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Sequential decisions in the Diamond-Dybvig banking model^{*}

Markus Kinaterer and Hubert János Kiss^{**}

Abstract

We study the Diamond-Dybvig model of financial intermediation (JPE, 1983) under the assumption that depositors have information about previous decisions. Depositors decide sequentially whether to withdraw their funds or continue holding them in the bank. If depositors observe the history of all previous decisions, we show that there are no bank runs in equilibrium independently of whether the realized type vector selected by nature is of perfect or imperfect information.

JEL classification numbers: C72, D82, G21.

Keywords: Bank Run, Imperfect Information, Perfect Bayesian Equilibrium.

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1 Introduction

Several banks and other financial institutions experienced sudden and massive withdrawals of deposits and other funding sources over the course of the recent financial crisis. Examples include the bank Northern Rock in the UK, the investment bank Bear Stearns in the US and the DSB Bank in the Netherlands. It is often claimed that in these episodes not only the deterioration of fundamental variables led to the run of withdrawals, but there was also an important self-fulfilling component to the behavior of depositors. Depositors may decide to rush to the bank to withdraw fearing that other depositors' withdrawal will cause the bank to fail. Depositors reacting to such fear may generate a self-fulfilling run. There is a substantial literature that studies the conditions under which such self-fulfilling bank runs arise in an economy with rational agents.

The seminal work by Diamond and Dybvig (1983) explains the occurrence of bank runs as the outcome of a simultaneous-move game and absent fundamental reasons. After each depositor observed her randomly drawn type (patient or impatient) and nothing else, she chooses to wait or to withdraw. Thereafter, those who withdraw contact the bank in a random order. If each patient depositor expects all other patient depositors to wait, then her best response is to wait as well. Thus, the unconstrained efficient allocation is implementable, that is, it is an equilibrium. However, if patient depositors have the opposite expectations, then it might be optimal for them to withdraw resulting in a bank run. Hence, in the classic Diamond-Dybvig framework, the efficient allocation of no bank run is not strongly implementable, that is, it is not the unique equilibrium outcome.

Green and Lin (2003) add to the Diamond-Dybvig framework aggregate uncertainty about liquidity needs. They assume that depositors know the order in which they have an opportunity to withdraw, they place less restrictions on the deposit contracts than Diamond and Dybvig (1983) and show that the unconstrained efficient allocation is strongly implementable.

This result opened up the question about the real nature of bank runs and suggested that special ingredients of the Diamond-Dybvig model are crucial to obtain bank runs. Ennis and Keister (2009b) show that the Green-Lin result ceases to hold when liquidity types are correlated among depositors, and the debate about the appropriate elements to explain bank fragility is still ongoing. We contribute to this debate by stressing the importance of observability of decisions. This is motivated by real-world bank runs (see Sprague (1910) and Wicker (2001)) and statistical data on run episodes. For instance, Kelly and O Grada (2000) study the behavior of depositors during the banking panics of 1854 and 1857 in New York. The depositors were mostly Irish immigrants, and the county of origin in Ireland was the most important factor to determine whether they withdrew or not. The authors explain this result arguing that immigrants from the same country tended to cluster in neighborhoods of their own and when they decided to withdraw or to wait, this information spread among them and prompted the observers to follow suit. Starr and Yilmaz (2007) use detailed data provided by a bank that suffered a run in Turkey in 2001. Depositors were grouped according to their deposit size and Starr and Yilmaz examine how the behavior of these groups depended on previous withdrawal hikes. The behavior of depositor groups of different sizes was responsive to actions of their peers, but not always to the observable behavior of depositors of other groups. In a recent study, Iyer and Puri (2011) investigate the underlying reasons for a run that affected an Indian bank in 2001. Their results highlight that a depositor's likelihood to run is increasing in the fraction of other people in his/her social network that have run. In all of these episodes, banks did not suffer from bad fundamentals, but runs were rather the outcome of a coordination failure among depositors. There is also growing experimental evidence (e.g., Schotter and Yorulmazer (2009) and Kiss et al. (forthcoming)) that attests to the idea that the degree of observability affects the likelihood of bank runs. Overall, these studies emphasize that understanding how observability

influences the existence of bank runs is of first order importance.

We study the finite-depositor version of the original Diamond-Dybvig model without aggregate uncertainty, that is, the number of patient and impatient depositors is commonly known. We assume that depositors contact the bank in an exogenously given fixed order to communicate whether to leave the money deposited or to withdraw it. Each depositor observes the decisions of preceding depositors.

First, we show that when liquidity types and actions are perfectly observed, then no bank run occurs and the unconstrained efficient allocation is strongly implementable. Our main contribution is to extend this result to the case when the sequence of liquidity types is imperfect information, that is, a depositor's liquidity type is her private information.

Under perfect information, our result is obtained by backward induction. A patient depositor observing a high number of waitings is sure that if she keeps her money in the bank, then her consumption is higher than if she withdraws it. Waiting dominates withdrawal for the last patient depositor if enough depositors before her waited. Anticipating this decision, the next to the last patient depositor's decision is of the same nature, and by moving backwards all patient depositors wait.

Under imperfect information, the liquidity type vector is randomly selected by nature and is unobserved by the depositors. Every depositor, as it is her turn to decide, observes previous decisions and forms beliefs about which vector was selected, or in other words, whether before her patient depositors waited and only impatient ones withdrew. Perfect Bayesian Equilibrium imposes a strong rationality criterion on the strategy profile and belief system. This enables us to obtain a unique prediction on depositors' behavior which coincides with the solution under perfect information. Patient depositors wait and impatient ones withdraw. This result is a consequence of depositors' backward looking beliefs combined with the anticipation of subsequent depositors' behavior by sequential rationality.

Although we focus on banks, run-like phenomena occur in other institutions and markets as well. During the recent world-wide financial and economic crisis different funds (money-market, hedge and pension) have been run by investors who withdrew their investment within a short amount of time (see, for example, Baba et al. (2009) and Duffie (2010)). Gorton and Metrick (forthcoming) analyze the run on the repo market during the panic of 2007-2008. After some minor modifications, our analysis applies to these markets and institutions as well.

Related literature

Table 1 classifies the related literature along two key dimensions: observed information and aggregate uncertainty about liquidity types. Four cases have been studied regarding the first dimension: i) nothing, ii) position in the line, iii) only withdrawals are observed and finally, iv) depositors know their position and observe previous decisions. As of aggregate uncertainty about liquidity needs, there are two groups of papers. While the canonical Diamond-Dybvig model and our paper assume a degenerate distribution of types, the Green-Lin tradition allows for non-degenerate distributions.¹ Within the models with aggregate uncertainty some assume that liquidity types are independent across depositors, while others allow for the correlation of types.

observation	Aggregate certainty		Aggregate uncertainty	
	nothing	Diamond and Dybvig (1983)	Peck and Shell (2003)	
	position		Green and Lin (2003), Ennis and Keister (2009b)	
	withdrawals		Ennis and Keister (2011)	
	position and previous decisions	this paper	Andolfatto et al. (2007)	

Table 1: Classification of the literature

Aggregate (un)certainty affects the efficient allocation in models. If the

¹Diamond and Dybvig (1983) touch upon the case with aggregate uncertainty, but do not solve it analytically.

liquidity needs in the population are uncertain, then the bank updates its beliefs about the distribution after each observed decision and reoptimizes the allocation. Withdrawing depositors obtain different payoffs depending on their position in the queue and the history. Without aggregate uncertainty, the unconstrained efficient allocation is independent of the depositors' choices since the number of different types is commonly known. This, in turn, determines the sensitivity of the bank to the aggregate demand for liquidity. Under the standard assumption of sequential service constraint, a bank that functions in an environment characterized by aggregate certainty realizes suddenly that aggregate withdrawal demand is too high. Until the number of withdrawals does not surpass the commonly known number of impatient depositors, the bank pays to withdrawing depositors the full amount of money specified by the unconstrained efficient allocation. This insensitivity opens up the possibility of bank runs. In contrast, when liquidity needs are uncertain, the bank reacts upon each observed piece of information.

Peck and Shell (2003) assume that depositors have no other information than their liquidity type. In their model, as in Diamond and Dybvig (1983), only depositors who wish to withdraw contact the bank and bank runs still constitute an equilibrium outcome. In another model with aggregate uncertainty, Green and Lin (2003) assume that each depositor contacts the bank during the early period, and that each has information about her position in the queue.² These changes lead to the absence of bank runs. However, in their model, the realization of liquidity needs is independent across depositors. This is crucial to obtain the no bank run results, as shown by Andolfatto et al. (2007) and Ennis and Keister (2009b). In Andolfatto et al. (2007), the bank informs each depositor of the complete history of actions taken by the preceding depositors. They prove that any allocation that is implementable is also strongly implementable. In a similar setup, Ennis and Keister (2009b)

²Without loss of generality, we can assume that the knowledge about the position is perfect (see Green and Lin (2000) and Ennis and Keister (2010)).

demonstrate that by placing a weak restriction on the correlation of types runs re-emerge as equilibrium outcomes. Ennis and Keister (2011) assume that the bank and the depositors observe withdrawals (but not waitings) as they occur. This environment admits partial bank runs. Thus, this strand of the literature studies the optimal contracts given the environment (especially the information available to the bank, but also other features, such as the correlation in liquidity types across depositors) and identifies conditions under which bank runs do not occur.

Since with aggregate uncertainty more information leads to the absence of bank runs, it is worth studying what happens when liquidity needs in the population are certain. Due to the insensitivity of the contract to aggregate demand for liquidity, in the models following Diamond and Dybvig (1983), bank runs represent an equilibrium outcome. However, these models assume that depositors have no other information than their own liquidity type.³ To our best knowledge, we are the first to study the case when depositors observe previous decisions as they occur. Observing the complete history of previous decisions is assumed by Andolfatto et al. (2007), although it is irrelevant there because a depositor reveals her type truthfully if all *subsequent* depositors do so and, consequently, her decision is independent of previous decisions. In our paper, a depositor's optimal choice depends on the history. Our result shows that in spite of the optimal contract being insensitive to the accumulating information about aggregate liquidity demand, information about all previous decisions is enough to eliminate bank runs as equilibrium outcome given the aggregate certainty assumption.

Notice that even though Diamond and Dybvig (1983) showed that an adequately designed suspension-of-convertibility clause prevents bank runs, our results are important for at least two reasons. First, such deposit freeze

³An exception is Gu (2011) who studies observability in form of a signal extraction problem in which depositors try to figure out whether the bank has fundamental problems or not. Thus, Gu's bank runs are not due to coordination failure but signal bad fundamentals.

policies are not explicit, they are not stipulated in deposit contracts, so depositors are unlikely to consider their effect when making withdrawal decisions. Second (and more importantly), Ennis and Keister (2009) show that due to time inconsistency when a run is underway, banking authorities will be more lenient when imposing suspension of convertibility than they would like depositors to believe *ex ante*. Hence, the credibility of the deposit freeze policy is questionable and for depositors it may seem optimal to participate in a bank run. Therefore, identifying factors that may mitigate the likelihood of bank runs in the canonical Diamond-Dybvig framework remains a relevant endeavor.

The remainder of the paper is organized as follows. Section 2 introduces the model which builds on the seminal paper by Diamond and Dybvig (1983). We extend the model by specifying the sequence of decisions and the information depositors have before making a decision. All results are derived in section 3, in particular, the equilibrium when the liquidity type vector is either perfectly or imperfectly observed. Section 4 concludes. All proofs are relegated to the Appendix.

2 The model

There are three time periods denoted by $t = 0, 1, 2$, and a finite set of depositors denoted by $I = \{1, \dots, N\}$, where $N > 2$. The consumption of depositor $i \in I$ in period $t = 1, 2$ is denoted by $c_{t,i} \in \mathbb{R}_+^0$, and her liquidity type by θ_i . It is a binomial random variable with support given by the set of liquidity types $\Theta = \{0, 1\}$. If $\theta_i = 0$, depositor i is called *impatient*, that is, she only cares about consumption at $t = 1$. If $\theta_i = 1$, depositor i is called *patient*. Depositor i 's utility function is given by

$$u_i(c_{1,i}, c_{2,i}, \theta_i) = u_i(c_{1,i} + \theta_i c_{2,i}).$$

It is assumed to be strictly increasing, strictly concave, twice continuously differentiable and to satisfy the Inada conditions. The relative risk-aversion

coefficient, $-c_i u_i''(c_i)/u_i'(c_i)$, is assumed to be strictly larger than 1, for all $c_i \in \mathbb{R}_+$, and all $i \in I$.

The number of patient depositors is assumed to be constant and given by $p \in \{1, \dots, N\}$ and the remaining depositors are impatient. The number of patient and impatient depositors is common knowledge.

Let $\Theta^N = \{0, 1\}^N$, and $\theta^N = (\theta_1, \dots, \theta_N)$ denote the sequence of depositors, also called liquidity type vector. The set of sequences of length N with p patient depositors is given by

$$\Theta^{N,p} = \{\theta^N \in \Theta^N : \sum_{i=1}^N \theta_i = p\}.$$

There are $\binom{N}{p}$ possible liquidity type vectors. One is selected randomly by a process which selects each of them with equal probability. The realized liquidity type vector is unobserved both by the depositors and by the bank.

At $t = 0$, each depositor $i \in I$ has one unit of a homogeneous good which she deposits in the bank. The bank has access to a constant-return-to-scale productive technology which pays a gross return of one unit for each endowment liquidated at $t = 1$, and a fixed return of $R > 1$ for each endowment liquidated at $t = 2$. It insures against the privately observed liquidity risk, which is only realized at $t = 1$, by offering a simple demand-deposit contract which offers to pay c_1^* to any depositor i who withdraws at $t = 1$ as long as the bank has funds, and the same pro rata share of funds available to all depositors who wait until $t = 2$.

2.1 The first-best allocation

First, the unconstrained efficient solution is derived. If a social planner observed each depositor's liquidity type, then she could maximize the sum of depositors' utilities with respect to $c_{1,i}$ and $c_{2,i}$ subject to a resource constraint and p . The first best allocation solves

$$\max_{c_{1,i}, c_{2,i}} (N - p)u_i(c_{1,i}) + pu_i(c_{2,i})$$

$$\text{s. t. } (N - p)c_{1,i} + \frac{p}{R}c_{2,i} = N.$$

The solution to this problem is

$$u'(c_1^*) = Ru'(c_2^*),$$

which, as in Diamond and Dybvig (1983), implies that $R > c_2^* > c_1^* > 1$. In the first-best allocation, all impatient depositors consume c_1^* in period 1, and all patient depositors c_2^* in period 2. Hence, patient depositors receive a higher consumption than impatient ones. The unconstrained efficient allocation offers liquidity insurance, because the amount of consumption received by a depositor who turns out to be impatient is higher than that in autarky.⁴

2.2 Mechanism, strategies and equilibrium concept

A sequential service constraint is assumed to hold, that is, at $t = 1$, the depositors contact the bank sequentially in the order given by θ^N , and the payment to any withdrawing depositor only depends on the history (as it will be specified below).

We consider a direct mechanism, in which each depositor's strategy is to announce a type. When type 0 or type 1 is announced, the depositor wishes to withdraw or wait, respectively. The mechanism is anonymous, that is, it does not depend on the depositors' indexes.

Each depositor i is assumed to observe the entire history of previous type announcements $s^{i-1} = (s_1, \dots, s_{i-1})$, where $s^{i-1} \in \Theta^{i-1}$. Depositor i 's strategy is conditional on the history and her type. It is defined as $\mathbf{s}_i : \Theta^{i-1} \times \Theta \rightarrow \Theta$. Depositor i announces a type from Θ , that is, $\mathbf{s}_i \in \{0, 1\}$ for all $i \in I$. Let $\mathbf{S} = \{0, 1\}^N$ be the game's strategy space, and let $\mathbf{s} \in \mathbf{S}$ be a strategy profile,

⁴In autarky, an impatient depositor earns the unit gross return at $t = 1$, while a patient depositor earns R at $t = 2$.

that is, $\mathbf{s} = (\mathbf{s}_1, \dots, \mathbf{s}_N)$. In order to emphasize depositor i 's strategy, \mathbf{s} is sometimes written as $(\mathbf{s}_i, \mathbf{s}_{-i})$.

Given strategy profile $\mathbf{s} \in \mathbf{S}$, depositor i 's consumption is specified by $c_i = (c_{1,i}; c_{2,i})$, where $c_{1,i} : \Theta^i \rightarrow \mathbb{R}_+^0$, and $c_{2,i} : \Theta^N \rightarrow \mathbb{R}_+^0$. The consumption of all depositors is feasible if $\sum_{i=1}^N (c_{1,i} + \frac{c_{2,i}}{R}) \leq N$. Depositor i 's period-1 consumption is then defined as

$$c_{1,i} = \begin{cases} c_1^*, & \text{if } s_i = 0 \text{ and } N - \sum_{j=1}^{i-1} s_j c_1^* \geq c_1^*, \\ y, & \text{if } s_i = 0 \text{ and } 0 < N - \sum_{j=1}^{i-1} s_j c_1^* < c_1^*, \\ 0, & \text{otherwise,} \end{cases}$$

where $y = N - \sum_{j=1}^{i-1} s_j c_1^*$. Thus, until the bank runs out of funds, any depositor who announces to be impatient receives a positive amount of consumption c_1^* or y .

Let $\eta \in \{0, \dots, p\}$ be the number of depositors who wait at $t = 1$, that is, each of them announces to be of type 1. Given $\eta = \sum_{i=1}^N s_i \geq 0$, all players who wait at $t = 1$, obtain the same consumption at $t = 2$, namely,

$$c_2(\eta) = \max\{0, \frac{R(N - (N - \eta)c_1^*)}{\eta}\}.$$

If $\eta = p$, that is, only impatient depositors withdraw at $t = 1$, then $c_2(\eta) = c_2^* > c_1^*$ and patient depositors enjoy a higher consumption than impatient ones.

The consumption in both periods depends on the strategy profile and determines each depositor's utility. For any $i \in I$ and any $\mathbf{s} \in \mathbf{S}$ this is denoted by $u_i(\mathbf{s})$. Thus, u_i is a mapping from \mathbf{S} to \mathbb{R}_+^0 . Let the tuple (I, \mathbf{S}, u) be the *bank run game*, where $u = (u_1, \dots, u_N)$.

Given the observed history s^{i-1} , and depositor i 's type θ_i , her strategy is determined. However, she does not observe the underlying type vector and both patient and impatient depositors may choose to withdraw. Therefore, she forms beliefs about the type vector that was selected by nature. Let

$\mu_i \equiv \mu_i(\theta^N \mid s^{i-1}, \theta_i)$ denote depositor i 's belief about the *true* type vector. The belief is conditional on the history and i 's type and together with a strategy profile defines a Perfect Bayesian Equilibrium.

Definition 1. *Given a bank run game. Then, strategy $\mathbf{s} \in \mathbf{S}$ and belief system $\mu = (\mu_1, \dots, \mu_N)$ are a Perfect Bayesian Equilibrium (PBE) if, and only if, for all $i \in I$, given θ_i , s^{i-1} and any $\tilde{\mathbf{s}}_i \in \{0, 1\}$,*

$$\sum_{\theta^N \in \Theta^N} \mu_i(\theta^N \mid s^{i-1}, \theta_i) u_i(\mathbf{s}) \geq \sum_{\theta^N \in \Theta^N} \mu_i(\theta^N \mid s^{i-1}, \theta_i) u_i(\tilde{\mathbf{s}}_i, \mathbf{s}_{-i}),$$

where $\mu_i(\theta^N \mid s^{i-1}, \theta_i)$ is consistent with Bayes' rule whenever possible.

A strategy profile and a system of beliefs are a PBE if, and only if, the strategy is sequentially rational for all players and the belief is consistent with the strategy (see Fudenberg and Tirole (1991) and Myerson (1997)).

3 Results

The simple demand-deposit contract defined above implements the first-best allocation (see Diamond and Dybvig (1983)). In case the unconstrained efficient allocation is the unique PBE outcome of the bank run game under the proposed mechanism, then it is called strongly implementable.

Definition 2. *Given a bank run game. Then, the unconstrained efficient allocation is strongly implementable under $\mathbf{s} \in \mathbf{S}$ by truth-telling if, and only if, in the unique PBE outcome of this bank run game, for all $i \in I$, $\mathbf{s}_i(s^{i-1}, \theta_i) = \theta_i$ and $\mu_i(\theta^N \mid s^{i-1}, \theta_i)$ is consistent with this strategy profile.*

Given p , it is possible to determine how many patient depositors have to wait in order for waiting to be an optimal strategy for each of them. In Lemma 1, one part of this threshold is derived,⁵ namely, the one (denoted

⁵The other part is a technical detail which is derived below in Proposition 1's proof.

as $\bar{\eta}$) such that $c_{2,i} > c_1^*$ for every patient depositor i . In case some patient depositor declares to be impatient, then the bank spends funds on her which it would otherwise have kept until period 2. Recall that η is the number of patient depositors that wait.

Lemma 1. *Given p , there is $1 \leq \bar{\eta}(p) \leq p$, such that for every patient depositor i , $c_{2,i}(\eta) < c_1^*$, for all $\eta \leq \bar{\eta}$, and $c_{2,i}(\eta) > c_1^*$, for all $\eta > \bar{\eta}$.*

The proof of Lemma 1 can be found in Appendix A.

3.1 The type vector is perfect information

Next, the benchmark case with perfect information is studied. Apart from commonly knowing the number of patient depositors p , the depositors also commonly know each depositor's type, or in other words, the type vector selected randomly by nature.

Impatient depositors have a strictly dominant strategy to always withdraw, and thus, $s_i(s^{i-1}, \theta_i = 0) = 0$. Conversely, the most important information for a patient depositor is her relative position among the patient depositors. By eliminating the uncertainty about the type vector we may apply standard backward induction to find the equilibrium in Proposition 1.

Proposition 1. *Given a bank run game. Suppose that the type vector is perfect information. Then, the unconstrained efficient allocation is strongly implementable by truth-telling.*

The proof of Proposition 1 can be found in Appendix B. In the proof it is shown that in the unique PBE, which is a subgame perfect equilibrium, each depositor reveals her type truthfully. Intuitively, the last patient depositor's decision is to wait in case sufficient preceding patient depositors waited such that period-2 consumption is at least as high for her as period-1 consumption. Anticipating this decision, the next to last patient depositor's decision is to wait, and by moving backward all patient depositors' decision is to wait.

Note that the concepts of PBE and subgame perfect equilibrium impose that each depositor decides rationally when it is her move based on the previous depositors' decisions and on the anticipated decision of subsequent depositors which is derived by sequential rationality and backward induction, respectively. Apart from the unique PBE derived in Proposition 1, the set of Nash Equilibria of this bank run game contains strategy profiles in which some or all patient depositors withdraw, and therefore, a bank run occurs. However, these equilibria are not subgame perfect and are eliminated when requiring the additional rigor of PBE or subgame perfectness.

3.2 The type vector is imperfect information

When the type vector is not observable, depositors cannot apply the previous reasoning. Given the observable history, a depositor forms beliefs about the type vector selected by nature and, by sequential rationality, anticipates how subsequent depositors behave. In this environment of imperfect information, sequential rationality plays a similar role as backward induction in games of perfect information (see Myerson (1997)). Before proving the general result, the difficulties that arise are illustrated in an example.

3.2.1 An example

We show in a detailed way how a unique PBE outcome arises in the following example with four depositors. Suppose that there are three patient depositors and all of them have to keep the money in the bank to make waiting worthwhile. There are four possible type vectors, since the impatient depositor may be in position 1, 2, 3 or 4, and each is equally likely to be selected.

Since the impatient depositor always withdraws, the argument focuses on the decision of patient depositors in the different positions. First, consider a patient depositor in the last position. She waits if she observes any history

containing at least two waitings. Otherwise, she withdraws. All other players anticipate player 4's behavior by sequential rationality.

A patient depositor in position 3 can observe four possible histories: $(0, 0)$, $(0, 1)$, $(1, 0)$ and $(1, 1)$. If she observes the first history, then she knows that she is better off to withdraw since a bank run occurs. In all other cases, she updates her belief, and since she is patient, she knows (or believes with probability 1) which of the four possible type vectors is realized. Given history $(1, 0)$ or $(0, 1)$, and in case she waits, then by sequential rationality, she anticipates that the last depositor is patient and waits. Therefore, she is better off to wait as well, and by doing so, induces the fourth depositor to wait. After observing the history $(1, 1)$ and given that she is patient, she waits and, by doing this, guarantees all patient depositors a higher period-2 consumption.

Consider now a patient depositor in position 2. She observes either a withdrawal or a waiting of the first depositor. In the first case, she knows (or believes with probability 1) which type vector is realized. By sequential rationality, she anticipates that the last two depositors are patient and wait if, and only if, she waits. Since by waiting her utility is strictly larger than by withdrawing she waits. If she observes that the first depositor waits, and given that she is patient, she eliminates the possible type vectors in which the first or second depositor is impatient. There are two type vectors left in which the first two depositors are patient and each was selected by nature with conditional probability of $\frac{1}{2}$. This is her updated belief about the realized type vector selected by nature. By sequential rationality, she anticipates that the last patient depositor will wait upon observing that she waited, independently of whether the last patient depositor is located in the third or fourth position. Therefore, she is strictly better off to wait and to enjoy a higher period-2 consumption than to withdraw. A similar argument applies to a patient depositor in position 1 who waits since she anticipates by sequential rationality that both other patient depositors will wait in a PBE.

Consider now the out of equilibrium behavior of a patient depositor. Obviously, a patient depositor in position 4 knows whether the observed history is compatible with PBE or not. If it is, then she waits, otherwise she withdraws. A patient depositor in position 3 observing the histories $(1, 0)$ or $(0, 1)$ believes to be on the equilibrium path and waits, as just derived, although both histories also arise when a patient depositor in position 2 or 1, respectively, decides to withdraw. However, in a PBE she follows her strategy as long as the observed history, and thus her belief, is consistent with the PBE strategy profile in which all patient depositors wait since any deviation from it yields her a lower payoff. Therefore, she withdraws if, and only if, she observes the history $(0, 0)$. Similarly, a patient depositor in position 2 after observing a withdrawal believes in equilibrium that the type vector selected by nature is the one in which the impatient depositor occupies the first position, and she waits, even if the withdrawal she observes were due to a patient depositor who deviates from PBE. Since her observation, and thus, her belief is consistent with the unique PBE outcome in this game she complies with her equilibrium strategy. Finally, a patient depositor in position 1, in a PBE, has a unique optimal decision to wait.

3.2.2 The general case

In general, the arguments in the previous example can be applied in order to obtain an equilibrium for any bank run game and the unique PBE outcome is always that no bank run occurs. By definition 2, this implies, that the unconstrained efficient allocation is strongly implementable.

Proposition 2. *Given a bank run game. Suppose that the type vector is private information. Then, the unconstrained efficient allocation is strongly implementable.*

The proof of Proposition 2 can be found in Appendix C. Intuitively, in the unique PBE outcome, a patient depositor believes to be on the equilibrium

path as long as there are $N - p$ or less withdrawals, that is, unless she observes a history which is incompatible with being on the equilibrium path. She waits and, by sequential rationality, anticipates that all other patient depositors behind her in the queue will wait as well. Since to wait yields each of them a strictly larger consumption, it is optimal to wait for each of them. This in turn induces all other patient depositors to wait. All impatient depositors withdraw and the efficient allocation is strongly implemented.

While this is the unique PBE outcome, there are several PBE which all are identical on the equilibrium path, though they differ on off equilibrium paths. To see this, consider any history in which the bank ran out of funds since there were too many withdrawals. Then, any depositor is indifferent to wait or to withdraw since her payoff is zero in any case, and any decision is optimal since there is no profitable deviation. Therefore, it is possible to construct multiple PBE that differ by the depositors' optimal behavior after the bank went bankrupt. However, on the equilibrium path such a history never occurs, and patient depositors always wait and impatient ones always withdraw, and the unique PBE outcome is that of no bank run.

Finally, note that other Bayesian Nash Equilibria exist. However, all of them are based on incredible threats. For example, for all depositors to withdraw is a Bayesian Nash Equilibrium of the bank run game which results in a bank run. However, in a PBE, each patient depositor upon being called to decide is better off to wait than to withdraw, unless she would receive a lower period-2 consumption. Since all patient depositors' reasoning is identical and each of them anticipates, by sequential rationality, that all subsequent patient depositors will wait as well, in a PBE, each of them is better off to wait. The rigor of PBE makes no bank run the unique outcome. Even after observing $N - p$ withdrawals, a patient depositor does not believe that some patient depositor before her withdrew.

4 Conclusion

Descriptions of bank runs suggest that the behavior of depositors depends crucially on the observed decisions of other depositors. Existing theoretical models in the Diamond-Dybvig tradition do not incorporate this idea, sequentiality is missing from these models.⁶ We attempt to make the first step to fill this gap and assume that depositors observe all previous actions. We show that bank runs do not occur in equilibrium, even though the type of preceding depositors is not observed. This result contrasts starkly with the findings of previous models. This suggests that the insensitivity of the Diamond-Dybvig contract to aggregate liquidity does not lead necessarily to bank runs being an equilibrium outcome. If all previous decisions are observed, in our model, bank runs are ruled out as equilibrium outcomes.

Two elements of the model seem to contribute to the absence of bank runs. First, aggregate certainty serves as a kind of coordination device to signal all patient depositors that it is in their best interest to wait, that is, it is commonly known that a bank run never occurs if all of them wait. Second, the sequential service constraint together with the perfect observability of previous decisions ensure that this is the unique PBE outcome. This equilibrium concept imposes strong rationality requirements on the depositors in terms of beliefs and sequential rationality. Relaxing these assumptions, that is, introducing elements of bounded rationality, restricting the observability of previous decisions or introducing aggregate uncertainty could restore bank run as an equilibrium outcome under certain conditions. We leave the study of these issues to future research.

⁶There are models that follow the spirit of Green and Lin (2003) and allow depositors to observe previous actions to a certain extent, such as Ennis and Keister (2011).

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Appendix A

Appendix A contains the proof of Lemma 1.

Proof. In order to derive the threshold value $\bar{\eta}$, a condition is found such that c_1^* is strictly smaller than period-2 consumption, that is,

$$c_1^* < \frac{R(N - (N - \eta)c_1^*)}{\eta}, \quad (1)$$

where the right-hand-side is period-2 consumption if η depositors wait at $t = 1$. Solving this inequality for η yields

$$\eta > \frac{RN(c_1^* - 1)}{c_1^*(R - 1)}. \quad (2)$$

Denote by $[x]$ the integer part of any $x \in \mathbb{R}$. Since η is a natural number, the previous condition becomes

$$\eta > \left\lceil \frac{RN(c_1^* - 1)}{c_1^*(R - 1)} \right\rceil \equiv \bar{\eta}. \quad (3)$$

The right-hand side of (3) defines the threshold value $\bar{\eta}$. This value is unique since the bank pays to every depositor who withdraws c_1^* , and therefore, loses funds monotonically. If there are too many withdrawals by patient depositors, then the bank can only pay out $c_{2,i} < c_1^*$ to every depositor i which waits until $t = 2$. If the number of patient depositors that wait η is not larger than $\bar{\eta}$, as derived in (3), then period-2 consumption is strictly below c_1^* . \square

Appendix B

Appendix B contains the proof of Proposition 1.

Proof. We show that under perfect information, in the unique PBE which in this case is the unique subgame perfect equilibrium each depositor reveals her type truthfully.

First, conditions are derived under which period-1 consumption is strictly larger than period-2 consumption and a patient depositor is better off to withdraw at $t = 1$, that is, she declares to be impatient, and does not reveal her type truthfully. Thereafter, the depositors' equilibrium strategies are derived and shown to be a PBE. Finally, uniqueness is established.

As shown in Lemma 1, if $\bar{\eta}$ or less patient depositors wait, then $c_{2,i}(\eta) < c_1^*$ and a patient depositor is better off to withdraw as long as the bank pays her c_1^* . However, at some point in time the bank cannot pay c_1^* any more, but rather has $0 < y < c_1^*$ of funds left which she would pay to the last depositor which declares to be impatient. Then, there are two possibilities. Either period-2 consumption is larger than or equal to y , or it is strictly smaller. In the first case, all patient depositors are better off to wait if, and only if, there is no more impatient depositor left in the queue (since she would withdraw y). If there is some impatient depositor left who would withdraw y , then the patient depositor is better off to take y at $t = 1$, rather than to get 0 at $t = 2$, and she withdraws all remaining funds. In this second case, the depositor whose turn it is, once the bank has left y of funds, declares to be impatient. Even a patient depositor would do this since y is strictly larger than her period-2 consumption in case she waits.

Now the complete strategy for all players is derived: an impatient depositor always withdraws and a patient depositor withdraws if, and only if, her period-1 consumption is strictly larger than her expected period-2 consumption. Otherwise, she waits. This is a subgame perfect equilibrium and a PBE of the bank run game if no depositor's deviation from this strategy profile

is profitable. Consider first any impatient depositor's deviation and suppose that she waits at $t = 1$. Then, she receives the same or a lower payoff since for her $\theta_i = 0$, and this deviation is not profitable. Consider now any patient depositor and that she withdraws instead of waiting. Then, her consumption is $c_1^* < c_2^*$, and this deviation is not profitable for her given that all other patient depositors wait under the proposed strategy profile. Similarly, she cannot deviate profitably if her strategy tells her to withdraw at $t = 1$ since too many depositors before her withdrew already. Then, she would receive 0 by waiting and at least the same amount by withdrawing and her deviation to wait is not profitable.

This subgame perfect equilibrium is found by backward induction. Since any impatient depositor has a strictly dominant strategy to withdraw at $t = 1$, the argument focuses on patient depositors. The last patient depositor waits if enough patient depositors waited since then her payoff is $c_{2,i} > c_1^*$ or $c_{2,i} > y$. The next to last patient depositor waits if enough patient depositors waited anticipating (by backward induction) that then also the last patient depositor is strictly better off to wait. This is true for all previous patient depositors. Finally, the first patient depositor waits, that is, all of them wait and each receives $c_2^* > c_1^*$. Since this subgame perfect equilibrium and PBE is the only one found by backward induction, it is unique. \square

Appendix C

Appendix C contains the proof of Proposition 2.

Proof. Suppose that each depositor's type is her private information, but that the number of patient depositors is commonly known. Then, the statement of Proposition 2 is shown as follows: first, a belief system and a strategy profile are derived such that the belief system is consistent with the strategy profile, and the strategy is sequentially rational given the depositors' beliefs. Thereafter, it is shown that no depositor's unilateral deviation from

the proposed strategy profile is profitable on the equilibrium path, and then, the depositors' out of equilibrium behavior is shown to be optimal given the proposed strategy profile. Finally, it is shown that no bank run is the unique equilibrium outcome.

Consider the following strategy profile: patient depositors wait and impatient depositors withdraw. Any impatient depositor's consistent belief is identical to that of a patient depositor in the same position and this is derived below. Impatient depositor i 's deviation would be to declare that she were patient. Then $c_{1,i} = 0$, and this deviation is not profitable since for her $\theta_i = 0$, that is, she does not value period-2 consumption. In other words, for her to withdraw is a dominant strategy.

From now on, consider only patient depositors. Consider first the last patient depositor in the queue (independently of the position she is assigned) and suppose that the depositors follow the proposed strategy profile. Then, she observes that all other $p - 1$ patient depositors located before her in the queue waited. She identifies (or believes with probability 1) which type vector was selected by nature—in particular, that she is the last patient depositor—and is strictly better off to wait rather than to withdraw since her consumption is $c_2^* > c_1^*$. In fact, she waits if, and only if, $c_{2,i} > c_1^*$ or $c_{2,i} > y$, in case the bank has left y of funds. Her deviation to declare that she is impatient in this case is not profitable since it would yield her a lower consumption.

Consider now the next to last patient depositor in the queue (independently of the position she is assigned) and suppose that the depositors follow the proposed strategy profile. Then, she observes $p - 2$ waitings and her conditional belief puts equal weight on all type vectors in which there is exactly one more patient depositor in each of the positions behind her in the queue. Thus, she believes with probability 1 that she is the second to last patient depositor. By sequential rationality, she anticipates the last patient depositor's decision to wait, and therefore, is better off to wait herself rather than

to withdraw. However, she waits if, and only if, $c_{2,i} > c_1^*$ or $c_{2,i} > y$, in case the bank has left y of funds. Her deviation to declare that she is impatient in this case is not profitable since it would yield her a lower consumption.

By induction, the same reasoning about a patient depositor's optimal decision and consistent belief applies to all other patient depositors (independently of the positions in the queue they are assigned). In particular, the first patient depositor decides to wait since she anticipates, by sequential rationality, that all other patient depositors will wait. Her consistent belief is the conditional probability that each type vector in which she is the first patient depositor in the queue is selected with equal probability. Thus, she believes with probability 1 that she is the first patient depositor in the queue called to make a decision. Her deviation to declare that she is impatient in this case is not profitable since it would yield her a lower consumption.

Consider now the out of equilibrium behavior and belief of any patient depositor. She believes to be on the equilibrium path as long as the history she observes does not indicate her the contrary. Suppose first that she observes $N - p$ or less withdrawals. Even if some of them were due to patient depositors that decided to withdraw, then she believes that all withdrawals are due to impatient depositors and that she is on the equilibrium path. Therefore, she complies with her PBE strategy and waits. Her decision is equivalent to the one she would take if she were on the equilibrium path and observed the same number of withdrawals, and therefore, she also forms the same beliefs and has no profitable deviation. If she observes strictly more than $N - p$ withdrawals, then she concludes that at least one patient depositor chose to withdraw. She withdraws if, and only if, her expected period-2 consumption is strictly smaller than her period-1 consumption, by sequential rationality, anticipating the decisions of subsequent depositors.

Finally, an impatient depositor's belief is identical to that of a patient depositor in the same position independently of whether she is on or off the equilibrium path. In any case, she always withdraws and has no profitable

deviation. Hence, every depositor believes (and this is consistent on the equilibrium path) that any depositor who withdraws is impatient and any depositor who waits is patient. Thus, the PBE is established. However, there are many PBE that differ on off equilibrium paths since any depositor is indifferent to wait or withdraw once the bank ran out of funds which on the unique equilibrium path never occurs.

In order to show that no bank run is the unique equilibrium outcome, suppose that there is some other PBE than the one just encountered. Then, a contradiction arises on the equilibrium path. Suppose that there is another PBE outcome such that, at $t = 1$, on the equilibrium path either an impatient depositor waits or a patient depositor withdraws, or both. Obviously, an impatient depositor cannot gain by declaring to be patient since she does not value period-2 consumption and receives $c_1^* > 0$ by declaring to be impatient. Consider next that on the equilibrium path a patient depositor withdraws at $t = 1$, although she observed $N - p$ or less withdrawals. As follows from above, her strategy is not profitable and she has a profitable deviation to wait. Thus, there is a unique optimal behavior for the depositors on the equilibrium path and this yields the unique PBE outcome.

In the unique PBE outcome, there is no bank run since on the equilibrium path, at $t = 1$, every patient depositor waits and every impatient one withdraws. Therefore, the unconstrained efficient allocation is strongly implementable. \square



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