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Coordination structures[*]

Alfonso Rosa-García and Hubert Janos Kiss[**]

Abstract

We study a coordination problem where agents act sequentially. Agents are embedded in an observation network that allows them to observe the actions of their neighbors. We find that coordination failures do not occur if there exists a sufficiently large clique. Its existence is necessary and sufficient when agents are homogenous and sufficient when agents differ and their types are private. Other structures guarantee coordination when agents decide in some particular sequences or for particular payoffs. The coordination problem embodied in our game is applied to the problems of revolts and bank runs.

Keywords: Social networks, coordination failures, multiple equilibria, revolts, bank runs.

JEL classification: C72, D82, D85, G21, Z13.

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1 Introduction

Coordination failures occur when agents fail to coordinate on an equilibrium in which they are better-off than in an alternative in which they end up. Jean-Jacques Rousseau presented this idea as an illustration of a problem which has become known as the stag-hunt game. In this game, there are two hunters who have to decide whether to hunt a stag or a hare. Hunting the stag is risky, because the payoff it yields depends on whether the hunters manage to coordinate. If they do so, then their payoff is higher than the one related to the safe choice of going for the hare. There are many socioeconomic situations in which this kind of coordination failures may occur (as in many Keynesian models (Cooper and John, 1988), bank runs (Diamond and Dybvig, 1983) or revolts (Chwe, 2000), among others).

We study coordination problems where the agents have to choose between a risky and a safe action. The safe action yields the highest payoff, except in the case in which the agent chooses the risky action and there are sufficiently many other agents choosing it as well. The number of agents required is given by a threshold. We assume that agents decide in a sequence, the order being exogenously determined. They are able to observe only the actions of those predecessors whom they are connected with. The objective of this paper is to study how observation should be structured in this class of games in order to ensure that efficient coordination is achieved.

The agents we study are embedded in a social network that allows to observe actions.[1] For simplicity, we focus on the undirected case, so if two agents are connected by an undirected link, the one who acts later observes the other agent's decision who is aware that her action will be observed.[2] The effect of a link, in this case, is to change the interaction between the agents, from simultaneous to sequential. We say that a network allowing the observation of actions is an *observation network*. Therefore, an observation network allows to model a situation where some of the agents decide simultaneously but others decide after observing the action of their predecessors.

In the class of games that we study, everybody choosing the safe action is an equilibrium of the simultaneous game which is Pareto-dominated by another equilibrium where everybody chooses the risky action. We look for observation networks that imply that the coordination game has a unique equilibrium in which all agents take their preferred risky action, independently of the order of decision. If this occurs for any possible payoffs that allow for the type of coordination failures that we study, we say that these observation networks are *coordination structures*. In particular, if this type of games are completely sequential (i.e., all agents observe all previous actions), it is easy to show that the efficient equilibrium is achieved.[3] So that, in the two extremes, we have that the complete network (i.e., everybody observes the action of everybody else) is a coordination structure, while the empty network (i.e., no action is observed) is not. Our aim is to characterize the minimal amount of observation that is required in order to have the first best as the unique

[1] Typically, social networks are used to model the interactions among agents. As far as we know, Choi et al. (2011) and Kiss et al. (2009) are the first papers that use networks to model the actions that agents are able to observe.

[2] A directed link between two agents would represent which one of the agents is able to observe the action of the other.

[3] This is the case since it is the unique subgame perfect equilibrium.

outcome of the game.

As a first example, we can imagine two individuals deciding whether to use a particular computer operating system. Suppose that in the case both choose Linux, they get the highest utility. If it is not the case, they prefer Windows rather than Linux. The question we want to answer is how much observation is required for guaranteeing coordination. Note that if they both decide simultaneously, the coordination on one or on the other operating system may be an equilibrium (as well as other equilibria in mixed strategies). However, if they act sequentially, the first individual would choose Linux knowing that, after observing it, the second one would adopt it as well. Therefore, an observation network where they are able to mutually observe each other, is sufficient for guaranteeing the successful coordination. A more general question would include several agents, who can potentially coordinate on different actions, and in which each agent requires a different number of coordinating people in order to prefer to play it.

A key network concept in our model is the one of *clique*. A clique is a group of agents completely connected among them. We find that, when agents are homogenous (i.e., all of them share the same risky action and threshold) the existence of a clique is necessary and sufficient to guarantee coordination: coordination failures can be sustained in equilibrium if such a clique does not exist, while in any observation network where the clique exists the unique equilibrium is the efficient one. In this case, the size of the clique is equal to the threshold, for any payoffs and for any order of decision. Therefore, for this case, any observation network including a clique with so many components as the threshold, is a coordination structure. Our general model studies the case of different agents with private types (i.e., with thresholds and preferred actions that are private information), and we find a sufficient condition on the size of a clique that guarantees that, for any payoffs and order of decision, only the efficient outcome can be sustained in equilibrium. For this case, however, it is not a necessary condition, and we find other structures that also guarantee the coordination. These structures are built by a higher number of nodes forming several cliques.

When agents are heterogenous and type is private information, we discuss also the case of observation networks that are sufficient to guarantee coordination for certain orders of decision or specific payoffs, and we name these networks quasi-coordination structures. In this context, we show with an example that more connections may generate coordination failures, a fact that is absent in coordination structures.

Our setup extends naturally to various examples. Revolts succeed if enough individuals join them and they can be modelled as coordination problems. Several papers (e.g. Granovetter, 1978 and Chwe, 1999 and 2000, among others) study what are the conditions for a revolt to succeed. Bank runs also may be viewed as coordination failures (Diamond and Dybvig, 1983). If many patient depositors try to withdraw, the bank suffers a bank run and it becomes optimal for all patient depositors to withdraw. Agents in these situations decide after observing the actions chosen by some predecessors, but ignoring also actions of some others. Our approach has the advantage of modelling simultaneous and sequential decisions in a unified framework.

Granovetter (1978) studies the case of collective action when individual participation depends on the existence of enough other participating people. He assumes that each previous action can be observed and analyzes how the distribution of thresholds must be so that a revolution occurs. In our model, we introduce the observation network that limits the number of previous actions that can be observed. In contrast with Granovetter, in our model the agents are called to participate just once and decide strategically: they take into account what they observe and also the strategies of the other players. In the model of Granovetter the agents observe all past actions, while our aim is precisely to characterize the minimal requirements on observation of actions in order to ensure the coordination on the risky action.

Our paper is mainly related to Chwe (1999, 2000), who analyzes how the structure of a communication network must be so that it enables revolts to succeed. The condition for coordination requires that agents know through the network that sufficiently many other agents are also willing to revolt. Chwe shows the key relevance of cliques in the formation of the local common knowledge that enables the revolts. Our aim is similar in the sense that we characterize also a family of networks under which coordination emerges. However, in the model by Chwe there exists local knowledge of types but complete uncertainty about the global distribution, while in our model we assume knowledge about the global distribution of types but uncertainty about its local configuration. This means that the network studied by Chwe allow that each agent learns the type of her neighbors, while our network does not allow that but only the knowledge of the actions previously chosen. We complement the results by Chwe showing that the coordination on the risky action is in fact the unique equilibrium under our conditions, while the conditions of Chwe guarantee that coordination is one of the equilibria.

Our research is also closely related to Choi et al. (2011), who show the impact of different observation networks on the coordination of agents in an experiment. Agents had to decide simultaneously during three rounds whether to contribute or not to a public good. They were embedded in a network that allowed them to observe the actions of their neighbors only if they were linked. They find that different structures have different impact on the possibilities of coordination. In their environment, they show that the agents, depending on their network position, decide to delay or to commit. By contrast, we study a different game where agents are called once to decide and therefore these effects are absent. In Kiss et al. (2009), the effect of having links in an observation network is analyzed in a bank run experiment. It is shown that links may promote coordination but also failures, depending on the particular action that is observed.

The observation of other agents' actions has an effect on coordination that has been studied in several strands of the economic literature. In herding models (e.g. Banerjee, 1992, or Bikhchandani, Hirshleifer and Welch, 1992) agents face a binary decision and receive a private signal about the quality of the alternatives before deciding. They also observe the actions of their predecessors. They use the information conveyed by these sources in order to choose the alternative that is expected to give the highest utility. In our setup, there is no uncertainty about the utility the alternatives yield, but the issue is to find the information structure that ensures that sufficiently many agents take their risky action and enjoy the highest possible utility. However, a similarity with herding arises when an agent takes into account that many other agents with low threshold may coordinate on their risky action. In this case, since the number of other agents choosing the risky action is sufficient, she will also choose the risky action, so a herd of risky action forms. However, notice that the reasons and mechanisms in herding papers are very different from ours. Costain (2007) shows in a model that nests global games and herding models that if most agents observe a few previous actions instead of playing a simultaneous-move game, then in the face of fundamental uncertainty multiplicity of outcomes is prevailing. Our paper - without considering fundamental uncertainty - asks how the social structure enabling observability of previous actions should be so that agents choose optimally and hence a unique outcome is obtained.

The rest of the paper is organized as follows. In the following section we define formally our model. In Section 3 we provide the results that allow us to determine the characteristics of the coordination structures. In section 4 we discuss the validity of our results as well as the case of structures that guarantee coordination under specific circumstances. Section 5 applies the model to riots, revolts and bank runs and section 6 concludes. Most of the proofs are relegated to the Appendix.

2 The model

2.1 The game

First we start with the general version of our model, in which each agent is endowed with her own particular risky and safe actions, as well as her threshold. Let be $N = \{1, 2, ..., n\}$ the set of agents. These agents are embedded in an *observation network* Γ that connects them. An observation network Γ is a collection of pairs ij such that if ij belongs to Γ, then agents i and j are linked and are able to observe each other's actions,[4] with $i, j \in N$. For convenience, we assume that an agent is always neighbor of herself, i.e. $ii \in \Gamma, \forall i \in N$.

[4] However, it is relevant only for the one who plays later.

We assume that the network is undirected, $ij \in \Gamma \iff ji \in \Gamma$ and common knowledge.[5] Agents that are linked are called neighbors. We define the set of neighbors of i as $N_i \subset N : \{\forall j : j \in N_i \leftrightarrow ij \in \Gamma\}$. A key network concept throughout the paper is the concept of clique. Given a network Γ a clique is a subset of agents $q \subset N$ such that they are completely connected, i.e. $\forall i, j \in q \rightarrow ij \in \Gamma$.

We name $A = \{\alpha, \alpha', \alpha'', ...\}$ the set of pure actions. Let $a_i \in A$ be the action chosen by agent i and $a = \{a_i : \forall i \in N\}$ the action profile, which contains the action chosen by every agent.

Payoffs of agents depend on their actions and actions of others and are given by a utility function u. We study generalizations of the stag-hunt game. Each agent is characterized by a risky action, say $\alpha \in A$, and a safe action, $\beta \in A$, as well as a threshold t. If at least t agents are choosing α, then the agent has the highest utility if she is choosing α. If less than t agents are choosing α, the individual prefers instead her safe action β:

$$u_i(\bar{a}') > u_i(\bar{a}'') > u_i(\bar{a}''') \tag{1}$$

$$\text{with } \left\{\bar{a}' : \left(\bar{a}'_i = \alpha, \sum_{j \in N} I_{\bar{a}'_j = \alpha} \geq t_i\right)\right\}, \{\bar{a}'' : \bar{a}''_i = \beta\}, \bar{a}''' \notin \{\bar{a}', \bar{a}''\} \tag{2}$$

where $I_{a_j=\alpha}$ is the indicator function that takes the value 1 if the agent chooses α and a_i is the action taken by agent i. Note that in this definition of the problem, the threshold t may vary across agents. Action β is the safe action, yielding a fixed utility independently of the other agents' choices. With these payoffs, typically, there exist Nash equilibria in pure strategies where the individuals choose their risky action, that Pareto-dominate other equilibria where all individuals choose their safe action.

At the beginning of the game, Nature reveals privately to each agent her risky and safe action as well as her threshold, which we name the type of the agent, $\tau_i = \{\alpha_i, \beta_i, t_i\} \in A^2 \times \mathbb{N}$. We assume that the number of agents of each type is fixed, common knowledge and given by k

$$\begin{aligned} k &: \left(A^2 \times \mathbb{N}\right) \rightarrow \mathbb{N} \\ k(\alpha, \beta, t) &= \#\{i : \tau_i = (\alpha, \beta, t)\} \end{aligned}$$

and therefore type is randomly assigned to each agent as in a sampling without replacement.

This is a strong assumption, since it means that the agents know exactly the number of agents of each type that exists, although they ignore the exact type of each agent (except the own type). A more standard

[5] The requirement of common knowledge about the network structure is strong. Results similar to those in the paper can be obtained with a less demanding condition: knowledge about the relations among the neighbors. We use a common knowledge structure because of its simplicity (a similar approach is used in Chwe, 2000).

assumption would be one where types are randomly drawn from a given distribution. However, we use our assumption because it allows us to focus on pure coordination problems, as explained later. We assume that Nature assigns types unconditionally, so that *ex ante* all agents can be of a given type with the same probability, given by k.

Consistency requires

$$\sum_{(\alpha,\beta,t)\in A^2\times\mathbb{N}} k(\alpha,\beta,t) = n. \tag{3}$$

Let denote by

$$K(\alpha,t) = \sum_{\substack{\beta \\ t'\leq t}} k(\alpha,\beta,t')$$

the number of agents whose risky action is $\alpha \in A$ and who have a threshold that is at most t.

We restrict our attention to situations in which coordination is possible, i.e.

$$\max K(\alpha,t) \geq \max\{t' : k(\alpha,\beta,t') > 0\}, \forall\alpha \in A$$

This condition implies that coordination is a rational outcome: the highest threshold of each risky action is smaller than the number of agents who prefer that action. This implies specifically that if all agents who prefer the risky action are choosing it, they are in fact behaving optimally. Our assumption of a fixed number of agents of each type allows us to study pure coordination problems. Since the number of agents of each type is known, we are restricting our attention to environments where coordination is always an equilibrium outcome: every perfectly rational agent would best respond choosing her own risky action if everyone else were choosing it also, and it is known that it is possible. Therefore, any equilibria different from the efficient one is always the result of a pure coordination failure. Our aim is to characterize the conditions over the observation network such that the efficient equilibrium is the only one possible.

2.1.1 The coordination game

Agents act in a consecutive manner according to the order of decision $\theta(N)$, which assigns a position to each agent. Let $\theta(N) = \{\theta_i : \forall i \in N\}$ such that $\theta_i \in \{1, 2, ..n\}$ and $\theta_i \neq \theta_j, \forall i, j \in N$. For instance, $\theta_i = 5$ indicates that agent i is the fifth to take the decision. Let $\Theta(N)$ denote the set of all possible orders of decision, which is equivalent to the set of possible permutations on $\{1, 2, ..n\}$. This order is randomly assigned by Nature according to some probability $P(\theta(N))$, which is assumed to be common knowledge: agents ignore the precise order of decision, but they know actions of their neighbors who have already decided. The order of decision and the observation network define the information set of each individual. Note that given this information, agents have partial information about their position. The agent knows exactly her position only if $P(\theta(N))$ is degenerated or if the agent is connected to every other individual. In this last case, the agent is able to infer perfectly her position because she knows exactly how many of the individuals have decided and how many have not. Whenever she is not linked with an individual, she cannot know whether the agents who are not connected with her have already decided or not.

For instance, an empty network corresponds to a simultaneous game, since nobody observes no other action: in this case, information sets only contain the private type of each individual. Links allow observation, transforming the game into a sequential one. In the other extreme case, the complete network, all the previous actions belong to the information set of the agent and she is able to infer perfectly her exact position in the order of decisions. Notice that the connections in the network and the position in the order of decision make agents heterogeneous.

A coordination game starts with Nature selecting the order of decision with the probability $P(\theta(N))$. We assume that this distribution is common knowledge (but not the realized order).[6] The network and the order of decision determine the information set of each agent, ψ_i. It includes the type of the agent and the actions played by those neighbors who have decided before the agent:

$$\psi_i = \{\tau_i, \{a_j : ij \in \Gamma; \theta_j < \theta_i; \theta_j, \theta_i \in \theta(N)\}\}. \tag{4}$$

where $\{a_j : ij \in \Gamma; \theta_j < \theta_i; \theta_j, \theta_i \in \theta(N)\}$ is the ordered set of actions of the preceding neighbors of i, in the order of decision selected by Nature. The knowledge of the network enters the information set by allowing the observation of neighbors' actions. The set of all possible information sets in which agent i could be is denoted by Ψ_i.

A strategy is a mapping from all possible information sets into actions. We allow for mixed strategies, so

$$s_i : \Psi_i \to \triangle\{A\}.$$

The complete history of the game up to agent i contains the order of decision, the actions up to agent θ_i and the type of each agent, and is defined as follows:

$$H_i = \left\{\theta(N); \{a_j, \forall j : \theta_j < \theta_i; \theta_j, \theta_i \in \theta(N)\}_{j \in N}; \{\tau_{j'}, \forall j' \in N\}\right\} \tag{5}$$

From this history, the agent i only knows her own type and the actions chosen by her neighbors who have already decided, but neither the type of the rest of individuals nor the actions of those individuals not connected to her. The belief of agent i is the probability that she assigns to each history that may have

[6] A natural extension of the model would endogenize the sequence. In this paper we analyze only the last stage of that extended version of the game. Since we allow for general $P(\theta)$, any possible result of the first stage could be represented by it.

occurred before she takes her decision, given her information set. Hence, the belief is defined as

$$\pi_i = \Pr\left(H_i \mid \psi_i\right) \geq 0 : \sum_{H_i} \Pr\left(H_i \mid \psi_i\right) = 1. \tag{6}$$

Now we are ready to define the coordination game.

Definition 1 *A coordination game is defined by* $N, \Gamma, A, k, P\left(\theta\left(N\right)\right)$ *and* $u\left(.\right)$ *that satisfies condition (1).*

A coordination game is defined by the observation network Γ, that connects the agents, the probability distribution over the different orders of play as well as the utility function that depends on the actions, which satisfies condition (1).

The following example clarifies each of the elements in a coordination game.

Example 1 *There are three agents,* $N = \{B, G, Y\}$*. All of them prefer Linux if at least two of them adopt it. Otherwise they would rather use Windows than being the only person using Linux. They are set in an observation network that links agents* B *and* G*, so* $\Gamma = \{BG\}$*. The action set for all of them is* $A = \triangle\{L, W\}$*,* L *denoting the choice of Linux (*W *denoting Windows). The utilities are given by:*

$$u_i(L, \sum_{j \in N} I_{a_j = L} \geq 2) = 4,$$

$$u_i(W) = 3,$$

$$u_i(L, \sum_{j \in N} I_{a_j = L} < 2) = 1.$$

The extensive form of the coordination game is as follows:

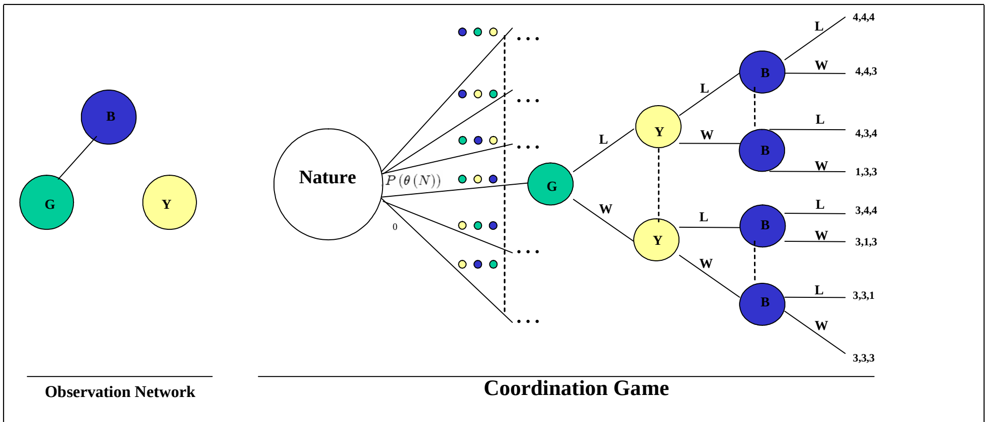


Figure 1

The coordination game associated with the network starts with Nature selecting one order of decision with probability $P(\theta(N))$, we illustrate the order GYB. This is the case in which $\theta(N) = \{3, 1, 2\}$ (remember that we have defined $N = \{B, G, Y\}$). Figure 1 shows that B is the third one who decides, and observes the action taken by G but not the action taken by Y, who is the second to decide. This is a case with

$$k = \begin{cases} 3 \textit{ for } (a, \beta, t) = (L, W, 2) \\ 0 \textit{ otherwise} \end{cases}$$

We use sequential equilibrium as the solution concept. A sequential equilibrium is defined by a profile of behavioral strategies in each information set and beliefs such that the strategies are best responses to the strategies of the other agents conditional on beliefs, and the beliefs are consistent with Bayes rule for some sequence of completely mixed strategies that converges to the equilibrium ones.

Definition 2 *Let (Σ', Π') be an assessment, a profile of behavioral strategies and beliefs for each player in each of her information sets. The assessment (Σ^*, Π^*) is a sequential equilibrium (Kreps and Wilson, 1983) if*

1. *(Σ^*, Π^*) is consistent:*

$$\begin{aligned} (\Sigma^*, \Pi^*) &= \lim_{n\to\infty} (\Sigma_n, \Pi_n), \{(\Sigma_n, \Pi_n)\} \subseteq \Phi^0, \\ \Phi^0 &= \left\{ \begin{array}{c} (\Sigma, \Pi) : \Sigma \textit{ is completely mixed and} \\ \Pi(x) = \frac{P^{\Pi}(x)}{P^{\Pi}(\psi(x))} \end{array} \right\} \end{aligned}$$

   where x is a decision node included in the information set $\psi(x)$, and $P^{\Pi}(x)$and $P^{\Pi}(\psi(x))$ are the probability assigned to x and to $\psi(x)$ respectively,by the system of beliefs Π.

2. *(Σ^*, Π^*) is sequentially rational:*

$$E\left(u_i | \Sigma^*_{-i}, \sigma^*_i\left(\bar{\psi}_i\right), \Pi^*, \bar{\psi}_i\right) \geq E\left(u_i | \Sigma^*_{-i}, \sigma_i\left(\bar{\psi}_i\right), \Pi^*, \bar{\psi}_i\right), \forall i \in N, \forall \bar{\psi}_i \in \psi_i$$

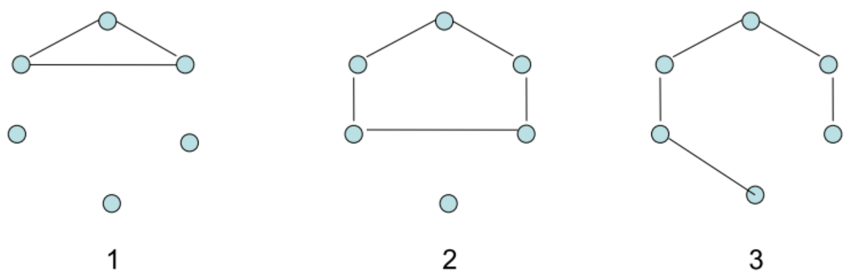
Our aim is to find the observation networks that lead to the Pareto-superior equilibrium in the coordination game defined by N, Γ, A and k, for any $\Theta(N), P(\theta(N))$ and $u(.)$.

Definition 3 *An observation network Γ is a coordination structure for N, A and k, if for all order of decision and utilities satisfying condition 1, $\forall P(\theta(N))$ and $\forall u(.)$, all agents take the risky action on the equilibrium path in any sequential equilibrium of the associated coordination game.*

If the number of agents who prefer each action is known, then Γ being a coordination structure implies that the unique equilibrium coincides with the efficient coordination. The observation network allows that some individuals in the game play sequentially instead of simultaneously, and as a consequence some Nash equilibria of the simultaneous version cannot be sustained as sequential equilibria. We can understand the concept of coordination structure as the social network such that, if it is known to exist, an agent not belonging to it would choose the risky action, since she can infer that the structure promotes the coordination on the efficient outcome.

3 Coordination Structures

We start our analysis with some illustrative examples of the simplest case with heterogenous individuals. Imagine a society formed by 6 agents, where 5 are willing to take the action α and the other agent always chooses β. We name her the "unwilling" agent. If types are private information, how are the coordination structures if agents that prefer α have threshold $t = 2$? Figure 2 shows some examples.



1 and 2 are coordination structures but 3 is not.

Figure 2

If there is an observation network as 1 or 2, in any sequential equilibrium, for any payoffs, and for any order selected by Nature, agents willing to take α choose it on the equilibrium path. Why? Since the agents have threshold $t = 2$, any of them would best respond by choosing α if she observes that a predecessor has chosen also α. In the network 1, the first agent in the clique who has to choose, knows that her action is observed by at least one agent willing to take the action (she is observed by 2 agents, and only 1 in the society does not want to choose α). Therefore, she responds by choosing α, since then the first best is obtained. The second agent who decides in the clique, when choosing, if she observes that the first one has not chosen α must believe with probability 1 that she was the unwilling agent, and assigns probability 1 to be observed by a willing agent. Therefore, on any equilibrium path, the willing agents in the clique choose α, and agents who does not belong to the clique best respond to these strategies by also choosing α. Note that every agent, belonging or not to the clique, chooses α in the unique equilibrium. And this occurs for any possible payoffs and order of decision. A similar argument can be applied in the network 2.

However, in network 3 a different equilibrium can be sustained. Imagine that everyone is playing β after observing β and that the utility of choosing α if nobody else chooses it is extremely low. If the sequence of decision goes from one extreme of the line to the other, these strategies can be sustained in equilibrium. If the first agent who decides is located at one of the two extremes and plays α, with positive probability she is observed by the unwilling agent, who would choose also β. Given the strategy, if the next individual who decides is a neighbor of that unwilling agent, she will choose β, and so on all the neighbors. In this case, therefore, the initial agent has a profitable deviation from α, since there is some probability of everybody choosing β. Thus, a coordination failure equilibrium can be sustained (although it is required for the result that the first agents who decide are in the extremes). Structure 3 does not guarantee that coordination failures do not emerge.

3.1 General result

The previous examples show some coordination structures for a very particular case: one where all the society shares the same risky action and threshold except for one individual. We have found some simple structures, as cliques, and some other complex structures. Here we explore if we can obtain more general results. We find that the existence of one sufficiently large clique guarantees the coordination.

Let us focus on the case of agents who share the same risky action α but with different thresholds t. Given our environment, an agent knows that there are sufficiently many people willing to take her own risky action in the clique if the clique is of a size at least $n - K(\alpha, t) + t$. This is the case since $K(\alpha, t)$ is the number of people in the society who prefer α with a threshold smaller or equal than t. The following lemma proves that, if there exists such a clique, the agent with threshold t chooses the risky action:

Lemma 1 *An agent of type $\{\alpha, \beta, t'\}, t' \leq t$ takes the risky action on any equilibrium path if there exists a clique of size $n - K(\alpha, t) + t$.*

Proof. *See Appendix* ■

In words, Lemma 1 says an agent with threshold t chooses the risky action in any sequential equilibrium if there exists a clique sufficiently large such that there are for sure in the clique at least t agents with the same risky action and a threshold smaller or equal than t. In the Appendix we prove that this is the case.

Because of Lemma 1, an agent of type (α, β, t) takes the risky action if there is a clique of size at least $n - K(\alpha, t) + t$. Suppose that such a clique does not exist, but there exists another smaller clique such that the agents with threshold $\bar{t} < t$ take the risky action α. Note that if the cardinal of this type of agents is higher than t, our initial agent will also choose the risky action in equilibrium. These insights are summarized in the following theorem:

Theorem 1 *An agent of type $\{\alpha, \beta, t\}$ takes the risky action on any equilibrium path if there exists a clique $q \subset N$ of size $\#q = q(t)$ where*

$$\begin{aligned} q(t) &= q_\alpha (r_\alpha (t)), \\ \textit{where } r_\alpha (x) &= \min \{N - K(\alpha, t) + t : \forall t \geq x\} \\ q_\alpha (x) &= \begin{cases} r_\alpha (x) & \textit{if } r_\alpha (y) < x \\ r_\alpha (y) & \textit{if } r_\alpha (y) \geq x \end{cases}, y = \max \{t : y < x\} \end{aligned}$$

where $q_\alpha (x)$ is constructed iteratively from the lowest threshold x for the action α.

Proof. *See Appendix.* ■

Theorem 1 shows that the existence of certain cliques guarantees that the agent takes the risky action in any sequential equilibrium. This result can be applied for environments where people differ in their objectives of coordination. The result of Lemma 1 is incorporated in $r_\alpha (x)$, which includes that an agent takes the risky action when the condition of the Lemma 1 is met by the agents with some threshold equal or larger than her threshold. The possibility that there are more than t agents choosing the risky action with a threshold smaller than t is incorporated in $q_\alpha (x)$, which is constructed recursively and says that, if there are more than t agents with a threshold smaller than $t' < t$, the size of the clique required by agents with threshold t is the same than the one required by the agents with threshold t'. The following example of a society of 25 agents who share the same risky action α but differ in their thresholds, and 1 agent who prefers β', illustrates the result.

Example 2 *Suppose a set of agents $N = \{1, 2, ..., 26\}$ who play a game of coordination. The type of agents* *is given by*

$$k(.,.,.)=\begin{cases}5 \; for \; (\alpha,\beta,2)\\ 5 \; for \; (\alpha,\beta,6)\\ 8 \; for \; (\alpha,\beta,10)\\ 2 \; for \; (\alpha,\beta,14)\\ 5 \; for \; (\alpha,\beta,21)\\ 1 \; for \; (\alpha',\beta,1)\\ 0 \; otherwise\end{cases}$$

We can compute here the size of the clique required by each agent in order to be sure that she chooses the risky action in any equilibrium:

t	$k(\alpha,\beta,t)$	$N-K(\alpha,t)$	$N-K(\alpha,t)+t$	$r_\alpha(t)$	$q_\alpha(r_\alpha(t))$
2	*5*	*21*	*23*	*18*	*18*
6	*5*	*16*	*22*	*18*	*18*
10	*8*	*8*	*18*	*18*	*18*
14	*2*	*6*	*20*	*20*	*18*
21	*5*	*1*	*22*	*22*	*22*

Table 1

If there is a clique of size 23*, agents with threshold* $t=2$ *know that there are at least 2 people with threshold 2 in the clique, and therefore, they are going to take the risky action. Agents with threshold at most 10 (*$t\leq 10$*) know that if there is a clique of size* 18*, it includes with certainty at least 10 agents with a threshold equal or smaller than 10. Therefore, those agents with threshold at most 10 in the clique coordinate on their efficient action in any equilibrium, and - since the network is common knowledge - also any other agent with the same threshold in the society will do so.*

Finally, there is an "amplifying" effect, that generates that agents with threshold 14 take the risky action also when there is a clique of size 18*. This occurs because this clique implies that all agents with a threshold smaller or equal than 10 take the risky action on any equilibrium path; and since there are 18 such agents, the effect of the clique affects also to those agents with threshold at most 18 (in this case, the 2 agents with threshold 14). However, the agents with threshold 21 will not necessarily follow suit. They require the existence of a sufficiently large clique to know that 21 agents will coordinate: in this case, a clique of size* 22*.*

Note that, in the example, if there is a clique of size 22, every individual chooses her risky action. This means that an observation network Γ including a clique of size 22 is, in fact, a coordination structure.

Nevertheless, if the highest threshold were 14 instead of 21, then a clique of size 18 would be a coordination structure. Therefore, we obtain a sufficient condition for the existence of a coordination structure, that follows directly from Theorem 1:

Corollary 1 *A network Γ is a coordination structure if there exists a subset s of agents that forms a clique such that*

$$\left\{ s \subset N : \#s = \max \left\{ q_\alpha \left(r_\alpha \left(t \right) \right) \right\}_{\forall \alpha \in A, \forall t} \right\}$$

Proof. *If there exists a clique of size $\left\{ s \subset N : \#s = \max \left\{ q_\alpha \left(r_\alpha \left(t \right) \right) \right\}_{\forall \alpha \in A, \forall t} \right\}$, given Theorem 1, for any agent in N there exists a sufficiently large clique such that she takes the risky action on any equilibrium path. Therefore, everyone takes the risky action and Γ is a coordination structure.* ■

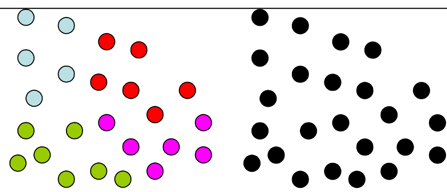
Agents of each type require a clique of a given size for choosing the risky action for sure. Corollary 1 says that if the largest of the required cliques exists, everyone chooses her risky action. And therefore Γ is a coordination structure. This can be applied when in the society there are groups of agents who differ in their risky action α. Under these conditions, Corollary 1 shows that if a sufficiently large clique exists, the unique equilibrium outcome is the Pareto-efficient one.

This result provides us with a sufficient condition to obtain coordination in the case of heterogenous agents with private types. It is interesting to note that the first agents to decide are not always those whose threshold is the lowest (see agents with $t = 6$ in the previous example). However, this does not impede coordination, because agents' choices are strategic and consequently even an agent with a high threshold takes the risky action if the observation network enables coordination. The ability to observe others' actions facilitates coordination. When people know that there is a sufficient number of other agents, and that they are observing each others' actions, they are able to signal their type to the rest of members of the group by choosing their preferred action. This happens only if the group is sufficiently large to ensure that enough people with the same type will observe the action: coordination failures are less likely when there are larger groups of people able to observe each other (even if they do not communicate). The assumption of mutual observability may become very demanding as the cardinal of people increases. Being this a limitation of the model, it can be likely in some environments, given the weak social interaction entailed by an observation network.

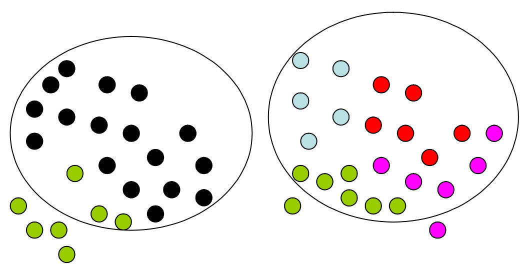
Figure 3 illustrates the case of a coordination structure when there are different risky actions (although for simplicity, we assume that all thresholds are the same). Suppose a society of 24 individuals who have to dress for a party. We assume that each one has a preferred color to dress, 7 of them preferring to dress in green, 6 in red, 6 in pink and 5 in blue. However, they prefer it only if at least 2 other people wear their preferred color. If not, they prefer to dress in black. Formally, this is a case where $N = \{1, 2, ..., 24\}$, $A = \{green, red, pink, blue, black\}$ and

$$k(\alpha, \beta, t) = \begin{cases} 7 \text{ if } (\alpha, \beta, t) = (green, black, 3) \\ 6 \text{ if } (\alpha, \beta, t) = (red, black, 3) \\ 6 \text{ if } (\alpha, \beta, t) = (pink, black, 3) \\ 5 \text{ if } (\alpha, \beta, t) = (blue, black, 3) \\ 0 \text{ otherwise} \end{cases}$$

Imagine that some of the people live in the same residence or building, so that all those who live there will see how the first one who goes to the party is dressed. In this situation, what observation networks are coordination structures?



If agents are isolated, each type could coordinate or not in their preferred action, as a result in some Sequential Equilibrium



Circles represent cliques. In the first case, there exists a clique of size 20. In any Sequential Equilibrium, agents who prefer green dress it, independently of if they belong or not to the clique; if the clique is of size 22, in any Sequential Equilibrium first best is obtained.

Figure 3

Theorem 1 shows that, if there is a clique of size 20 (i.e., if 20 of the individuals live in the same building), every agent whose preferred action is green chooses it in any sequential equilibrium. It is not necessary true for the other agents. A clique of size 22 is a coordination structure, since the agents who prefer blue are those who require the highest clique, and it is of size 22. If such an observation structure exists, in any order of decision, for any payoffs satisfying condition 1, every agent chooses her risky action on the equilibrium path and the first best is obtained: all agents will dress their favorite colors. That is, if at least 20 people live in the same building so that they will be able to observe how each one dress for the party, our result states that every person who prefers green will dress it in any equilibrium, even if she does not live in the building. If 22 people live there, everybody would dress her preferred color. So that the clique with 22 people is a coordination structure. It is important to remember that we are analyzing a situation where agents do not communicate. Our conditions are sufficient if the individuals in the clique are able just to observe what the others finally wear.

The clique required by Theorem 1 and Corollary 1 may be very large, which is typically not likely to occur. However, the kind of social interaction that we are studying is mild: a link between two subjects only implies that they are able to observe their actions. This can be simply the case of living in the same neighborhood. Imagine the case of revolts. People may prefer go to the streets if enough other people are going to revolt. We require these actions to be observable. One can imagine that a city would be in fact a clique that connects all its citizens, since in a city, each agent would observe if other agents are in the streets. Our result implies that if there es a sufficiently large clique, coordination emerges. In the revolt environment, we would say that if there is a sufficiently large city, where people are able to observe each other, we expect that revolt occurs. We argue that this mechanism can give some insights about the cities being places where revolts generally start. In fact, we are saying that if people know that there is a sufficiently large city, where there are for sure sufficiently many people willing to revolt, everyone would decide to revolt, even people who are not in the city, and even people who require high thresholds for participating. Given that information is private, the agents take the decision although they do not know which of their neighbors are willing to choose it. We show that in this situation the unique outcome is the efficient one when a sufficiently large clique exists. The key point is to know that there are sufficiently many people willing to revolt.

3.2 Coordination structures with smaller cliques

An agent that has to coordinate with a part of the society in order to maximize utility chooses her risky action if she knows that there is a sufficiently large clique, as determined by Theorem 1. But note that it is not a necessary condition: it is possible to find structures with cliques of smaller sizes that also guarantee the coordination on the risky action for any payoffs and order of decision. Figures 4 and 5 illustrate the case when there is 1 agent in the society who will play always the safe action and the required threshold is $t = 2$ or $t = 3$ respectively. Here we discuss how these structures can be constructed. We focus on the simplest case, when there is a group of agents with the same risky action α, the same threshold t and the rest of the society chooses always the safe action β. We name n_α the cardinal of agents who prefer α if at least t agents choose it and n_β the cardinal of agents who always choose β. Abusing the notation we say that an agent is of type α or of type β respectively. In this kind of situations, following Corollary 1, a clique of size $n_\beta + t$ is a coordination structure. In fact, any larger structure including a clique of size $n_\beta + t$ is a coordination structure. But what about structures formed by smaller cliques?

What do we know about any sequential equilibrium, for any order and payoffs? Any equilibrium strategy profile must include that an agent of type α chooses the action α in all those information sets where she observes $t-1$ actions α. For the simplest case, when $t=2$, an agent of type α chooses it if she knows that she is followed by another agent of the same type. If in the society there is only one agent of type β, $n_\beta = 1$, an agent of type α who has two neighbors who have not played yet will choose action α in equilibrium. The simple structure where we know that there will be a initial player with two neighbors is a triangle, the clique of size 3. For this case, one clique of size 3 is sufficient. In fact, it is just a special case of Lemma 1.

For this case, there other coordination structures, as depicted in Figure 4. A square is also a coordination structure: an agent of type α has two neighbors and will choose α; consistent beliefs imply that, if she does not play α, then she must be the type β agent with probability 1. The square is an example of coordination structure formed by cliques of size 2 for the case $t=2$, $n_\beta = 1$.

For the case of $t=2$, $n_\beta = 2$ each agent requires to have 3 successors in order to play the risky action if being of type α. In a clique of size $\#q = t + n_\beta = 4$, every agent has 3 followers, and therefore if the first one plays β, consistent beliefs imply that she is a type β agent. In order to obtain a coordination structure, we need agents who have 3 successors. In this way, an agent of type α will choose α in any sequential equilibrium, because having 3 successors she knows that at least 1 is of type α. It can be obtained with a clique of size 4 or with other structures with more nodes and connections but cliques of smaller size. In Figure 4 we have drawn different coordination structures when $t=2$ and the number of agents of type β is between 1 and 4. for the case of $t=2$, we obtain coordination structures if there are sufficiently many agents with $n_\beta + 1$ neighbors.

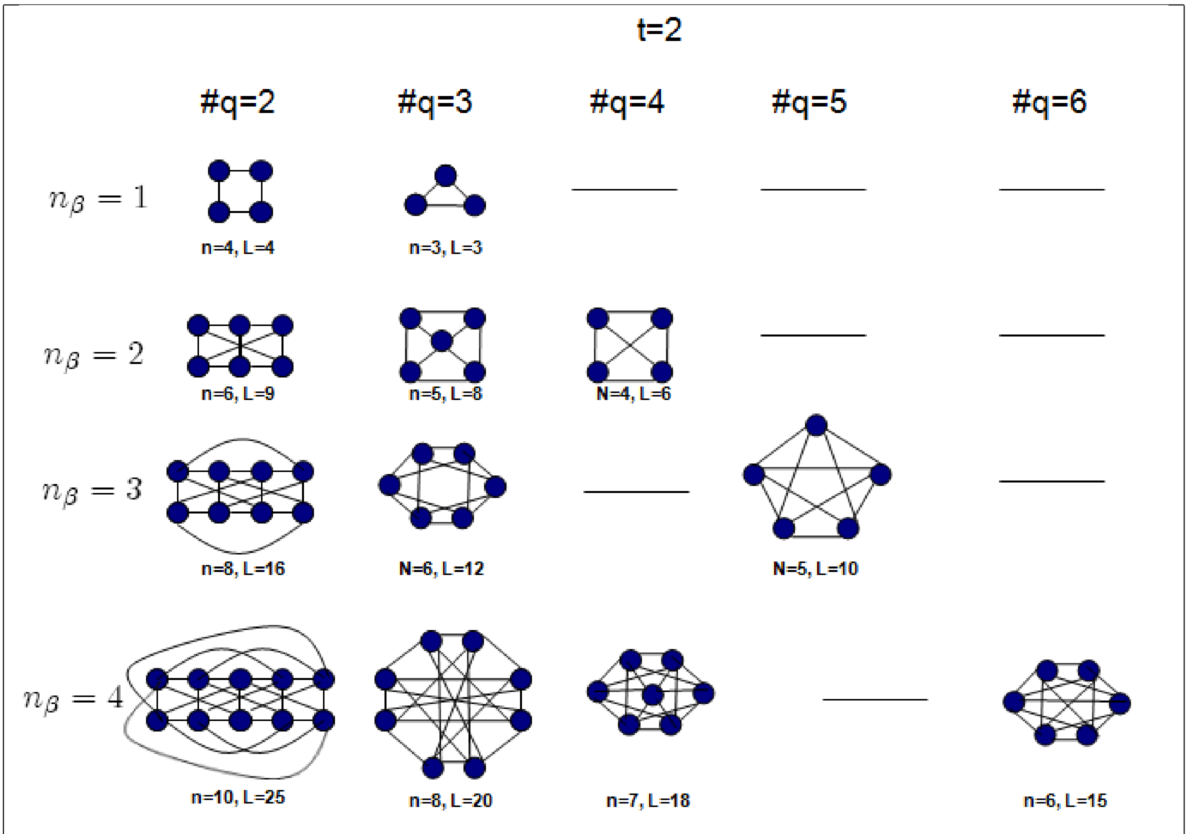


Figure 4

When coordination is more difficult, that is, when more agents are required to obtain coordination, coordination structures become more complex. In Figure 5 we have drawn the coordination structures formed by cliques of size 3 for the case when $t = 3$ and $n_\beta = 1$ and $n_\beta = 2$. In these cases, cliques of size 4 and 5 would be sufficient, following Corollary 1. But structures become much more complex when using smaller cliques. In the simplest case, when $t = 3$ and $n_\beta = 1$, a square with all corners linked with a "center" implies that, if the center is the first who plays, she will choose α if she is of such a type.[7] To obtain a coordination structure, we must ensure that each of the agents is the center of a square. Figure 5 shows how a network where it occurs looks like. When $n_\beta = 2$, the center of an hexagon whose opposite corners are connected would also choose α if she is of such type and is the first to play. Therefore, we can construct a coordination structure if we guarantee that each node is the center of an hexagon of that type. Note that in

[7] Note that in a square with all corners connected to a center the largest clique is of size 3.

such an hexagon, the highest clique is of size 2, and therefore, if agents connect to some center, the highest clique remains of size 3.

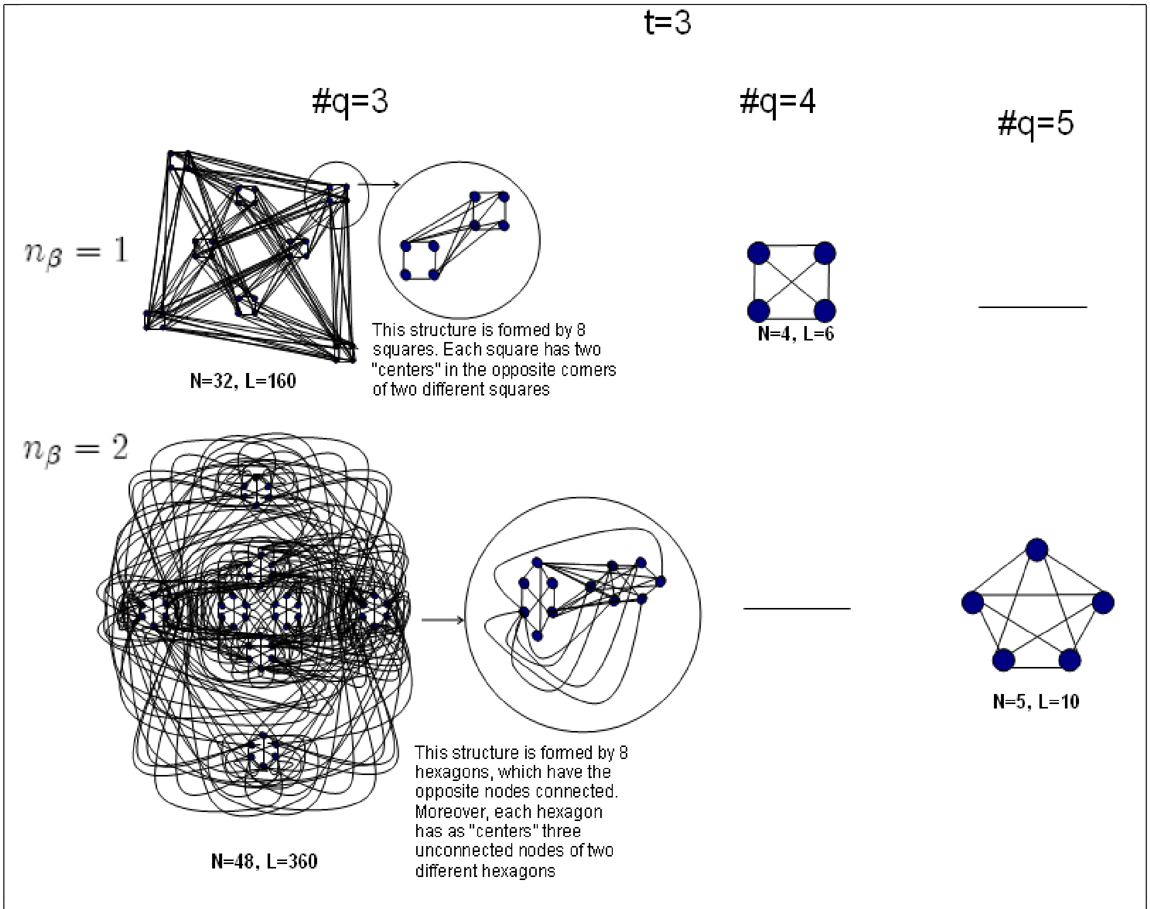


Figure 5

Figure 5 illustrates the complexity that is required to have a coordination structure when the size of the cliques is limited. Corollary 1 guarantees the existence of a coordination structure with relatively low requirements in terms of number of nodes and connections. Moreover, it guarantees the existence of some coordination structure when there are sufficiently many agents for the coordination: if $t \leq F_a(t)$, that is, if there are enough agents for coordination, a complete network always contains a clique of size $n - F_a(t) + t$, and we can guarantee that, at least the complete network, is a coordination structure.

3.3 Homogenous case

We name homogeneous the case in which all the agents share the same risky action and threshold t. Importantly in this case, every agent knows that the rest of players has the same threshold as she has. The threshold t represents the number of agents that are required to take the risky action in order to make it the optimal decision. In this version of the problem, agents face a binary decision. Let $A = \{\alpha, \beta\}$ be the pure action set. We name α the risky and β the safe action, meaning that agents' highest payoff is obtained when choosing α if sufficiently many other agents choose it also. The preferences that we study are represented as follows:

$$u_i \left(a_i = \alpha, \sum_{j \in N} I_{a_j = \alpha} \geq t \right) > u_i \left(a_i = \beta \right) > u_i \left(a_i = \alpha, \sum_{j \in N} I_{a_j = \alpha} < t \right), \tag{7}$$

where $I_{a_j=\alpha}$ is the indicator function that takes the value 1 if the agent choose α and a_i is the action taken by agent i. Action β is the safe action, yielding a fixed utility independently of the other agents' choices. Note that the utility does not depend on the position in the network or the order of decision, it is only a function of the actions taken by the agents.

One can imagine this setup as the generalized version of the example in the Introduction. In a large department, everybody may prefer Linux (the risky action), but only if enough other agents adopt it as well. Otherwise, they prefer the safe option and use Windows.

For this homogenous case, we find a necessary and sufficient condition:

Proposition 1 *In the homogenous case with threshold t, an observation network Γ is a coordination structure if and only if there exists a subset of agents $\{q \subset N : \#q = t\}$ that forms a clique.*
Proof. *See Appendix.* ■

A clique is sufficient by an argument of backward induction. If there is a clique of size t, in any order the last agent in the clique chooses the risky action if she observes $t-1$ risky actions. Agent in position $t-1$ in the clique best responds to a strategy of this type by choosing the risky action if she observes $t-2$ risky actions, and so on. Therefore, any agent in the clique in position r among those that also belong to the clique, chooses the risky action when observing $r-1$ risky actions. Thus, the first agent in the clique chooses it, and any agent out of the clique best responds to these strategies by choosing the risky action, since there are t agents who choose it. Everyone chooses the risky action on the equilibrium path. We prove that the clique is necessary by constructing an equilibrium assessment in which, on the equilibrium path, everyone chooses the safe action when the clique does not exist. If everyone believes that everyone else is going to choose the safe action, everyone choosing the safe action can be sustained in equilibrium, if the clique does not exist. This is the case because an initial agent cannot know if her action will be observed by sufficiently many people who are also mutually observing their actions in order to be sure that the first best is obtained. Note also that any structure that contains the clique is also a coordination structure. In this sense, a clique of size t can be said to be minimal and sufficient.

Figure 6 illustrates the importance of the clique. In this example, there are two different network structures of $n = 8$, and it is required that at least 3 agents take the risky action so that it becomes profitable.

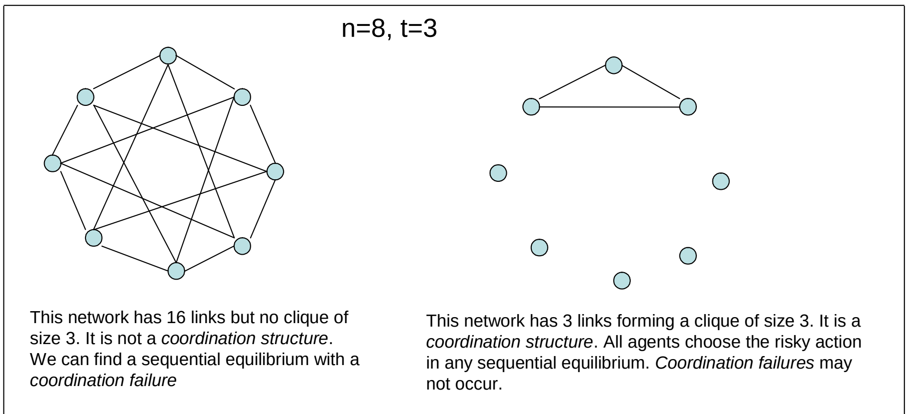


Figure 6

The network structure on the left has 16 connections but no clique of size 3. Given our result, in the structure one the left, coordination failures are possible. In fact, they may occur with any sequence of decision as well as for arbitrarily close payoffs. However, in any sequential equilibrium in the structure on the right, the Pareto-dominant equilibrium is the only one that emerges.

4 Discussion

4.1 Relevance of cliques: consistency

Our results support the importance of cliques in the emergence of coordination from a new point of view. Cliques, usually understood as the representation of groups, enable coordination because the agents are able to signal their decision in those structures. Importantly for our results, it holds for any utility function (it does not depend on u): it is not possible to find a coordination failure equilibrium if the clique does exist, and in the homogenous case, it is possible always if the clique does not exist. This has relevant implications.

Imagine the case of a society deciding whether to revolt. Our homogenous case would describe a society where everyone wants to revolt and it is known, but a certain number of actually revolting people is required to make the uprising successful. In this situation, in a simultaneous set up (i.e., an empty network in our model), both situations are equilibria, everyone or nobody revolting. In principle, the equilibrium that emerges would depend on the relative payoffs.[8] For instance, if the punishment after a failed revolt is very large, we would expect that people do not revolt. However, once the clique exists, relative payoffs do not matter, and everyone would choose the risky action even if the punishment is very large. This intuition makes clear that dictators are interested in destroying such type of cliques, since punishment *per se* would not matter.

How important are the assumptions that we use for this result? Is it consistent? Our model requires several strong assumptions. First of all, our results rely on the perfect rationality of the agents. In particular, most of the results proceed from backward induction arguments. However, experimental and empirical evidence (i.e. on the centipede game and other related experiments) show that many people do not use backward induction. Another weakness is the requirement of knowing the distribution of types and the network structure, which are very demanding conditions.

However, we argue that there are several arguments that support the consistency of our results and that the existence of a clique really plays a crucial role. With respect to the previous literature, Chwe (2000) obtained a result that is connected with ours (we detail this connection in a later section). Importantly, he shows that the existence of cliques has a key role in generating the common knowledge that allows coordination (among other results). We assume common knowledge and show that cliques play also a crucial role for obtaining coordination as the *unique* expected outcome.

One of the main points is the independence of the result with respect to utilities: if the clique exists and the benefit of coordination is arbitrarily small, the unique equilibrium implies that coordination emerges; if the clique does not exist, even if many agents are able to observe each other, and even if the benefit of coordination is arbitrarily large with respect to the failure, it is possible to construct an equilibrium characterized by coordination failure.

The clique is sufficient for coordination for any possible order of decision, but it is also necessary in the homogenous case. In such a environment, no matter how the structure or the payoffs are, no matter who starts to play, the existence of the clique guarantees the efficient coordination, and the non-existence of the clique guarantees the existence of a coordination failure. In fact, the existence of the clique is necessary

[8] This kind of simultaneous situations have been succesfully analyzed with *global games* (Carlsson and Van Damme, 1993). These models show the importance of the relative riskyness of each equilibrium in determining which one is selected.

and sufficient for any order of decision: there is no structure in which the coordination failure may not be sustained, even if the first agent who decides is observed by many other people (but not forming a clique of size t).

The existence of a sufficiently large clique guarantees the coordination of agents given any payoffs and any order of decision. Now we discuss the case of structures that guarantee the coordination for particular payoffs or order of decisions (a case that we name quasi-coordination structures).

4.2 Quasi-coordination structures

Up to now, we have analyzed how are the structures in which coordination problems do not occur for any payoffs and for any order of decision. However, in particular situations, it is possible also that the unique equilibrium is the one where everybody chooses her risky action with other structures, for particular payoffs and/or sequences of decisions. In this subsection, we study this situation. These are interesting cases since, posibly, the observation structure cannot be changed, but modifying the payoff or stablishing efficiently the order of decision may help to reach the first best. We focus, as previously, on the simplest coordination problem in a heterogenous society: a society formed by n_α agents. These agents prefer α over the safe action β if at least t agents are choosing α. There are also n_β agents who always choose action β.

The agents who face the coordination problem are homogenous with respect to their threshold, so that the utility of agents of type α can be reduced to the payoffs $u_{\alpha,\geq t} > u_\beta > u_{\alpha,<t}$, that represent the utility they get if choose α or β, depending on whether finally there are less than t people choosing α or not. Since we deal now with specific payoffs, we assume that agents are expected utility maximizers.

4.2.1 Quasi-coordination structures for specific payoffs

We say that a network Γ is a quasi-coordination structure for specific payoffs if, for any possible order of decision, any agent chooses always her risky action on the equilibrium path, under certain restrictions over the payoffs. The following proposition shows that an isolated clique of size $\#q = t$ can be sufficient. An isolated clique $q \subset N$ is a set of individuals such that $\forall i \in q, ij \in \Gamma \iff j \in q$. It occurs when the benefit of the risky action is sufficiently high.

Proposition 2 *In any sequential equilibrium, any agent of type α chooses α on the equilibrium path, for any order of decision, if there is an isolated clique $q \subset N$ of size $\#q = t$ when*

$$\frac{u_{\alpha,\geq t} - u_{\alpha,<t}}{u_\beta - u_{\alpha,<t}} > \frac{(n_\alpha + n_\beta - 1)!\,(n_\alpha - t - 1)!}{(n_\alpha + n_\beta - t - 1)!\,(n_\alpha - 1)!} = \frac{1}{\tilde{p}} \text{ and } n_\alpha > t$$

Proof. *See Appendix.* ■

This proposition reveals that, even when types are uncertain, a clique of size t may be sufficient for coordination. This is true if $u_{\alpha,\geq t}$ and $u_{\alpha,<t}$ are sufficiently large with respect to u_{β}. Under these circumstances, an agent who is not connected to the clique assigns a sufficiently high probability to the event that there are t agents of type α in the clique who are going to coordinate (in the Appendix we prove that it is the case). In fact, $\tilde{p}$ is the probability of having t agents of type α in the clique conditional on having one agent of type α out of the clique. Therefore, any agent who is not connected to the clique best responds by choosing α, and everyone chooses the risky action on the equilibrium path. In general, when there are larger cliques, the probability of having sufficiently many agents of type α' in the clique increases, and there is a larger set of utilities under which coordination would emerge for any order of decision. This reasoning connects with Lemma 1: when there is a clique of size $n - K(\alpha, t) + t$, it includes for sure a set of t agents with threshold t. Since it occurs with probability 1, every agent chooses the risky action for general payoffs $u_{\alpha,\geq t} > u_{\beta} > u_{\alpha,<t}$.

Non-isolated case: more connections may destroy coordination Interestingly, more connections can potentially generate coordination failures. Our general results of coordination structures show some kind of networks such that, if the structure exists, coordination emerges always, independently of whether there are more connections or not. It is not the case with quasi-coordination structures for specific payoffs. Note that Proposition 2 requires the existence of an *isolated* clique. When it exists, nobody knows what is occurring in the clique, agents believe that coordination occurs with sufficient probability, and they take the risky action. But if agents observe what occurs in the clique, with positive probability there are in the clique agents not of type α. These agents will play β. The agents who do not belong to the clique but observe these actions know that there are other agents observing the clique who are of type α. And they will decide simultaneously (i.e., neither observing nor being observed by others) with the rest of agents of type α. In this case, under some sequences of decision, agents of type α would choose the safe action on some equilibrium path (specifically, when they play after agents in the clique, and agents in the clique are not of type α). In Figure 7 we present two different networks. For the case in which $n_{\alpha} = 4$, $n_{\beta} = 3$, we have that a clique of size 3 includes 3 agents of type α with probability $p = \frac{4}{35}$. An agent of type α who does not belong to the clique believes that it is formed by agents of type α with probability $\tilde{p} = \frac{1}{20}$.[9] If utilities are given by $u_{\alpha,\geq t} = 21, u_{\beta} = 1, u_{\alpha,<t} = 0$, the agents of type α who are not connected to the clique in the network on the left choose the risky action in any equilibrium, and agents in the clique best respond to it

[9] It is the probability of the 3 agents in the clique being of type α conditional on one agent out of the clique being of type α. Note that in such circumstances, there are 3 agents of type α and 3 agents of type β that can be in the clique, and the probability of everyone being of type α is given by $\tilde{p} = \frac{3}{6} \cdot \frac{2}{5} \cdot \frac{1}{4} = \frac{1}{20}$

by also choosing α, even if they observe that some agent in the clique does not choose α.

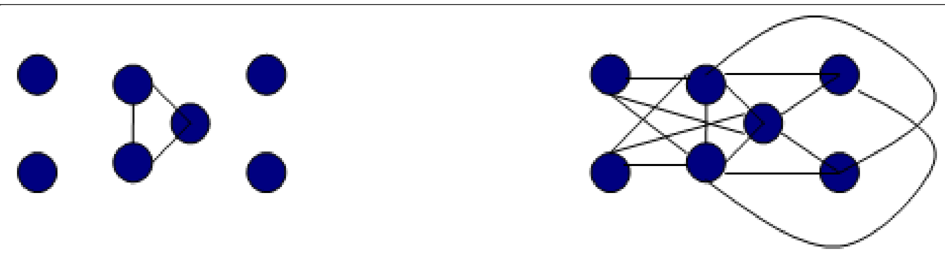


Figure 7

However, in the figure on the right all the agents are connected to the clique. If the sequence is such that the agents in the clique are the first to decide and they are of type β, all agents out of the clique know that they are playing simultaneously with the rest of agents of type α and coordination may fail in equilibrium, just by assuming that in such information set anyone of them chooses β. In this sense, more connections, more information, may be detrimental for coordination.[10]

4.2.2 Quasi-coordination structures for specific orders of decision

We have shown that smaller networks may be quasi-coordination structures for specific payoffs. Now we study the case of quasi-coordination structures that guarantee the efficient equilibrium for certain sequences of decision but general payoffs. We show some examples, using the same framework that in the rest of this subsection. Suppose that there are $n_\alpha = 2$ and $n_\beta = 1$ individuals, and that threshold is $t = 2$. The central agent in a segment of three nodes chooses the risky action in this context. Therefore, if in the order of decision the central agents choose first and there are two segments of three individuals, the risky action is played by everyone. Figure 8 illustrates some examples for the cases in which $t = 2$ or $t = 3$ and $n_\beta = 1$ or $n_\beta = 2$. Tese networks are quasi-coordination structures for any possible payoffs if the nodes with an assigned position play in that position, and for any order of decision of the nodes for those we have not written a position. If the order of decision is known, since at least t agents will choose α, any agent of type α in the society who is not included in those structures chooses it as well.

[10] It is not the case for coordination structures, but it may occur with the quasi-coordination structures for certain payoffs.

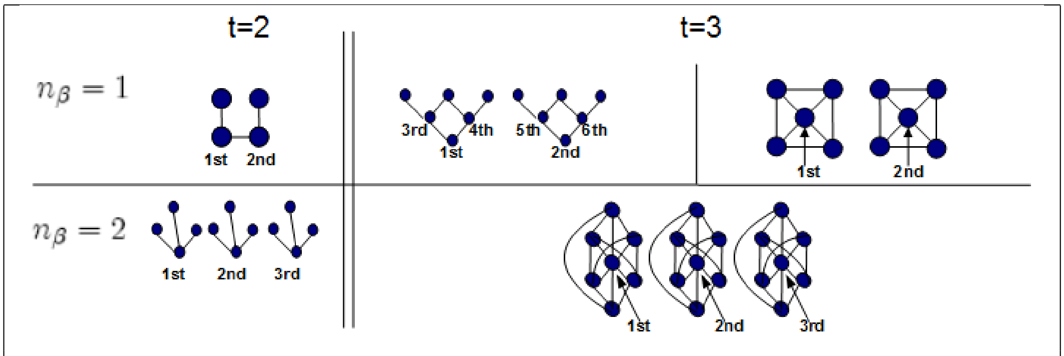


Figure 8

Quasi-coordination structures for specific orders of decision can be especially relevant if we think of an extended version of the coordination game where we would endogenize the order of decisions. Specifically, the conditions in which the extended model would have a unique efficient equilibrium can be explored. If it is the case, we could obtain the efficient coordination with less demanding requirements over the social structure.

5 Applications

In this section we relate our model to the minimal sufficient networks in Chwe (1999, 2000), the threshold models by Granovetter (1978) and apply it to a bank run model à la Diamond and Dybvig (1983).

5.1 Networks that allow the revolts are sufficient for them

The aim of this paper is very close to the one by Chwe (1999, 2000), as explained in the introduction. Our work characterizes the type of structures where coordination emerges among agents when the network enables observability of actions and type distribution is known, although type may be private information. Chwe characterized how the structure of a communication network must be in order to allow coordination. Both approaches stress the importance of cliques to achieve this coordination.

In the model of Chwe there is a set of agents who have to decide whether to revolt (r, the risky action) or not (s, the safe action). Agents are of type willing (w) or unwilling (x). An agent of the willing type prefers the risky action to the safe one if all the other agents are choosing the risky action; an agent of the unwilling type, always prefers the safe action. Utilities of agents of willing type are assumed to be supermodular in the action of the others, i.e., the difference in utility for an agent of type w between r and s is increasing in the number of other agents who choose r: for an agent of type w, the action of revolting is increasingly attractive when there are more people participating in the revolt.

The agents are embedded in a communication network. When two agents are connected through a directed link $i \to j$ (that we represent as ij), it means that i "talks" to j. This communication process allows in practice that the private type of i is revealed to j. The network structure is common knowledge, so that if two connected agents talk to each other, say i and j are connected and talk to k, this means that k knows the types of i and j, as well as that they talk between themselves and know their respective types. In this sense, the network generates "local common knowledge".

In this environment, Chwe studies the structure of the minimal sufficient networks for coordination. The main concern in his work is how is a communication network that enables coordination independently of the agent's beliefs. Chwe analyzes in which situations would all the agents take part in the revolt for the case in which every agent is of the willing type. He studies in which case each agent has incentives to revolt if she only has information about her neighbors. Chwe shows that the minimal networks that enable coordination can be described by a set of cliques in sequence, such that there exists some leading cliques who decide to revolt and who are followed by other cliques. This type of structure allows people to recognize that there are sufficiently many people willing to revolt, who at the same time know that there are sufficiently many people willing to revolt.

The concept of sufficient network of Chwe requires that revolting is a possibility *for any* distribution of the unknown types. In particular, in a minimal sufficient network, an agent knows the types of those neighbors that talk to him and this allows him to know that there are enough people for successful coordination (given her preferences): a minimal sufficient network in the sense of Chwe guarantees the existence of one equilibrium where agents decide to revolt, i.e., take their risky action. Given their information, agents know that every agent has information about the existence of enough people willing to participate, and they best respond by choosing the risky action if anyone else chooses the risky action.

The aim in our paper is to characterize the set of structures in which, when we assume that the existence of that equilibrium is known, only the efficient outcome can be sustained in a (sequential) equilibrium. In our set up the revolt equilibrium (the Pareto efficient one) always exists: our assumption is that agents know that there are enough people for coordination, because either everyone of them is homogenous or because they know the fixed number of agents of each type, which allows the coordination. In this sense, the cases that we study are specifically the situations that allow coordination in the sense of Chwe. We add conditions over the observation network that generate that coordination becomes the unique equilibrium outcome.

How related are then the sufficient networks by Chwe and the coordination structures of the present work? Applying our concepts of sequential decision, we show now that if the communication network which is sufficient in the sense of Chwe exists and everyone is of the willing type, given that information of types is restricted to those transmitted by the network, all agents choose the risky action on any sequential equilibrium, independently of the order of decision. This result holds when observation of actions occurs in the same way as communication transmission:

Proposition 3 *Suppose a set of agents N embedded in a sufficient network Γ such that everyone is willing to revolt in the sense of Chwe (2000). Suppose that Nature selects an order of decision according to $P(\theta(N))$ and that if $ij \in \Gamma$, a_i is in the information set of j if $i < j$. Then every agent chooses to revolt in any weak perfect Bayesian equilibrium.*[11]

Proof. *See Appendix.* ■

In a sufficient network, there are "leading cliques" where in some equilibria any agent in the clique best responds to the action of revolting by also revolting. There are other cliques whose members "are talked" by the participants in the leading clique. Those "followers" best respond by revolting to the action of members in their clique and in the leading clique.

The result by Chwe states that an equilibrium exists where everyone revolts (if everybody is willing to do so). The communication structure reveals private types to agents in the leading clique. Therefore, any of the members of the clique knows that there are sufficiently many people for coordination in such a clique. For any order, we know that the last one who decides in the clique would best respond to anyone else choosing to revolt by also revolting. And we can do the backward induction argument used for the sufficiency part of the result of coordination structures. For any order, in equilibrium, agents in cliques which follow the leading clique have to best respond to strategies of agents in the leading clique which imply that they revolt. And therefore, it can be shown that every agent will revolt in a sufficient network.

Our results reinforce those by Chwe, showing that coordination must emerge when cliques exist, with independence of the relative payoffs. Revolts are not only a possibility, they become the unique possibility.

5.2 Riots are not necessarily initiated by the most radical people

Granovetter (1978) modeled the conditions required for the emergence of collective behavior starting from an individual distribution of preferences. In his model, agents decide to join a riot depending on how many agents are taking part in it. Given a distribution of those thresholds, Granovetter studied how many agents

[11] This implies that it is also chosen in any sequential equilibrium, the equilibrium concept used across the paper.

actually take part in the revolt. Interestingly, two very similar societies could generate completely different outcomes.

Agents decide in a dynamic context. Initially, the agents who have threshold 0 go to the streets. The threshold represents the number of people that an agent requires to join the riot. People with threshold 0 (or very low) are the initial "instigators". They want to participate in the revolts independently of others, and when they are on the streets, other people whose willingness is not so high, also decide to participate. This generates a cascade effect that determines how many people is finally in the streets. Let $h(t)$ be the number of people with threshold t and $H(t) = \sum_{k=0}^{t} h(k)$. In the model by Granovetter, the number of people that joins the riots is given by $n^* = \max\{H(t) : k \leq H(k), \forall k \leq t\}$. The reason is that people with a certain threshold joins the riots when they observe that at least as many people as their threshold is taking part in it.

This process of collective behavior also shows that two very similar societies in their microstructure (preference distribution, as stated by $h(t)$) can generate very different aggregated behavior. Granovetter illustrates this fact showing two similar societies formed by 100 citizens.[12] In one of them each agent i has threshold i for $i \in \{1, 2, ..., 100\}$. The other society is exactly the same but agent i has threshold 2. In the first society, everyone takes part in the riots: the agent 1 starts, then the agent 2 (who observes 1 people revolting), then the agent 3 (who observes 2 people already revolting), and so on. In the second society, nobody revolts, since there exists no "initial demonstrator". Granovetter emphasizes two points: two societies which are basically identical can generate outcomes hugely different; and the key role that initial demonstrators play in generating social movements.

Based on Granovetter's assumptions, we study the effect of strategic behavior when the agents have information about the aggregate distribution of types. Connecting the case studied by Granovetter with our model, we assume that agents are called once to decide by Nature in some exogenously order and that the distribution of type $h(t)$ is common knowledge. Since in the model of Granovetter agents respond to the actions of their predecessors, we assume that all actions are known. In our terminology, we would say that a complete observation network Γ is connecting the agents. Under this circumstances, we ask how many agents take part in the revolts, and if there is some difference depending on whether initial agents called to decide are those with low or high thresholds. The following Proposition answers these questions:

Proposition 4 *For any order of decision, each agent i with a threshold $t_i \leq \max\{t : H(t) \geq t\}$ chooses the risky action on the path of any sequential equilibrium.*

[12] The original threshold model by Granovetter establishes decisions with respect to the proportion of the citizens that take part. We use a version were thresholds depend on the amount of agent that participate. Both models are equivalent.

Proof. *This Proposition follows from Lemma 1, for a unique risky action when there exists a complete observation network.* ■

In line with our results over coordination structures, the revolt emerges for any order in which agents are called to decide. When threshold distribution is known, all the agents take the risky action on the equilibrium path, independently of the order in which they are called to decide: in this sense, collective action is not necessarily initiated by those agents with the lowest threshold, while it is the case when distribution is unknown. Therefore, we can infer from the model that unexpected collective actions, that is, those that occur when people did not know about the existence of sufficiently many willing people, must be initiated by a set of initial "instigators" (as named by Granovetter), who start the process and that generate that people realize that there exist sufficiently many people willing to take part. But when it is known that there are sufficiently many people willing to take part, i.e. the existence of a certain distribution is known although people do not know which threshold each particular individual has, we show that social movements are initiated by any of the willing people, independently of her threshold.

Another important difference between ignoring or knowing the distribution is the number of people that take part in the revolts. When the distribution is unknown, people go to the streets only if there is a certain number of previous instigators, and the number of total people who participate is given by the first crossing point between the type distribution and the 45^o line, when we plot the distribution $H(t)$ against thresholds, as shown by Granovetter. This is the case because before that crossing point, people observe that an number of people higher than their threshold is participating in the revolt. But the people with higher thresholds do not observe that and stay at home. When distribution is known, all the people who know that in the society there are sufficiently many other people willing to take part in the revolts, would in fact take part (and even if they are the first to decide and do not observe anything). In the graph, the number of people would be given by the last point where distribution $H(t)$ where over the 45^o line. Figure 10 illustrates this with an example:

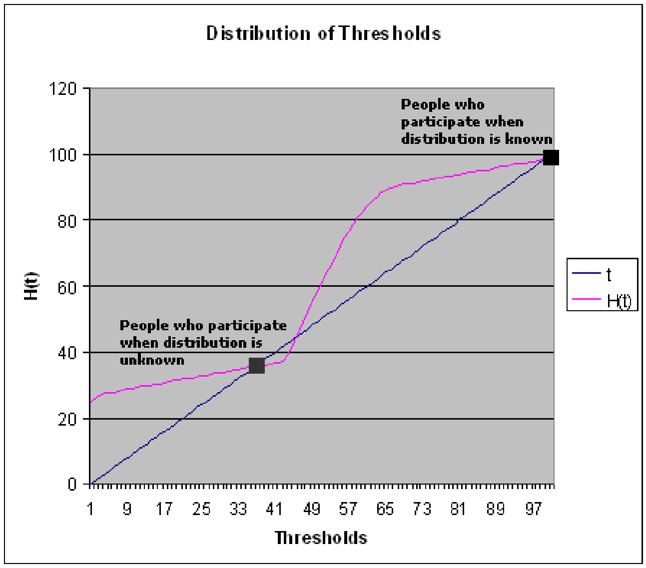


Figure 9. Example of distribution of thresholds and number of people who participate in the revolts when distribution is known or unknown.

5.3 Local banks may be immune to runs

In this section we adapt our general setup to the canonical model of depositor decision by Diamond and Dybvig (1983). We find how connected a given society must be in order to prevent bank runs as the result of coordination failures. The required clique is decreasing in the profitability of the long-term investment. This possibility of observing other depositors' actions is not likely in large banks but it is possible in local banks in a given community (such as some small thrifts in the US or Cajas Rurales in Spain). In such situation, it is likely that decisions on whether withdraw the money or not from the bank are observed in the local community. Therefore, since the community may act as an observation network that guarantees coordination, the result suggests that small local banks may be less likely to suffer bank runs as coordination failures, but they may be more susceptible to fundamental problems.[13]

5.3.1 The environment

Let $N = \{1, 2, ..., n\}$ denote the set of depositors. There are three time periods denoted by $t = 0, 1, 2$ and depositor i's consumption in period t is denoted by $c_{t,i} \in \mathbb{R}_+$. Depositors may be patient and impatient.

[13] Obviously, bank runs may occur as the result of problems with the fundamentals of the bank.

Impatient depositors only care about consumption at $t = 1$ whereas patient depositors value consumptions at $t = 1, 2$. Depositor i's utility function is given by

$$u_i(c_{1,i}, c_{2,i}, \lambda_i) = u_i(c_{1,i} + \lambda_i c_{2,i}). \tag{8}$$

If $\lambda_i = 1$ (0), depositor i is (im)patient. The utility is strictly increasing, strictly concave, twice continuously differentiable and satisfies the Inada conditions. The relative risk-aversion coefficient $-c_i u_i''(c_i)/u_i'(c_i) > 1$, for every $c_i \in \mathbb{R}_+$, and all $i \in N$.

The number of patient depositors is assumed to be constant and given by $p \in [1, n]$. The remaining depositors are impatient. The number of patient and impatient depositors is common knowledge.

At $t = 0$, each depositor $i \in N$ has one unit of a homogeneous good which she deposits in the bank. The bank has access to a constant-return-to-scale productive technology which pays a gross return of one unit for each endowment withdrawn at $t = 1$, and a fixed return of $R > 1$ for each endowment withdrawn at $t = 2$.

The bank acts in the interest of the depositors and tries to maximize their expected utility. If the bank could observe each depositor's consumption preferences, then she would be able maximize the sum of depositors' utilities with respect to $c_{1,i}$ and $c_{2,i}$ subject to a resource constraint and p. The optimization problem is the following:

$$\begin{array}{l} \max_{c_{1,i}, c_{2,i}} \ (n-p)u_i(c_{1,i}) + pu(c_{2,i}) \ \text{ s. t.} \\ (n-p)c_{1,i} + [pc_{2,i}/R] = n. \end{array} \tag{9}$$

The solution to this problem is

$$u_i'(c_{1,i}^*) = Ru_i'(c_{2,i}^*), \tag{10}$$

which - as in Diamond and Dybvig (1983) - implies that $R > c_{2,i}^* > c_{1,i}^* > 1$. This is the unconstrained efficient allocation. The bank insures against the privately observed liquidity risk, which is only realized at the beginning of $t = 1$, by offering a simple demand-deposit contract that implements the unconstrained efficient allocation. The simple demand-deposit contract offers to pay $c_{1,i}^*$ to any depositor i who withdraws at $t = 1$ as long as the bank has funds. Any patient depositor i who waits until $t = 2$ receives a pro rata share of the funds available then. Let $\eta \in [0, p]$ be the number of depositors who wait at $t = 1$. Given η, depositor i's consumption at $t = 2$, if she waits is

$$c_{2,i}(\eta) = \begin{cases} \max\{0, \frac{R(N-(N-\eta)c_{1,i}^*)}{\eta}\} \text{ if } \eta > 0 \\ 0 \text{ if } \eta = 0 \end{cases} \tag{11}$$

If $\eta = p$, that is, only impatient depositors withdraw at $t = 1$, then $c_{2,i}(\eta) = c^*_{2,i}$ and patient depositors enjoy a higher consumption than impatient ones. However, if η is too low, then to withdraw at $t = 1$ is better also for patient depositors since to wait until $t = 2$ yields them strictly less than $c^*_{1,i}$: if the number of patient depositors who keep the money in the bank is below $\bar{\eta}$, a threshold value for η, then their period-2 consumption is strictly below $c^*_{1,i}$.

Lemma 2 *There is* $1 \leq \bar{\eta} \leq p$ *such that for all* $i \in N$,

$$\begin{gathered} c_{2,i}(\bar{\eta}-1) < c^*_{1,i}, \text{ for any } \eta \leq \bar{\eta}-1, \text{ and} \\ c^*_{1,i} \leq c_2(\bar{\eta}), \text{ for any } \eta \geq \bar{\eta}. \end{gathered} \tag{12}$$

Based on the equality of $c^*_1 = c_2(\bar{\eta})$ we obtain that

$$\bar{\eta} = \frac{Rn(c^*_1 - 1)}{c^*_1(R-1)}.$$

The value of $\bar{\eta}$ is the threshold for patient depositors. Notice that this threshold is the same for all of them. Hence, in this application there are two types: $\tau(0, 1)$ and $\tau(1, \bar{\eta})$. That is, impatient depositors always choose to withdraw, independently of what other depositors do, whereas patient depositors prefer to wait if at least $\bar{\eta}$ other depositors wait.

5.3.2 Information and decisions

At the beginning of period 1 liquidity types (patient or impatient) are realized privately. Let $\Lambda^N = \{0, 1\}^N$ and $\lambda^N = (\lambda_1, ..., \lambda_N)$ denote the type vector of the depositors that satisfies $\sum_{i=1}^{N} \lambda_i = p$. After the realization of types, depositors contact the bank sequentially at $t = 1$ according to the order of decision given by $\theta(N)$.[14] The depositors are embedded in an *observation network* Γ. Depositors choose if they want to withdraw (action 0) or to keep their money deposited (action 1) and they observe the choices of their neighbors who precede them.[15] Depositor i's information set is defined as

$$\psi_i = \{\tau_i, \theta(N), \{a_j : ij \in \Gamma; \theta_j < \theta_i; \theta_j, \theta_i \in \theta(N)\}\}. \tag{13}$$

[14] We define here $\theta(N)$ only to conform to the original setup. Knowing the exact order of decision is not necessary to obtain the result.

[15] We use "to keep the money deposited" and "to wait" as synonyms. We assume that depositors observe this action following Green and Lin (2003).

Note that depositor i observes previous actions enabled by the network, but she does not observe types. A strategy is a mapping from all possible information set into actions. We allow for mixed strategies, so

$$s_i : \Psi_i \rightarrow \triangle \{0, 1\}.$$

The bank pays immediately to those who choose to withdraw. Consumption in period 2 is determined by equation 11.

The bank does not know the network and consequently cannot condition the payment to depositors on it. The bank has to respect the sequential service constraint, so the bank cannot make depositors wait and condition payment on information which is not available at the time the depositor wants to withdraw.

We find that bank runs as coordination failure do not occur in equilibrium if the depositors are sufficiently connected.

Proposition 5 *In the finite-depositor version of the Diamond-Dybvig model, if there is a clique of size at least* $(n-p) + \frac{Rn(c_1^*-1)}{c_1^*(R-1)}$, *then there is no bank run in any sequential equilibrium.*
Proof. *The result follows from Theorem 1, given that* $\frac{Rn(c_1^*-1)}{c_1^*(R-1)}$ *is the threshold such that a patient agent prefers the risky action of not to withdraw over the safe one.* ■

The efficient payoffs that generate the multiplicity of equilibria allowing for a bank run, imply the coordination in the no run equilibrium if the depositors are sufficiently connected. This result shows that small, local banks, whose depositors are able to observe each other, should not fall prey to bank runs as the result of coordination failures. Deposit insurance has been proposed as one of the most effective mechanisms to avoid this undesirable bank runs. Our result suggests that small, local banks, may be less likely to suffer them just because of the social configuration of their depositors.

6 Conclusion

We introduce in this paper the notion of *observation network* to model the type of social structure that allows agents to observe each others' actions. We characterize the structure of such networks that yield that agents coordinate on the efficient equilibrium in games related to the generalization of the stag-hunt game. We provide necessary and sufficient conditions on the size of the clique such that, if it exists, the efficient coordination emerges as the unique outcome, and the social network is considered a *coordination structure*. We apply our model to revolutions and bank runs, but it also applies to any other situation where coordination failures may emerge, as problems of product adoption. We find that the existence of cliques play a crucial role in guaranteeing the coordination of the agents, and so our analysis naturally complements the one by Chwe (1999, 2000).

We study situations in which it is known that coordination is possible, so the problems that we study are pure problems of coordination. With this aim, we focus on the case in which the number of agents of each type is known. All agents know that efficient coordination is a possible outcome, and the networks that we characterize avoid the problem of coordination failure. A different approach would occur if we assumed that types are randomly drawn from some distribution. In this situation, agents would not know with certainty if there are sufficiently many agents for getting coordination. The role of an observation structure in such circumstances is out of the scope of this paper, but is an interesting line of future research.

The unique equilibrium prediction when the observation network meets our conditions is an issue that can be tested. We have carried out some related experiments for the bank run environment (Kiss, Rodríguez-Lara and Rosa-García, 2009) that provide mixed evidence on the issue: although some structures generate more coordination, agents are affected also by the particular observations. However, more evidence is required to be sure on the effects of observation networks on the behavior.

The model invites us to explore many issues that we have not discussed in this article. Although our agents decide sequentially, they decide just once, so dynamic considerations are not studied and they play relevant roles in many coordination issues, as those that we study (bank runs and revolts). In this context, it is also likely that social learning plays a role in the emergence of the efficient equilibrium. Another interesting issue is to study the incentives to form the network. Would agents choose a social structure that may lead to a coordination failure? And if so, under which conditions?

Moreover, some of our assumptions are too restrictive. For instance, some of the actions in real life may be only observed if they are chosen. In the case of bank runs, typically the action of withdrawing is observed but the action of waiting is not. In the case of revolts, the action that is observed is typically the one of taking part, but not staying at home. Note that in the first case the observed action is the safe one and in the second is the risky one. Other issue is what occurs with other coordination problems, as for instance for generalizations of the battle of sexes. Also free-riding may be relevant: how would the results change if once the sufficient number of people chooses the risky actions, the rest of agents does not have incentives to take it.

These related issues are of special interests. We require in our analysis that agents have access to a large number of information, and to be able to observe the precise order in which actions are chosen. New internet social networks, such as Facebook and Twitter are a way of communication and they allow to show own actions to other agents, so that each time more individuals are informed about the actions carried out by other individuals. This can generate several effects, including those that our results postulate, that coordination on efficient situations may be facilitated. We have done a first approach to this kind of topics (Kiss and Rosa-García, 2011) analyzing how the coordination is facilitated when agents get information through internet social media in contrast with traditional social media. We find that social media are a better way of coordination and may facilitate revolutions, as it has been argued in the recent Arab revolts. Up to which point this is really a relevant effect and if its relevance can be applied to the related extensions of the analysis of coordination structures is still an open question.

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8 Appendix

Lemma 1: An agent of type $\{\alpha, \beta, t'\}, t' \leq t$ takes the risky action on any equilibrium path if there is a clique of size $n - K(\alpha, t) + t$

Proof. Suppose that there is a clique q of size $\#q = n - K(\alpha, t) + t$.

A history up to agent i is defined by the order of decision (θ), a type for each agent τ, and the sequence of actions chosen by the predecessors of i, $a^i = \{a_{\theta_k} : \theta_k \in [1, 2, ..., \theta_i - 1]\}$.

From now on and without loss of generality, we focus on agents that belong to the clique and name them according to their order of decision among the individuals in the clique. Therefore, agent i is the agent who decides in the i^{th} position among those that belong to the clique. For simplicity, we say that an agent j is of type $\bar{\tau}$ if $\tau_j = \{\alpha, \beta, t''\}, t'' \leq t$.

The information set of agent i is formed by the type of i and the actions that she observes. For $i \in q$, we define as $a^{i,q}$ the actions observed by i chosen by agents in the clique and as $a^{i,-q}$ the actions observed by i chosen by agents who do not belong to the clique. The information set of i is given by $\varphi_i = \left\{\tau_i, \left\{a^{i,-q}\right\}, \left\{a^{i,q}\right\}\right\}$, where $a^{i,-q} = \{a_j : j \in N_i, \theta_j < \theta_i, j \notin q\}$ and $a^{i,q} = \{a_j : j \in N_i, \theta_j < \theta_i, j \in q\}$.

Let define as $\bar{\Phi}(\sigma)$ the set of information sets such that

$$\varphi_i \in \bar{\Phi}(\sigma) \Leftrightarrow \exists r : \begin{cases} i \in q \ \ (1) \\ \tau_i = \bar{\tau} \ \ (2) \\ \#\{a_j = \alpha : j \in q, j < i\} = r \ \ (3) \\ \#q - i \geq t - r - 1 \ \ (4) \\ \sigma_j(\varphi_j) = \alpha, \forall \varphi_j : \left\{j \in q, j < i, \left\{a^{j,q} | \varphi_i\right\} \in \varphi_j, \tau_j = \bar{\tau}, a_j \neq \alpha\right\} \ \ (5) \\ \exists h_i \rightsquigarrow \varphi_i : P(a_j|\sigma) > 0, \forall \{a_j \in h_i : a_j \neq \alpha \cup j \notin q\} \ \ (6) \end{cases}$$

Let us briefly explain how are the information sets included in $\bar{\Phi}(\sigma)$. In those information sets, agent i is of type $\bar{\tau}$ and observes a number r of actions α chosen by predecessors in the clique. After her there are in the clique more agents ($\#q - i$) than the risky actions required in order to obtain her first best if she chooses the risky action ($t - r - 1$). Moreover, given the strategy σ, any of the safe actions observed by i in the clique cannot be chosen by an agent of type $\bar{\tau}$, because any predecessor j of i, if is of type $\tau_j = \bar{\tau}$, would have chosen α after observing the sequence of actions in the clique in any possible information set. Finally, there exists a history such that the actions observed out of the clique or in the clique which differ from α are chosen with positive probability given σ.

Note that in these information sets any consistent belief assigns probability 0 to any history in which the actions different from α are chosen by agents of type $\bar{\tau}$. If φ_i occurs with positive probability (i.e., φ_i is on the equilibrium path) the actions different from α cannot be chosen by agents of type $\bar{\tau}$ given (5). If φ_i occurs with probability 0, given that we are using the concept of sequential equilibrium, the beliefs must be dynamically consistent with σ. The condition (6) requires the existence of a history in which those actions which do not correspond to actions α observed in the clique are chosen with positive probability by agents from a type different of $\bar{\tau}$. Because of condition (5), moreover, any agent of type $\bar{\tau}$ in the clique would have chosen α in any possible information set consistent with φ_i. These two conditions together imply that there exists no history with positive belief in which there are agents of type $\bar{\tau}$ choosing any of the actions different from α observed by i in the clique. Therefore, any possible belief consistent with σ assigns probability 0 to those actions being chosen by agents of type $\bar{\tau}$. The maximum number of agents of type $\bar{\tau}$ who are in the clique and have already chosen equals r, the number of actions of type α observed by i. But it implies that there are $(N - F(\bar{\tau}) - i + r + 1)$ or less agents of a type different from $\bar{\tau}$ after i in the clique. This means that i has at least $t - r - 1$ successors of type $\bar{\tau}$ in the clique with probability 1.

Now we prove that, if agent i is in an information set $\varphi_i \in \bar{\Phi}(\sigma)$ with $r \leq t-2$ and she chooses α, she expects with probability 1 that the next agent of type $\bar{\tau}$ will be also in an information set that belongs to $\bar{\Phi}(\sigma)$ for any equilibrium strategy.

Let j be the following agent of type $\tau_j = \bar{\tau}$. For simplicity, let be j the position of the agent among those in the clique. The agent i has $N - F(\bar{\tau}) + t - i$ successors in the clique. With probability 1 she believes that at least $t-r-1$ are of type $\bar{\tau}$. Therefore, she believes with probability 1 that $j \in [i+1, N - F(\bar{\tau}) + r + 2]$.[16] Suppose that i chooses α in $\varphi_i \in \bar{\Phi}(\sigma)$. [17] Note that i expects that for j the conditions 1 to 4 that define an information set that belongs to $\bar{\Phi}(\sigma)$ hold with probability 1. First, take into account that j will observe at least $r' = r+1$ actions α in the clique. Therefore:

1. $j \in q$

2. $\tau_j = \bar{\tau}$

3. $\#\{a_{j'} = \alpha : j' \in q, j' < j\} = r' \geq r+1$

4. $\#q - j \geq \#q - (N - F(\bar{\tau}) + r + 2) = N - F(\bar{\tau}) + t - (N - F(\bar{\tau}) + r + 2) = t - r - 2 \geq t - r' - 1$

Suppose now that σ is an equilibrium strategy. Suppose, moreover, that agent k is in an information set $\varphi_k \in \bar{\Phi}(\sigma)$ and that $\sigma_{k'}(\varphi_{k'}) = \alpha, \forall k' > k, \varphi_{k'} \in \bar{\Phi}(\sigma)$. Then, if k plays α, the following agent of type $\bar{\tau}$ is expected to be in an information set of type $\bar{\Phi}(\sigma)$ with probability 1. Note that in this case, conditions (5) and (6) also hold:

5. $\sigma_k(\varphi_k) = \alpha, \forall \varphi_k : \left\{k \in q, k < j, \left\{a^{k,q} | \varphi_j\right\} \in \varphi_k, \tau_j = \bar{\tau}, a_j \neq \alpha\right\}$ (5), this holds if k chooses α, even if $\sigma_k(\varphi_k) \neq \alpha$, since if k chooses α her action is not required to satisfy this condition.

6. $\exists h_i \rightsquigarrow \varphi_i : P(a_j|\sigma) > 0, \forall \{a_j \in h_i : a_j \neq \alpha \cup j \notin q\}$, which still holds for k' if it holds for k, since that history must exist for those actions different from α observed in the clique.

Therefore, we have proved that, if individual k is in an information set $\varphi_k \in \bar{\Phi}(\sigma)$ and k chooses α, and moreover $\sigma_{k'}(\varphi_{k'}) = \alpha, \forall k' > k, \varphi_{k'} \in \bar{\Phi}(\sigma)$, then she expects with probability 1 that the following

[16] Note that given (3) and that $r \leq t-2$, we have that $N - F(\bar{\tau}) + r + 2 \geq i + 1$.

[17] Importantly, we are not saying that $\sigma(\varphi_i) = \alpha$. So here we are also studying the case of a possible deviation of i with respect to σ.

individual of type $\bar{\tau}$ is also in an information set of type $\bar{\Phi}(\sigma)$. We prove now that that in an equilibrium strategy in such a situation, agent k plays α. Suppose that $\sigma_k(\varphi_k) = \alpha$. Agent k has no profitable deviation: the following agent of type $\bar{\tau}$ is expected to choose α and there are enough agents who, with probability 1, are expected to choose α, so that the expected payoff is the maximum one (more than t' individuals choose α with probability 1). It is not possible to sustain that $\sigma_k(\varphi_k) \neq \alpha$, since there is a profitable deviation: if k chooses α the following individuals of type $\bar{\tau}$ are expected to be in information sets belonging to $\bar{\Phi}(\sigma)$, who by assumption choose α, and therefore k would increase her payoff by deviating.

But note that in an equilibrium strategy, $\sigma_{k'}(\varphi_{k'}) = \alpha, \varphi_{k'} \in \bar{\Phi}(\sigma), \forall r > t-2.$, so that by backward induction we have that, for any agent i who is in an information set $\varphi_i \in \bar{\Phi}(\sigma)$, she believes with probability 1 that the following individual of type $\bar{\tau}$ is in an information set of type $\bar{\Phi}(\sigma)$ if she chooses α and σ is an equilibrium strategy. Under this circumstances, we have also proved that the best response in such an information set is to choose α. So that, in equilibrium, we have that $\sigma_i(\varphi_i) = \alpha, \forall \varphi_i \in \bar{\Phi}(\sigma)$.

The question now is if the set $\bar{\Phi}(\sigma)$ is empty or not. But take any equilibrium path and the first individual of type $\bar{\tau}$ who chooses in the clique. Her information set belongs to $\bar{\Phi}(\sigma)$ since $(1-6)$ hold. Therefore, her successors of that type will play with probability 1 the action α. Since there are in the clique at least t individual of that type, it is a best response to any agent of type $\bar{\tau}$ to choose the action α, independently of if she belongs to the clique or not. *Q.E.D.* ■

Theorem 1: An agent of type $\{\alpha, \beta, t\}$ takes the risky action on any equilibrium path if there exists a clique $q \subset N$ of size $\#q = q(t)$ where

$$\begin{aligned} q(t) &= q_\alpha(r_\alpha(t)), \\ \text{where } r_\alpha(x) &= \min\{N - K(\alpha, t) + t : \forall t \geq x\} \\ q_\alpha(x) &= \begin{cases} r_\alpha(x) \text{ if } r_\alpha(y) < x \\ r_\alpha(y) \text{ if } r_\alpha(y) \geq x \end{cases}, y = \max\{t : y < x\} \end{aligned}$$

where $q_\alpha(x)$ is constructed iteratively from the lowest threshold x for the action α.

Proof. Take an agent of type t. Lemma 1 implies that any agent with threshold t chooses also the risky action r if there exists a clique of size $r_\alpha(t) = \min\{N - K(\alpha, t) + t : \forall t \geq x\}$ (note that it is the smallest size of clique required for thresholds higher than t). Note that if the agents with the threshold immediately lower to t (that we note by t_{-1}) are $K(t_{-1}) \geq t$, any agent with threshold t takes the risky action if there exists a clique sufficiently as large as required by agents of threshold t_{-1}. This possibility is incorporated by $q_\alpha(x)$. ■

Proposition 1: In the homogenous case with threshold t, an observation network Γ is a coordination structure if and only if there exists a subset of agents $\{s \in N : \#s = t\}$ that forms a clique.

Proof. Sufficiency: An observation network Γ is a coordination structure if there exists a subset of agents $\{s \in N : \#s = t\}$ that forms a clique.

We prove this by backward induction. Let $\{\sigma^* (\varphi), \Pi_i (H_i|\varphi)\}$ be an assessment that defines a sequential equilibrium. Let $\{q \subset N : \#q = t\}$ be a set of t agents completely connected under Γ, i.e. a clique of size t. From now on, agent $i, i = 1, 2, .., t$ is the agent who is in the i^{th} position in the clique according to $\theta(n)$. In any sequential equilibrium assessment $\{\sigma^* (\varphi), \Pi_i (H_i|\varphi)\}$, the agent $i = t$ chooses α in any information set φ_t in which she observes $t-1$ risky actions by her predecessors in the clique. Agent in position $i = t-1$ best responds by choosing α in any information set φ_{t-1} where she observes $t-2$ risky actions by her predecessors in the clique. Applying this idea recursively, agent i in the clique best responds to the equilibrium strategy by choosing α in those information sets φ_i where she observes $i-1$ risky actions chosen by her predecessor in the clique. Then the agent $i = 1$ chooses the risky action although she does not observe anything. And thus, in any sequential equilibrium, the t agents in the clique choose the risky action on the equilibrium path. Any agent who does not belong to the clique best responds by also choosing α. Therefore, if there exists a clique of size t, every agent chooses her risky action on the equilibrium path.[18] Q.E.D.

Necessity: An observation network Γ is a coordination structure only if there exists a subset of agents $\{s \in N : \#s = t\}$ that forms a clique.

For proving this part of the result, we propose a strategy and a consistent belief that define a sequential equilibrium where the strategy profile implies that all the agents choose the safe action on the equilibrium path.

First, let define $Q_{\Gamma,k} \subset P(N)$ where $P(N)$ is the power set of N, as the set of cliques of size k in Γ:

$$q \in Q_{\Gamma,k} \Longleftrightarrow \begin{cases} q \subset N \\ \#q = k \\ \forall i, j \in q \rightarrow ij \in \Gamma \end{cases}$$

For the information set φ_i of the agent i, we define $Q^-_{\Gamma,k,\varphi_i} \subset Q_{\Gamma,k}$ as the set of cliques of size k in Γ such that the agent i does not observe the action of any agent belonging to the clique,

$$q \in Q^-_{\Gamma,k,\varphi_i} \Longleftrightarrow \begin{cases} q \in Q_{\Gamma,k} \\ \forall j \in q \rightarrow j \in \{\{N \backslash N_i\} \cup \{j' \in N_i : \theta_j \geq \theta_i|\varphi_i\}\} \end{cases}$$

We define $N_{i,<\theta_i,\varphi_i} \subset N_i$ as the set of neighbors of i with an order of decision previous to i given the

[18] The strategy in equilibrium implies to choose the risky action over the equilibrium path for arbitrary beliefs. This means that, in fact, we have proved also that there is a unique equilibrium path in any Weak Perfect Bayesian Equilibrium, which is a much softer equilibrium concept.

information set φ_i,

$$j \in N_{i,<\theta_i,\varphi_i} \Leftrightarrow j \in N_i \cap \{j : \theta_j < \theta_i | \varphi_i\}$$

For the information set φ_i and the clique $q \in Q^{-}_{\Gamma,k,\varphi_i}$, we define as $\#\alpha_{q,\varphi_i}$ the number of actions of type α that belong to φ_i, and that are taken by agents who are neighbors of all members of q, plus the cardinal of q:

$$\#\alpha_{q,\varphi_i} = \#q + \#\left\{a_j = \alpha : \left\{k \in q, j \in N_{i,<\theta_i,\varphi_i}\right\} \rightarrow jk \in \Gamma\right\}$$

For the agent i, we define as $\bar{\varphi}_i^r$ the set of information sets in which the maximal $\#\alpha_{q,\varphi_i}$ is r:

$$\varphi_i \in \bar{\varphi}_i^r \Leftrightarrow \max\left\{\#\alpha_{q,\varphi_i}\right\} = r$$

We prove now that $\sigma^*(\varphi), \Pi^*(H|\varphi)$ where

$$\begin{aligned}
\sigma_i^*(\varphi_i) &= \begin{cases} \beta \text{ if } \varphi_i \in \bar{\varphi}_i^r : r < t \\ \bar{\sigma}_i(\varphi_i) \text{ if } \varphi_i \in \bar{\varphi}_i^r : r \geq t, \text{ for some optimal } \bar{\sigma}_i \end{cases} \\
\Pi_i^*(h_i|\varphi_i) &= \begin{cases} 0 \text{ if } h_i : \#\{a_j \in h_i : a_j = \alpha, j \notin N_i\} > 0 \text{ and } \varphi_i \in \bar{\varphi}_i^r : r < t \\ \bar{\pi} \text{ otherwise} \end{cases}
\end{aligned}$$

defines a sequential equilibrium for an appropriate $\bar{\sigma}_i(\varphi_i), \Pi_i(h_i|\varphi_i)$ in which the safe action is played on the equilibrium path when when there is no clique of size t in Γ.

We show, *first*, that this strategy defines a path where everybody takes the safe action. If there is no clique of size t, any agent who does not observe any risky action is in an information set $\bar{\varphi}_i^r < t$. This is the case because in such case we have that $\#\left\{a_j = \alpha : \forall k \in q, j \in N_{i,<\theta_i,\varphi_i} \rightarrow jk \in \Gamma\right\} = 0$ and $\#\alpha_{q,\varphi_i} = \#q$, which is smaller than t, for any clique $q \subset \Gamma$. Therefore any agent who does not observe any action α plays β according to $\sigma^*(\varphi), \Pi^*(H|\varphi)$, and therefore β is played by everyone along the equilibrium path.

Second, note that $\varphi_i \in \bar{\varphi}_i^r : r < t$ are information sets where the agent observes $t-2$ or less actions of type α. This is the case because the agent herself is a clique of size 1 who observes all the actions she observes, and therefore these information sets occur when a maximum of $r-1 < t-1$ actions of type α are observed. Choosing β in those information sets is an optimal decision if it is expected that nobody else is going to play α and nobody else has played it (this last statement occurs with probability 1 according to Π^*).

Third, we show that the strategy is a best response to $\sigma^*_{-i}(\varphi)$ given $\Pi^*(H|\varphi)$. If the agent i plays the safe action her expected payoff is

$$E_{h_i}\left(u_i\left(\beta, \sigma^*_{-i}\right) | \Pi_i^*(h_i|\varphi_i), \varphi_i \in \bar{\varphi}_i^r : r < t\right) = u_i(a_i = \beta)$$

We focus on the information sets $\varphi_i \in \bar{\varphi}_i^r : r < t$. In those information sets, the beliefs assign probability 0 to other actions of type α being chosen by predecessors of i who have not been observed. Take now the agent j who plays after i. We show that she will play β according to $\sigma^*(\varphi), \Pi^*(H|\varphi)$. Suppose that $r \leq t-2$. Since i is in an information set $\varphi_i \in \bar{\varphi}_i^r : r < t$ and she believes that there is no predecessor who has played α and is not observed by i, any successor j is, at maximum, in an information set $\varphi_j \in \bar{\varphi}_j^r, r = t-1$, and plays β. Suppose that $r = t-1$. Here we can have 2 different cases, depending on whether i belongs to the clique, $i \in q$, such that any agent in the clique observes the same $r - \#q$ actions or i may not belong to it, $i \notin q$. Suppose that $i \notin q$. In such a case, the first agent who decides, j, in that clique will be in an information set $\varphi_j \in \bar{\varphi}_j^r : r < t$, since not all the agents in the clique observe the action of i (since $i \notin q$). Finally, suppose that $i \in q$. But then, since $\varphi_i \in \bar{\varphi}_i^r : r = t-1$ and $i \in q$, if i chooses α, the rest of agents in q will be still in an information set $\varphi_j \in \bar{\varphi}_j^r : r = t-1$. This is the case because they observe one risky action more than i but the clique has one individual less. Therefore, they would play β according to $\sigma^*(\varphi), \Pi^*(H|\varphi)$. This means that given the beliefs and the strategies, if the agent plays α nobody else is expected to play α and the expected payoff is

$$\begin{aligned} E_{h_i}\left(u_i\left(\alpha, \sigma_{-i}^*\right)|\Pi_i^*\left(h_i|\varphi_i\right), \varphi_i \in \bar{\varphi}_i^r : r < t\right) &= u_i\left(a_i = \alpha, \sum_{j \in N} I_{a_j = \alpha} < t\right) < \\ &< u_i\left(a_i = \beta\right) \end{aligned}$$

And therefore playing β is a best response to $\bar{\sigma}_{-i}, \Pi_i\left(h_i|\varphi_i\right)$ in the information sets $\varphi_i \in \bar{\varphi}_i^r : r < t$.

Fourth, the belief is dynamically consistent in the information sets $\varphi_i \in \bar{\varphi}_i^r : r < t$. Suppose that any agent j in an information set $\varphi_j \in \bar{\varphi}_j^r : r < t$ plays the completely mixed strategy $(\varepsilon, 1-\varepsilon)$, representing the probability of playing (α, β). Note that this strategy converges to $\sigma_j^*\left(\bar{\varphi}_j^r\right)$ when $\varepsilon \to 0$. Since the first agent who decides is in an information set $\varphi_j \in \bar{\varphi}_j^r : r < t$, and that all agents play β the subsequent agents are also in an information set of type $\bar{\varphi}^r : r < t$, the belief that assigns probability 0 to any non-observed agent having chosen β is consistent with those strategies.

Up to now, we have proved that the strategy is optimal over all the information sets in which $\varphi_i \in \bar{\varphi}_i^{k,m} : k < t - m$. In the other information sets, the agents would choose any best response; however, optimality of the equilibrium strategy on the equilibrium path is independent of what occurs in those information sets. Note that the existence in those information sets of the equilibrium is guaranteed by standard arguments. Since we allow for mixed strategies, the equilibrium in those information sets could be the same that in a reduced version of the game where Nature chooses directly α instead of the agents in the information sets $\varphi_i \in \bar{\varphi}_i^r : r < t$.

Thus, a coordination failure may be sustained, *Q.E.D.* ■

Proposition 2: In any sequential equilibrium, any agent of type α chooses α on the equilibrium path, for any order of decision, if there is an isolated clique of size $q \subset N, \#q = t$ if

$$\frac{u_{\alpha,\geq t} - u_{\alpha,<t}}{u_\beta - u_{\alpha,<t}} > \frac{(n_\alpha + n_\beta - 1)!\,(n_\alpha - t - 1)!}{(n_\alpha + n_\beta - t - 1)!\,(n_\alpha - 1)!}$$

Proof. Suppose that

$$\frac{u_{\alpha,\geq t} - u_{\alpha,<t}}{u_\beta - u_{\alpha,<t}} > \frac{(n_\alpha + n_\beta - 1)!\,(n_\alpha - t - 1)!}{(n_\alpha + n_\beta - t - 1)!\,(n_\alpha - 1)!}$$

Let us define

$$\tilde{p} = \frac{(n_\alpha + n_\beta - t - 1)!\,(n_\alpha - 1)!}{(n_\alpha + n_\beta - 1)!\,(n_\alpha - t - 1)!}$$

Then we have that

$$\begin{aligned} &\rightarrow \quad (u_{\alpha,\geq t} - u_{\alpha,<t}) \cdot \tilde{p} > u_\beta - u_{\alpha,<t} \\ &\rightarrow \quad \tilde{p} \cdot u_{\alpha,\geq t} + (1 - \tilde{p}) \cdot u_{\alpha,<t} > u_\beta \end{aligned}$$

and let $\{\sigma^*(\varphi), \Pi^*(H|\varphi)\}$ be an assessment that defines a sequential equilibrium. $\Pi_i(H_i|\varphi)$ are dynamically consistent and therefore they are obtained from $\sigma^*(\varphi)$ whenever possible. The probability that there are t agents of type α in a clique of size q, $\#q = t$, is given by

$$P(\tau_j = \alpha, \forall j \in q) = \frac{(n_\alpha + n_\beta - t - 1)!\,(n_\alpha - 1)!}{(n_\alpha + n_\beta - 1)!\,(n_\alpha - t - 1)!} = \tilde{p}$$

For simplicity, we name from now on agents in the clique q with her position in the sequence selected by Nature among the agents in the clique. So agent $i = 1$, $1 \in q$ is the first agent who decides in the clique and agent $i \in q$ is the i^{th} who decides in the clique. The probability of having a sequence of $t - i$ agents of type α in the last $t - i$ positions in the clique, conditional on the first i^{th} individuals being of type α is given by

$$\begin{aligned} \tilde{p}_i &= \frac{((n_\alpha - i) + n_\beta - (t - i) - 1)!\,((n_\alpha - i) - 1)!}{((n_\alpha - i) + n_\beta - 1)!\,(n_\alpha - i - (t - i) - 1)!} = \\ &= \frac{(n_\alpha + n_\beta - t - 1)!\,(n_\alpha - 1)!}{(n_\alpha + n_\beta - 1)!\,(n_\alpha - t - 1)!} = \tilde{p} \end{aligned}$$

Agent t in the clique, if she observes $t-1$ predecessors choosing α, chooses also α in any equilibrium if she is of type $\tau_t = \alpha$. Suppose that the agent $t - 1 \in q$ has a belief $\tilde{\pi}_{t-1}$ such that the probability of being followed by an agent of type α is $p_\alpha^{t-1} \geq \tilde{p}$. Her best response $\varphi_{t-1} = \tilde{\varphi}_{t-1}$ to an information set where she observes $t - 2$ actions of type α by her predecessors conditional on π_{t-1} is α, since her expected payoff of choosing α is

$$\begin{aligned} Eu\left(\alpha, \tilde{\varphi}_{t-1}, \tilde{\pi}_{t-1}\right) &\geq p_\alpha^{t-1} \cdot u_{\alpha,\geq t} + \left(1 - p_\alpha^{t-1}\right) \cdot u_{\alpha,<t} \\ &\geq \tilde{p} \cdot u_{\alpha,\geq t} + (1 - \tilde{p}) \cdot u_{\alpha,<t} \geq u_\beta \end{aligned}$$

Take an agent $j \notin q$ of type α who does not observe anything, apart from her type $\varphi_j = \{\tau_j = \alpha\}$. The system of equilibrium beliefs Π^* must be consistent applying Bayes rule to the equilibrium strategies $\sigma^*(\varphi)$. A best response of agent $j \notin q$ who does not observe anything but her type must respond assigning probability $\tilde{p}$ to the event of agent $i \in q$ in the clique being in an information set such that she is of type α, she observes $i-1$ actions of type α and is followed by $t-i$ agents of type α. This is the case because Bayes rule requires that beliefs are consistent with the strategies, and therefore, the ex-ante probability must be consistent with the system of beliefs. Hence, agent $j \notin q$ assigns probability $\tilde{p}$ to the event of all agents in the clique being of type α and choosing it according to $\{\sigma^*(\varphi), \Pi^*(H|\varphi)\}$. The best response ogr the agents who do not observe anything is therefore α if they are of such type, since $\tilde{p} \cdot u_{\alpha,\geq t} + (1-\tilde{p}) \cdot u_{\alpha,<t} \geq u_{\beta}$. Agents who observe some actions must assign probability 0 in her consistent beliefs to the event of an action which is not α being chosen by an initial agent of type α. This means that posterior agents either assign a higher probability to the event of being there t agents in the clique choosing α (if they do not observe actions of type α) or require less agents in the clique being of type α (if they observe actions of type α). And therefore, in equilibrium, any agent who does not observe actions in the clique best responds by choosing α. Since the clique is isolated, every agent chooses α on the equilibrium path and agents in the clique best respond to it by also choosing α, in any information set. ■

Proposition 3: Suppose a set of agents N embedded in a sufficient network Γ such that everyone is willing to revolt in the sense of Chwe (2000). Suppose that Nature selects an order of decision according to $P(\theta(N))$ and that if $ij \in \Gamma$, a_i is in the information set of j if $i < j$. Then every agent chooses to revolt in the path of any weak perfect Bayesian equilibrium.

Proof. We briefly state the model by Chwe (2000). There is a finite set of agents $N = \{1, 2, ..., n\}$. There are two types, w (willing to revolt) and x (unwilling). Each person i chooses an action $a_i \in \{r, s\}$ that states for revolting (r, the risky action) and not (s, the safe action). Utility of each person depends on own type and the full profile of actions. If a person is of type x, action s is a dominant strategy. If a person is of type w, her utility is supermodular, i.e., the difference in utility between r and s is increasing in the number of people who chooses to revolt. A person of type w prefers the action r if everyone else is choosing r. A network Γ is a collection of pairs such that if $ji \in \Gamma$ it means that person j talks to i, i.e. i knows the utility function of j. Γ is common knowledge. Γ is a sufficient network if when everyone is willing to revolt then there exists an equilibrium where everyone revolt, regardless of the belief over the type of unobserved agents. This means that there exists an equilibrium where everyone revolts even if the agent believes that all those non-observed agents are of type s. Let t_i be such that the agent i gets a higher utility by choosing r than s if at least $t_i - 1$ other agents choose r. Suppose that Nature calls to decide to the agents according to $P(\theta(N))$ and that agent i observe the action chosen by j if and only if $ji \in \Gamma$ and $\theta_j < \theta_i$.

Chwe shows that, if Γ is sufficient, there is a sequence of cliques that cover N. These cliques are hierarchically connected such that there is a leading clique in which agents talk to each other, and everyone talks to the following cliques, and so on. Agents in the leading clique best responds to the rest of the agents in the leading by choosing r if they choose r. This means that the clique contains sufficiently many people willing to take r. Since agents in the clique know that because they talk to each other and know their types, by Proposition 1 they choose r in any order of decision. The clique in the following position is formed by agents that are talked by all agents in the leading clique, and therefore know the types in the leading clique and in the following clique. They best respond to agents in the leading clique and to the agents in the following clique by choosing r if they choose r. Since in equilibrium all the agents in the leading clique choose r and there are sufficiently many agents in the following clique for making r optimal, we can apply again Proposition 1 to the agents in this clique, conditioning on the fact that agents in the leading clique choose r. We can apply recursively this argument for all agents in N to show that every of them chooses r for any $P(\theta(N))$. *Q.E.D.* ■

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