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Multi-product Firms and Business Cycle Dynamics

Antonio Minniti and Francesco Turino*

Abstract

Recent empirical evidence provided by Bernard *et al.* (2010) and Broda and Weinstein (2010) shows that a significant share of product creation and destruction in U.S. industries occurs within existing firms and accounts for a relevant share of aggregate output. In the present paper, and consistently with this evidence, we relax the standard assumption of mono-product firms that is typically made in dynamic general equilibrium models. Building on the work of Jaimovich and Floetotto (2008), we develop an RBC model with multi-product firms and endogenous markups to assess the implications of the intra-firm extensive margin on business cycle fluctuations. In this environment, the procyclicality of product creation emerges as a consequence of strategic interactions among firms. Because of the proliferation effect induced by changes in product scope, our model embodies a quantitatively important magnification mechanism of technology shocks.

Keywords: Multi-product Firms, Business Cycles, Firm Dynamics.

JEL codes: E32, L11.

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1 Introduction

Multi-product enterprises dominate production activity in the U.S. economy; according to Bernard *et al.* (2010), 39 percent of firms produce more than one product and generate, on average, 87 percent of U.S. manufacturing output. These firms vary their product mixes with higher intensity and frequency than do single-product firms. Despite this empirical evidence, multi-product firms have so far received relatively little attention in macroeconomics. Only recently has a growing body of international trade literature incorporated multi-product firms into models of general equilibrium to address issues related to trade liberalization (see Bernard *et al.* 2006; Nocke and Yeaple 2006; Eckel and Neary 2010; Mayer *et al.* 2010).[1] These contributions show that intra-firm adjustments within multi-product firms are not limited to changes in the scale of production; in fact, the choice of product range adds a new "extensive" margin of firm adjustment that plays an important role in economy-wide shocks such as trade liberalization.

In this paper, we embed multi-product firms in a real business-cycle (RBC) model with imperfect competition and endogenous entry to assess the role of the intra-firm "extensive" margin in the analysis of economic fluctuations at the business-cycle frequency. By proposing a model for multi-product firms, we fill a gap in the theory. In fact, a common feature of the RBC literature is that firms producing differentiated brands are mono-product firms. This assumption may be a useful simplification as a first approximation in the study of business-cycle fluctuations. However, because most industries are characterized by the presence of economies of scope, modeling firms as single-product entities is often restrictive. Moreover, under the assumption of mono-product firms, firm creation and destruction is equivalent to market entry and exit of products. This is somewhat unsatisfactory in light of the empirical research by Bernard *et al.* (2010) and Broda and Weinstein (2010), who point out that a significant share of product creation and destruction in U.S. industries occurs within the boundaries of the firm. In particular, Bernard *et al.* (2010) show that product switching exerts a substantial influence on firm activity because product creation and destruction account for an important share of overall production. The authors find that the gross contributions of adding and dropping products to the evolution of aggregate manufacturing output are as large as the gross contributions of firm entry and exit.[2] The importance of product creation and destruction is also confirmed by the work of Broda and Weinstein (2010), who show that, across product groups, almost one third of the growth rate of consumption expenditures is reflected in the growth rate of expenditure shares in new product varieties.[3]

In the present paper, we build on the recent work of Jaimovich and Floetotto (2008) and link the endogenous behavior of firms with the business-cycle properties of the economy by taking firms' strategic interactions into account. We thus depart from the standard Dixit-Stiglitz structure by assuming that, in choosing their product ranges and pricing strategies, companies behave as oligopolists and not as monopolistic competitors. As a result, in our model, each firm co-ordinates its pricing decisions by internalizing demand linkages between the varieties it produces, taking into account that a price reduction for one of its products reduces sales of all other goods in its product line. This is the so-called *cannibalization* effect, which is a distinguishing feature of multi-product firms. Moreover, because companies are large and produce a non-negligible set of varieties, they also take into account the effects of their pricing decisions on the industry's price index while taking the prices of all other competitors as given. Firms attempt to increase their market share through product proliferation. We assume that a variety-level fixed production cost bounds the company's product range. Firms enter instantaneously in each period until all

[1] In recent decades, the field of industrial organization has devoted considerable attention to the study of multi-product firms, mainly by using partial equilibrium analysis. Important contributions on this topic include Brander and Eaton (1984), Shaked and Sutton (1990), Anderson and Palma (1992) and, more recently, Ottaviano and Thisse (1999) and Allanson and Montagna (2005).

[2] By using data at the 5-digit SIC-level for all U.S. manufacturing firms, Bernard *et al.* (2010) document the importance of product adding and dropping within firms. They show that 54% of firms change their mix of products within a 5-year census period (representing 89% of firms weighted by output) with 25% of firms both adding and dropping at least one product.

[3] Broda and Weinstein (2010) measure products at the finest possible level of product disaggregation, namely the product barcode. Their analysis is based on data that include all purchases of products with barcodes by a representative sample of U.S. consumers.

profit opportunities are exhausted; they pay a fixed period-by-period production cost and earn zero profits in every period. Given that firms are multi-product, market structure is then endogenously determined by the entry and exit decisions of individual producers as well as by their optimal choices of product scope.

In our model, technology shocks not only affect the entry and exit of firms, but also generate product switching with companies that add and drop products over the business cycle. In this respect, we find that firms' product scopes increase during a macroeconomic expansion. This finding is in agreement with empirical evidence provided by Broda and Weinstein (2010), who show that net product creation is strongly procyclical. This feature of our model depends crucially on the assumption that product markets are oligopolistically competitive. In fact, we prove that introducing multi-product firms in the canonical Dixit-Stiglitz framework with monopolistic competition has the counterfactual implication that the product scope does not change in response to a technology shock, and thus product switching is predicted to be acyclical. The reason is that the procyclicality of the product range results from the fact that firms use the product scope as a tool to relax price competition, and this property only holds true in a strategic set-up. The intuition is as follows. When product markets are oligopolistically competitive, a company chooses its optimal product scope by taking into account the effects of this choice on both its own and all other firms' pricing decisions. Because increasing the number of varieties produced raises each firm's own price and reduces other firms' prices, we show that companies try to mitigate competition by under-expanding their product scope with respect to a situation of monopolistic competition in which firms behave in a non-strategic manner. However, the strategic effect of the product scope becomes less important as the number of companies increases. In fact, with a very large number of incumbents, this effect vanishes, and the market structure converges towards monopolistic competition. As a result, the incentive of each company to create new product varieties rises with the number of competitors in the market. Consequently, a positive technology shock induces new firms to enter the market and, at the same time, prompts incumbents to increase the number of varieties that they produce. We call this mechanism the *proliferation* effect. In a model of monopolistic competition with multi-product firms, such a mechanism is not operative because (in the context of this market structure) the strategic effect of the product scope is absent. In such a case, we show that firms' product ranges are determined only by exogenous parameters and are not affected by technology shocks.

A few remarks on the *proliferation* effect are in order. First, in our model, adjustments along the intra-firm extensive margin act as a sort of barrier to entry over the business cycle. By comparing our model with an alternative set-up with mono-product firms, we find that the number of new firms entering the market after a positive technology shock is substantially lower when the economy is populated by multi-product firms. This low number is due to changes in firms' product ranges absorbing part of the increase in demand that results from a positive shock. Thus, profit opportunities are exhausted more quickly when firms are multi-product than when they are mono-product.

Second, in a framework with multi-product firms and oligopolistic competition, product range adjustments at the firm level amplify the effects of technology shocks at the aggregate level. In this environment, product diversity induces increasing returns at the aggregate level. Thus, for a given amount of productive factors, output is larger for greater varieties of goods produced in equilibrium. In canonical models of monopolistic competition, either because of the "love-for-variety" property (Chatterjee and Cooper 1993 and Bilbiie *et al.* 2007) or because of returns to specialization (Devereux *et al.* 1996), this channel affects aggregate fluctuations through the process of firms' entry and exit. In addition, when the economy is populated by multi-product firms, product diversity has implications for aggregate fluctuations through variations in product scope. In this case, the product space depends not only on the number of active firms but also on the product range chosen by each of them. Consequently, any shock that fosters firm entry or expands firms' product ranges (or both) also affects aggregate output by increasing the number of available goods in the overall economy. We show that the combined effect of this channel with firms' entry and exit and with countercyclical markup variations provides a quantitatively important endogenous amplification mechanism. By calibrating our model to the U.S. economy, we find that output volatility increases by 103% relative to the canonical RBC framework with perfect competition. This result indicates that, in our model, the amount of volatility of technology shocks required to account for the same fluctuations in actual data is necessarily lower than the amount required in standard RBC models. In other words, introducing multi-product firms in an environment with endogenous entry and oligopolistic competition substantially strengthens the weak internal propagation mechanism of the canonical RBC model.

Finally, because of the stronger endogenous amplification mechanism, we show that our model outperforms the standard RBC framework with respect to the implied second-moment properties of key macroeconomic aggregates. Most notably, in our model, volatilities of output and investment essentially match the data.

This paper is related to an extensive body of literature in macroeconomics that analyzes business cycle fluctuations with imperfect competition. Among the early contributions on this subject, Rotemberg and Woodford (1999) find that collusion can generate countercyclical markups, while Galí (1994) reaches a similar conclusion with a model in which variations in the composition of demand lead to variations in markups. Both Chatterjee and Cooper (1993) and Devereux *et al.* (1996) analyze the effects of entry and exit on business-cycle dynamics and show that net business formation is procyclical. Firms' strategic interactions (with the implications stressed above) distinguish our model from these papers in which the market structure is of monopolistic competition. More recent contributions in the field include Bilbiie *et al.* (2007) and Jaimovich and Floetotto (2008). The first paper studies the role of product creation in propagating business cycle fluctuations in a model of monopolistic competition and sunk entry costs; the authors analyze the contributions of intensive and extensive margins (i.e., changes in production of existing goods and in the range of available goods) to the economy's responses to changes in aggregate productivity. We differ from this paper in two main respects. First, in Bilbiie *et al.* (2007), each product unit is interpreted as a product line within a multi-product firm; due to the assumption of a continuum of goods, firm boundaries are left unspecified without concern for strategic interactions. In our paper, we explicitly model multi-product firms by taking into account strategic interactions within and across firms. Second, the source of cyclical movements in markups is different in the two models. In Bilbiie *et al.* (2007), countercyclical markups are due to demand-side pricing complementarities; in our paper, countercyclical markups are related to variations in the number of competitors and occur due to supply-side considerations.[4] Jaimovich and Floetotto (2008) develop an RBC model with mono-product firms and endogenous entry by taking firms' strategic interactions into account. The authors show that oligopolistic behavior generates an important channel of shock magnification that operates through an endogenous mechanism of markup variation. In the present paper, we complement and extend the analysis of Jaimovich and Floetotto (2008) by proposing an RBC model with multi-product firms. In our paper, oligopolistic behavior gives rise to countercyclical markups and procyclical product ranges. The latter represent an additional channel that strengthens the endogenous amplification mechanism that is embodied in the model with mono-product firms.

The rest of the paper is organized as follows. Section 2 presents the model, while Section 3 solves it. Section 4 describes the calibration and the main dynamic implications of the model. Finally, Section 5 provides conclusions. The calculation details are provided in the Appendix.

2 The model

In this section we present a baseline framework that captures the key features of multi-product firms in a stochastic dynamic general equilibrium setup. The specific framework that we develop is a stochastic version of the conven-

[4] Another important difference between the two models is that the number of firms is a state variable in Bilbiie *et al.* (2007) because entry is subject to a sunk entry cost and a time-to-build lag. In our baseline model, there is no sunk entry cost; we assume a period-by-period, zero-profit condition so that firms enter instantaneously in each period until all profit opportunities are exploited. In Section 4.3.1, however, we study the richer dynamic problem with sunk entry costs. This alternative assumption does not alter our results, and, in particular, the magnification mechanism remains quantitatively significant.

tional neoclassical growth model augmented with an imperfectly competitive structure of product markets, firms' entry/exit decisions and endogenous choices of product scope. By using this framework, we analyze the consequences of adjustments that occur along the *intra-firm* extensive margin (the number of varieties produced) on both business-cycle fluctuations and firm dynamics.

2.1 Households

The economy is composed of a continuum of identical households whose mass is normalized to one. The representative household has preferences over consumption and leisure with utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[log(C_t) - \xi \frac{H_t^{1+\chi}}{1+\chi} \right], \tag{1}$$

where C_t and H_t denote, respectively, consumption and hours worked by the household at time t, $\beta \in (0,1)$ is the subjective time discount factor, $\chi \geq 0$ is the inverse of Frisch labor supply elasticity, E_t denotes the mathematical expectations operator conditional on information available at time t, and $\xi > 0$ is a preference parameter controlling for the disutility of labor.

At time t, there are M_t multi-product firms that produce differentiated goods. We assume that C_t is a consumption aggregator that combines all of the product varieties from multi-product firms:

$$C_t = \left[\sum_{i=1}^{M_t} x_t^C(i)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{2}$$

where $\theta > 1$ denotes the elasticity of substitution between any two firms and $x_t^C(i)$ is a composite consumption good that collects all the products from company i. Thus:

$$x_t^C(i) = \left[\sum_{j=1}^{N_t(i)} x_t^C(i,j)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}. \tag{3}$$

In Eq. (3), $x_t^C(i,j)$ denotes the purchase of product j manufactured by firm i, $N_t(i)$ is the number of varieties produced by firm i, and $\gamma > 1$ is the elasticity of substitution among these differentiated goods. Product varieties are grouped into nests, with goods within a nest being produced by the same firm. In what follows, we assume that $\gamma > \theta$, which means that the degree of substitutability between varieties within nests is larger than the degree of substitutability between nests.[5] This assumption is consistent with the empirical evidence provided by Broda and Weinstein (2010).

The representative household holds one asset, the capital stock K_t, which is assumed to evolve over time according to the following law of motion:

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{4}$$

where $\delta \in (0,1)$ is the rate of depreciation of the capital stock, and I_t denotes the investment aggregator that combines all of the investment goods produced by the multi-product firms:

$$I_t = \left[\sum_{i=1}^{M_t} x_t^I(i)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}. \tag{5}$$

[5]This is usually referred to as the *market segmentation* case. This industry configuration arises when nest $i \in [1, M]$ corresponds to firm i, which produces N_i close substitute varieties of the good. The theoretical literature (see Brander and Eaton 1984) has also proposed an alternative case, which is denoted *market interlacing.* Under this industry configuration, each nest $i \in [1, M]$ is occupied by N_i firms and consists of varieties produced by different firms; in this case, each manufacturer produces less closely related products.

As before, $x_t^I(i)$ is a composite investment good that aggregates all of the products from company i:

$$x_t^I(i) = \left[\sum_{j=1}^{N_t(i)} x_t^I(i,j)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \tag{6}$$

where $x_t^I(i,j)$ is the purchase of the investment good j manufactured by firm i.

The representative household supplies labor services per unit of time and rents capital to firms. The labor and capital markets are perfectly competitive so that households take as given the wage rate w_t paid per unit of labor services and the rental rate r_t paid per unit of capital. In addition, the representative household is entitled to the receipt of pure profits from the ownership of firms, Π_t. The flow budget constraint is then given by the following equation:

$$\sum_{i=1}^{M_t} \sum_{j=1}^{N_t(i)} p_t(i,j) \left[x_t^C(i,j) + x_t^I(i,j) \right] \leq w_t H_t + r_t K_t + \Pi_t, \tag{7}$$

where $p_t(i,j)$ is the price of variety j produced by firm i.

To solve for the consumption optimization problem, we use a three-stage utility maximization procedure. In the first stage, households choose consumption and investment goods within a nest. In the second stage, expenditure is instead allocated across nests. Finally, in the third stage, households choose consumption and leisure over time.

Let us analyze consumption decisions first. In the first stage, the representative household maximizes $x_t^c(i)$ subject to the expenditure constraint on the products of firm i, $\sum_{j=1}^{N_t(i)} p_t(i,j) x_t^C(i,j) \leq e_t^C(i)$. Thus, we obtain the demand function for consumption good j in nest i:

$$x_t^C(i,j) = \frac{e_t^C(i)}{p_t(i,j)^{\gamma} q_t(i)^{1-\gamma}}, \tag{8}$$

where $q_t(i) = \left[\sum_{j=1}^{N_t(i)} p_t(i,j)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$ is the price index corresponding to firm i. In the second stage, the representative household maximizes C_t subject to the budget constraint on composite goods, $\sum_{i=1}^{M_t} q_t(i) x_t^C(i) \leq e_t^C$; this yields:

$$x_t^C(i) = \frac{e_t^C}{q_t(i)^{\theta} q_t^{1-\theta}}, \tag{9}$$

where $q_t = \left[\sum_{i=1}^{M_t} q_t(i)^{1-\theta} \right]^{\frac{1}{1-\theta}}$ is the aggregate consumption price index. Because $e_t^C(i) = q_t(i) x_t^C(i)$, by using Eqs. (8) and (9), we can write the consumption demand of variety j produced by firm i as:

$$x_t^C(i,j) = \frac{e_t^C}{q_t^{1-\theta} p_t(i,j)^{\gamma} q_t(i)^{\theta-\gamma}}.$$

Following a similar procedure, the aggregate demand for investment good j produced by firm i reads as:

$$x_t^I(i,j) = \frac{e_t^I}{q_t^{1-\theta} p_t(i,j)^{\gamma} q_t(i)^{\theta-\gamma}},$$

where e_t^I is the total expenditure on investment goods. Thus, the aggregate demand for variety j produced by firm i, $x_t(i,j)$ can be expressed as:

$$x_t(i,j) = \frac{e_t}{q_t^{1-\theta} p_t(i,j)^{\gamma} q_t(i)^{\theta-\gamma}}, \tag{10}$$

where $e_t \equiv e_t^C + e_t^I$ denotes total expenditure for consumption and investment goods at date t. As can be observed, the demand for each individual variety depends negatively on its price and positively on both the nest-level and industry-level price indexes.

In the third stage, the representative household chooses sequences of consumption C_t and hours worked H_t to maximize the inter-temporal utility function (1) under the law of motion for capital (4) and the life-time budget constraint (7). Notice that by virtue of equation (10), the latter can be rewritten as follows:

$$e_t = q_t\left(C_t + I_t\right) \leq w_t H_t + r_t K_t + \Pi_t.$$

The first-order conditions for an interior maximum are then given by the following two equations:

$$\xi H_t^{\chi} = \frac{w_t}{q_t C_t}, \tag{11}$$

$$\frac{1}{C_t} = \beta E_t \left\{ \frac{1}{C_{t+1}} \left[(1-\delta) + \frac{r_{t+1}}{q_{t+1}} \right] \right\}. \tag{12}$$

Under the assumption of a perfectly competitive labor market, Eq. (11) describes the supply of hours for working activities, while Eq. (12) is the well-known Euler equation that provides the inter-temporal optimality condition for consumption.

2.2 Firms

The economy is characterized by a finite number of multi-product firms. During each period t, entry and exit of multi-product firms into and out of the existing market is determined by a zero-profit condition. Incumbent firms face the same production technology by using capital and labor as inputs. In addition to a fixed production cost per variety, we assume that there exists a firm-level fixed cost that must be paid regardless of the size of the firm's product range. Thus, the overall production of a typical firm i that produces $N_t(i)$ varieties at time t can be written as:

$$\sum_{j=1}^{N_t(i)} x_t(i,j) = \sum_{j=1}^{N_t(i)} \left[z_t k_t(i,j)^{\alpha} h_t(i,j)^{1-\alpha} - \phi_v \right] - \phi_f, \tag{13}$$

where $k_t(i,j)$ and $h_t(i,j)$ denote, respectively, the amount of capital and labor employed by firm i in producing variety j, ϕ_v is the fixed cost per variety, ϕ_f is the firm-level fixed cost and $\alpha \in [0,1]$. We assume that z_t is an economy-wide productivity shock at time t, the log of which follows a stationary AR(1) process with persistence parameter $\varrho \in (0,1]$ and a normally distributed innovation, ε_t, with a mean of zero and a standard deviation of σ_ε.

We model firms' decisions as a two-stage game. In the first stage, firms choose the number of varieties to produce and determine the sizes of their product ranges. In the second stage, they act as Bertrand-Nash competitors in the product market. The model is solved by backward induction using the subgame Nash perfect-equilibrium concept.

2.3 Pricing

When setting the price of each product, a multi-product firm does not take the price of the other varieties that it produces as given. Rather, it internalizes demand linkages between the varieties within its nest. A firm takes into account that a price reduction for one of its product negatively affects the sales of all other varieties that it produces. This effect is called the *cannibalization* effect. Moreover, because companies are not small relative to the size of the market and they produce a non-negligible set of varieties, they take the aggregate price index into consideration when making a pricing decision.[6]

[6] Appendix A provides the derivation of the elasticity of the price index with respect to the price of a product variety. In the same Appendix, we determine the demand elasticities of a good in response to variations in its own price and other goods' prices within the same nest.

A typical firm i chooses a pricing rule for each variety within its nest to maximize profits:

$$\pi_t(i) = \sum_{j=1}^{N_t(i)} \left[p_t(i,j)x_t(i,j) - w_t h_t(i,j) - r_t k_t(i,j)\right]. \tag{14}$$

As shown in Appendix B, a Nash equilibrium in prices emerges when each firm i charges the same price for all of the product varieties within its nest:

$$p_t(i,j) = p_t(i) = mc_t \frac{[\theta - (\theta-1)\epsilon_t(i)]}{(\theta-1)[1-\epsilon_t(i)]}, \quad \forall j \in [1, N_t(i)]. \tag{15}$$

In this equation, $mc_t \equiv w_t^{1-\alpha} r_t^{\alpha} / \left[z_t(1-\alpha)^{1-\alpha}\alpha^{\alpha}\right]$ is the marginal cost of producing one more variety and $\epsilon_t(i)$ is firm i's market share:

$$\epsilon_t(i) \equiv \frac{q_t(i)x_t(i)}{e_t} = \left[\frac{q_t(i)}{q_t}\right]^{1-\theta} = \frac{N_t(i)^{\frac{1-\theta}{1-\gamma}} p_t(i)^{1-\theta}}{\sum_{i=1}^{M_t} N_t(i)^{\frac{1-\theta}{1-\gamma}} p_t(i)^{1-\theta}}, \tag{16}$$

where $x_t(i) \equiv x_t^C(i) + x_t^I(i)$ denotes the total demand for consumption and investment goods produced by firm i.

2.4 Product scope

In the first stage of the game, firms anticipate the subsequent price competition and determine their product ranges by playing a Nash game. Thus, when choosing a product scope, a firm takes the number of competitors and their product ranges as given. As shown in Appendix C, the gross profits of firm i can be expressed as:

$$\pi_t(i) = e_t \epsilon_t(i) L_t(i) - mc_t \left[N_t(i)\phi_v + \phi_f\right], \tag{17}$$

where $L_t(i) \equiv [p_t(i) - mc_t]/p_t(i) = 1/[\theta - \epsilon_t(i)(\theta-1)]$ is the Lerner index of market power. Firm i maximizes profits with respect to $N_t(i)$ by taking into account the strategic effect of the product range decision, that is, the effect of its product range choice on both its own and all other firms' pricing decisions. Now, the first-order condition for an interior maximum can be written as:

$$\frac{e_t \epsilon_t(i) L_t(i)}{N_t(i)} \eta_t(i) = mc_t \phi_v, \tag{18}$$

where $\eta_t(i) = \theta L_t(i) \cdot [\partial \epsilon_t(i)/\partial N_t(i)] \cdot [N_t(i)/\epsilon_t(i)]$ is the elasticity of the variable profits of firm i with respect to the size of its product range, $N_t(i)$. The left-hand side of Eq. (18) provides the benefit of expanding the product range by one unit: opening a new product line increases the firm's market share and leads to higher profits. However, a greater number of products involves higher proliferation costs. In fact, the right-hand side of Eq. (18) represents the cost of adding one more variety to the nest.

3 Symmetric rational expectations equilibrium

From now on, we restrict our attention to symmetric equilibria. In solving the model, we first examine product- and firm-level variables, and then we turn to aggregate variables.

3.1 Product- and firm-level variables

Under the assumption of symmetry, each company produces the same number of varieties and presents the same market share, that is, $N_t(i) = N_t$, and $\epsilon_t(i) = 1/M_t$ for every $i \in [1, M_t]$. This implies that the price of each product variety boils down to:

$$p_t = mc_t\mu(M_t) = mc_t\frac{[(M_t-1)\theta+1]}{(\theta-1)(M_t-1)}, \tag{19}$$

where $\mu(M_t)$ denotes the mark-up ratio that is a decreasing function of the number of firms.

Using the fact that $q_t(i) = \left[\sum_{j=1}^{N_t(i)} p_t(i,j)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$, and substituting $x_t^C(i)$ and $x_t^I(i)$, respectively, from (3) and (6) into (16), output per product (*intensive margin*) is written as:

$$x_t = \frac{e_t}{p_t N_t M_t}, \tag{20}$$

and, consequently, output per firm can be easily expressed as $x_t N_t = e_t/(p_t M_t)$.

Each firm chooses its optimal product scope by taking into account the effect of its product range choice on both its own and all other firms' pricing decisions. As shown in Appendix C, in our strategic set-up, firms use the product range as a tool to relax price competition. The reason is that an increase in the number of a firm's varieties raises the firm's own price and reduces other firms' prices. Consequently, to mitigate price competition in the second stage of the game, firms contract their product range in the first stage, reducing the variety offered. The elasticity of variable profits with respect to the size of each firm's product scope measures the extent to which firms under-expand their product scope with respect to a situation of monopolistic competition in which firms' strategic interactions are neglected:

$$\eta_t = \frac{(\theta-1)}{(\gamma-1)}\left[\frac{\theta(M_t-1)^2}{M_t\theta\left(M_t-1\right)+\theta-1}\right].$$

As can be easily ascertained, the term in the square brackets (which is smaller than one) is an increasing function of the number of firms, M_t. Intuitively, the strategic effect of the product scope becomes less important as M_t increases so that the incentive to create new varieties increases with the number of firms. When the latter becomes very large, the term in the square brackets tends toward one, and the elasticity η_t tends toward $(\theta-1)/(\gamma-1)$, as in monopolistic competition. Now, replacing η_t from the above equation into Eq. (18) and then imposing symmetry, the number of varieties per firm (*intra-firm extensive margin*) can be written as:

$$N_t = \frac{e_t\theta\left(M_t-1\right)}{\phi_v\left(\gamma-1\right)p_t\left[M_t\theta(M_t-1)+\theta-1\right]}. \tag{21}$$

Finally, using (19) and (20) into (17) and requiring that firms make zero profits, we obtain:

$$x_t N_t\left[\mu(M_t)-1\right] = \phi_v N_t + \phi_f. \tag{22}$$

This condition, which relates the number of firms (*inter-firm extensive margin*) to the output per product and the product scope, must hold in every period.

3.2 Aggregate variables

In a symmetric equilibrium, $h_t(i,j) = h_t$ and $k_t(i,j) = k_t$ for every $i \in [1, M_t]$ and $j \in [1, N_t(i)]$. Then, aggregate hours and aggregate capital are equal to $H_t = M_t N_t h_t$ and $K_t = M_t N_t k_t$, respectively. The total number of firms M_t can be found by combining technology (13) with the zero-profit condition (22):

$$M_t = z_t K_t^{\alpha} H_t^{1-\alpha}\frac{\mu(M_t)-1}{\mu(M_t)\left(\phi_v N_t+\phi_f\right)}. \tag{23}$$

Now, the clearing of the goods market requires that in equilibrium $Y_t \equiv C_t + I_t$, where Y_t denotes the aggregate level of production. The latter can be easily found by plugging (3) into (2) and (6) into (5) after using Eqs. (22)

Table 1: **Calibration**

Parameter	Value	Description
β	.9902	Subjective discount factor
α	0.25	Capital Elasticity of output
χ	0.77	Inverse of Frish elasticity
δ	0.025	Capital depreciation rate
ξ	10.63	Preference parameter
z	1	Long-Run rate of technology
γ	11.5	Within-brand elasticity of substitution
θ	10	Across-firm elasticity of substitution
ϕ_f	0.057	Fixed cost
ϕ_v	0.003	Fixed cost
ϱ	0.9702	Persistence of technology shocks
σ_ε	0.0072	Standard error of innovations in the rate of technology

and (23) to simplify:

$$Y_t = z_t K_t^\alpha H_t^{1-\alpha} \frac{M_t^{1/(\theta-1)} N_t^{1/(\gamma-1)}}{\mu(M_t)}. \tag{24}$$

In what follows, we use the price p_t as the *numéraire* and set it to one; thus, we can rearrange Eq. (19) and write the marginal cost of production mc_t as $1/\mu(M_t)$. By using this normalization, the equilibrium wage rate, the rental rate and the price index, respectively, read as follows:

$$w_t = \frac{(1-\alpha)z_t}{\mu(M_t)} K_t^\alpha H_t^{-\alpha}, \tag{25}$$

$$r_t = \frac{\alpha z_t}{\mu(M_t)} K_t^{\alpha-1} H_t^{1-\alpha}, \tag{26}$$

$$q_t = M_t^{\frac{1}{1-\theta}} N_t^{\frac{1}{1-\gamma}}. \tag{27}$$

4 Results

In this section, we first calibrate the model to the U.S. economy, and then we analyze the model's dynamics at the firm and aggregate levels.

4.1 Calibration

The model has a total of 12 parameters, $\omega = \{\beta,\ \chi,\ \xi,\ \theta,\ \gamma,\ z,\ \delta,\ \alpha,\ \phi_v,\ \phi_f,\ \varrho,\ \sigma_\varepsilon\}$. Because many of these are standard in the business-cycle literature, we use common values. Specifically, we calibrate the model to the U.S. economy assuming that each period corresponds to a quarter. We thus set the subjective discount factor, β, equal to 0.99, which implies a yearly nominal interest rate of 4 percent. Following Ravn *et al.* (2006), we set the labor income share (wH/qY) to 0.75, the investment share (I/Y) to 0.18, and the Frisch elasticity of labor supply to 1.3. These restrictions imply that the capital elasticity of output in production, α, is 0.25, the capital depreciation rate, δ, is 0.025, and the inverse of the Frisch elasticity of labor supply, χ, is roughly 0.77. As in Prescott (1986), the preference parameter ξ is chosen to ensure that, in the steady state, the representative consumer devotes 1/4 of his time to labor activities. As concerns stochastic productivity, we normalize the long-run rate of technology, z, to 1, and, following King and Rebelo (1999), we set the autocorrelation coefficient, ϱ, and the standard deviation of technology shocks, σ_{ε}, to 0.97 and 0.0072, respectively.

Turning to firm-related parameters $\{\gamma, \theta, \phi_v, \phi_f\}$, we set the across-brand elasticities of substitution, θ, to 10. This value is consistent with the empirical evidence provided by Cogley and Sbordone (2008) for the U.S. economy and is intermediate among values typically used in the RBC literature. The within-brand elasticity of substitution, γ, is set equal to 11.5 in accordance with the empirical evidence provided by Broda and Weinstein (2010) for multi-product firms in the U.S. economy. More specifically, the value assigned to γ corresponds to the median of the distribution of the estimated elasticities obtained by Broda and Weinstein (2010) using data for 122 product groups in the U.S. economy.[7] To assign values to fixed cost parameters $\{\phi_f, \phi_v\}$, we introduce two calibration restrictions. First, as in Jaimovich and Floetotto (2008), we assume that the long-run aggregate markup, μ, takes the value of 1.3.[8] Then, using the expression for the mark-up ratio, $\mu = [(M-1)\,\theta + 1] \,/\, [(\theta - 1)\,(M-1)]$, we can find the number of firms, M, that (given the value assigned to θ) causes the equilibrium mark-up rate, μ, to be equal to 1.3:

$$M = 1 + \frac{1}{\mu(\theta - 1) - \theta}. \tag{28}$$

As a second restriction, we assign a value of 3.5 to the long-run product scope, N. According to the evidence reported by Bernard *et al.* (2010), this number corresponds to the average number of products produced by manufacturing firms in the U.S. economy over the period 1987-1997.[9] The introduction of this second restriction is necessary because in our framework, unlike in models of mono-product firms, knowing the equilibrium number of firms does not allow us select values for the fixed-cost parameters. In fact, in a model with multi-product firms, the zero-profit condition (23) involves the two fixed-cost parameters, ϕ_f and ϕ_v, and the equilibrium product scope, N. Thus, although the latter can be determined by using (21), we still do not have sufficient information to simultaneously determine ϕ_f, and ϕ_v. This requires the introduction of another calibration restriction in addition to (28). By using the latter and the restriction $N = 3.5$ in combination with equations (21) and (23), we set the fixed cost parameters ϕ_v and ϕ_f to 0.0031 and 0.0568, respectively. This implies that, in the steady state, sunk fixed costs absorb approximately 20% of the gross production of each firm. Table 1 summarizes the set of calibrated parameters.

4.2 Firm-level Dynamics

We now examine the dynamics at the firm level and compare our model's implications with those of two different scenarios. In the first, firms are mono-product and behave as oligopolists as in the model of Jaimovich and Floetotto (2008) (Figure 1). For this economy, we retain the parameterization used in the multi-product-firm model, but we adjust the value of fixed-cost parameters to ensure that the long-run number of active firms, M, is equal in the two economies.[10] By doing this, we remove any long-run effects associated with the number of active firms; thus, any difference in firm dynamics across the two economies can be directly imputed to short-term adjustments in the intra-firm extensive margin (the number of products per firm). This allows us to more clearly disentangle the impact of firms' product scopes on entry/exit decisions over the course of the business cycle. In the second scenario, firms are still multi-product but behave as monopolistic competitors (Figure 2). Unlike in the first case, computations based on this economy are obtained by retaining the same parameterization as that summarized in Table 1. Thus, comparing our model with this alternative economy allows us to assess the overall impact on firm-level dynamics of introducing firms' strategic interactions. The impulse-response functions summarize the dynamic

[7]See Table 8 (page 717) in Broda and Weinstein (2010).

[8]This value is intermediate among those typically used in RBC models with monopolistically competitive product markets. For instance, Rotemberg and Woodford (1999) assume an average markup of 20%, while Bilbiie *et al.* (2007) assume a markup of 36%.

[9]See Table 1 (page 79) in Bernard *et al.* (2010).

[10]More precisely, the fixed cost per variety, ϕ_v, has been set equal to zero, while the firm-level fixed cost, ϕ_f, has been adjusted to ensure that the long-run number of active firms, M, is equal in the two economies with and without multi-product firms.

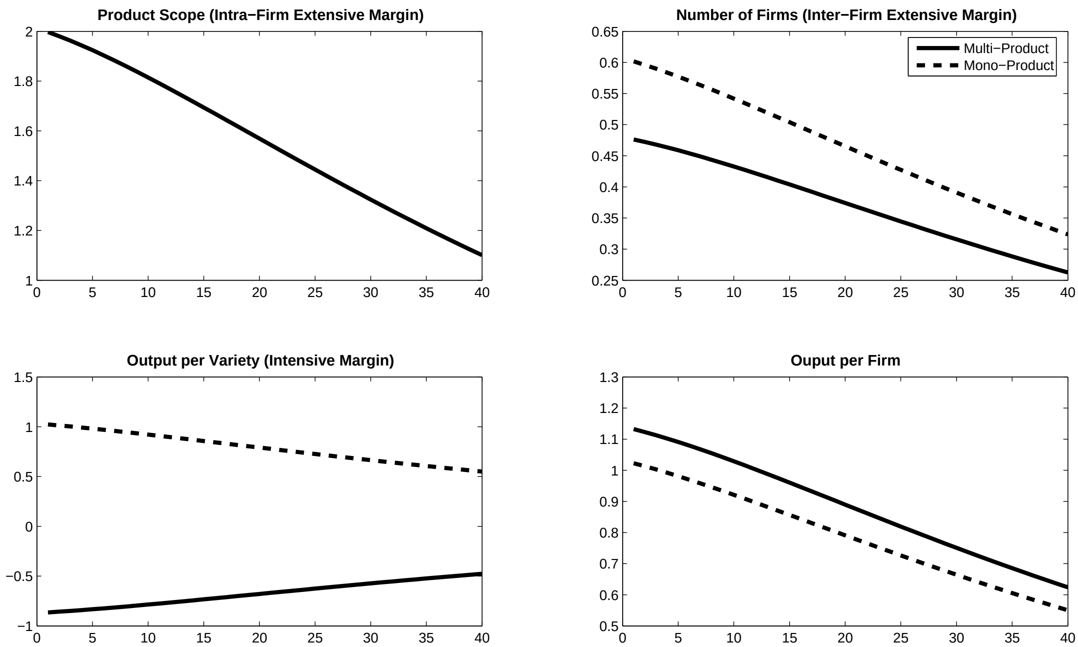

Figure 1: **Firm-level Dynamics: Multi-product vs Single-product Firms**. The picture shows the responses of a subset of firm-level variables to a 1% increase in the rate of technology. Impulse responses are measured in terms of percentage deviations from the steady state. Horizontal axes display the number of quarters after the shock.

responses of a selected variable to a 1% technology shock. The upper panels of Figures 1 and 2 depict the responses of the inter-firm extensive margin and the intra-firm extensive margin, respectively. We separate the two extensive margins because technology shocks not only affect the entry decisions of firms but may also create opportunities for companies to introduce new goods into the market. The lower-left panels show the responses of the intensive margin (output per variety) to the technology shock. Finally, the lower-right panels depict the dynamic responses of total output per firm. The continuous lines refer to the firm dynamics in our model, while the dashed lines denote those of the two alternative scenarios.

4.2.1 Impulse-response functions

In Figure 1, we plot the firm-level dynamics of our model and compare them with those obtained in a set-up in which firms are mono-product and behave as oligopolists. In this alternative scenario, each company takes into account the effects of its own price decisions on the aggregate price index (as in Jaimovich and Floetotto 2008). The upper-left panel of Figure 1 shows the dynamic response of the product scope in our model: a positive technology shock causes an expansion in firms' product ranges. This is in agreement with the empirical evidence reported in Broda and Weinstein (2010), where it is shown that product creation is strongly procyclical. This feature of the model is also interesting in light of the empirical evidence provided by Bernard *et al.* (2010) that highlights the importance of the extensive margin at the firm level.[11] As explained more extensively in Appendix C, the procyclicality of the product scope in our model is a direct consequence of the strategic effect of the product range. The intuition is as follows. An expansionary technology shock increases the aggregate demand for consumption and

[11] In particular, these authors find that roughly two-thirds of the average product's output is produced by existing firms. The remaining part is almost evenly split between firms that are adding or dropping products and entering or exiting firms. Thus, this evidence suggests that the inter-firm and intra-firm extensive margins are equally important for explaining firm-level dynamics.

investment goods and fosters the entry of new firms. Now, because the strategic effect of the product scope becomes less important as the number of firms increases, the entry of new companies increases the incentive to introduce new product lines. This results in an expansion in the size of each firm's product range.

The upper-right panel of Figure 1 shows that an expansionary technology shock has a positive impact on firm entry both when companies are multi-product and when they are single-product. This finding has important implications for markup variations. In fact, because the markup ratio is a decreasing function of the number of firms, a positive technology shock induces the entry of new companies, which decreases the average markup. In the single-product-firm model, the intra-firm extensive margin is missing, and the dynamic response of the inter-firm extensive margin is even stronger. Now, because the two models are calibrated to ensure that the long-run number of active firms is equal in the two scenarios, the mono-product-firm model is characterized by a larger number of competitors throughout the transition to the steady state. Intuitively, a larger number of products per firm increases the burden of the proliferation costs that incumbents must incur. Consequently, in the model with multi-product companies, the maximum number of firms that the market can accommodate with non-negative profits is smaller.

The lower-left panel of Figure 1 shows that output per variety responds positively to a technology shock when firms are mono-product and negatively when firms are multi-product. The reason is that output per variety coincides with output per firm in a mono-product-firm model. In our framework, when companies expand the sizes of their product ranges, the quantity of each variety produced decreases. Due to the *cannibalization* effect, multi-product firms reduce the production of their individual varieties when expanding their product scopes; in fact, the opening of a new product line cannibalizes sales of existing products.

Finally, the lower-right panel shows that output per firm responds positively to a technology shock in both models. Interestingly, the impact is more pronounced when firms are multi-product. This finding indicates that the increase in the intra-firm extensive margin is so large that it not only dominates the decrease in the intensive margin, but it also makes output per firm expand more in multi-product firms than it does in single-product ones.

In Figure 2, we compare our model with a different set-up in which firms are multi-product but behave as monopolistic competitors. As the upper-right panel of this figure shows, an expansionary technology shock has a positive impact on the number of firms in both models. The response is larger under monopolistic competition; in the latter case, the adjustment to a shock occurs only along the inter-firm extensive margin. In fact, as shown by the upper-left panel and lower-left panels, respectively, of Figure 2, the product scope and output per variety do not respond to a technology shock when firms behave as monopolistic competitors. Now, the intra-firm extensive margin is not working because the strategic effect of the product scope, which is responsible for the procyclicality of the product range, is missing in a model where firms' strategic interactions are neglected. Moreover, there is no adjustment along the intensive margin because an expansionary technology shock fosters the entry of new companies without modifying the size of each firm's product range or the price of each product variety. As shown in Appendix D, output per variety, $x_t = e_t/(M_t N_t p_t)$, remains constant because the number of firms, M_t, varies proportionally with the aggregate demand for consumption and investment goods, e_t. Finally, the lower right panel of Figure 2 shows that the response of output per firm to a technology shock is nil under monopolistic competition; this follows from the constancy of both the product scope and output per variety.

A clear picture emerges from the above analysis: in and of itself, the presence of multi-product firms is of limited interest for the analysis of firm-level dynamics. Instead, introducing firms' strategic interactions is fundamental to addressing both the countercyclicality of markups and the procyclicality of the product range.

4.3 Aggregate Dynamics

We now study the implications of multi-product firms for aggregate dynamics. Our aim in this section is to assess the contribution of the intra-firm extensive margin to the economy's response to changes in the technology shock, z_t. As in the previous section, we address this issue by performing several numerical experiments that compare

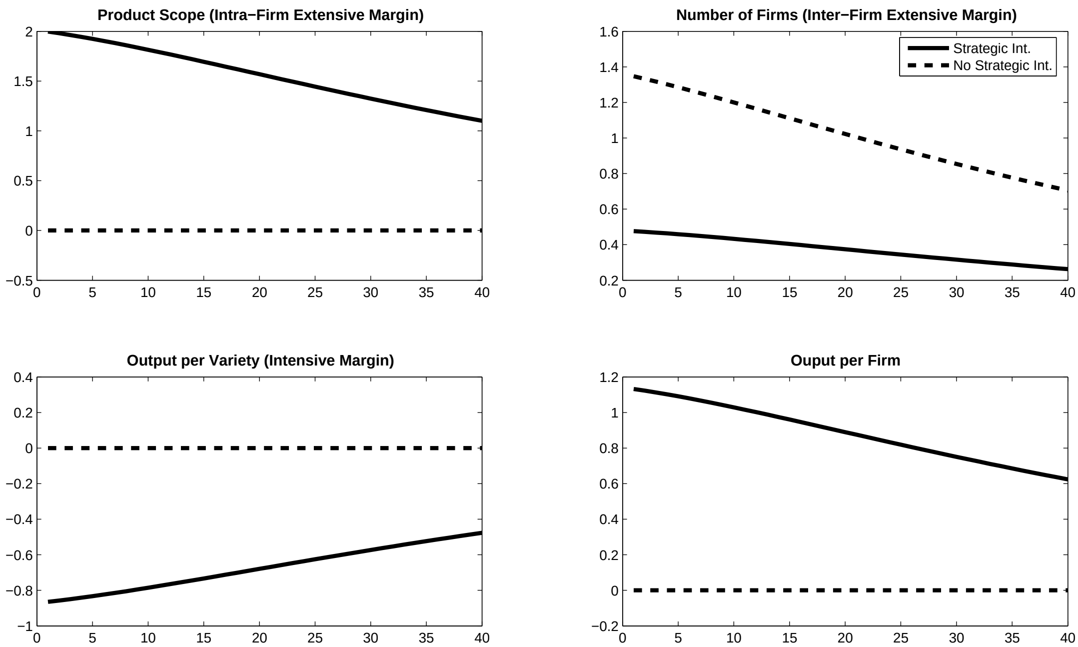

Figure 2: **Firm Dynamics: the role of Strategic Interaction**. The picture shows the responses of a subset of firm-level variables to a 1% increase in the rate of technology. Impulse responses are measured in terms of percentage deviations from the steady state. Horizontal axes display the number of quarters after the shock.

the properties of our model with those of other alternative setups, including the canonical RBC model with perfect competition. The results are summarized in Figure 3, where we display the impulse response functions of selected endogenous variables, and in Tables 2 and 3, where we evaluate the strength of the endogenous amplification mechanism (Table 2) and the ability of our model to replicate selected business cycle properties of U.S. data (Table 3). Model-based second-order moments involved in the computation of the statistics reported in Tables 2 and 3 are estimated using the following procedure. First, we draw 5000 sequences of technology shocks, each 200 periods in length. Second, we use the model to generate artificial series for selected endogenous variables. Finally, we detrend the artificial data with the HP-filter ($\lambda = 1600$), use the resulting series to compute second-order moments, and then take the mean of these moments over the 5000 replications. Additionally, in Table 2, we report statistics for the baseline case ($\mu = 1.3$) and for alternative values of the long-run average markup, μ, to evaluate how sensitive our results are to alternative calibrations.

4.3.1 The Endogenous Amplification Mechanism

We begin the analysis by testing whether adjustments along the intra-firm extensive margin operate, in aggregate, as a mechanism that boosts the response of output to exogenous technology shocks, z_t. Answering this question is important for evaluating whether this new margin of adjustment at the firm level strengthens the weak internal amplification mechanism that is embedded in the canonical RBC model with perfect competition.[12] To this end, we first define an appropriate measure of total factor productivity (TFP). Formally, given Eq. (24), the *effective*

[12]This is a well-known deficiency of the basic RBC model; the interested reader is referred to Cogley and Nason (1995) for further details.

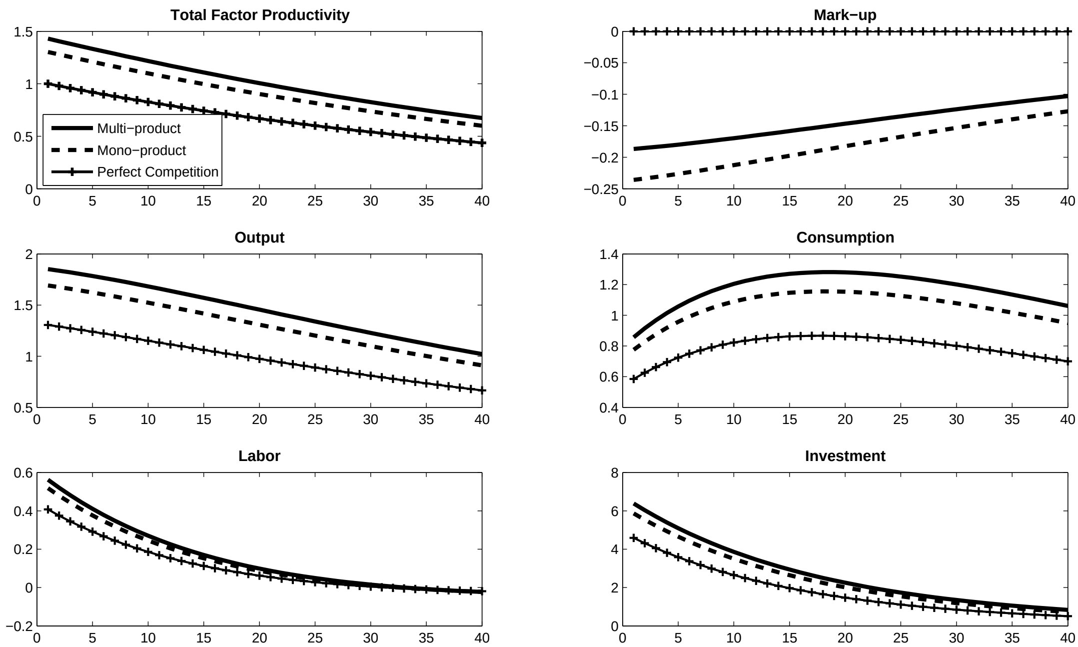

Figure 3: **Aggregate Dynamics**. The picture shows the responses of a subset of aggregate variables to a 1% increase in the rate of technology. Impulse responses are measured in terms of percentage deviations from the steady state. Horizontal axes display the number of quarters after the shock.

rate of TFP in our model can be simply defined as follows:[13]

$$TFP_t = \frac{Y_t}{H_t^{1-\alpha} K_t^{\alpha}} \equiv z_t \left(\frac{M_t^{\frac{1}{\theta-1}} N_t^{\frac{1}{\gamma-1}}}{\mu(M_t)} \right). \qquad (29)$$

In addition to the purely exogenous rate of technology, z_t, movements in TFP are also driven by an endogenous component, which is the term in brackets in Eq. (29). This factor captures the nature of the magnification mechanism that is embedded in a model with multi-product firms, endogenous entry and imperfectly competitive product markets. Accordingly, there are two sources of potential endogenous amplification: oligopolistic competition and product diversity. The first factor affects the effective TFP by inducing countercyclical markup variations through the process of firms' entry and exit (see the second panel of Figure 3). However, this source of amplification is not related to the multi-product nature of the firm because it works similarly in a model of oligopolistic competition with mono-product firms (see Jaimovich and Floetotto 2008). A positive technology shock creates new profit opportunities that induce firm entry. The increase in M_t, in turn, leads to a decline in the aggregate average markup, $\mu(M_t)$. Now, with lower markups, producers need to sell more goods to recover fixed costs.[14] Because TFP is measured only in terms of actual sales, the decline in the average markup is accompanied by an increase in the effective TFP.

The second source of potential endogenous amplification is related to the property that product diversity at the firm level induces increasing returns at the aggregate level. This is a direct consequence of the assumption that households' preferences exhibit the "love-for-variety" property. The mechanism is the same as the one through

[13] We use the word *effective* to distinguish total factor productivity from the purely exogenous technology shock, z_t.

[14] This follows from the zero-profit condition (22), which implies that the ratio of fixed costs to sales, $(\phi_f + \phi_v N_t)/(N_t x_t)$, being equal to $\mu(M_t) - 1$, increases in the average markup, $\mu(M_t)$.

which the process of firms' entry and exit amplifies the effects of technology shocks in a model with mono-product firms (see Chatterjee and Cooper 1993 and Bilbiie *et al.* 2007).[15] Intuitively, if households' preferences display the "love-for-variety" property, this means that a larger range of available products raises utility. Because each entrant introduces a new differentiated good into the market, when the number of active firms increases, households derive larger utility from a given nominal spending amount; thus, the price index of the variety basket declines. Given that the latter is the relevant price for households, the process of firms' entry also shifts the schedules of both aggregate labor and saving supply to the right.[16] Therefore, with an unexpected boost in the rate of technology, the resulting positive net-entry response encourages households to work harder and to accumulate more capital, thus amplifying the effect of the original shock.[17] In addition to the entry and exit channel, product diversity in the presence of multi-product firms has implications for aggregate fluctuations through variations in product scope. In such an environment, the product space depends not only on the number of active firms but also on the product range chosen by each of them. Consequently, any shock that fosters firm entry or expands firms' product ranges (or both) also boosts the aggregate level of output by expanding the set of available goods in the overall economy. This effect is captured by the two terms $M_t^{\frac{1}{\theta-1}}$ and $N_t^{\frac{1}{\gamma-1}}$ in Eq. (29), which provide the contribution of the inter-firm and intra-firm extensive margins to the effective TFP, respectively.

The endogenous amplification that is induced by the combined effects from both product diversity and oligopolistic competition is evident in the impulse response functions. Focusing on the first panel of Figure 3, we see that the response of the effective TFP (continuous line) substantially deviates from technology shocks (line with the + marker), being larger both at the impact (about 45% larger) and during the transition back to the steady state. The first row of Table 2 quantifies the relative strength of this effect by comparing the volatilities of technology shocks, σ_z^2, and effective TFP, σ_{TFP}^2. Because the ratio $\sigma_z^2/\sigma_{TFP}^2$ measures the marginal contribution of the exogenous factor z_t to the overall TFP volatility, the reported statistic provides information on how strongly the response of TFP deviates from technology shocks.[18] As shown in the first column of Table 2, in our model, the estimates of $\sigma_z^2/\sigma_{TFP}^2$ range from a minimum of 0.40 to a maximum of 0.67, meaning that the effective rate of TFP is between 49% and 150% more volatile than the technology shock (depending on the value chosen for the markup ratio, μ). Thus, a large fraction of TFP volatility can be explained by endogenous components in our framework, thereby indicating that the multi-product-firm model embodies a quantitatively important endogenous amplification mechanism.[19]

Exactly as in the case of a purely exogenous technology shock, changes in the effective rate of TFP translate into aggregate demand and supply fluctuations by directly affecting the relevant prices for both households and firms, w_t/q_t and r_t/q_t. It follows, therefore, that the boosting effect on TFP is transmitted to the rest of the economy through the usual channels.[20] As a consequence, output, consumption, investment, and hours worked are all more responsive in the economy with multi-product firms than in the canonical RBC model (as shown in panels 4-6 of Figure 3). The amplification effect appears to be particularly strong with respect to aggregate consumption and output; in both cases, it induces a response to a technology shock that is much larger than its counterpart in the

[15] Devereux *et al.* (1996) obtain the same result in an alternative set-up where households derive utility from a homogeneous consumption good produced by a competitive sector that aggregates intermediate goods using a CES production function. In such an environment, the mechanism that relates the process of entry and exit to economic fluctuations is driven by increasing returns to specialization at the aggregate level instead of the "love-for-variety" property.

[16] To ascertain this, notice that with mono-product firms, the labor supply is still defined as in Eq. (11) but with the price index q_t equal to $M_t^{1/(1-\theta)}$. *Ceteris paribus*, a larger number of active firms shifts the labor supply schedule to the right, and thus households are more willing to work. A similar conclusion holds true for saving supply.

[17] For further details on this mechanism, see Chatterjee and Cooper (1993).

[18] In detail, indicating the endogenous component in (29) with ω_t, the log-deviation of TFP with respect to the steady state can be written as $\hat{TFP}_t = \hat{z}_t + \hat{\omega}_t$. This implies that $\sigma_{TFP}^2 = \sigma_z^2 + \sigma_\omega^2 + 2Cov(\hat{\omega}_t, \hat{z}_t)$, where $Cov(\cdot)$ denotes the covariance operator. Hence, σ_z^2 captures the marginal contribution of the exogenous technology shock, while $\sigma_\omega^2 + 2Cov(\hat{\omega}_t, \hat{z}_t)$ measures the contribution of endogenous components to TFP volatility.

[19] By denoting the endogenous component in the effective TFP with $\hat{\omega}_t$, it is possible to prove that $Cov(\hat{\omega}_t, \hat{z}_t) > 0$. Thus, it follows from the previous footnote that the fraction of TFP volatility explained by endogenous components can be simply computed as 1-$\sigma_z^2/\sigma_{TFP}^2$.

[20] Dividing both (25) and (26) by q_t yields (i) $w_t/q_t = \alpha TFP_t(K_t/H_t)^{(1-\alpha)}$ and (ii) $r_t/q_t = (1-\alpha)TFP_t(H_t/K_t)^{\alpha}$. These equations are identical to those that would be obtained in the canonical RBC model but with TFP_t replaced with z_t.

Table 2: **Evaluating the endogenous magnification mechanism.**

Ratios	Oligopolistic Competition			Monopolistic Competition	Forward-Looking Entry		
	$\mu = 1.2$	$\mu = 1.3$	$\mu = 1.4$	$\mu = 1.3$	$\mu = 1.2$	$\mu = 1.3$	$\mu = 1.4$
$\sigma_z^2/\sigma_{TFP}^2$	0.67	0.52	0.40	0.80	0.67	0.52	0.40
$\sigma_y^2/\sigma_{y,RBC}^2$	1.56	2.02	2.56	1.32	1.56	2.02	2.55
$\sigma_{TFP}^2/\sigma_{TFP,MONO}^2$	1.10	1.20	1.29	1	1.10	1.21	1.28
$\sigma_y^2/\sigma_{y,MONO}^2$	1.09	1.20	1.28	1	1.10	1.20	1.27
$\sigma_\mu^2/\sigma_{\mu,MONO}^2$	0.45	0.63	0.76	1	0.45	0.62	0.75
$\sigma_m^2/\sigma_{m,MONO}^2$	0.45	0.63	0.76	1	0.45	0.62	0.75

Note: Variances of TFP, output (Y), net business formation (M), and markup (μ) are computed as follows. First, we take 5000 sequences of draws of technology shocks, each 200 periods in length. Second, we use the model to generate artificial series for Y, TFP, M and μ. Finally, we use the HP-filter ($\lambda = 1600$) with the log of each series x_t in order to extract fluctuations at the business cycle frequency, compute their standard deviations, and then take the mean of these statistics over the 5000 simulations. The subscripts MONO and RBC respectively refer to computations performed by using the model with mono-product firms, and with the RBC model with perfect competition. Forward-Looking Entry refers to computations based on a model in which entry is a forward-looking decision by firms, as in Bilbiie *et al.* (2007)

perfectly competitive economy. As concerns output, such an effect implies that in our framework, relative to the canonical RBC model, output volatility increases by 57% when $\mu = 1.2$, by 103% in the baseline calibration $\mu = 1.3$, and by 158% when μ is set equal to 1.4 (see the second row of Table 2). Hence, in our model, because of the endogenous amplification mechanism, the amount of volatility of technology shocks required to account for the same fluctuations in actual data is necessarily lower than that required by the standard RBC model.

To evaluate the contribution of the multi-product nature of firms to the above amplification effect, we now compare our framework with the alternative economy in which firms behave as oligopolists and are single-product, as in the model of Jaimovich and Floetotto (2008). Figure 3 performs such a comparison in terms of impulse response functions and shows that TFP, output and its components are all the more responsive when firms are multi-product rather than mono-product (compare continuous versus dashed lines). The third and fourth rows of Table 2 quantify this effect and show that, with adjustments along the intra-firm extensive margin, TFP and output volatilities increase by 9% when $\mu = 1.2$, by 20% when $\mu = 1.3$ and by 29% when $\mu = 1.4$. Thus, the presence of multi-product firms strengthens the endogenous amplification mechanism embodied in a model with endogenous entry and oligopolistic competition. Such a result is entirely driven by the *proliferation* effect, as both markup and firm entry are substantially less responsive when firms are multi-product rather than when they are mono-product (see the fifth and sixth rows of Table 2). The response of the intra-firm extensive margin is therefore so important that it not only compensates the effect of both the total number of firms and the markup but also strengthens the responses of both effective TFP and output in our set-up. It is worth emphasizing, however, that this result depends crucially on the presence of strategic interactions among firms. Shutting off such a channel (for instance by assuming monopolistically competitive producers) implies no differences between the response of the economy with and without multi-product firms (as shown in the second column of Table 2). We believe that our paper complements and extends the results provided in Jaimovich and Floetotto (2008) regarding the endogenous amplification mechanism that results from the interaction between net business formation and countercyclical markups. Our analysis shows that introducing multi-product firms in their environment makes the endogenous propagation mechanism even stronger.

Finally, in the third column of Table 2, we also report computations based on an alternative formulation in which the entry decision is forward looking (as in the model of Bilbiie *et al.* (2007)). In this environment, each prospective entrant decides whether to enter the market based on the post-entry present values of the firm, which

Table 3: **Second-Order Moments.**

$Variable(X)$	$\sigma(X)$			$\sigma(X)/\sigma(Y)$			$Corr(X_t, X_{t-1})$			$Corr(X_t, Y_t)$		
Y	1.81	1.21	1.72	1	1	1	0.84	0.70	0.70	1	1	1
C	1.35	0.58	0.84	0.75	0.48	0.49	0.8	0.75	0.75	0.88	0.96	0.95
I	5.3	4.22	5.88	2.93	3.50	3.43	0.87	0.69	0.68	0.8	0.98	0.98
H	1.79	0.38	0.52	0.99	0.31	0.30	0.88	0.68	0.68	0.88	0.97	0.97

Note: for each statistic we report (i) data to the left; (ii) computations based on the perfectly competitive economy in the center; (iii) computations based on the model with multi-product firms to the right.

depend on all future profit flows and on the exogenous exit probability.[21] Relative to our baseline specification in which firms' entry is governed by a period-by-period zero-profit condition, such an alternative framework captures a well-known property of the U.S. data: the coexistence between countercyclical markup movements and procyclical aggregate profits. However, relying on this alternative structure rather than on our baseline set-up does not in any respect modify the main conclusions of our paper. As can be seen by comparing the first and the third columns of Table 2, the results obtained in the two formulations are very similar. In particular, the magnification mechanism remains quantitatively important when entry is a forward-looking decision. We interpret this as a robustness check for our results.

4.3.2 Second-Order Moments

To further evaluate the properties of our framework, we compute standard second-order moments of our artificial economy for GDP, consumption, investment and hours worked and compare them with those obtained in actual data and in the standard RBC model. The results are summarized in Table 3, where, for each statistic, the first number is the empirical moment implied by the U.S. data (taken from King and Rebelo 1999), the second number is the moment obtained in the canonical RBC model with perfect competition, and the third number is the moment implied by our model. As a direct consequence of the stronger endogenous amplification mechanism, we see that the model with multi-product firms outperforms the RBC framework in terms of volatility of all of the endogenous variables. Most notably, variances of output and investment essentially match the data, as is evident from the first column of Table 3. However, our model faces the same difficulties as the canonical RBC model: (i) relative to GDP, consumption and hours worked are not sufficiently volatile (second column of Table 3); (ii) in comparison with the data, autocorrelations are generally too low (see the third column); and (iii) all of the endogenous variables are too strongly correlated with output, as shown by the fourth column of Table 3. Overall, we view the enhanced ability to replicate the second-order moments in the real data as a success of our model.

5 Conclusions

Recent empirical work by Bernard *et al.* (2010) and Broda and Weinstein (2010) indicates that a significant share of product creation and destruction in the U.S. industries occurs within existing firms and accounts for an important share of aggregate output. Consistent with this empirical evidence, in this paper, we relax the standard assumption of mono-product firms that is typically made in dynamic general equilibrium models. Building on the paper of Jaimovich and Floetotto (2008), we develop an RBC model with multi-product firms and endogenous strategic interactions. Oligopolistic behavior gives rise to countercyclical markups and procyclical product range: an expansionary shock to technology fosters firm entry and leads to an expansion in firms' product ranges. Due

[21]See Appendix F for a detailed presentation of this specification of our model.

to the proliferation effect induced by changes in product scope, we show that our model embodies a quantitatively important amplification mechanism.

The model has a number of potentially interesting extensions. First, the framework could be extended to an open-economy context to deal with optimal coordination of monetary and fiscal policy and international business cycle issues. Given the omnipresence and empirical importance of multi-product firms in the global economy, such an extension would be of great interest in the analysis of the relationship between international trade and aggregate macroeconomic fluctuations across countries.

Another important extension would be to introduce nominal rigidities in the form of sticky prices and to study the implications of multi-product firms for the analysis of inflation dynamics. Because pricing and product scope decisions by firms are interrelated, adjustments along the intra-firm extensive margin might affect the magnitude of price changes and thus the properties of inflation dynamics. Moreover, when prices are rigid, firms may react to market changes by using their product ranges, thereby potentially affecting the aggregate implications of nominal rigidities on both resource allocation and inflation dynamics. These issues are the focus of our future research.

A Price and demand elasticities

In this Appendix, we first derive the elasticity of the price index with respect to the price of a product variety. Then, we determine the demand elasticities of a good in response to variations of its own price and other goods' prices within the same nest.

Transforming the demand function $x_t(i,j)$ (10) into logarithms yields the following:

$$\ln x_t(i,j) = (\theta - 1)\ln q_t + \ln e_t - \gamma \ln p_t(i,j) - (\theta - \gamma)\ln q_t(i).$$

By using the fact that $q_t(i) = \left[\sum_{j=1}^{N_t(i)} p_t(i,j)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$, it is easy to show that $\partial \ln q_t(i)/\partial \ln p_t(i,j) = [p_t(i,j)/q_t(i)]^{1-\gamma}$. Thus, the effect of $p_t(i,j)$ on the market price index q_t can be expressed as:

$$\frac{\partial \ln q_t}{\partial \ln p_t(i,j)} = \frac{\partial \ln q_t}{\partial \ln q_t(i)} \frac{\partial \ln q_t(i)}{\partial \ln p_t(i,j)} = \left[\frac{q_t(i)}{q_t}\right]^{1-\theta} \left[\frac{p_t(i,j)}{q_t(i)}\right]^{1-\gamma}.$$

Using this result, we can write the two demand elasticities as follows:

$$\frac{\partial \ln x_t(i,j)}{\partial \ln p_t(i,j)} = -\gamma - (\theta - \gamma)\left[\frac{p_t(i,j)}{q_t(i)}\right]^{1-\gamma} + (\theta - 1)\left[\frac{q_t(i)}{q_t}\right]^{1-\theta} \left[\frac{p_t(i,j)}{q_t(i)}\right]^{1-\gamma}, \tag{A.1}$$

$$\frac{\partial \ln x_t(i,k)}{\partial \ln p_t(i,j)} = -(\theta - \gamma)\left[\frac{p_t(i,j)}{q_t(i)}\right]^{1-\gamma} + (\theta - 1)\left[\frac{q_t(i)}{q_t}\right]^{1-\theta} \left[\frac{p_t(i,j)}{q_t(i)}\right]^{1-\gamma}, \text{ for } k \neq j. \tag{A.2}$$

B Pricing

In this Appendix, we determine the optimal pricing strategy; this corresponds to solving the second stage of the game. Firm i maximizes profits (14) under the constraint (13). The Lagrangian for this maximization problem is as follows:

$$\mathcal{L} = \sum_{j=1}^{N_t(i)} [p_t(i,j)x_t(i,j) - w_t h_t(i,j) - r_t k_t(i,j)] + \lambda \left\{ \sum_{j=1}^{N_t(i)} \left[z_t k_t(i,j)^\alpha h_t(i,j)^{1-\alpha} - \phi_v\right] - \phi_f - \sum_{j=1}^{N_t(i)} x_t(i,j) \right\}.$$

The optimality conditions are:

$$\frac{\partial \mathcal{L}}{\partial p_t(i,j)} = 0 \Rightarrow x_t(i,j) + \sum_{j=1}^{N_t(i)} [p_t(i,j) - \lambda] \frac{\partial x_t(i,j)}{\partial p_t(i,j)} = 0, \tag{B.1}$$

$$\frac{\partial \mathcal{L}}{\partial h_t(i,j)} = 0 \Rightarrow -w_t + (1-\alpha)\lambda z_t k_t(i,j)^\alpha h_t(i,j)^{-\alpha} = 0, \tag{B.2}$$

$$\frac{\partial \mathcal{L}}{\partial k_t(i,j)} = 0 \Rightarrow -r_t + \alpha \lambda z_t k_t(i,j)^{\alpha-1} h_t(i,j)^{1-\alpha} = 0. \tag{B.3}$$

Combining Eqs. (B.2) with (B.3), we obtain the Lagrange multiplier λ amounting to:

$$\lambda = \frac{w_t^{1-\alpha} r_t^\alpha}{z_t (1-\alpha)^{1-\alpha} \alpha^\alpha},$$

which corresponds to the marginal cost of producing one more variety, mc_t. Using (B.2) and (B.3) and replacing λ

with mc_t, the total cost of production by firm i boils down to:

$$\sum_{j=1}^{N_t(i)} [w_t h_t(i,j) + r_t k_t(i,j)] = \sum_{j=1}^{N_t(i)} mc_t z_t k_t(i,j)^{\alpha} h_t(i,j)^{1-\alpha} = mc_t \left\{ \sum_{j=1}^{N_t(i)} [x_t(i,j) + \phi_v] + \phi_f \right\}.$$

Consequently, firm i's profits are written as:

$$\sum_{j=1}^{N_t(i)} [p_t(i,j)x_t(i,j) - mc_t x_t(i,j)] - mc_t [N_t(i)\phi_v + \phi_f]. \tag{B.4}$$

Using Eqs. (A.1) and (A.2) into Eq. (B.1) and replacing λ with mc_t, we obtain the following:

$$x_t(i,j) - \frac{\gamma x_t(i,j)}{p_t(i,j)} [p_t(i,j) - mc_t] - \sum_{k=1}^{N_t(i)} \frac{x_t(i,k)}{p_t(i,j)} [p_t(i,k) - mc_t] \left\{ \theta - \gamma - (\theta - 1) \left[\frac{q_t(i)}{q_t}\right]^{1-\theta} \right\} \left[\frac{p_t(i,j)}{q_t(i)}\right]^{1-\gamma} = 0.$$

Substituting for $x_t(i,j)$ using (10) in the above equation yields:

$$e_t \left[\frac{q_t(i)}{q_t}\right]^{1-\theta} - \gamma e_t \left[\frac{q_t(i)}{q_t}\right]^{1-\theta} \frac{[p_t(i,j) - mc_t]}{p_t(i,j)} = \sum_{k=1}^{N_t(i)} x_t(i,k) [p_t(i,k) - mc_t] \left\{ \theta - \gamma - (\theta - 1) \left[\frac{q_t(i)}{q_t}\right]^{1-\theta} \right\}. \tag{B.5}$$

The right-hand side of this equation is the same for all $j \in [1, N_t(i)]$. We conclude that firm i charges the same price for all of the product varieties that it produces, that is, $p_t(i,j) = p_t(i)$ for all $j \in [1, N_t(i)]$.

In what follows, we solve Eq. (B.5) for $p_t(i)$. Before proceeding, we must first observe that the term $[q_t(i)/q_t]^{1-\theta}$ in Eq. (B.5) amounts to firm i's market share $\epsilon_t(i) \equiv q_t(i)x_t(i)/e_t$, where $x_t(i) \equiv x_t^C(i) + x_t^I(i)$ denotes total demand of consumption and investment goods produced by firm i. This can be easily verified by using Eq. (9). Now, because $q_t(i) = \left[\sum_{j=1}^{N_t(i)} p_t(i,j)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$ and $q_t = \left[\sum_{i=1}^{M_t} q_t(i)^{1-\theta}\right]^{\frac{1}{1-\theta}}$, firm i's market share $\epsilon_t(i)$ can be written as:

$$\epsilon_t(i) \equiv \left[\frac{q_t(i)}{q_t}\right]^{1-\theta} = \frac{N_t(i)^{\frac{1-\theta}{1-\gamma}} p_t(i)^{1-\theta}}{\sum_{i=1}^{M_t} N_t(i)^{\frac{1-\theta}{1-\gamma}} p_t(i)^{1-\theta}}. \tag{B.6}$$

Then, using Eqs. (10) and (B.6) into Eq. (B.5) and rearranging terms yields:

$$p_t(i) = mc_t \frac{[\theta - (\theta - 1)\epsilon_t(i)]}{(\theta - 1)[1 - \epsilon_t(i)]}. \tag{B.7}$$

As can be seen, Eq. (B.7) coincides with Eq. (15) in the text.

C Product scope

In this Appendix, we determine firms' optimal product scope. To proceed, we rearrange Eq. (B.4) to write firm i's gross profits as a function only of the market share $\epsilon_t(i)$. Substituting $x_t(i,j)$ from (10) into (B.4) and then using $q_t(i) = \left[\sum_{j=1}^{N_t(i)} p_t(i,j)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$ and (B.6) to simplify yields:

$$\pi_t(i) = e_t \frac{\epsilon_t(i)[p_t(i) - mc_t]}{p_t(i)} - mc_t [N_t(i)\phi_v + \phi_f].$$

By using the definition of the Lerner index of market power, $L_t(i) \equiv [p_t(i) - mc_t]/p_t(i) = 1/[\theta - \epsilon_t(i)(\theta - 1)]$, in the above equation, we can write the gross profits of firm i as:

$$\pi_t(i) = e_t \epsilon_t(i) L_t(i) - mc_t [N_t(i)\phi_v + \phi_f].$$

Differentiating $\pi_t(i)$ with respect to $N_t(i)$ yields the first order condition:

$$\frac{\partial \pi_t(i)}{\partial N_t(i)} = \frac{e_t \epsilon_t(i) L_t(i)}{N_t(i)} \eta_t(i) - mc_t \phi_v = 0, \tag{C.1}$$

where $\eta_t(i) = \theta L_t(i) \cdot [\partial \epsilon_t(i)/\partial N_t(i)] \cdot [N_t(i)/\epsilon_t(i)]$ is the elasticity of variable profits of firm i with respect to the size of its product range, $N_t(i)$. In what follows, we first calculate $\eta_t(i)$; then, we use this result in (C.1) and determine the optimal firms' product scope. Now, differentiating $\epsilon_t(i)$ from Eq. (B.6) with respect to $N_t(i)$, we obtain:

$$\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = \frac{(\theta - 1)}{(\gamma - 1)} \frac{\epsilon_t(i)}{N_t(i)} - (\theta - 1)\epsilon_t(i) \left[\frac{1}{p_t(i)} \frac{\partial p_t(i)}{\partial N_t(i)} - \frac{1}{q_t} \frac{\partial q_t}{\partial N_t(i)} \right]. \tag{C.2}$$

The first term on the right hand side of Eq. (C.2) gives the increase in the market share resulting from the opening of a new product line when firms' strategic interactions are neglected (monopolistic competition). In such a case, the Lerner index of market power $L_t(i)$ is equal to $1/\theta$ so that the elasticity of variable profits with respect to the level of product proliferation amounts simply to $(\theta - 1)/(\gamma - 1)$. Given the assumption that $\theta < \gamma$, this elasticity is lower than one. This feature of the model reflects the *cannibalization* effect: the opening of a new product line reduces the sales of the firm's existing varieties so that the introduction of a new variety leads to a less than proportionate increase in variable profits. The second term on the right side of Eq. (C.2) captures the *strategic* effect of the product scope, that is, the effect of the firm's product range choice on both its own and all other firms' pricing decisions. As we will see shortly, the partial derivatives $\partial p_t(i)/\partial N_t(i)$ and $\partial q_t/\partial N_t(i)$ in the square brackets of Eq. (C.2) are respectively positive and negative; thus, the *strategic* effect of the product range decision always leads to a contraction in firms' product scope with respect to monopolistic competition.

To proceed with the calculation of $\partial \epsilon_t(i)/\partial N_t(i)$, we first compute the partial derivative $\partial q_t/\partial N_t(i)$. Because $q_t = \left[\sum_{k=1}^{M_t} N_t(k)^{\frac{1-\theta}{1-\gamma}} p_t(k)^{1-\theta} \right]^{\frac{1}{1-\theta}}$, after some rearrangement, we can express $\partial q_t/\partial N_t(i)$ as:

$$\frac{\partial q_t}{\partial N_t(i)} = q_t^{\theta} \left[\sum_{k=1}^{M_t} N_t(k)^{\frac{1-\theta}{1-\gamma}} p_t(k)^{-\theta} \frac{\partial p_t(k)}{\partial N_t(i)} + \frac{1}{1-\gamma} N_t(i)^{\frac{1-\theta}{1-\gamma}-1} p_t(i)^{1-\theta} \right]. \tag{C.3}$$

Now, we show that the first term in the square brackets in the above equation is equal to zero. In fact, by using Eq. (B.7), we write the inverse of the markup as:

$$\frac{p_t(i)}{p_t(i) - mc_t} = \theta - \epsilon_t(i)(\theta - 1).$$

Summing the above equation over i yields:

$$\sum_{i=1}^{M_t} \frac{p_t(i)}{p_t(i) - mc_t} = 1 + (M_t - 1)\theta,$$

which indicates that the sum of inverse markups over all firms is independent of firms' product scopes. Differentiating the above equation with respect to $N_t(i)$, we obtain:

$$\sum_{k=1}^{M_t} \frac{mc_t}{[p_t(k) - mc_t]^2} \frac{\partial p_t(k)}{\partial N_t(i)} = 0.$$

Under symmetry, this equation boils down to $(M_t - 1)\partial p_t(k)/\partial N_t(i) + \partial p_t(i)/\partial N_t(i) = 0$. Using this result in (C.3) and imposing $p_t(k) = p_t$ and $N_t(k) = N_t$ for every $k \in [1, M_t]$, the first term in the square brackets of Eq. (C.3) cancels out, and we can write $\partial q_t/\partial N_t(i)$ as:

$$\frac{\partial q_t}{\partial N_t(i)} = -q_t^{\theta} \frac{q_t(i)^{1-\theta}}{(\gamma - 1)\, N_t(i)}.$$

As anticipated, the partial derivative $\partial q_t/\partial N_t(i) < 0$ so that an increase in the level of product proliferation $N_t(i)$ reduces the aggregate price index q_t. Plugging $\partial q_t/\partial N_t(i)$ back into (C.2) yields:

$$\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = \frac{(\theta - 1)}{(\gamma - 1)} \frac{\epsilon_t(i)}{N_t(i)} [1 - \epsilon_t(i)] - (\theta - 1) \frac{\epsilon_t(i)}{p_t(i)} \frac{\partial p_t(i)}{\partial N_t(i)}. \tag{C.4}$$

Now, we need to compute the partial derivative $\partial p_t(i)/\partial N_t(i)$. Thus, we differentiate $p_t(i)$ from Eq. (B.7) with respect to $N_t(i)$ to obtain:

$$\frac{\partial p_t(i)}{\partial N_t(i)} = \frac{mc_t}{(\theta - 1)\,[1 - \epsilon_t(i)]^2} \frac{\partial \epsilon_t(i)}{\partial N_t(i)}.$$

Replacing $\partial p_t(i)/\partial N_t(i)$ into Eq. (C.4) and then using Eq. (B.7) to simplify yields:

$$\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = \frac{(\theta - 1)}{(\gamma - 1)} \frac{\epsilon_t(i)}{N_t(i)} \frac{[1 - \epsilon_t(i)]^2\, [\theta - \epsilon_t(i)\,(\theta - 1)]}{\{\theta\,[1 - \epsilon_t(i)] + (\theta - 1)\, \epsilon_t(i)^2\}}. \tag{A.14}$$

Because $\partial \epsilon_t(i)/\partial N_t(i) > 0$, we obtain that $\partial p_t(i)/\partial N_t(i) > 0$, which indicates that increasing the number of a firm's varieties raises the firm's own price. In light of these results, one can easily comprehend the reason why the *strategic* effect of the product range decision leads to a contraction of the product scope. When firms' strategic interactions are neglected, an increase in the level of proliferation $N_t(i)$ raises the market share, $\epsilon_t(i)$, without affecting the price, $p_t(i)$, which remains constant. In our strategic set-up, firms proliferate less because they use the product range as a tool to relax price competition. According to Eq. (B.7), price $p_t(i)$ is positively related to the market share $\epsilon_t(i)$. Thus, an increase in the level of proliferation $N_t(i)$ raises the market share $\epsilon_t(i)$, which then elevates price $p_t(i)$. Because $(M_t - 1)\partial p_t(k)/\partial N_t(i) + \partial p_t(i)/\partial N_t(i) = 0$, the increase in $p_t(i)$ is accompanied by a decrease in other firms' prices. Consequently, firm i tries to mitigate price competition in the second stage of the game by under-expanding its product scope in the first stage.

To conclude, we replace $\epsilon_t(i) = 1/M_t$ into Eq. (C.5), and then we use this equation to derive the elasticity of variable profits of a typical firm i with respect to the size of its product range:

$$\eta_t = \frac{(\theta - 1)}{(\gamma - 1)} \left[\frac{\theta (M_t - 1)^2}{M_t \theta\,(M_t - 1) + \theta - 1} \right].$$

Finally, plugging η_t into the first-order condition (C.1) and then using (B.7) to simplify, we write the optimal product scope as:

$$N_t = \frac{e_t \theta\,(M_t - 1)}{\phi_v\,(\gamma - 1)\, p_t\, [M_t \theta (M_t - 1) + \theta - 1]}.$$

As can be easily ascertained, the elasticity of variable profits with respect to the level of product proliferation is an increasing function of the number of firms, M_t. When the latter becomes very large, this elasticity tends to $(\theta - 1) / (\gamma - 1)$, as in the case of monopolistic competition. Intuitively, the larger the number of firms, the smaller the impact that a change in the market share $\epsilon_t(i)$ has on the price $p_t(i)$ (see Eq. (B.7)). Because the *strategic* effect of the product scope becomes less and less important as M_t increases, the incentive to create new varieties increases with the number of firms. This feature of the model is responsible for the procyclicality of the product scope. Intuitively, a positive technology shock $z_t > 0$ is equivalent to an expansion of the market because it increases the aggregate demand of consumption and investment goods, e_t. This, in turn, fosters the entry of new firms into the product market. Because the elasticity of variable profits with respect to the level of product proliferation is an increasing function of M_t, the increase in the equilibrium number of firms is accompanied by an expansion of the product scope.

D Monopolistic competition

In this Appendix, we solve the model of monopolistic competition when firms are multi-product. Following a similar procedure to that used in Appendix C, we write the gross profits of firm i as:

$$\pi_t(i) = e_t\epsilon_t(i)L_t(i) - mc_t\left[N_t(i)\phi_v + \phi_f\right].$$

Because firms' strategic interactions are neglected under monopolistic competition, the optimal price is a constant mark-up over marginal cost:

$$p_t(i) = mc_t\frac{\theta}{(\theta-1)}. \tag{D.1}$$

The Lerner index of market power $L_t(i)$ is equal to $1/\theta$; consequently, firm i's profits can be written as:

$$\pi_t(i) \quad = \quad e_t\frac{\epsilon_t(i)}{\theta} - mc_t\left[N_t(i)\phi_v + \phi_f\right].$$

We now differentiate the above equation with respect to $N_t(i)$. Because the *strategic* effect of the product scope is absent, the elasticity of variable profits with respect to the level of product proliferation is equal to $(\theta-1)/(\gamma-1)$. Solving the first-order condition under symmetry and then using (D.1) to simplify yields:

$$N_t = \frac{e_t}{\phi_v\left(\gamma-1\right)p_tM_t}. \tag{D.2}$$

Substituting this result back into firm i's profits and replacing mc_t with $p_t\left(\theta-1\right)/\theta$, we obtain the following:

$$\pi_t(i) = \frac{e_t\left(\gamma-\theta\right)}{\left(\gamma-1\right)M_t\theta} - p_t\frac{(\theta-1)}{\theta}\phi_f.$$

Under free entry, firms enter until their profits fall to zero. Setting $\pi_t(i) = 0$, and solving for M_t yields:

$$M_t = \frac{e_t\left(\gamma-\theta\right)}{p_t\left(\gamma-1\right)\left(\theta-1\right)\phi_f}. \tag{D.3}$$

The equilibrium number of varieties produced by firm i can be found by substituting (D.3) into (D.2):

$$N_t = \frac{\phi_f}{\phi_v}\frac{(\theta-1)}{(\gamma-\theta)}.$$

Finally, by replacing N_t and M_t into $x_t \equiv e_t/\left(M_tN_tp_t\right)$, we easily obtain output per firm:

$$x_t = \phi_v\left(\gamma-1\right).$$

As can be seen, N_t and x_t are determined only by exogenous parameters. A positive technology shock $z_t > 0$ increases the aggregate demand of consumption and investment goods, e_t, and fosters the entry of new firms into the product market, M_t. Now, as Eq. (D.3) shows, M_t varies proportionally to e_t. Consequently, under monopolistic competition, the product range and output per variety are not affected by the technology shock.

E Forward-looking entry decision

We have so far assumed that the process of entry/exit is governed by the static zero-profit condition. In this Appendix, we modify this hypothesis by assuming that entry is a forward-looking decision, as in Bilbiie *et al.* (2007). The rest of the model is left unchanged.

In every period, there are M_t incumbents producing multiple products and an unbounded mass of prospective entrants. Entering firms are forward looking and thus correctly anticipate their future profits, $\pi_t(i)$, and the probability, δ_m, of exiting the market. Therefore, entrants in period t compute their expected post-entry value, $v_t(i)$, as:

$$v_t(i) = E_t \sum_{s=t}^{\infty} (1-\delta_m)^{s-t} r_{t,s} \pi_s(i),$$

where $r_{t,s}$ is the stochastic discount factor. There is a sunk entry cost, ψ, expressed in terms of units of output. The equilibrium equates the present discounted value of future profits (expressed in real terms) to the upfront entry cost:

$$v_t(i)/q_t = v_t/q_t = \psi, \; \forall i \in [1, M_t].$$

The assumption made on the timing of entry and exit implies that the number of incumbents at time t is given by:

$$M_t = (1-\delta_m) M_{t-1} + M_t^e,$$

where M_t^e is the number of firms entering the market at time t. Incumbents produce goods for both investment and consumption purposes via a Cobb-Douglas technology. We assume that the same production function is used for the creation of new firms. Therefore, the technology of setting up M_t^e new firms is given by:

$$\psi M_t^e = z_t \left(H_t^e\right)^{1-\alpha} \left(K_t^e\right)^{\alpha},$$

where H_t^e and K_t^e are the amounts of labor and capital, respectively, devoted to the creation of new firms.

In this environment, households may invest both in capital and in new firms and receive pure profits from the firms they own. Therefore, the flow budget constraint of a representative household is:

$$\sum_{i=1}^{M_t} \sum_{j=1}^{N_t(i)} p_t(i,j) \left[x_t^C(i,j) + x_t^I(i,j)\right] + v_t M_t^e \leq w_t H_t + r_t K_t + M_t \pi_t.$$

The first-order conditions for the household's optimal plan are described by Eqs. (8)-(12) and augmented by the first-order condition with respect to M_t^e:

$$\frac{v_t - \pi_t}{q_t} = \beta \left(1-\delta_m\right) E_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-1} \frac{v_{t+1}}{q_{t+1}} \right\}.$$

The market clearing condition in the goods sector is $Y_t^g \equiv C_t + I_t$, where Y_t^g denotes total production in this sector.

Because $e_t = q_t(C_t + I_t)$, we easily obtain the following:

$$Y_t^g \equiv C_t + I_t = \frac{e_t}{q_t} = \frac{M_t N_t x_t p_t}{q_t}.$$

Normalizing the price p_t to 1 and, then, substituting for $N_t x_t$ using (13), the above equation can be rewritten as:

$$Y_t^g = \frac{z_t (K_t^g)^\alpha (H_t^g)^{1-\alpha} - (\phi_v N_t + \phi_f) M_t}{q_t},$$

where K_t^g and H_t^g denote, respectively, the total amounts of capital and labor employed in the goods sector. Making similar substitutions into Eq. (14) and then plugging in w_t and r_t respectively from Eqs. (25) and (26), individual profits expressed in real terms, π_t/q_t, can be written as:

$$\begin{aligned}
\frac{\pi_t}{q_t} &= \frac{1}{q_t M_t}\left[z_t (K_t^g)^\alpha (H_t^g)^{1-\alpha} - (\phi_v N_t + \phi_f) M_t - w_t H_t^g - r_t K_t^g\right] \\
&= \frac{1}{M_t}\left\{\left[1 - \frac{1}{\mu_t(M_t)}\right] z_t \frac{(K_t^g)^\alpha (H_t^g)^{1-\alpha}}{q_t} - \frac{(\phi_v N_t + \phi_f) M_t}{q_t}\right\} \\
&= \left[1 - \frac{1}{\mu_t(M_t)}\right] \frac{Y_t^g}{M_t} - \frac{\phi_v N_t + \phi_f}{\mu_t(M_t) q_t}.
\end{aligned}$$

Aggregate output, capital and labor in the overall economy are given, respectively, by:

$$\begin{aligned}
Y_t &= Y_t^g + \frac{v_t}{q_t} M_t^e, \\
K_t &= K_t^g + K_t^e, \\
H_t &= H_t^g + H_t^e.
\end{aligned}$$

Because capital and labor receive the same return in both sectors, the following condition must hold in equilibrium:

$$\frac{K_t^e}{H_t^e} = \frac{K_t^g}{H_t^g}.$$

Numerical experiments based on this alternative set-up are performed by calibrating the model with the same restrictions of the baseline multi-product-firm model (see Section 4.1). Additionally, as in Bilbiie *et al.* (2007), we set the exit probability, δ_m, to 0.025, while the sunk entry-cost, ψ, is pinned down from the steady-state equilibrium as a result of our calibration restrictions.

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