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Frictional and Non Frictional Unemployment in Models with Matching Frictions

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Frictional and Non Frictional Unemployment in Models with Matching Frictions[*]

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Abstract

This paper uses a model with a matching function in the labor market, where matches last for one period, to obtain the amount of frictional and non frictional (rationed/disequilibrium) unemployment for different standard wage-setting rules when there are matching frictions. We also compute the frictional and non frictional unemployment rate for two economies characterized by different labor market institutions, namely the Spanish and US economies. The empirical analysis takes into account two types of micro-foundations of the matching function: coordination failure and mismatch due to heterogeneity in the labor market. The empirical findings for Spain suggest that approximately half of all unemployment is due to job rationing and the other half to frictional and mismatch problems. However, the rationing unemployment rate for the US economy represents, two thirds of all unemployment on average, while frictional and mismatch problems account for only a third.

Keywords: matching frictions, frictional unemployment, disequilibrium unemployment.

JEL classification numbers: E24, O41.

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1 Introduction

The depth and persistence of the current crisis has fundamentally affected the US and euro area economies in the following ways: i) banks are lending less money to businesses and consumers, ii) unemployment is rising and has reached a very high level in the US and the euro area iii) there is more social spending and less revenue, which in turn leads to a high government deficit and debt levels. All these problems are closely related, making it perhaps difficult to solve one problem by isolating it from the rest. However, in this paper we focus on the problem of unemployment. We believe that the unemployment problem in Italy, Greece, Ireland, Portugal and Spain is crucial to escape from this crisis. More specifically, the high unemployment rate in Spain is the most important problem in the Euro zone. The unemployment rate in Spain was last reported at 26.02 percent in the fourth quarter of 2012. More importantly, projections are very pessimistic. The last report on the unemployment rate released by the OECD forecasts a rise to 26.9% in 2013. However, one major problem the country faces to reduce unemployment is ascertaining the origin of aggregate unemployment. Thus, the aim of this paper is to introduce a new method to disentangle the unemployment rate by origin. More specifically, we decompose unemployment under frictional and rationing unemployment into a large number of relevant wage setting mechanisms that appear in the theoretical literature.

The reason for this breakdown is that in models with frictional unemployment, one can always ask the following question: as wage setting exists in these models, that is, the labor market is not competitive, how much employment is due to the wage being set above the competitive wage and how much is due to friction. In order to answer this question, the model must be analyzed using the same wage-setting rule and eliminating frictions.

Michaillat (2011) does so, obtaining that with the standard surplus sharing rule, if one eliminates frictions, the wage set is the competitive wage, which then yields that all the employment obtained in these models is frictional[1]. In order to have non frictional (rationed) unemployment, the wage setting-rule must be changed. Michaillat (2011) also shows that with rigid wages, that is, with an exogenous given wage rule, eliminating frictions and maintaining this wage setting-rule results in non frictional unemployment. Moreover, he analyzes how frictional and non frictional unemployment changes with this wage setting rule when there are technological shocks.

This paper follows Michaillat's procedure, developing some aspects not present in his models. First, in his paper eliminating frictions means not employing a matching function that implies no frictions, but making the cost of opening a vacancy equal to zero.

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[1]Even with constant or decreasing marginal product of labor.

However, this circumstance results in these frictions in the labor market not disappearing in his model. Unlike, Michaillant (2011), we take the alternative approach of using a matching function that eliminates frictions and maintains a positive cost for opening a vacancy[2]. Second, we adapt the model to the case where matches last for only one period. This case appears in Bean and Pissarides (1993) and in our opinion it has two advantages over the model in which only some employed workers lose their jobs. First, it is directly comparable with the standard model of the labor market without frictions, with a labor demand and a labor supply function and, second, considering this case, there can be changes in employment over time, thereby avoiding the assumption of a constant employment rate.

As in Michaillat (2011), we show that with rigid wages, one normally obtains non frictional unemployment, but the problem of using this wage rule is that one does not know where wages came from. We complete our presentation by analyzing four additional wage-setting rules: setting the wage according to the marginal product of labor, according to labor productivity and finally assuming that there is a union in the labor market that sets the wage at firm level and at a more centralized level. We obtain that, depending on the specific wage-setting rule assumed, there are cases where there is either only frictional or only non frictional unemployment.

Finally, we propose a means of analyzing how to compute frictional and non frictional unemployment rates based on a model with rigid wages and compute it for the Spanish and US labor markets. The empirical findings for Spain suggest that about half of all unemployment is due to job rationing and the other half to frictional and mismatch problems. However, the rationing unemployment rate in the US economy represents on average two thirds of all unemployment, while frictional and mismatch problems account for only one third.

The remainder of the paper is organized as follows. In the next section we present a model to determine the amount of employment in a labor market with frictions. Additionally, we obtain frictional and non frictional unemployment taking into account a simply exogenous given wage. In the interest of clarity, the appendix of the article includes the study of the four wage setting mechanisms mentioned previously. The third section is devoted to computing the frictional-rationing unemployment rate for Spain and the USA. The fourth and final sections presents some comments and concluding remarks.

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[2]For example, Mortensen and Pissarides (2001) show the role of the cost of recruiting on the one hand and hiring and training costs on the other. More recently, Brown et. al (2011) explain that vacancy posting costs are incurred before firms and workers make contact, whereas the hiring cost is incurred by the firm after contact is made. In this paper, we focus on the cost of recruiting or vacancy posting cost.

3

2 Theoretical Framework

In the first place, this section presents the theoretical framework for formalizing the amount of employment in a labor market with frictions in regard to a labor market without frictions. In order to address these results, we derive the number of vacancies opened (announced) by a firm and solve profit maximization with respect to employment, expressly taking into account the cost of opening a vacancy.

In this section our aim is to fully characterize aggregate unemployment and disentangle it into frictional and rationing unemployment. In order to accomplish this goal, we must assume a wage formation mechanism. Therefore, in the next subsection we review a large number of relevant wage formation mechanisms that appear in the theoretical literature (the appendix details some of these mechanisms), and break down the aggregate unemployment rate into frictional and rationing unemployment.

2.1 Employment in a labor market with frictions.

In this subsection, we describe the key features of the labor market matching framework with frictions relative to the case in which there are no frictions. As in Bean and Pissarides (1993), we assume a labor market where matches last for one period. The reason for choosing this approach is because it is easy to perform a direct comparison with the standard labor market, when there are no frictions, characterized by the usual labor demand and supply functions. The matching function is

$$\bar{L} = m(\bar{L}^d, \bar{L}^s = N) \tag{1}$$

where $\bar{L}$ is the aggregate employment flow (units of labor employed in a period) and $\bar{L}^d$ denotes aggregate labor demand or vacancies opened by firms at the beginning of a period[3]. Finally, let $\bar{L}^s$ be the aggregate labor supply or unemployment at the beginning of a period, which we assume is inelastic and equal to population N. Unemployment at the end of the period is $U = N - \bar{L}$ and unfilled vacancies at the end of the period are $V_U = \bar{L}^d - \bar{L}$. We omit time subscripts when they are not necessary and assume discrete time.

When there are frictions in the labor market, the properties of the matching function are[4] $\frac{\partial m}{\partial \bar{L}^d} > 0$, $\frac{\partial m}{\partial N} > 0$, $\frac{\partial^2 m}{\left(\partial \bar{L}^d\right)^2} < 0$, $\frac{\partial^2 m}{(\partial N)^2} < 0$, $m(0, N) = m(\bar{L}^d, 0) = 0$, $m(\infty, N) = N$, $m(\bar{L}^d, \infty) = \bar{L}^d$ and, for positive numbers, $\bar{L} = m(\bar{L}^d, N) < \min(\bar{L}^d, N)$. In a labor market without frictions the matching function is $\bar{L} = \min(\bar{L}^d, N)$.

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[3] We identify labor demand with vacancies is because matches last for one period. In a general situation, vacancies are the difference between labor demand and people already employed.

[4] For more detail, see, Bean and Pissarides (1993).

One key element of this analysis is to ascertain the relationship between the number of vacancies opened and the number of workers employed by the firm. We begin by obtaining the following inverse matching function for $\bar{L}^d$ from expression (1)

$$\bar{L}^d = m^{-1}(\bar{L}, N) \tag{2}$$

which yields the number of aggregate vacancies needed, for a given labor supply, in order to obtain $\bar{L}$ matches (units of labor employed). In this case, the inverse matching function has the following properties $\frac{\partial m^{-1}}{\partial \bar{L}} > 0$, $\frac{\partial m^{-1}}{\partial N} < 0$, $m^{-1}(0, N) = 0$, $m^{-1}(N, N) = \infty$, $m^{-1}(\bar{L}, \infty) = \bar{L}$.

There are three possible outcomes from this expression that take into account two elements: whether or not there are frictions and whether or not there is an excess of supply or demand in the labor market. Case (i) is characterized by the presence of frictions and the inverse matching function $\bar{L}^d$ is defined by the expression (2), as indicated above. If there are no frictions, then there are also two possibilities, namely Case (ii) under an excess of demand in the labor market, $\bar{L}^d = m^{-1}(\bar{L}, N)$ is any number $\bar{L}^d \geqq \bar{L}$ if $\bar{L} = N$ and Case (iii) under an excess of supply in the labor market, whereby $\bar{L}^d = m^{-1}(\bar{L}, N) = \bar{L}$ if $\bar{L} < N$, that is, there is unemployment.

We assume infinite firms within the interval $[0; 1]$ where the subscript i represents the firm level. We consider that if a firm wants to employ L_i units of labor, knowing the inverse matching function, the number of vacancies the firm opens (announces) L_i^d will be proportional to the total number of vacancies required to employ $\bar{L}$ units of labor, that is:

$$L_i^d = \frac{\bar{L}^d}{\bar{L}} L_i = \frac{m^{-1}(\bar{L}, N)}{\bar{L}} L_i \tag{3}$$

Of course, if there are frictions, L_i^d is a unique number and $L_i^d > L_i$ because $\bar{L}^d > \bar{L}$. If there are no frictions and $\bar{L} < N$, then $m^{-1}(\bar{L}, N) = \bar{L}$ and if a firm wants to employ L_i units of labor, the number of vacancies opened is L_i^d, which in the case of (3) will be equal to L_i. When $\bar{L} = N$, even with an aggregate number of vacancies $\bar{L}^d > \bar{L}$, we obtain $\bar{L}$ units of labor employed and assume that the firm sets $L_i^d = \frac{\bar{L}^d}{\bar{L}} L_i$.

It is important to emphasize that this relationship between announced vacancies (labor demand), L_i^d, and units of labor employed L_i is known by the firm. Thus, the optimization problem of the representative firm to choose L_i (employment) and K_i (capital) is given by

$$AF(K_i, L_i) - wL_i - \gamma_0 L_i^d - (r + \delta)K_i = AF(K_i, L_i) - wL_i - \gamma_0 \frac{m^{-1}(\bar{L}, N)}{\bar{L}} L_i - (r + \delta)K_i$$

where w is the real wage, γ_0 the cost of opening a vacancy, r the interest rate and δ the depreciation rate. Finally, we assume that $AF(K_i, L_i)$ is a neoclassical production

5

function where A is total factor productivity. The first order condition for (optimal) employment in firm i yields

$$AF_L(K_i, L_i) = w + \gamma_0 \frac{m^{-1}(\bar{L}, N)}{\bar{L}}$$

assuming $\gamma_0 = \gamma w$, it is possible to rewrite the above expression as follows[5]:

$$AF_L(K_i, L_i) = \left[1 + \gamma \frac{m^{-1}(\bar{L}, N)}{\bar{L}}\right] w = \left[1 + \frac{\gamma}{\frac{\bar{L}}{m^{-1}(\bar{L}, N)}}\right] w. \qquad (4)$$

The term L_i is the amount of employment that the firm wants to have and, thus, the number of vacancies (labor demand) that the firm announces is $L_i^d = \frac{m^{-1}(\bar{L}, N)}{\bar{L}} L_i$. Note that if the cost of vacancies γ is equal to 0, then the amount of employment chosen by the firm is given by $AF_L(K_i, L_i) = w$. This expression is the usual equation for labor demand without frictions except that now, in order to obtain this amount of employment, the firm sets labor demand using $L_i^d = \frac{m^{-1}(\bar{L}, N)}{\bar{L}} L_i$.

Now, assuming that all firms are equal, $L_i = \bar{L} = L$ and $K_i = K$ the amount of aggregate employment in a model with frictions in the labor market L_F is given by:

$$AF_L(K, L_F) = \left[1 + \gamma \frac{m^{-1}(L_F, N)}{L_F}\right] w \qquad (5)$$

This expression implies that the labor demand in a labor market with frictions is $L_F^d = m^{-1}(L_F, N)$.

In a labor market with frictions there is always unemployment, $L_F < N$, for a positive wage because we know, from the matching function that, $L_F = m(L_F^d, N) < \min(L_F^d, N) \leqq N$ for a real value of L_F^d. This value is obtained, as mentioned above, using the inverse matching function for a positive value of employment. When the wage is equal to zero firm i wants infinite employment but, knowing that $m(\infty, N) = N$, the maximum amount of employment will be N, so the firm asks for infinite vacancies, obtaining N units of labor and N is the aggregate amount of labor. Figure 1 represents, L_F and L_F^d in a labor market with frictions.

The qualitative impact of exogenous variables on L_F in a labor market with frictions can be expressed as:

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[5]One can compare this equation to the standard equation obtained when only a constant proportion q of workers lose their jobs ($q = 1$ when matches last for one period), assuming steady state for $\theta = \frac{V}{U}$ and $m(\theta) = \frac{M(V;U)}{V}$ (Cahuc and Zylberberg (2004) equation (30), chapter 9):

$$AF_L(K, L) = w + \frac{\gamma(r + q)}{m(\theta)}.$$

The assumption that matches last for one period makes the dynamic program of the firm static.

$$\frac{\partial L_F}{\partial w} = -\frac{-\left[1 + \gamma\frac{m^{-1}(L,N)}{L}\right]}{AF_{LL} - \gamma\frac{m^{-1}(L,N)}{L^2}\left[\frac{\partial m^{-1}}{\partial L}\frac{L}{m^{-1}} - 1\right]} \tag{6}$$

that is < 0 as long as $\frac{\partial m^{-1}}{\partial L}\frac{L}{m^{-1}} > 1$.

$$\frac{\partial L_F}{\partial N} = -\frac{-\left[\frac{w\gamma}{L}\frac{\partial m^{-1}}{\partial N}\right]}{AF_{LL} - \gamma\frac{m^{-1}(L,N)}{L^2}\left[\frac{\partial m^{-1}}{\partial L}\frac{L}{m^{-1}} - 1\right]} \tag{7}$$

that is > 0 as long as $\frac{\partial m^{-1}}{\partial L}\frac{L}{m^{-1}} > 1$.

$$\frac{\partial L_F}{\partial \gamma} = -\frac{-\left[1 + w\frac{m^{-1}(L,N)}{L}\right]}{AF_{LL} - \gamma\frac{m^{-1}(L,N)}{L^2}\left[\frac{\partial m^{-1}}{\partial L}\frac{L}{m^{-1}} - 1\right]} \tag{8}$$

that is < 0 as long as $\frac{\partial m^{-1}}{\partial L}\frac{L}{m^{-1}} > 1$.

$$\frac{\partial L_F}{\partial A} = -\frac{F_L}{AF_{LL} - \gamma\frac{m^{-1}(L,N)}{L^2}\left[\frac{\partial m^{-1}}{\partial L}\frac{L}{m^{-1}} - 1\right]} \tag{9}$$

that is > 0 as long as $\frac{\partial m^{-1}}{\partial L}\frac{L}{m^{-1}} > 1$.

The condition $\frac{\partial m^{-1}}{\partial L}\frac{L}{m^{-1}} > 1$ means that the elasticity of the inverse matching function with respect to the wage is greater than one. That is, in order to increase employment by a given %, there must be a larger % increase in labor demand. The same occurs, for example, with the Cobb-Douglas matching function $L = m(L^d, N) = B(L^d)^\varphi(N)^{1-\varphi}$ with $0 < \varphi < 1$. In this case, the inverse matching function is $L^d = m^{-1}(L, N) = B^{-\frac{1}{\varphi}}L^{\frac{1}{\varphi}}N^{1-\frac{1}{\varphi}}$ with $\frac{\partial m^{-1}}{\partial L}\frac{L}{m^{-1}} = \frac{1}{\varphi} > 1$.

By contrast, in a labor market without frictions $L = \min(L^d, N)$, if there is an excess of supply in the labor market $L^d < N$, then $L^d = m^{-1}(L, N) = L$. Replacing this expression in (5) yields the usual condition for computing labor demand when there are no frictions and the firm does not take into account the matching friction in its program when opening a vacancy has a cost.

$$AF_L(K, L) = [1 + \gamma]w. \tag{10}$$

If the previous equation gives an amount of labor $\tilde{L}$ such that $\tilde{L} < N$, we therefore have that the amount of non frictional employment is equal to $L_{NF} = \tilde{L} = L_{NF}^d < N$. When there is an excess of demand of labor in the labor market, it must be that $L^d \geqq N$ and then $L = \min(L^d, N) = N$, in this case we also must have $\tilde{L} \geqq N$. In summary, in a labor market without frictions, when $\tilde{L} < N$ then $L_{NF} = \tilde{L} = L_{NF}^d < N$ and when $\tilde{L} \geqq N$ then $L_{NF} = N < L_{NF}^d$.

Finally, we have that $L_F < \tilde{L}$[6] and, as observed above, for a positive wage $L_F < N$,

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[6]This is because $AF_L(K, L_F) = \left[1 + \gamma\frac{m^{-1}(L_F, N)}{L_F}\right]w > [1 + w] = AF_L(K, \tilde{L})$, that is, $AF_L(K, L_F) >$
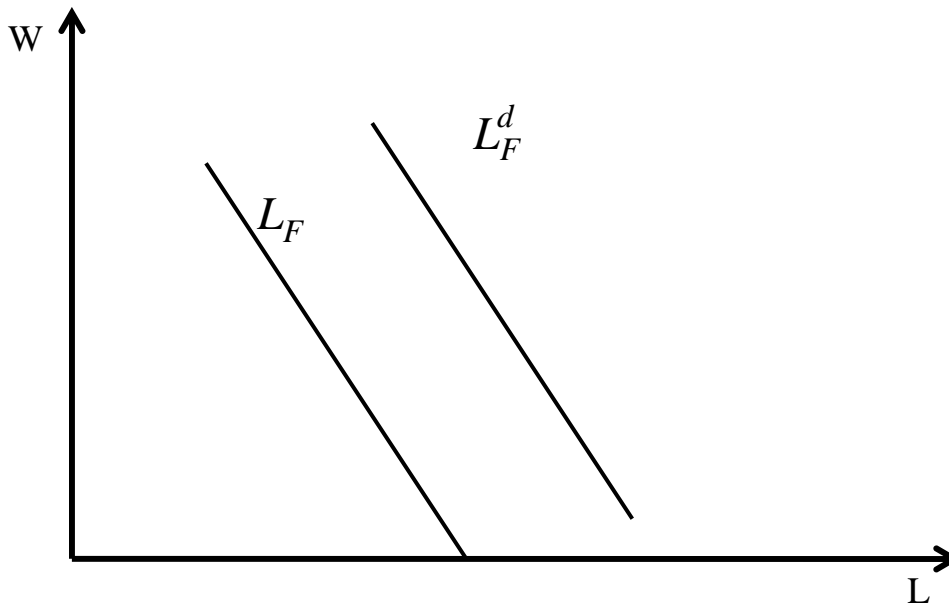
FIGURES

Fig. 1 A picture of L_F and L_F^d

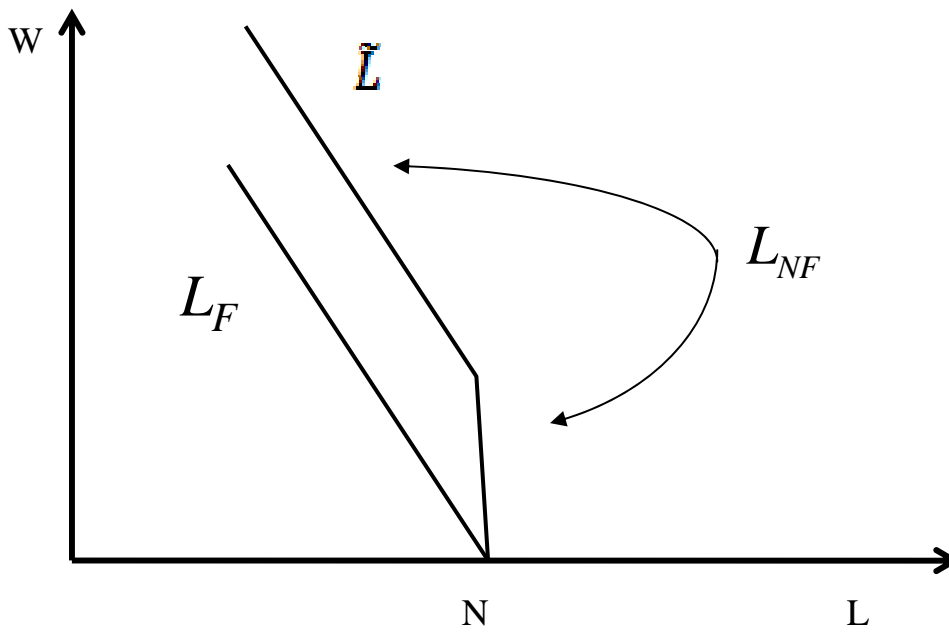Fig. 2 A picture of L_F and L_F^d

which means that for a positive wage $L_F < L_{NF}$ and for zero wages $L_F = L_{NF}$. Figure 2 presents the pictures of L_F and L_{NF}, in terms of the wage in a labor market with and without frictions[7].

2.2 Wage setting and frictional and non frictional unemployment

The next step in our analysis consists of analyzing frictional and non frictional unemployment under different wage setting rules. From the results presented above, it is clear that in a competitive labor market with frictions, the competitive wage is zero because for a positive wage,there is always unemployment and the unemployed people accept a lower wage. When the wage is zero, we have full employment, because the firm asks for infinite vacancies. Of course, if there are no frictions, we have a positive competitive wage. In order to have unemployment in a labor market with frictions, we must assume that the labor market is not competitive and that somebody sets the wage. When there is a specific wage equation, the intersection between the employment equation in a labor market with frictions, L_F, and the wage equation yields the equilibrium amount of employment and the wage (L^*, w^*). In this case, the amount of unemployment (at the end of the period) is given by $U = N - L^*$ and the unemployment rate in a labor market with frictions is $u = \frac{N-L^*}{N}$.

Following a similar procedure, the intersection between the employment equation in a labor market without frictions, L_{NF}, and the wage equation gives the equilibrium amount of employment and the wage in the same labor market without frictions (L_{NF}^*, w_{NF}^*). Then, the amount of non frictional (rationed/disequilibrium) unemployment[8] is given by $U_{NF} = N - L_{NF}^*$ and the unemployment rate in a labor market without frictions is defined by the expression $u_{NF} = \frac{N-L_{NF}^*}{N}$. Thus, if it turns out that $L_{NF}^* > L^*$ then we say that frictions in the labor market produce the amount of frictional unemployment $U_F = L_{NF}^* - L^*$ and the frictional unemployment rate $u_F = \frac{L_{NF}^*-L^*}{N}$. Of course, we have that $U = U_{NF} + U_F$ and $u = u_{NF} + u_F$. Figure 3 illustrates these concepts for a typical wage equation with a positive slope. The following step is to decompose the unemployment rate into frictional and non frictional unemployment for some standard wage setting rules that appear in the literature.

The first wage-setting rule analyzed is that used in the next section: an exogenous given wage: $w = \bar{w}$ (Blanchard and Galí (2011), Hall (2005), Schimer (2004), Michaillat

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$AF_L(K, \tilde{L})$ and this is only possible if $L_F < \tilde{L}$.

[7]It is difficult to prove that $L_F^d < \tilde{L}$.

[8]Alternatively, we may define the equilibrium amount of employment without frictions as the intersection between L_F^d and the wage equation, a misleading definition in our opinion because L_F^d is the labor demand of the firm when there are frictions.

8

(2011)[9]). In particular, the equilibrium amount of employment in a labor market with frictions L^* is given by

$$AF_L(K, L^*) = \left[1 + \gamma \frac{m^{-1}(L^*, N)}{L^*}\right] \bar{w} \tag{11}$$

having that $L^* < N$ and that, as we saw in the previous section, $\frac{\partial L_F}{\partial A} > 0$ as long as $\frac{\partial m^{-1}}{\partial L} \frac{L}{m^{-1}} > 1$, meaning that with this wage-setting rule technological shocks produce fluctuations in employment when there are frictions. The equilibrium amount of employment in a labor market without frictions L_{NF}^* is given by the $\tilde{L}$ such that

$$AF_L(K, \tilde{L}) = [1 + \gamma] \bar{w} \tag{12}$$

if $\tilde{L} < N$ and by N if $\tilde{L} \gtreqqless N$.

From the previous section we know that for the same given wage $\bar{w}$, $L_{NF}^* > L^*$ and with this wage-setting rule there is always frictional unemployment. Moreover, if the wage set $\bar{w}$ is greater than the competitive wage of a labor market without frictions, that is $\tilde{L} < N$, we have also rationed unemployment. Figure 4 illustrates this case. Summarizing, in a labor market with frictions this wage-setting rule always generates frictional unemployment and, depending on the wage set, it may generate non frictional (rationed) unemployment.

In this context, a decrease in A implies a reduction in L^* (if $\frac{\partial m^{-1}}{\partial L} \frac{L}{m^{-1}} > 1$) and L_{NF}^* (if $L_{NF}^* = \tilde{L} < N$), in other words, a negative supply shock increases both the unemployment rate $\frac{N-L^*}{N}$ and also the non frictional unemployment rate $\frac{N-L_{NF}^*}{N}$. In this case, the effect on the frictional unemployment rate is given by:

$$
\begin{aligned}
\frac{\partial u_F}{\partial A} &= \frac{1}{N}\left[\frac{\partial L_{NF}^*}{\partial A} - \frac{\partial L^*}{\partial A}\right] = -\frac{F_L}{AF_{LL}} - -\frac{F_L}{AF_{LL} - \gamma \frac{m^{-1}(L,N)}{L^2}\left[\frac{\partial m^{-1}}{\partial L} \frac{L}{m^{-1}} - 1\right]} = \\
&= \frac{F_L\left(\gamma \frac{m^{-1}(L,N)}{L^2}\left[\frac{\partial m^{-1}}{\partial L} \frac{L}{m^{-1}} - 1\right]\right)}{AF_{LL}\left(AF_{LL} - \gamma \frac{m^{-1}(L,N)}{L^2}\left[\frac{\partial m^{-1}}{\partial L} \frac{L}{m^{-1}} - 1\right]\right)} > 0
\end{aligned}
$$

as long as $\frac{\partial m^{-1}}{\partial L} \frac{L}{m^{-1}} > 1$, that is, a reduction in A or a negative supply shock decreases the frictional unemployment rate as shown in Michaillat (2011).

The problem with using this wage rule is that the wage is set exogenously. In order to avoid this, one must specify a wage-setting rule. The most frequently used wage-setting rules used in the literature are, among others: setting the wage according to the marginal product of labor or, according to labor productivity or assuming that there is a union in the labor market that sets the wage at firm level or at a higher centralized

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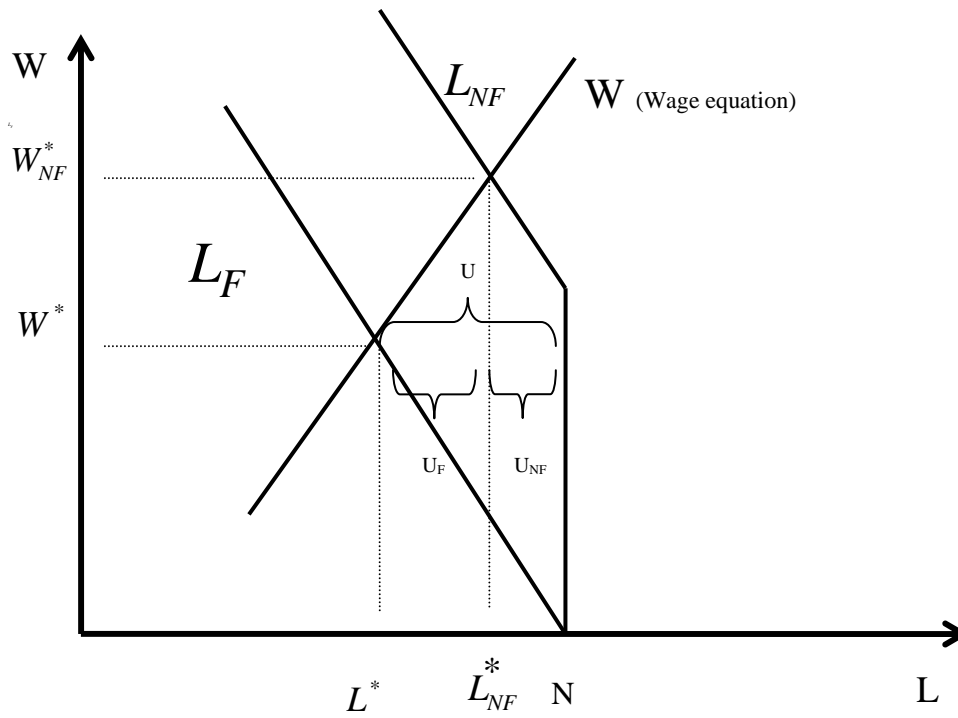[9]Michaillat uses $w = \bar{w}A^\eta$ where $\eta < 1$.

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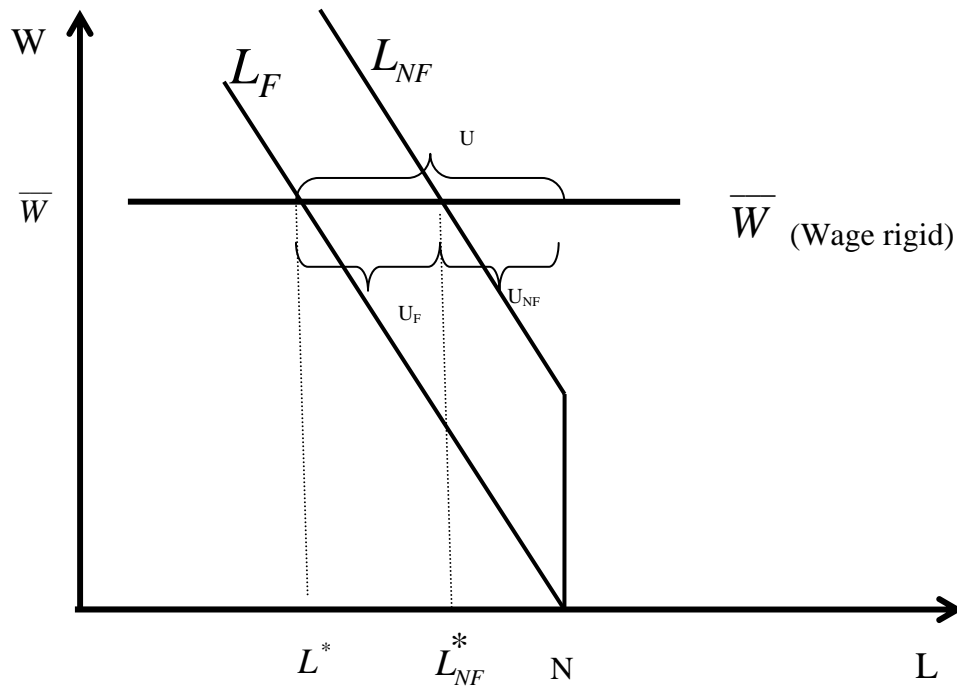Fig. 3 A typical picture of the labor market shown U_F and U_{NF}

Fig. 4 U_{NF} and U_{NF} with a rigid wage.

level. There is a detailed analysis of all these cases in the appendix. The main results can be summarized as follows: if the wage is set according to the marginal product of labor (Ericksson (1997), there is always frictional unemployment. Moreover, when an unique non frictional unemployment equilibrium exists, there is full employment, meaning that all unemployment is frictional. The appendix also discusses the case of individual wage setting according to marginal productivity of labor and firms acting as Stackelberg leaders (Bean and Pissarides (1993)).

In another case, the wage is set according to labor productivity (see, for example, Nickell (1999) and Raurich and Sorolla (2011)). Assuming a Cobb-Douglas production function, we also obtain full employment when a unique non frictional unemployment equilibrium exists, which implies that all unemployment is frictional.

Finally, we consider that there is a union in the labor market that acts as a Stackelberg leader and knows the employment equation of the firm, in which case we have firm level wage setting. We also analyze the (aggregate) employment equation, in which case we have national wage setting. Assuming a Cobb-Douglas production function, the results show that the firm level wage-setting system results in all unemployment being rationed, while all unemployment is frictional in the case of national level wage-setting.

In short, the presence of frictional unemployment when there are matching frictions depends on how the wage is set. We have obtained situations where all unemployment is frictional, or there is no frictional unemployment whatsoever. Consequently, if the intention is to compute the frictional unemployment rate, we must specify which wage equation is assumed.

3   Computing the frictional unemployment rate

This section presents a method for evaluating the quantitative decomposition of the aggregate unemployment rate into two broad categories, namely frictional unemployment due to coordination failure and mismatch and rationing (non frictional, non Walrasian, disequilibrium) unemployment.

The first component of frictional unemployment is characterized by the existence of search and matching frictions between workers and firms in the labor market. More specifically, frictional unemployment is due to a coordination problem between job applications, made by workers and vacant jobs and to mismatch. Some examples of coordination problems can be found in Pissarides (1979), while Blanchard and Diamond (1994) compare vacant jobs to urns and job applications to balls.

The second component is the mismatch unemployment defined by the literature as a situation in which there is a degree of maladjustments between labor demand and

labor supply or, more precisely, between vacant jobs and unemployed workers [10]. In agreement with this general definition, different empirical measures have emerged that try to measure the mismatch in several dimensions: regions, skills, sectors, occupations etc. Researchers have performed many empirical studies on many countries and different periods of time to measure these dimensions of mismatch unemployment. Moreover, the have used at least four differents approaches to do so. In the book entitled *Mismatch and Labour Mobility* edited by (Padoa-Shioppa, 1991), several authors examined the problem of mismatch using alternative indicators. For example, in Chapter 8 Freeman analyzes a concept of mismatch associated with sort-run sectorial shocks in the US labor market. In Chapter 5 Bentolila and Dolado measure the mismatch in the Spanish labor market taking into account a disequilibrium model. In Chapter 11 Abraham defines the indicator of mismatch focusing on the idea that frictional unemployment is unavoidable, while in Chapter 2 Jackman, Layard and Savouri measure mismatch in terms of the NAIRU. All of the previously mentioned methods of measuring mismatch have weaknesses, but the main shortfall comes from the fact that there is still no unique definition of the concept of mismatch. There is, however, a relevant consensus in the literature regarding mismatch as a temporary phenomenon, associated with sector specific shocks, or a more persistent and continuous phenomenon across occupations and between regions, as well as across skills[11].

The second category consists of unemployment due to wage rigidity above market-clearing level, in which wages are set by an economic agent (e.g. firms, workers, or the government). The concept of job rationing due to wage setting has been a popular topic in the economic literature for a long time[12].

Why is it important to decompose the unemployment rate into these two components? Breaking down the observed rate of unemployment is very important for policy makers since it allows them to apply one type of policy or another. For example, the findings of theoretical and empirical research suggest that reducing labor mobility costs, providing training programs to increase worker skills or reducing unemployment benefits improves frictional unemployment. However, fiscal policy can not improve it. The dramatic increase in unemployment in recent years has reopened the debate over the causes of unemployment. Thus, while Kocherlakota (2010) claims that the increase in unemployment is associated with an increase in mismatch, Krugman(2010) explains that the problem lies in weak aggregate labor demand.

First of all, let us define the unemployment rate u as usual, that is,

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[10]For a survey on aggregate matching function studies and microfoundations, see for example, Petrongolo and Pissarides (2001) and Stevens (2007).

[11]Nickell draws this important conclusion in the final remarks of Mismatch and Labour Mobility (Padoa-Shioppa, 1991).

[12]Recent research has turned its attention to fixed wages to explain the higher unemployment rate. See, for example, Blanchard and Galí (2010), Galí (2010) and Michaillat (2011).

11

$$u = \frac{N - L^*}{N}$$

where N and L^* are the active labor force and the observed employment level.

Indeed, we can assume that the decomposition of the unemployment rate into the non frictional (disequilibrium/rationed) unemployment rate u_{NF} and the frictional unemployment rate u_F is:

$$u = \frac{N - L^*_{NF}}{N} + \frac{L^*_{NF} - L^*}{N} = u_{NF} + u_F. \tag{13}$$

In order to calculate the frictional unemployment rate, which is not directly observable, we only need to compute non frictional employment L^*_{NF} which, if we assume that there is non frictional unemployment, $L^*_{NF} < N$, is given by the expression (10):

$$AF_L(K, L^*_{NF}) = [1 + \gamma] w.$$

Assuming a Cobb-Douglas production function $Y = AK^\alpha L^{1-\alpha}$, we obtain the following expression for L^*_{NF}:

$$L^*_{NF} = \left[\frac{[1 + \gamma] w}{(1 - \alpha) A} \right]^{-\frac{1}{\alpha}} K. \tag{14}$$

Substituting (14) in (13) yields the following expression to compute the frictional unemployment rate:

$$u_F = \frac{\left[\frac{[1+\gamma]w}{(1-\alpha)A} \right]^{-\frac{1}{\alpha}} K - L^*}{N}. \tag{15}$$

The fundamental disadvantage of this method is that we need data on w_t, A_t, K_t, α and γ. Michaillat (2011) uses this method assuming a production function without capital. We must stress at this point that we assume wage rigidity to guarantee that the observed wage w_t is the one that will prevail when there are no frictions.

A similar methodology can be used to compute observed employment L^*, using (5), we get:

$$AF_L(K, L^*) = \left[1 + \gamma \frac{m^{-1}(L^*, N)}{L^*} \right] w,$$

This expression can be rewritten, assuming a Cobb-Douglas production function, as:

$$L^* \left[1 + \gamma \frac{m^{-1}(L^*, N)}{L^*} \right]^{\frac{1}{\alpha}} = \left[\frac{w}{(1 - \alpha) A} \right]^{-\frac{1}{\alpha}} K. \tag{16}$$

We also assume a Cobb-Douglas functional form for the matching function with

12

constant returns $L = m(L^d, N) = B(L^d)^\varphi (N)^{1-\varphi}$, where B stands for a scale parameter that captures the efficiency of the matching technology and φ is the elasticity of employment with respect to labor demand. The inverse of the matching function is $L^d = m^{-1}(L, N) = B^{-\frac{1}{\varphi}} L^{\frac{1}{\varphi}} N^{1-\frac{1}{\varphi}}$ and, thus, L^* is given by[13]:

$$L^* \left[1 + \gamma B^{-\frac{1}{\varphi}} \left(\frac{L^*}{N} \right)^{\frac{1}{\varphi}-1} \right]^{\frac{1}{\alpha}} = \left[\frac{w}{(1-\alpha)A} \right]^{-\frac{1}{\alpha}} K. \tag{17}$$

Once we have an explicit form for L^* and L^*_{NF} the frictional unemployment rate can be computed using (15). As mentioned above, the problem with this method is that we need data on w_t, A_t, K_t, α, γ and, additionally, φ.

Nevertheless, there is a way of avoiding the use of some of the data mentioned above, namely by considering another way of calculating frictional unemployment:

$$u_F = \frac{L^*_{NF} - L^*}{N} = \frac{L^*_{NF} - L^*}{L^*} \frac{L^*}{N} = \left[\frac{L^*_{NF}}{L^*} - 1 \right] \frac{L^*}{N}. \tag{18}$$

If $L^* < N$, then L^* may be expressed as (17), whereas L^*_{NF} is given by (14).

Substituting these two expressions in the equation above and assuming wage rigidity, we obtain[14]:

$$u_F = \left[\left[\frac{1 + \gamma B^{-\frac{1}{\varphi}} \left(\frac{L^*}{N} \right)^{\frac{1}{\varphi}-1}}{1 + \gamma} \right]^{\frac{1}{\alpha}} - 1 \right] \frac{L^*}{N}. \tag{19}$$

Thus, we can compute the frictional unemployment rate using data on the level of employment L^*, and the active labor force N. Additionally, we also need to know a few parameters related to the matching function (B, φ), the cost of opening a vacancy proportional to the wage γ and, finally, the labor income share of national income α.

As mentioned previously, we distinguish two types of micro foundations in the matching function, based on coordination failure and mismatch due to heterogeneity in

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[13] At the empirical level, the Cobb-Douglas function is the most widely accepted specification, although at theoretical level it is one of the most controversial points in the literature. A great deal of research addresses the topic of microfoundations in the Cobb-Douglas function (see, for example, Petrongolo and Pissarides (2001) or Stevens (2007)). The assumption of constant returns to scale is supported empirically by Pissarides (1986) and Blanchard and Diamond (1989).

[14] There is also an alternative method for evaluating the frictional/structural unemployment rate. When there are frictions we can compute labor demand following the expression $L^d_t = L^*_t + V_t$ where V_t are the unfilled job vacancies. Substituting the definition above into the expression (16) we obtain:

$$u_F = \left[\left[\frac{1 + \gamma(1 + \frac{V_t}{L^*})}{1 + \gamma} \right]^{\frac{1}{\alpha}} - 1 \right] \frac{L^*}{N}.$$

The problem with this method is having a good proxy of the stock of job vacancies from numerous countries at one point in time.

the labor market[15]. The existence of coordination failures yields overcrowding in some jobs and no applications in others. As a result, not all job-seeker and job vacancy pairs are matched at any point in time. In this case, imperfect information and search activities relating to the matching process produce frictional unemployment. We assume that during the contact stage of the matching process, all workers simultaneously send one application (N), but only a proportion L of workers get a job[16]. Therefore, we assume in our model a value of parameter B equal to $\frac{L}{N}$[17].

Additionally, we incorporate the concept of mismatch between vacant jobs and unemployment workers in our measure of unemployment. More specifically, we calculate the mismatch component of unemployment for the entire economy taking into account that mismatch would reduce effective unemployment in the contact technology between job seekers and vacancies. Hence, our specification of the total effective labor force is therefore $\hat{N} = L + \lambda U$, where the parameter $\lambda, 0 \leq \lambda \leq 1$, measures the percentage of the unemployed that have been seeking a job for less than one year[18]. In other words, we are excluding long-term unemployed (one year or more) from the labor force[19].

This approach taken to measure mismatch does not directly capture the concept of mismatch that appears in the literature commented previously. Instead, it provides a measure of the relevant consequences that stem from mismatch long-term unemployment. These issues are particularly relevant for policy analysis, as our measure does not depend on a particular index of mismatch used to measure the imbalance in skills or between regions.

Therefore, this study *simultaneously* considers the two sources of frictions most commonly used in the labor market literature. Thus, in the case in which λ is equal to zero, we will only have frictional unemployment due to coordination failures u_{FCF}, whereas if λ is positive, we will have two sources of frictions. Taking into account both sources, we replace the term B with $\frac{L}{N}$ and the labor force with the effective labor force $\hat{N}$ in the inverse matching function. As a result, we obtain the following expression for

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[15]See Pissarides and Petrongolo (2001) and Stevens (2007) for a recent survey.

[16]We assume that the matching process has two stages: the contact stage and the selection stage. (See Brown et. al (2011) for more details).

[17]We also explore another possibility for estimating the efficiency of the matching technology. More specifically, we can solve parameter B from the matching function $B = \frac{L}{N}\left(\frac{N}{(L^d)}\right)^{\varphi}$ and compute this expression directly based on the data availabe on vacancies, unemployment, the active labor force and the observed employment level given an estimated value of φ.

The average value of B computed following this procedure for Spain in the period from 1980 to 2011, does not differ significantly from assuming B=L/N. However, average frictional unemployment is around 0.25%. This low value could be due to how difficult it is to ascertain the total number of vacancies in an economy. See, for example Antolin (1994).

[18]In this respect, our model does somewhat resemple that referred to as a "ranking" by Petrongolo when considering a different way of treating short-term and long term unemployment in the matching function.

[19]In the context of matching frictions, an increase in mismatch between workers and vacancies leads to a rise in the duration of unemployment. See, for example Petrongolo and Pissarides (2001).

the **frictional** unemployment rate

$$u_{F2S} = \left[\left[\frac{1 + \gamma \left(\frac{N}{\hat{N}} \right)^{\frac{1}{\varphi}} \frac{\hat{N}}{L}}{1 + \gamma} \right]^{\frac{1}{\alpha}} - 1 \right] \frac{L^*}{N}, \quad (20)$$

where u_{FCF} implies $N = \hat{N}$ in the previous expression. The most appealing feature of this approach is that it distinguises between **frictional** unemployment rate, u_{F2S}, and rationed unemployent, $u - u_{F2S}$, and we can also decompose *frictional unemployment into coordination failures and mismatch unemployment,* where frictional unemployment due to coordination failures is given by u_{FCF} and mismatch unemployment by $u_{F2S} - u_{FCF}$.

We apply our analysis to the Spanish and US labor markets and construct indicators of frictional unemployment. We have chosen these two countries because they represent two labor markets in which unemployment behaves completely differently. Moreover, there are strong institutional differences, such as the level at which wage bargaining takes place: central, sectorial or firm level. It should also be noted that this method for measuring frictional/non frictional unemployment is very general and may therefore be used for any country. Finally, it is important emphasize that the fit obtained by the method proposed in this section will yield a more accurate decomposition if we improve the estimation of parameters φ and γ.

3.1 Results of the decomposition for Spain

Table 1 presents the parameter values used to decompose the unemployment rate for Spain.

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TABLE 1

PARAMETER VALUES, QUARTERLY DATA

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Parameter	Value	Description	Source/Target
α	0.5	Share of labor	
φ	0.15	Beveridge elasticity	Estimated Value
γ	0.05	Vacancy cost	Michaillant(2011)
B	L/N	Matching function scale	Failure of Coordination
γ/q	0.1	Flow cost of recruiting	Pissarides (2009)
		Data, Mean Values	
u	16,53%	Unemployment	Data BDREMS/INE
u_{LT}	7,95%	Long Term Unemployment	INE
L/N	0.83	Labor force participation	INE
$\frac{m^{-1}(L^*,N)}{L^*}$	2	Duration of a vacancy	Data BDREMS/INE

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15

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We employ quarterly data covering the period 1980 to 2011. The data on the variables we use (the unemployment rate, the vacancy rate, etc.) were obtained from two sources. The number of vacancies per quarter is provided by the REMSDB macroeconomic database compiled to simulate and calibrate the Rational Expectations Model (REMS) for the Spanish economy. The rest of the data comes from the Quarterly National Accounts (SQNA) and the Encuesta de Población Activa (Labor Force Survey) provided by the Instituto Nacional de Estadística (INE; National Statistical Institute).

In order to compute the frictional/mismatch unemployment rate, we must calculate or estimate some parameters. The parameter α is calculated for every year as the ratio of total compensation of employees over gross domestic product. According to our calculations, the average labor share estimated is equal to 0.49. Moreover, we also estimate a simple Beveridge Curve for the Spanish economy to obtain an estimated coefficient of the elasticity of unemployment, with respect to the vacancy rate, equal to -0.18. This estimated coefficient is consistent with the range of values estimated in other more extensive studies of the Spanish labor market[20].

Taking into account the relationship between the matching function and the Beveridge curve, we obtain an estimated coefficient of the elasticity of employment with respect to labor demand $\hat{\varphi} = 0.15$[21].

We set the value of the vacancy posting cost to $\gamma = 5\%$ of annual labor costs per worker[22]. This value is in line with the calibration by Sala and Silva (2009), who find that the hiring cost of new hired workers represents 2% of the wage. However, other studies focusing on the Spanish labor market estimate a unit hiring cost of between 10 percent and 16 percent of the gross annual wage of a permanent worker (see, for instance, Alonso-Borrego et. al (2006)). More recently, Aguirregabiria and Alonso Borrego (2009) estimated hiring costs for temporary and permanent workers using a panel of 2356 Spanish manufacturing firms. They found that hiring costs are similar and that values range from 10% to 18% of workers annual salaries. We consider the most conservative of these scenarios and set this cost at 5%, due to the fact that we assume that it reflects the time and money involved in a screening process to opening a vacancy.

Finally, given the data on labor demand and employment and the parameter $\hat{\varphi}$ estimated we obtain, on average, the expected duration of a vacancy in the sample, $\frac{m^{-1}(L^*,N)}{L^*}$, equal to 2. Consequently, the expected recruitment cost per worker, $\gamma\frac{m^{-1}(L^*,N)}{L^*}$, is approximately equal to 10% of quarterly worker compensation in accordance with Sala and Silva (2009) for Spain and comparable to other calibrations for the US economy.

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[20] For more details on this issue, see for example, Antolin (1994 and 1995).

[21] For more details on this relationship, see for example, Dixon et. al. (2012). This estimated value is in line with the estimates by Burda and Wyplosz (1994) and Fonseca and Muñoz (2003).

[22] If we increase the value of this parameter, we obtain a higher frictional unemployment rate.

16

Figure 9 displays the observed unemployment rate of the Spanish labor market over the period of study and our computation of frictional, u_{F2S}, and coordination failure unemployment, u_{FCF}. The top line in Figure 9 represents the trajectory of the observed unemployment rate, while the middle line represents the evolution of frictional unemployment u_{F2S} and, finally, the bottom line represents the estimated level of coordination failure unemployment u_{FCF}. The pattern of these figures highlights several points of interest. Firstly, average *coordination failure* unemployment remained very low at 1.62% over the period from 1980 to 2010. Secondly, average *mismatch* unemployment ($u_{F2S} - u_{FCF}$) is quite high at 6.5%. Thus, the sum of the results obtained, taking into account the values of the parameter mentioned above, show an average frictional unemployment rate of around 8.2%. Therefore, the remainder of total unemployment is due to wage rigidity, more specifically over 8.3%. Thirdly, it is worthwhile stressing that the mismatch unemployment rate registers a markedly pro-cyclical pattern over the entire period considered. This result appears consistent with the evidence on estimated mismatch provided in Bentolila and Dolado (1991) for the Spanish labor market. Fourthly, as shown in Figure 10, the weighting of the frictional unemployment rate in regard to unemployment records a considerable rise during periods of economic recession. According to our estimations, the weighting of the frictional/mismatch unemployment rate peaked at 60% during two periods, more specifically between 1985 and 1989 and 1994 and 1995.

Recently, using a model with multiple submarkets in the labor market with search frictions and adjustment costs, Herz and Rens (2011) pointed out that the fluctuations in overall structural unemployment looked very similar to the fluctuations in the overall unemployment rate[23].

We check the robustness of the results to (i) a change in the vacancy cost γ, and (ii) a change in the elasticity of employment with respect to labor demand $\hat{\varphi}$. We recalculated the decomposition of the unemployment rate between frictional and rationing unemployment rate. In general, a higher vacancy cost or lower elasticity implies a rise in frictional unemployment.

We should point out that the policy implications of all these results can be used to guide reforms aimed at combating unemployment by understanding the origin of the aggregate unemployment rate.

3.2   Results of the decomposition for the USA

Table 2 lists the values of the parameters used to decompose the unemployment rate for the US economy.

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[23]See, for instance Abraham and Katz (1986).

TABLE 2
PARAMETER VALUES, QUARTERLY DATA

Parameter	Value	Description	Source/Target
α	0.55	Share of labor	Data BDREMS/INE
φ	0.06	Beveridge elasticity	Estimated Value
γ	0.08	Vacancy cost	Silva and Toledo(2009)
B	L/N	Matching function scale	Failure of Coordination
γ/q	0.1	Flow cost of recruiting	Pissarides (2009)
		Data, Mean Values	
u	6,36%	Unemployment	OECD (MEI)
u_{LT}	0.70%	Long Term Unemployment	OECD (MEI)
L/N	0.93	Labor force participation	OECD (MEI)
$\frac{m^{-1}(L^*,N)}{L^*}$	1.20	Duration of a vacancy	Data OECD(MEI)

The analysis covers the period dating from 1980Q1 till 2011Q3. The data are published by the OECD Main Economic Indicators (MEI) database. More specifically, GDP and labor income data come directly from the United States Department of Commerce, Bureau of Economic Analysis, while the labor force statistics are provided by the United States Population & Labor Force US Bureau of Labor Statistics.

Using the previous approach, we calculate the parameter $\alpha = 0.55$ and an estimated coefficient of the elasticity of unemployment, with respect to a vacancy rate equal to -0.06, which implies an estimated parameter of $\hat{\varphi} = 0.06$. It is not surprising that there are some differences in light of the fact that we use a different measure of vacancies, that our data cover a different period, and that we have not taken into account that the slope of the Beveridge curve varies substantially over time[24]. Finally, we also computed the expected recruitment cost per worker, $\gamma\frac{m^{-1}(L^*,N)}{L^*}$, as follows. The duration of a vacancy $\frac{m^{-1}(L^*,N)}{L^*}$, on average, is computed using the expression (20). Moreover, we target γ to generate a per worker hiring cost $\gamma\frac{m^{-1}(L^*,N)}{L^*}$ approximately equal to 10% of quarterly worker compensation. This implies a value of γ approximately equal to 8% of the worker's quarterly wage. It is worth noting that this recruitment cost per worker is within the range of the estimated values obtained by the literature. For instance, Michaillat (2011) set a flow cost of recruiting equal to 0.32 per month, equivalent approximately to a quarterly value of 0.10. In the same line, Shimer (2005) and Pissarides (2009) chose a cost of approximately 0.15 of a worker´s quarterly wage [25].

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[24]Our data on unfilled job vacancies are based on the number of help-wanted advertisements published in the classified sections of newspapers and collected by the Conference Board. Recently, numerous empirical studies of the U.S. labor market have used the Job Openings and Labor Turnover Survey (JOLTS) provided by the Bureau of Labor Statistics (BLS) of the U.S. Department of Labor. For an interesting discussion on these issues see, for instance, Yashiv (2006).

[25]Michaillant (2011) estimated the per-period cost of opening a vacancy at 0.098 of a worker's

Figure 11 shows, the evolution of the overall unemployment rate and the frictional and coordination failure unemployment rates in the US. It is worth stressing three aspects of the data analyzed. First, the average overall unemployment rate is approximately three times less than in the case of the Spanish labor market. More specifically, the rate in the United Sates is 6.4%. Second, the average coordination failure unemployment rate is around 0.9%, while mismatch unemployment stood at around 1.6% during the period considered. These results can be at least partly attributed to the lower proportion of long-term unemployment in the US labor market. Third, mismatch unemployment also has an important cyclical component[26]. Fourth, as shown in Figure 12, the weighting of the frictional unemployment rate in regard to unemployment is over 37%. However, this percentage has risen by over 86% in the past 3 years. This rapid growth in the weighting of the frictional unemployment rate shows the seriousness of the current recession and the important challenge faced by policy makers in the US.

We also analyze the robustness of the decomposition of the unemployment rate to changes in the parameters γ and $\hat{\varphi}$. The changes in the rate of frictional unemployment are in the same line as those analyzed in the Spanish case.

4   Conclusions

The depth and persistence of the current crisis has fundamentally affected the USA and the euro area economy in the following ways: i) banks are lending less money to businesses and consumers, ii) unemployment is rising and has reached a very high level in the US and the euro area iii) there is more social spending and less revenue, which in turn leads to high government deficit and debt levels. All these problems are strongly related and it is perhaps difficult to solve one problem by isolating it from rest of problems. However, in this paper we focus on the problem of unemployment. We believe that the unemployment problem in Italy, Greece, Ireland, Portugal and Spain is crucial in order to escape from this crisis. More specifically, the high unemployment rate in Spain is the most important problem in the Euro zone. The unemployment rate in Spain was last reported at 26.02 percent in the fourth quarter of 2012. More importantly projections are very pesimistic. The last report on the unemployment rate released by the OECD forecasts a rise to 26.9% in 2013. However, one major problem to reduce unemployment is ascertaining the origin of aggregate unemployment. Thus, the aim of this paper is to introduce a new method for decomposing the unemployment rate by origin. More specifically, we decompose unemployment into frictional and rationing unemployment taking into account a large

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wage.Moreover, he estimated the value of parameter B=0.93. This value is the same as that considered in this paper taking into account the approximation L/N.

[26]These results are in line with the work by Abraham and Katz (1986) and Herz and Rens (2011), among others.

number of relevant wage setting mechanisms that appear in the theoretical literature. This study has shown that the existence of frictional unemployment, when there are matching frictions, depends heavily on how wages are set. We have obtained situations where all unemployment is frictional or there is no frictional unemployment. These findings suggest that, in general, if we wish to compute the frictional unemployment rate, we have to specify which wage equation is assumed.

From an empirical perspective, we compute the frictional and non frictional unemployment rates based on a model with rigid wages for the Spanish and US labor markets. The empirical findings for Spain suggest that about half of all unemployment is due to job rationing and the other half due to frictional and mismatch problems. However, in the case of the US economy, the rationing unemployment rate represents, on average, two thirds of all unemployment, while frictional and mismatch problems only account for a third. Moreover, our approach allows us not only to estimate the contribution of frictional unemployment to total unemployment, but also to decompose frictional unemployment into two relevant sources: coordination failure and mismatch.

Although our analysis is subject to some inaccurancies, we hope to contribute to a coherent view of the debate regarding the nature and likely remedies of the current state of unemployment in Spain. Several improvements to the method for decomposing the unemployment rate are possible, but simplicity is also an advantage.

20

5 Appendix

1a) Individual wage setting according to marginal product of labor (Ericksson (1997)[27]). This is the version of the usual wage setting rule of matching models for the case where matches last for only one period. In this context, the surplus of a match for the worker is the wage w (assuming zero unemployment benefit)[28]. On the other side, the surplus of a match for the firm is the marginal product of labor provided by this worker minus the wage. That is, $AF_L(K, L) - w$, because the vacancy cost is paid regardless of whether or not the job is filled. Therefore, in a Nash bargaining solution the wage maximizes $(w)^\beta (AF_L(K, L) - w)^{1-\beta}$, which gives the wage equation:

$$w = \beta AF_L(K, L). \tag{21}$$

In this case, the equilibrium amount of employment in a labor market with frictions L^* is given by:

$$1 = \beta \left[1 + \gamma \frac{m^{-1}(L^*, N)}{L^*} \right]$$

having that $L^* < N$ and that L^* does not depend on A. This means that, under this wage-setting rule, technological shocks do not produce fluctuations in employment when there are frictions. Assuming $L = m(L^d, N) = B(L^d)^\varphi (N)^{1-\varphi}$, then $m^{-1}(L, N) = B^{\frac{1}{\varphi}} L^{\frac{1}{\varphi}} N^{1-\frac{1}{\varphi}}$ and $L^* = B^{\frac{1}{\varphi-1}} \left[\frac{1}{\gamma} \left(\frac{1}{\beta} - 1 \right) \right]^{\frac{\varphi}{1-\varphi}} N$.

In a labor market without frictions $\tilde{L}$ is given by:

$$AF_L(K, \tilde{L}) = [1 + \gamma] w,$$

from where we obtain:

$$w = \frac{1}{[1 + \gamma]} AF_L(K, \tilde{L}),$$

which, combined with the wage equation (21) yields:

$$\beta = \frac{1}{[1 + \gamma]}$$

this meaning that if $\beta = \frac{1}{[1+\gamma]}$, the wage and the downward part of the employment equation without frictions L_{NF} coincide completely and we have multiplicity of equilibria.

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[27]In fact, Ericksson sets the wage equal to the marginal product of labor plus the vacancy cost saved (see equation 5 of his paper).

[28]In models where only some workers lose their jobs and under a constant probability of leaving unemployment, the surplus of a match for the worker is equal to the difference between the sum of all income throughout his or her employed life minus the sum of all income throughout his or her unemployed life. Logically, this procedure yields a more complex wage setting rule.

If $\beta < \frac{1}{[1+\gamma]}$ the wage equation is below $\tilde{L}$, meaning that in the non frictional equilibrium, we have full employment. Therefore, under this wage setting rule, all unemployment is frictional. Figure 5 illustrates this situation. Finally, if $\beta > \frac{1}{[1+\gamma]}$ there is non frictional unemployment equilibrium because $w = \beta A F_L(K, L)$ and L_{NF} never cross.

1b) Individual wage setting according to marginal productivity of labor and firms acting as Stackelberg leaders (Bean and Pissarides (1993)).

In a seminal paper, Bean and Pissarides (1993) consider the wage setting equation described in the previous section, but this time assuming that the firm decides employment knowing this wage setting rule.

Thus, the firm considers that $w = \beta A F_L(K_i, L_i)$, while the variables γ_0, $\bar{L}$ and N are given and chooses L_i (employment) and K_i in order to maximize:

$$AF(K_i, L_i) - wL_i - \gamma_0 L_i^d - (r+\delta)K_i \tag{22}$$
$$= AF(K_i, L_i) - \beta A F_L(K_i, L_i)L_i - \gamma_0 \frac{m^{-1}(\bar{L}, N)}{\bar{L}} L_i - (r+\delta)K_i,$$

the first-order condition for this problem yields:

$$(1-\beta)AF_L(K_i, L_i) = \beta F_{LL}(K_i, L_i)L_i + \gamma_0 \frac{m^{-1}(\bar{L}, N)}{\bar{L}}, \tag{23}$$

which implies that, in equilibrium, the amount of employment in a labor market with frictions is given by:

$$(1-\beta)AF_L(K, L^*) = \beta F_{LL}(K, L^*)L^* + \gamma_0 \frac{m^{-1}(L^*, N)}{L^*}. \tag{24}$$

Then we have that

$$\frac{\partial L^*}{\partial A} = -\frac{(1-\beta)F_L - \beta A F_{LL}L^*}{AF_{LL} - \beta A F_{LLL}L^* - \gamma \frac{m^{-1}(L,N)}{L^2}\left[\frac{\partial m^{-1}}{\partial L}\frac{L}{m^{-1}} - 1\right]} > 0 \tag{25}$$

as long as $F_{LLL} > 0$ and $\frac{\partial m^{-1}}{\partial L}\frac{L}{m^{-1}} > 1$.

In a labor market without frictions employment is given by L_{NF}^* such that

$$(1-\beta)AF_L(K, L_{NF}^*) = \beta F_{LL}(K, L_{NF}^*)L_{NF}^* + \gamma_0, \tag{26}$$

or

$$(1-\beta)AF_L(K, L_{NF}^*) - \beta F_{LL}(K, L_{NF}^*)L_{NF}^* = \gamma_0, \tag{27}$$

and there is unemployment if $L_{NF} < N$. Summarizing, in a labor market with frictions this wage setting rule always generates frictional unemployment and may generate non
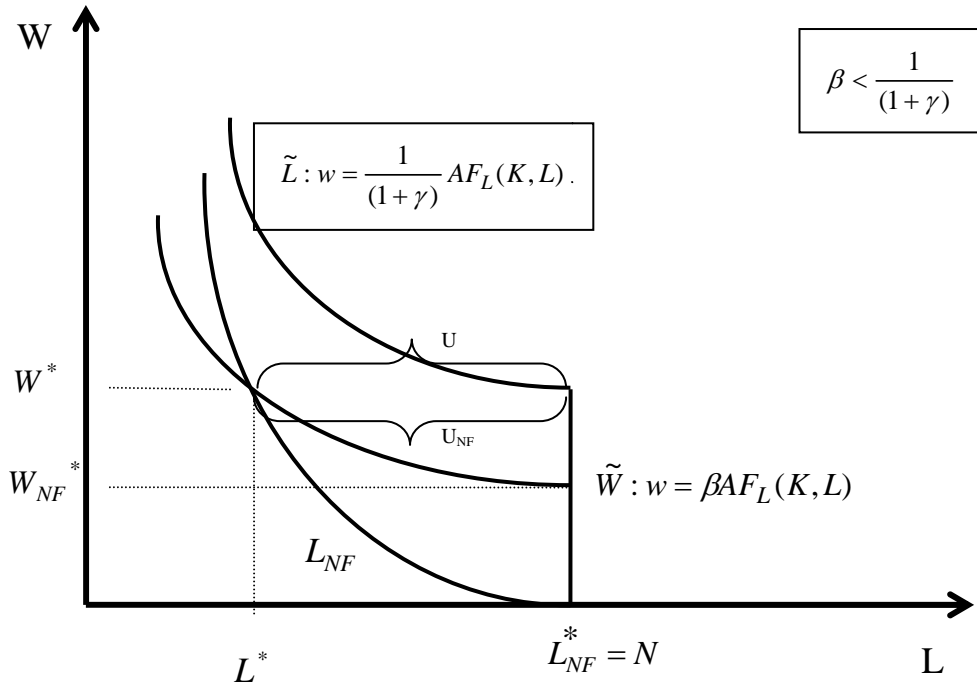
Fig. 5 U_F and U_{NF} when setting wages according to marginal product.

frictional (rationed) unemployment. In this case, it is not possible to provide the usual picture of the labor market because the firm, acting as a Stackelberg leader, chooses a point of the wage curve, so L_F and L_{NF} are not formally defined.

2) Collective wage setting according to labor productivity (Nickell (1999) and Raurich and Sorolla (2011)).

In this model, we assume collective bargaining, meaning that either all workers are employed or not. The surplus for all workers of being employed is wL. The surplus for the firm that employs these workers is $AF(K, L) - wL$. In a Nash bargaining solution, the wage chosen maximizes the following expression $(wL)^\beta (AF(K, L) - wL)^{1-\beta}$ which gives:

$$w = \beta \frac{AF(K, L)}{L}, \tag{28}$$

a wage equation that, when combined with the employment equation of a labor market with frictions, yields:

$$AF_L(K, L) = \left[1 + \gamma \frac{m^{-1}(L, N)}{L}\right] \beta \frac{AF(K, L)}{L}$$

and, then, employment in a labor market with frictions is given by:

$$F_L(K, L^*) \frac{L}{F(K, L^*)} = \beta \left[1 + \gamma \frac{m^{-1}(L^*, N)}{L^*}\right],$$

having that, of course, $L^* < N$ and that L^* does not depend on A. This result suggests that under the wage-setting rule, technological shocks do not produce fluctuations in employment when there are frictions.

In a labor market without frictions, the intersection between $\tilde{L}$ and the wage equation is $\tilde{L}^*$ and is provided by the following expression:

$$F_L(K, L^*) \frac{L}{F(K, L^*)} = \beta \left[1 + \gamma\right],$$

and there is unemployment if $\tilde{L}^* < N$, that is, in this case $L_{NF}^* = \tilde{L}^* < N$ which does not depend on A. Figure 6 provides a picture of this case. The problem with the previous equation is that, with a Cobb-Douglas production function $Y = AK^\alpha L^{1-\alpha}$, we obtain the following expression $F_L(K, L) \frac{L}{F(K,L)} = 1 - \alpha$ meaning that when $1 - \alpha = \beta [1 + \gamma]$ both equations coincide, resulting in multiplicity of equilibria. Once again, as in case 2.a, when $1 - \alpha < \beta [1 + \gamma]$, the wage equation is below $\tilde{L}$ and there is full employment, meaning that all unemployment is frictional.

3) Union monopoly model at firm level.

Assuming a Cobb-Douglas production function $Y = AK^\alpha L^{1-\alpha}$, in this model it is easy to show that the employment in firm i, L_i, when there are frictions is given by:
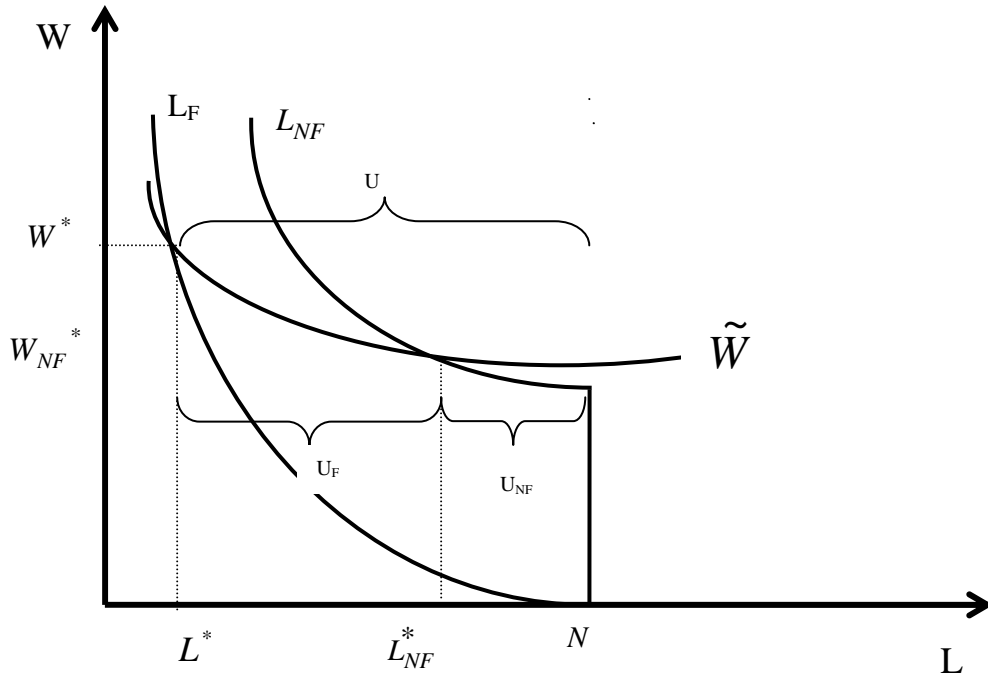
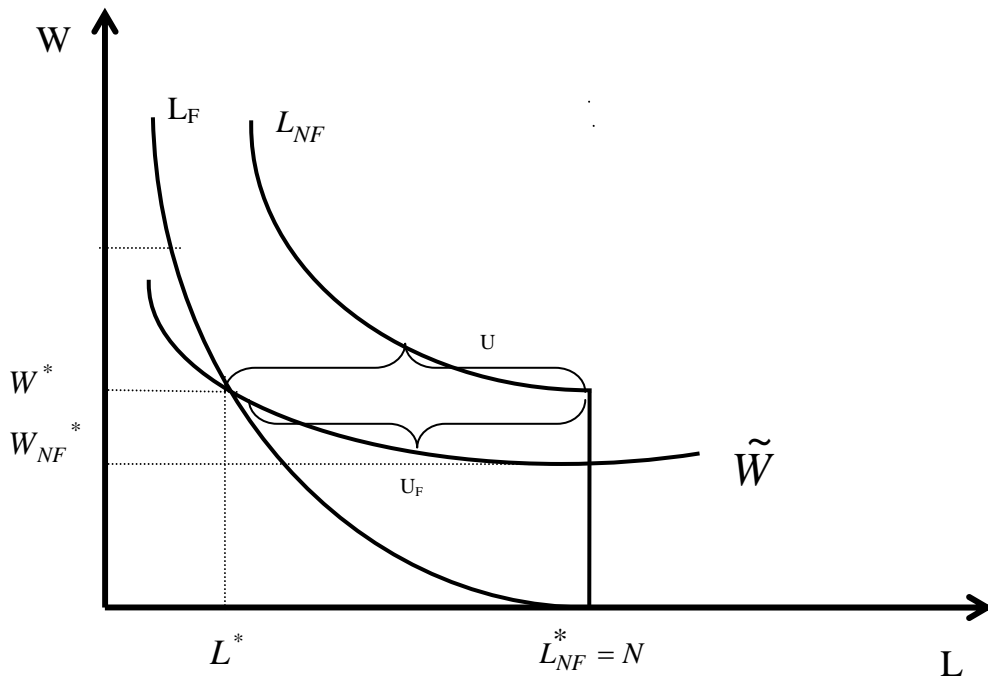Fig.6 U_F and U_{NF} when setting wages according to labour productivity (Y/L).

Fig.6 U_F and U_{NF} when setting wages according to labour productivity (Y/L).

$$L_i = \left[\frac{\left[1 + \gamma \frac{m^{-1}(\bar{L}, N)}{\bar{L}} \right] w_i}{(1-\alpha)A} \right]^{-\frac{1}{\alpha}} K_i$$

and without frictions by:

$$L_i = L_i^d = \left[\frac{[1+\gamma] w_i}{(1-\alpha)A} \right]^{-\frac{1}{\alpha}} K_i,$$

ϵ, being in both cases the elasticity with respect to the wage equal to $\frac{1}{\alpha}$. Now, if the ith union maximizes $(w_i - r)L_i$, where r is the alternative labor income, we obtain the wage-setting rule:

$$w_i = \frac{r}{1 - \frac{1}{\epsilon}} = \frac{r}{1-\alpha}.$$

If the alternative labor income is $r = \frac{\bar{L}}{N}w_e + \frac{(N-\bar{L})}{N}s$, where w_e is the alternative wage and s the unemployment benefit we have:

$$w_i = \frac{r}{1-\alpha} = \frac{\frac{\bar{L}}{N}w_e + \frac{(N-\bar{L})}{N}s}{1-\alpha}.$$

Finally, we solve the general equilibrium in the labor market by assuming $s = \sigma w_e$ and $w_i = w_e$. This implies that the amount of employment L is given by:

$$1 = \frac{\frac{L}{N} + \frac{(N-L)}{N}\sigma}{1-\alpha}$$

and, thereby, the employment rate with or without frictions is the same[29] :

$$\frac{L^*}{N} = \frac{L_{NF}^*}{N} = \frac{1-\sigma-\alpha}{1-\sigma} = 1 - \frac{\alpha}{1-\sigma} < 1.$$

Accordingly, if there are frictions there is unemployment and the employment rate is equal to the employment rate when there are no frictions. For this reason, all unemployment is non *frictional* and, in a labor market with frictions, the wage will be lower. In other words, in a labor market with frictions this wage-setting rule does not generate frictional unemployment and there is non frictional (rationed) unemployment. Figure 7 represents the labor market for this case. Note that the employment rate does not depend on A.

4) Union monopoly model at national level.

In a labor market with frictions, the program of the national union is to choose L[30] in

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[29]If wages are set at the national level, the alternative income is the unemployment benefit s. If the union maximizes the postulated utility function then we have $w = \frac{s}{1-\alpha}$, which is a way of justifying an exogenous wage if the union does not take into account how the unemployment benefit is financed.

[30]Given the relationship between w and L, it is the same to choose w or L.

order to maximize wL[31], where the wage is given by (5). In this model, the union chooses L in order to maximize:

$$\frac{AF_L(K,L)L}{\left[1+\gamma\frac{m^{-1}(L,N)}{L}\right]},$$

which yields the first-order condition:

$$\left[\frac{F_{LL}(K,L^*)L^*}{F_L(K,L^*)}+1\right]\left[\frac{1}{\gamma\frac{m^{-1}(L^*,N)}{L^*}}+1\right]=\left[\frac{\partial m^{-1}}{\partial L}\frac{L^*}{m^{-1}}-1\right],$$

having that, of course, $L^* < N$. Note also that the amount of employment does not depend on A. When the production and matching functions are Cobb-Douglas, the amount of employment in a labor market with frictions is:

$$L^* = \left[\gamma\left(\frac{1-\alpha}{\left(\frac{1}{\varphi}-1\right)-(1-\alpha)}\right)\right]^{\frac{\varphi}{1-\varphi}}B^{\frac{1}{1-\varphi}}N.$$

In a labor market without frictions, where the wage is given by (10), the program of the union is to choose L in order to maximize wL through the following expression:

$$\frac{AF_L(K,L)L}{[1+\gamma]},$$

which yields this first-order condition:

$$\frac{F_{LL}(K,L^*_{NF})L^*_{NF}}{F_L(K,L^*_{NF})}=1,$$

When the production function is Cobb-Douglas, labor demand is $\left[\frac{[1+\gamma]w}{(1-\alpha)A}\right]^{-\frac{1}{\alpha}}K$ and, therefore, we obtain that $wL = \frac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{[1+\gamma]^{\frac{1}{\alpha}}w^{\frac{1-\alpha}{\alpha}}}K$. In this case, the union chooses the competitive wage, meaning that there is full employment in a labor market without frictions and then all unemployment is frictional under this wage-setting rule. In other words, in a labor market with frictions and a Cobb-Douglas production function, this wage-setting rule, always generates frictional unemployment and there is no non frictional (rationed) unemployment. Figure 8 illustrates the labor market for a Cobb-Douglas production function in this case.

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[31] This expression maximizes $(1-t)wL + s(N-L)$, taking into account that $s = \frac{twL}{N-L}N$. If the union does not take this into account, the relationship yields $w = \frac{s}{1-\frac{1}{\epsilon}}$ and, with constant elasticity, we have a rigid wage.
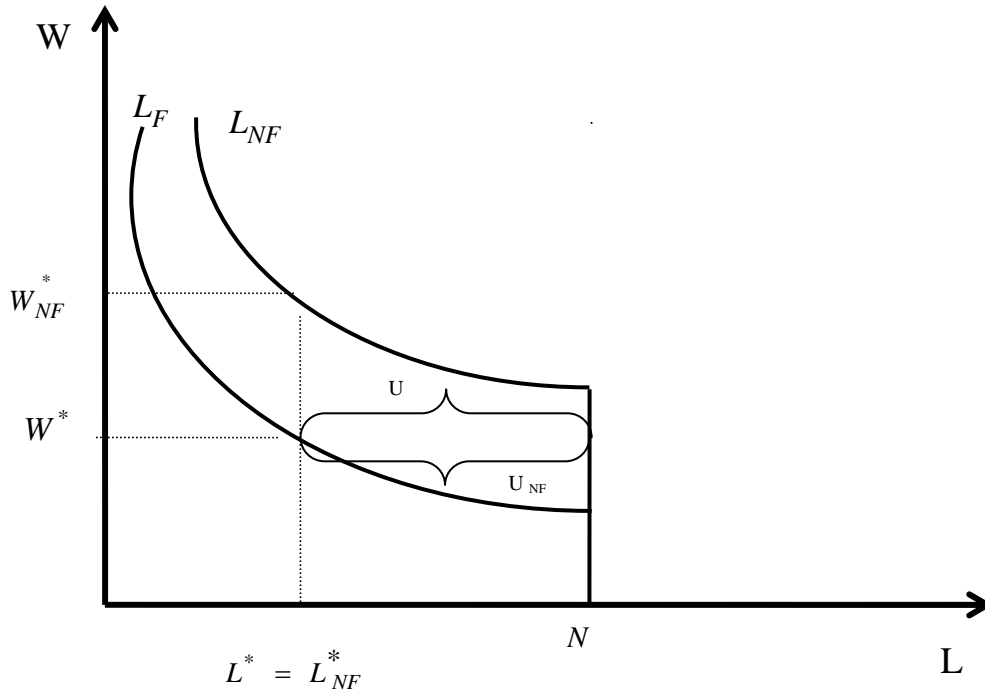
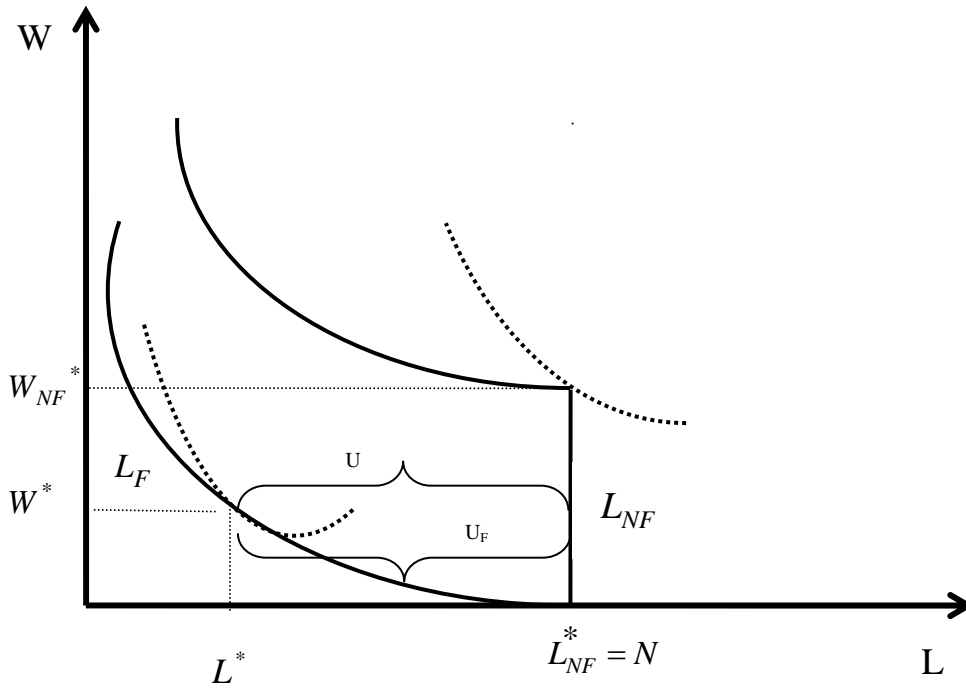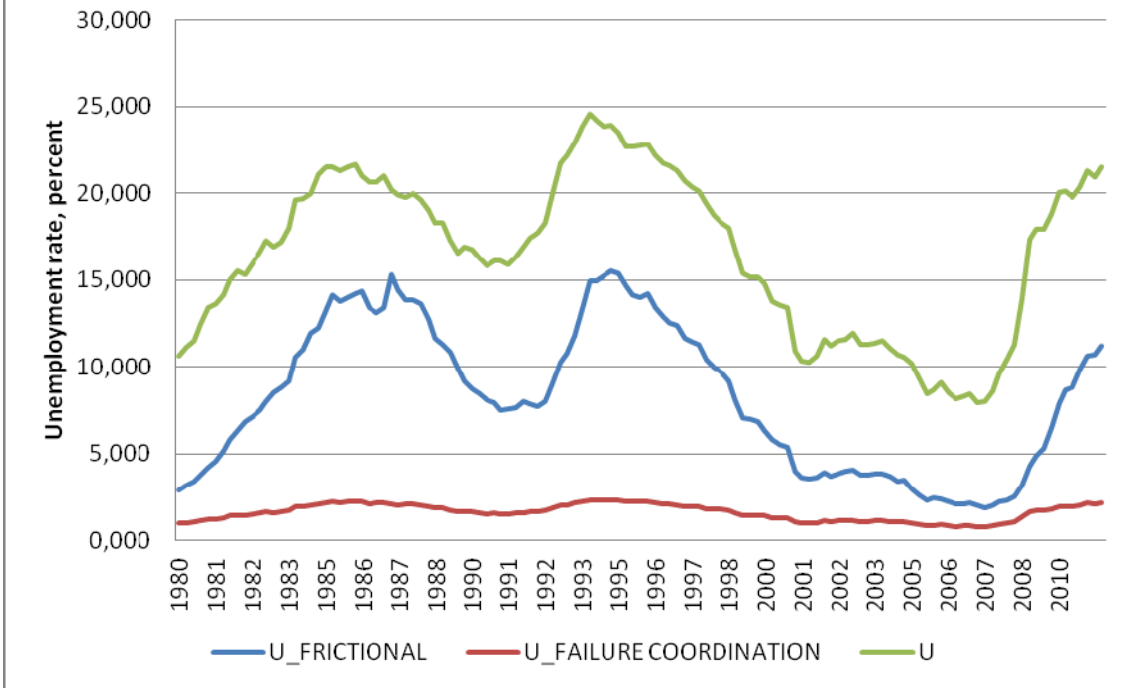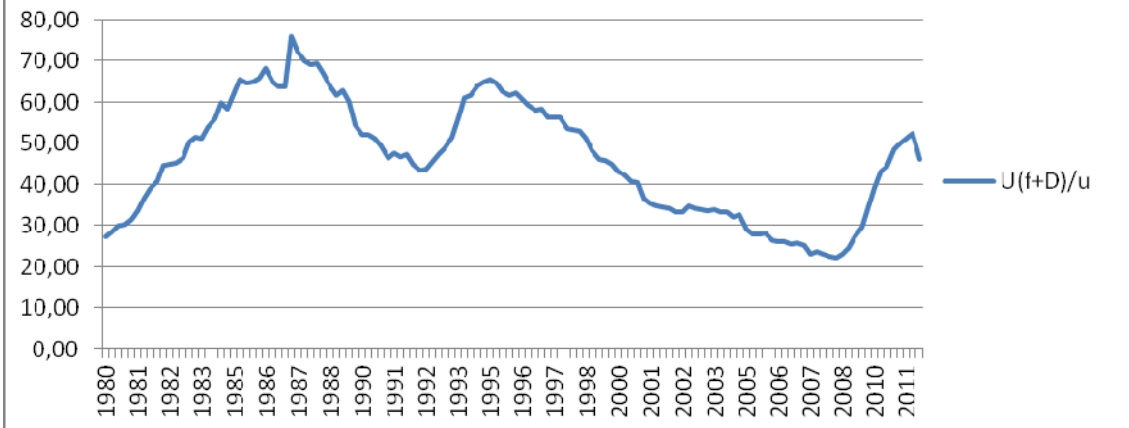Fig. 7 U_F and U_{NF} when a union at the firm level set the wage.

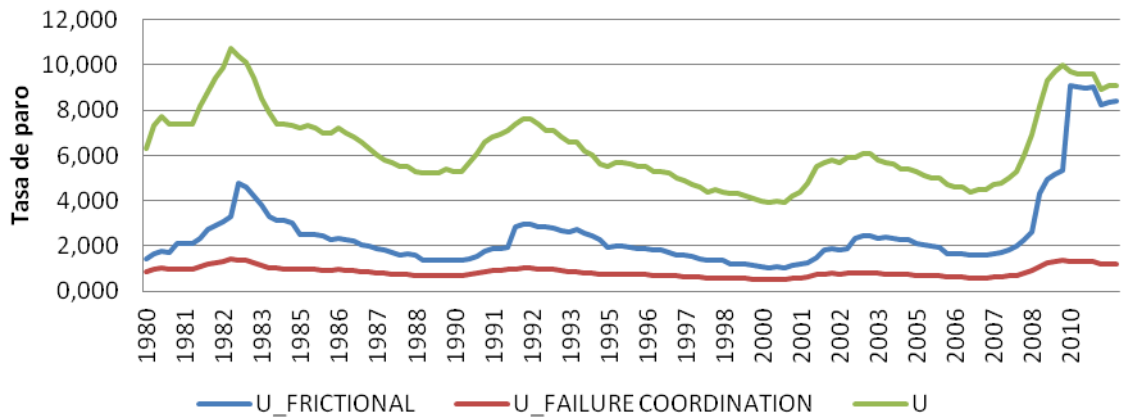Fig. 8 U_F and U_{NF} when a union at the national level set the wage.
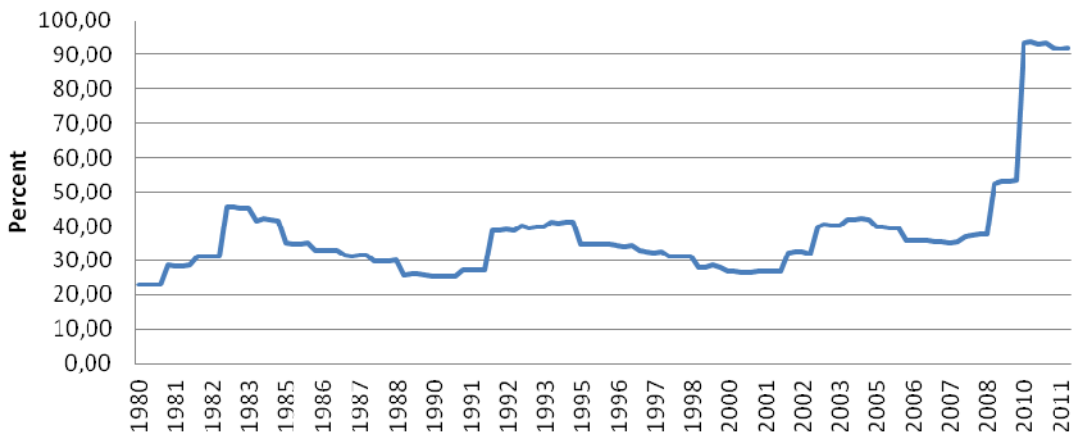
Figure 9. Decomposition unemployment rate for Spain

Legend: U_FRICTIONAL — U_FAILURE COORDINATION — U

Figure 10. Weight of frictional versus unemployment. Spain

Legend: U(f+D)/u

Figure 11. Decomposition unemployment rate for USA.

Figure 12. Wheight of the frictional versus unemployment. USA

References

[1] Abraham, K.G. (1991). "Mismatch and Labour Mobility: Some final remarks", in Shioppa, F.P. (comp.): Mismatch and Labour Mobility, Cambridge University Press, Cambridge, Chapter 11, 453-480.

[2] Abraham, K. G. and Katz L.F. (1986). "Cyclical unemployment: Sectorial Shifts or Aggregate Disturbances?", *Journal of Political Economy*, 94(3): 507-522.

[3] Alonso-Borrego, C., Fernández Villaverde J. and Galdón-Sanchez (2006). "Evaluating Labor Market Reforms: A General Equilibrium Appoach" *unpublished manuscript.*

[4] Aguirregabiria, V. and Alonso-Borrego C. (2009). Labor Contracts and Flexibility: Evidence from a Labor Market reforms in Spain", Working Paper 09-18. Economic Series. Universidad Carlos III de Madrid.

[5] Antolín, P. (1994). "Unemployment Flows and Vacancies in Spain", IVIE, Working Paper (EC) nº 5.

[6] Antolín, P. (1995). "Movilidad Laboral, Flujos de Desempleo, Vacantes y Comportamiento en la Búsqueda de Empleo en el Mercado de Trabajo Español", *Moneda y Crédito*, 201: 49-86.

[7] Bean, C. and Pissarides, C.A. (1993). "Unemployment, Consumption and Growth", *European Economic Review* 37: 837-54.

[8] Bentolila, S. and Dolado J.J. (1991). "Mismatch and Internal Migration in Spain, 1962-1986", in Shioppa, F.P. (comp.): Mismatch and Labour Mobility, Cambridge University Press, Cambridge, Chapter 5, 182-236.

[9] Blanchard, O. J. and Diamond (1989). "The Beveridge Curve", *Brookings Papers on Economic Activity*, 1:1-60

[10] Blanchard, O. J. and Diamond (1994). "Ranking, Unemployment Duration, and Wages", *Review of Economic Studies*, 61(3): 417-434.

[11] Blanchard, O. J. and Galí, J. (2010). "Labor Markets and Monetary Policy:A New-Keynesian Model with Unemployment", *American Economic Journal: Macroeconomics* 2 (2): 1-30.

[12] Brown A.J., Merkl C. and Snower D.J. (2011). "An Incentive Theory of Matching", Kiel Institute Working Papers, 1512.

[13] Burda, M. and Wyplosz, C. (1994). "Gross Worker and Job Flows in Europe", *European Economic Review*, 38 (6): 1287-1315.

21

[14] Cahuc, P. and Zylberberg, A. (2004). Labor Economics, *The MIT Press*.

[15] Choi, S. and Rios-Rull J.V. (2009). "Understanding the Dynamics of Labor Share: The Role of Noncompetitive Factor Prices", *Annals of Economics and Statistics* 95/96:251-278.

[16] Dixon, R., Lim G. C. and Freebairn J. (2012). "Regional Beveridge Curves: A latent Variable Approach", *Regional Studies*, first article 1-16

[17] Eriksson, C. (1997). "Is There a Trade-Off between Employment and Growth?",*Oxford Economic Papers* 49: 77-88.

[18] Fonseca, R. and Muñoz, R. (2003). "Can the Matching Model Account for Spanish Unemployment?", *Investigaciones Económicas*, 27(2): 277-308.

[19] Freeman R.B. (1991). "Labour market tightness and the Mismatch between Demand and Supply of Less-educated Young Men in the United States in the 1980s", in Shioppa, F.P. (comp.): Mismatch and Labour Mobility, Cambridge University Press, Cambridge, Chapter 8,360-385 .

[20] Galí J. (2000). "Monetary Policy and Unemployment", NBER Working Paper n° 15871.

[21] Hagedorn, M. and Manovskii I. (2008). "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited", *American Economic Review*, 98(4):1692-1706.

[22] Hall, R.E. (2005)."Employment Fluctuations with Equilibrium Wage Stickiness", *American Economic Review*, 95 (1): 50-65.

[23] Herz, B. and Rens T. (2011). "Structural Unemployment". *unpublished manuscript.*

[24] Jackman, R.A., Layard, R. and Savouri, S. (1991). "Mismatch: A Framework for Thought" , in Shioppa, F.P. (comp.): Mismatch and Labour Mobility, Cambridge University Press, Cambridge, Chapter 2, 44-101.

[25] Kocherlakota, N. (2010). Inside the FOMC. Speech in Marquette, MI, on August 17 as president of the Federal Bank of Minneapolis.

[26] Krugman, P. (2010). Structure of excuses. September 27, New York Times.

[27] Michaillat, P. (2011). "Do Matching Frictions Explain Unemployment? Not in Bad Times", *American Economic Review*, forthcoming.

22

[28] Nickell, S. (1999). "Product markets and labour markets", *Labour Economics*, 6, 1-20.

[29] Petrongolo, B. and Pissarides, C.A. (2001). "Looking into the Black Box: A Survey of the Maching Function", *Journal of Economic Literature*, 39: 390-431.

[30] Pissarides, C.A. (1979). "Job Matching with State Employment Agencies and Random Search", *Economic Journal*, 89: 818-833.

[31] Pissarides, C.A. (1986). "Unemployment and Vacancies in Britain", *Economic Policy*, 1(3): 499-559.

[32] Raurich, X. and Sorolla, V. (2011). "Labour Productivity, Wages, Employment and Growth", *unpublished manuscript*.

[33] Sala, H. and Silva J.I. (2009). "Flexibility at the Margin and Labour Market Volatility: The Case of Spain", *Investigaciones Económicas*, vol. XXXIII (2):145-178.

[34] Shimer, R. (2004). "The Consequences of Rigid Wages in Search Models", *Journal of the European Economic Association*, 2: 469-479.

[35] Silva, J.I. and Toledo M. (2009). "Labor Turnover Costs and the Cyclical Behavior of Vacancies and Unemployment" *Macroeconomic Dynamics*, 13, supplement S1: 76-96.

[36] Stevens, M. (2007). "New Microfoundations for the Aggregate Matching Function", International Economic Review", 48 (3): 847-868.

[37] Yashiv, E. (2006). "Evaluating the Performance of the Search and Matching Model, *European Economic Review*, 50: 909-936