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Why people reach intermediate agreements? Axiomatic and strategic justifications*

José-Manuel Jiménez-Gómez**

Abstract

As Roemer (1986) points out, things become more interesting once we include information. In this paper, following the line started by Jiménez-Gómez and Marco-Gil (2008), we define both a lower and an upper bounds on awards in the framework of the *Lorenz-Bifocal Bankruptcy Problem (Gadea et al. (2010))*, which is an extended bankruptcy problem enriched with a *Commonly Accepted Equity Principles* set and the idea of treat everybody as evenly as possible (Dutta and Ray (1989) and Arin (2007), among others). Moreover, we contribute with the definition of the *Lorenz Double Boundedness Recursive procedure*, which consists on the recursive imposition of both bounds, providing a natural way of justifying the convex combination of bankruptcy rules. Specifically, we retrieve the midpoint of extreme and opposite well known ways of distributing the resource. Finally, we complete our analysis from the strategic viewpoint, obtaining similar results.

Keywords: bankruptcy problems, lower bound, upper bound, duality, recursivity.
JEL Classifications: C71, D63, D71.

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1. Introduction.

When agents face the problem of distributing a scarce resource, they can face it from a double viewpoint: awards and losses. In the former case we focus on the amount of awards that we get. For instance, nowadays, most of the governments have assured an amount of money deposited on the people's bank accounts. In the latter case, we are worry about the quantity of incurred losses, or, in other words, we concern on the highest level of awards that we can obtain. For example, and returning to the previous illustration, some governments have established an upper bound on the bank managers' salaries. Therefore, it appears in a 'natural' way that, in these situations, each agent should receive, not only, at least, a minimum amount according to all the rules which satisfy some accorded equity properties, but also, no more than the maximum amount provided by all these rules.

On the other hand, in Lerner's [17] words: *"People seem to have a strong desire to believe in a just world."* In this sense, as Schokkaert and Overlaet [24] points out, in the vast literature on distributive justice we can find two streams: the philosohers-economists, and the psycologists-sociologists. The former line, which is more theoretical, defines formal models trying to gather an acceptable interpretation of fair distributions. The latter, more informal and descriptive, tries to explain the way in which people perceives fairness, and their behavior when facing distribution problems. However, *Fairness* hardly leads to a single viewpoint: the same distribution problem faced by two different societies, may, almost certainly, lead to the use of different distributional rules (Moulin [20], Schokkaert and Overlaet [24], Young [30], among others). In this sense, trying to throw light on this issue, Roemer [23] says: *"Things become more interesting once we leave the restricted welfarist of bargaining framework and include information on resources, preferences, needs, skills and so on."*

To this respect, Gadea-Blanco et al. [11] and Jiménez-Gomez [15] join all the previous ideas to analyze thoroughly the consequences of enriching the classical model of rationing with a third element, P, called *Legitimate Principles* set, composed of basic ethical principles commonly accepted by a society to resolve a concrete family of such problems. Moreover, following Arin [2] and Dutta and Ray [10], among others, they consider the idea that the general desirable social goal is to treat everybody as evenly as possible, captured by the *Lorenz criterion* (Lorenz [18]).

In this paper, by using as starting point the *Lorenz-Bifocal Bankruptcy Problems*, we propose the simultaneous combination of both a lower and an upper bounds on awards, named *Lorenz Double Boundedness Recursive Process.* This idea, although with differences in the procedure, has been introduced in bargaining problems by Marco et al. [19] with the definition of the *Unanimous-Concession* mechanism. At a first step, their process guarantees to each agent the minimum amount according to a set of agreed solutions, which would be the disagreement point at the following step, and straightforwardly determine the maximum amount that each agent can receive, named the ideal point.

Moreover, we show that the rule so obtained coincides with the average of the *Lorenz-Focal rules*. This result has two consequences. Firstly, we provide a new justification of the convex combination of two extreme and opposite ways of distributing the endowment. And, secondly, we obtain a new method which is invariant to the viewpoint (gains and losses) used to ration the resources. This means, that our new approach treats symmetrically the problem of 'what is available' and 'what is missing', i.e., the rule so obtained is *Self-Dual*.

Finally, we apply these results to three different *Legitimate Principles* sets, providing new basis for the average of old bankruptcy rules.

The paper is organized as follows: Section 2 presents the preliminaries. Section 3 provides our new approach. Section 4 gathers our main results and applies them on different *Legitimate Principles* sets. Section 5 analyzes the strategical model. Section 6 summarizes our conclusions. Finally, the Appendices gathers technical proofs.

2. Preliminaries.

A **bankruptcy problem** is a pair $(E, c) \in \mathbb{R}_+ \times \mathbb{R}^n_+$, where E denotes the endowment and c is the vector of each agents' claim, c_i, for each $i \in N$, $N = \{1, ..., i, ..., n\}$, such that the agents' aggregate demand is higher than the endowment, $\sum_{i \in N} c_i > E$.

For notational convenience, $\mathcal{B}$ will denote the set of all bankruptcy problems; C the sum of the agents' claims, $C = \sum_{i \in N} c_i$; and L the total amount of losses to distribute among the agents, $L = C - E$.

In this context, a rule is a function, $\varphi : \mathcal{B} \to \mathbb{R}^n_+$, such that for each $(E, c) \in \mathcal{B}$, (a) $\sum_{i \in N} \varphi_i(E, c) = E$ (efficiency) and (b) $0 \leq \varphi_i(E, c) \leq c_i$ for each $i \in N$ (non-negativity and claim-boundedness).

Particularly, we focus on the following rules. The first one recommends equal awards to all claimants subject to no-one receiving more than her claim.

The **Constrained Equal Awards** rule, *CEA*, (Maimonides 12th Century, among others) recommends, for each $(E, c) \in \mathcal{B}$, the vector $(\min\{c_i, \mu\})_{i \in N}$, where μ is chosen so that $\sum_{i \in N} \min\{c_i, \mu\} = E$.

Next rule provides for each problem the awards that the *Constrained Equal Awards* rule recommends for $(E, c/2)$, when the endowment is less than the half-sum of the claims. Otherwise, each agent first receives her half-claim, then the *Constrained Equal Awards* rule is re-applied to the residual problem $(E - C/2, c/2)$.

Piniles' rule, *Pin*, (Piniles [22]) provides, for each $(E, c) \in \mathcal{B}$, the vector $(CEA_i(E, c/2))_{i \in N}$, if $E \leq C/2$; and $(c_i/2 + CEA_i(E - C/2, c/2))_{i \in N}$, if $E \geq C/2$.

The following rule is inspired by the *Uniform* one (Sprumont [25]), a solution to the problem of fair division when the preferences are single-peaked. It makes the minimal adjustment in the formula of the *Uniform* rule, taking the half-claims as the peaks and guaranteeing that awards are ordered in the same way as claims are.

The **Constrained Egalitarian** rule, *CE*, (Chun et al. [5]) chooses, for each $(E, c) \in \mathcal{B}$, the vector $(CEA_i(E, c/2))_{i \in N}$, if $E \leq C/2$; and $(\max\{c_i/2, \min\{c_i, \delta\}\})_{i \in N}$, if $E \geq C/2$, where δ is chosen so that $\sum_{i \in N} CE_i(E, c) = E$.

Given a rule φ, for each $(E, c) \in \mathcal{B}$ and each $i \in N$, its dual, φ^d, assigns losses in the same way as φ assigns gains (Aumann and Maschler [1]), $\varphi_i^d(E, c) = c_i - \varphi_i(L, c)$.

In this regard, we obtain that the *Constrained Equal Losses* rule, *CEL,* (Aumann and Maschler [1]), the *Dual of Piniles'* rule, *DPin*, and the *Dual of Constrained Egalitarian* rule, *DCE*, are dual of the *CEA*, the *Pin* and the *CE* rules, respectively.

Finally, the starting point of our next analysis is the extended problem proposed by Gadea-Blanco et al. [11], called *Lorenz-Bifocal Bankruptcy Problem*, which is based on the idea that a society requires both as general social goal to treat everybody as evenly as possible, and that the allocation satisfies a *Legitimate Principles* set.

On the one hand, following Arin [2], Dutta and Ray [10], Cowell [6] and Lambert [16], and with the aim of determining the two focal distribution rules, the discrepancy for sharing the resources is considered by means of the existence of two fixed rules based on the *Lorenz criterion*. Two focus which gather the two most egalitarian rules according to extreme and opposite ways of facing bankruptcy problems: gains and losses. We call these two focus **Lorenz-Focal rules,** which are the *Lorenz-Gains* and *Lorenz-Losses Maximal rules* satisfying P_t, denoted by LGM^{P_t} and LLM^{P_t}, respectively. Formally:

Let $A = \{x \in \mathbb{R}^N : x \geq 0\}$, and for each vector $x \in A$, we denote by $\Pi(x)$ the vector that results from x by permuting the coordinates in such a way that $\Pi_1(x) \leq \Pi_2(x) \leq ... \leq \Pi_n(x)$. Let $x, y \in \mathbb{R}^N$, we say that x **Lorenz dominates** y, denoted by $x \succ_L y$, if $\Pi_1(x) \geq \Pi_1(y)$, $\Pi_1(x) + \Pi_2(x) \geq \Pi_1(y) + \Pi_2(y)$, and so on, with at least one strict inequality. Note that, given $x, y \in \mathbb{R}^N$, we do not impose on these vectors the condition $\sum_{i \in N} x_i = \sum_{i \in N} y_i$ in order to apply the *Lorenz* domination criterion (see Arin [2]).

If the partial sums are equal for $\Pi(x)$ and $\Pi(y)$, the two vectors, x and y are said to be **Lorenz equivalent**, denoted by $x \sim_L y$.

Moreover, a vector $x \in A$ is **Lorenz Maximal** if there is no other vector $y \in A$ such that, $y \succ_L x$. And, particularly, given a set $S \subseteq A$, a vector $x \in S$ is **Lorenz Maximal** in S if there is no other vector $y \in S$ such that, $y \succ_L x$.

Finally, applying the *Lorenz-criterion* on the two focus, gains and losses, that 'naturally' arises in bankruptcy problems, we can define the concepts of *Lorenz-Gains* and *Lorenz-Losses* domination.

Given two bankruptcy rules f and g, we say that f Lorenz-gains dominates g if for each $(E,c) \in \mathcal{B}$, $f(E,c) \succ_L g(E,c)$. And a bankruptcy rule f is **Lorenz-Gains Maximal, LGM,** if there is no other g such that, for each $(E,c) \in \mathcal{B}$, $g(E,c) \succ_L f(E,c)$. Analogously, f Lorenz-losses dominates g if for each $(E,c) \in \mathcal{B}$, $(c - f(E,c)) \succ_L (c - g(E,c))$. And a bankruptcy rule f is **Lorenz-Losses Maximal, LLM**, if there is no other g such that, for each $(E,c) \in \mathcal{B}$, $(c - g(E,c)) \succ_L (c - f(E,c))$.

On the other hand, we consider that a society agrees in a set of basic properties or principles on which the distribution of the resource must be made in base of a *Legitimate Principles* set. That is, these are problems where all the admissible rules must satisfy the '*Commonly Accepted Equity Principles'* set.

Therefore, the *Lorenz-Bifocal Bankruptcy Problem* is defined as follows.

A **Lorenz-Bifocal Bankruptcy Problem**, LB_{P_t}, is a triplet $LB_{P_t} = (B, LGM^{P_t}, LLM^{P_t})$ where $B = (E,c) \in \mathcal{B}$, P_t is a fixed set of principles on which a particular society has agreed, and both LGM^{P_t} and LLM^{P_t} are the Lorenz-Gains and Lorenz-Losses Maximal rules satisfying P_t for each $B \in \mathcal{B}$, respectively.

Henceforth, P denotes the set of all subsets of properties of rules. Each $P_t \in P$ represents a specific society which will always apply such principles for solving its problems; $\mathcal{LB}_P$ denotes the set of all *Lorenz-Bifocal Bankruptcy Problem,* Φ the set of all rules and $\Phi(LB_{P_t})$ be the subset of *Lorenz-Bifocal Admissible rules* in P_t.

So that, a *Lorenz-Bifocal Admissible rule* will be a rule satisfying P_t which recommends an allocation between the two *Lorenz-Focal rules.* That is,

A **Lorenz-Bifocal Admissible rule** on $\mathcal{LB}_P$ is a function $\varphi : \mathcal{LB}_P \to \mathbb{R}^n$, such that for each *Lorenz-Bifocal bankruptcy* problem $LB_{P_t} \in \mathcal{LB}_P$, φ is a rule satisfying P_t, associating for each $i \in N$ a part of the resources satisfying

$$\begin{aligned} \min\{LGM_i^{P_t}(E,c), LLM_i^{P_t}(E,c)\} &\leq \varphi_i(E,c) \\ &\leq \max\{LGM_i^{P_t}(E,c), LLM_i^{P_t}(E,c)\}. \end{aligned}$$

3. The model.

In this section, using the concept of the *P-Safety* (Jiménez-Gómez and Marco-Gil [13]), we define a lower and an upper bound on awards, named the *Lorenz P-Safety* and *Lorenz*

P-Ceiling, respectively. Particularly, the former corresponds with the smallest amount that each agent could get according to the application of the *Lorenz-Bifocal Admissible rules.* The latter ensures that each agent's awards are confined to the maximum amount among the *Lorenz-Bifocal Admissible rules.* Formally:

Definition 3.1. *Given $LB_{P_t} \in \mathcal{LB}_P$, the* **Lorenz P-Safety,** *Ls, is for each $i \in N$,*
$$Ls_i(LB_{P_t}) = \min\left\{LGM_i^{P_t}(E,c), LLM_i^{P_t}(E,c)\right\}.$$

Definition 3.2. *Given $LB_{P_t} \in \mathcal{LB}_P$, the* **Lorenz P-Ceiling,** *Lc, is for each $i \in N$,*
$$Lc_i(LB_{P_t}) = \max\left\{LGM_i^{P_t}(E,c), LLM_i^{P_t}(E,c)\right\}.$$

As we have mentioned, bankruptcy problems has been rewritten for losses by using the idea of duality. In this regard, when focusing on losses, the starting point of our model will be the *Dual Lorenz-Bifocal Bankruptcy problems.* That is, given $P_t \in P$ and $(E,c) \in \mathcal{B}$, we consider the set of dual properties of P_t, the problem of distributing the losses (L,c) and the dual bankruptcy rules.

Given a *Lorenz-Bifocal Bankruptcy Problem,* LB_{P_t}, the *Dual Lorenz-Bifocal Bankruptcy problem,* denoted by $(LB_{P_t})^d$, is a triplet $(LB_{P_t})^d = (B^d, \left(LGM^{P_t}\right)^d, \left(LLM^{P_t}\right)^d)$ where $B^d = (L,c) \in \mathcal{B}$, P_t is a fixed set of principles on which a particular society has agreed, and both $\left(LGM^{P_t}\right)^d$ and $\left(LLM^{P_t}\right)^d$ are the *Lorenz-Gains* and *Lorenz-Losses Maximal* rules satisfying P_t for each $B^d \in \mathcal{B}$, respectively.

Note that, whenever the accorded properties are *Self-Dual*, if we focussed on losses instead of awards, an alternative way of understanding this limit would be as a lower bound on losses, i.e., for each $(LB_{P_t}) \in \mathcal{LB}_P$, the *Lorenz P-Ceiling*, for each $i \in N$, is the smallest loss incurred for each agent by all the *Lorenz-Bifocal Admissible rules* in P_t,

$$\begin{aligned} Lc_i(LB_{P_t}) &= c_i - \min\left\{LGM_i^{P_t}(L,c), LLM_i^{P_t}(L,c)\right\} \\ &= c_i - Ls_i\left((LB_{P_t})^d\right). \end{aligned}$$

Therefore, the *Lorenz P-Safety* and the *Lorenz P-Ceiling* are dual, a fact that will be used later on.

To this regard, as we have seen, there are everyday situations in which a society obtains an agreement on the properties which define the way of rationing the endowment, and then decides that each agent should receive, at least, a minimum amount, and, at most, a maximum amount according to all the *Lorenz-Bifocal Admissible* rules. With these ideas in mind, we define the *Lorenz Double Boundedness Recursive Process* as the procedure in which at each step, every agent's claim is truncated by her *Lorenz P-Ceiling*, and each of them receives her *Lorenz P-Safety*.

Definition 3.3. *Given $m \in \mathbb{N}$, the* ***Lorenz Double Boundedness Recursive Process****, $LDBR^m$, associates for each $LB_{P_t} \in \mathcal{LB}_P$ and each $i \in N$,*

$$\begin{gathered}\left[LDBR(LB^m_{P_t})\right]_i = Ls_i(LB^m_{P_t}),\\ \text{where } LB^m_{P_t} \equiv \left((E^m, c^m), LGM^{P_t}, LLM^{P_t}\right),\\ (E^1, c^1) \equiv (E, c) \text{ and for } m \geq 2,\\ E^m \equiv E^{m-1} - \sum_{i\in N} Ls_i(LB^{m-1}_{P_t}).\\ c^m_i = Lc_i(LB^{m-1}_{P_t}) - Ls_i(LB^{m-1}_{P_t}).\end{gathered}$$

According to this process, an agent will get at the first step her *Lorenz P-Safety* of the original problem. At the second step, we redefine a residual problem, in which the endowment consists on the remaining resources and each agent's claim is truncated by her *Lorenz P-Ceiling*, and then adjusted down by the amount received. Then each agent receives her *Lorenz P-Safety* of this residual problem, and so on. We can easily see that this process is not always efficient, but when it does, we call it the *Lorenz Double Recursive rule.* Formally:

Definition 3.4. *The* ***Lorenz Double Recursive*** *rule, φ^{LDR}, associates for each $LB_{P_t} \in \mathcal{LB}_P$ and each $i \in N$, $\varphi^{LDR}_i(LB_{P_t}) = \sum\limits_{m=1}^{\infty} \left[LDBR(LB^m_{P_t})\right]_i$, whenever*

$$\sum_{i\in N}\left(\sum_{m=1}^{\infty}\left[LDBR(LB^m_{P_t})\right]_i\right) = E.$$

Note that this rule will be a *Lorenz-Bifocal Admissible* rule whenever it fulfills the *Legitimate Principles* set on which the *Lorenz Double Boundedness Recursive Process* is based. A fact that, as we will see later, cannot be always guaranteed.

4. Main results and applications.

This section presents a general result and its application on three *Legitimate Principles* sets.

Firstly, next theorem establishes that, whenever the properties selected are *Self-Dual*, the final allocation provided by the *Lorenz Double Recursive rule* will correspond with the average of the lower and upper bounds for each *Lorenz-Bifocal Bankruptcy Problem*, i.e., the *Lorenz P-Safety* and the *Lorenz P-Ceiling*.

Theorem 4.1. *For each $LB_{P_t} \in \mathcal{LB}_P$, such that P_t is Self-Dual, then for each $i \in N$,*

$$\varphi_i^{LDR}(LB_{P_t}) = \frac{Lc_i(LB_{P_t}) + Ls_i(LB_{P_t})}{2}.$$

Proof. The proof of this result is based on two lemmas and a remark.

The first lemma shows that, in any step $m \in \mathbb{N}$, $m > 1$, the sum of the *Lorenz P-Safety* and the *Lorenz P-Ceiling* coincides with the sum of the claims.

Lemma 4.2. *For each $LB_{P_t} \in \mathcal{LB}_P$, such that P_t is Self-Dual, and $m \in \mathbb{N}, m > 1$,*

$$\sum_{i \in N} \left[Lc_i(LB_{P_t}^m) + Ls_i(LB_{P_t}^m) \right] = C^m.$$

Proof. Let $LB_{P_t} \in \mathcal{LB}_P$, such that P_t is *Self-Dual* and $m \in \mathbb{N}, m > 1$. Note that for each $i \in N$, and each *Lorenz-Bifocal Admissible rule*, φ,

$$\min \left\{ LGM_i^{P_t}(E,c), LLM_i^{P_t}(E,c) \right\} \leq \varphi_i \leq \max \left\{ LGM_i^{P_t}(E,c), LLM_i^{P_t}(E,c) \right\}.$$

Then, by the definitions of the *Lorenz P-Safety* and the *Lorenz P-Ceiling*, these two rules define the *Lorenz P-Safety* and the *Lorenz P-Ceiling* of each agent for a set of properties P_t, i.e.,

$Ls_i(LB_{P_t}) = \min \left\{ LGM_i^{P_t}(E,c), LLM_i^{P_t}(E,c) \right\}$, and

$Lc_i(LB_{P_t}) = \max \left\{ LGM_i^{P_t}(E,c), LLM_i^{P_t}(E,c) \right\}$.

Moreover, by their dual relation, for each agent we are adding the two *Lorenz-Focal rules*. So next expression comes straightforwardly.

$$\sum_{i \in N} \left[\frac{Lc_i(LB_{P_t}^m) + Ls_i(LB_{P_t}^m)}{2} \right] = E^m.$$

Finally, we know that

$$
\begin{aligned}
E^m &= E^{m-1} - \sum_{i \in N} Ls_i(LB_{P_t}^{m-1}) = \\
&= \sum_{i \in N} \left[\frac{Lc_i(LB_{P_t}^{m-1}) + Ls_i(LB_{P_t}^{m-1})}{2} \right] - \sum_{i \in N} Ls_i(LB_{P_t}^{m-1}) = \\
&= \sum_{i \in N} \left[\frac{Lc_i(LB_{P_t}^{m-1}) - Ls_i(LB_{P_t}^{m-1})}{2} \right] = C^m/2,
\end{aligned}
$$

by the definition of the *Lorenz Double Boundedness Recursive Process*. **q.e.d.**

The following remark is a direct consequence of Lemma 4.2 and it says that for each *Lorenz-Bifocal Bankruptcy Problem*, and at any step $m \in \mathbb{N}, m > 1$, the half of the claims sum at every step of the *Lorenz Double Boundedness Recursive Process* coincides with both the endowment and the total loss at every step of the process.

Remark 1. *For each* $(LB_{P_t}) \in \mathcal{LB}_P$*, such that* P_t *is Self-Dual and* $m \in \mathbb{N}, m > 1$, $E^m = L^m = C^m/2$.

Proof. Let $LB_{P_t} \in \mathcal{LB}_P$, such that P_t is *Self-Dual* and $m > 1 \in \mathbb{N}$. We know that, $L^m = C^m - E^m$. By Lemma 4.2, $E^m = C^m/2$. Therefore, $L^m = C^m - C^m/2 = C^m/2$. **q.e.d.**

The second lemma says that, each agent's claim at each step different of the initial one coincides with sum of both the lower and upper bound on awards.

Lemma 4.3. *For each* $LB_{P_t} \in \mathcal{LB}_P$*, such that* P_t *is Self-Dual and* $m > 1 \in \mathbb{N}$*,*

$$c_i^m = Lc_i(LB_{P_t}^m) + Ls_i(LB_{P_t}^m).$$

Proof. Let $LB_{P_t} \in \mathcal{LB}_P$, such that P_t is *Self-Dual*, for each $i \in N$, and each $m > 1 \in \mathbb{N}$, by Remark 1 we know that for $m > 1 \in \mathbb{N}$, $L^m = E^m$, so, $Ls_i(LB_{P_t}^m) = Ls_i((LB_{P_t})^d)$. By duality $Lc_i(LB_{P_t}^m) = c_i^m - Ls_i(\left(LB_{P_t}^m\right)^d) = c_i^m - Ls_i(LB_{P_t}^m)$,then, $c_i^m = Lc_i(LB_{P_t}^m) + Ls_i(LB_{P_t}^m)$. **q.e.d.**

Proof of Theorem 4.1.

Let $LB_{P_t} \in \mathcal{LB}_P$, such that P_t is *Self-Dual*, for each $i \in N$, and each $m \in \mathbb{N}$,

$$\varphi_i^{LDR}(LB_{P_t}) = Ls_i(LB_{P_t}) + \sum_{m=2}^{\infty} Ls_i(LB_{P_t}^m).$$

By the definition of the *Lorenz Double Boundedness Recursive Process*,

$$\begin{aligned} \sum_{m=2}^{\infty} c_i^m &= \sum_{m=2}^{\infty} \left[Lc_i(LB_{P_t}^{m-1}) - Ls_i(LB_{P_t}^{m-1}) \right] = \\ &= Lc_i(LB_{P_t}) + \sum_{m=2}^{\infty} Lc_i(LB_{P_t}^m) - Ls_i(LB_{P_t}) - \sum_{m=2}^{\infty} Ls_i(LB_{P_t}^m). \end{aligned}$$

By Lemma 4.3,

$$\sum_{m=2}^{\infty} c_i^m = \sum_{m=2}^{\infty} \left[Lc_i(LB_{P_t}^m) + Ls_i(LB_{P_t}^m) \right].$$

So,

$$\begin{aligned} Lc_i(LB_{P_t}) + \sum_{m=2}^{\infty} Lc_i(LB_{P_t}^m) - Ls_i(LB_{P_t}) - \sum_{m=2}^{\infty} Ls_i(LB_{P_t}^m) = \\ = \sum_{m=2}^{\infty} \left[Lc_i(LB_{P_t}^m) + Ls_i(LB_{P_t}^m) \right]. \end{aligned}$$

Thus,

$$\sum_{m=2}^{\infty} Ls_i(LB_{P_t}^m) = \left(Lc_i(LB_{P_t}) - Ls_i(LB_{P_t}) \right)/2.$$

Therefore,

$$\begin{aligned} \varphi_i^{LDR}(LB_{P_t}) &= Ls_i(LB_{P_t}) + \frac{Lc_i(LB_{P_t}) - Ls_i(LB_{P_t})}{2} \\ &= \frac{Ls_i(LB_{P_t}) + Lc_i(LB_{P_t})}{2}. \end{aligned}$$ **q.e.d.**

Therefore, the next result, which follows straightforwardly from Theorem 4.1, shows that the *Lorenz Double Recursive rule* for P_t can be defined as the average of the associated *Lorenz-Focal rules.*

Proposition 4.4. *For each $LB_{P_t} \in \mathcal{LB}_P$, such that P_t is Self-Dual, then*

$$\varphi^{LDR}(LB_{P_t}) = \frac{LGM^{P_t}(E,c) + LLM^{P_t}(E,c)}{2}.$$

A direct consequence of the above proposition is that the *Lorenz Double Recursive rule* not only is always well defined, i.e. satisfies efficiency, non-negativity and claim-boundedness, but it also satisfies *Self-Duality* (Yeh and Thomson [27]), which means that it provides the same allocation of the endowment when distributing awards or losses.

Moreover, with this result we retrieve the convex combination of the two extreme *Lorenz-Focal rules*. Particularly, the *Lorenz Double Recursive rule* proposes the mid-point between the two rules which represent extreme and opposite ways of sharing awards among claimants according to the imposed requirements. So, in other words, it could be said that the rationing of the endowment obtained by the recursive double imposition of the *Lorenz P-Safety* and the *Lorenz P-Ceiling* neither favor nor hurts to any agent in particular. Following Yeh and Thomson [27]:

> *'When two rules express opposite viewpoints on how to solve a bankruptcy problem, it is natural to compromise between them by averaging'.*

Finally, next remark provides a generalization of previous results.

Remark 2. *For any problem such that there exist two Focal rules, which are dual each other, the final allocation provided by the Double Recursive rule will correspond with the average of these two Focal rules.*

Note that in this case we consider the *Double Recursive* rule as the procedure were each agent receives the smallest amount according to these focus and her claim is truncated by the highest amount recommended by both of them.

4.1. Applications.

In this context, we consider three possible choices of the *'Commonly Accepted Equity Principles'* set that a society could require on the rules, taking into account that the introduced properties have been understood by many authors as minimal requirements of fairness (see for instance Thomson [26]).

Specifically, we consider the following *Legitimate Principles* sets,

P_1 =*{Efficiency, Claim-boundedness and Non-Negativity},*
$P_2 = P_1 \cup$*{Resource Monotonicity, Super-Modularity and Midpoint Property},* and
$P_3 = P_1 \cup$*{Resource Monotonicity* and *Midpoint Property}.*

Let us dwell on the meaning of the proposed principles.

Resource Monotonicity (Curiel et al. [7], Young [31], among others) demands that if the endowment increases, then all individuals should get at least what they received initially.

Resource Monotonicity: for each $(E, c) \in \mathcal{B}$ and each $E' \in \mathbb{R}_+$ such that $C > E' > E$, then $\varphi_i(E', c) \geq \varphi_i(E, c)$, for each $i \in N$.

Order Preservation (Aumann and Maschler [1]) requires respecting the ordering of the claims: if agent i's claim is at least as large as agent j's claim, he should receive and loss at least as much as agent j, does respectively.

Order Preservation: for each $(E, c) \in \mathcal{B}$, and each $i, j \in N$, such that $c_i \geq c_j$, then $\varphi_i(E, c) \geq \varphi_j(E, c)$, and $c_i - \varphi_i(E, c) \geq c_j - \varphi_j(E, c)$, that is $l_i(E, c) \geq l_j(E, c)$.

A *Super-Modular* rule (Dagan et al. [8]) allocates each additional dollar in an 'order preserving' manner. In other words, when the endowment increases, agents with higher claims receive a greater part of the increment than those with lower claims.

Super-Modularity: for each $(E, c) \in \mathcal{B}$, all $E' \in \mathbb{R}_+$ and each $i, j \in N$ such that $C > E' > E$ and $c_i \geq c_j$, then $\varphi_i(E', c) - \varphi_i(E, c) \geq \varphi_j(E', c) - \varphi_j(E, c)$.

Midpoint Property (Chun, Schummer and Thomson [5]) requires that if the estate is equal to the sum of the half-claims, then all agents should receive their half-claim.

Midpoint Property: for each $(E, c) \in \mathcal{B}$ and each $i \in N$, if $E = C/2$, then $\varphi_i(E, c) = c_i/2$.

At this point, Lorenz comparisons of bankruptcy rules from the gains viewpoint can be found in Bosmans and Lauwers [3] and in Thomson [28]. This fact together duality let us to know the *Lorenz P-Safety* and the *Lorenz P-Ceiling* for $P_t \in \{P_1, P_2, P_3\}$. That is, (see Jiménez-Gómez [15]) the *Lorenz-Focal rules* that mark out the region of *Lorenz-Bifocal Admissible rules* for P_1, P_2, and P_3, are the pairs (CEA, CEL), $(Pin, Dpin)$, and (CE, DCE), respectively. So, *Lorenz-Bifocal Bankruptcy Problems* for each of these principles sets are well-defined, being their elements triplets, such that, for each $(E, c) \in \mathcal{B}$,

$$\begin{aligned} LB_{P_1} &= ((E, c), CEA, CEL), \\ LB_{P_2} &= ((E, c), Pin, Dpin), \\ LB_{P_3} &= ((E, c), CE, DCE). \end{aligned}$$

Graphically, Figures 4.1, 4.2, 4.3 and 4.4 represent the two bounds for bi-personal problems with $P_t \in \{P_1, P_2, P_3\}$.

Particularly, the black line represents the estate (E). The blue and the green lines show the *Lorenz-Focal* rules marking out the area of all the admissible path of awards satisfying the properties in P_1, P_2, and P_3. That is, the green and the blue solid lines are the *CEL* the *CEA* rules for P_1, the *DPin* and *Pin* rules for P_2, and the *DCE* and the *CE* rules for P_3, respectively.

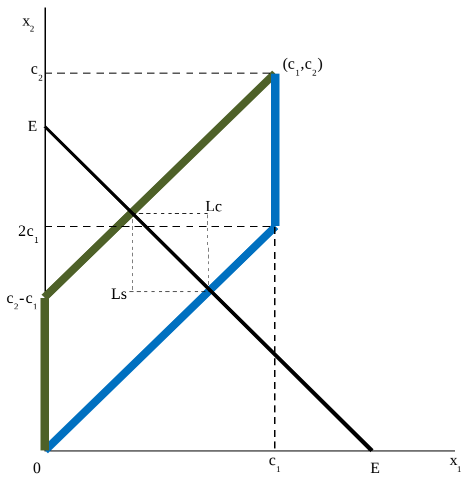

Figure 4.1: Lorenz P-Safety and Lorenz P-Gain for P_1.

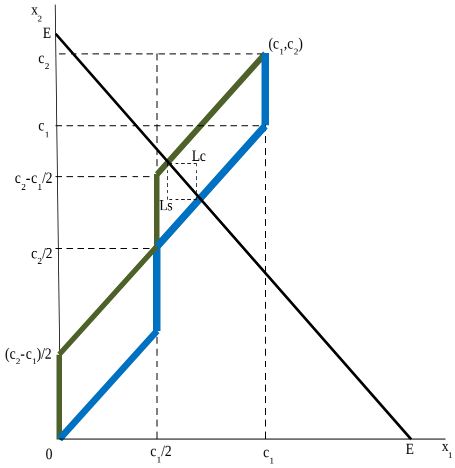

Figure 4.2: Lorenz P-Safety and Lorenz P-Gain for P_2.

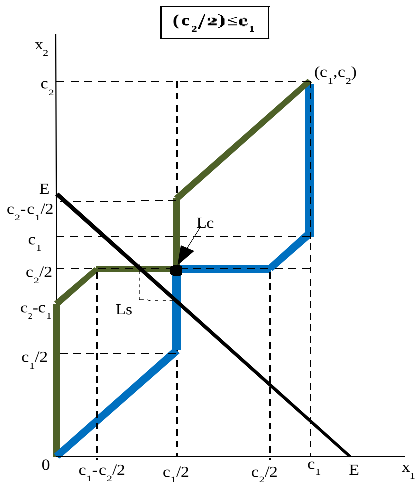

Figure 4.3: Lorenz P-Safety and Lorenz P-Gain for P_3 (case a).

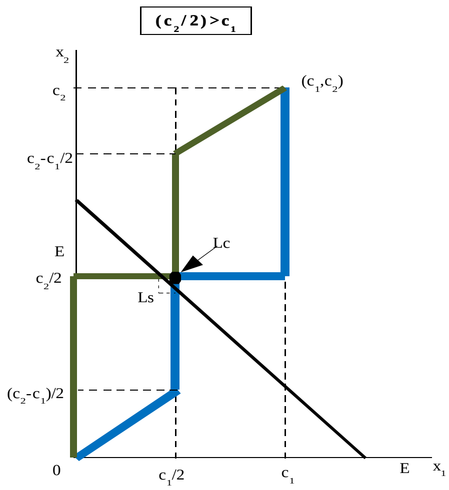

Figure 4.4: Lorenz P-Safety and Lorenz P-Gain for P_3 (case b).

Concluding this section, we apply the result in Theorem 4.1 to the three different *Legitimate Principles* sets mentioned previously.

Corollary 4.5. *For each* $LB_{P_1} \in \mathcal{LB}_P$, $LB_{P_1} = ((E,c), CEA, CEL)$, *the Lorenz Double Recursive rule is the average of the Constrained Equal Awards and the Constrained Equal Losses rules.*

Corollary 4.6. *For each* $LB_{P_2} \in \mathcal{LB}_P$, $LB_{P_2} = ((E,c), Pin, DPin)$, *the Lorenz Double Recursive rule is the average of Piniles' and the Dual of Piniles' rules.*

Corollary 4.7. *For each* $LB_{P_3} \in \mathcal{LB}_P$, $LB_{P_3} = ((E,c), CE, DCE)$, *the Lorenz Double Recursive rule is the average of the Constrained Egalitarian and the Dual Constrained Egalitarian rules.*

Note that these corollaries imply that the allocation proposed by our new procedure is *Lorenz-Bifocal Admissible* with $P_t \in \{P_1, P_2, P_3\}$, since, by Yeh and Thomson [27], the *Lorenz Double Recursive rule* preserves *Resource Monotonicity, Super-Modularity* and the *Midpoint* property.

However, the *Lorenz Double Recursive rule* fails some properties, even when the two *Lorenz-Focal rules* fulfill them (Yeh and Thomson [27]), such that *Composition Down* (Moulin [21]), *Composition Up* (Young [30]), and *Consistency* (Young [30]). Then, it is not possible to ensure that the final allocation were *Lorenz-Bifocal Admissible* when uses at the same time and recursively the *Lorenz P-Safety* and the *Lorenz P-Ceiling.* For example, we can see that if we add to the set P_1 the property of *Consistency,* the two *Lorenz-Focal rules* remain the *Constrained Equal Awards* and the *Constrained Equal Losses* rules, but the average of these *Lorenz Admissible* rules does not satisfy one of the initial properties. Then, the natural question is '*Would a society have any argument to apply this new rule?'* The answer could be affirmative since, although a society could have reasons to not agree on applying the result of the *Lorenz Double Recursive rule*, since it may fail some properties, this way of distribute the endowment has been defended by many authors as a natural way to agree on a 'middle' allocation between two extreme ways of rationing (see, for instance, Yeh and Thomson [27]). Moreover, another reason to use this method is that we know exactly the result of the procedure and the properties which are satisfied by the final allocation.

5. An strategic view.

Next, we analyze the previous model from a strategical point of view. Concretely, we define a new mechanism which combines the philosophy of the *Diminishing Claims* (Chun [4]) and the *Unanimous Concessions* (Herrero [12]) procedures, using the fact that they are dual.

This new method, named the *Double Concessions* procedure, says that since agents have chosen their preferred rules, if at the initial step there is no agreement, at the second step, each agent receives the smallest amount among all the proposed at step 1. (*Unanimous Concessions).* Now, we redefine the residual bankruptcy problem, in which the endowment is the leftover resources, and the claims are truncated by the maximum amount recommended *(Diminishing Claims)* by all the suggested rules and adjusted down by the amounts just given. Then, the procedure is again applied until an agreement is reached. If this is not the case, the solution will be the limit of the procedure if it is feasible, and zero otherwise. Formally:

Definition 5.1. ***Double Concessions procedure***, *du:*

Let $LB_{P_t} \in \mathcal{LB}_P$. At the first stage, each agent chooses a rule $\varphi^i \in \Phi(LB_{P_t})$. The proposal of the Double Concessions procedure, $du\left[\varphi, LB_{P_t}\right]$ is obtained as follows:

[Step 1] If all agents agree on $\varphi(LB_{P_t})$, then $du\left[\varphi, LB_{P_t}\right] = \varphi LB_{P_t}$. Otherwise, go to next step.

[Step 2] Let us define

$Ls_i\left(LB_{P_t}\right) = \min_{j \in N} \varphi_i^j\left(LB_{P_t}\right),$

$Lc_i\left(LB_{P_t}\right) = \max_{j \in N} \varphi_i^j\left(LB_{P_t}\right),$

$c^2 = Lc\left(LB_{P_t}\right) - Ls\left(LB_{P_t}\right),$

$E^2 = E - \sum_{i \in N} Ls_i\,(LB_{P_t})$, and
$LB^2_{P_t} = \left(\left(E^2, c^2\right), LGM^{P_t}, LLM^{P_t}\right)$.
Now, if all agents agree on $\varphi(LB^2_{P_t})$, then $du\,[\varphi, LB_{P_t}] = Ls\,(LB_{P_t}) + \varphi\left(LB^2_{P_t}\right)$. Otherwise, go to next step.

[Step $m+1$] Let us define
$Ls_i\left(LB^m_{P_t}\right) = \min_{j \in N} \varphi^j_i\left(LB^m_{P_t}\right),$
$Lc_i\left(LB^m_{P_t}\right) = \max_{j \in N} \varphi^j_i\left(LB^m_{P_t}\right),$
$c^{m+1} = Lc\left(LB^m_{P_t}\right) - Ls\left(LB^m_{P_t}\right),$
$E^{m+1} = E^m - \sum_{i \in N} Ls_i\left(LB^m_{P_t}\right)$, and
$LB^{m+1}_{P_t} = \left(\left(E^{m+1}, c^{m+1}\right), LGM^{P_t}, LLM^{P_t}\right)$.
Now, if all agents agree on $\varphi\left(LB^{m+1}_{P_t}\right)$, then
$du\,[\varphi, LB_{P_t}] = \sum_{k=1}^{m} Ls\left(LB^k_{P_t}\right) + +\varphi\left(LB^{m+1}_{P_t}\right)$. Otherwise, go to next step.

[Limit case] Compute $\lim_{m \to \infty} \sum_{k=1}^{m} Ls\left(LB^k_{P_t}\right)$. If it converges to an allocation, x, such that $\sum_{i \in N} x_i \leq E$, $du\,[\varphi, LB_{P_t}] = x$. Otherwise, $du\,[\varphi, LB_{P_t}] = 0$.

From now on, let $\Gamma^{du}_{LB_{P_t}}$ denote the game induced by the *Double Concessions* procedure when agents acts strategically, in which the set of players is N, the strategies for each agent are rules in $\Phi\,(LB_{P_t})$ and the payoffs are the sum of the amounts received by each agent in each step $m \in \mathbb{N}$. That is,

$$\Gamma^{du}_{LB_{P_t}} = \left\{ N, \left\{\varphi^i \in \Phi(LB_{P_t})\right\}_{i=1}^{n}, \left\{ \sum_{k=1}^{m} Ls_i\left(LB^k_{P_t}\right) \right\}_{i=1}^{n} \right\},$$

where m denotes the step where the agreement is reached, and ∞ otherwise.

Next theorem shows the main result when applying the *Double Concessions* procedure in P_t.

Theorem 5.2. *In any Nash equilibrium induced by the game $\Gamma^{du}_{LB_{P_t}}$, such that P_t is Self-Dual, each agent receives the amount given by the average of the two Lorenz-Focal rules.*

Proof.- Let $LB_{P_t} \in \mathcal{LB}_P$, such that P_t is *Self-Dual.*

By Theorem 4.1 we know that whenever each claimant's *Lorenz P-Safety* and *Lorenz P-Ceiling* corresponds with the amount recommending by one of the two *Lorenz-Focal rules*, then all the agents receive the amount given by the average of the two *Lorenz-Focal rules.*

Moreover, by the definition of the *Double Concessions* procedure can easily note that the *Lorenz-Gains Maximal* rule is a weakly dominant strategy for the smallest agent, and the *Lorenz-Losses Maximal* rule is a weakly dominant strategy for the highest claimant. Thus, for each $i \in N$,

$$\begin{aligned} Ls_i(LB^m_{P_t}) &= \min_{\varphi\in\Phi(LB_{P_t})} \left\{LGM_i^{P_t}, LLM_i^{P_t}\right\}, \text{ and} \\ Lc_i(LB^m_{P_t}) &= \max_{\varphi\in\Phi(LB_{P_t})} \left\{LGM_i^{P_t}, LLM_i^{P_t}\right\}. \end{aligned}$$

Therefore, by Theorem 4.1,

$$\begin{aligned} du\left[\varphi, LB_{P_t}\right] &= \lim_{m\to\infty}\sum_{k=1}^{m} Ls\left(LB^k_{P_t}\right) = Ls_i(LB_{P_t}) + \sum_{m=2}^{\infty} Ls_i(LB^m_{P_t}) \\ &= \frac{Ls_i(LB_{P_t}) + Lc_i(LB_{P_t})}{2}. \end{aligned}$$ **q.e.d.**

Consequently, next results are straightforwardly obtained by applying Theorem 5.2.

Corollary 5.3. *In any Nash equilibrium induced by the game* $\Gamma^{du}_{LB_{P_1}}$, *each agent receives the amount given by the average of the Constrained Equal Awards and the Constrained Equal Losses rules.*

Corollary 5.4. *In any Nash equilibrium induced by the game* $\Gamma^{du}_{LB_{P_2}}$, *each agent receives the amount given by the average of Piniles' and the Dual of Piniles' rules.*

Corollary 5.5. *In any Nash equilibrium induced by the game* $\Gamma^{du}_{LB_{P_3}}$, *each agent receives the amount given by the average of the Constrained Egalitarian and the Dual Constrained Egalitarian rules.*

As pointed out in the previous section, the convex combination of rules preserves *Resource Monotonicity*, *Super-Modularity* and the *Midpoint* properties (Yeh and Thomson [27]). Hence, as theses corollaries show, our new procedure is *Lorenz-Bifocal Admissible* for $P_t \in \{P_1, P_2, P_3\}$, while applying independently the *Diminishing Claims* and the *Unanimous Concessions* fail *Resource Monotonicity* (see Jiménez-Gómez [14]).

6. Conclusions.

In this paper we have defined a new method for distributing the endowment, using the idea of recursively guaranteeing 'fair' minimum and maximum amounts to each agent in bankruptcy problems, named the *Lorenz Double Recursive rule.*

In this context, our main result states that the *Lorenz Double Recursive rule* corresponds with the average of the two extreme *Lorenz-Bifocal Admissible rules*, implying that our new rule retrieves an allocation which coincides with the midpoint between the rule that favors the highest claimant and the other one that favors the lowest agent, i.e., we obtain new justifications for the convex combination of rules. Then, we particularize this procedure to some *Legitimate Principles* sets proposed which have been interpreted by many authors as 'basic' requirements, recovering the average of old and well-known rules. Moreover, we have shown that our process does not guarantee that the final allocation were admissible.

Finally, we have obtained similar results when applying this methodology in the strategical framework. In this line, we define a new mechanism, named the *Double Concessions* procedure, combining the *Diminishing Claims* (Chun [4]) and the *Unanimous Concessions* (Herrero [12]) procedures. Thus, we have justified from an axiomatic and a non-cooperative point of view the convex combination of rules.

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