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Loyalty Discounts*

Ugur Akgun and Ioana Chioveanu**

Abstract

This paper considers the use of loyalty inducing discounts in vertical supply chains. An upstream manufacturer and a competitive fringe sell differentiated products to a retailer who has private information about the level of stochastic demand. We provide an analysis of the market outcomes when the manufacturer uses two-part tariffs (2PT), all-unit discounts (AU) and market share discounts (MS). We show that retailer's risk attitude affects manufacturer's preferences over these three pricing schemes. When the retailer is risk-neutral, it bears all the risk and all three schemes lead to the same outcome. When the retailer is risk-averse, 2PT performs the worst from manufacturer's perspective but it leads to the highest total surplus. For a wide range of parameter values (but not for all) the manufacturer prefers MS to AU. By limiting retailer's product substitution possibilities MS makes the demand for manufacturer's product more inelastic. This reduces the amount (share of profits) the manufacturer needs to leave to the retailer for the latter to participate in the scheme.

Keywords: vertical contracts, loyalty discounts, private information, market share discounts.

JEL Classification: J42, J12, J13.

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1 Introduction

A loyalty discount is the practice that implicitly or explicitly makes discounts conditional on the share of a buyer's purchases made from a supplier within a given period. The discount is typically applied in a rollback format where once a buyer qualifies it receives a discount on all purchases in the period not only on those above the target. This type of discount is in most cases difficult to link to particular instances of economies of scale. The latter can occur at overall production level or in fulfilling a specific order, but they are unlikely to relate to total purchases of a customer over a period. While loyalty inducing programs directed to final consumers have rarely raised competition concerns[1], the use of rollback rebates in wholesale markets has frequently come under antitrust scrutiny in recent years.[2]

Understanding the motive for the use of these rebates poses a challenge to economics and policy design. As with related practices of vertical price control and exclusive dealing, firms' use of loyalty discounts has the potential to be both procompetitive and anticompetitive. The major concern with rollback loyalty rebates is that a supplier with substantial market power sets a low price conditional on exclusive (or nearly exclusive) dealing, with the effect that a market is foreclosed to a rival competitor.[3] However, a discount that is fundamentally competitive could be defined as a loyalty discount because it induces the buyer to purchase more from one supplier and less from another. There are, moreover, several plausible reasons for the use of loyalty discounts other than exclusion.

In order to assess the impact of such practices on competition and consumer surplus it is important to disentangle the underlying motivations. This paper studies the private and social incentives for the use of such contracts under demand uncertainty. In the analysis of vertical chains the tension between efficient surplus extraction and maximization of surplus is thoroughly studied as a principal-agent problem where the retailer has private information related to uncertainty. In contrast to the principle-agent literature where different risk attitudes of the two parties play a central role, previous work in the present context assumes that both upstream and downstream firms are risk-neutral. It is quite plausible that a manufacturer that deals with many retailers in different local markets (potentially subject to uncorrelated shocks) behave as risk-neutral. However, it is much less likely that a retailer would agree to bear all the market risk by signing a contract which aims to induce certain level of purchases at no additional cost to the manufacturer. In effect, the current analysis suggests that the differences

[1] Probably most familiar to consumers are the "frequent flyer" schemes promoted by airlines and related programs run by supermarkets, cafés, bookstores, or credit card issuers.

[2] The case law related to loyalty inducing rebate programs is developing faster than the economic analysis of the practices. Comprehensive overviews of relevant antitrust cases in US and Europe are presented in Mills (2004) and the OFT report 804 (2005).

[3] Lately, European and North American case law have focused on whether loyalty discounts can serve as an exclusionary device that would violate Article 82 of EC Treaty or Section 2 of the Sherman Act. In addition, firms' use of loyalty discounts in the distribution of their products has also been attacked as unlawful primary line price discrimination under the Robinson Patman Act and EC law (Art. 82 (c)).

in attitude towards risk across the vertical levels can explain emergence of different types of loyalty inducing contracts. In particular, it focuses on two-part tariffs (2PT) and two types of rollback discounts with quantity or market share targets. The latter are named all-unit (AU) and market share (MS) discounts, respectively. We show that under uncertainty, if the retailer is infinitely risk averse, the manufacturer strictly prefers MS and AU to 2PT. Using a linear demand system, we also show that, for a wide range of parameters, it strictly prefers MS to AU,[4] and welfare is highest under two-part tariffs. Private incentives for the use of MS are driven by their ability to induce the retailer to act on a target share that reduces the demand elasticity of manufacturer's product. Furthermore, MS limit product substitution while they allow the retailer to use private information and respond to actual market conditions. Even if implementation of MS requires costly monitoring of rival sales, there is a non-trivial range of costs for which the supplier might still strictly prefer them to 2PT or AU. The importance of the retailer's risk attitude is indicated by the fact that, under risk neutrality, the manufacturer is indifferent between two-part tariffs, rollback market share and quantity discounts. The retailer bears all risk and purchases the product at marginal cost.

The present article offers an alternative motivation for offering certain loyalty discounts which, in view of regulatory authorities, are implemented due to foreclosure/exclusion reasons. It assesses the effects of the various trading forms on competition and welfare.

Despite the ubiquity of this practice, the theoretical literature in economics did not address rollback loyalty discounts specifically until quite recently. Most of economists' attention was captured by related practices like exclusive dealing or incremental units discounts (Bernheim and Whinston, 1998). Comparisons of different rebate schemes, or of alternative contracts between buyers and suppliers are scarce in the economic literature. Such analysis is important to understand the conditions that lead to the emergence of a specific type of rebate or trading form.

Recent research has identified market conditions under which the use of rollback discounts improves surplus transfer from retailers to manufacturers.[5] Under complete information, Inderst and Shaffer (2010) show that market-share contracts allow a dominant supplier to dampen competition between the retailers, extract more profit than two-part tariffs or own-supplier contracts, and deliver the joint-profit maximizing outcome. Hence, even absent exclusionary concerns, market share contracts may harm consumers. Marx and Shaffer (2004) propose a rent-shifting rationale for quantity discounts when two upstream sellers sequentially contract with one downstream firm. Surplus extraction is better if contracts depend on both seller's quantities. Exclusion is desirable only if the rival is inefficient.

[4] This also informs on the relative private desirability of incremental unit discounts (that do not rollback to inframarginal units once the target is reached) since they cannot improve upon 2PT under our information/risk setting.

[5] In a different vein, Mills (2004) proposes an efficiency-based rationale. He argues that market-share discounts may induce merchandising effort on retailers' part. Consumers exposed to merchandising are able to make better purchase decisions. Market foreclosure is unlikely to occur and welfare is increased.

Under asymmetric information, rollback discounts may solve or alleviate the adverse selection problem. Kolay, Shaffer and Ordover (2004) show that a menu of rollback quantity discounts generates higher upstream profits than a menu of two-part tariffs in a bilateral monopoly setting, but its welfare effects depend on demand parameters. Majumdar and Shaffer (2009) report that by conditioning a discount on both quantity and market share thresholds the manufacturer can improve upon rollback quantity discounts. The share threshold reduces the retailer's informational rent by decreasing the attractiveness of concealing a high demand state. The authors identify a condition under which the suggested contracts can replicate the full information outcome.

This article shows that, under incomplete information, there are private incentives for the use of loyalty discounts when a risk neutral supplier offers non-contingent contracts to a risk averse retailer. Although we address different vertical contracts and market structure, our information setting is similar to Rey and Tirole (1986). They highlight the delegation problem under uncertainty as an essential driver of private incentives to use vertical restraints and analyze three types of contracts, two-part tariffs (competition), exclusive territories and resale price maintenance. The interplay of monopoly power exploitation and insurance to secure contracting underlies the private desirability of the contracts. Private and social incentives may not be aligned.

Section 2 lays out the model and introduces the contracts under certainty. Section 3 examines the contracts under uncertainty and retailer risk aversion. Section 4 considers retailer risk neutrality and discusses a number of extensions. Some concluding remarks are collected in the last section. All proofs missing from the text are relegated to the appendix.

2 Model

Consider a vertically-related industry where a manufacturer and a competitive fringe operate at the upstream level. The fringe produces an imperfect substitute of the product supplied by the manufacturer. The manufacturer and the competitive fringe supply their products in many independent and identical markets each served by a different monopolist retailer.

We are interested in the relative performances of standard non-linear pricing contracts under uncertainty and risk aversion. We model each retailer as facing an uncertain local demand and require that the retailer and the manufacturer sign their contract prior to the resolution of the uncertainty. More precisely the order of moves is as follows. First, the manufacturer offers a contract to the retailer. If the retailer rejects, it cannot sell the manufacturer's good. Second, the demand is determined and the retailer chooses the quantities of the two products to purchase. Third, the retailer sells the products to the final consumers at the market clearing prices for the chosen quantities. The manufacturer is assumed to be risk-neutral, as it operates in many independent markets with uncorrelated shocks. The retailer is assumed to be risk averse.

Only non-contingent contracts which avoid arbitrage opportunities across local markets, are considered. That is, the manufacturer's contract offer is such that at equilibrium the retailer chooses to purchase at the same wholesale price regardless of the realization of the uncertainty. If alternatively the manufacturer's offer induced the retailer to buy at different wholesale prices for different realizations of uncertainty, retailers that receive different demand shocks could profitably trade with each other.

In each local market, the retailer purchases the goods and resells them to the final consumers without incurring additional costs. The retailer buys the competitively supplied product at marginal cost. Prior to the retailer's final quantity choices, the retailer and the manufacturer sign a non-linear contract that stipulates the terms and conditions of purchase. Three types of contracts are considered: standard two-part tariffs, quantity discounts and market share discounts. All specify a unit price (w) and a non-negative franchise fee (F). Quantity discounts offer a rebate off-the-list price for all the units purchased once a *quantity* threshold is met. The market share contract specifies a discount that also applies to all units. However, a *share* of retailer's purchases must be made from the manufacturer in order to qualify for the rebate. The total outlay of a retailer who signs a standard two-part tariff is given by $C^{2PT}(q_1) = wq_1 + F$ for $q_1 \geq 0$. An AU contract stipulates two wholesale prices ($w_H > w_L$) and a quantity target (q^T) to qualify for the discounted price:

$$C^{AU}(q_1) = \begin{cases} w_H q_1 + F \text{ for } q_1 < q_1^T \\ w_L q_1 + F \text{ for } q_1 \geq q_1^T \end{cases}.$$

An MS contract stipulates two wholesale prices ($w_H > w_L$) and a share target, $\tau \in (0,1)$:

$$C^{MS}(q_1) = \begin{cases} w_H q_1 + F \text{ for } q_1 < \tau(q_1 + q_2) \\ w_L q_1 + F \text{ for } q_1 \geq \tau(q_1 + q_2) \end{cases}.$$

The marginal costs of production for both products are assumed to be zero. The inverse demand system for the differentiated goods is given by $P_1(q,\theta)$ and $P_2(q,\theta)$, where $q = (q_1, q_2)$ is the vector of chosen quantities. $P_i(q) \in C^1$ and $\partial P_i/\partial q_i < 0$ whenever $P_i(q) > 0$ for $i = 1, 2$. The parameter θ is a discrete random variable which captures potential demand uncertainty common to both products. It takes with probability p a low value (θ_L) and with probability $(1-p)$ a high value (θ_H). Let $E(\theta)$ be the expectation of θ and $P_i(0,\theta_L) > 0$. Shocks in different downstream markets are assumed to be iid.

The retailer chooses q_1 and q_2 to maximize its profits, $\pi(q,\theta) = R(q,\theta) - wq_1 - F$, where $R(q,\theta) = P_1(q,\theta)q_1 + P_2(q,\theta)q_2$ is its revenue. We assume that, for a given w, $\pi(q) \in C^2$ and is strictly concave and submodular ($\partial^2\pi/\partial q_1 \partial q_2 < 0$). The retailer's outside option consists of selling only the competitively supplied variety. Then, if the retailer rejects the manufacturer's offer it chooses q_2 to maximize $R_0(q_2,\theta) = P_2(0,q_2,\theta)q_2$. Let $R_0^*(\theta)$ be the maximal profit of a retailer which only sells good 2.

A risk neutral retailer accepts to sign the contract if its expected profit exceeds the expected value of its outside option. In contrast, an extremely (or infinitely) risk averse retailer signs the contract only if its profits under the low (worst) demand realization ($\theta = \theta_L$) are higher or equal than the value of the outside option for that demand scenario.

Under a deterministic demand the objectives of the manufacturer in designing a vertical contract are maximization of the surplus in the vertical chain and extraction of this surplus. Market power in vertical chains leads to a conflict between maximization and extraction of the surplus in the chain: surplus maximization might fail due to double marginalization.[6] Two-part tariffs are an efficacious way to avoid this problem: The product is passed downstream at upstream marginal cost and rent extraction is performed through the franchise fee. With deterministic demand, two-part tariffs and discounts based on quantity or market share thresholds defined as above are all equally effective tools of replicating the integrated firm's solution.[7]

Proposition 1 *Under deterministic demand, two part tariffs, all-unit quantity and market share discounts are equivalent both from manufacturer's and social planner's viewpoints. They all maximize surplus in the vertical chain.*

3 Uncertainty with a Risk-Averse Retailer

The vertical contracts need to cater for an additional objective if the demand is uncertain and the retailers are risk averse: *insurance provision to the retailers*. Achieving this objective deteriorates the ability of these contracts to eliminate the tension between maximization and extraction of the surplus. Our principal focus is to understand how the relative performance of these contracts are affected by the risk attitude of the retailers. In section 4, we show that the relative performance of these contracts under uncertainty is still the same when the retailers are risk neutral.

An infinitely risk averse retailer accepts a contract only if under a low demand realization the contract provides it with profits weakly greater than its outside option. This requirement implies that under high demand the manufacturer cannot absorb all surplus via the franchise fee and any attempt to extract more from the retailer requires a wholesale price above the marginal cost. Consequently, the unit price charged exceeds the level that maximizes surplus in the vertical chain. Intuitively, for a given p, a larger difference $\theta_H - \theta_L$ makes the retailer's participation constraint more restrictive and increases the manufacturer's need to absorb surplus via the unit price. Hence, the fact that a risk averse retailer requires some insurance to sign the contract, leads to double marginalization even when two-part tariffs are used.

Under a standard two part tariff, the retailer chooses q_1 and q_2 to maximize

$$\pi_{2PT} = P_1(q, \theta)q_1 + P_2(q, \theta)q_2 - wq_1 - F.$$

[6] This intuition goes back to Spengler (1950).

[7] Table 1 summarizes the equilibrium outcomes under complete information for the linear demand example in Section 3.

The optimal output choices, $q_1^*(w,\theta)$ and $q_2^*(w,\theta)$, satisfy the first-order conditions:

$$\frac{\partial P_1}{\partial q_1}q_1 + \frac{\partial P_2}{\partial q_1}q_2 + P_1 = w \quad (1)$$

$$\frac{\partial P_1}{\partial q_2}q_1 + \frac{\partial P_2}{\partial q_2}q_2 + P_2 = 0. \quad (2)$$

Let $R^*(w,\theta) = R(q_1^*, q_2^*, \theta)$ be the optimal second stage revenue. Concavity and submodularity of the retailer's profits imply that $\partial q_1^*/\partial w_{2PT} < 0$ and $\partial q_2^*/\partial w_{2PT} > 0$. Under a two-part tariff, double marginalization makes the retailer substitute away from manufacturer's product in favor of the competitively supplied variety.

The upstream manufacturer sets w_{2PT} and F_{2PT} to maximize

$$U = pw_{2PT}q_1^*(w_{2PT},\theta_L) + (1-p)w_{2PT}q_1^*(w_{2PT},\theta_H) + F_{2PT}$$

$$\text{subject to } R^*(w_{2PT},\theta_L) - w_{2PT}q_1^*(w_{2PT},\theta_L) - R_o^*(\theta_L) - F_{2PT} \geq 0.$$

Notice that the outside option (R_o^*) is independent of w_{2PT} and the maximand is increasing in F_{2PT}, thus the constraint binds at the optimum. By the envelope theorem, (1) and (2) we have that the optimal unit price, w_{2PT}^*, satisfies

$$(1-p)(q_1^*(w_{2PT}^*,\theta_H) - q_1^*(w_{2PT}^*,\theta_L) + w_{2PT}^*\frac{\partial q_1^*(\theta_H)}{\partial w}) = -pw_{2PT}^*\frac{\partial q_1^*(\theta_L)}{\partial w}. \quad (3)$$

If a rollback discount does not induce the retailer to act on the threshold, then the retailer's quantity choices will still be governed by (1) and (2). Consequently, a rollback discount can improve upon a 2PT only by inducing the retailer to act on the threshold. If a retailer facing an all-unit quantity discount acts on the threshold then it optimizes by choosing the quantity of the competitively supplied product to be sold with the threshold quantity for the manufacturer's product. For example, if the scheme induces to act on the target when demand is low

$$\widehat{q}_2(q_1^T,\theta_L) = \max_{q_2}[R(q_1^T,q_2,\theta_L) - w_L q_1^T - F]$$

where q_1^T is the quantity threshold that qualifies for the discounted unit price w_L. Let $\widehat{R}(q_1,\theta)$ denote the optimal second stage revenue when the retailer sells q_1 units of the manufacturer's product.

Next proposition establishes that when the retailer is infinitely risk-averse there exists an all-unit quantity discount which the manufacturer strictly prefers to the optimal 2PT. In particular, the manufacturer can make the retailer buy the same quantity as in the optimal 2PT at a higher price when the demand is low.

Proposition 2 *Under demand uncertainty, with an infinitely risk-averse retailer, the upstream manufacturer strictly prefers all-unit quantity discounts to two-part tariffs.*

Proof. The manufacturer's profits with the optimal 2PT are given by:

$$U_{2PT}^* = pw_{2PT}^*q_1^*(w_{2PT}^*,\theta_L) + (1-p)w_{2PT}^*q_1^*(w_{2PT}^*,\theta_H)$$
$$+R^*(w_{2PT}^*,\theta_L) - w_{2PT}^*q_1^*(w_{2PT},\theta_L) - R_0^*(\theta_L).$$

Consider an all-unit quantity discount which offers the rebated price $\widehat{w}_{AU} = w^*_{2PT}$ if $q_1 \geq q_1^T = q_1^*(w_{2PT}, \theta_L)$ and a fee $F_{AU} = R^*(w_{2PT}, \theta_L) - (w_{2PT} + \epsilon)q_1^*(w_{2PT}, \theta_L) - R_0^*(\theta_L)$. Notice that $U_{AU}(\widehat{w}_{AU}, q_1^T) = U_{2PT}$. The marginal variation in supplier profits when increasing w_{AU} evaluated at $\widehat{w}_{AU}$ is given by :

$$\frac{\partial U}{\partial w_{AU}} = (1-p)(q_1^*(w^*_{2PT}, \theta_H) - q_1^*(w^*_{2PT}, \theta_L) + w^*_{2PT}\frac{\partial q_1^*(\theta_H)}{\partial w}). \tag{4}$$

By (3) it follows that

$$\frac{\partial U}{\partial w_{AU}} = -pw^*_{2PT}\frac{\partial q_1^*(\theta_L)}{\partial w_{AU}} > 0.$$

Then, there exists $\epsilon > 0$ such that $U_{AU}(w^*_{2PT} + \epsilon, q_1^*(w^*_{2PT}, \theta_L)) > U^*_{2PT}$. ■

Let us now consider rollback market share discounts. Under this type of contract, the retailer qualifies for an off-the-list price discount if at least a percentage τ of its purchases are made from the manufacturer. If such a contract induces the retailer to act on the share target, it restricts retailer's ability to shift away from supplier's product in favor of the competitively supplied product. This reduces the market for substitutes of manufacturer's product and allows the retailer to act against a more inelastic demand and charge a higher unit price. A similar effect is achieved in the case of a negative demand shock by an all-unit quantity discount. However, when demand is high, the optimal AU contract does not decrease the elasticity of manufacturer's product, and then higher wholesale price decreases its sales volume. In addition, even when the retailer acts on the target, market share contracts allow the retailer to adjust its purchase to market conditions, unlike quantity discounts. But, MS contracts require costly monitoring of retailer's purchases, as the threshold depends on rival sales,too.

A retailer which is acting exactly on the share threshold chooses q_1 and q_2 to maximize

$$P_1(q, \theta)q_1 + P_2(q, \theta)q_2 - wq_1 - F,$$
$$\text{subject to } q_1 = \tau(q_1 + q_2).$$

Let $s = \tau/(1-\tau)$ (note that $\tau \in (0,1) \Rightarrow s < \infty$). Then the constraint requires that $q_1 = sq_2$. Substituting the constraint, it follows that, if the retailer acts exactly on the threshold, the quantity of good 2, q_2^{**}, maximizes:

$$\pi_{MS} = P_1(sq_2, q_2, \theta)sq_2 + P_2(sq_2, q_2, \theta)q_2 - wsq_2 - F.$$

The first order condition of the maximization problem is:

$$\frac{\partial P_1}{\partial q_1}s^2q_2 + \frac{\partial P_1}{\partial q_2}sq_2 + (P_1 - w)s + \frac{\partial P_2}{\partial q_1}sq_2 + \frac{\partial P_2}{\partial q_2}q_2 + P_2 = 0. \tag{5}$$

Proposition 3 *Under demand uncertainty, with an infinitely risk-averse retailer, the upstream manufacturer strictly prefers rollback market share discounts to two-part tariffs.*

In order to understand the relative private desirability of different loyalty inducing schemes (AU vs. MS) and the social incentives for their use, we focus on a linear demand example which allows to pin down closed form solutions.

3.1 Linear demand system and welfare analysis

Let the preferences of the representative consumer be given by a quadratic utility:

$$U(q_1, q_2) = (a+\theta)q_1 + (a+\theta)q_2 - \frac{1}{2}(q_1^2 + 2\beta q_1 q_2 + q_2^2),$$

where $\beta \in (0,1)$ measures product differentiation. Table 2 summarizes all equilibrium outcomes when the retailer is risk averse for the linear demand example.

Let us consider first a two-part tariff. The optimal second stage quantity and profits are given by:

$$q_1^* = \frac{(1-\beta)(a+\theta) - w}{2(1-\beta^2)} \text{ and } q_2^* = \frac{(1-\beta)(a+\theta) + \beta w}{2(1-\beta^2)}, \tag{6}$$
$$\pi^*(\theta) = \frac{2(1-\beta)a(a-w) + w^2}{4(1-\beta^2)} + \theta\frac{2a-w}{2(1+\beta)} + \theta^2\frac{1}{2(1+\beta)} - F.$$

The retailer's outside option is $R_o^*(\theta) = (a+\theta)^2/4$. Hence, the retailer's participation constraint in this case requires that $\pi^*(\theta_L) \geq R_o^*(\theta_L)$.

The upstream manufacturer's equilibrium choices and profits are, respectively:

$$w^{2PT} = (1-\beta)(1-p)(\theta_H - \theta_L),\ F^{2PT} = \frac{(1-\beta)[a-(1-p)(\theta_H-\theta_L)+\theta_L)]^2}{4(1+\beta)}, \text{ and}$$
$$U^{2PT} = \frac{(1-\beta)[(a+\theta_L)^2 + (1-\beta)(1-p)^2(\theta_H-\theta_L)^2]}{4(1+\beta)}.$$

The social planner is assumed to be risk neutral and, hence, takes into consideration the expected welfare and consumer surplus.

Consider a quantity discount. The optimal AU induces only the low type to act on threshold, while the high type purchases above the threshold. It follows that

$$\widehat{R}(\theta_L) = \frac{(a+\theta_L)^2}{4} - (1-\beta^2)(q_1^T)^2 + (1-\beta)(a+\theta_L)q_1^T.$$

The constraint is binding and, at equilibrium,

$$q_1^T = \frac{(a+\theta_L)(1+p) - (1-p)(\theta_H-\theta_L)}{2(1+\beta)(1+p)},\ w^{AU} = \frac{(1-\beta)(\theta_H-\theta_L)}{1+p} \text{ and}$$
$$F^{AU} = \frac{(1-\beta)[a-(\theta_H-2\theta_L)]^2}{4(1+\beta)} + \frac{p(1-\beta)(\theta_H-\theta_L)[a-(\theta_H-2\theta_L)]}{2(1+\beta)(1+p)}.$$

As in the case of a two-part tariff, risk aversion and the related insurance required by the retailer to participate in the contract, push up the wholesale price above the level that maximizes total surplus. However, as shown in Proposition 2, a discount off-the-list price when total purchases are above $\overline{q}_1^T$, allows the upstream manufacturer to absorb more surplus than a 2PT:

$$U^{AU} = \frac{(1-\beta)(1+p)(a+\theta_L)^2 + (1-\beta)(1-p)(\theta_H-\theta_L)^2}{4(1+\beta)(1+p)}. \tag{7}$$

Although the wholesale price involves a higher mark-up than in the case of a 2PT ($w^{AU} > w^{2PT}$), when demand is low, the retailer purchases more of manufacturer's product than under a 2PT due to the incentives provided by the mechanism.

However, private and social incentives are not aligned: Expected total welfare and expected consumer surplus are lower under AU than under 2PT,

$$W^{AU} - W^{2PT} = -\frac{(1-p)(1-\beta)p^2(2+3p)(\theta_H-\theta_L)^2}{8(1+p)^2(1+\beta)} < 0 \text{ and} \tag{8}$$

$$CS^{AU} - CS^{2PT} = -\frac{(1-\beta)p^2(2-p-p^2)(\theta_H-\theta_L)^2}{8(1+p)^2(1+\beta)} < 0. \tag{9}$$

This welfare result is driven by the following quantity changes. In contrast to a 2PT, an AU makes a retailer who is facing a negative shock to cut down on the competitively supplied product in order to increase the quantity of the substitute product and qualify for the discount. Formally, $\widehat{q}_2(w^{AU},\theta_L) - q_2^*(w^{2PT},\theta_L) = -\beta(1-p)p(\theta_H-\theta_L)/[2(1+p)(1+\beta)] < 0$. On the other hand, as the AU discounted wholesale price is higher than the 2PT one, the high type purchases less of the manufactured product, $q_1^*(w^{AU},\theta_H) < q_1^*(w^{2PT},\theta_H)$.[8]

Finally, let us consider a market share contract. A retailer acting exactly on the threshold chooses optimally only the quantity of the competitively supplied product.[9] That is,

$$q_2^{**} = \frac{a+\theta+s(a+\theta-w)}{2(1+2\beta s+s^2)} \text{ and } \pi^{**}(w,F,\theta) = \frac{[a+\theta+s(a+\theta-w)]^2}{4(1+2\beta s+s^2)} - F.$$

A market share discount could induce the retailer to act on the threshold under both demand realizations, only under one of them, or under none of them. It turns out that the optimal MS induces both types to act on the share threshold and choose optimally only the quantity of the competitively supplied product. The supplier chooses w, s and F to maximize

$$w_{MS}s\frac{a+E(\theta)+s(a+E(\theta)-w_{MS})}{2(1+2\beta s+s^2)} + F_{MS} \text{ subject to } \pi^{**}(w_{MS},F_{MS},\theta_L) \geq \frac{(a+\theta_L)^2}{4}.$$

The constraint is binding and it follows that, at equilibrium,

$$s^{MS} = 1,\ w^{MS} = 2(1-p)(\theta_H-\theta_L) \text{ and}$$
$$F^{MS} = \frac{[a+\theta_L-(1-p)(\theta_H-\theta_L)]^2}{2(1+\beta)} - \frac{(a+\theta_L)^2}{4}.$$

As in the case of two-part tariffs and all-unit discounts, the participation constraint of the risk averse retailer leads to a wholesale price above upstream marginal cost. The resulting upstream profits are given by:

$$U^{MS} = \frac{(1-\beta)(a+\theta_L)^2 + 2(1-p)(\theta_H-\theta_L)^2}{4(1+\beta)}.$$

The optimal MS induces retailers to act on threshold regardless of the shock, as it allows for more flexible downstream choices.

Expected total welfare and expected consumer surplus are lower under MS than under 2PT:

$$W^{MS} - W^{2PT} = -\frac{1}{8}(1-p)(\theta_H-\theta_L)[2(a+\theta_L)+3(1-p)(\theta_H-\theta_L)] < 0 \text{ and}$$
$$CS^{MS} - CS^{2PT} = -\frac{1}{8}(1-p)(\theta_H-\theta_L)[2(a+\theta_L)+(1-p)(\theta_H-\theta_L)] < 0.$$

[8] Notice, however, that $q_1^{AU}(\underline{\theta}) > q_1^{2PT}(\underline{\theta})$ and $q_2^{AU}(\overline{\theta}) > q_2^{2PT}(\overline{\theta})$.

[9] When acting on the share threshold, the retailer actually chooses optimally the purchases of only one product. The purchases of the substitute product are determined by the share requirement. We let the retailer choose q_2.

Proposition 4 *Under demand uncertainty, with an infinitely risk-averse retailer and linear demand, expected total welfare and consumer surplus are highest under 2PT. Expected consumer surplus is lowest under MS. For $p^2 < (1+\beta)/2$, from both private and social viewpoints, market share discounts outperform all-unit discounts.*

The intuition underlying Proposition 4 is related to the delegation problem in Rey and Tirole (1986). The manufacturer pursues to fully exploit monopoly power in the vertical chain and to offer insurance to the retailer under uncertainty. In our model, the fact that the retailer is a multiproduct firm affects both upstream objectives. An integrated monopoly optimally uses market power in the vertical structure. It passes the product downstream at marginal cost and, under uncertainty, it chooses $q_1 = q_2 = (a+\theta)/[2(1+\beta)]$. The retail quantity responds to the uncertainty, and the share of manufacturer's product is constant across states. When dealing with a risk averse retailer, in order to provide insurance, the manufacturer cannot extract the incremental surplus from the retailer through the franchise fee, and is forced to sell its product above marginal cost. With a 2PT, the retailer chooses $q_1^{2PT} = \frac{a+\theta-(1-p)(\theta_H-\theta_L)}{2(1+\beta)} < \frac{a+\theta}{2(1+\beta)}$ and $q_2^{2PT} = \frac{a+\theta+(1-p)\beta(\theta_H-\theta_L)}{2(1+\beta)} > \frac{a+\theta}{2(1+\beta)}$. The quantities respond to the uncertainty. Due to the higher unit price, the share of manufacturer's product is lower, $\tau^{2PT}(\theta) = q_1^{2PT}/(q_1^{2PT}+q_2^{2PT}) < 50\%$ as the retailer purchases more of the substitute product (in addition, $\tau^{2PT}(\theta_L) < \tau^{2PT}(\theta_H)$). When demand is low, the AU scheme induces the retailer to act on threshold. This limits retailer's ability to cut down the share of manufacturer's product when facing a negative shock ($\tau^{AU}(\theta_L) = \frac{q_1^T}{q_1^T+q_2^{AU}} > \tau^{2PT}(\theta_L)$). However, this comes at a cost, as $\tau^{AU}(\theta_H) < \tau^{2PT}(\theta_H)$. Finally, MS allows retailer's choices to respond to the uncertainty, but it prevents the retailer from buying more of the substitute product when confronted with a higher unit price, $q_1^{MS} = q_2^{MS} = \frac{a+\theta-(1-p)(\theta_H-\theta_L)}{2(1+\beta)}$. Although, $q_1^{MS} = q_1^{2PT}$, retailer's demand for q_1 is more inelastic when acting on the threshold and this allows the manufacturer to absorb more surplus by charging a higher wholesale price. Insurance provision prevents the manufacturer from passing the product downstream at marginal cost and this causes a loss of efficiency. Nevertheless, a share based target allows him to restore the vertically integrated share of purchases ($\tau_{MS} = 50\%$). Moreover, the share of manufacturer's product is invariant across demand realizations.

4 Uncertainty with a Risk-Neutral Retailer

The retailer makes quantity choices after observing the realized demand, therefore the second stage optimization problems presented in the previous section still apply. But, as contracts are agreed upon before the resolution of uncertainty, a different risk attitude changes the first stage optimization problem. When the manufacturer faces a risk neutral retailer, the participation constraint requires retailer's expected profit to be at least equal to retailer's expected outside option.

Under a 2PT, the upstream manufacturer chooses w and F to maximize

$$pwq_1^*(w,\theta_L) + (1-p)wq_1^*(w,\theta_H) + F$$

subject to

$$p(R^*(w,\theta_L) - wq_1^*(w,\theta_L)) + (1-p)(R^*(w,\theta_H) - wq_1^*(w,\theta_H)) - F \geq pR_o^*(\theta_L) + (1-p)R_o^*(\theta_H).$$

The constraint is increasing in the franchise fee and, then, the supplier chooses the unit price to maximize $pR^*(w,\theta_L) + (1-p)R^*(w,\theta_H)$. It follows that the optimal unit price w_{RN} satisfies the first order condition

$$p(\frac{\partial R(\theta_L)}{\partial q_1}\frac{\partial q_1^*(\theta_L)}{\partial w} + \frac{\partial R(\theta_L)}{\partial q_2}\frac{\partial q_2^*(\theta_L)}{\partial w}) + (1-p)(\frac{\partial R(\theta_H)}{\partial q_1}\frac{\partial q_1^*(\theta_H)}{\partial w} + \frac{\partial R(\theta_H)}{\partial q_2}\frac{\partial q_2^*(\theta_H)}{\partial w}) = 0 \quad (10)$$

Using (1) and (2), by envelope theorem, (10) becomes $pw_{RN}\frac{\partial q_1^*(\theta_L)}{\partial w} + (1-p)w_{RN}\frac{\partial q_1^*(\theta_H)}{\partial w} = 0$, and $\frac{\partial q_1^*(\theta)}{\partial w} < 0$ implies that, at equilibrium,

$$w_{RN} = 0 \text{ and } F_{RN} = E_\theta(R^*(0,\theta)) - E_\theta(R_0^*(\theta)). \quad (11)$$

Let us consider an all-unit quantity discount which induces the retailer to act on the quantity target only under a low demand. Then, the supplier chooses w, q_1^T and F to maximize

$$pwq_1^T + (1-p)wq_1^*(w,\theta_H) + F$$

subject to

$$p(\widehat{R}(q_1^T,\theta_L) - wq_1^T) + (1-p)(R^*(w,\theta_H) - wq_1^*(w,\theta_H)) - F \geq pR_0^*(\theta_L) + (1-p)R_0^*(\theta_H).$$

The constraint is increasing in the franchise fee and, then, the supplier chooses w and q_1^T to maximize $p\widehat{R}(q_1^T,\theta_L) + (1-p)R^*(w,\theta_H)$. It follows that the optimal unit price w_{RN} satisfies the first order condition $(1-p)(\frac{\partial R(\theta_H)}{\partial q_1}\frac{\partial q_1^*(\theta_H)}{\partial w} + \frac{\partial R(\theta_H)}{\partial q_2}\frac{\partial q_2^*(\theta_H)}{\partial w}) = 0$. By a similar argument as in the case of a 2PT, it follows that the optimal unit price and franchise fee are given by (11). In addition, $q_1^T = \arg\max p\widehat{R}(q_1^T,\theta_L) = q_1^*(0,\theta_L)$. The optimal 2PT and AU result in the same output levels.

Finally, consider a market-share discount which induces the retailer to act on the threshold always. Then, the supplier chooses w, s and F to maximize

$$pwsq_2^{**}(w,s,\theta_L) + (1-p)wsq_2^{**}(w,s,\theta_H) + F$$

subject to

$$p(R^{**}(w,s,\theta_L) - wsq_2^{**}(w,s,\theta_L)) + (1-p)(R^{**}(w,s,\theta_H) - wsq_2^{**}(w,s,\theta_H)) - F \geq$$
$$pR_0^*(\theta_L) + (1-p)R_0^*(\theta_H).$$

The constraint is increasing in the franchise fee and, then, the supplier chooses w and s to maximize $pR^{**}(w,s,\theta_L) + (1-p)R^*(w,s,\theta_H)$. Then, the unit price satisfies

$$p(\frac{\partial R}{\partial q_1}s\frac{\partial q_2^{**}(\theta_L)}{\partial w} + \frac{\partial R}{\partial q_2}\frac{\partial q_2^{**}(\theta_L)}{\partial w}) + (1-p)(\frac{\partial R}{\partial q_1}s\frac{\partial q_2^{**}(\theta_H)}{\partial w} + \frac{\partial R}{\partial q_2}\frac{\partial q_2^{**}(\theta_H)}{\partial w}) = 0. \quad (12)$$

From (6), using envelope theorem, (13) becomes $p\frac{\partial q_2^{**}(\theta_L)}{\partial w}sw + (1-p)\frac{\partial q_2^{**}(\theta_H)}{\partial w}sw = 0$. Then, the optimal unit price and franchise fee are given by (11).

This analysis proves the following result.

Proposition 5 *Under demand uncertainty, with a risk-neutral retailer, the upstream monopolist is indifferent between two-part tariffs, market-share and all-unit discounts. Private and social incentives are aligned.*

The welfare results directly follows from the outcome equivalence. All generate the same consumer surplus. The retailer's incremental profits are strictly positive under a positive shock, but the retailer incurs in losses when demand is low. If the implementation of a market share contract requires the manufacturer to engage in costly monitoring, this contract should not be observed in practice. Table 1 summarizes the equilibrium outcomes when the retailer is risk neutral for the linear demand example.

4.1 Two-part Tariffs and Incremental-Unit Discounts

We analyzed two-part tariffs, all-unit and market share discounts. Yet there are other type of contracts that allow to eschew double marginalization in vertical chains. As first pointed out by Buchanan (1952), incremental-unit discounts (off-the-list price discounts that apply only to the additional units purchased above a specified quantity target) can also be used to transfer rents to upstream manufacturer. Moreover, Gabor (1955) shows that incremental-unit discounts obtain the same outcome as two-part tariffs under certainty. Kolay, Shaffer and Ordover (2004) extend this result to incomplete information and contingent contracts. We consider non-contingent contracts and both retailer risk aversion and risk neutrality under uncertainty. In this section we discuss some relevant equivalence between two-part tariffs and incremental-unit discounts in our setting that allow us to inform on the private and social incentives for their use when compared to rollback discounts.

Under uncertainty, with non-contingent contracts it can be shown that: i) if the retailer is infinitely risk averse, the profit maximizing two-part tariff is outcome equivalent to the profit maximizing incremental-unit discount; ii) if the retailer is risk neutral, there exists a two-part tariff that is weakly better than the profit maximizing incremental-unit discount from manufacturer's viewpoint. The converse of ii) does not hold. A risk neutral retailer signs a 2PT whenever his expected incremental profits are non-negative. Thus, optimal 2PT tariff for the manufacturer would require the retailer to have expected incremental profits equal to zero. Then, under a low demand realization, the retailer's ex-post incremental profits are negative. No incremental-unit discount can achieve this outcome since the retailer would choose not to purchase the manufacturer's good under low demand realization if he cannot make non-negative incremental profits ex-post.

Whenever the retailer is infinitely risk averse, the 2PT and the IU are outcome equivalent. When the retailer is risk neutral, it can be shown that the profit maximizing 2PT is at least

as good as the profit maximizing IU from manufacturer's viewpoint. Under uncertainty, with non-contingent contracts, if the retailer is risk neutral (averse), then the upstream manufacturer weakly (strictly) prefers AU and MS to incremental-unit discounts.

5 Conclusions and Extensions

We have shown that the risk attitude of the retailers can play a crucial role in the form of contract and loyalty discount applied by the manufacturer. It has been shown that in vertical relations, manufacturer's preference over the contracts is affected by their rent extraction and risk sharing properties. This result carries on to the case of loyalty discount schemes. A novel driver of our results, in a setting where upstream manufacturer competes with a competitive fringe, is contract's ability to affect product substitution.

A market share contract that induces a buyer to act on the share threshold limits retailer's ability to substitute away manufacturer's product when facing a relatively higher unit price. In addition, market share contract provides a higher degree of flexibility to the retailer due to the fact that an absolute share threshold is achievable at many different quantity levels. That is, the retailer can still obtain the discount in a bad season as all of its sales would be low. When the retailer is risk averse, this allows the manufacturer to guarantee the participation to the contract at a lower cost than in a 2PT and, for a wide range of parameters, than in an all-unit discount. However, there is conflict between social and private incentives for the use of the contract, as total welfare and consumer surplus are highest under 2PT.

When the retailer is risk-neutral the ability to charge a higher wholesale price does not play a role. The retailer bears all the risk and buys the product at marginal cost. The manufacturer and the social planner are indifferent between 2PT, AU and MS.

Amongst possible extensions are generalizations in three directions. The two products sold by the retailer may eventually be vertically differentiated. It is interesting to see if the results extend to more general downward sloping demand functions, or to more general utility functions of the risk averse retailer.

6 Appendix A: Proofs

Proof of Proposition 3. Let $s^* = \frac{q_1^*(w_{2PT}^*,\theta_L)}{q_2^*(w_{2PT}^*,\theta_L)} \leq \frac{q_1^*(w_{2PT}^*,\theta_H)}{q_2^*(w_{2PT}^*,\theta_H)} = s^H$. Then upstream profits under an market share contract with share target $s < s^H$ and rebated price $\widehat{w}_{MS} = w_{2PT}^*$can be written as

$$U(s, w_{2PT}) = pw_{2PT}q_1^{**}(s, w_{2PT}, \theta_L) + (1-p)w_{2PT}q_1^*(w_{2PT}, \theta_H) +$$
$$R(q_1^{**}(s, w_{2PT}, \theta_L), q_2^{**}(s, w_{2PT}, \theta_L), \theta_L) - w_{2PT}q_1^{**}(s, w_{2PT}, \theta_L).$$

Notice that $q_i^{**}(s^*, w_{2PT}, \theta_L) = \begin{cases} q_i^*(w_{2PT}, \theta_L) \text{ for } s \leq s^* \\ q_i^{**}(s, w_{2PT}, \theta_L) \text{ for } s > s^* \end{cases}$ for $i = 1, 2$, and $U(s^*, w_{2PT}) = U^{2PT}$. Moreover, $\frac{\partial U(s,w_{2PT})}{\partial s} = \frac{\partial R}{\partial q_1}\frac{\partial q_1^{**}}{\partial s} + \frac{\partial R}{\partial q_2}\frac{\partial q_2^{**}}{\partial s} - (1-p)w_{2PT}\frac{\partial q_1^{**}}{\partial s}$. Evaluating this expression at s^* where $\frac{\partial R}{\partial q_1} = w_{2PT}$ and $\frac{\partial R}{\partial q_2} = 0$ it follows that $\frac{\partial U(s,w_{2PT})}{\partial s} \mid_{s=s^*} = pw_{2PT}\frac{\partial q_1^{**}}{\partial s}$.
Implicit function theorem and concavity of the profit function imply that $sign\frac{\partial q_2^{**}}{\partial s} = sign\frac{\partial^2 \pi_{MS}}{\partial q_2 \partial s}$. From (6),

$$\frac{\partial^2 \pi_{MS}}{\partial q_2 \partial s} = s^2 q_2^2 \frac{\partial^2 P_1}{\partial q_1^2} + 2sq_2\frac{\partial P_1}{\partial q_1} + sq_2^2\frac{\partial^2 P_1}{\partial q_2 \partial q_1} + q_2\frac{\partial P_1}{\partial q_2} +$$
$$sq_2\frac{\partial P_1}{\partial q_1} + (P_1 - w) + sq_2^2\frac{\partial^2 P_2}{\partial q_1^2} + q_2\frac{\partial P_2}{\partial q_1} + q_2^2\frac{\partial^2 P_2}{\partial q_1 \partial q_2} + q_2\frac{\partial P_2}{\partial q_1}. \quad (13)$$

Evaluating (7) at w_{2PT}^*, given that $sq_2\frac{\partial P_1}{\partial q_1} + (P_1 - w) + q_2\frac{\partial P_2}{\partial q_1} = 0$, the expression becomes
$\frac{\partial^2 \pi}{\partial q_2 \partial s} = s^2 q_2^2 \frac{\partial^2 P_1}{\partial q_1^2} + 2sq_2\frac{\partial P_1}{\partial q_1} + sq_2^2\frac{\partial^2 P_1}{\partial q_2 \partial q_1} + q_2\frac{\partial P_1}{\partial q_2} + sq_2^2\frac{\partial^2 P_2}{\partial q_1^2} + q_2^2\frac{\partial^2 P_2}{\partial q_1 \partial q_2} + q_2\frac{\partial P_2}{\partial q_1} =$
$q_1^2\frac{\partial^2 P_1}{\partial q_1^2} + 2q_1\frac{\partial P_1}{\partial q_1} + q_1q_2\frac{\partial^2 P_1}{\partial q_2 \partial q_1} + q_2\frac{\partial P_1}{\partial q_2} + q_1q_2\frac{\partial^2 P_2}{\partial q_1^2} + q_2^2\frac{\partial^2 P_2}{\partial q_1 \partial q_2} + q_2\frac{\partial P_2}{\partial q_1} =$
$q_1[q_1\frac{\partial^2 P_1}{\partial q_1^2} + q_2\frac{\partial^2 P_2}{\partial q_1^2} + 2\frac{\partial P_1}{\partial q_1}] + q_2[q_1\frac{\partial^2 P_1}{\partial q_2 \partial q_1} + \frac{\partial P_1}{\partial q_2} + q_2\frac{\partial^2 P_2}{\partial q_1 \partial q_2} + \frac{\partial P_2}{\partial q_1}] < 0$
The last inequality follows from concavity and submodularity of the profit function. Notice that $\frac{\partial^2 \pi}{\partial q_1^2} = [\frac{\partial^2 P_1}{\partial q_1^2}q_1 + \frac{\partial^2 P_2}{\partial q_1^2}q_2 + 2\frac{\partial P_1}{\partial q_1}] < 0$ is a necessary condition for a negative semidefinite Hessian under a 2PT and $\frac{\partial^2 \pi}{\partial q_1 \partial q_2} = [\frac{\partial^2 P_1}{\partial q_2 \partial q_1}q_1 + \frac{\partial P_1}{\partial q_2} + \frac{\partial^2 P_2}{\partial q_1 \partial q_2}q_2 + \frac{\partial P_2}{\partial q_1}] < 0$ under strategic substitutability. Under the latter assumption, $\frac{\partial q_2}{\partial s} < 0 \Rightarrow \frac{\partial q_1}{\partial s} > 0$.

■

6.1 Linear Example

2PT: Total surplus (gross utility of the representative consumer) and consumer surplus (utility net of consumer's expenditure) for $\theta \in \{\theta_L, \theta_H\}$ are given by:

$$W^{2PT}(\theta) = \frac{6a^2 - 2a(1-\beta)(1-p)(\theta_H - \theta_L) + (1-p)^2(\theta_H - \theta_L)^2}{8(1+\beta)}$$
$$+\frac{\theta[6a - 2(1-\beta)(1-p)(\theta_H - \theta_L)]}{4(1+\beta)} + \frac{3\theta^2}{4(1+\beta)} \text{ and}$$
$$CS^{2PT}(\theta) = \frac{2a^2 - 2a(1-\beta)(1-p)(\theta_H - \theta_L) + (1-\beta)(1-p)^2(\theta_H - \theta_L)^2}{8(1+\beta)}$$
$$+\frac{\theta[2a - (1-\beta)(1-p)(\theta_H - \theta_L)]}{4(1+\beta)} + \frac{\theta^2}{4(1+\beta)}.$$

7 Appendix B: Tables

Table 1	Certainty Case			Uncertainty Risk Neutrality		
Contract	2PT	AU	MS	2PT	AU	MS
q_1	$\frac{a}{2(1+\beta)}$			$\frac{a+\theta}{2(1+\beta)}$		
q_2	$\frac{a}{2(1+\beta)}$			$\frac{a+\theta}{2(1+\beta)}$		
P_1	$\frac{a}{2}$			$\frac{a+\theta}{2}$		
P_2	$\frac{a}{2}$			$\frac{a+\theta}{2}$		
R	$\frac{a^2}{4}$			$R_L = -\frac{(1-\beta)(1-p)(\theta_H-\theta_L)(2a+\theta_L+\theta_H)}{4(1+\beta)} + \frac{(a+\theta_L)^2}{4}$ $R_H = \frac{(1-\beta)p(\theta_H-\theta_L)(2a+\theta_L+)}{4(1+\beta)} + \frac{(a+\theta_H)^2}{4}$		
w	0			0		
$F = U$	$\frac{a^2(1-\beta)}{4(1+\beta)}$			$\frac{(1-\beta)[(1-p)(a+\theta_H)^2+p(a+\theta_L)^2]}{4(1+\beta)}$		
$CS/E(CS)$	$\frac{a^2}{4(1+\beta)}$			$\frac{p(a+\theta_L)^2+(1-p)(a+\theta_H)^2}{4(1+\beta)}$		
$W/E(W)$	$\frac{3a^2}{4(1+\beta)}$			$\frac{3[p(a+\theta_L)^2+(1-p)(a+\theta_H)^2]}{4(1+\beta)}$		

Table 2: Uncertainty and Infinite Risk Aversion			
Contract	2PT	AU	MS
q_1	$\frac{a+\theta-(1-p)(\theta_H-\theta_L)}{2(1+\beta)}$	$q_1^L = \frac{(1+p)(a+\theta_H)-2(\theta_H-\theta_L)}{2(1+\beta)(1+p)}$ $q_1^H = \frac{(1+p)(a+\theta_H)-(\theta_H-\theta_L)}{2(1+\beta)(1+p)}$	$\frac{a+\theta-(1-p)(\theta_H-\theta_L)}{2(1+\beta)}$
q_2	$\frac{a+\theta+\beta(1-p)(\theta_H-\theta_L)}{2(1+\beta)}$	$q_2^L = \frac{(1+p)(a+\theta_L)+\beta(1-p)(\theta_H-\theta_L)}{2(1+\beta)(1+p)}$ $q_2^H = \frac{(1+p)(a+\theta_H)+\beta(\theta_H-\theta_L)}{2(1+\beta)(1+p)}$	$\frac{a+\theta-(1-p)(\theta_H-\theta_L)}{2(1+\beta)}$
P_1	$\frac{a+\theta+(1-\beta)(1-p)(\theta_H-\theta_L)}{2}$	$p_1^L = \frac{(1+p)(a+\theta_L)+(1-\beta)(1-p)(\theta_H-\theta_L)}{2(1+p)}$ $p_1^H = \frac{(1+p)(a+\theta_H)+(1-\beta)(\theta_H-\theta_L)}{2(1+p)}$	$\frac{a+\theta+(1-p)(\theta_H-\theta_L)}{2}$
P_2	$\frac{a+\theta}{2}$	$\frac{a+\theta}{2}$	$\frac{a+\theta+(1-p)(\theta_H-\theta_L)}{2}$
R_L	$\frac{(a+\theta_L)^2}{4}$	$\frac{(a+\theta_L)^2}{4}$	$\frac{(a+\theta_L)^2}{4}$
R_H	$\frac{(1-\beta)(\theta_H-\theta_L)[2(a+E(\theta))-(\theta_H-\theta_L)]}{4(1+\beta)} + \frac{(a+\theta_H)^2}{4}$	$\frac{(1-\beta)(\theta_H-\theta_L)[(2a+\theta_H)(1+p)^2-(3+4p)(\theta_H-\theta_L)]}{4(1+\beta)(1+p)^2} + \frac{(a+\theta_H)^2}{4}$	$\frac{(\theta_H-\theta_L)[(1-\beta)(2a+\theta_H+\theta_L)-4(1-p)(\theta_H-\theta_L)]}{4(1+\beta)} + \frac{(a+\theta_H)^2}{4}$
w	$(1-\beta)(1-p)(\theta_H-\theta_L)$	$\frac{(1-\beta)(\theta_H-\theta_L)}{1+p}$	$2(1-p)(\theta_H-\theta_L)$
F	$\frac{(1-\beta)[a+\theta_L-(1-p)(\theta_H-\theta_L)]^2}{4(1+\beta)}$	$\frac{(1-\beta)[a+\theta_L-(\theta_H-\theta_L)][(1+p)(a+\theta_L)-(1-p)(\theta_H-\theta_L)]}{4(1+\beta)(1+p)}$	$\frac{2[a+\theta_L-(1-p)(\theta_H-\theta_L)]-(1+\beta)(a+\theta_L)^2}{4(1+\beta)}$
U	$\frac{(1-\beta)[(a+\theta_L)^2+(1-p)^2(\theta_H-\theta_L)^2]}{4(1+\beta)}$	$\frac{(1-\beta)[(1+p)(a+\theta_L)^2+(1-p)(\theta_H-\theta_L)^2]}{4(1+\beta)(1+p)}$	$\frac{(1-\beta)(a+\theta_L)^2+2(1-p)^2(\theta_H-\theta_L)^2}{4(1+\beta)}$
$E(W)$	$\frac{A}{8(1+\beta)} + \frac{3p(a+\theta_L)^2+3(1-p)(a+\theta_H)^2}{4(1+\beta)}$	$\frac{A}{8(1+\beta)} - \frac{(1-p)(1-\beta)p^2(2+3p)(\theta_H-\theta_L)^2}{8(1+p)^2(1+\beta)}$ $+\frac{3p(a+\theta_L)^2+3(1-p)(a+\theta_H)^2}{4(1+\beta)}$	$\frac{A}{8(1+\beta)} - \frac{(1-p)(\theta_H-\theta_L)[2(a+\theta_L)+3(1-p)(\theta_H-\theta_L)]}{8}$ $+\frac{3p(a+\theta_L)^2+3(1-p)(a+\theta_H)^2}{4(1+\beta)}$
$E(CS)$	$\frac{B}{8(1+\beta)} + \frac{p(a+\theta_L)^2+(1-p)(a+\theta_H)^2}{4(1+\beta)}$	$\frac{B}{8(1+\beta)} - \frac{(1-p)p^2(2-p-p^2)(\theta_H-\theta_L)^2}{8(1+p)^2(1+\beta)}$ $+\frac{p(a+\theta_L)^2+(1-p)(a+\theta_H)^2}{4(1+\beta)}$	$\frac{B}{8(1+\beta)} - \frac{(1-p)(\theta_H-\theta_L)[2(a+\theta_L)+(1-p)(\theta_H-\theta_L)]}{8}$ $+\frac{p(a+\theta_L)^2+(1-p)(a+\theta_H)^2}{4(1+\beta)}$
$A = (1-\beta)(1-p)(\theta_H-\theta_L)(-2a+\theta_L-3E(\theta))$, $B = (1-\beta)(1-p)(\theta_H-\theta_L)[2a+\theta_L+E(\theta)]$			

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