



WP-AD 2011-24

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Edita / Published by: Instituto Valenciano de Investigaciones Económicas, S.A.

Depósito Legal / Legal Deposit no.: V-4185 -2011

Impreso en España (diciembre 2011) / Printed in Spain (December 2011)

Optimal tax enforcement under prospect theory^{*}

Amedeo Piolatto and Gwenola Troitin^{**}

Abstract

Prospect Theory (PT) has become the most credited alternative to Expected Utility Theory (EUT) as a theory of decision under uncertainty. This paper characterizes the optimal income tax and audit schemes under tax evasion, when taxpayers behave as predicted by PT. We show that the standard EUT results keep holding under PT, under even weaker conditions. Under fair assumptions on the reference income and on the utility function of taxpayers, we show that the optimal audit probability function is non-increasing and the optimal tax function is nondecreasing and concave.

Keywords: Tax evasion; Income tax enforcement; Prospect theory.

JEL codes: D81; H26; K42.

* For their comments, we are grateful to Jukka Pirttilä, Matthew Rablen, Matti Tuomala, Alain Trannoy and to the anonymous referees. Piolatto acknowledges financial support from IVIE, the Spanish Ministry of Science and Innovation (grant ECO2009-12680), the Barcelona GSE Research Network and the Generalitat de Catalunya (grant 2009 SGR 102).

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1. Introduction

In most countries, income tax administrations rely on income self-reports from taxpayers. Thus, they have an incentive to misreport their income, in order to reduce their income tax liability. Losses to public budgets from tax evasion are indeed significant: the US Internal Revenue Service (IRS), for example, estimates the tax gap in 2001 at USD 345 billion, i.e., almost 16% of the total tax revenue (IRS, 2006). The main tools in the hand of the tax administration to limit possible misbehaviour are to audit taxpayers and to verify the information provided. Audits being costly,¹ the tax administration generally selects the reports to be audited. Assessed misreporting can result in penalties and fines; the setting up of penalty schemes also satisfies an objective of both horizontal and vertical equity among taxpayers. On top of tax rates, an optimal tax policy includes therefore an audit strategy and a scheme of penalties.

This gives rise to interesting questions, that we aim at analysing, about the optimal audit strategies and penalty schemes, and the nature of interactions between tax rates and audit strategies. For that, we need to take into account taxpayers' attitude toward risk and uncertainty. Although the Expected Utility Theory (EUT) has been considered for long time the most convenient framework, there is a growing consensus about the need of an alternative theory of the agents' behaviour under uncertainty.² Pioneered by Kahneman and Tversky (1979), prospect theory (PT) has become one of the most prominent alternatives to EUT, and it is widely used in empirical research.³ According to prospect theory, the carrier of utility is the difference between the final level of income and a determined reference income, and not the final level of income (as suggested by EUT). Agents think of gains and losses relative to this reference point: this phenomenon is known in cognitive sciences as the *framing effect*.⁴ The utility function, convex for gains and concave for losses, expresses the *loss aversion* phenomenon: individuals care generally more about potential losses than potential gains. Prospect Theory is nowadays commonly used in cognitive sciences and has become one of the standards in the Behavioural Economics literature.

The recent literature on taxation highlights problems in using the EUT setting for tax evasion decision issues, because it contradicts the empirical evidence in several ways. In particular, with a reasonable degree of risk aversion, it overestimates the willingness of agents to misreport their income, therefore, it predicts more tax evasion than what really occurs. Furthermore, under the assumption of Decreasing Absolute Risk Aversion (DARA), it predicts that an increase in the tax rate leads to a decrease in tax evasion.⁵ As a consequence, we observe a growing interest for Prospect Theory within the taxation literature. Kanbur et al. (2008) study the optimal non-linear taxation under Prospect Theory, and show that the standard Mirrlees (1974) results' are modified in several interesting ways. Dhimi and al-Nowaihi (2007) apply prospect theory to the taxpayers' decision to evade taxes, and show that predictions are both quantitatively and qualitatively more in line with the empirical evidence than under EUT. In Dhimi and al-Nowaihi (2010) the tax rate is endogenous: one main finding is that the best description of the data is obtained by combining taxpayers behaving according to PT and the government acting as predicted by EUT.

To the best of our knowledge, ours is the first attempt to analyse the optimal audit scheme under the more realistic assumption that agents behave according to Prospect

¹One main source of cost is the wage and the formation of the tax administration agents. Also, there are incentive problems related to the corruptibility of auditors (Hindriks et al. 1999).

²See, for instance, Mirrlees (1997).

³See, for example, Yaniv (1999), Camerer (2000) or Camerer and Loewenstein (2003).

⁴For more on that, see Tversky and Kahneman (1981).

⁵See Yitzhaki (1974) for the linear case, and Trannoy and Trotin (2010) for the non-linear case.

Theory. For the case of EUT, Cremer and Gahvari (1996) focus on the moral hazard problem occurring when the labour supply choice is endogenous. The EUT-works that are closer to ours are probably Chander and Wilde (1998) and Chander (2004, 2007). In the former, they characterise the optimal tax schedule in the presence of enforcement costs and clarify the nature of the interplay between optimal tax rates, audit probabilities and penalties for misreporting. In particular, under the (rather strong) assumption of risk neutral expected-utility-maximiser taxpayers, they show that the optimal tax function must generally be increasing and concave. This because a progressive tax function implies stronger incentives to misreport and thus it calls for larger audit probabilities. Chander (2004, 2007) studies the same issues for the case of risk averse taxpayers, when the incentive to misreport is weaker. By introducing a measure of aversion to large risks, he shows that the optimal tax function is increasing and concave if the taxpayer's aversion to such large risks is decreasing with income.

Our paper extends the optimal tax enforcement literature, considering agents that behave according to prospect theory. Reference dependence being a crucial element in prospect theory, we need to define a general reference income. The most natural choice is to restrict our attention, setting as a lower-bound the legal income (i.e., the after-tax disposable income, under no tax evasion) and as an upper-bound the pre-tax income. Our paper shows that Chander and Wilde (1998)'s results hold under a set of less restrictive assumptions when agents behave according to prospect theory, as opposed to EUT. In particular, we depart from the strong assumption of risk neutrality of taxpayers and show that the optimal audit probability function is always non-increasing. Concerning the optimal tax function, we show that it is always non-decreasing and concave when the pre-tax income is the used as a reference; nevertheless, for the same result to hold when the reference income is the legal one, we need to impose a further restriction: we show that a sufficient condition is to have Decreasing Prospect Risk Aversion (DPRA).

The paper is organised as follows. The next section describes a general model of income tax enforcement under prospect theory and introduces the definition of an optimal tax and audit scheme. Section 3 and 4 solves the model using as the reference income respectively the legal income and the pre-tax income. Section 5 concludes.

2. The model

Taxpayers income w of a taxpayer is a random variable with distribution function g , defined over the interval $[0, \bar{w}]$, with $\bar{w} > 0$. The tax administration knows g but not w . Following prospect theory, when the taxpayer sends a message $x \in [0, \bar{w}]$ to the tax administration about his income, he compares possible outcomes relative to a certain reference income R rather than to the final status.⁶ Hence, the taxpayer defines *gains* the outcomes producing an income above R and *losses* if the resulting income is below R . Each taxpayer's reference income R depends on his initial income:

$$R = R(w) \in [0, w]. \quad (1)$$

The choice of R embeds at once what the taxpayer considers to deserve (or the price he is willing to pay for public goods), expressed by the tax rate function. It also embeds the characteristics of the cheating game to which he subjects himself by not declaring his whole income, expressed by the audit probability and the penalty functions. Following prospect theory, the utility function $u(x - R)$ is assumed to be:

- (i) continuous on $\mathbb{R}$, twice continuously differentiable on $\mathbb{R}^*$ and equal to zero in zero:
 $u(0) = 0$,

⁶See, for example, Kahneman and Tversky (2000).

- (ii) increasing, convex for losses and concave for gains: $u' > 0$ on $\mathbb{R}^*$, $u'' > 0$ on $\mathbb{R}_-^*$ and $u'' < 0$ on $\mathbb{R}_+^*$ (*Diminishing marginal sensitivity*),
- (iii) steeper for losses than for gains: $u'(-k) > u'(k)$ for $k \in \mathbb{R}_+^*$ (*Loss aversion*).

Figure 1 represents a typical utility function.

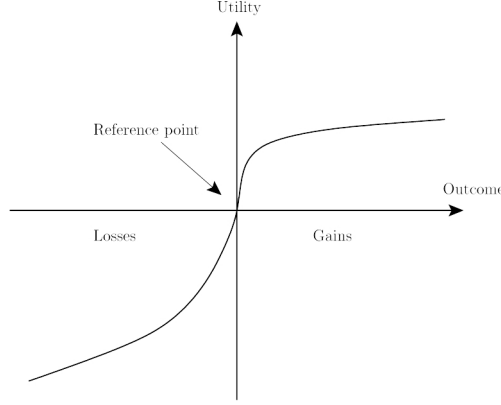


Figure 1: Utility of an outcome

The tax administration sets a mechanism up consisting of a set $X \subset \mathbb{R}_+^*$ of messages, a twice continuously differentiable tax function $t : X \rightarrow \mathbb{R}_+$, a probability function $p : X \rightarrow [0, 1]$ and a penalty function $f : X \times [0, \bar{w}] \rightarrow \mathbb{R}_+$. A taxpayer sending the message $x \in X$ is audited with probability $p(x)$. For an initial income w , he pays $t(x) \in [0, w]$ if no audit occurs and $f(w, x) \in [t(x), w]$ if an audit occurs.⁷ The associated payment function for the taxpayers is defined by:

$$r(w, x) = (1 - p(x))t(x) + p(x)f(w, x), \text{ for all } (w, x) \in [0, \bar{w}] \times X. \quad (2)$$

Audits are assumed to be costly, c being the cost for an audit. Others things being equal, the tax administration then prefers smaller audit probabilities to reduce audit costs.

To be feasible, a mechanism must satisfy that a taxpayer can send a message implying a payment not larger than his initial income. We define *direct revelation mechanisms* those that satisfy the following requirements:

- (i) *First feasibility requirement*: For all $w \in [0, \bar{w}]$, the set of feasible messages $X(w) = \{x \in X, t(x) \leq w\}$ contains at least one element and for all $x \in X(w)$, $f(w, x) \leq w$.
- (ii) *Second feasibility requirement*: The maximisation problem of the taxpayer:

$$\max_{x \in X(w)} [(1 - p(x))u(w - t(x) - R(w)) + p(x)u(w - f(w, x) - R(w))]$$

has a solution for all $w \in [0, \bar{w}]$.

The revelation principle can be applied to this setting.⁸ Hence, we restrict our attention to *truthful direct revelation mechanisms*. Taking into account the feasibility

⁷We assume that when audits occur, the true income of the taxpayer is observed without error.

⁸This states that for each direct revelation mechanism, there exists an equivalent one (from both the tax administration and the taxpayers perspective) under which it is optimal for each taxpayer to report income truthfully. Without loss of generality, the attention can then be confined to these truthful mechanisms.

requirements mentioned above, they are composed of schemes (t, p, f) such that for all $w \in [0, \bar{w}]$:

- (i) $t(w) \leq w$,
- (ii) $f(w, x) \leq w$, for all $x \in X(w)$,
- (iii) $(1 - p(w))u(w - t(w) - R(w)) + p(w)u(w - f(w, w) - R(w)) \geq$
 $(1 - p(x))u(w - t(x) - R(w)) + p(x)u(w - f(w, x) - R(w))$, for all $x \in X(w)$.

The third condition is the *incentive constraint*, which implies that the utility of the taxpayer is maximised if he reports his income truthfully.

With a truthful direct revelation mechanisms, the payment function for the taxpayers is defined by:

$$r(w) = (1 - p(w))t(w) + p(w)f(w, w), \text{ for all } w \in [0, \bar{w}]. \quad (3)$$

The objective of the tax administration is to maximise its revenue net of audit cost:

$$\max_{r, p} \left[\int_0^{\bar{w}} r(w)g(w)dw - c \int_0^{\bar{w}} p(w)g(w)dw \right]. \quad (4)$$

Denote by F the set of truthful direct revelation mechanisms. Following Chander and Wilde (1998), a scheme (t, p, f) is *efficient* in F if there is no other scheme $(t', p', f') \in F$ such that $p' \leq p$, $r' \geq r$ and $r' \neq r$ or $p' \neq p$, where r and r' are the payment functions corresponding to (t, p, f) and (t', p', f') . It is impossible to decrease the audit probability of a taxpayer and to (weakly) increase the total taxes and fines proceeds without increasing someone else's audit probabilities; similarly, it is impossible to raise the proceeds for some income levels and to (weakly) decrease the corresponding audit probabilities, without lowering proceeds for some other levels.

Notice that this notion of efficiency being independent of the density function, a scheme is efficient irrespectively of g . An optimal scheme maximises the tax administration's total revenue, net of audit costs, and by definition it is efficient.

3. When the reference income is the legal income

Through this section, we use the legal after-tax income as the reference income: $R(w) = w - t(w)$. The legal income has the specificity that the taxpayer is always in the domain of gains, unless he pays more than what he is legally required to do. The incentive constraint thus becomes:

$$p(w)u(t(w) - f(w, w)) \geq (1 - p(x))u(t(w) - t(x)) + p(x)u(t(w) - f(w, x)), \quad (5)$$

for all $x \in X(w)$.

This incentive constraint can be weakened, as expressed by the following lemma.

Lemma 1. *The incentive constraint (5) is equivalent to:*

$$(1 - p(x))u(t(w) - t(x)) + p(x)u(t(w) - w) \leq 0, \text{ for all } x \in X(w). \quad (6)$$

Proof. See the Appendix. □

F is the set of all schemes (t, p) that satisfy the conditions (i) and (ii) for a truthful direct revelation mechanism and the new incentive constraint (6). An efficient scheme in F is now a scheme (t, p) for which there is no other scheme $(t', p') \in F$ such that $t' \geq t$, $p' \leq p$ and $t' \neq t$ or $p' \neq p$.

The following monotonicity and concavity results hold for any efficient (henceforth optimal) scheme.

Lemma 2. *A scheme $(t, p) \in F$ is efficient in F only if the incentive constraint for each income level $w \in [0, \bar{w}]$ is binding at some $x \in X(w)$.*

Proof. See the Appendix. □

Proposition 3. *A scheme $(t, p) \in F$ is efficient in F only if t is non-decreasing and p is non-increasing in w .*

Proof. See the Appendix. □

Lemma 4. *If for all $\hat{w} \in [0, \bar{w}]$, there exists an affine function $l_{\hat{w}}$ on $[0, \bar{w}]$ such that for all $w \in [0, \bar{w}]$, $l_{\hat{w}}(w) \geq t(w)$ and $l_{\hat{w}}(\hat{w}) = t(\hat{w})$, then t is concave.*

Proof. See the Appendix. □

The incentive constraint (6), when binding, requires the agents' utility to be the same when declaring all their income and when not. This can be seen as a lottery implying gambling the legal income $w - t(w)$ for a possible gain which depends on the gap between the two tax levels $t(w) - t(x)$.

Let define a risk aversion measure by:

$$M(k) = -\frac{u''(k)}{(u'(k))^2}, \text{ for all } k \in \mathbb{R}^*.^9 \quad (7)$$

This measure slightly differs from the classical Arrow-Pratt risk aversion measure in expected utility theory. In that it can take both be negative and positive values. The taxpayer is risk averse for gains (u is concave) and risk seeker for losses (u is convex).

The following condition concerns how the taxpayer takes his tax evasion decision. The utility function u satisfies *decreasing prospect risk aversion* (DPRA) in $p \in]0, 1[$, on $\mathbb{R}_+^*$, if z is increasing in $y \in \mathbb{R}_+^*$, at a non-decreasing rate, where z is implicitly defined by equation (8):

$$(1 - p)u(y) + pu(-z) = 0. \quad (8)$$

In the setting of prospect theory, z is always increasing with y . In addition, the convexity condition generally holds. This is the case, for instance, of the power utility function derived from that of Tversky and Kahneman (1992) used to describe the behaviour of individuals under risk:

$$u(k) = \begin{cases} k^\alpha & \text{if } k \geq 0, \\ -\mu(-k)^\beta & \text{if } k < 0, \end{cases} \quad (9)$$

where $0 < \alpha \leq \beta < 1$, and $\mu > 1$ because of loss aversion.¹⁰

More formally, the convexity condition is equivalent to:

$$(1 - p) | M(-z) | \geq p | M(y) |, \text{ where } y \text{ and } z \text{ are defined by (8)}. \quad (10)$$

Weighted by the probability coefficients, the risk seeking behaviour defined by M in $-z$ must be higher than the risk aversion in y . Applying that to the tax evasion decision, in expected terms, the risk seeking behaviour in case of loss must be larger than the risk aversion in case of gain. The values of p being usually very close to zero, this condition is relatively weak and easily holds.

The concavity result of Chander and Wilde (1998) holds under this setting, at least when the utility function of taxpayers satisfies DPRA.

⁹This measure can also be defined on 0 in this manner: $M(0_-) = -\frac{u''(0_-)}{(u'(0_-))^2}$ and $M(0_+) = -\frac{u''(0_+)}{(u'(0_+))^2}$.

¹⁰More precisely, from experimental motives, it suggests that $\alpha = 0.88$ and $\mu = 2.25$.

Proposition 5. *If u satisfies DPRA, then a scheme $(t, p) \in F$ is efficient in F only if t is concave.*

Proof. See the Appendix. □

4. When the reference income is the initial income

In this section we consider the initial income as the reference income: $R(w) = w$. This extreme case corresponds to an extremely tax-averse taxpayer. This implies that any payment to the tax administration always lies in the loss domain and that therefore taxpayers are rent-seekers. The incentive constraint becomes:

$$(1 - p(w))u(-t(w)) + p(w)u(-f(w, w)) \geq (1 - p(x))u(-t(x)) + p(x)u(-f(w, x)),$$

for all $x \in X(w)$. (11)

Similarly to what we did for the previous case (see Section 3, Lemma 1), the incentive constraint can be weakened.

Lemma 6. *The incentive constraint is equivalent to:*

$$u(-t(w)) \geq (1 - p(x))u(-t(x)) + p(x)u(-w), \text{ for all } x \in X(w). \quad (12)$$

F is now the set of all schemes (t, p) that satisfy the conditions (i) and (ii) for a truthful direct revelation mechanism and the incentive constraint (12). The notion of efficiency is the same as for when the reference income is the legal one.

Again, optimal schemes are characterised by the following monotonicity and concavity results.

Lemma 7. *A scheme $(t, p) \in F$ is efficient in F only if the incentive constraints for each income level $w \in [0, \bar{w}]$ are binding at some $x \in X(w)$.*

Proof. The proof, similar to the one for Lemma 2, is available from the authors, upon request. □

Proposition 8. *A scheme $(t, p) \in F$ is efficient in F only if t is non-decreasing and p is non-increasing.*

Proof. See the Appendix. □

Proposition 9. *A scheme $(t, p) \in F$ is efficient in F only if t is concave.*

Proof. See the Appendix. □

Results are very similar to those of Section 3. However, under the current framework, we do not need additional assumptions about the shape of the utility function to ensure that the tax function of a revenue maximising scheme is concave. This comes directly from the convexity of the utility function, all its arguments being negative.

A priori, the most natural restriction for the reference income is to be not lower than the legal one and not higher than the initial one ($w - t(w) \leq R(w) \leq w$). Indeed, this corresponds to the case of a taxpayer whose final income will exceed the legal income while remaining below the initial one. The taxpayer derives an obvious disutility from paying the legal tax. Following the reasonings for the legal income and the initial income, it can be proved that, in a revenue maximising framework, (a) the probability function is non-increasing, (b) the tax function is non-decreasing, and (c) the interval on which the utility function is convex is larger when the reference income increases. Therefore, the conditions for the tax function to be concave become less restrictive.

Concluding remarks

This paper characterises the optimal income tax and audit schemes when tax evasion decisions of taxpayers verify prospect theory. It advances the theory of risk aversion in prospect theory by introducing a useful risk aversion measure in this setting.

We conclude that the penalty for misreporting should take an extreme value at the optimum. Although, this is not observed in practice, lowering this optimal penalty would only reinforce our results, because incentives for misreporting would be stronger and it would be harder to design a progressive tax function.

The analysis is restricted to the case of a government maximising its revenue, net of audit costs. As shown in Chander and Wilde (1998) in an expected utility theory setting, an optimal scheme can be net revenue maximising (but it is not always), if the government maximises a social welfare function with some redistributive purposes. It would be an interesting issue for future researches to study if this is the case under the setting that we presented.

Appendix A

Proof of Lemma 1. We can weaken the constraint by rising $f(w, x)$. This is possible as long as $f(w, x) < w$, and up to $f(w, x) = w$, for which equations (5) and (6) are equivalent.

In addition, the function $\phi(t) = p(w)u\left(\frac{t-r}{p(w)}\right) - (1-p(x))u(t-t(x)) - p(x)u(t-w)$ is increasing with t when t is smaller but sufficiently close to r , for all $r > 0$. Then, $f(w, w)$ can decrease and $t(w)$ can rise, while keeping constant the payment $r(w)$, as long as $f(w, w) > t(w)$. The conditions in (5) and (6) are then equivalent when $f(w, w) = t(w) = r(w)$. $\square$

Proof of Lemma 2. Suppose that $w \in [0, \bar{w}]$ exists such that for all $x \in X(w)$, the following inequality holds: $(1-p(x))u(t(w)-t(x)) + p(x)u(t(w)-w) < 0$.

Because u is increasing, t' such that $t'(w) > t(w)$, $t'(x) = t(x)$ for all $x \in X(w) \setminus \{w\}$ and

$$(1-p(x))u(t'(w)-t'(x)) + p(x)u(t'(w)-w) < 0,$$

can then be considered. This contradicts the efficiency of (t, p) in F . $\square$

Proof of Proposition 3.

- Suppose that there exists $w, w' \in [0, \bar{w}]$ such that $w < w'$ and t is decreasing on $[w, w']$. According to Lemma 2, there exists $x' \in X(w')$ such that the incentive constraint (6) for w' is binding at x' . By the incentive constraint (6) for w ,

$$(1-p(x'))u(t(w)-t(x')) + p(x')u(t(w)-w) \leq 0,$$

and, because u is increasing,

$$(1-p(x'))u(t(w')-t(x')) + p(x')u(t(w')-w') < 0.$$

This contradicts the fact that for w' , (6) is binding at x' .

- According to (6), for all $x \in X$, for all $w \in [0, \bar{w}]$ such that $x \in X(w)$,

$$p(x) \geq \frac{u(t(w)-t(x))}{u(t(w)-t(x)) - u(t(w)-w)}.$$

Then, (t, p) being efficient,

$$p(x) = \sup_{w > t(x)} \frac{u(t(w)-t(x))}{u(t(w)-t(x)) - u(t(w)-w)}.$$

t is non-decreasing, p is thus non-increasing. If there exists $x \in X$ which does not belong to any $X(w)$, $w \in [0, \bar{w}]$, then, according to (6), $p(z) = 0$ for all $z \geq x$. $\square$

Proof of Lemma 4. Let there be some $\hat{w} \in [0, \bar{w}]$.

- The slope of $l_{\hat{w}}$ is $t'(\hat{w})$. Indeed, for all $w \in [0, \bar{w}]$, the first order Taylor expansion of t near $\hat{w}$ is:

$$t(w) = t(\hat{w}) + t'(\hat{w})(w - \hat{w}) + r_1(w), \text{ with } r_1(w) \ll w - \hat{w} \text{ (when } w \rightarrow \hat{w}\text{)}.$$

Since $l_{\hat{w}}$ is an affine function which crosses t in $\hat{w}$, $l_{\hat{w}}(w) = t(\hat{w}) + \lambda(w - \hat{w})$. For all $w \in [0, \bar{w}]$, $l_{\hat{w}}(w) \geq t(w)$, then:

$$\lambda(w - \hat{w}) \geq t'(\hat{w})(w - \hat{w}) + r_1(w).$$

For all $w > \hat{w}$, $\lambda \geq t'(\hat{w}) + r_0(w)$, with $r_0(w) \ll 1$, then $\lambda \geq t'(\hat{w})$,
for all $w < \hat{w}$, $\lambda \leq t'(\hat{w}) + r_0(w)$, with $r_0(w) \ll 1$, then $\lambda \leq t'(\hat{w})$, then $\lambda = t'(\hat{w})$.

- For all $w \in [0, \bar{w}]$, $l_{\hat{w}}(w) = t(\hat{w}) + t'(\hat{w})(w - \hat{w})$. In addition, the second order Taylor expansion of t near $\hat{w}$ is:

$$t(w) = t(\hat{w}) + t'(\hat{w})(w - \hat{w}) + t''(\hat{w})\frac{(w - \hat{w})^2}{2} + r_2(w),$$

with $r_2(w) \ll (w - \hat{w})^2$.

$l_{\hat{w}}(w) \geq t(w)$, then $t''(\hat{w})\frac{(w - \hat{w})^2}{2} + r_2(w) \leq 0$, then $t''(\hat{w}) \leq 0$. This is verified for all $\hat{w} \in [0, \bar{w}]$, t is then concave on $[0, \bar{w}]$.

□

Proof of Proposition 5. Let there be some $\hat{w} \in [0, \bar{w}]$. Since (t, p) is efficient, according to Lemma 2, it exists some $\hat{x} \in [0, \bar{w}]$ such that $t(\hat{x}) \leq \hat{w}$ and $(1 - p(\hat{x}))u(t(\hat{w}) - t(\hat{x})) + p(\hat{x})u(t(\hat{w}) - \hat{w}) = 0$. Three cases arise from the value of $p(\hat{x})$.

- *First case:* $p(\hat{x}) = 0$, then $u(t(\hat{w}) - t(\hat{x})) = 0$, then $t(\hat{w}) = t(\hat{x})$. In addition, according to the incentive constraints (6), for all $w \in [0, \bar{w}]$, $u(t(w) - t(\hat{x})) \leq 0$, then $t(w) \leq t(\hat{x})$. The (constant) affine function $l_{\hat{w}}(w) = t(\hat{x})$ satisfies the assumptions of Lemma 4.
- *Second case:* $p(\hat{x}) = 1$, then $u(t(\hat{w}) - \hat{w}) = 0$, then $t(\hat{w}) = \hat{w}$. Then, since $t(w) \leq w$, for all $w \in [0, \bar{w}]$, the affine function $l_{\hat{w}}(w) = w$ satisfies the assumptions of Lemma 4.
- *Third case:* $0 < p(\hat{x}) < 1$, since u satisfies DPRA, the curve $C_{p(\hat{x})}$ defined by:

$$(1 - p(\hat{x}))u(y) + p(\hat{x})u(-z) = 0$$

is increasing and convex in the coordinate system $(0, y, z)$. Denote by $\hat{\Phi}$ the associated function and let there be some $\hat{z} \in [0, \bar{w}]$. Denote by $\hat{y}$ the real number such that $\hat{\Phi}(\hat{y}) = \hat{z}$. The tangent to $C_{p(\hat{x})}$ at $\hat{y}$ in $(0, y, z)$ is below itself. Denote by $\hat{k}$ the function associated to the tangent:

$$\hat{k}(y) = a(y - \tilde{y}), \text{ with } \hat{k}(\hat{y}) = \hat{\Phi}(\hat{y}) = \hat{z}, \tilde{y} \in [0, \bar{w}], \text{ and } a > 0.$$

For all $z \in [0, \bar{w}]$ such that $(1 - p(\hat{x}))u(y) + p(\hat{x})u(-z) \leq 0$, $\hat{k}(y) \leq \hat{\Phi}(y) \leq z$, because u is increasing.

Consider $z = w - t(w)$, $\hat{z} = \hat{w} - t(\hat{w})$, $\hat{y} = t(\hat{w}) - t(\hat{x})$ and $y = t(w) - t(\hat{x})$, $(1 - p(\hat{x}))u(y) + p(\hat{x})u(-z) \leq 0$ according to (6) and $(1 - p(\hat{x}))u(\hat{y}) + p(\hat{x})u(-\hat{z}) = 0$, then the affine function:

$$l_{\hat{w}}(w) = \frac{w + a(t(\hat{x}) + \hat{y})}{a + 1}$$

satisfies the assumptions of Lemma 4.

This is verified for all $\hat{w} \in [0, \bar{w}]$, t is then concave on $[0, \bar{w}]$, according to Lemma 4. $\square$

Proof of Proposition 8.

- Let us suppose that there exists $w, w' \in [0, \bar{w}]$ such that $w < w'$ and t is decreasing on $[w, w']$. According to Lemma 7, there exists $x' \in X(w')$ such that the incentive constraints (12) for w' are binding at x' . But according to the incentive constraints (12) for w ,

$$u(-t(w)) \geq (1 - p(x'))u(-t(x')) + p(x')u(-w).$$

u being increasing and t being decreasing on $[w, w']$, the following function is increasing on $[w, w']$:

$$\psi(v) = u(-t(v)) - (1 - p(x'))u(-t(x')) - p(x')u(-v).$$

Then, $\psi(w') > \psi(w) > 0$, which contradicts the fact that the constraints (12) for w' are binding at x' .

- According to (12), for all $x \in X$, for all $w \in [0, \bar{w}]$ such that $x \in X(w)$,

$$p(x) \geq \frac{u(-t(x)) - u(-t(w))}{u(-t(x)) - u(-w)}.$$

Then, (t, p) being efficient,

$$p(x) = \sup_{w > t(x)} \frac{u(-t(x)) - u(-t(w))}{u(-t(x)) - u(-w)}.$$

t is non-decreasing, p is thus non-increasing. If there exists $x \in X$ which does not belong to any $X(w)$, $w \in [0, \bar{w}]$, then, according to (12), $p(z) = 0$ for all $z \geq x$. $\square$

Proof of Proposition 9. Let there be some $\hat{w} \in [0, \bar{w}]$. Since (t, p) is efficient, according to Lemma 7, it exists some $\hat{x} \in [0, \bar{w}]$ such that $t(\hat{x}) \leq \hat{w}$ and $u(-t(\hat{w})) = (1 - p(\hat{x}))u(-t(\hat{x})) + p(\hat{x})u(-\hat{w})$. Three cases arise from the value of $p(\hat{x})$.

- *First case:* $p(\hat{x}) = 0$, then $u(-t(\hat{w})) = u(-t(\hat{x}))$, then $t(\hat{w}) = t(\hat{x})$. In addition, according to (12), for all $w \in [0, \bar{w}]$, $u(-t(w)) \geq u(-t(\hat{x}))$, then $t(w) \leq t(\hat{x})$. The (constant) affine function $l_{\hat{w}}(w) = t(\hat{x})$ satisfies the assumptions of Lemma 4.
- *Second case:* $p(\hat{x}) = 1$, then $u(-t(\hat{w})) = u(-\hat{w})$, then $t(\hat{w}) = \hat{w}$. Then, since $t(w) \leq w$, for all $w \in [0, \bar{w}]$, the affine function $l_{\hat{w}}(w) = w$ satisfies the assumptions of Lemma 4.
- *Third case:* $0 < p(\hat{x}) < 1$, then, u being convex on $\mathbb{R}_+^*$, for all $w \in [0, \bar{w}]$,

$$u(-t(w)) \geq (1 - p(\hat{x}))u(-t(\hat{x})) + p(\hat{x})u(-w) \geq u(-(1 - p(\hat{x}))t(\hat{x}) - p(\hat{x})w),$$

then, $t(w) \leq l_{\hat{w}}(w)$, where $l_{\hat{w}}$ is the affine function defined by $l_{\hat{w}}(w) = (1 - p(\hat{x}))t(\hat{x}) + p(\hat{x})w$.

In addition, following the incentive constraints (12), the expected utility for the initial income $\hat{w}$ is maximized by $\hat{x}$. The payment when declaring $\hat{x}$ is then lower than the one when declaring truthfully, that is:

$$r(\hat{w}, \hat{x}) = (1 - p(\hat{x}))t(\hat{x}) + p(\hat{x})\hat{w} \leq r(\hat{w}) = t(\hat{w}),$$

then $t(\hat{w}) = l_{\hat{w}}(\hat{w})$.

This is verified for all $\hat{w} \in [0, \bar{w}]$, t is then concave on $[0, \bar{w}]$, according to Lemma 4. $\square$

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