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a political economy model with altruistic agents and retirement concerns

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Financing public education: a political economy model with altruistic agents and retirement concerns

Amedeo Piolatto[*]

Abstract

Public services provision depends on tax proceeds. The tax rate to finance public school is chosen through majority voting. Under the monotonicity condition implying that the preferred tax rate is decreasing in income, the literature predicts that the median voter is decisive and poor agents form a coalition against rich agents. I show that this does not occur and a coalition of the type "ends against the middle" occurs if agents care about others' level of education. I use a OLG model, in which adults are altruist and retirees' pension depends on average education (used as a proxy for productivity).

1 Introduction

The 2008 financial crisis further sharpened public budget concerns, calling for a reduction of the deficit and a reform of the welfare system. This may be problematic for politicians caring about re-election. Reforms must be sustainable, i.e., a majority of voters must support them. This paper provides a positive analysis of the voters' preferred level of public investment in education, when the public authority provides two different goods: education and social security.

Education and the retirement system share several features: both are publicly provided private goods,[1] they are often used for redistribution purposes, they tend to be financed through the income tax and they receive full attention from the media at any attempt of reform. Despite those similarities, few papers consider simultaneously the provision of the two goods: Kaganovich and Zilcha (1999) focuses on the optimal (in terms of growth) allocation of tax proceeds between education and social security. Similarly, Pecchenino and Pollard (2002) and Zhang and Zhang (2004) study the impact of social security and education on growth, the first through a theoretical model and the second through a cross-country regression. They all conclude that, at least in the long run, education has a positive effect on growth and, consequently, a more generous pension system is sustainable.

The literature on education is broad; spanning from its consequences on growth and spillovers (Romer 1986, Lucas Jr. 1988, Gradstein and Justman 1997), to the signaling role on the labour market (Spence 1973), or to its role in the reduction of conflicts (Lott 1990, Usher 1997, Gradstein 2000).

The literature on the political economy of education is the closest to this paper (Epple and Romano 1996a, 1996b, 1998, Glomm and Ravikumar 1998, Chen and West 2000, Cohen-Zada and Justman 2003, Gradstein and Justman 2005, Dur and Glazer 2008, and Piolatto 2010). It considers education as a consumption good and studies the optimal consumption behaviour; agents attend either a publicly financed school or a (costly) private one and vote over the tax rate to finance public education. The wills of the majority of voters are used as a proxy for the public authority decision. Although departing from the way representative democracies work, the fact that voters influence the decisional process (e.g., through strikes, referenda or the election process) and the interest in education of the public opinion explains why this is common in the literature.

From Stiglitz (1974), it is known that the median voter theorem does not apply because preferences for education are not single peaked in the presence of a dual (public-private) system. To overcome this technical difficulty, it is common to

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[1] I disregard the discussion of why education, being not a public good, is publicly financed. Readers may refer to Lochner and Moretti (2001), Checchi (2003), or Cai and Treisman (2004).

introduce a monotonicity assumption (implying a Single Crossing Property analogous to the Spence-Mirrlees condition). The two most common assumptions, defined as Slope Rising in Income (SRI) and Slope Decreasing in Income (SDI), imply that the preferred tax rate (conditional on attending public school) is respectively increasing or decreasing in income.[2] For instance, under SDI a higher income corresponds to a lower most preferred tax rate: if a voter is in favour of no-tax, so does anyone richer; when a voter's preferred tax rate is positive, all poorer voters prefer a larger one. The central difference behind the two assumptions lays in the relative weight of two components of public education. An increase in the tax rate implies both more redistribution and better quality of the public service. When the former element dominates the latter, preferences show the SDI property and viceversa.

The empirical literature cannot clarify which between SDI and SRI is more realistic. Epple and Romano support the SRI assumption (e.g., in Epple and Romano 1998 and Epple et al. 2004), claiming that the resulting "ends against the middle" equilibrium corresponds to what empirically observed. On the opposite, Justman among others (see Gradstein, Justman, and Meier 2005) supports the SDI assumption on the base of some studies of the shape of the utility function. My paper shows that the two claims are compatible, with the equilibrium under SDI that, under many circumstances, is of the "ends against the middle" type. It also shows that the common result that the median voter is pivotal under SDI is not robust to the introduction of education externalities. When voters are aware of those externalities, the equilibrium coalition regroups heterogeneous social classes and the median voter is not decisive. Furthermore, SDI is no longer sufficient to guarantee the existence of an equilibrium.

It is common to assume that citizens care about their children education and that this is sufficient to justify the desire for public education; this doesn't explain the evidence that also some voters without children are in favour of it. This interest may be motivated by direct (e.g., impact of others' education on wages or pension) or indirect (e.g., altruism, social stability, growth, increase in the quality of life) reasons. My contribution differs from the previously cited political economy literature in that it integrates those two features. I propose a positive analysis of the public provision of education in which voters are aware of the positive return (through pensions) of the educational system and they are altruistic. I focus on the types of coalition that form when voting over the tax to finance public school.

To feature the concern for social security and for the level of education of society, I propose a model à la Epple and Romano, in an overlapping generations

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[2]Technically, it is a condition on the sign of the derivative of the marginal rate of substitution between education and the numeraire: non-positive under SDI and non-negative under SRI.

(OLG) society in which agents' utility depends on others' education: i) agents are altruistic, and ii) population's average instruction affects agents' utility. Introducing an OLG structure allows to account for the intergenerational externalities (retired people's welfare depends on new generations' productivity). I introduce the concern for average education through pensions, but results would not be qualitatively affected when considering, for example, public services' quality increase or social frictions' reduction in more educated societies.

The structure of the paper is as follows: section 2 describes the model, section 3 presents some preliminary results, and section 4 describes the possible equilibria. Section 5 provides some comparative statics results, while section 6 concludes.

2   The model

Three cohorts (students, adults and retirees) live at the same time. There is no population growth: each adult has one child. At the end of each period, adults retire and children become adults. Education is compulsory and both publicly and privately provided: the two are mutually exclusive. The quality of education, modelled as per-student expenditure, is denoted by X_P for the public sector and X_R for the private one. $\overline{X}$ is the average quality of education, measured in terms of the cohort's average spending for instruction.

Public education is financed through a proportional tax (t) on income (ω) and its access is free. I assume the quality of education in the public sector to be homogeneous over the country.[3] Private schools are costly, and students can choose the level of quality to buy.[4]

Adults vote over the universal tax rate to finance public education, and choose the instruction that their child receives (public or private). For private school students, the decision includes the share of budget devoted to education; the remaining is used to consume the numeraire b. When deciding, they are concerned by their children's education and their own consumption of the numeraire (both in the current and in the next period of their life). As previously discussed, adults care about others' education too: this enters into the model through the presence of both a direct (through altruism) and an indirect (through pensions) positive externality of others' education. Altruism is modelled through a utility function that depends positively on average education (and thus on the tax rate that finances public school). Its impact may increase in income.[5]

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[3] A notable exception is the U.S.A., where public school is financed through local taxes on real estates, thus richer areas have a higher per-student expenditure.

[4] It can be a person choosing amongst different schools, or a single school proposing extra services (e.g., elective classes, gym, library, extra-curricular activities) à la carte.

[5] There may be several rationales for that (as discussed in Dur and Teulings 2001): richer

Assumption 1 (Altruism) *Agents are altruistic, i.e., they care about the average level of education (regardless of the consequences that this has on their own income). Altruism enters the utility function through $V : \Re^2 \to \Re$ depending both on $\overline{X}$ and on individuals' income ω_z.*

The average income of each generation is assumed to be a positive function of the education received: increasing the tax rate increases the average educational level and also agents' future income. This affects retirees' pension, via the social security system.[6]

Their utility function, W_z, assumed to be separable, is

$$W_z(X, b_z, b_{z+1}, \overline{X}, \omega_z) = U(X, b_z) + G(b_{z+1}) + V(\overline{X}, \omega_z) \tag{1}$$

where U is the utility during adulthood (depending on consumption and owns children's school quality), G is the utility of consumption when retired, and V is the altruistic component. Adults face three trade-offs: 1) consumption of the numeraire versus their child's education, 2) current versus future consumption, and 3) own consumption versus average education of society (that comes from the altruistic component of the utility function).

For the sake of simplicity, people do not discount future. Income, endogenous and time dependent, in period z is distributed on the interval $(\omega_{\min}, \omega_{\max})_z$, with density f_z and cumulative distribution F_z. The average income is $\overline{\omega}_z = \int_{\omega_{\min}}^{\omega_{\max}} \omega_z f(\omega_z) d\omega_z$; ω_{med} is the median. Besides the tax to finance public education, a tax of rate s is levied to finance pensions: adults' disposable income is $\beta\omega$, where $\beta = [1 - t - s]$). Retirees' only source of income is given by their pension, which is not taxed.

The pension system is of the type Pay-As-You-Go consisting, similarly to Casamatta et al. (2000b), of a contributory and a noncontributory part. The pension system is mixed: in period $z+1$ a retiree receives $\alpha s \cdot \omega_z + [1 - \alpha]s \cdot \overline{\omega}_{z+1}$. That is: he receives a fraction $\alpha \in [0, 1]$ of the tax for pension ($s \cdot \omega_z$) paid in period z (contributory or Bismarkian component), and a share $[1 - \alpha]$ of the average contribution $s \cdot \overline{\omega}_{z+1}$ paid by current workers (noncontributory component); both s and α are exogenous.[7]

Assumption 2 (Regularity conditions) *Goods are normal (income elasticity takes values in the interval $(0, 1)$), functions U, G and V are of class C^2 or higher,*

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people have lower marginal utilities of consumption; income and education are often positively correlated, as well as education and the importance attributed by people to education; richer people tend to care more about social stability (and education has an impact on it); finally education can be a cheaper and preferable way to redistribute among social classes.

[6]We obtain analogous results considering an indirect effect of education on the quality of life, such as an increase in public goods.

[7]See Casamatta et al. (2000b) for a model on retirements with vote over s.

U is increasing and concave both in X and b, and there are no cross effects between X and ω $\left(i.e., \frac{\partial^2 U}{\partial \omega \partial X} = 0\right)$. G is increasing and concave in consumption and V is increasing and concave in average education quality.

Assumption 2 has an important consequence on the impact of a change in the tax rate:

Lemma 1 (Aggregate expenditure for education) *Aggregate educational expenditure is increasing in the tax rate t: $\frac{\partial}{\partial t}\left(t\overline{\omega}_z + \int_{\omega_{\min}}^{\omega_{\max}} X_R(\omega_z)d\omega_z\right) \geq 0$*

Proof. See Appendix A. ■

Corollary 1.1 *Education is measured in terms of expenditure. This implies that average education is increasing in the tax rate.*

Corollary 1.2 *Altruism is increasing in the tax rate: $\frac{dV(t_z,\omega)}{dt_z} \geq 0$: by the chain rule, V increases in average educational expenditure and thus in the tax level.*

Assumption 3 (Relation income-education) *Adults' income ω is an increasing and concave function of the average level of education: $\frac{\partial \omega}{\partial \overline{X}} \geq 0$ and $\frac{\partial^2 \omega}{\partial \overline{X}^2} \leq 0$.*

Assumption 3 allows us to capture the presence of some positive externalities of agents' education on productivity.

Corollary 3.1 *Given assumption 3 and corollary 1.1, we conclude that $\overline{\omega}_z$ is a function of t_{z-1}, with $\partial \overline{\omega}_z/\partial t_{z-1} > 0$ and $\partial^2 \overline{\omega}_z/\partial t_{z-1}^2 \leq 0$. Henceforth, we replace the notation $\overline{\omega}_{z+1}$ by $\overline{\omega}(t_z)$, to stress the intertemporal relation.*

Remark: Income is endogenous: it depends on t. Given that I do not need a specific density function and I do not compute the steady state level of t, I do not need to study how income changes through time (which would require to assume a specific functional form for the utility, and to model the production side of the economy). Even though the density function might change over time, and so its extremes ($\omega_{\min}, \omega_{\max}$), model's results are not affected, since they only concern the way coalitions are formed, not the tax rate absolute value. The consequences of a generalised increase in income are a priori unknown, even keeping prices constant. The assumptions on the preferred tax (SRI and SDI) only concern the preferred tax of an agent, *ceteris paribus*.[8] When others' income level changes and the tax rate remains constant, tax proceeds (per student expenditure) rise. If utility functions are homothetic, the preferred tax rate remains constant, otherwise it might increase or decrease through time.

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[8]In particular, when others' contribution is constant.

Assumption 4 (SDI) *The monotonicity condition Slope Decreasing in Income (SDI) holds for function U: the marginal rate of substitution between instruction and the numeraire is decreasing in income $\left(\frac{\partial MRS_{X,b}}{\partial \omega} \leq 0 \right)$.*

Population is normalised to one and the share of students attending a public school is n_p. From the government budget constraint, the quality of public school X_P is equal to

$$X_P = \frac{t\overline{\omega}}{n_p} \tag{2}$$

Adults' consumption of the numeraire is equal to $b_z = \beta\omega - X_R$ (disposable income net of the expenditure for private education). To stress that numeraire's consumption is different for agents attending a private school, for the numeraire consumption level in period z I use the notation $b_z = \beta\omega$ when $X_R = 0$ (agents attending public school) and $b_{z,R} = \beta\omega - X_R$ when $X_R \neq 0$.

Equation 1, defining the lifetime utility of an agent, can be rewritten as follows

$$W_z =$$

$$U(X_P, b_z)+ \quad G(b_{z+1})+ \quad V(t_z, \omega) \text{ for } X_R = 0 \tag{3a}$$

$$U(X_R, b_{z,R})+ \quad G(b_{z+1})+ \quad V(t_z, \omega) \text{ for } X_R \neq 0, \tag{3b}$$

where 3a represents the utility of an agent whose child attends a public school, while 3b is for the others.

All relevant decisions are taken by adults. They choose their offsprings' kind of school (public or private), they vote over the tax rate t and (if their children attend a private school) they choose how to share their budget between progenies' school tuition and the numeraire. Retirees consume all their pension to buy the numeraire good, which implies that a retiree's consumption of numeraire in period $z+1$ is given by $b_{z+1} = \alpha s \cdot \omega_z + [1-\alpha]s \cdot \overline{\omega}(t_z)$.

Lemma 2 *The third derivatives of functions U and G with respect to their arguments are positive, that is: $\frac{\partial^3 U}{\partial X^3} \geq 0$, $\frac{\partial^3 U}{\partial b^3} \geq 0$, $\frac{\partial^3 G}{\partial b^3} \geq 0$.*

Proof. See Appendix B. ∎

3 Preliminary results

I first introduce some results from Glomm and Ravikumar (1998) and Epple and Romano (1996a) that I use subsequently. In subsections 3.1 and 3.2 I analyse respectively the behaviour of parents that are preferring public and private school, and determine their preferences over the tax rate.

Adults' choice between public and private education only depends on the first part of the utility function (U), which shows the same properties as the utility function in Epple and Romano (1996a) and Glomm and Ravikumar (1998). I determine the preferred consumption choice for an agent, for a given tax rate. I conclude that: i) an income threshold $\hat{\omega}(t)$ exists,[9] such that agents prefer public school if and only if $\omega < \hat{\omega}$, while for $\omega = \hat{\omega}$ they are indifferent; ii) the threshold $\hat{\omega}$ determines a unique equilibrium.

The maximisation problem of an agent whose child attends a private school is:

$$\max_{X_R} W_z = U(X_R, b_{z,R}) + G(b_{z+1}) + V(t_z, \omega) \tag{4}$$
$$s.t.\ b_{z,R} = (1 - t - s)\omega - X_R$$
$$b_{z+1} = \alpha s \cdot \omega_z + [1 - \alpha]s \cdot \overline{\omega}_{z+1}$$

The level of X_R maximising (4) depends on t and it solves the equation: $\frac{\partial U}{\partial X_R} = \frac{\partial U}{\partial b}$. I define $X_R^* = \arg\max(W_z)$. The value for t also defines the numeraire consumption of adults choosing public education, this being all their disposable income.

Lemma 3 (From Epple and Romano (1996a)) *Agents prefer private schooling, if and only if $U(X_R^*, b_{z,R}) > U(X_P, b_z)$. Given t, the level of income $\hat{\omega}(t)$ that makes an agent indifferent between the alternatives is unique. For any income $\omega > \hat{\omega}(t)$, $U(X_R^*, b_{z,R}) > U(X_P, b_z)$; vice versa, if $\omega < \hat{\omega}(t)$, then $U(X_R^*, b_{z,R}) < U(X_P, b_z)$.*

Proof. See Appendix C. ∎

The number of students in public school, n_p, depends on the distribution of income:[10] $n_p = F(\hat{\omega})$.

Lemma 4 (From Glomm and Ravikumar (1998)) *For all $t \in (0, 1)$, it exists a unique n_p that solves $n_p = F(\hat{\omega})$. It can be observed that $\hat{\omega}$ is decreasing in n_p, while increasing in $\overline{\omega}$ and in t.*

Proof. See Appendix D. ∎

The first property $\left(\frac{\partial \hat{\omega}}{\partial n_p} < 0\right)$ comes from congestion: increasing the number of students in public school (given the tax rate), the per-student available resources decreases. A change in the tax rate influences $\hat{\omega}$ through two channels: it increases the total resources for public education, and it reduces agents' disposable income: numeraire's consumption falls and its marginal utility increases. Out of equilibrium, a further (indirect) effect of a tax increase provokes an increase in n_p, which causes a reduction of $\hat{\omega}$.

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[9] When possible, I use the shorter notation $\hat{\omega}$

[10] See Glomm and Ravikumar (1998) for more details and properties of n_p.

3.1 The behaviour of voters that, in equilibrium, prefer public school ($\omega < \hat{\omega}$)

The utility function of agents whose child attends public school (i.e., $\omega < \hat{\omega}$) is given by (3a), which can be rewritten as:

$$W_z = U\left(\frac{t\overline{\omega}}{n_p}, \beta\omega\right) + G(\alpha s \cdot \omega_z + [1 - \alpha]s \cdot \overline{\omega}(t_z)) + V(t_z, \omega) \tag{5}$$

The maximisation of this expression with respect to t_z yields the following first order condition:

$$\omega_z \frac{\partial U}{\partial b} = \frac{\overline{\omega}_z}{n_p}\frac{\partial U}{\partial X} + k\frac{\partial\overline{\omega}(t_z)}{\partial t_z}\frac{\partial G}{\partial b} + \frac{\partial V}{\partial t_z} \tag{6}$$

where factor $k = s[1 - \alpha]$ relates retirees' pension in time $z + 1$ to adults' average income in $z + 1$, which depends on the current tax rate t_z. It is an equity measure, i.e., it computes how much the society is redistributing, and it is equal to 0 for $\alpha = 1$ (purely Bismarckian/contributory system). A decrease in k implies a reduction in adults' marginal benefits of increasing the current education tax. Equation (6) implicitly defines $t^*(\omega)$, the preferred tax rate of an agent of income ω, equating the marginal cost and the marginal benefit of increasing the tax. The left hand side represents the loss of utility caused by a reduction in the consumption of the numeraire during the current period. The right hand side includes the additional utility generated by i) the higher quality of education that the own child receives (first term), ii) the increase in future consumption, through the increase in pensions (second term), iii) the altruistic attitude component, for which the average education level matters (last term).

Note that if pensions are not related to the educational level (i.e., the pension system is purely contributory or education doesn't affect wages) and agents are not altruistic ($\frac{\partial V}{\partial t_z} = 0$), we are back to the model of Epple and Romano (1996a).

Equation (6) can be rewritten as:

$$\omega_z \frac{\partial U}{\partial b} - \frac{\overline{\omega}_z}{n_p}\frac{\partial U}{\partial X} = k\frac{\partial\overline{\omega}(t_z)}{\partial t_z}\frac{\partial G}{\partial b} + \frac{\partial V}{\partial t_z} \tag{7}$$

The right hand side accounts for the second period consumption and the benevolence attitude. Always positive, it is a function of the Bismarckian factor, of the impact of t on average income, of the importance of the second period consumption and of altruism. The left hand side regroups the effects of a tax rate change on the first-period utility. It takes negative values in the unrealistic case of $\frac{\partial U/\partial X}{\partial U/\partial b} > \frac{n_p\omega_z}{\overline{\omega}}$, which is a condition on the marginal rate of substitution between education and current consumption, occurring if the marginal utility of education is so large that agents prefer not to consume the numeraire in the first period of life.

Proposition 1 *For a voter whose child is attending a public school, the preferred tax changes with income as follows:*

$$\frac{\partial t^*}{\partial \omega} = -\frac{-\frac{\partial U}{\partial b} - \beta \omega_z \frac{\partial^2 U}{\partial b^2} + k\alpha s \frac{\partial \overline{\omega}(t_z)}{\partial t_z} \frac{\partial^2 G}{\partial b^2} + \frac{\partial^2 V}{\partial t \partial \omega}}{\omega_z^2 \frac{\partial^2 U}{\partial b^2} + \left[\frac{\overline{\omega}_z}{n_p}\right]^2 \frac{\partial^2 U}{\partial X^2} + k\frac{\partial^2 \overline{\omega}(t_z)}{\partial t_z^2} \frac{\partial G}{\partial b} + \left(k\frac{\partial \overline{\omega}(t_z)}{\partial t_z}\right)^2 \frac{\partial^2 G}{\partial b^2} + \frac{\partial^2 V}{\partial t^2}}. \tag{8}$$

The sign of (8), given by

$$sign\left(\frac{\partial t^*}{\partial \omega}\right) = sign\left(-\frac{\partial U}{\partial b} - \beta \omega_z \frac{\partial^2 U}{\partial b^2} + k\alpha s \frac{\partial \overline{\omega}(t_z)}{\partial t_z} \frac{\partial^2 G}{\partial b^2} + \frac{\partial^2 V}{\partial t \partial \omega}\right), \tag{9}$$

depends on the altruism factor: altruism absent, it would always be negative.

Proof. See Appendix E. ∎

The SDI assumption (implying $-\frac{\partial U_z}{\partial b} - \beta\omega\frac{\partial^2 U_z}{\partial b^2} \leq 0$) in Epple and Romano (1996a) ensures that the preferred tax is decreasing in income; here we have two additional terms that contribute to determine if the preferred tax is increasing in income or not. Taking into account the effects of the generosity component, the preferred tax rate is increasing in income if:

$$\frac{\partial^2 V}{\partial t \partial \omega} \geq \left(\frac{\partial U}{\partial b} + \beta \omega_z \frac{\partial^2 U}{\partial b^2}\right) - k\alpha s \frac{\partial \overline{\omega}(t_z)}{\partial t_z} \frac{\partial^2 G}{\partial b^2}$$

and decreasing otherwise. On the right hand side, both the bracket and the other term are positive: the intuition is that the first period dis-utility from an increase in t is increasing in income (elements in the bracket) and the second period utility variation for a tax increases is decreasing in income (elements outside the bracket).

Define $\breve{\omega}$ as the income for which i.e., $\frac{\partial t^*(\breve{\omega})}{\partial \omega} = 0$:

$$\left[\frac{\partial^2 V}{\partial t \partial \omega}\right]_{\omega=\breve{\omega}} = \left[\frac{\partial U}{\partial b} + \beta \omega_z \frac{\partial^2 U}{\partial b^2} - k\alpha s \frac{\partial \overline{\omega}(t_z)}{\partial t_z} \frac{\partial^2 G}{\partial b^2}\right]_{\omega=\breve{\omega}}$$

. Then the second derivative of (6) with respect to ω establishes if the preferred tax rate is concave or convex in income, thus if $\breve{\omega}$ is a minimum or a maximum. The second order condition is:

$$k \cdot (\alpha s)^2 \frac{\partial \overline{\omega}(t_z)}{\partial t_z} \left[\frac{\partial^3 G}{\partial b^3}\right] - 2\beta \frac{\partial^2 U}{\partial b^2} - \beta^2 \omega \frac{\partial^3 U}{\partial b^3} + \frac{\partial^3 V}{\partial t \partial \omega^2}. \tag{10}$$

When equation 10 is positive, the preferred tax is convex in income, while it is concave when 10 is negative. By lemma 2, the first two terms are always positive, the third is always negative; we have imposed no restrictions on the shape of V, so that the fourth term can be either positive or negative. The shape of the preferred tax depends on: i) the impact on agents utility of the numeraire consumption

(both in their first and second period of life) and ii) the shape of function V (how income affects altruism). Both new elements (with respect to Epple and Romano (1996a)) affect the derivative sign.[11]

The sign of 10 depends on the relative weight of each component, it is extremely complicated to give a clear intuition, without analysing the many different combinations determining a constant sign of the equation. I solve the model under for the alternative conditions of preferred tax concave (the sign of 10 is negative, as depicted in figure 1, left chart) and convex (10 is positive, as in figure 1, right chart) in income.
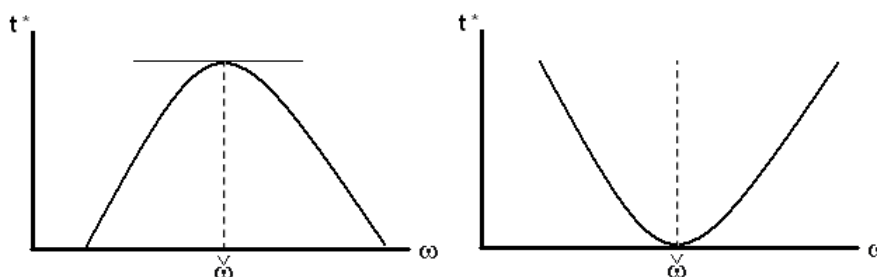

Figure 1: Preferred tax concave (left) and convex (right) in income

3.2 The behaviour of voters that, in equilibrium, prefer private school ($\omega > \hat{\omega}$)

In Epple and Romano (1996a), parents of private school students are always in favour of no tax. The additional factors introduced (pension concerns and altruism) affect their behaviour. The utility function of agents whose children (at equilibrium) attend private school (i.e., $\omega > \hat{\omega}$) is given by 3b, which can be rewritten as:

$$W_z = U(X_R^*, \beta\omega - X_R^*) + G(\alpha s \cdot \omega_z + [1 - \alpha]s \cdot \overline{\omega}(t_z)) + V(t_z, \omega) \qquad (11)$$

From the F.O.C. we derive the preferred tax rate, defined implicitly as:

$$\omega\frac{\partial U}{\partial b} = k\frac{\partial \overline{\omega}_z}{\partial t_z}\frac{\partial G}{\partial b} + \frac{\partial V}{\partial t_z}. \qquad (12)$$

By the envelope theorem, a change in t does not affect X_R^*. Compared to (6), in (12) one term misses: the marginal cost of decreasing consumption in the first per-

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[11]Although I assume that the SDI assumption holds, the condition on the concavity/convexity of the preferred tax do not use this assumption. The only role of the SDI assumption is to allow easily comparisons with similar papers and to simplify some demonstrations (using some results from Epple and Romano (1996a)); the model results are not affected.

iod equals the sum of consumption benefits in the second period and the additional utility connected with benevolence.

Given income, the preferred tax is smaller if one chooses a private school: The $\frac{\overline{\omega}}{n_p}\frac{\partial U}{\partial X}$ term is missing, which means that people do not care about the quality of public school *per se.*. Moreover, numeraire consumption is lower (because of the school tuition to be paid), thus the marginal cost of an additional reduction in consumption is larger. In fact, $\beta\omega - X_R^* < \beta\omega$ (where $\beta\omega$ is the consumption of numeraire of people attending public school), thus the marginal cost of the tax $(\omega\frac{\partial U}{\partial b})$ for adults choosing a private school for their offsprings is larger than if they had chosen a public school.

Proposition 2 *For voters whose child attend a private school, the preferred tax changes with income as follows:*

$$\frac{\partial t^*}{\partial \omega} = -\frac{-\frac{\partial U}{\partial b} - \beta\omega_z \frac{\partial^2 U}{\partial b^2} + k\alpha s \frac{\partial \overline{\omega}(t_z)}{\partial t_z}\frac{\partial^2 G}{\partial b^2} + \frac{\partial^2 V}{\partial t \partial \omega}}{\omega^2 \frac{\partial^2 U}{\partial b^2} + k\frac{\partial^2 \overline{\omega}(t_z)}{\partial t_z^2}\frac{\partial G}{\partial b} + k^2 \frac{\partial^2 G}{\partial b^2} + \frac{\partial^2 V}{\partial^2 t}}. \tag{13}$$

Altruism absent, the preferred tax is always decreasing in income.

Proof. See Appendix F. ■

Qualitatively, the shape of t^* is the same for agents with income $\omega > \hat{\omega}$ and $\omega < \hat{\omega}$, and it is represented in figure 1.

3.3   Comparison

I compare previous results, starting from the shape of the function determining the preferred tax and the value of stationary points.

Proposition 3 *The sign of 8 and 13 is the same. While the preferred tax rate depends on voters school choice, its sign does not. For all voters, t^* is either concave or convex in income and the stationary point $\breve{\omega}$ is the same.*

Proof. See Appendix G. ■

The preferred tax by an agent changes with income. According to the shape of the altruism function V (if it is increasing or decreasing in income), and to its relative impact on the utility function, the preferred tax can be concave or convex in income. The previous proposition says that the concavity of t^* does not depend on adults school choice for their child nor it does $\breve{\omega}$, the stationary point (maximum or minimum) of the function. Absent altruism, the preferred tax would be always decreasing in income; pension concerns affect the value of the preferred tax and possibly the slope of the function but, under the SDI assumption, it does not modify the sign of the first derivative, which remains negative.

It would be useful to know if $\breve{\omega}$ is greater than $\hat{\omega}$, but this can be computed only if we assume a specific functional form for V and restrict the values of some parameters of the model. The value of the preferred tax depends on pensions, on altruism and on the school choice of voters.

Proposition 4 *The preferred tax rate is always larger for agents if their children attend public school than otherwise.*
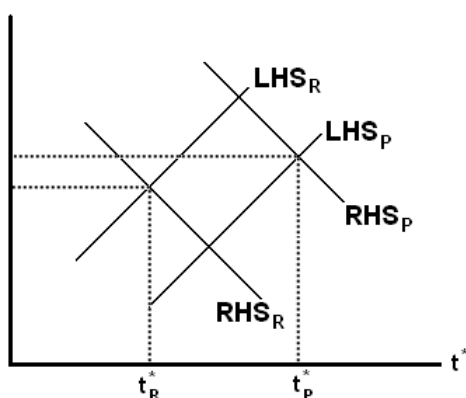
Proof. See Appendix H. ∎

Figure 2: Preferred tax under public and private regime

4   Equilibrium

Without additional assumptions on the shape of the utility functions, in Epple and Romano (1996a) the median voter is always decisive. Their equilibrium is clearly not robust to the introduction of altruism and pension concerns. It is not possible to solve the extended model and to derive general properties, computing the equilibrium tax rate in a closed form, without introducing specific assumptions on the functional forms. It is nevertheless possible to solve the model graphically, which allows us to describe the type of equilibrium and to forecast the composition of coalitions (verifying, for instance, if coalitions are homogeneous in income, as it is in Epple and Romano 1996a).

Why the preferred tax rate can be either concave or convex in income? In this model we observe many contrasting effects: for $\omega > \hat{\omega}$ (private school costumers) the direct effect of a tax is always negative (it is only a cost). Indirect effects are positive (through the change on the average level of education and income). An increase in t induces a rise in consumption when retired; the impact is larger for the poorest agents, if the pension system is highly Beveridgean, also rich agents are in

favour of a large tax rate (it is the unique way to have a high level of consumption in the last period of their life). The benevolent attitude induces all voters to be in favour of positive tax rates. The willingness to smooth consumption over time (intertemporal elasticity of substitution) and to balance the marginal utility of the numeraire and of education (intra-goods elasticity of substitution), and the degree of benevolence of the society, all contribute to determine the degree of convexity.

I firstly consider the case in which the preferred tax is concave in income, then I move to the opposite case. By definition, half of the population would prefer an increase in the equilibrium tax, while the other half would prefer to decrease it. Figure 3 helps to distinguish all possible equilibria: the preferred tax rate (depending on income) is depicted for both cases of public school and private school attendance. Since we observe a discontinuity in the preferred tax rate for the income level $\omega = \hat{\omega}$ (the income of the agent indifferent between public and private school), the equilibrium might depend on the position of $\hat{\omega}$ relative to $\breve{\omega}$. From left to right, the first line of figure 3 depicts: a possible shape of agents' utility functions (depending on their schooling choice), and the two possible equilibria when $\hat{\omega} \geq \breve{\omega}$. The bottom line, instead, consider the three cases in which $\hat{\omega} \leq \breve{\omega}$.
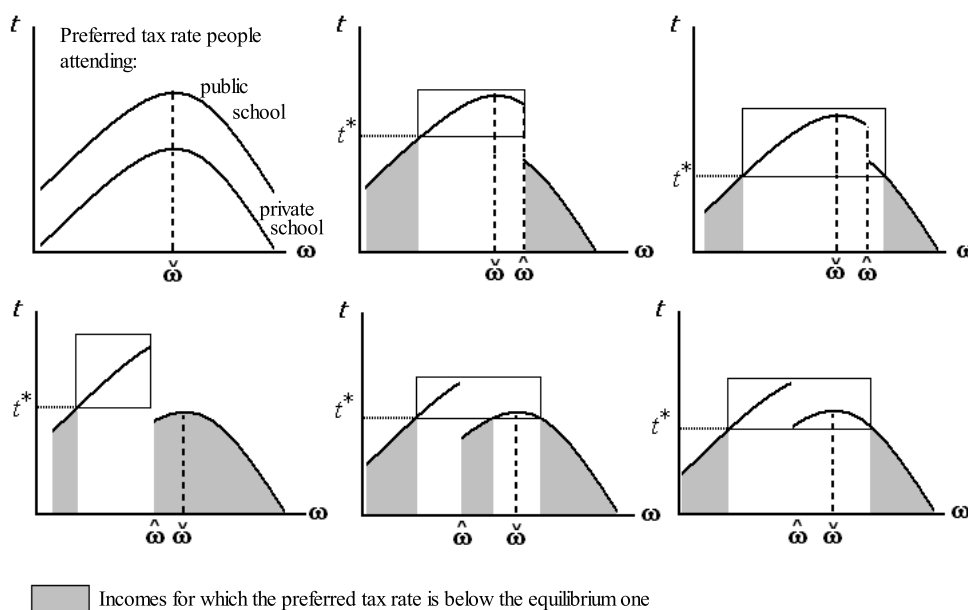

Figure 3: The concave case - different equilibria

To anticipate the possible coalitions, we should assemble all agents with the highest preferred tax rate, up to form a group of half of the population. Coloured areas, in the graphs, correspond to levels of income for which households prefer a reduction in the tax rate. Rectangles identify the preferred tax rates of those agents that are in favour of an increase in the tax rate. The intersection point(s) of

the rectangle and of the preferred tax chart(s) allow(s) to identify the Condorcet winner(s) that, a priori, is different from the preferred tax rate by the indifferent, the median voter, and $\breve{\omega}$.[12]

We start with the concave case when $\hat{\omega} \geq \breve{\omega}$. The largest preferred tax rate is the one of $\breve{\omega}$. The coalition of voters asking for an increase in the tax rate includes agents with income close to $\breve{\omega}$. The population density function affects the composition of the coalition, which includes (the richest) people choosing public school (top centre picture in figure 3), and possibly some agents preferring to have their children attending a private school (top right picture).[13] This depends on whether at least half of voters i) choose public school and ii) have a preferred tax rate larger than all those who choose a private one. The coalition is of the "ends against the middle" kind, with agents having an income sufficiently close to $\breve{\omega}$ that form a coalition facing the richest and poorest agents in society.

When $\hat{\omega} < \breve{\omega}$ (bottom row in the figure) the indifferent voter has the largest preferred tax. Amongst people attending private schools the preferred tax is increasing in a first time and decreasing for $\omega > \hat{\omega}$, the poorest agents preferring private school (i.e., $\omega \in (\omega, \hat{\omega})$) might join the middle class coalition or the other one. The left and right pictures depict two situations analogous to those described in the previous paragraph. The bottom-left and bottom-right graphs, more precisely, depict the cases in which a standard "ends against the middle" equilibrium occurs, the difference is that in the left case only public school costumers ask for an increase in the tax while in the right graph even the poorest private school costumers (driven by the willingness to increase their future consumption) vote for an increase in the tax rate. The reason why the richest people in the country are in favour of a tax reduction comes from the fact that the marginal cost for them of an increase in the tax rate is very high, due to their gross income. The centre-bottom picture is a limit case, in which the half of population willing to have an increase in the tax rate includes the richest voters choosing public school and a subset (which is not the poorest one) of agents choosing private education.

Reasons are diverse why people with income $\omega > \hat{\omega}$ may join the coalition, this explain the variety in results. When those in favour of an increase in the tax are the poorest agents in society, this can be due, on top of altruism and pension concerns, by the fact that an increase in public school quality may be sufficient for them to move from private to public education. Contrary, in the case depicted in the bottom-centre graph, the poorest people with $\omega > \hat{\omega}$ do not want any further

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[12]Remind that the Condorcet winner is the preferred tax rate, winning all pairwise comparisons; the indifferent voter is the one that reaches the same level of satisfaction choosing the private or the public school for his child.

[13]In this case, the coalition is composed by the richest public-school consumers and the poorest private-school consumers.

reduction in numeraire consumption when adults (having a large marginal utility of consumption), while altruism and the willingness to consume more when retired induce some richer people to be in favour of an increase in the tax rate.

Proposition 5 (Equilibrium in the concave case) *Two equilibria, of the type "ends against the middle", are possible under concavity of the preferred tax. The richest and poorest agents in the society ask for a reduction of the tax rate, while the middle class prefers an increase in it; among voters preferring private schools, the poorest ones might, or not, join the middle-class coalition.*

Proof. See Appendix I. ■

When the preferred tax rate is convex in income (i.e., equation 10 is positive), the preferred tax rate and equilibria are depicted in Figure 4. The top-left chart superposes the preferred tax rate of an agent, as a function of income, depending on the chosen kind of school. Equilibrium is computed in the same way as before: we decrease the tax rate up to the point in which half of voters prefer a larger tax rate and the other half asks for a decrease of it.

Four types of equilibrium can occur. We separate the case of $\hat{\omega} < \breve{\omega}$ (depicted in top-centre and top-right pictures of figure 4) and of $\hat{\omega} < \breve{\omega}$, for which all four depicted equilibria are possible. In all cases, the preferred tax rate is decreasing for the poorest agents in society, while increasing for the richest; the poorest agents in the economy always belong to the coalition asking for an increase in the tax rate.
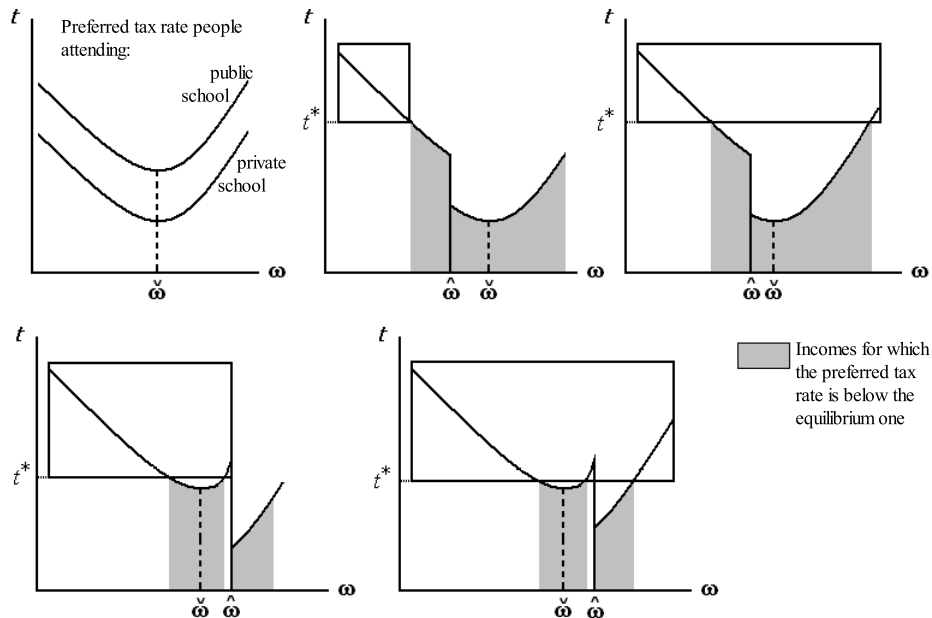

Figure 4: The convex case - different equilibria

Regardless of the relation between $\hat{\omega}$ and $\breve{\omega}$, two equilibria are always possible. In the top-centre picture, the poorest half of the population coincides with the half

of population with the largest preferred tax rate. This implies that a coalition of the poorest faces a coalition of the richest agents in the economy. In the top-right picture, the equilibrium is similar, but the richest agents in the economy also prefer a large tax rate; the very richest join the coalition of those in favour of a rise in the tax rate. Two other equilibria (depicted in the bottom line of picture 4) can only occur when $\hat{\omega} > \breve{\omega}$ (the income corresponding to the smallest preferred tax rate denotes a voter whose child attends a public school). The poorest agents always attend public school; together with the richest students in public school, they demand for a higher tax rate. The richest voters may join (bottom-left picture) this coalition or the one including all the other voters (bottom-right picture).

The poorest voters are moved by the willingness to have a good public service and to profit of intra-temporal redistribution. The richest students in public school ask for an increase in the tax to have more inter-temporal redistribution. Within those asking for a tax reduction, agents with income close to $\breve{\omega}$ are particularly concerned by the low consumption of numeraire when adults, and they do not profit enough from redistribution to agree on a further reduction in current consumption. The poorest voters in private school as well prefer an increase in current consumption to an increase in their pension. The richest voters might join either coalition depending on which factor is more relevant between current consumption and the join impact of altruism and consumption smoothing.

Proposition 6 *Four different equilibria can occur: either the poorest voters ask for higher tax rates and oppose the richest, or the equilibrium is of the type "ends against the middle". In both cases, the richest voters with children attending public schools may join either coalition.*

Proof. See Appendix J. ■

It is not possible to give more analytical details on the thresholds and conditions determining the different equilibria: the proposed extension of the model makes it much more realistic but also it makes less clear the composition of different coalitions and it becomes impossible to predict the preferred tax. Many scenarios are feasible. We are sure that a majority voting tax exists under the introduced assumptions. Only specifying the utility function and all parameters, it is possible to be more precise concerning the equilibrium.

5   Comparative statics

Despite the limitations of this model due to the difficulty to restrict the set of feasible outcomes, it is possible to forecast the effects of a change in some parameters. I concentrate on the effects of a change in the Bismarckian factor α both

on agents' preferred tax given income, and on the equilibrium number of agents attending public school n_p.

Concerning n_p, we know that $n_p = F(\hat\omega)$. The effect on n_p of a change in α is:

$$\frac{\partial n_p}{\partial \alpha} = \frac{\partial n_p}{\partial \hat\omega} \cdot \frac{\partial \hat\omega}{\partial t} \cdot \frac{\partial t}{\alpha}$$

The first term $(\frac{\partial n_p}{\partial \hat\omega})$ is clearly positive. By lemma 1 of Glomm and Ravikumar (1998) we also know that $\frac{\partial \hat\omega}{\partial t} > 0$. We conclude that the effect of a change in α on n_p depends on the sign of $\frac{\partial t}{\alpha} = \frac{\partial \left(s[1-\alpha] \frac{\partial \bar\omega(t_z)}{\partial t_z} \frac{\partial G}{\partial b} \right)}{\partial \alpha}$

$$\frac{\partial t}{\alpha} = \underbrace{s \frac{\partial \bar\omega}{\partial t}}_{positive} \left(\underbrace{-\frac{\partial G}{\partial b}}_{negative} + \underbrace{s[\omega - \overline\omega(t_z)][1-\alpha]\frac{\partial^2 G}{\partial b^2}}_{negative \ if \ \omega > \overline\omega(t_z); \ positive \ otherwise} \right) \qquad (14)$$

The first term in bracket represents the fact that an increase in α leads to a more Bismarkian (contributive) pension system and thus the incentive to have a high tax decreases. The second term in parenthesis (i.e., $s[\omega - \overline\omega](t_z)[1-\alpha]\frac{\partial^2 G}{\partial b^2}$) depends on redistribution: the higher α the less redistribution we have. When α is low, the only way to increase consumption in the second period (regardless of income) is to increase the tax rate, even though this implies more redistribution. The higher is α, the lower is redistribution and the higher is the pension of households with income above the average. This means that the marginal benefit from an increase in the tax declines for all agents that loose money from redistribution. Rephrasing, the first term accounts for the fact that with a large α the value of pension increases and there is less need for redistribution, while the second term considers the fact that when α is low, redistribution is less effective, in some sense the rate of return of investing in education of future generations is lower (intergenerational redistribution is lower).

For agents with income below the average, we observe two opposite driving forces: on the one side the reduction in α reduces the "power of the channel (pension)" through which the tax increases consumption in the second period (e.g., with $\alpha = 1$ increasing t does not increase pensions). On the other side, for $\alpha < 1$, to compensate the reduction in the weight of the redistributive component, agents are willing to increase the tax rate. The second effect dominates the first one and thus the second term of the bracket is positive. The overall effect is negative for $\omega > \overline\omega$ and unknown for $\omega < \overline\omega$.

I used Matlab to draw the preferred tax and to show the effects of a change in α and in σ (the coefficient of relative risk aversion). For that, I use Glomm and Ravikumar (1998)'s (section 3.1) functional form for U, and assume that $V(t,\omega) = \frac{1}{3}t^{-\frac{2}{3}}\sqrt{\omega}$ and $\overline\omega(t_{z-1}) = kt_{z-1}\overline\omega(t_{z-2})$.

From

$$W = \frac{1}{1 - \sigma} \left\{ X^{1-\sigma} + b_z^{1-\sigma} \right\} + \frac{1}{1 - \sigma} \left\{ b_{z+1}^{1-\sigma} \right\} + V(t_z, \omega) \qquad (15)$$

we obtain

$$X_R^* = \frac{\beta \omega}{2}$$

and

$$b_{z,R} = \frac{\beta \omega}{2}$$

The equilibrium condition $U(X_R^*, c) = U(X_P, b)$ is:

$$2 \left[\frac{\beta \omega}{2} \right]^{1-\sigma} = [\beta \omega]^{1-\sigma} + \left[\frac{t \overline{\omega}}{n_p} \right]^{1-\sigma}$$

Solving the equation we obtain

$$\hat{\omega} = \frac{t}{1 - t - s} \frac{\overline{\omega}}{n_p} \left[\frac{1}{2^\sigma - 1} \right]^{\frac{1}{1-\sigma}}$$

And finally $n_p = F(\hat{\omega}(t, \overline{\omega}, n_p)) = F\left(\frac{t}{1-t-s} \frac{\overline{\omega}}{n_p} \left[\frac{1}{2^\sigma - 1} \right]^{\frac{1}{1-\sigma}} \right)$
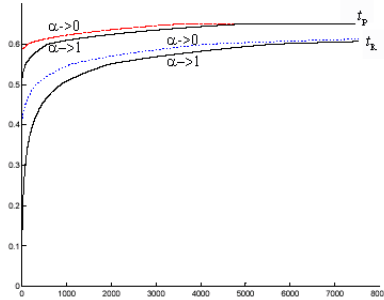

Figure 5: t as a function of income for different levels of α

Figure 5 shows the change in the preferred tax rate due to a change in α both for households choosing private education (the two lowest lines) and for those choosing the public one. In both cases the dashed line (representing α close to zero) is always above the solid line (representing α close to one). The reason for that is quite obvious: the tax has an effect on pensions only via the beveridgean component. When $\alpha = 1$, all tax effects on pensions disappear.

A low elasticity of substitution (high σ) means that agents are more willing to smooth consumption over time and among goods, thus they are favourable to a higher tax. On the opposite, a decrease in σ means that households are less prone to substitute current numeraire consumption for future consumption and for education: the preferred tax rate is lower. In the simulation it happens that for high levels of σ (as well as for very low ones) the tax is concave in income,

while it is convex for intermediate levels.

The simulation confirmed that an increase in the average income causes an increase in the preferred tax level, especially among the poorest agents. This result is even more evident for low levels of α. An additional result of the simulation is that the preferred tax decreases when s increases.

Comparative statics results allow us to see that (ceteris paribus) having intergenerational redistribution can affect both the preferred tax by an agent and the distribution of people between public and private school. With respect to Epple and Romano (1996a), those concerns may lead to both an increase in agents preferred tax and in the number of students attending public school. Absent intergenerational redistribution ($\alpha = 1$), $\frac{\partial t}{\alpha} < 0$: a more redistributive society prefers a larger investment on education. This effect seems to be confirmed by western countries' experience. Per-capita public investments for education in scandinavian/nordic countries (known to be highly redistributive) is larger[14] than, for instance, the United States.

6   Conclusions

Epple and Romano (1996a) consider a society in which adults transfer part of their income to new generations to provide instruction. They assume, as I do, that all voters have a child in school age. This is clearly not realistic in western societies where fertility is particularly low. Assuming SDI, Epple and Romano (1996a) capture the fact that the marginal effect on agents' utility of an increase in the tax rate is larger through the channel of redistribution than through the channel of goods consumption. Therefore, the poorest voters (that benefit from redistribution) are in favour of large tax rates even at the price of low levels of consumption of the numeraire. On the opposite, richer voters (among those attending public school) prefer a smaller level of consumption of education since it implies also less redistribution and larger levels of consumption of the numeraire.

In my model, I account for two additional reasons why a voter may be in favour of a tax to finance education, regardless of having children of school age.[15] Those two elements are altruism and selfish interest. In my model, the selfish attitude takes the form of money intergenerational redistribution, through pensions: adults live for two periods and their pension depends also on society's productivity (and thus on the level of education). Even people without children have an interest in

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[14]Public investment on education in Denmark, Sweden, Norway and Iceland is between 7% and 8%, while in the U.S.A. it is at 4.8% (Crell, 2008).

[15]It would be possible to rewrite both models, considering that agents who are considered to prefer private school are in fact agents without children, who are consuming another private good.

having a more educated society, because this implies less social problems, more technological and scientific progress (eventually leading to more infrastructures, services for the elders, new medical treatments, etc.). Moreover, western societies attribute great importance to education, and citizens often believe that everyone should be given the same chances and be allowed to study. For those reasons this model is far more realistic than Epple and Romano (1996a), considering the existence of different forces driving the decision to vote for the tax financing public education.

In Epple and Romano (1996a), the monotonicity assumption (SDI) is sufficient to conclude that the median-income voter is pivotal, his most preferred tax rate is the majority voting equilibrium, and two coalitions opposes in equilibrium: one composed by the poorest agents in the economy and the other by the richest ones. Introducing the additional elements had an obvious cost for the model, which becomes much less tractable. Analytical results cannot be obtained unless we introduce more restrictions on the parameters. Nonetheless, it is possible to solve the model graphically, reducing possible equilibria to four. Whereas in Epple and Romano (1996a) the median voter is always the Condorcet winner,[16] in this model this is only one amongst the four possible results, occurring under specific assumptions, otherwise a middle-class coalition opposes, at equilibrium, a coalition of the poorest and richest voters ("ends against the middle" equilibrium). Finally, on top of the known "ends against the middle" equilibrium, in my model it is possible to observe two other equilibria in which, with respect to standard equilibria, some agents with income close to the one of the indifferent voter deviate and join a different coalition. The presence of this group of "switching voters", is due to the the fact that agents with income close to the indifference voter are very sensitive to a change in the tax rate, so that, according to the level of redistribution and their level of altruism, they may have interest in modifying their behaviour. Furthermore, allowing the preferred tax to be concave or convex, instead of being monotonic in income, implies that the density function of population matters for the equilibrium, affecting the composition of the winning coalition, that may include both people attending private and public school.

Epple and Romano (1996a)'s result are consistently affected by the introduction of the two additional elements that I proposed, in terms of i) the number of people attending public school (n_p may increase or decrease), ii) the equilibrium value of the tax rate (t as well can either increase or decrease), iii) the type of equilibrium (while they predict that a coalition of rich people faces a coalition of poor voters, my model shows that it is more likely to have the middle class having opposite

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[16]All voters poorer than the median voter ask for an increase in the tax rate and the others for a decrease

needs than the others) and finally iv) the identity of the pivotal voter.

One strength of this model is that it is more general and realistic than the previous ones, and can thus be applied to a broader class of situations (Epple and Romano (1996a) framework being just one of them). The main weakness is that it does not provide precise results without adding further restrictions.

A possible (and interesting) direction for future work could be to use empirical data on the distribution of income and on the pension system to reduce possible frameworks and study more in detail those scenarios that are more likely to occur.

Once restricted the set of feasible outcomes, one natural extension of this model would be to make it more flexible on the pension side. In particular, Epple and Romano (1996a) and Casamatta et al. (2000a) would be really "integrated" allowing agents to vote also over the tax s that finances pensions and over the Bismarkian factor α.

Appendix

A   Proof of Lemma 1

A rise in the tax rate implies a reduction in disposable income for all agents. Those attending private school adjust their consumption of education according to their income elasticity of demand. By the normality ($\epsilon_{\omega,X} > 0$) but not luxury ($\epsilon_{\omega,X} < 1$) of X (Assumption 2), the reduction in consumption is lower than the extra tax collected. Since all the collected tax is used to finance public education, aggregate expenditure in education is positively correlated with the tax rate t.

B   Proof of Lemma 2

Given that the first derivatives are positive ($\frac{\partial U}{\partial X} \geq 0$, $\frac{\partial U}{\partial b} \geq 0$, $\frac{\partial G}{\partial b} \geq 0$) and the second derivatives are negative ($\frac{\partial^2 U}{\partial X^2} \leq 0$, $\frac{\partial^2 U}{\partial b^2} \leq 0$, $\frac{\partial^2 G}{\partial b^2} \leq 0$), third derivatives have to be positive. To prove that, consider that a negative second derivative implies that the first derivative is a decreasing function. Then, a decreasing and concave function with an unbounded domain necessarily crosses, at some point, the horizontal ax, taking then negative values. This contradicts the fact that, for any value of the variable, the first derivative is always positive. Then a necessary condition for the first derivative to be always positive, when the second derivative is negative, is that the third derivative is positive (thus the first derivative is a decreasing and convex function).

C   Proof of Lemma 3

Since (4) depends on X_R only through its first component, the framework is the same as in Epple and Romano (1996a). See Epple and Romano (1996a), pp. 300-304 and in particular lemma 1 (and its corollary 1) and lemma 2 for the proof.

D   Proof of Lemma 4

See proposition 2 and lemma 1 in Glomm and Ravikumar (1998).

E   Proof of Proposition 1

Equation 8 is obtained by deriving 6 with respect to ω (using the implicit function theorem). Its sign only depends on the numerator, since the denominator is always

negative but becomes positive because of the minus in front of the fraction. In equation 9, the third term is always negative, by assumption 4, the sum of the two first terms is also negative, thus only altruism might reverse the sign of the equation.

F   Proof of Proposition 2

Use the implicit function theorem on equation 12 to compute the derivative of the optimal tax with respect to income. The denominator is always negative. The sign, as in the previous section, depends only on the numerator. Assumption 4 is sufficient to ensure that the sign of the two first terms in the numerator is always negative.

G   Proof of Proposition 3

The numerator of both 8 and 13 is the same; differences in the denominator determines a change in the value of t^* but not in the sign. The stationary point $\breve{\omega}$ coincides with the point in which the numerator equals zero, thus it is the same for both types of agent.

H   Proof of Proposition 4

Denote $\tilde{t}$ the tax that maximises the utility of an agent of income $\tilde{\omega}$ when her/his child attends a private school; by equation 12 it has to be that $\omega \frac{\partial U}{\partial b} = k \frac{\partial \overline{\omega}(t_z)}{\partial t} \frac{\partial G}{\partial b} + \frac{\partial V}{\partial t}\Big|_{\omega=\tilde{\omega};t=\tilde{t}}$ (i.e., the marginal cost and benefit of the tax are equal). Suppose that an agent with equal income prefers his child to attend a public school. Compared to the previous agent, the larger consumption of numeraire implies that the left hand side (LHS) of the equation (the marginal cost) decreases. Instead, the marginal benefit (right hand side - RHS) is larger, because it includes, for people whose children attend a public school, the extra term corresponding to the marginal benefit in period z of an increase in the tax rate. As a consequence, $\tilde{t}$ cannot be the equilibrium tax for this other agent and $\omega \frac{\partial U_z}{\partial b} < k \frac{\partial \overline{\omega}_z}{\partial t} \frac{\partial U_{z+1}}{\partial b} + \frac{\partial V}{\partial t} + \frac{\overline{\omega}}{n_p} \frac{\partial U_z}{\partial X}\Big|_{\omega=\tilde{\omega};t=\tilde{t}}$. Since LHS is increasing in t and RHS is decreasing, the equilibrium tax must be higher (see figure 2).

I   Proof of Proposition 5

Picture 3 and its explanation serve as a proof. As a general idea, the preferred tax rate function is defined by two concave segments, each being concave. This implies that both very poor and very rich people are in favour of low tax rates, while the middle class prefers larger tax rates. Due to the discontinuity of the function, different scenarios can occur, according to the identity of the indifferent voter. The poorest people preferring private school, may join the coalition asking for a reduction in the tax rate.

J   Proof of Proposition 6

Picture 4 and its explanation serve as a proof. The idea is that the preferred tax rate function is defined as two discontinuous segments, each convex. According to the population density function and the value of the function near the discontinuity point, the coalition may include, besides the poorest agents in the population, also the richest agents attending public school and the richest agents attending private school.

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