functional forms of consumer distribution, we generalise the result, considering individual, discrete consumers, deciding on their location on Hotelling's line in a non-cooperative way. Agglomeration emerges as a non-cooperative equilibrium location of consumers. Furthermore, equilibrium locations imply high transportation costs. No restriction is required concerning the functional form of transport costs except that they increase with distance. Throughout the paper, we use the equilibrium concept of *subgame perfection.*

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[1] Like, for example, the works of Hotelling (1929), Salop (1979), D'Aspremont *et al.* (1979), *etc.*

[2] See, for example, Eaton and Lipsey (1975).

[3] A long list of works is reviewed in Krugman (1993).

[4] More precisely, in García *et al.* (1995b) it was assumed that $Beta_{(a,b)}$ is the distribution function of consumers and make the parameters a and b subject to consumer choice, endogenising - indirectly - consumers' distribution.

Together with the standard assumptions of increasing returns due to production, distribution and/or transportation at a large scale, the strategic advantage of *being where the rest of the population is* implies another source of incentives for consumers to agglomerate. While firm location is exogenously given, the analysis presented here can be extended to allow for endogenous firm location. However, the question concerning maximum or minimum differentiation becomes less interesting when consumer population tends to agglomerate. The implication of our result for the long run (not studied here) is that another, more general principle will determine firms' location and this is: *firms will try to locate near consumers, while strategic consumers would rather avoid being too close to any particular firm.*

2. HOTELLING'S LINE WITH ONE CONSUMER

In this section we study the location choice of one consumer. We assume that she is the only one who is interested in buying one unit of a homogeneous -in all other characteristics except for that represented on the line under consideration- good sold by two firms, ***A*** and ***B***, located at some positive distance ***AB*** from each other[5]. The consumer can locate at any point on ***AB***[6], taking into consideration that, in the stage that follows her location-decision stage, each firm will set a price (net of constant marginal costs) to maximise its own profits, taking the other firm's price-strategy as given.

The consumer chooses a location ***X*** that maximises her utility given by

(1) $$U_X = R - P_I - t_{(XI)}$$

where ***R*** is a reservation price defined as the maximum expenditure that the consumer is willing to undergo for the purchase of a unit of the good. Any expenditure higher than ***R*** implies a

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5 We know that with linear transportation costs the equilibrium location of the two firms implies minimum differentiation, but, as shown by D'Aspremont *et al.* (1979), the location of the two firms at the same point is not a stable equilibrium, given that any firm can find it profitable to deviate from that point. Furthermore, quadratic transportation costs make firms to adopt maximum differentiation. Therefore, our assumption concerning a positive distance between the two firms is compatible with both results, given that we require *some* distance between the two firms but not necessarily the maximum or the minimum that yields positive profits, avoiding any special assumptions concerning the limits of the space under consideration.

6 In this assumption, we anticipate the main implication of the result presented here, according to which, the consumer will avoid any location that allows any of the two sellers enjoy the benefits from *monopolising* her location. In any case, the extension of our analysis to the case of many firms, spread over the whole space under consideration, would indicate that the location of a consumer *outside* the segment defined by the positions of two firms would be *inside* another segment, defined by another pair of firms, except for the case of the *marginal* firm.

negative utility and, therefore, the consumer prefers not to consume the good at all. P_I is the price -net of a constant marginal production cost which is assumed to be equal to zero[7]- set by firm ***I***, which is one of ***A*** and ***B*** and is such that

$$(2) \qquad P_I + t_{(XI)} \leq P_J + t_{(XJ)}$$

where $J \neq I$ and $t_{(XI)}$, $t_{(XJ)}$ are the costs that the consumer has to pay in order to transport a unit of the good from firm locations ***I*** and ***J***, respectively, to the consumer location ***X***. We do not impose any restriction on $t_{(x)}$, except that $\partial t_{(x)} / \partial x > 0$. In other words, we require that transportation costs are an increasing function of the distance between the firm and the consumer. It will be convenient to denote locations with capital characters (***X***) and distances with small (***x***) and assume that the length of ***AB*** is ***1***, with firm ***A***'s location being ***0*** and firm ***B***'s location being ***1***. Then, if ***I***=***A*** and ***J***=***B***, equations (1) and (2) can be simplified setting $t_{(XA)} = t_{(x)}$ and $t_{(XB)} = t_{(1-x)}$. Let the consumer be on ***X***, with $XA < XB$ or $x < 1/2$ (Figure 1).

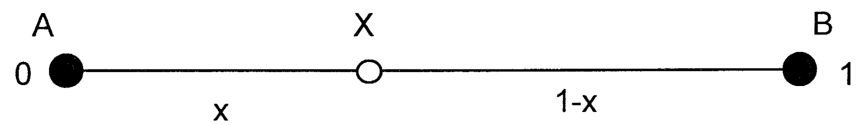

FIGURE 1: Hotelling's Line with One Consumer.

Then it is easy to obtain the following lemma:

Lemma 1. *If the consumer location is* ***X*** *such that* $x \leq 1/2$, *Bertrand-Nash prices in the one-consumer case imply an expenditure (price plus transportation costs) of the minimum between* ***R*** *and* $t_{(1-x)}$.

Proof: 1) Let us first suppose that ***R*** is sufficiently high, so that it does not affect the firms' pricing decisions. Like in other price competition models, it is straighforward that firms are involved in price cuts, because with an infinitely lower price than one's rival, the consumer can be *reached* and positive profits can be earned. Then, firm ***A***, which is the one located nearest to the consumer, realises that there is a positive price which its rival ***B*** cannot *beat* unless it sets a negative price. In fact, firm ***A***'s price that *beats* any positive price of ***B*** is equal to the difference

[7]In García *et al.* (1995c) we extend the analysis to a monopsony supplied by two firms whose production costs are different.

between the transportation costs that the consumer has to pay in order to transport a unit of the good from the location of each firm to her own location. That is, $P_A = t_{(1-x)} - t_{(x)}$.

Then we can calculate the expenditure (price plus transportation costs) of the consumer $P_A + t_{(x)} = t_{(1-x)} - t_{(x)} + t_{(x)} = t_{(1-x)}$.

2) If $R < t_{(1-x)}$, firm A monopolises the consumer's location and sets a price that allows the consumer buy one unit of the good. This price allows for the payment of the corresponding transportation costs and is, therefore, given by $P_A = R - t_{(x)}$, yielding total expenditure -by definition- equal to R. ***QED***

The implication of this lemma is that the consumer, after all, is asked to spend either all the amount she is willing to pay for purchasing a unit of the good, or an amount which is equal to the transportation costs that correspond to her distance from the firm located furthest from X. The intuition behind this result is that locations near one of the two firms, although implying lower transportation costs, imply a strategic disadvantage for the firm located far from the location of the consumer. The firm closest to the consumer has, then, an increased power which is expressed in the possibility of setting higher than competitive prices and -in the case of a low R- to extract all the income that the consumer is willing to pay for the purchase of a unit of the good, except for the income that is spent on transportation costs.

Assuming that the consumer locates on X_0 that minimises total expenditure subject to the restriction that she can buy one unit of the good from the firm whose price, increased by the corresponding transportation costs, is lower, Lemma 1 leads to the following proposition:

Proposition 1. *The optimum location of one consumer on Hotelling's line is X_0 at a distance from A equal to $x_0 = 1/2$ if $t_{(1/2)} \leq R$. If $t_{(1/2)} > R$, then $x_0 = x_1$, or $x_0 = x_2$, where $x_1 < 1/2 < x_2$ and such that $t_{(x_1)} \leq R$ and $t_{(1-x_2)} \leq R$.*

Proof: The proof follows Lemma 1 when R is sufficiently high so that it does not affect pricing strategies. According to Lemma 1, the expenditure of the consumer is equal to the transportation cost that corresponds to the largest distance between the consumer and a firm. Therefore, the consumer has to minimise that distance and this is achieved on X_0 at a distance $x_0 = 1/2$ from 0 (the location of firm A). If $R < t_{(1/2)}$ then the consumer will locate either at a distance x_1 from 0 (near firm A), or at x_2 from 0 (near firm B) that satisfy the restriction that R is not lower than the transportation costs corresponding to the distance of the consumer from her nearest firm. ***QED***

If $R < t_{(1/2)}$ and the consumer chooses a location near firm A, at a distance x_1 from it as defined in Proposition 1, firm A's price and profit is given by $P_A = \Pi_A = R - t_{(x_1)}$. In that case, firm B cannot *reach* the consumer. The consumer is indifferent among all those locations between 0 and a location X_1^0 at a distance x_1^0 such that $t_{(x_1^0)} = R$. This is because, in any case, the consumer's total expenditure will be R. The implication of this observation is that the consumer's decision may lead to a very broad range of outcomes implying different levels of social efficiency. On one extreme, the location of the consumer on 0 implies zero transportation costs and maximum private profits equal to R. On the other extreme, the location X_1^0 implies maximum transportation costs equal to R and zero private profits. The conclusions drawn here with respect to a location near firm A hold in exactly the same way for a location near firm B.

If $R \geq t_{(1/2)}$, Bertrand-Nash equilibrium prices will be $P_A = P_B = 0$, yielding profits $\Pi_A = \Pi_B = 0$.

In other words, if the consumer's budget constraint is not binding, her location decision will lead to maximum transportation costs and minimum private profits.

3. HOTELLING'S LINE WITH n CONSUMERS

In this section, we extend our analysis to the case of n identical[8] consumers. The consumers choose simultaneously their location on the linear segment defined by the positions of two firms A and B. A strategy profile $K=(x_1,..,x_i,..,x_n)$ describes the decision of consumer i to locate on X_i at a distance x_i from 0. All other assumptions and notation of the previous section are kept unchanged.

Let us first suppose that $R \geq t_{(1/2)}$. Starting from the equilibrium location of one consumer (Proposition 1), we will show that any new consumer will prefer to locate in the middle of the

[8] The term implies that the utility of each one of the consumers is given by equation (1).

linear segment defined by the positions of the two firms. It is convenient to show, first, the following:

Lemma 2. *If the **n-th** consumer location is **X** such that* $x \leq 1/2$ and $R \geq t_{(1-x)}$, *while **n-1** consumers are located in the middle of the line **AB**, Bertrand-Nash prices imply an expenditure for the **n-th** consumer (price plus transportation costs) of at least* $t_{(1-x)}$.

Proof: We derive a positive price strategy of firm ***B*** (the one furthest from the *n*-th consumer), for which a further price-cut by firm ***A*** aiming at gaining those on the middle of the line is not profitable. This price ***P*** is such that $Pn \leq t_{(1-x)} - t_{(x)}$, and gives us the minimum price that firm ***B*** would charge with a positive probability implying that firm ***A***'s best respond is also the minimum that will be charged with a positive probability too.

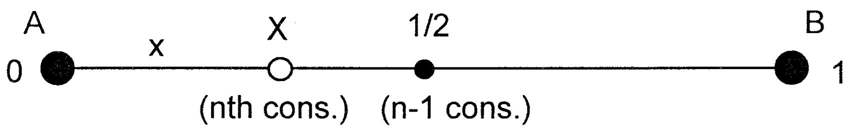

FIGURE 2: Hotelling's Line with ***n*** Consumers.

This procedure determines the minimum prices that the two firms will set in a mixed strategy equilibrium and the corresponding minimum expenditure for the ***n***-th consumer $t_{(1-x)}$. ***QED***

This leads us to the following proposition:

Proposition 2. *If* $\boldsymbol{R} \geq t_{(1/2)}$, *then* $\boldsymbol{x_i}$**=1/2** *is an equilibrium location for **n** consumers.*

Proof: Straightforward from lemma 2, given that the ***n***-th consumer's deviation from the middle of the line, when all other consumers are located there, is unprofitable. Then proposition 2 follows, like proposition 1 from lemma 1. ***QED***

If all consumers are located in the middle of the segment, like in the case of one consumer (Proposition 1), Bertrand-Nash prices and profits are $P_A = P_B = \Pi_A = \Pi_B = 0$, while total transport costs are high, considering the fact that locations near the firm from which the consumer purchases the good imply lower expenditure on transportation costs.

As indicated above, the middle of the segment is an equilibrium location only if consumers can afford paying the transportation costs that correspond to the distance between their location and the location of any of the two firms. That is, if $t_{(1/2)} \leq R$. Otherwise, consumers will have to locate at a distance from one of the two firms that makes the transportation of one unit of the good from the firm to the consumer location feasible. However, we will show that the agglomeration of all consumers at any other location, except for that in the middle of the line, cannot emerge as an equilibrium location strategy of ***n*** consumers, unless consumers agglomerate exactly ***on*** firms' locations.

Consider a location ***Y*** between the middle of the segment and the location of firm ***A***. Let all consumers be located on ***Y*** (Figure 3).

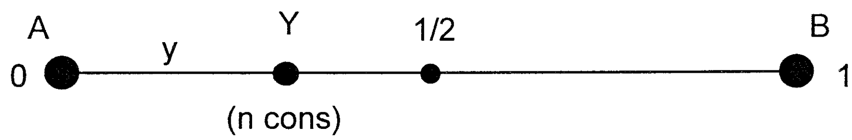

FIGURE 3: Hotelling's Line with ***n*** Consumers (the case of asymmetric agglomerations).

We will show that the ***n***th consumer will deviate from the location ***Y*** to another, closer to the location of firm ***A***, in order to save transportation costs. Then we obtain the following proposition:

Proposition 3. *The location of all consumers on **Y** at a distance **y** such that* $1/2 > y > 0$ *is not an equilibrium location for **n** consumers.*

Proof: The location of all consumers at ***Y*** gives firm ***A*** the possibility to set a positive price equal to $P_A = t_{(1-y)} - t_{(y)}$ that cannot be *beaten* by any positive price of its rival, yielding profits $\Pi_A = nP_A = n(t_{(1-y)} - t_{(y)})$. The ***n***th consumer considers moving closer to firm ***A***. The location of the ***n***th consumer at ***Y****, such that $y^* < y$ leads the consumer to lower expenditure due to lower transportation costs, provided that firm ***A*** will not respond to the new location of the consumer with a higher price. Let the situation that follows the decision of the ***n***th consumer to locate on ***Y**** be depicted in Figure 4.

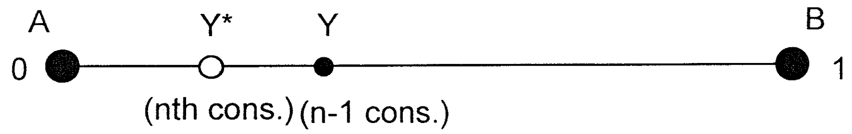

FIGURE 4: Hotelling's Line with ***n*** Consumers (deviation from an asymmetric agglomeration).

Firm A will set a higher price $\boldsymbol{P_A^*} > \boldsymbol{P_A}$, if it is more profitable for it to *abandon* its ***n-1*** clients in order to extract a higher profit from selling to the ***n***th consumer alone. This new price is given by $\boldsymbol{P_A^*} = \boldsymbol{t}_{(1-y^*)} - \boldsymbol{t}_{(y^*)}$ and it will be preferred to $\boldsymbol{P_A}$ if $\Pi_A^* > \Pi_A$, that is, $\boldsymbol{t}_{(1-y^*)} - \boldsymbol{t}_{(y^*)} > \boldsymbol{n}(\boldsymbol{t}_{(1-y)} - \boldsymbol{t}_{(y)})$, or:

(3)
$$\boldsymbol{n} < \frac{\boldsymbol{t}_{(1-y^*)} - \boldsymbol{t}_{(y^*)}}{\boldsymbol{t}_{(1-y)} - \boldsymbol{t}_{(y)}}$$

If the condition in (3) is satisfied, the expenditure of the consumer at ***Y**** becomes $\boldsymbol{G}_{Y^*} = \boldsymbol{t}_{(1-y^*)}$ which is strictly higher than $\boldsymbol{G}_Y = \boldsymbol{t}_{(1-y)}$, given that $1 - \boldsymbol{y}^* > 1 - \boldsymbol{y}$ and the assumption that $\partial t_{(x)} / \partial x > 0$. Then, the ***n***th consumer's objective becomes to locate as close to firm A as possible, without inducing a price increase. In other words, minimise $\boldsymbol{y}^*$ subject to the restriction

(4)
$$\boldsymbol{n} \geq \frac{\boldsymbol{t}_{(1-y^*)} - \boldsymbol{t}_{(y^*)}}{\boldsymbol{t}_{(1-y)} - \boldsymbol{t}_{(y)}}$$

That is, the necessary condition for a price increase by firm A to be profitable is not satisfied. If ***n=1***, this cannot be true and, therefore, the only consumer on the line never finds it profitable to move towards one of the two firms (Proposition 1). However, we see that, for $\boldsymbol{n} \geq 2$ and small deviations of the ***n***th consumer from *Y*, it happens that $\frac{\boldsymbol{t}_{(1-y^*)} - \boldsymbol{t}_{(y^*)}}{\boldsymbol{t}_{(1-y)} - \boldsymbol{t}_{(y)}} \approx 1 < 2$. Furthermore, for large ***n***, the condition is satisfied for an even broader range of deviations from *Y*.

Therefore, $\forall \boldsymbol{n} \geq 2$, $\exists y^* < y$ such that firm A will not set $\boldsymbol{P_A^*} > \boldsymbol{P_A}$ and the ***n***th consumer will spend less on transportation costs without experiencing any loss due to a price increase. Then, the location of all consumers on ***Y*** is not an equilibrium. ***QED***

Proposition 3 indicates that as n increases, the importance of the nth consumer for firm A becomes less and any deviation from the location, where the other consumers are, is not punished with higher prices. The difference between this result and the one obtained in Proposition 2 lies in the fact that, when consumers are located in the middle of the segment, firms earn zero profits and any deviation from that location by any consumer gives the possibility to one of the two firms to earn strictly positive profits. This characteristic of the set up presented here is responsible for the fact that agglomeration in the middle emerges as an equilibrium location strategy, while other agglomerations are not sustainable as an equilibrium, unless they satisfy the conditions described below, in Proposition 4.

In Proposition 3, we have seen that there is always an incentive for a consumer to move closer to the firm from which she purchases the good. However, such a deviation is not always possible. This is true for the locations A and B. For technical reasons related with the proof of Proposition 3, we have not considered yet the possibility of agglomerations at the locations A and B. The next proposition refers to that possibility.

Proposition 4. *The location of all consumers on* A *or* B *emerges as an* equilibrium location of n consumers iff $n \to \infty$.

Proof: Let all consumers be located at A. Suppose now that the nth consumer considers locating on X, at a distance x from A (Figure 5).

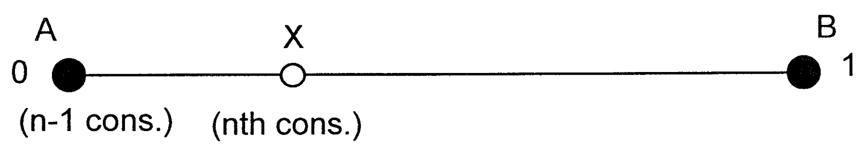

FIGURE 5: Hotelling's Line with n Consumers (agglomeration on the extremes of the line).

The deviation is justified if it causes firm A to lower its price. In fact, if all consumers are located on A, $P_A = t_{(1)}$ is a strictly positive price of firm A which firm B cannot *beat*. The price that can make firm A *reach* the consumer located at X is given by $P_A^* = t_{(1-x)} - t_{(x)} < P_A$. The two prices imply expenditures for the nth consumer given, respectively, by G_A and G_X and are such that $G_A = t_{(1)} > G_X = t_{(1-x)}$. Therefore, the *deviating* consumer will spend less at X if firm A finds it profitable to lower its price, in order to *reach* X. This happens if $(n-1)P_A < nP_A^*$. That is, if

$(n-1)t_{(1)} < n(t_{(1-x)} - t_{(x)})$, or

$$(5) \qquad \frac{t_{(1)}}{t_{(1-x)} - t_{(x)}} < \frac{n}{n-1}$$

For ***n=1***, the condition is satisfied and, therefore, the deviation away from ***A*** is always profitable. The same would be true for deviations from ***B*** and this is what we have already seen in Proposition 1.

However, $lim_{n\to\infty} \frac{n}{n-1} = 1$ and $t_{(1)} - t_{(0)} > t_{(1-x)} - t_{(x)}$ imply that, for a very large ***n***, individual deviations from ***A*** cannot be profitable. Then, the agglomeration of consumers on ***A*** can be an equilibrium location strategy for $n \to \infty$. ***QED***

4. THE CASE OF COMPLEMENTARY PRODUCTS AND COMPETITION ON A PLANE WITH ONE STRATEGIC CONSUMER

We consider now the case of a two-dimensional space. There are two pairs of firms (***A1, B1*** and ***A2, B2***), selling two perfectly complementary[9] products. Each pair of firms defines a linear segment (like the competition-on-a-line case presented in earlier sections). Producers of the same product compete in the standard price-setting fashion, as assumed in earlier sections. Like in the one-dimension-one-product case, a consumer will choose her location to minimise her expenditure subject to the restriction that she buys ***one*** unit of each product[10].

First, we use Lemma 1 to derive the Iso-Utility curves for a strategic consumer on the plane. According to the Lemma, a unique consumer located at ***X***, whose distance from the two producers of a good are given respectively by ***x*** and ***y*** spends for a unit of the good an amount that equals $\mathbf{max\{t_{(x)},t_{(y)}\}}$. It is straightforward that this is also true in a two-dimensional market. Under the assumption that the reservation price of the consumer is high enough, so that the purchase of one unit of the good is feasible, the Iso-Utility curves coincide with the set of Iso-Expenditure curves implied by the application of Lemma 1 in the case of a two-dimensional market. In Figure 6, ***L*** represents such an Iso-Expenditure curve for a consumer who considers locating at a certain point in a two-dimensional market with two firms located at two points -***A,***

[9] *Perfectly complementary* implies here that, like in standard spatial competition models, consumer utility becomes *zero* for any consumption bundle for which the quantity of any of the two products is *zero*.

[10] We assume that the reservation price of the consumer is high enough, so that the optimum location is always feasible.

B- competing in the supply of a homogeneous -in all characteristics, except for location of purchase- product.

Iso-Expenditure or Iso-Utility curves are *lenses* obtained from the intersection of two circles, each one having its center at the location of each producer. This follows from the fact that along ***L*** the consumer spends the same transportation cost for a unit of the good from the firm located furthest from her location, the upper part of the lense corresponding to transportation from ***B*** and the lower part to transportation from ***A***. The expenditure function of the consumer has a global minimum in the middle of the segment defined by the two firms' locations ***M***. From this and the monotonocity of the transportation cost function, it follows that the further an *Iso-Utility* contour lies from ***M,*** the lower the utility of the consumer. Any movement of the consumer along ***L*** leaves consumer utility unaffected. Infinitely small movements around the point of tangency of ***L*** and a tangent on both directions of the tangent has no effect on consumer utility. In this case, the vertical line on ***AB*** passing from ***M*** is denoted by ***Mm*** and the point of its intersection with ***L*** is denoted by ***K***. Then, moving from ***K*** along points on the left of ***Td*** implies lower utility levels. The same is true for movements from ***K*** on the directions ***Mm*** and ***Td*** as indicated by the arrows. We use these properties to provide a graphical demonstration of the following proposition:

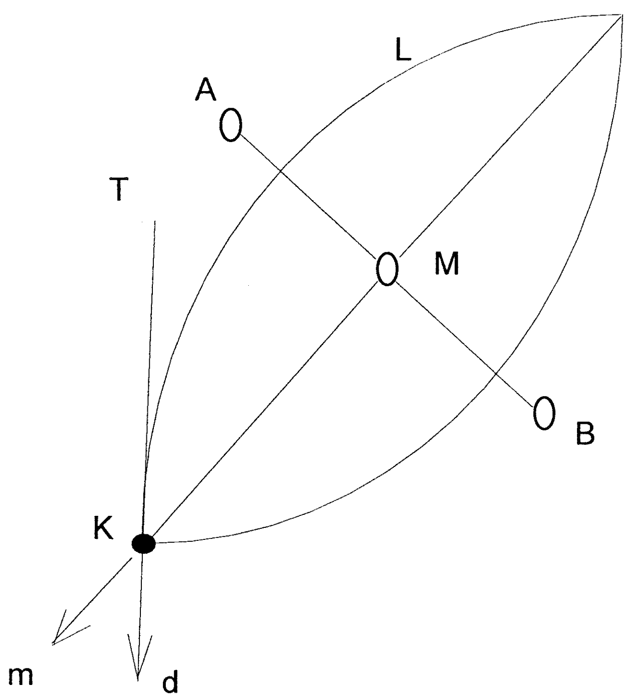

FIGURE 6: Iso-Utility curve (***L)*** for a strategic consumer under competition between two sellers (***A, B***) of a product on a plane.

Proposition 5: *Let A_iB_i denote a linear segment whose middle point is* $\boldsymbol{M_i}$ *and a straight line* $\boldsymbol{M_im_i}$ *which is vertical to A_iB_i on M_i. For* **i** *being* ***1*** *or* ***2****, where* ***1*** *and* ***2*** *are two perfectly complementary products and A_i, B_i, the locations of the two price-setting competitors in the supply of* **i***, the optimal location of a consumer will be a point on* $\boldsymbol{M_1OM_2}$*, where* ***O*** *is the point of intersection of* $\boldsymbol{M_1m_1}$ *and* $\boldsymbol{M_2m_2}$*.*

Proof: Let us suppose that in a market like the one described by the proposition, we can make a consumer located at a point on $\boldsymbol{M_1OM_2}$ better off by departing from her original location. It is straightforward that moving either on the direction denoted by $\boldsymbol{M_1m_1}$ or on the direction denoted by $\boldsymbol{M_2m_2}$ should be ruled out, as it implies increasing both the expenditure on good *1* and for good *2*. On the contrary, moving along the *Iso-Utility* contour ***L2*** on the direction indicated by the arrow, along the points ***a****,* ***b****,* ***c***, reduces the expenditure corresponding to the purchase of the first product, leaving the expenditure on the second product unchanged. Then, a point ***K*** is determined as the intersection of ***L2*** and $\boldsymbol{M_1m_1}$.

Consider, now the point $K \in M_1m_1$ and the possibility of reducing the consumer's expenditure on the second product, leaving the expenditure on the first product unchanged. There should be a tangent of ***L1*** on ***K,*** so that an infinitely small movement on the direction denoted by ***Kd*** would leave the expenditure on the first product unchanged, reducing the expenditure required on the second product by moving towards an *Iso-Utility* curve closer to $\boldsymbol{M_2}$. The existence of a tangent ***Kd*** on ***K***, like the one depicted in the figure, contradicts a basic feature of the *Iso-Utility lenses*: that is, there are *two* lines which are tangent on the *Iso-Utility lense* ***L1***. Any departure from ***K*** along ***Kd*** implies moving towards the exterior of ***L1***. Therefore, moving from ***K*** towards the interior of ***L2*** will, necessarily increase expenditure on the first product. In other words, $\boldsymbol{M_1OM_2}$ defines a frontier along which it is impossible to decrease expenditure on one good without increasing expenditure on the other. ***QED***

5. CONCLUSIONS

We have studied the location of a number of discrete, strategic consumers on Hotelling's line as a non-cooperative one-shot game. The location of two firms on the line has been assumed to imply some exogenously determined positive distance between them. Agglomeration was shown to emerge as an equilibrium outcome of the game. Consumers will, in general agglomerate in the middle between the two firms, although this situation implies high transportation costs. Consumer gains from increased competition between the two suppliers compensate losses due to high transportation costs. However, the budget constraint of the consumer may make the location in the middle impossible. Other locations -except for that in the middle of the line between the two firms- have been shown not to be sustainable as equilibrium locations for consumer agglomeration. An exception to this result is found if the number of consumers tends to infinity. In that case, consumer agglomeration on the location of one of the two firms may emerge as an equilibrium.

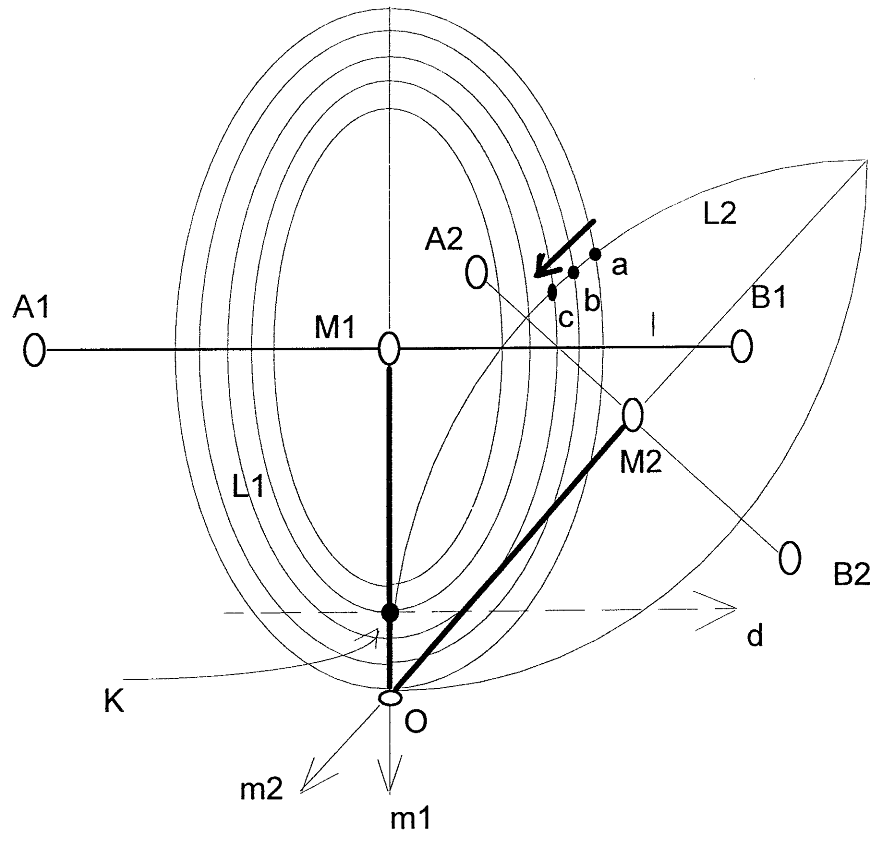

FIGURE 7: *Locus* of potential optimal locations (***M1OM2***) for a consumer under competition between two pairs of sellers (***A1, B1*** and ***A2, B2***) of two perfectly complementary products ***1*** and ***2*** on a plane.

In general, minimisation of transportation costs is the exception rather than the rule. Especially for low values of the reservation price of consumers, the possibility of completely inefficient outcomes with all consumer income spent on transportation costs and with zero consumer and producer surplus cannot be ruled out. With a reservation price sufficiently high to pay for the costs of transporting a unit of the good from firms' locations to the middle of the line, producer surplus is zero. On the alternative points of attraction of consumer agglomeration, that is the locations of the two firms, transportation costs are zero and producer surplus is positive. In general, when consumers are *rich* enough, their strategic behaviour leads them to the maximum distance from both suppliers and leaves them with a positive surplus.

In a more realistic set up with a two-dimensional space and two pairs of sellers of two perfectly complementary products, minimisation of transportation costs recovers its importance in determining optimum consumer locations. In fact, this result indicates that the interaction of two factors determines the optimal location of a consumer. On one hand, the *intensity-of-competition* effect of agglomeration attracts the population near the middle of the segment defined by each pair of competitors. On the other hand, minimisation of transportation costs requires being not too far from firms' locations. The interplay between these two factors determines the optimum location of a consumer on the plane.

Beyond the standard sources of economies of agglomeration (increasing returns due to large scale production, distribution, or transportation), we suggest that the strategic advantage of an individual consumer from being where many consumers are -avoiding being too close to any firm, is an extra feature of spatial competition that induces consumer concentration.

With respect to the interpretation of the line as an abstraction of a space of characteristics[11], the results presented here go in the same direction as the results presented in the literature on product diversity[12]. There, diversity of preferences and the role of the *marginal consumer* are shown to be of great importance. Here, the endogenisation of consumer location tells the same story but from the consumer's point of view. Consumers not only want to be where other consumers are, but they prefer to keep at equal distance from different suppliers so that the segment of the market that they represent cannot be *monopolised* by any firm. Low income consumers cannot be as strategic as they would like to be and this is the only case in which they really care about transportation costs. Otherwise, they avoid being too close to any supplier, even if this implies high transport costs.

Despite the simplicity of the set up, the results may be extended beyond the strictly locational interpretation of spatial competition models. Especially, when consumers are firms

[11] See Horstmann and Slivinski (1985).

[12] See especially Novshek and Sonnenschein (1979) and Deneckere and Rothschild (1992).

purchasing inputs, in which case consumption is an important factor in the determination of their location, the set up presented here may suggest an additional attractor of consumer location. With respect to the non locational interpretation of our results, we should bare in mind that, although it is easier to imagine a strategic monopsonist deciding on her location on a line than on her *ideal variety*, monopsonists often take measures to guarantee the survival of more than one of their suppliers in order to maintain intense competition among them. Among such measures, we mention *Two-Vendors Policy*, a strategy adopted by large-scale Japanese manufacturers aiming at establishing long-term relations with *two* suppliers, in order to avoid giving too much power to one of them alone. Such contracts are similar to locating not too close to any of the two firms, even in the case that one of them might be closer to the consumer's original ideal variety.

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