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Beneficial consumer fraud*

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Abstract

This paper discusses the role of prices as signals in a static two-sided asymmetric information monopolistic model. We demonstrate the existence of equilibria involving fraud even if consumers' private information about quality is made arbitrarily precise. Our main result is that higher levels of fraud attempted by low-quality sellers may be beneficial to prospective buyers via a decrease in the market partial-pooling price. Furthermore, more precise private information may harm consumers. These findings suggest that standard consumer policies based on the provision of accurate information and the promotion of honesty in the market place may reduce consumers' welfare.

Keywords: Consumer Protection, Consumer Fraud, Asymmetric information, Price Signalling.

JEL classification: D18; D82; D42; L15; L51.

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1 Introduction

Information asymmetries about the quality of products or services between consumers and traders create incentives to attempt fraud. Information provided by independent third parties such as consumer report magazines and institutional warnings are the traditional tools employed to allegedly enhance consumer protection from unfair business practices. Nonetheless, if despite this information, consumers can only assess the quality of the good imperfectly and prices play a signalling role, a supplier of a low quality product might have an incentive to influence the buyer's perception about the quality of his product by distorting his¹ pricing decision. If the consumer is fooled into purchasing a low quality product at a price above the full information price for this item, then the attempted fraud by the seller becomes successful and we say that the buyer becomes a victim of fraud.

Consumer fraud is a common phenomenon nowadays and it is well documented in different surveys.² The results suggest that around 15% of the adult U.S. population are victims of fraud each year. There is a wide variety of experience (and credence) goods for which fraud is a relevant problem. Some

¹For ease of exposition, we refer to a seller as “he” and to a consumer as “she”.

²For instance, refer to the ones sponsored by the National Institute of Justice (NIJ) in 1991, by the Federal Trade Commission (FTC) in 2003, and by the American Association of Retired Persons (AARP) in 1998.

examples are second hand objects³, illicit drugs⁴, gas saving devices⁵ or even fish.⁶

Governments are concerned about protecting consumers against fraud and recognize the need of developing consumer law and implementing public policies in order to achieve this goal. Consumer protection is thought to generate a healthy economy based on trust and confidence that enhances consumers' welfare. Such goals are expressed in the communication from the Commission of the European Communities "Empowering consumers, enhancing their

³The lemons suggested by Arkelof (1970) in his classical paper about the adverse selection problem.

⁴Illicit drugs are frequently sold diluted and/or adulterated. The lack of correlation between purity and price per raw gram can be taken as evidence of the existence of fraud. Refer to Reuter and Caulkins (2004) and Simon and Burns (1997).

⁵Canada's Competition Bureau recently warned Canadian consumers to be aware of false claims about so-called gas saving devices that supposedly improve fuel efficiency, reduce harmful emissions and reduce repair costs on a vehicles engine. These devices are advertised in garages and accredited installation centers, in newspapers, on the radio and Internet and range in price from \$100 to \$600. The Bureau was unaware of any credible scientific evidence that could demonstrate that such products can significantly improve fuel efficiency.

⁶Lower-quality and less expensive fish are often mislabeled as desirable species for financial gain, an activity called seafood fraud. For example, a piece of sushi sold as the luxury treat white tuna usually turns out to be Mozambique tilapia or even escolar, a much cheaper fish that can cause severe gastrointestinal distress. Recent studies using DNA bar coding techniques have found that seafood may be mislabeled at restaurants and stores as often as 25 to 70 percent of the time for fish like red snapper, wild salmon, Atlantic cod, tuna and grouper among others. Refer to reports about investigations carried out by Globe (Abelson and Daley (2011) and Daley and Abelson (2011)); by Consumer Reports Magazine in 2011; by Oceana (Warner (2011)); and by the U.S. Government Accountability Office (GAO) in 2009.

welfare, effectively protecting them”. This communication refers to the development and adoption of the E.U. Consumer Policy Strategy during the period 2007-2013. The main purpose of this paper is to show that building up consumers’ confidence by reducing market fraud or providing more accurate private information to consumers may indeed harm consumers.

We assume that two exogenously given qualities are offered in a market for an experience good (Nelson (1970)). The seller is a monopolist and a price setter. Consumers have access to a *noisy private signal* about the quality of the good without incurring any cost. In our model, the level of fraud determines the precision of the *public signal* (the price) set strategically by the seller. Ceteris paribus, the less fraud, the more important price is as a signal of quality and potential buyers can make more informed choices. From customers’ perspective, an increase in the precision of the endogenous public information is costly via a higher equilibrium price for high-quality products. We show that the benefit to consumers of a more precise public information is outweighed by the price distortion for some parameter values. As a result, more precise information provided by the supplier is beneficial from the seller’s point of view but it hurts consumers in expected terms. Hence, devoting effort to building up trust and confidence by promoting honesty in two-sided asymmetric markets with monopolistic power may go against the objective of enhancing consumers’ welfare.

If as the efficient market hypothesis holds there is a positive relationship between the equilibrium prices and the precision of the imperfect private information owned by consumers, the a priori costless exogenous private information provision becomes costly to consumers. We show that if this is the case, the value of private quality information can be very well negative to con-

sumers. Furthermore, even if the efficient market hypothesis did not hold, due to the supply-side structure of the market, it is immediate that the revelation of perfect information about quality (regardless of its revealing source) deprives consumers of any informational rent. In turn, this prevents consumers from appropriating any gains from trade. All these facts imply that the empowerment of consumers by a policy favoring accurate information may be harmful for them.

If the main objective of consumer protection policies is to enhance consumers' welfare, other consumer policy approaches, such as a combination of a better private information provision and price regulation strategies, such as the imposition of a price ceiling, might be superior to simply promoting honesty in the market and providing accurate information.

Our paper is organized as follows. A brief review of related works in the literature is provided in section 2. The model is formalized in section 3. Section 4 carries out the equilibrium analysis and presents the central results of the paper. Section 5 discusses policy implications. All proofs are in the appendix.

2 Literature Review

The analysis of fraud has received little attention by the industrial organization literature on price as a signal of quality. Its main focus is on examining the informational content of prices and therefore, it restricts attention to the existence of pure strategy separating equilibria that survive selection criteria. Two close two-sided asymmetric models that combine hidden action and hidden information are Hertzendorf (1993) and Bester and Ritzberger (1999). In Hertzendorf (1993) model, a monopolist can signal its quality to consumers

through its selection of price and advertising. Advertising is stochastic because consumers may see fewer advertisements than were originally purchased. However, the supplier controls the amount of information about her product that is available to potential buyers. In our model, the information structure is not completely endogenous. The buyer has access to a public signal set by the seller and a costless private signal provided by an independent party. In Bester and Ritzberger (1999), an informed monopolist sets the price and uninformed buyers may infer quality from the price or acquire private information. The private test for quality fully reveals the true quality but it is costly. As a result, buyers are either perfectly informed or completely uninformed about the quality of the product when they take their purchasing decisions. Their information structure is more extreme than ours. The results are that for small costs of information acquisition, prices reveal information imperfectly. As the information cost vanishes, the prices become perfectly revealing, approaching the full information levels.⁷ We find a similar result in our model if the information structure is modified in such a way that a sufficiently large fraction of consumers have access to perfect information. If all consumers have access to imperfect private information, the results are that the level of fraud in the market vanishes as the private information is made arbitrarily precise. However, the equilibrium prices do not necessarily approach the full information levels. The slightest decrease in the information precision favors the existence

⁷Unfortunately, it is not possible to solve explicitly for the equilibrium and the authors rely on a numerical example to perform comparative statics. They find that the relationship between the high-quality seller's price and the information cost (as it vanishes) is not monotonic. In principle, the high-quality seller's price could be higher or lower than its full information level.

of equilibria that do not exist under full information.

The literature on the moral hazard aspects of the choice of quality has focused on the analysis of fraud. Our analysis is quite distinct from this literature. In our model, quality is exogenously given and we focus on signalling considerations. An alternative literature which has also focused on fraud (referred to as expert cheating) includes research work on credence goods. With credence goods, consumers cannot judge actual quality either before or after purchase (Darby and Karni (1973)). Most of the existing literature on credence goods (see Pitchik and Schotter (1987 and 1993), Taylor (1995), Fong (2005) and Dulleck and Kerschbamer (2006)) assumes that the consumer is either completely informed or completely uninformed about the nature of her problem. An exception is Hyndman and Ozerturk (forthcoming) who introduce non-identifiable heterogeneously informed consumers. The authors show that when a positive fraction of consumers observe a noisy but informative signal about the seriousness of the problem and the expert cannot distinguish between informed and uninformed consumers, there is a unique equilibrium outcome in which the expert is always truthful to all types of consumers. A noisy quality signal, which is imperfectly correlated with the true quality of the good, is also introduced in our model and customers' types are non-identifiable by the seller. We prove the existence of equilibria involving fraud and show that these fraudulent equilibria exist even if consumers' private information is made arbitrarily precise.

Our paper also contributes to the literature on the value of information. Schlee (1996) analyzes a model in which both consumers and a monopolistic producer are uncertain about the quality of the goods they exchange. His model is technically different from ours. The differences are that quality in-

formation is public and there are no information asymmetries. The author then abstracts from signalling considerations. He shows that consumers may sometimes prefer less public information about product quality.⁸ We demonstrate that a similar result extends to a two-sided asymmetric information signalling model. Our finding is important from consumer policy's perspective. The author also provides an example where private information about quality may hurt consumers facing a monopolist price setter. However, his example is constructed under the assumption that trade is not desirable in the low-quality state under full information. We obtain a similar finding in a signalling setting⁹ under the assumption that trade is always desirable under full information.

Finally, Amstrong, Vickens and Zhou (2009) discuss other possible undesirable effects of consumer protection policies in a setting with no potential for signalling (they analyze markets of homogenous product qualities with search costs). Using a parsimonious oligopolistic model with price dispersion, the authors show that imposing a price cap or allowing consumers to opt out of advertising reduces the endogenous proportion of consumers who are more informed about price deals in the market, encouraging firms to offer high prices and harming consumers.

⁸The author identifies two properties of the cost functions (it is assumed that the cost of production is independent of quality and it is strictly increasing in quantity) that lead to a negative value of information for consumers: increasing returns to scale and "sufficiently" convex marginal costs.

⁹Gal-Or (1988) investigates the value of information to firms in a signalling duopolistic model and she also shows that more precise incomplete information about cost may hurt firms.

3 The Model

Our modeling approach is in the spirit of Voorneveld and Weibull (2004). Consider the simplest static monopolistic two-sided asymmetric information setting. Two exogenously given qualities (high quality and low quality respectively) are offered in the market: $q_H > q_L > 0$. We model quality as an experience attribute and it is assumed to be seller's private information. Consumers are ex ante identical and do not interact strategically with each other. This allows us to consider each potential buyer in isolation. The seller has one unit of the good for sale and the buyer is willing to purchase at most one unit. Prior to purchase, the buyer obtains without cost a private and imperfectly informative binary signal about the quality of the product. Importantly, the seller can control the buyer's informational environment only via his take-it-or-leave-it price offer. In other words, the seller's ability to manipulate the consumer's beliefs is limited by the consumer's private information. Both seller and buyer are assumed to be risk neutral and expected utility maximizers. We formalize the market interactions between the seller and the buyer as a signaling game with the following stages:

- (1) Nature draws a type θ for the seller from the set of feasible types: $\Theta = \{L, H\}$ according to a commonly known probability distribution fully described by $Pr(H) = \pi \in (0, 1)$. In addition, Nature picks the signal realization s from the set of feasible signals $S = \{L, H\}$, according to the following structure: $Pr(s = \theta|\theta) = \delta \forall \theta \in \Theta$ and $Pr(s = \theta'|\theta) = 1 - \delta \forall \theta' \neq \theta$ and $\theta, \theta' \in \Theta$ where $\frac{1}{2} < \delta < 1$. The number δ is interpreted as the probability of observing the correct signal realization according to

quality. It denotes the precision¹⁰ of the customer's private signal. Its value is common knowledge. Given this information structure, the types of seller and buyer are imperfectly correlated. A more accurate signal is associated with better information in the sense of Blackwell (1951).

- (2) The seller observes θ and then, publicly posts a price, $p \in \mathbf{R}$. A pure strategy for the seller is a pair $\mathbf{p} = (p_L, p_H) \in \mathbf{R}^2$ of prices where p_θ is the price charged by each seller-type that nature might draw. Allowing mixed strategies, we can then write the seller's strategy as: $\phi : \Theta \rightarrow D$, where D is the set of density distribution functions on \mathbf{R} , and the probability that type θ seller charges any given price p is denoted by $\phi_\theta(p)$.
- (3) The potential customer observes the price-signal pair (p, s) (but not θ) and then she updates her beliefs as to which type of seller she faces. Let $\mu : \mathbf{R} \times \mathbf{S} \rightarrow [0, 1]$ be the buyer's posterior belief that the unit at hand is of high quality if offered an item at price p and the signal realization s is observed. Based on this belief, the consumer then chooses whether to or not to buy the good. Allowing mixed strategies for the buyer, her strategy can be represented by $b : \mathbf{R} \times \mathbf{S} \rightarrow [0, 1]$, being $b(p, s)$ the probability that she accepts to trade at price p if the signal realization s is observed.
- (4) If trade takes place, the seller serves the forthcoming demand at the posted

¹⁰It could be interpreted as the consumer's ability to understand and process available information to assess the quality of the good. In general, consumers may differ in their ability to process information about quality. Other specifications of the information structure that account for this heterogeneity are analyzed in section 4.

price p and the players' *ex-post* utilities are given by:

$$u_b = v_\theta - p$$

$$u_\theta = p - w_\theta$$

where $v_\theta > w_\theta \geq 0$ are respectively the buyer and seller's valuations (reservation prices) of a given item of quality $\theta \in \Theta$. We assume that $v_H > v_L$ and normalize $w_L = 0$. The ex-ante expected valuation of the good is denoted by $\bar{v} \equiv \pi v_H + (1 - \pi)v_L$. Both agents' utilities are also normalized to zero in the no-trade case. These values imply that trade is desirable under perfect information.

4 Main Results

4.1 Equilibrium Analysis

We solve this game by using the notion of (weak) Perfect Bayesian equilibrium (PBE) as the solution concept. Note that accepting all price offers satisfying $p \leq v_L$ with certainty and rejecting all price offers such that $p > v_H$ is a customer's best response irrespectively of the signal realization observed.

Under perfect information, the buyer is not be willing to purchase a low quality unit at any price exceeding her valuation for this item and the seller is aware of this fact. The imperfect ability of the consumer to assess the quality of the object might lead her to occasionally be fooled into purchasing low-quality products at a price above her valuation. Throughout the paper, selling a low quality object at price higher than the buyer's valuation for a low

quality object is referred to as *fraud*.¹¹

Lemma 4.1. *Assume $\delta \in (\frac{1}{2}, 1)$. The candidates for separating equilibria are pure-strategy equilibria with $p_L = v_L$ and $p_H \geq v_H$.*

In any separating equilibrium, the low-quality seller reveals himself by quoting the same price as under full information and the high-quality seller posts a price at least as high as the full information monopoly price.

Corollary 4.2. *Assume $\delta \in (\frac{1}{2}, 1)$. None of the candidate separating equilibria exhibits fraud.*

Separating equilibria are truthful equilibria in the sense that the seller never attempts fraud. However, note that if trade takes place, the seller is always able to fully extract the buyer's surplus. Honest behavior by the seller ensures the buyer a zero utility in equilibrium. Because fraud is not attempted, the buyer's utility is always non-negative but the perfect revealing prices prevent the buyer from earning any of the realized gains from trade. This result holds even if the buyer's private information is made arbitrarily precise but it is sensitive to the specification of the information structure considered.

Suppose instead that a positive fraction (α) of consumers had access to perfect information and the remaining consumers are uninformed. The informed consumers would disregard any public signal revealed by the seller in equilibrium since it would add no useful information in assessing quality. As a result, separating equilibria can be supported for prices strictly lower than

¹¹In the English dictionary by Oxford University Press, fraud is defined as the action or an instance of deceiving somebody in order to make money. Alternatively, it is defined as a thing that is not what is claimed to be.

the buyers' valuation for high quality items.¹² On one hand, there is a positive externality caused by the presence of few perfectly informed consumers which results in the impossibility of the high quality seller to extract the entire consumer's surplus. Consequently, the gains from trading high quality items are shared by both the buyer and the seller. But as the fraction of perfectly informed consumers increases, the utility of all types of buyers decreases due to the increase in the equilibrium price charged by the high quality seller. Thus, the positive externality vanishes in the limit, as α tends to $1 - \frac{v_L}{v_H}$.

Almost all pooling equilibria are fraudulent equilibria in the sense that the seller with the low quality items attempts fraud. It can be easily proven that these equilibria exist for all parameter values. But since the level of fraud is at its maximum level and it does not depend on the accuracy of the buyer's private information, we find these equilibria not interesting for our comparative statics exercise. Throughout the paper, we restrict attention to "hybrid equilibria" defined as mixed-strategy equilibria in which the low quality seller uses a price randomization strategy and the high quality seller uses a pure strategy. The seller with the low-quality good reveals himself by quoting a low price p_L with a positive probability strictly lower than one and he imitates any price p_H that the high-quality seller quotes in equilibrium with the complementary probability.

¹²Formally, if $\alpha \in \left(1 - \frac{v_L}{w_H}, 1 - \frac{v_L}{v_H}\right)$, there exist a unique separating equilibria with $p_H < v_H$ that satisfies the Intuitive Criterion. The equilibrium high price is given by $p_H = \frac{v_L}{1-\alpha}$ and the expected utility of the buyer is $u_b = \pi \left(v_H - \frac{v_L}{1-\alpha}\right)$. In the same range of α -values there exist hybrid equilibria (refer to definition (4.1) and lemma (4.3) in the main text) for each $p_H \in P'(\alpha)$ where $P'(\alpha) \equiv \left\{p \in R \mid p \in (\bar{v}, v_H) \ \& \ p \geq \max \left\{ \frac{v_L}{1-\alpha}, v_H - \frac{v_L}{\alpha} \right\} \right\}$. The buyer's ex-ante utility is given by $u_b = \pi(v_H - p_H)\alpha$.

Definition 4.1. Suppose $\delta \in (\frac{1}{2}, 1)$. A hybrid equilibrium is a mixed-strategy equilibrium in which the high quality seller chooses a high price and the low quality seller randomizes between the high price and a low price, that is, $\phi_H^*(p_H) = 1$ and $\phi_L^*(p_L) = 1 - \phi_L^*(p_H)$ where $\phi_L^*(p_H) \in (0, 1)$.

Lemma 4.3. Assume $\delta \in (\frac{1}{2}, 1)$. The candidates for hybrid-equilibria are partial-pooling equilibria with $p_L = v_L$ and $p_H \in (\max\{v_L, w_H\}, v_H)$.¹³

As in any separating equilibrium, the low-quality seller partially reveals himself by quoting the same price as under full information.

Corollary 4.4. Assume $\delta \in (\frac{1}{2}, 1)$. All candidate hybrid equilibria exhibit fraud.

Although in this static setting there are no explicit reputation concerns, hybrid equilibria reflect an implicit opportunity cost associated to fraud for the low quality seller. In any hybrid equilibrium, the buyer accepts a low price offer with certainty and rejects a high price offer with a probability which is dependent on the signal realization observed. Thus, the low quality seller faces a trade-off between charging the low price and obtaining a low payoff, or charging the high price which yields a high payoff with probability strictly lower than one. This implicit opportunity cost associated to fraud captures the essence of reputation concerns by limiting the extent to which the seller can deceive the buyer.

A PBE consists of beliefs and strategies satisfying two conditions: (i) given the players' beliefs, their strategies are sequentially rational; and (ii) at information sets on the equilibrium path, beliefs are determined by Bayes' rule and

¹³We slightly abuse notation here because the high price could in principle take a value equal to w_H if $w_H > v_L$.

the players' equilibrium strategies. At the buyer's information set, the system of consumer beliefs must be Bayes-consistent:

$$\mu(p, s) = \begin{cases} \frac{\pi\delta}{\pi\delta+(1-\pi)(1-\delta)\phi_L^*(p_H)} & \text{if } p = p_H \text{ \& } s = H \\ \frac{\pi(1-\delta)}{\pi(1-\delta)+(1-\pi)\delta\phi_L^*(p_H)} & \text{if } p = p_H \text{ \& } s = L \\ 0 & \text{if } p = v_L \\ \text{Arbitrary} & \text{if } p \notin \{v_L, p_H\} \end{cases} \quad (1)$$

It can be easily shown that there exists a continuum of hybrid equilibria parameterized by p_H .

Proposition 4.5. *Assume $\delta \in (\frac{1}{2}, 1)$. For each $p_H \in P(\delta)$ there exists a hybrid equilibrium characterized by the following strategies:*

$$\begin{aligned} \phi_H^*(p) &= \begin{cases} 1 & \text{if } p = p_H \\ 0 & \text{otherwise} \end{cases} \\ \phi_L^*(p) &= \begin{cases} \left(\frac{\pi}{1-\pi}\right) \left(\frac{1-\delta}{\delta}\right) \left(\frac{v_H-p_H}{p_H-v_L}\right) & \text{if } p = p_H \\ 1 - \left(\frac{\pi}{1-\pi}\right) \left(\frac{1-\delta}{\delta}\right) \left(\frac{v_H-p_H}{p_H-v_L}\right) & \text{if } p = v_L \\ 0 & \text{otherwise} \end{cases} \\ b^*(p, H) &= \begin{cases} 1 & \text{if } p \leq v_L \text{ or if } p = p_H \\ 0 & \text{otherwise} \end{cases} \\ b^*(p, L) &= \begin{cases} 1 & \text{if } p \leq v_L \\ \frac{1}{\delta} \left(\frac{v_L}{p_H} - (1-\delta)\right) & \text{if } p = p_H \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{where } P(\delta) \equiv \left\{ p \in (v_L, v_H) : \max \left\{ w_H, \frac{\left(\frac{\pi}{1-\pi}\right)\left(\frac{1-\delta}{\delta}\right)v_H+v_L}{\left(\frac{\pi}{1-\pi}\right)\left(\frac{1-\delta}{\delta}\right)+1} \right\} \leq p \leq \frac{v_L}{1-\delta} \right\}.$$

Note that the set $P(\delta)$ is non-empty for all $\delta \in (\frac{1}{2}, 1)$ if $\bar{v} \leq 2v_L$. Otherwise, it is non-empty for all $\delta \in [\underline{\delta}, 1)$ where $\underline{\delta}$ is the highest root that satisfies $\max \left\{ w_H, \frac{(\frac{\pi}{1-\pi})(\frac{1-\delta}{\delta})v_H + v_L}{(\frac{\pi}{1-\pi})(\frac{1-\delta}{\delta}) + 1} \right\} = \frac{v_L}{1-\delta}$. The set $P(\delta)$ guarantees that the seller, irrespective of his type, does not have incentives to deviate (by outright cheating the customer in the case of a low-quality seller and by outpricing himself in the case of the high-quality seller).

The proposition can be easily proven using the least favorable system of beliefs in information sets off the equilibrium path.¹⁴ The equilibrium price posted by the high quality seller exactly equals the expected valuation of the customer who observes the low signal realization given the equilibrium level of fraud ($\phi_L^*(p_H)$) in the market. As a result, the customer who observes the low signal realization is indifferent between accepting or rejecting trade at the high price. In turn, her equilibrium randomization strategy makes the seller who has a low quality unit indifferent between charging either price.

The ex-ante equilibrium probability of trade for low-quality items when they are offered at the high price is given by $t_L^*(p_H) = (1-\delta) + \delta b^*(p_H, L) = \frac{v_L}{p_H}$ whereas the equilibrium probability of trade for high quality-items is given by $t_H^* = \delta + (1-\delta)b^*(p_H, H) = 1 - (\frac{1-\delta}{\delta}) \left(1 - \frac{v_L}{p_H}\right)$. As expected, the higher is p_H , the lower are these trading probabilities. Despite a lower probability of transaction, a higher p_H increases the high quality supplier's expected utility due to the higher mark-up: $u_H^* = \left[1 - (\frac{1-\delta}{\delta}) \left(1 - \frac{v_L}{p_H}\right)\right] (p_H - w_H)$.

Interestingly, the level of fraud is decreasing in the value of the high price. The higher is p_H , the less often the low-quality seller cheats her cus-

¹⁴That is, the buyer believes that the item is of low quality with probability one if $p \notin \{v_L, p_H\}$. These pessimistic beliefs support the largest set of PBE outcomes.

tomers in order to keep serving both types of customers' demands. Otherwise, the buyer who observed a low signal would strictly prefer to reject trade at the high price. The *incidence of fraud* is defined as the customer's ex ante probability of becoming a victim of fraud. Its equilibrium value is given by $\Phi^* = (1 - \pi)\phi_L^*(p_H)t_L^*(p_H) = \pi \left(\frac{1-\delta}{\delta}\right) \left(\frac{v_H-p_H}{p_H-v_L}\right) \frac{v_L}{p_H}$. Given that the seller cheats his customers less often and the buyer with a low signal realization accepts trade at p_H less frequently, we conclude that the higher is p_H , the lower the incidence of fraud. This positive effect on the incidence of fraud dominates the negative effect due to a higher suffered loss in the unfortunate case of becoming a victim of fraud. As a result, the higher p_H , the lower the equilibrium expected loss due to fraud: $ELoss = \Phi^*(p_H - v_L) = \pi \left(\frac{1-\delta}{\delta}\right) \left(\frac{v_H}{p_H} - 1\right) v_L$.

On the other hand, the lower equilibrium level of fraud implies that some transactions that were previously offered at the high price and that were rejected by the customer with the low signal realization, are now offered at the low price and they are accepted by any customer with certainty. This leads to a higher equilibrium ex-ante trade probability for low-quality units: $t_L^* = 1 - \phi_L^*(p_H)(1 - t_L^*(p_H)) = 1 - \left(\frac{\pi}{1-\pi}\right) \left(\frac{1-\delta}{\delta}\right) \left(\frac{v_H}{p_H} - 1\right)$, and to larger realized gains from trade:

$$\begin{aligned} W^* &= \pi t_H^*(v_H - w_H) + (1 - \pi)t_L^*v_L = \\ &= \pi(v_H - w_H) + (1 - \pi)v_L - \pi \left(\frac{1 - \delta}{\delta}\right) \left(v_H - w_H - v_L + \frac{v_L w_H}{p_H}\right) \end{aligned}$$

Despite of less consumer fraud, equilibria with higher values of p_H are associated with lower ex-ante (and ex-post) customers' expected utility levels:

$$Eu_b^* = \pi t_H^*(v_H - p_H) + \Phi^*(v_L - p_H) + (1 - \pi)(1 - \phi_L^*(p_H))(v_L - v_L) = \pi \left(\frac{2\delta - 1}{\delta}\right) (v_H - p_H) \quad (2)$$

The explanation of this result is intuitive. The negative effect due to a lower equilibrium probability of trade for the high-quality item and the lower consumer surplus obtained from any such transaction dominates the positive effect from a decrease in the expected loss due to fraud. Despite a larger number of market transactions taking place at the low price, these transactions have no impact on the consumer's wellbeing. This is because the low quality seller is able to extract the entire consumer's surplus when he offers his product at the low price.

Corollary 4.6. *Assume $\delta \in (\frac{1}{2}, 1)$. Hybrid equilibria with $p_H \in P(\delta)$ can be indexed by p_H . The higher is p_H , the lower are the level of fraud and the level of consumers' wellbeing.*

Clearly, a policy aimed at building up an economy based on trust and confidence by promoting honesty does not enhance consumers' welfare, but merely reduces it by increasing the price charged for the high quality items, harming consumers.

4.1.1 Robustness

An insight offered by the model is that the presence of an exogenous *informative* signal, even if its accuracy level is slight, is crucial in determining whether higher levels of fraud are beneficial to consumers. If consumers were completely uninformed, a continuum of partial-pooling equilibria in the spirit of the above hybrid equilibria could be supported if the high price exceeded the ex-ante valuation of the good.¹⁵ The equilibrium level of fraud would be

¹⁵Otherwise, pooling equilibria can be supported in this range of parameter values.

given by $\phi_L^* = \left(\frac{\pi}{1-\pi}\right) \left(\frac{v_H - p_H}{p_H - v_L}\right)$ and the uninformed consumer's randomization strategy would be $b_u^*(p_H) = \frac{v_L}{p_H}$. As before, equilibria characterized by a higher partial-pooling price would exhibit less fraud. However, the potential increase in the consumer's ex ante utility from a lower expected loss due to fraud would be exactly offset by the higher price distortion. This same result is obtained in our model for a second class of hybrid equilibria (not characterized in any proposition). In this second class of hybrid equilibria, the equilibrium level of fraud chosen by the low-quality seller $\left(\phi_L^* = \left(\frac{\pi}{1-\pi}\right) \left(\frac{\delta}{1-\delta}\right) \left(\frac{v_H - p_H}{p_H - v_L}\right)\right)$ is such that the expected valuation of the customer who observes the more favorable signal coincides with the high price posted by the high-quality seller. As a result, the buyer with the high signal realization agrees to trade at this price with probability $b^*(p_H, H) = \frac{v_L}{1-\delta} \frac{1}{p_H}$ making the seller who has a low quality unit indifferent between charging either price. A necessary condition for its existence is $p_H \geq \frac{v_L}{1-\delta}$. This condition guarantees that the low-quality seller does not deviate by outright cheating his customer. As a result, these second class equilibria are characterized by higher high prices than our first class equilibria. In the first (second) class of equilibria, the buyer with the low (high) signal realization is sometimes fooled into purchasing low-quality products at the high price, resulting in a monetary loss, but she is exactly compensated from this loss by obtaining positive gains from her purchases of high-quality products. Her expected utility is zero in equilibrium. The crucial difference between the two classes of equilibria resides on the type of consumer who is not made indifferent between accepting or rejecting trade at the high price. In the first class of hybrid equilibria, the buyer who observes a favorable signal always accepts trade at the high price because the observation of a high sig-

nal realization makes her be strictly more optimistic about the quality of the product than her counterpart. She does obtain gains from trade because of the positive correlation between the types of the buyer and the seller,. Instead, in the second class of equilibria, the buyer with the low signal realization does not obtain any gains from trade because she never accepts trade at the high price and all her surplus is extracted when a transaction takes place at the low price. As a result, the consumer's ex-ante utility is zero in all hybrid equilibria with $p_H > \frac{v_L}{1-\delta}$.

Our result is robust to other specifications of the information structure. Suppose that the potential buyer observes an informative signal of precision δ with probability α and uninformative signal with the complementary probability. This information structure accounts for heterogeneity among consumers in the degree of signal precision. Here, α can be interpreted as the fraction of buyers who are active information seekers. After all, few consumers take notice of information provided in warnings or consumer reports. Under these assumptions, for each $p_H \in P'(\delta)$, where $P'(\delta) \equiv \{p \in (\bar{v}, v_H) : \max\{w_H, \frac{(\frac{\pi}{1-\pi})(\frac{1-\delta}{\delta})v_H+v_L}{(\frac{\pi}{1-\pi})(\frac{1-\delta}{\delta})+1}\} \leq p \leq \frac{v_L}{1-\alpha\delta}\}$, the hybrid characterization specified in proposition (4.5) can be supported in equilibrium $\forall \alpha \in (0, 1)$ and for all $\delta \in (\frac{1}{2}, 1)$. In addition, for each $p_H \in P''(\delta)$, where $P''(\delta) \equiv \left\{p \in (\bar{v}, v_H) : \max\left\{w_H, \frac{v_L}{1-\alpha\delta}\right\} \leq p \leq \frac{v_L}{(1-\delta)\alpha}\right\}$, and all $\alpha \in (0, 1)$ and $\delta \in (\frac{1}{2}, 1)$, there exists a hybrid equilibrium characterized by the following strategies: $\phi_L^* = \left(\frac{\pi}{1-\pi}\right) \left(\frac{v_H-p_H}{p_H-v_L}\right)$ and $b^*(p_H, L) = 0 < b_u^*(p_H) = \left[\frac{v_L}{p_H} - \alpha(1-\delta)\right] \left(\frac{1}{1-\alpha}\right) < b^*(p_H, H) = 1$. In the latter equilibria, the expected loss due to fraud is invariant to information parameters but the equilibrium trading probability for high quality units is increasing in both the fraction of informed consumers and the precision of the signal. As a result, the buyer's and the seller's utilities

are also increasing in both parameters. The working channel through which higher levels of consumer fraud result beneficial to buyers is exactly the same as under our original information structure. This holds true even if we allow a fraction of consumers to become perfectly informed about the quality of the good on sale. This implies that our finding does not depend on how information is sorted among consumers, that is, it does not depend on the dissemination of information.

4.1.2 Refinements

The out-of-equilibrium beliefs play an important role in sustaining equilibria. Given the multiplicity of hybrid equilibria analyzed in the previous section, we proceed to apply some of the refinements suggested in the literature in order to prune the set of outcomes. It can easily be shown that the “Equilibrium Domination Test” and the “Intuitive Criterion” suggested by Cho and Kreps (1987) have no cutting power over in this model so we impose stronger restrictions on the beliefs off the equilibrium path. Our model is a monotonic signalling game with only two types of sender, so that criteria D1, Universal Divinity (criterion D2) and Never a Weak Best Response are all equivalent (Cho and Sobel, 1990).

Criterion D1 states that if the set of buyer’s best responses that make a seller of type θ willing to deviate to p is strictly smaller than the set of buyer’s best responses that make type θ' willing to deviate, then the buyer should believe that type θ' is infinitely more likely to deviate to p than type θ is.

This criterion was developed in the context of pure signalling games with one receiver. In our model, the receiver is the buyer who is of one of two

possible types. Thus, we extend the idea of criterion D1 to this setting. The out-of-equilibrium beliefs of the buyers are mappings from the observed vector of price and signal realization to the distribution of sender's types. To evaluate the reasonability of a system of beliefs supporting an equilibrium, we impose the restriction $\mu(p, H) \geq \mu(p, L) \forall p \in R$. This means that after observing an out-of-equilibrium price p , the buyer with the high signal realization is at least as optimistic about the quality of the good on sale as the buyer who observes the low signal realization.

Requiring the hybrid equilibria to be immune to D1 alleviates up to some degree the multiplicity problem.

Proposition 4.7. *Hybrid equilibria indexed by a high price p_H satisfying $u_H^* \geq v_L \left(1 - \frac{w_H}{v_H}\right)$ survive criterion D1.*

Criterion D1 effectively increases the lower-bound of the high price support. However, its power is limited and the multiplicity of equilibria persists.

4.2 The Efficient Market Hypothesis and Information

Better private information owned by consumers reduces the degree of information asymmetry between the two sides of the market. *Ceteris paribus*, trade as well as the utilities of both agents and the realized gains from trade in equilibrium are monotonic in the quality of private information in our model. More precise private information also decreases the equilibrium level of fraud and the incidence of fraud in the market so that the endogenous public information that reaches the consumer in equilibrium is also more precise, enhancing consumers' welfare.

The purpose of this section is to show that a policy based on the provision of better private information may make consumers worse off if there is a strictly positive relationship between the equilibrium value of the high price and the accuracy of the buyer's private information.

From proposition 4.5, the upper-bound and the lower-bound of the high price support are (weakly) increasing and decreasing respectively in the quality of private information. The less noise, the more values of the high price can be supported in equilibrium and the less often is the high price selected by the low-quality seller. In the financial markets literature, the efficient market hypothesis holds that prices aggregate all relevant private information.¹⁶ This implies that equilibrium prices, and not just the level of fraud, should also reveal the precision of the private information owned by consumers. Under this hypothesis, equilibrium prices should be a function of this precision. It seems reasonable to assume that the less noise (the lower the information gap between the seller and the consumer), the higher the degree of market power (indexed by p_H) captured by the high-quality supplier in equilibrium. Under this assumption, the equilibrium value of the high price is an increasing function of the precision of the private information that reaches the buyer: $p_H = p(\delta)$ and $p'(\delta) > 0$. More precise private information is now implicitly costly to the consumer because it is accompanied in equilibrium by an increase in the price posted by the high quality seller, reducing the surplus of the consumer from her purchases of high quality products. If the increase in the equilibrium high price due to a higher precision is sufficiently large, ie. if $p'(\delta) > \frac{v_H - p(\delta)}{\delta(2\delta - 1)}$, a policy based on the provision of better private information

¹⁶In this setting, the return distribution of an asset can be interpreted as quality.

will then harm consumers.

To illustrate this discussion, consider the highest feasible equilibrium high price $p_H = \frac{v_L}{1-\delta}$ for all $\delta \in \left(\frac{1}{2}, 1 - \frac{v_L}{v_H}\right)$. This hybrid equilibrium price is a strictly increasing and convex function of δ and it is the unique hybrid equilibrium high price that can be supported under the assumption of a fraction δ of perfectly informed consumers (see footnote (12)). The ex-ante consumer's utility is then given by $Eu_b^* = \pi \left(\frac{2\delta-1}{\delta}\right) \left(v_H - \frac{v_L}{1-\delta}\right)$, which is also concave in δ for all $\delta \in \left(\frac{1}{2}, 1 - \frac{v_L}{v_H}\right)$. Let δ_0 denote the lowest root that satisfies condition $v_H = v_L \left(\frac{1-2\delta(1-\delta)}{(1-\delta)^2}\right)$. This value maximizes the buyer's ex-ante utility. Since $\delta_0 \in \left(\frac{1}{2}, 1 - \frac{v_L}{v_H}\right)$, more precise private information is harmful for consumers if $\delta \in \left(\delta_0, 1 - \frac{v_L}{v_H}\right)$.

5 Policy Implications

Governments are concerned about consumers' welfare and about protecting them from unfair commercial practices. They recognize the need of developing and implementing consumer policies in order to achieve this goal. Some examples are the operation of the new Consumer Protection Co-operation (CPC) regulation¹⁷, the Unfair Commercial Practices (UCP) directive¹⁸, the European Consumer Centre (ECC) Network and the 2007 communication from the

¹⁷Regulation (EC) No 2006/2004 of the European Parliament and of the Council of 27 October 2004 on cooperation between national authorities responsible for the enforcement of consumer protection laws (OJ L 364, 9.12.2004, p.1).

¹⁸Directive 2005/29/EC of the European Parliament and of the Council of 11 May 2005 concerning unfair business-to-consumer commercial practices in the internal market (OJ L 149, 11.6.2005, p.22).

Commission of the European Communities “Empowering consumers, enhancing their welfare, effectively protecting them”. The Commission Communication cites:

“Confident, informed and empowered consumers are the motor of economic change as their choices drive innovation and efficiency... EU Consumer policy can address problems that individuals lack the capacity to tackle. It ensures goods and services are safe and that markets are fair and transparent, so that consumers can exercise informed choice and rogue traders are excluded... The Commission will have three main objectives in the period 2007-2013: (1) To empower EU consumers. Putting consumers in the driving seat benefits citizens but also boosts competition significantly. Empowered consumers need real choices, accurate information, market transparency and the confidence that comes from effective protection and solid rights; (2) To enhance EU consumers’ welfare in terms of price, choice, quality, diversity, affordability and safety. Consumer welfare is at the heart of well-functioning markets; (3) To protect consumers effectively from the serious risks and threats that they cannot tackle as individuals. A high level of protection against these threats is essential to consumer confidence”.

A preference for policies that favor accurate information provision is revealed by the Commission in this communication. The regulators’ presumption is that through more accurate information, consumers are able to make better informed choices, building up a health economy based on trust and confidence. In our model, an improvement in the quality of the buyer’s private information decreases the information asymmetry between consumers and the seller. But as it had been shown in the previous subsection, the value of private information about quality may be very well negative for some parameter values if it leads to a sufficiently increase in the equilibrium partial-pooling

price. Hence, a better information provision could harm consumers' welfare, contradicting the second main objective of the commission.

The need for action to increase consumer confidence in the retail side of the internal market has also been expressed by the commission. Policies that stop dishonest practices and exclude rogue traders from the market, for example by streamlining the prosecution of fraud, are favored by the Commission. In our model, separating equilibria encompass honest behavior by businesses as the low-quality seller *always* charges the buyer's willingness to pay for low quality units in these equilibria. Equilibrium prices are perfectly revealing and therefore, the noisy private signal that reaches the consumer is ignored in equilibrium. The perfect information about quality revealed in equilibrium by the seller and the seller's ability to commit to a leave-it-or-take-it offer allows the seller to extract the entire consumer's surplus. As a result, none of the gains from trade accrue to the consumer in equilibrium. Instead, the development of fraud in hybrid equilibria favors the persistence of the two-sided asymmetric information in the market. In turn, this incomplete information limits the extent to which the high-quality seller can appropriate the gains from trade by lowering the equilibrium price for such units. Consequently, the presence of fraud in the market enhances consumers' welfare given that the consumer now enjoys a positive ex-ante expected utility in equilibrium. From consumers' point of view, this result casts doubt on the desirability of promoting honesty in this type of markets. Hence, although dishonest business practices might undermine the consumer's confidence in the market, these practices might be willingly tolerated by prospective buyers if suppliers with high quality products charge lower prices.

References

- [1] Akerlof, G., (1970), “The Market for Lemons: Quality Uncertainty and the Market Mechanism”, *Quarterly Journal of Economics* 84, 488-500.
- [2] Abelson, J., and Daley, B., (2011), “On the Menu, But Not On Your Plate”, *Boston Globe*, October 23 2011.
- [3] Daley, B., and Abelson, J., (2011), “From Sea to Sushi Bar, a System Open to Abuse”, *Boston Globe*, October 24 2011.
- [4] Amstrong, H., Vickens, J., and Zhou, J., (2009), “Consumer Protection and the Incentive to Become Informed”, *Journal of the European Economics Association* 77, 399-410.
- [5] Blackwell, D., (1951), “Comparison of Experiments” in J. Neymand, ed., *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability* (Berkeley: University of California Press), 93-102.
- [6] Bester, H., and Ritzberger, K., (2001), “ Strategic Pricing, Signalling, and Costly Information Acquisition”, *International Journal of Industrial Organization* 19, 1347-1361.
- [7] Cho, I.-K., and Kreps, D., (1987), “Signalling Games and Stable Equilibria”, *Quarterly Journal of Economics* 102, 179-221.
- [8] Cho, I.-K., and Sobel, J., (1990), “Strategic Stability and Uniqueness in Signalling Games”, *Journal of Economic Theory* 50, 381-413.
- [9] Consumer Reports, (2011), “Mystery Fish”, *Consumer Reports Magazine*, December 2011.

- [10] Darby, M., and Karni, E., (1973), “Free Competition and the Optimal Amount of Fraud”, *Journal of Law and Economics* 16, 67-88.
- [11] Dulleck, U., and Kerschbamer, R., (2006), “On Doctors, Mechanics and Computer Specialists: The Economics of Credence Goods”, *Journal of Economic Literature* 44, 5-42.
- [12] Fong, Y.F., (2005), “When Do Experts Cheat and Whom Do They Target?”, *RAND Journal of Economics* 36, 113-130.
- [13] Gal-Or, E., (1988), “The Advantages of Imprecise Information”, *RAND Journal of Economics* 19, 266-275.
- [14] U.S. Government Accountability Office, (2009), “Seafood Fraud: FDA Program Changes and Better Collaboration among Key Federal Agencies Could Improve Detection and Prevention”, GAO-09-258, February 2009.
- [15] Hertzendorf, Mark N., (1993), “I’m not a High-Quality Firm- but I Play One on TV”, *RAND Journal of Economics* 24, 236-247.
- [16] Hyndman, K. and Ozerturk, S., Forthcoming, “Consumer Information in a Market for Expert Services”, *Journal of Economic Behavior and Organization*.
- [17] Nelson, P., (1970), “Information and Consumer Behavior”, *Journal of Political Economy*, 78, 311-329.
- [18] Oceana, (2011), “Bait and Switch: How Seafood Fraud Hurts Our Oceans, Our Wallets and Our Health”, Oceana’s Report, May 25 2011.

- [19] Pitchik, C., and Schotter, A., (1987), “Honesty in a Model of Strategic Information”, *American Economic Review* 77, 1032-36.
- [20] Pitchik, C., and Schotter, A., (1993), “Information transmission in regulated markets”, *Canadian Journal of Economics*, 26, 815-29.
- [21] Reuter, P., and Caulkins, J.P., (2004), “Illegal “Lemons”: Price Dispersion in Cocaine and Heroine Markets”, *Bulletin on Narcotics*, vol. LVI, Nos. 1 and 2.
- [22] Schlee, E., (1996), “The Value of Information about Product Quality”, *RAND Journal of Economics* 27, 803-815.
- [23] Simon, D., and Burns, E., (1997), “The Corner: a Year in the Life of an Inner-City Neighborhood”, New York, Broadway Books.
- [24] Taylor, C.R., (1995), The Economics of Breakdowns, Checkups, and Cures, *Journal of Political Economy* 103, 53-74.
- [25] Voorneveld, M., and Weibull, J.W., (2004), “Prices and Quality Signals”, Working Paper 551, Department of Economics, Stockholm School of Economics.
- [26] Warner, K., (2011), “Seafood Fraud Found in Boston-Area Supermarkets”, *Oceana’s Report*, October 24 2011.

Appendix

Proof of Lemma 4.1. At any information set (p_L, s) , the customer believes that the item is of low quality with certainty. If a price $p_L > v_L$ is offered, the buyer's unique best response is not to purchase the product regardless of the signal realization observed. Given that there is no trade with certainty, the low quality seller's utility is zero. But he could deviate to a lower price $p' < v_L$, sell the product with certainty and obtain positive rents. A contradiction. Then $p_L \leq v_L$.

In equilibrium, it must be the case that $b^*(v_L, s) = 1 \forall s \in S$. We prove this statement by contradiction. Suppose not. Then, the low quality seller could guarantee a profit arbitrarily close to v_L by setting a price p' sufficiently close to v_L given that any price offer p with $p < v_L$ is accepted by the buyer with certainty. But then, his expected payoff could be increased from p' to p'' by deviating to a price $p'' \in (p', v_L)$. A contradiction. We conclude that $b^*(v_L, s) = 1 \forall s \in S$ which implies $p_L = v_L$. In any separating equilibrium $p_L \neq p_H$. Hence, $p_H > v_L$.

We prove that $p_H \geq v_H$ by contradiction. At any information set (p_H, s) , the customer believes that the item is of high quality with certainty. If $p_H < v_H$, the buyer's unique response is to always purchase the product independently of the signal realization. But then, the low quality seller would prefer to deviate and set always $p_H > v_L$ guaranteeing a strictly greater payoff for himself.

Proof of Lemma 4.3. By the argument used in the previous proof, we have that $b^*(v_L, s) = 1 \forall s \in S$ which implies $p_L = v_L$ and $p_H \geq v_L$.

In any hybrid equilibrium $p_L \neq p_H$. Hence, $p_H > v_L$. Charging a price $p_H \geq v_H$ results in no trade given that the buyer's unique best response is not to purchase the product regardless of the signal realization observed. As a result, the seller would earn a zero payoff if he charged this price. But then, the low quality seller would prefer to deviate and charge v_L guaranteeing a strictly positive payoff equal to $v_L > 0$ for himself. Hence, $p_H < v_H$. Finally, note that a high quality seller is not willing to sell his item at a price $p_H < w_H$ since, if he did so, he would get a negative payoff. Thus, $p_H \geq w_H$.

Proof of Proposition 4.7. Formally, let $MBR(\mu_L, p)$ be the set of mixed best responses for the buyer who observes signal $s = L$ to price p for beliefs $\mu(p, L) \in [0, 1]$. Similarly, let $MBR(\mu_H, p)$ the set of mixed best responses for the buyer who observes signal $s = H$ to price p for beliefs $\mu(p, H) \in [0, 1]$. Let $MBR(\mu, p) \equiv MBR(\mu_H, p) \times MBR(\mu_L, p)$ where $\mu \equiv (\mu_H, \mu_L) \in [0, 1]^2$. Fix a vector of equilibrium payoffs u_θ^* for the sender. Define $D(\theta, \Theta, p)$ to be the buyer's set of mixed-strategy best responses to price p and beliefs concentrated on Θ that make type θ strictly prefer p to his equilibrium strategy:

$$D(\theta, \Theta, p) = \bigcup_{\mu: \mu \in [0, 1]^2} \{(b(p, H), b(p, L)) \in MBR(\mu, p) : u_\theta^* < u_\theta(p, (b(p, H), b(p, L)))\} \quad (3)$$

where,

$$u_\theta(p, (b(p, H), b(p, L))) = [\delta b(p, \theta) + (1 - \delta)b(p, \theta')](p - w_\theta)$$

and let $D^0(\theta, \Theta, p)$ be the set of mixed best responses that make type θ exactly indifferent.

A type θ is deleted for strategy p under criterion D1 if there is a type $\theta' \in \Theta$ such that

$$\{D(\theta, \Theta, p) \cup D^0(\theta, \Theta, p)\} \subset D(\theta', \Theta, p) \quad (4)$$

where the symbol \subset denotes strict inclusion.

Having deleted a type for strategy p , we can impose further restrictions on the buyer's beliefs and the best responses to those beliefs. This criterion is a strengthening of the Intuitive Criterion as type θ is removed by the latter criterion only if the sets $D(\theta, \Theta, p)$ and $D^0(\theta, \Theta, p)$ are empty.

The restriction $\mu(p, H) \geq \mu(p, L) \forall p \in R$ implies another restriction: $b(p, H) \geq b(p, L)$. The seller's equilibrium payoffs are $u_H^* = \left[1 - \frac{1-\delta}{\delta} \left(1 - \frac{v_L}{p_H}\right)\right] (p_H - w_H)$ and $u_L^* = v_L$. Note that an equilibrium necessary condition for type H seller not willing to deviate is $u_H^* \geq v_L - w_H$.

For each $p \leq u_H^*$ or $p > v_H$, $D(H, \Theta, p) = \{\emptyset\}$ whereas for each $u_H^* < p \leq v_H$:

$$D(H, \Theta, p) = \{(b(p, H), b(p, L)) \in [0, 1]^2 : b(p, H) \geq b(p, L) \ \& \ \delta b(p, H) + (1 - \delta)b(p, L) > \frac{u_H^*}{p - w_H}\} \quad (5)$$

Similarly, for each $p \leq v_L$ or $p > v_H$, $D(L, \Theta, p) = \{\emptyset\}$ whereas for each $v_L < p \leq v_H$:

$$D(L, \Theta, p) = \{(b(p, H), b(p, L)) \in [0, 1]^2 : b(p, H) \geq b(p, L) \ \& \ (1 - \delta)b(p, H) + \delta b(p, L) > \frac{v_L}{p}\} \quad (6)$$

If a type H seller were deleted for a given strategy $p > v_L$, the deviation should be interpreted as coming from type L . This induces the buyer to strictly reject trade at p , which is an equilibrium response. Therefore, the hybrid equilibrium would survive criterion D1. Only if a type L were deleted for some strategy $\max\{v_L, w_H\} < p \leq v_H$, the hybrid equilibrium would fail this criterion. A type L is deleted for a strategy p if the unique solution to the following system:

$$\delta b_H + (1 - \delta)b_L = \frac{u_H^*}{p - w_H} \quad (7)$$

$$(1 - \delta)b_H + \delta b_L = \frac{v_L}{p} \quad (8)$$

satisfies $b_H < b_L$. Straightforward calculations reveal that those hybrid equilibria characterized by a high price p_H satisfying $u_H^* < v_L \left(1 - \frac{w_H}{v_H}\right)$ fail criterion D1.



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