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Common-property, public infrastructure and rent dissipation in the long-run[*]

Ramón J. Torregrosa-Montaner [**]

Abstract

By means of an overlapping generations model we study some long-run steady state features prompted by free-access public capital which enters a constant-returns-to-scale production function, dissipating its return among the private factors. The public investment is funded by lump-sum taxes in both younger and older generations. Our main conclusion is that the utility of an individual living in the long run steady state equilibrium may decline, even when the private capital-labor ratio increases, as a consequence of both an increase in per-capita public investment, and a shift of the tax burden from the younger to the older generation.

Keywords: Public capital, rent dissipation, long-run steady state.

JEL classification numbers: E69, H49.

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1 Introduction

There is a broad consensus among economists that free-access public infrastructure plays an important role in supporting economic activity. The explanation for this is that this kind of capital stock, such as roads, streets, bridges, water and sewer treatment systems, etcetera, contributes to enhance the production possibility set, lowering production costs and making the remaining private production factors more productive. However, as happens with private capital stock in the long run, the impact of changes in public infrastructure upon economic activity and welfare may depend on the stock of public capital already in place in such a way that the larger the stock of public capital the lower the impact of additions to this stock.[1] In other words, the over- or under-accumulation of public capital stock in the long-run generates differents effects on endogenous variables when public investment policy changes. In adition, the starting point to tackle when dealing with public infrastructure, considered as an input, is how it enters the aggregated production function. When public capital influences multifactor productivity and constant returns to scale only affect private inputs, zero-profits are reached under competitive conditions. Meade (1952) claimed this case as "atmospheric" public input and more recently it has been referred to as pure or factor-augmenting public input. In contrast, when public capital enters a constant-returns-to-scale production function there are decreasing returns to scale in the private factors. This is the so-called profit-augmenting or unpaid-factor case, and it implies that when private factors are hired at their marginal products, economic profits emerge as a consequence of Euler's formula.

The solutions proposed for maintaining the equilibrium zero-profit condition under price equal marginal cost in the unpaid-factor case have been diverse and range from assuming that the public-capital-profit is fully appropriated by the government (Basu, 1987); is shared by government and consumers (Keen and Marchand, 1997); is charged by a rental on its use (Pestiau, 1974); or is captured by one of the factors (Kellermann, 2007). However, Feehan and Batina (2007) propose a different solution for this feature by considering that the public-capital-profit dissipates among private factors in such a way that, at zero profit equilibrium, the prices of private factors are set above their marginal products. This makes the public input equivalent to a common property resource in such a way that, since it is provided on a free-access basis, firms over-hire private factors, making an inefficient use of resources. Feehan and Batina apply this characterization to deriving the optimal design of the private factor taxes that are employed to finance a public input, by using a static-partial-equilibrium model where both the interest rate and the amount of private capital stock are exogenous.

This paper extends Feehan and Batina's price rule to a dynamic general equilibrium model where both the interest rate and the private capital stock are endogenously determined. For that purpose we are going to consider an overlapping generations model similar to the one used by Pestiau (1974), where

[1] Empirical research as that of Demetriades and Mamuneas (2000), Romp and de Haan (2007) and Pereira and Andraz (2012), supports this idea.

individuals live two periods and the aggregated linear-homogeneous production function depends on labour, private capital and free-access public capital which is financed by taxes in both the younger and older generation, as in Bierwag, Grove and Kang (1969). Since this framework allows us to characterize the optimal golden-age path amounts of both private and public capital stocks, we can build a theoretical background where to study how the under- or over-accumulation of public capital stock in the long-run steady state affects changes in public investment. In particular, provided that the public policy instruments available in our model are the weight of the tax burden borne by each generation and the public investment policy, our main concern will be to study the long-run steady state effects of changes in these policy instruments on private capital stock and welfare. As we will see, our main conclusions are linked to certain conditions under which over- and under- accumulation of public capital stock is considered.

The paper is structured as follows. Section 2 presents the model and the equilibrium concept. Section 3 states our main conclusions in the long-run steady state and, finally, Section 4 includes some final remarks and comments.

2 The model

2.1 Households

Let us consider an overlapping-generations economy where the population on date $t \geq 1$ is given by L_t and it grows at an exogenous rate $n > -1$ so that

$$L_t = (1+n)L_{t-1}, \tag{1}$$

with L_0 given. Each individual lives for two periods. During the first period, when individuals are young, they work, and in the second period, when they are old, they are retired from the labour force. An individual born in period t is endowed with one unit of labour, which is supplied inelastically, and saves during youth in order to consume when he is old.

The representative household is characterized by the utility function

$$u(c_t^y, c_{t+1}^o), \tag{2}$$

where c_t^y denotes the consumption of the agent at time t when young, and c_{t+1}^o is the consumption of the agent at time $t+1$ when old. Let us assume that both c_t^y and c_{t+1}^o are normal commodities.

The representative household maximizes its utility subject to the intertemporal budget constraints

$$c_t^y + S_t = W_t - T_t^y$$
$$c_{t+1}^o = (1 + r_{t+1})S_t - T_{t+1}^o$$

Where S_t is saving, W_t is the wage, T_t^y and T_{t+1}^o are lump-sum taxes when young and old respectively, and r_{t+1} is the interest rate. In this trend, the consolidated budget constraint is

$$c_t^y + \frac{c_{t+1}^o}{1+r_{t+1}} = W_t - T_t^y - \frac{T_{t+1}^o}{1+r_{t+1}} \tag{3}$$

From the first order condition of the problem of maximizing (2) subject to (3) we hold that

$$\frac{\partial u/\partial c_{t+1}^o}{\partial u/\partial c_t^y} = \frac{1}{1+r_{t+1}}, \tag{4}$$

which yields the solution

$$c_t^y(W_t, T_t^y, T_{t+1}^o, r_{t+1}), c_{t+1}^o(W_t, T_t^y, T_{t+1}^o, r_{t+1}),$$

so that

$$S_t(W_t, T_t^y, T_{t+1}^o, r_{t+1}) = W_t - T_t^y - c_t^y(W_t, T_t^y, T_{t+1}^o, r_{t+1}). \tag{5}$$

From the previous assumption of normality in consumption in both periods $\frac{\partial c_t^y}{\partial W_t} > 0$ and $\frac{\partial c_{t+1}^o}{\partial W_t} > 0$, which implies that

$$\frac{\partial S_t}{\partial W_t} = 1 - \frac{\partial c_t^y}{\partial W_t} > 0. \tag{6}$$

In addition, by writing consumption when young in equilibrium as

$$c_t^y(m_t, r_{t+1}),$$

where, $m_t = W_t - T_t^y - \frac{1}{1+r_{t+1}} T_{t+1}^o$. Exploiting Equation (5), it is fairly easy to prove that

$$\begin{aligned} \frac{\partial S_t}{\partial T_t^y} &= -\frac{\partial S_t}{\partial W_t} < 0, \\ \frac{\partial S_t}{\partial T_{t+1}^o} &= \frac{1}{1+r_{t+1}} \left(1 - \frac{\partial S_t}{\partial W_t}\right) > 0, \end{aligned} \tag{7}$$

a property which shows the different effect that taxes in younger and older generations have on savings. On the one hand, an increase in taxes when young decrease savings due to the decrease in present disposable income, on the other, an increase in taxes when old increases savings because the representative individual expects to pay higher taxes in the future. In turn, regarding the variation of savings with respect to the interest rate, we will assume that $\frac{\partial c_t^y}{\partial r_{t+1}} \leq 0$, which means that the substitution effect dominates the income effect, assumption necessary to keep the stability of the model as long as $\frac{\partial S_t}{\partial r_{t+1}} = -\frac{\partial c_t^y}{\partial r_{t+1}} \geq 0$ (Galor and Ryder, 1989).

2.2 Aggregated production function

In every period, the private consumption good is produced by a technology that uses three factors: private capital, public capital and labour, denoted by K_t, G_t and L_t respectively and produces a simple aggregated output Y_t in period t. For sake of simplicity we assume that both private and public capital fully depreciate in each period. The technology displays constant returns to scale and is represented by a linearly homogeneous Cobb-Douglas production function

$$Y_t \equiv F(K_t, G_t, L_t) = [H(K_t, G_t)]^{\alpha} L_t^{1-\alpha},$$

where H is also linearly homogeneous in K_t and G_t.[2] Let $k_t = K_t/L_t$ and $g_t = G_t/L_t$ be the private and the public capital-labour ratios, and $f(k_t, g_t)$ the output-labour ratio so that

$$f(k_t, g_t) = [h(k_t, g_t)]^{\alpha}.$$

By applying Euler's formula this function is homogeneous of degree $0 < \alpha < 1$, that is :

$$\alpha f(k_t, g_t) = f_k k_t + f_g g_t. \tag{8}$$

In addition, since $f(k_t, g_t)$ exhibits decreasing returns,

$$f_i > 0, \quad f_{ii} < 0, \quad i = k_t, g_t. \tag{9}$$

Where f_i and f_{ii} are the first and the second partial derivatives of f with respect to $i = k_t, g_t$. Moreover, according with Heijdra and van der Ploeg (2002, pp 634) and Agénor (2013), we assume that public and private capital are complementary factors.[3]

$$f_{ij} > 0, \quad i, j = k_t, g_t; \quad i \neq j.$$

2.3 Prices

The problem of considering public capital as a common-property resource is that, under a linearly homogeneous production function in all inputs, price equal marginal product is no longer an equilibrium, to see that let us recall Euler´s formula

$$F(K_t, G_t, L_t) - F_K K_t - F_L L_t = F_G G_t > 0.$$

Thus, the marginal product-price rule generates a profit which leads to more entry and increasing output. To ammend this problem Feehan and Batina

[2] This assumption is in regard to the private-factors-price formation rule considered in the next subsection. Under a linearly homogeneous non-nested production function it is not possible to maintain a clear cut sign for the variation in the rate of return of private capital with respect to changes in both private and public capital stock.

[3] Empirical literature also suports this assumption, see Pereira and Andraz (2012).

(2007) consider that public capital return $F_G G_t$ is dissipated among private factors in such a way that the equilibrium is restored until private factors are hired up to their marginal products

$$R_t = \frac{F_G Y_t}{Y_t - F_G G_t}, W_t = \frac{F_W Y_t}{Y_t - F_G G_t}, \tag{10}$$

where $R_t = 1 + r_t$ is the rate of return of the private capital. This is the standard commons equilibrium; note that at that point profit is eliminated, that is $Y_t = R_t K_t + W_t L_t$, which results from a higher demand of private factors and an increase in prices since, in our model, private factors are supplied inelastically in each period

To express the rate of return of the private capital in per capita (and elasticity) terms will be useful along the paper thus, rearranging terms, Equation (10) can be written as

$$R_t = \frac{f_k f(k_t, g_t)}{f(k_t, g_t) - f_g g_t} = \frac{f_k}{1 - \eta_g}, \tag{11}$$

Where $\eta_g = f_g g_t / f \in (0, 1)$ is the elasticity of aggregated output with respect to public capital. We see that, according to (11), the demand of private capital is higher than those of the competitive equilibrium, since under this price rule the rate of return of private capital is higher than its marginal product for every level of k. In addition, as long as Equation (11) depends on total product and both marginal products, it is advisable to study how it varies with changes in both private and public capital-labour ratio stock.

Proposition 1 $\frac{\partial R_t}{\partial g_t} > 0$ *and* $\frac{\partial R_t}{\partial k_t} < 0$.

Proof: see Appendix 1.

Regarding the wage, let us express it in per capita terms as

$$W_t = f(k_t, g_t) - R_t k_t. \tag{12}$$

Note that, unlike with the competitive case, here the change in the wage when the private capital-labour ratio changes, does not have a clear-cut (positive) sign. This is due to the distortion introduced by the dissipation of public capital return in the private factors. In fact, by partially deriving Equation (12) with respect to k_t, we have:

$$\frac{\partial W_t}{\partial k_t} = -\frac{\partial R_t}{\partial k_t} k_t - \eta_g R_t. \tag{13}$$

The first term is similar to the usual effect that positively links changes in the wage with changes in the private capital-labour ratio, but the second term, which represents the distortion induced by the public capital return dissipation in private capital return, acts by reducing the wage.

2.4 Government

There are two policy instruments: the level of public investment on date t, $I_t^G = G_{t+1}$, and the level of taxes levied on each generation. The Government cannot issue debt in such a way that its policy has to fulfil the usual budget constraint, given by:

$$I_t^G = T_t^y L_t + T_t^o L_{t-1},$$

Dividing by L_t, taking into account (1), we can express the Government's budget constraint in per capita terms as

$$i_t^g = T_t^y + \frac{T_t^o}{1+n},$$

where public investment in per capita terms is given by

$$i_t^g = (1+n)g_{t+1}.$$

Substituting public investment, the consolidated Government's budget constraint can be written as

$$T_t^y + \frac{T_t^o}{1+n} = (1+n)g_{t+1}. \tag{14}$$

In what follows we are going to consider a constant per-capita public investment policy so that $g_t = g \quad \forall t$. In addition, as in Bierwag, Grove and Kang (1969), let us define T_t as the total tax revenue achieved in period t, so that $T_t^y = (1-\tau)T_t$ where $0 \leq \tau \leq 1$, is the share of tax revenue borne by the older generation. It is straightforward that $(1-\tau)T_t + \frac{1}{1+n}T_t^o = T_t$, then $T_t^o = (1+n)\tau T_t$, which, substituting in (14), allow us to determine the amount of taxes of each generation which would fulfil the Government´s budget constraint:

$$\begin{aligned} T_t^y &= (1-\tau)(1+n)g, \\ T_t^o &= \tau(1+n)^2 g. \end{aligned} \tag{15}$$

2.5 Equilibrium

Given initial private and public capital-labour ratios (k_0, g_0), an equilibrium is a sequence of allocations $\left\{c_t^y, c_{t+1}^o, k_t, g_t\right\}_{t=0}^{\infty}$, factor prices $\{W_t, R_t\}_{t=0}^{\infty}$, and lump sum taxes $\{T_t^y, T_t^o\}_{t=0}^{\infty}$ such that:

(i) Given the factor prices and the taxes, the allocation solves the maximization problem of each consumer;

(ii) Given the allocation, the factor prices are consistent with the firms' profit maximization;

(iii) The market for the consumption commodity clears at every date; and

(iv) $T_t^y + \frac{T_t^o}{1+n} = (1+n)g_{t+1}$ and $S_t = (1+n)k_{t+1}$.

Notice that the accumulation expression for the private capital-labour ratio provided by the notion of equilibrium can be written, taking into account (1), as follows:

$$S_t(W_t, T_t^y, T_{t+1}^o, r_{t+1}) = (1+n)k_{t+1}. \tag{16}$$

3 Private capital accumulation and welfare in steady-state

Let us study how changes in policy instruments τ and g affect the main variables of the model in the long-run steady state, that is when both the private and public capital-labour ratio remain constant, $k_t = k$ and $g_t = g \quad \forall t$. Therefore, in steady-state the equilibrium, Equation (16) can be written as

$$S(W, T^y, T^o, r) = (1+n)k. \tag{17}$$

As our results are going to be related to the position of our equilibrium with respect to the optimal golden-age path, let us define k^{gold} and g^{gold} to be the levels of private and public capital-labour ratios which fulfil the production golden rule $f_h = 1+n$, $h = k, g$,[4] in such a way that when $h < h^{gold}$ ($f_h > 1+n$) we claim that there is an under-accumulation of (private or public) capital-labour ratio, and when $h > h^{gold}$ ($f_h < 1+n$) there is an over-accumulation. Note that under the Feehan and Batina's (2007) price rule while $r > n$ does not imply necessarily an under-accumulation of private capital-labour ratio in long run equilibrium, $r < n$ does imply an over-accumulation of it.

Therefore, let us study the changes in the long-run steady state private capital-labour ratio that stem from an exogenous change in the weight of tax burden borne by each generation, holding the per-capita public investment constant. By taking the total derivative of (17) with respect to τ, exploiting Equations (7) and (13),

$$\frac{dk}{d\tau} = g\frac{(1+n)}{D(\cdot)}\left(\frac{\partial S}{\partial W} + \left(\frac{1+n}{1+r}\right)\left(1 - \frac{\partial S}{\partial W}\right)\right),$$

where

$$D(\cdot) = 1 + n - \left(\frac{\partial S}{\partial r} - \frac{\partial S}{\partial W}k\right)\frac{\partial R}{\partial k} + \eta_g(1+r)\frac{\partial S}{\partial W} \tag{18}$$

is positive in order to ensure the local stability, $\frac{dk_{t+1}}{dk_t} < 1$, at the private capital accumulation steady state equilibrium (note that $\frac{\partial r}{\partial k} = \frac{\partial R}{\partial k}$). In turn, provided that $\frac{\partial S}{\partial r} > 0$, $\frac{\partial S}{\partial W} > 0$ and according to Proposition 1, a more than sufficient condition for $D(\cdot) > 0$ is that $\frac{\partial S}{\partial r} - \frac{\partial S}{\partial W}k > 0$. In what follows we are going to assume this both to ensure the stability of the model and for convenience in further results. This leads us to the following proposition:

[4] See Appendix 2.

Proposition 2 *The amount of the long-run equilibrium private capital-labour ratio will be increased by increasing the share of tax burden borne by the older generation.*

The proof is straightforward in view of (6) and (18). Thus, since $\frac{dk}{d\tau} > 0$, it follows immediately that $\frac{dR}{d\tau} = \frac{\partial R}{\partial k}\frac{dk}{d\tau} < 0$ due to Proposition 1. This result is similar, but not identical[5], to a standard proposition of the life-cycle models and is consistent with the fact that savings increase the heavier the tax burden imposed on the older generation, stated in Equation (7). Therefore, if we are to maximize the long-run steady state private capital-labour ratio we should impose the entire tax burden on the older generation.

On the other hand, regarding the effects of a change in the weight of tax burden borne by each generation on the utility of an individual living in the long-run steady state holding the per-capita public investment constant, from (2), let us define the indirect utility function

$$V(W, r, T^y, T^o) = u[c^y(W, r, T^y, T^o), c^o(W, r, T^y, T^o)], \tag{19}$$

Taking its total derivative with respect to τ exploiting the first order condition for the individual's equilibrium (4), we obtain

$$\frac{dV}{d\tau} = \frac{\partial u}{\partial c^y}\left[\left(\frac{\partial c^y}{\partial W} + \frac{1}{1+r}\frac{\partial c^o}{\partial W}\right)\frac{dW}{d\tau} + \left(\frac{\partial c^y}{\partial r} + \frac{1}{1+r}\frac{\partial c^o}{\partial r}\right)\frac{dR}{d\tau} + \dots\right.$$

$$\left.\dots + \left(\frac{\partial c^y}{\partial T^y} + \frac{1}{1+r}\frac{\partial c^o}{\partial T^y}\right)\frac{dT^y}{d\tau} + \left(\frac{\partial c^y}{\partial T^o} + \frac{1}{1+r}\frac{\partial c^o}{\partial T^o}\right)\frac{dT^o}{d\tau}\right].$$

But, according to (3), $\frac{\partial c^y}{\partial W} + \frac{1}{1+r}\frac{\partial c^o}{\partial W} = 1$, $\frac{\partial c^y}{\partial r} + \frac{1}{1+r}\frac{\partial c^o}{\partial r} = \frac{S}{1+r}$, $\frac{\partial c^y}{\partial T^y} + \frac{1}{1+r}\frac{\partial c^o}{\partial T^y} = -1$ and $\frac{\partial c^y}{\partial T^o} + \frac{1}{1+r}\frac{\partial c^o}{\partial T^o} = \frac{-1}{1+r}$. Therefore:

$$\frac{dV}{d\tau} = \frac{\partial u}{\partial c^y}\left[\frac{dW}{d\tau} + \frac{S}{1+r}\frac{dR}{d\tau} - \frac{dT^y}{d\tau} - \frac{1}{1+r}\frac{dT^o}{d\tau}\right].$$

Finally, taking into account Equations (11), (12), (15) and (17) we can write

$$\frac{dV}{d\tau} = \frac{\partial u}{\partial c^y}\left[-\eta_g(1+r)\frac{dk}{d\tau} + \left(\frac{r-n}{1+r}\right)\left((1+n)g - k\frac{dR}{d\tau}\right)\right]. \tag{20}$$

Since $\frac{dk}{d\tau} > 0$ and $\frac{dR}{d\tau} < 0$, the long-run level of utility is negatively affected by the fall in the return of the private capital-labour ratio due to its increase, and ambiguously affected as regards the difference between the interest rate in equilibrium r, which represents the marginal substitution rate between present and future consumption, and the exogenous population growth rate n, which

[5] For instance, in the basic model with (only) private capital stock, non-distortionary prices and outstanding government debt of Bierwag, Grove and Kang (1969), the amount of the long-run equilibrium private capital-labour ratio increases (decreases) by increasing the share of tax burden borne by the older generation, according as $r > n$ ($r < n$). In our case this effect is independent of this (private capital stock) dynamic efficient condition.

represents the rate at which savings have to grow to maintain the steady state private capital-labour ratio. Thus, a clear-cut sign for $\frac{dV}{d\tau}$ depends on its second part such that the following proposition can be made:

Proposition 3 *The long-run utility level will be decreased by imposing a heavier tax burden on the older generation whenever $r \leq n$.*

The proof is straightforward in view of Proposition 2 and Equation (20).

From Proposition 3 we have that in our model the utility of the representative individual living in the long-run steady state may decline as a consequence of a shift of the tax burden from the younger to the older generation, whenever the equilibrium interest rate is not higher than the rate at which it should have to grow along the balanced growth path. This is because indirect utility is measured in terms of present value and the particular and sufficient conditions stated in Proposition 3 work to increase the opportunity cost of present consumption. Note that the opposite condition ($r > n$) is not enough to assert an increase in the long-run utility level of the individual. At this point let us note that this result is similar but not identical to that of Bierwag, Grove and Kang (1969), where the long-run utility level increases by imposing a heavier tax burden on the older generation whenever $r > n$, but the reverse does not imply a decrease in the long-run utility level. This is because of the way in which public capital stock enters the production function in our model. Since the first part of Equation (20) reflects the negative effect that the increase in the private capital-ratio has on the equilibrium wage, due to the distortion exerted by the dissipation of public capital return, the condition $r > n$ is no longer enough to increase the long-run utility level. In this trend,$r \leq n$ becomes sufficient to hold a clear-cut (negative) sign in this derivative. In turn, this sufficient condition means that an increase in the weight of tax revenue borne by the older generation requires an increase in current savings so that, if the opportunity cost of consumption when young is higher than the rate at which savings have to grow in the balanced growth path, the substitution of present consumption by future consumption leads to a fall in welfare. In addition, note that $r \leq n$ also implies that $f_k < 1 + n$, which corresponds with an over-accumulation of the steady state amount of private capital-labour ratio.

Analogously, let us consider the changes in the long-run steady state of the private capital-labour ratio that stem from an exogenous change in the per-capita public investment policy, given the distribution of tax burden between generations. Taking the total derivative of (17) with respect to g, we find that

$$\frac{dk}{dg} = \frac{1}{D}\left(f_g\frac{\partial S}{\partial W} + \left(\frac{\partial S}{\partial r} - \frac{\partial S}{\partial W}k\right)\frac{\partial R}{\partial g} + \frac{\partial S}{\partial T^y}\frac{dT^y}{dg} + \frac{\partial S}{\partial T^o}\frac{dT^o}{dg}\right).$$

Taking into account (7), (15) and rearranging terms, the above expression can be written as

$$\frac{dk}{dg} = \frac{1}{D}\left[(f_g - (1-\tau)(1+n))\frac{\partial S}{\partial W} + \left(\frac{\partial S}{\partial r} - \frac{\partial S}{\partial W}k\right)\frac{\partial R}{\partial g} + ... \right. \tag{21}$$

$$\left. ... + \left[1 - \frac{\partial S}{\partial W}\right]\frac{(1+n)^2}{1+r}\tau\right].$$

The variation in the steady state private capital-labour ratio due to changes in the per-capita public investment policy is here split into three parts. The second and third part are positive, and therefore the ambiguity of the sign of Equation (21) depends on the net return of the public capital stock in the steady state equilibrium. The following Proposition states a sufficient condition upon which (21) has a clear-cut sign.

Proposition 4 *The amount of the private capital-labour ratio will be increased by increasing the per-capita public investment policy whenever* $f_g \geq (1-\tau)(1+n)$.

The proof is straightforward in view of (21).

As a corollaries of Proposition 4 we find that, on the one hand, when $0 \leq \tau < 1$, $f_g - (1+n) \geq 0$ becomes a more than sufficient condition for Equation (21) to be positive. This is equivalent to the case in which all the tax burden is borne by the young generation, $\tau = 0$. On the other hand, when $\tau = 1$, that is, when all the tax burden is borne by the older generation, Equation (21) becomes unequivocally positive.

Therefore, when public capital stock is financed by both generations its net return affects savings directly by means of the wages and taxes when young. This means that in such a case a minimum net return of public capital is required to increase the private capital-labour ratio by increasing per-capita public investment. For instance, when the marginal product of public capital stock is higher than its growth rate along a balanced growth path, which is equivalent to a under-accumulation of public capital-labour ratio for the long-run steady state, the effect on the long-run steady state amount of private capital-labour ratio of an increase in the per-capita public investment policy is positive. In the extreme case in which public capital stock is financed only by taxes on the older generation, an increase in such taxes increases savings, because the younger generation expects to pay more taxes in the future, with the direct consequence of an increase in private capital investment. On the other hand, a fall in the amount of the private capital-labour ratio as a consequence of an increase in the amount public capital-labour ratio would be possible in the case of a higher enough over-accumulation of public capital-labour ratio in the steady-state equilibrium. The fact that $\tau = 0$ would help in this case as long as the higher the taxes for the young generation the lower the savings with the direct consequence on private capital investment. Moreover, let us assess the effects of such a policy on the return of private capital in equilibrium. The derivative of private capital return with respect to g is:

$$\frac{dR}{dg} = \frac{\partial R}{\partial g} + \frac{\partial R}{\partial k}\frac{dk}{dg}. \tag{22}$$

This Equation shows the particular effect that the price distortion, exerted by the dissipation of public capital returns between the price of the remaining private production factors, causes in our model. On the one hand, changes in the per-capita public capital investment policy has the standard opposite effect on the private capital return by means of changes in the amount of the private capital-labour ratio. On the other hand, there is a partial positive effect which directly affects the private capital return. In fact, this feature allows the possibility of both the return of private capital and the private capital-labour ratio to change positively with changes in the public per-capita investment policy whenever $0 < \frac{dk}{dg} \leq \Psi$, where $\Psi = -\frac{\partial R/\partial g}{\partial R/\partial k} = \frac{f_g\left[(1-\alpha)\eta_k^{f_g}+\eta_k^2\right]}{f_k\left[(1-\eta_k)\eta_k-(1-\alpha)\eta_k^{f_k}\right]} > 0$ (see Appendix 1). In addition, in view of Equation (22) and Proposition 1, it is fairly easy to see that private capital return changes positively with changes in the per-capita public investment policy whenever $\frac{dk}{dg} \leq 0$. Thus, exploiting Equations (18) and (21) we have:

$$\begin{aligned} \frac{dR}{dg} &= \frac{1}{D}\left[\left(1+n+\eta_g(1+r)\frac{\partial S}{\partial W}\right)\frac{\partial R}{\partial g}+... \right. \\ &\left. \left((f_g-(1-\tau)(1+n))\frac{\partial S}{\partial W}+\left[1-\frac{\partial S}{\partial W}\right]\frac{(1+n)^2}{1+r}\tau\right)\frac{\partial R}{\partial k}\right]. \end{aligned} \quad (23)$$

That is, $\frac{dR}{dg}$ can be expressed as a function of the partial changes that the per-capita public investment and the private capital-labour ratio cause in the private capital return. The coefficient of the first partial change is positive which prompts the first part of (23) to be positive since $\frac{\partial R}{\partial g} > 0$. On the other hand, the coefficient of the second partial change does not have a clear-cut sign and it can be negative under some reasonable conditions which depends on the over-accumulation of per-capita public capital-labour ratio. The following Proposition is devoted to stating other conditions upon which $\frac{dR}{dg}$ has a clear-cut sign.

Proposition 5 *The private capital return will be increased by increasing the per-capita public investment policy whenever $\tau = 0$ and $f_g \leq 1+n$.*

The proof is straightforward in view of (23).

As we can see, Proposition 5 states that a sufficient condition for an increase in private capital return, as a consequence of an increase in the per-capita public investment policy, is that the marginal product of public capital stock must be no higher than its growth rate along the balanced growth path. As it has been stated earlier, this is equivalent to an over-accumulation of public capital-labour ratio for the long-run steady state. Note that according to Equation (21) the over-accumulation of public capital-labour ratio is a necessary (but not sufficient) condition for the negativeness of $\frac{dk}{dg}$. In this trend Proposition 5 states that private capital return can rise as a consequence of an increase in the per-capita public investment policy whenever the effect on the long-run steady state amount of private capital-labour ratio is not positive enough to compensate the partial effect of this policy on the return of private capital. On the other hand, this effect is as clear as the investment policy is entirely financed by the younger generation due to the fact that when public capital stock is funded only by taxes on the younger generation, an increase in such taxes would decrease savings, because the younger generation expects to pay lower taxes in the future, with the direct consequence of an increase in private capital return.

Finally, regarding the effects of a change in the per-capita public investment policy on the utility of an individual living in the long-run steady state, let us assess the total derivative of (19) with respect to g, exploiting the first order condition for the individual's equilibrium (4),

$$\frac{dV}{dg} = \frac{\partial u}{\partial c^y}\left[\left(\frac{\partial c^y}{\partial W} + \frac{1}{1+r}\frac{\partial c^o}{\partial W}\right)\frac{dW}{dg} + \left(\frac{\partial c^y}{\partial r} + \frac{1}{1+r}\frac{\partial c^o}{\partial r}\right)\frac{dR}{dg} + ...\right.$$

$$\left. ... + \left(\frac{\partial c^y}{\partial T^y} + \frac{1}{1+r}\frac{\partial c^o}{\partial T^y}\right)\frac{dT^y}{dg} + \left(\frac{\partial c^y}{\partial T^o} + \frac{1}{1+r}\frac{\partial c^o}{\partial T^o}\right)\frac{dT^o}{dg}\right].$$

But, according to Equation (3), $\frac{\partial c^y}{\partial W} + \frac{1}{1+r}\frac{\partial c^o}{\partial W} = 1$, $\frac{\partial c^y}{\partial r} + \frac{1}{1+r}\frac{\partial c^o}{\partial r} = \frac{S}{1+r}$, $\frac{\partial c^y}{\partial T^y} + \frac{1}{1+r}\frac{\partial c^o}{\partial T^y} = -1$ and $\frac{\partial c^y}{\partial T^o} + \frac{1}{1+r}\frac{\partial c^o}{\partial T^o} = \frac{-1}{1+r}$,

$$\frac{dV}{dg} = \frac{\partial u}{\partial c^y}\left[\frac{dW}{dg} + \frac{S}{1+r}\frac{dR}{dg} - \frac{dT^y}{dg} - \frac{1}{1+r}\frac{dT^o}{dg}\right].$$

On the other hand, taking into account Equation (12), $\frac{dW}{dg} = f_g - \eta_g(1+r)\frac{dk}{dg} - \frac{dR}{dg}k$, and Equations (15) we have

$$\frac{dV}{dg} = \frac{\partial u}{\partial c^y}\left[f_g - (1+n)\left[1 + \tau\left(\frac{n-r}{1+r}\right)\right] + ...\right. \tag{24}$$

$$\left. ... + \left(\frac{n-r}{1+r}\right)k\frac{dR}{dg} - \eta_g(1+r)\frac{dk}{dg}\right].$$

As we can see, the sign of (24) is ambiguous in general as long as the signs of its first and third part are negatively related. The following Proposition states a sufficient condition upon which (24) has a clear-cut sign.

Proposition 6 *The utility of an individual living in the long-run will be decreased by increasing the per-capita public investment policy whenever* $\tau = 0$, $f_g \leq 1+n$, *and* $n \leq r$.

Proof: see Appendix 3.

From Proposition 6 we hold that in our model the utility of the representative individual living in the long-run steady state may decline as a consequence of a rise in the per-capita public investment. This is because indirect utility is measured in terms of present value and the particular and sufficient conditions work in the direction to increase the opportunity cost of present consumption. On the one hand, the entire tax burden is borne by the younger generation which reduces the present disposable income affecting welfare in equilibrium and, thus, consumption when young. Secondly, the marginal product of public capital stock must be no higher than its growth rate along a balanced growth path, which means that there is an over-accumulation of public capital-labour ratio for the long-run steady state. Note that, according to the former requirement and Proposition 5, in such a case the return of private capital rises with increases in the per-capita public investment policy. Thirdly, the opportunity cost of consumption when young has to be greater than the rate at which savings has to grow along the balanced growth path, a condition which does not necessarily mean an under-accumulation of private capital-labour ratio in our model.

4 Final comments

In this paper we have used an overlapping generations model to analyse some long-run steady state effects prompted by free-access public capital when it enters a constant-returns-to-scale aggregated production function. The model allow us to characterize the optimal golden-age path amounts of both per-capita private and public capital ratios in such a way that we can understand how the under- or over-accumulation of public capital stock in the long-run steady state affects our results. To characterize the rent dissipation phenomenon arising from the free-access public capital basis, we follow Feehan and Batina's (2007) price rule, which makes the public capital stock equivalent to a common property resource, giving rise to inefficiency. In addition, public policy consists of providing a certain amount of per-capita public investment which is funded by means of taxes in both the younger and older generations. As a consequence of that, we obtained results which concern both changes in the share of the tax burden, by shifting it from the younger to the older generation, holding public investment constant, and changes in the constant per-capita public investment policy, by increasing the public capital-labour ratio. In the first case a shift in the tax burden from the younger to the older generation increases the amount of the long-run steady state private capital-labour ratio. Such a change may decrease the utility of an individual living in the long-run steady state whenever the equilibrium interest rate is no higher than the rate at which it should have to grow along the balanced growth path, a situation that corresponds to an over-accumulation of private capital-labour ratio. Although these results are similar to the standard propositions of the life-cycle models, they differ in certain important details, according to the way in which public capital stock enters the production function in our model.

In addition, regarding the case of changes in the per-capita public investment policy, the distortion exerted by the dissipation of the public capital return among the remaining private factor prices may allow for positive changes in both the private capital return and its capital-labour ratio by enhancing the public capital-labour ratio. Moreover, the over- or under-accumulation of long-run public capital-labour ratio with respect to that of the optimal golden-age path plays an important role in the per-capita public investment policy. In this trend, our main results are that a long-run public capital-labour ratio under-accumulation is sufficient to give rise to an increase in private capital-labour as a consequence of increases in per-capita public investment; meanwhile, an over-accumulation is necessary to give rise to an increase in private capital returns and a decrease in the utility of an individual living in the long run, even when such a policy may enhance the private capital-labour ratio. This last conclusion depends on the fact that the entire tax burden has to be borne by the younger generation, and the equilibrium interest rate has to be greater than the rate at which savings has to grow along the balanced growth path.

Appendix

1. Proof of Proposition 1.

Let us introduce the following notation for the elasticities:

$$\begin{aligned} \eta_i &= f_i \frac{i}{f}, \quad i = k_t, g_t. \\ \eta_j^{f_i} &= f_{ij} \frac{j}{f_i}, \quad i, j = k_t, g_t. \end{aligned} \tag{25}$$

From (9) it is fairly easy to see that

$$\eta_i > 0, \quad \eta_i^{f_i} < 0, \quad \eta_j^{f_i} > 0, \quad i, j = k_t, g_t; \quad i \neq j.$$

On the other hand, by dividing Equation (8) by f we hold that

$$0 < \eta_k + \eta_g = \alpha < 1, \tag{26}$$

and, partially deriving Equation (8) with respect to k_t (g_t), and dividing by f_k (f_g) we have

$$\eta_g^{f_k} + \eta_k^{f_k} = \alpha - 1 < 0, \tag{27}$$

and

$$\eta_k^{f_g} + \eta_g^{f_g} = \alpha - 1 < 0. \tag{28}$$

by making the same operation with respect to g_t.

To prove that $\frac{\partial R_t}{\partial g_t} > 0$, let us take the partial derivative of (11) with respect to g (subindex t is dropped),

$$\frac{\partial R_t}{\partial g_t} = \frac{(f_{kg} f + f_k f_g)(f - g f_g) - f_k f (f_g - f_g - g f_{gg})}{(f - g f_g)^2},$$

rearranging terms and taking the notation in terms of the elasticities (25)

$$\frac{\partial R_t}{\partial g_t} = \frac{f_g}{k_t (1 - \eta_g)^2} \left[\eta_k^{f_g} (1 - \eta_g) + \eta_k \eta_g^{f_g} + (1 - \eta_g) \eta_k \right],$$

taking into account (26) it can be written as

$$\frac{\partial R_t}{\partial g_t} = \frac{f_g}{k_t(1-\eta_g)^2}\left[\eta_k^{f_g}(1-\alpha+\eta_k) + \eta_k\eta_g^{f_g} + (1-\alpha+\eta_k)\eta_k\right],$$

rearranging terms

$$\frac{\partial R_t}{\partial g_t} = \frac{f_g}{k_t(1-\eta_g)^2}\left[\eta_k^{f_g}(1-\alpha) + \eta_k(\eta_k^{f_g} + \eta_g^{f_g}) + (1-\alpha)\eta_k + \eta_k^2\right],$$

exploiting (28) and clearing

$$\frac{\partial R_t}{\partial g_t} = \frac{f_g}{k_t(1-\eta_g)^2}\left[(1-\alpha)\eta_k^{f_g} + \eta_k^2\right] > 0. \blacksquare$$

To prove that $\frac{\partial R_t}{\partial k_t} < 0$, let us take the partial derivative of (11) with respect to k (subindex t is dropped),

$$\frac{\partial R_t}{\partial k_t} = \frac{(f_{kk}f + f_k^2)(f - gf_g) - f_k f(f_k - gf_{gk})}{(f - gf_g)^2},$$

rearranging terms and taking the notation in terms of the elasticities (25)

$$\frac{\partial R_t}{\partial k_t} = \frac{f_k}{k_t(1-\eta_g)^2}\left[\eta_k^{f_k}(1-\eta_g) + \eta_k\eta_g^{f_k} - \eta_k\eta_g\right],$$

taking into account (26) it can be written as

$$\frac{\partial R_t}{\partial k_t} = \frac{f_k}{k_t(1-\eta_g)^2}\left[\eta_k^{f_k}(1-\alpha) + \eta_k(\eta_k^{f_k} + \eta_g^{f_k}) - \eta_k\eta_g\right],$$

exploiting (27), and rearranging terms

$$\frac{\partial R_t}{\partial k_t} = \frac{f_k}{k_t(1-\eta_g)^2}\left[\eta_k^{f_k}(1-\alpha) - (1-\alpha+\eta_g)\eta_k\right],$$

finally, taking into account (26) it can be written as:

$$\frac{\partial R_t}{\partial k_t} = \frac{f_k}{k_t(1-\eta_g)^2}\left[(1-\alpha)\eta_k^{f_k} - (1-\eta_k)\eta_k\right] < 0. \blacksquare$$

2. Optimal golden-age path

In order to assess the optimal golden-age path we act analogously to the case of two factors. Thus, let us compute the path which maximizes the lifetime utility of the representative household subject to the economy-wide steady-state resource constraint:

$$\begin{aligned} & \max u(c^y, c^o) \\ \text{s.t. } c^y + \frac{1}{1+n}c^o &= f(k,g) - (1+n)k - (1+n)g. \end{aligned}$$

The first-order conditions for the optimal golden-age path consist of the steady-state resource, the so-called biological-interest-rate consumption golden rule

$$\frac{\partial u/\partial c^o}{\partial u/\partial c^y} = \frac{1}{1+n},$$

and the production golden rule which defines the dynamic efficiency (Diamond, 1965) for both private and public capital-labour ratio,

$$\begin{aligned} f_k &= 1+n, \\ f_g &= 1+n. \end{aligned}$$

Both the biological-interest-rate consumption golden rule and the production golden rule are analytically independent (Samuelson, 1968), in such a way that the optimum consumption pattern and the optimality in the division of output among generations may be not held simultaneously.

3. Proof of Proposition 6.

By taking $\tau = 0$, taking into account (18) and (21) and operating, Equation (24) can be written as:

$$\begin{aligned} \frac{dV}{dg} &= \frac{\partial u}{\partial c^y} \left\{ \left(\frac{n-r}{1+r} \right) k \frac{dR}{dg} + ... \right. \\ & ... + \frac{1}{D} \left[(f_g - (1+n)) \left(1 + n - \left(\frac{\partial S}{\partial r} - \frac{\partial S}{\partial W} k \right) \frac{\partial R}{\partial k} \right) - ... \right. . \\ & \left. \left. ... - \eta_g (1+r) \left(\frac{\partial S}{\partial r} - \frac{\partial S}{\partial W} k \right) \frac{\partial R}{\partial g} \right] \right\} \end{aligned}$$

Since $\left(\frac{\partial S}{\partial r} - \frac{\partial S}{\partial W} k \right) > 0$, the third part of the equation is negative, the second part is negative whenever $f_g - (1+n) \leq 0$, which, according to proposition 5, is also sufficient for $\frac{dR}{dg} > 0$, thus the first part of the equation is not positive if $n \leq r$.■

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