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Plurality versus proportional electoral rule: study of voters' representativeness*

Amedeo Piolatto**

Abstract

Thinking of electoral rules, common wisdom suggests that proportional rule is more fair, since all voters are equally represented: at times, it turns out that this is false. I study the formation of both Parliament and Government; for the composition of the former I consider plurality and proportional rule; for the formation of the latter, I assume that parties play a non-cooperative game à la Rubinstein. I show that, unless parties are impatient to form a Government, proportional electoral rules translate into a more distortive distribution of power among parties than plurality rule; this happens because of the bargaining power of small parties during Government formation.

JEL Classification: C71, D72, H1, P16.

Keywords: electoral systems, proportional rule, plurality rule, voters' representation.

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“Thus there is an inherent conflict between two goals. The ideals of democracy and equality require as proportional representation as possible while efficient government often requires less proportional representation” (Laakso and Taagepera (1981), p. 107).

1 Introduction

Electoral systems differ in government efficiency and representativeness. Representativeness, is the system capability to produce laws in line with voters’ will. A system is perfectly representative if parties’ power is proportional to their share of votes. Efficiency is the capability to produce well structured and coherent laws, wasting as few resources as possible. Efficiency is beyond the scope of this work; I focus on representativeness, measuring the difference between party’s power and share of votes under different voting rules.

The link between voters’ preferences and the legislative outcome is extremely complex. Under proportional rule, Parliament’s composition perfectly reflects parties’ shares of vote. Common wisdom suggests that, better reflecting voters’ preferences, proportional systems are the most equitable.¹ A main drawback of the proportional rule is the greater instability and the increase in the laws production time, compared with majority voting systems.² From 2006 to 2008, Mr. Prodi led the Italian government; a small pivotal party succeeded in heavily influencing his activity; I shows that common wisdom is misleading: plurality rule, at times, better reflects voters’ preferences.

Two filters possibly intervene (see Figure 1): the electoral system (filter 1) and the coalition formation (filter 2). Any but the proportional rule implies a distortion in Parliament composition. The coalition formation process also produces distortions: some parties are excluded from the government; in addition, the distribution of power amongst parties differs from the distribution of seats in Parliament, because of parties’ bargaining power and pivotal position. The two distortions determine voters’ total misrepresentation of preferences. When of opposite sign, the two distortions compensate, possibly cancelling out.

Several papers were devoted to each distortion separately,³ I analyse the whole electoral process, from elections to government formation. Within the (possibly infinite) set of electoral rules, I restrict my attention to purely proportional and

¹See, for instance, Douglas (1923).

²See Laakso and Taagepera (1981), Nurmi (1981) and Schofield (1981).

³Besides the well known paper from Taagepera and Shugart (1989), Morelli (2004) and Kestelman (1999) offer a clear and succinct review of the literature on electoral rules’ distortions. Instead, Baron and Ferejohn (1989), Snyder Jr., Ting, and Ansolabehere (2005) and Kalandrakis (2006) study some properties of the coalition formation game.

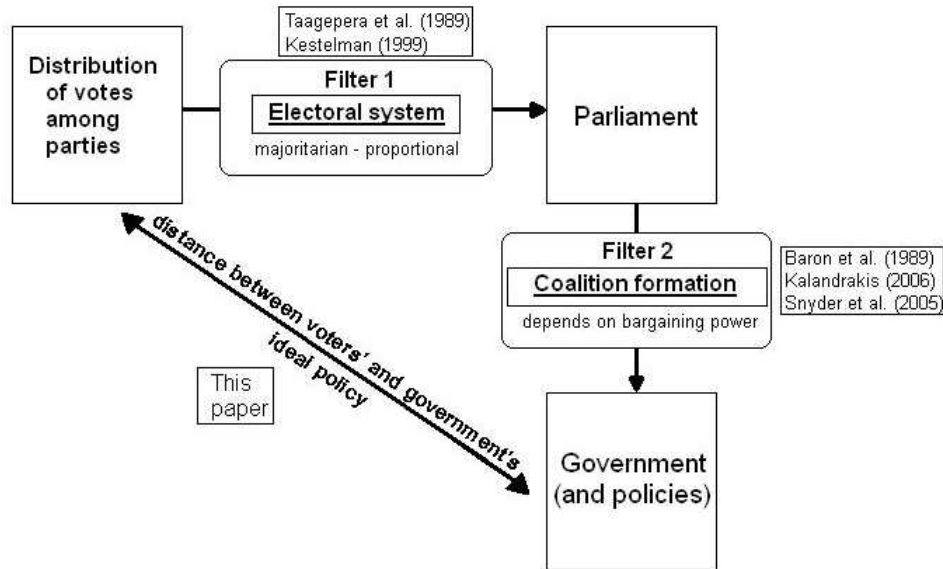


Figure 1: The government formation process

plurality rules, which inspired most western democracies. I compute the misrepresentation of voters' preferences, defined as the difference between parties' power and received share of votes. I derive the conditions under which a rule reflects citizens' preferences better than another.

I find that small parties' political power is more than proportional to their seats share, due to their pivotal position. The distortion in favour of small parties decreases if parties are impatient to form a coalition to rule the country. The distribution of seats under plurality rule is favourable to big parties. Pushing in opposite directions, the distortions compensate; when their magnitude is similar, voters' preferences are better represented under plurality rule. I conclude that majority voting is preferable when parties are patient, while proportional rule is more adequate when parties are impatient.

Section 2 describes the model. Section 3 illustrates results and discusses the consequences of relaxing some assumptions. Section 4 illustrates the model using 2006 and 2008 Italian elections data. The last section concludes.

2 The model

The political process starts with the elections, negotiations occur once the distribution of seats in Parliament is known. Parties try to form a coalition controlling the majority of seats; when they succeed, they share the power. The government can rule the country if supported by a majority: at any time, the share of power must satisfy parties' participation constraint.

Parties in the winning coalition derive office and ideological benefits.⁴ Forming a coalition means to agree on the political program and the share of economic benefits. For expositional convenience, I consider that the winning coalition shares a budget and self-interested parties only care about their share.⁵

Shares depend on bargaining power; they are the outcome of an either cooperative or non-cooperative game. Cooperative coalition theory suits if players form the grand coalition, maximise total joint profit, and share benefits according to a *value*.⁶ Non-cooperative theory suits when players maximise own payoffs given others' best response. This usually occurs if some agents find it convenient to deviate from the cooperative equilibrium. For parties, belonging to the government is necessary to obtain a positive payoff; if a coalition controls the majority of seats, there is no interest in enlarging it. It is reasonable to expect that parties act non-cooperatively.

Coalition formation starts choosing a party, called 'formateur', in charge of leading negotiations.⁷ If a single party controls the majority of seats, it rules the country alone. The way a formateur is chosen is usually not determined by the constitution. Bigger parties have larger probabilities of being selected. Baron and Ferejohn (1989) and Austen-Smith and Banks (1988) propose alternative ways of attaching parties the probability of being formateur. Austen-Smith and Banks (1988) suppose that parties' shares determine the probability of being a successful formateur; the largest party is chosen first, in case of failure, the second largest is chosen and so forth. Instead, Baron and Ferejohn (1989) directly attach to parties a probability of being a successful formateur equal to their seats shares. When the formateur is successful, it always belongs to the winning coalition. The two procedures give similar results; I follow Baron and Ferejohn (1989)'s approach that, common in the theoretical literature,⁸ performs well in empirical tests.⁹

I consider a country with 3 parties and 3 groups of homogeneous citizens indexed by i ; c_i denotes groups' relative size, i.e., the proportion of votes of a party;

⁴Ideological benefits are the right to implement the preferred policy. Office benefits are direct and indirect monetary benefits including, for instance, the possibility to choose public expenditure in strategic sectors. Some papers concentrate on office (e.g., Riker (1962) or Baron and Ferejohn (1989)) or ideological (e.g., Schofield (1986)) benefits. Austen-Smith and Banks (1990) analyse both separately, assuming they are orthogonal. In Sened (1996) the two elements are really amalgamated.

⁵Alternatively, think a) of orthogonal projects to finance, with each party interested in one, or b) that parties fix the time devoted to law proposals, with power representing the capability to pursue owns' agenda.

⁶A *value* is a function allocating payoffs in a unique way and respecting some required axioms. The Shapley (1953) and Owen (1977, 1981) value are the most well known. They assign players a power proportional to the expected value of their marginal contributions to all possible coalitions.

⁷For a detailed explanation of the role of the formateur and how it is chosen see Diermeier and Merlo (2004).

⁸See Baron and Diermeier (2001) or Diermeier, Eraslan, and Merlo (2007).

⁹See Diermeier and Merlo (2004).

$c_1 + c_2 + c_3 = 1$. Without loss of generality, I order groups by their size, thus $1 > c_1 \geq c_2 \geq c_3 > 0$. I assume the number of parties to remain unchanged when the electoral rule changes; I do not consider ideological restrictions during the coalition formation.¹⁰ Party i 's political program maximises the utility voters in group i . Vector $e = (e_1, e_2, e_3)$ denotes parties' share of seats. An electoral system is seen as a function F transforming parties' share of votes into shares of seats, i.e., $e = F(c)$. I focus on two systems: proportional and majority voting (also called "plurality rule").¹¹

Assumption 1 (No standing-alone) *No party ever obtains the majority of seats: thus, $e_1 < 0.5$ and, a fortiori, $c_1 < 0.5$.*¹²

The grey area in Figure 2 shows the possible combinations of e_2 and e_3 respecting the ordering $0.5 > e_1 \geq e_2 \geq e_3 > 0$.

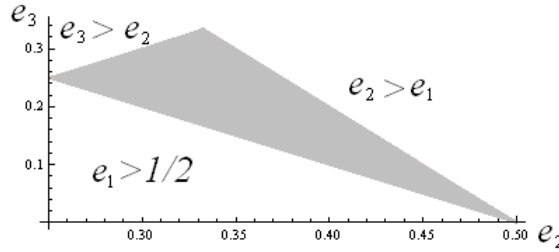


Figure 2: Possible combinations of e_2 and e_3 .

Assumption 2 (Constant coalition value) *The amount of resources to allocate is constant. The bargaining issue boils down to the "sharing a dollar" problem, where each party (and its voters) is only interested in its share of total budget.*

When no party secures a majority, the bargaining phase begins. A coalition S is the result of an agreement between two parties on how to share the budget. Let Z denote the set of all feasible allocations (i.e., $Z = \{z \in \mathfrak{R}_+^3 : \sum_{i=1}^3 z_i \leq 1\}$), z_i is the budget share of party i . Agents' utility, linear in z_i , is independent of z_j (i.e., $U_i(z) = z_i$). A winning coalition has to be supported by at least half of Parliamentarians. $D \subseteq 2^{\{1,2,3\}}$ is the set of all possible winning coalitions. With

¹⁰Fixing the number of parties is as a short term assumption. Ideological restrictions are not an issue; the model can be interpreted as the process occurring when ideologically close parties are negotiating. The exclusion of a party can be reproduced by rescaling seats shares.

¹¹Under a proportional rule, parties' share of seats equals the share of votes received, i.e., $e_i = c_i$. Under plurality rule, the country is divided into Q districts (one for available seat); in each district, the candidate receiving more votes wins: $e_i = \frac{Q_i(c)}{Q}$, where $Q_i(c)$ is the number of districts in which party i secured the relative majority of vote casts.

¹²Since non proportional electoral rules favour big parties, $c_1 > 0.5$ would imply $e_1 \geq 0.5$.

three parties, D does not depend on e : any pair of parties can secure a majority. Given the asymmetry among parties implied by the special role of the formateur, coalitions (i, j) and (j, i) are different; henceforth, the first element in a coalition denotes the formateur.

At time $t = 0$ a party, called formateur, is randomly chosen. The formateur should form a coalition $S \in D$; it proposes a vector z which should be approved by the parties in S .¹³ If z is accepted by S the game ends: the government is formed and the budget is shared according to z . Otherwise, in the next period a formateur (possibly the same one) is randomly chosen and the game continues until an agreement is reached. I use the notation z_i^j to indicate the i^{th} element of vector z when j is the formateur.

Assumption 3 (Recognition probability) *As in Baron and Ferejohn (1989), the recognition probability π_i of being a successful formateur equals party i 's share of seats (i.e., $\pi_i = e_i$).*

To model the bargaining game, I follow Kalandrakis (2006) and Snyder Jr., Ting, and Ansolabehere (2005), both considering a game à la Rubinstein-Ståhl (Rubinstein (1982)). The continuation value, v , is defined as the vector of the next period expected utility of parties. I only focus on stationary proposal strategies involving no delay: in each period a party behaves the same way when formateur and proposes a share vector such that, without delay, all parties belonging to the proposed coalition accept.

Assumption 4 *Parties discount the future (they care about the time needed, after the elections, to form a government). The patient rate, $\delta < 1$, is the same for all parties.*¹⁴

The utility in time t of a share z_i in period $t + k$ is given by $U_i(z, k) = \delta^k z_i$; parties' outside opportunity is zero. Party i 's continuation value $v_i = \delta \sum_{h \in D} \pi_h z_i^h$, is the discounted expected utility of i , conditional on forming a coalition in the subsequent period; the uncertainty concerns the formateur's identity: the value of v depends on recognition probabilities.

Given the vector of seats shares e and the time discount factor δ , a game is denoted by $\Gamma(\delta, e)$. When an agent is formateur, its action consists in proposing a division $z^i \in Z$ of the budget, the others' action space consists in accepting the formateur's proposal or not. A no-delay, Stationary, Subgame Perfect, Pure Strategy (SSPPS) equilibrium for game $\Gamma(\delta, e)$ is a set z^i of stationary strategies

¹³To have a winning coalition, all parties in S should obtain a positive share. There is no reason to leave a positive share to parties outside the coalition.

¹⁴This is a simplifying assumption, allowing to have a lighter notation and simpler computations. Actually, results depend only on the smallest party's δ .

and of acceptance strategies. A SSPPS equilibrium requires the share of all parties in the coalition to be such that $z_i \geq v_i$;¹⁵ existence of SSPPS Nash equilibrium is not an issue for game $\Gamma(\delta, e)$ by the arguments of Banks and Duggan (2000). Other equilibria may exist; I focus on the stationary ones in pure strategy.

Voter preferences. Preferences cannot be directly observed, people’s vote can. If voters act strategically, we cannot deduce their preferences from their vote; a change in the electoral system may affect the voting strategy. I assume sincere voting; this event is not empirically irrelevant: for instance, Hooghe, Maddens, and Noppe (2006) found empirical evidence that after the last voting rule change in Belgium there were no significant changes in aggregate voters’ behaviour, they interpreted it as a signal of myopic/sincere voting. Some arguments can justify voters’ myopic behaviour under majority voting, for instance, parties’ policy vectors can be orthogonal or sufficiently different for a voter not to consider two parties as substitutes. Furthermore, it is costly to be informed about politics (programmes, performances, the electoral system, other voters’ expected behaviour) and voters may prefer to vote sincerely. Assuming sincere voting allows inferring voters’ preferences from votes.

Measuring misrepresentation. The model goal is to relate voters’ preferences to government policies.¹⁶ Given voters’ choice, I determine the winning coalition and compute parties’ shares. I measure misrepresentation through M^y , where $y = \{PR; MV\}$ denotes the electoral system, with PR=proportional rule and MV=majority voting and compare rules’ representativeness.

The electoral system determines the number of seats a party controls, thus, it affects the probability at which a coalition forms and parties’ budget shares. Let $\Pr(S)$ be coalition S ’s probability to form, thus, of a formateur to be successful:

$$M^y = \sqrt{\sum_{i=1}^3 \left(\sum_{S \in D} \Pr(S) \cdot z_i^S - c_i \right)^2} \quad (1)$$

represents the Euclidean distance between parties’ expected budget share and the optimal one. i.e., Equation 1 computes the distance between parties’ expected power (discounted for the probability of forming each possible coalition) and their share of votes, c_i .

Equation 1 takes large values when parties are either under or over-represented. Over-representation occurs when a party, being pivotal for a coalition, obtains a larger share of benefits than the share of population it represents.

To compute M^y , both c_i and e_i are necessary.¹⁷ The relation between c_i and e_i

¹⁵For more details on that, see Kalandrakis (2006), p. 444.

¹⁶Policies are interpreted as parties’ resources, which are proportional to parties’ power/share z_i .

¹⁷To compute $\Pr(S) \cdot z_i^S$, e_i is required.

depends on the electoral system. Taagepera and Shugart (1989) show that, properly choosing parameter τ , every electoral system can be approximated through a function $e_i = F(c_i) = \frac{c_i^\tau}{\sum_{j=1}^n c_j^\tau}$. For the proportional rule, $\tau = 1$ and $c_i = e_i$. Under plurality rule, the share of seats depends on the geographical distribution of voters' preferences over districts; for plurality single-member district systems and a normal population, it is usually considered that $\tau \approx 3$ (Qualter (1968)), from which the name 'cube rule'.¹⁸ According to Taagepera and Shugart (1989), $\tau = 2.5$ better suits modern western societies with plurality single-member district systems, while $\tau = 8$ would be a better approximation for the USA actual system. In the literature, the original cube rule (with $\tau = 3$) is usually assumed to be a good approximation for the two party case and also to fit data for the three party case; the precision of this measure falls when the number of parties increases.

Assumption 5 (Cube rule) *To compute the share of seats of a party under majority voting, I assume that we observe the aggregate number of votes for each party (c_i) and the cube rule holds, thus $e_i = \frac{c_i^3}{\sum_{j=1}^3 c_j^3}$.*¹⁹

From the previous assumption, Equation 1 can be rewritten as follows:

$$M^{PR} = \sqrt{\sum_{i=1}^3 \left(\sum_{j=1}^3 c_j z_i^j - c_i \right)^2} \quad (2a)$$

$$M^{MV} = \sqrt{\sum_{i=1}^3 \left(\sum_{j=1}^3 \frac{c_j^\tau}{c_1^\tau + c_2^\tau + c_3^\tau} z_i^j - c_i \right)^2} \quad (2b)$$

3 Model results

In this section, after deriving parties' power share, I compute voters' misrepresentation (Equation 1) under proportional and plurality rule. I show that the distortion under the former is larger than under the latter when δ is large enough.

A coalition of two out of three parties can always secure a majority. The formateur compares its utility in each possible coalition to choose the other member. Ex ante, eight scenarios may occur, depending on the identity of the formateur.

Proposition 1 (Sharing rule) *In equilibrium, coalitions always include only two parties. Formateur i proposes to party j its continuation value v_j , and zero*

¹⁸J. P. Smith formulated this relation in 1909, in a report to the British Royal Commission on electoral systems. Duverger (1954) developed and made it famous. For a discussion on it and its drawbacks, see Riker (1982), Blau (2001) or Rogowski and Kayser (2002).

¹⁹All the results can be replicated with different values of τ , to fit a given country's electoral system and geographical distribution of preferences among voters. Section 4.1 discusses on that.

to the other one. The share vector $z^i = (z_i^i; z_j^i; z_x^i)$ takes the following values $(1 - z_j^i; \frac{\delta}{1-\delta e_i}(e_j z_j^j + e_x z_x^x); 0)$. Party x , excluded from the coalition, receives 0; party j obtains the present value of what it would get (in discounted expected terms) in the next period.

Proof. There is no reason to form a coalition with more than one party, since the value of a coalition is constant and 2 parties are sufficient to control the majority of seats. The formateur, being residual claimant, minimises party j 's share (z_j), proposing the minimum value that a party would accept. The cheapest price that a party accepts is its next period discounted profit, i.e., its continuation value $v_j = \delta(e_i z_j^i + e_j z_j^j + e_x z_x^x)$. From $z_j^i = v_j = \delta(e_i z_j^i + e_j z_j^j + e_x z_x^x)$, $z_j^i = \frac{\delta}{1-\delta e_i}(e_j z_j^j + e_x z_x^x)$ directly follows. ■

Proposition 2 (Minimal winning coalition) *In the SSPPS equilibrium, the formateur always forms a coalition with the smallest other party.²⁰ The ex-ante unique equilibrium coalitions are $\{(1,3), (2,3), (3,2)\}$, where the first element of a pair denotes the formateur. Equilibrium shares depend on the formateur: a priori, $z_j^i \neq z_i^j$.*

Proof. In the appendix. ■

Corollary 2.1 (Probability of forming a coalition) *From Assumption 3 on recognition probabilities, the probability $Pr(S)$ of coalition $S = (i, j)$ is: $Pr(1, 3) = e_1$, $Pr(2, 3) = e_2$ and $Pr(3, 2) = e_3$.*

Corollary 2.2 *Table 1 summarises equilibrium parties' shares z^i .*

$z^1 = \left(\frac{(1-\delta)(1-\delta e_3)}{1-\delta+\delta^2 e_1 e_3}; 0; \frac{(1-\delta e_2 - \delta e_3)\delta e_3}{1-\delta+\delta^2 e_1 e_3} \right)$
$z^2 = \left(0; \frac{(1-\delta)(1-\delta e_3)}{1-\delta+\delta^2 e_1 e_3}; \frac{(1-\delta e_2 - \delta e_3)\delta e_3}{1-\delta+\delta^2 e_1 e_3} \right)$
$z^3 = \left(0; \frac{(1-\delta)\delta e_2}{1-\delta+\delta^2 e_1 e_3}; \frac{(1-\delta)(1-\delta e_2) + \delta^2 e_1 e_3}{1-\delta+\delta^2 e_1 e_3} \right)$

Table 1: The equilibrium vectors z .

Proposition 2 means that small parties are “cheaper”; the formateur prefers them to form a coalition. From Table 1, the discount factor δ plays a key role in the budget share. The formateur is residual claimant: it pays to partners their continuation value, which is increasing in patience.

In Snyder Jr., Ting, and Ansolabehere (2005), players' equilibrium shares are all the same, because the authors solve for one of the mixed strategies equilibria

²⁰This result is in line with the empirical evidence that parties tend to form minimal winning coalitions, to reduce coordination costs and increase efficiency. Theoretical (i.e., Riker (1962)) and empirical (i.e., Martin and Stevenson (2001)) studies confirm it.

(players compete on z_i , to belong to the winning coalition; the formateur can extract more surplus from them). In my model, *a priori*, shares are different for each party. When δ is close to one (parties are patient), the formateur is forced to let almost all the share to the other party.

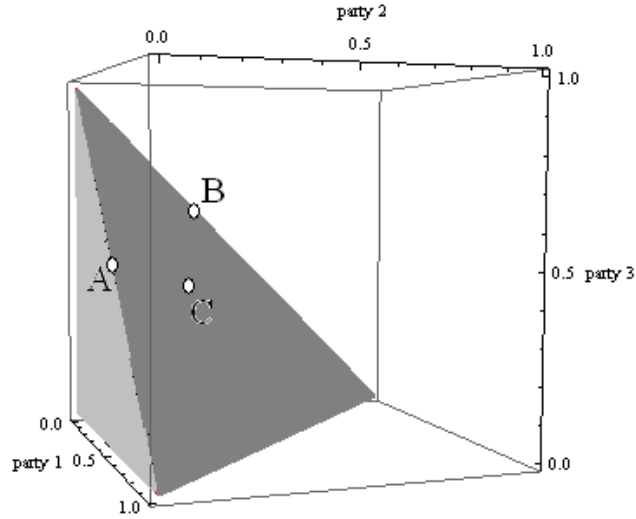


Figure 3: Coalitional space Z

Figure 3 shows feasible combinations of budget share amongst 3 parties. Each axe represents one party's share of votes and of budget. The simplex dark side is set Z ; points A and B are examples of budget shares for (1,3) and for (2,3). Point C is an example of an optimal point.²¹ Points A and B depend on δ , point C does not; the larger δ , the further A and B are from the formateur's axe. By Proposition 1, the equilibrium share always lies on a vertex (one party receives 0).

Using Table 1 in Equation 1, I compute voters' misrepresentation under the proportional (M^{PR}) and plurality (M^{MV}) rule. Equation 1 becomes:

²¹Optimal in the sense that each party's budget share equals its share of votes.

$$\begin{aligned}
M^y = & \left[\left(e_1 \frac{(1-\delta)(1-\delta e_3)}{1-\delta+\delta^2 e_1 e_3} - c_1 \right)^2 + \right. \\
& \left(e_2 \frac{(1-\delta)(1-\delta e_3)}{1-\delta+\delta^2 e_1 e_3} + e_3 \frac{(1-\delta)\delta e_2}{1-\delta+\delta^2 e_1 e_3} - c_2 \right)^2 + \\
& \left. \left(\frac{(e_1+e_2)(1-\delta e_2-\delta e_3)\delta e_3}{1-\delta+\delta^2 e_1 e_3} + \frac{e_3(1-\delta)(1-\delta e_2)+\delta^2 e_1 e_3}{1-\delta+\delta^2 e_1 e_3} - c_3 \right)^2 \right]^{0.5}, \tag{3}
\end{aligned}$$

where each of the three main elements in the expression is a function of the distances between a party share of votes and its power.

Under proportional rule, seats and votes shares are the same; Equation 3 (misrepresentation) becomes

$$\begin{aligned}
M^{PR} = & \frac{1}{[1-\delta+\delta^2 c_1 c_3]^2} \left[((1+\delta)\delta c_1 c_3)^2 + \right. \\
& \left. \delta^2 c_1^2 ((\delta c_1 - 1 - \delta) c_3 - 2)^2 + (\delta c_2 c_3 (2 + \delta c_1 - 2\delta))^2 \right]^{0.5}. \tag{4}
\end{aligned}$$

Using the cube rule (Assumption 5) to obtain the relation between c_i and e_i , under majority voting, Equation 3 becomes

$$\begin{aligned}
M^{MV} = & \left[[c_3^3 [(\delta-1)[(c_3^3+c_2^3)c_3^3\delta+\sigma^2]-\delta c_3^6(1+\delta)]-c_3\sigma x \right]^2 + \\
& [c_2^3 [(1-\delta)\sigma^2-2c_3^6\delta^2]-c_2\sigma x]^2 + \\
& [c_3^3 [c_1^3\sigma\delta+2(c_1^3+c_2^3)c_3^3\delta^2+(1-\delta)\sigma^2]-c_3x\sigma]^2 \Big]^{0.5} \frac{1}{(\sigma x)^2}, \tag{5}
\end{aligned}$$

where $\sigma = \sum_{i=1}^3 c_i^3$ and $x = (1-\delta)\sigma^2 + c_1^3 c_3^3 \delta^2$.

The difference in misrepresentation between the two electoral systems, denoted $MM = M^{PR} - M^{MV}$, is:

$$\begin{aligned}
MM = & \frac{1}{[1-\delta+\delta^2 c_1 c_3]^2} \left[((1+\delta)\delta c_1 c_3)^2 + \right. \\
& \left. \delta^2 c_1^2 ((\delta c_1 - 1 - \delta) c_3 - 2)^2 + (\delta c_2 c_3 (2 + \delta c_1 - 2\delta))^2 \right]^{0.5} \\
& - \left[[c_3^3 [(\delta-1)[(c_3^3+c_2^3)c_3^3\delta+\sigma^2]-\delta c_3^6(1+\delta)]-c_3\sigma x \right]^2 + \\
& [c_2^3 [(1-\delta)\sigma^2-2c_3^6\delta^2]-c_2\sigma x]^2 + \\
& [c_3^3 [c_1^3\sigma\delta+2(c_1^3+c_2^3)c_3^3\delta^2+(1-\delta)\sigma^2]-c_3x\sigma]^2 \Big]^{0.5} \left(\frac{1}{\sigma x} \right)^2. \tag{6}
\end{aligned}$$

$MM > 0$ means that the difference between parties' expected and optimal share is larger under proportional than plurality rule; when Equation 6 is positive, plurality rule better represents voters' preferences (i.e., each party's expected budget share is closer to its share of votes).

Proposition 3 (Role of the discount factor) *Regardless of the distribution of seats among parties, two thresholds exist ($\underline{\delta} \approx 0.108, \bar{\delta} \approx 0.780$) for the discount factor such that: a) majority voting is preferable when $\delta > \bar{\delta}$, and b) proportional rule is preferable when $\delta < \underline{\delta}$. When the value of δ is within the two thresholds, parties' relative share of seats determines which voting system is preferable.*

Proof. Solving for δ , plurality rule is preferable if and only if $MM > 0$; the inequality can only be solved numerically. Figure 4 describes the behaviour of Equation 6, showing its shape for six values of δ . The horizontal axe depicts party 3 seats share (e_3), while e_2 is on the depth axe. The dark surface corresponds to the zero plan; the light surface depicts Equation 6. Given δ and the combinations of seat shares, the light surface is above the dark one if the majority voting is preferable. For instance, for $\delta = 0.78$ and $\delta = 0.98$, the light surface is above the dark one regardless of the distribution of seats among parties. ■

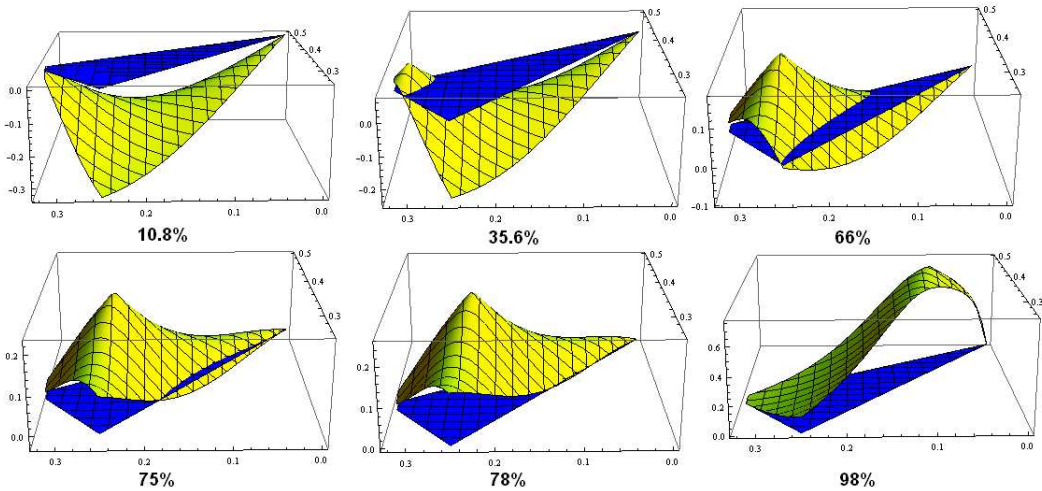


Figure 4: The impact of δ

Proposition 3 states that either system is always preferred outside the interval $(0.108, 0.780)$; within it, the relative share of seats determines which system is preferred. After elections a coalition forms: the budget share the formateur lets to its partner equals partner's continuation value. When δ is small enough (parties are impatient), it is cheap to persuade a partner: indeed, when δ tends to 0, the formateur's share tends to one and parties' expected utility tends to their share of

seats; than, the best electoral system is such that $c_i = e_i$, i.e., it is the proportional one.²² Since filter 2 (Figure 1) disappears for δ going to zero, there is no reason to distort the mechanism at filter-one level.

The share the formateur has to leave equals its partner's discounted expected earning. When δ gets larger (parties are patient), a distortion appears at filter two level: the formateur has to leave to its partner a larger portion of future earnings; small parties' expected share becomes greater than their share of received votes. Plurality rule distorts election results in the opposite direction (reducing small parties share of seats); when δ is large enough ($\delta > 0.78$), filter two distortion is large and the majority rule desirable. When $\delta \in (0.108, 0.78)$, small parties' bargaining power is limited: according to parties' relative seats share, plurality rule distortion may be larger than what necessary to counter-balance the coalition formation distortion (i.e., the distortion at filter 1 level induced by plurality rule is too large compared to the one at filter 2 level). For $\delta \in (0.108, 0.78)$ and c_3 close to zero the proportional rule is preferable while, for c_3 and c_2 big enough, majority voting is better than proportional rule.

3.1 Relaxing some assumptions

This subsection is meant to briefly discuss the impact of some assumptions. Limiting parties to 3 allows to obtain closed form results. It is possible to solve the model for $n > 3$ parties, but additional restrictions should be introduced. Firstly, it would be necessary to model formateur's trade-off between increasing the number of parties and forming a coalition including larger parties. Proposition 2 may not hold: forming a coalition with the smallest party may be not enough to secure the majority. According to the kind of coalition that is formed, the thresholds for δ would change but qualitatively results would be the same.

Relaxing Assumption 1 can lead to two scenarios: if a party controls the majority of seats regardless of the electoral system, all rules are equally representative of voters' preferences. Additional, non trivial, analysis is required for the case in which one party controls the majority alone only under one regime.

If (contrary to Assumption 2) the coalition's value depends on the identity of partners (e.g., because of ideological affinities), coalition formation would integrate this. The same considerations as for the number of parties would hold. I excluded this case, because results would be assumption driven. As mentioned, having total incompatibility between two parties is equivalent to consider the model with $n - 1$

²²From an analytical point of view: $\delta \rightarrow 0$ implies $z_j^i \rightarrow 0$, thus $z_i^i \rightarrow 1$ and $EU_i(z_i^S, 0) = \sum_{h \in D} \pi_h z_i^h \rightarrow \pi_i$. Since $\pi_i = e_i$, to minimise the difference between the share of votes (c_i) and the expected share of budget for a party ($EU_i(z_i^S, 0)$) we need $e_i = c_i$, which is the case under the proportional electoral system.

parties.

Concerning the common value of δ (Assumption 4), the discount factor of the non-formateur party determines parties' share; all results can be extended, replacing δ by the discount factor of the non-formateur party in the winning coalition. The value of δ must always be strictly smaller than 1 to ensure that the solution of the problem exists and is unique.

Section 4.1 discusses the role of Assumption 5. In particular, it explains the role of τ and how results change for $\tau < 3$.

4 A model illustration with Italian elections' data

In Italy, over the two last legislatures, the smallest parties in the winning coalitions had very much power compared to their seats share in Parliament. In this section I use Italian elections results to clarify the model.²³

Italian Parliament has two houses: the lower house and the Senate. All adults (older than 18 years old) can vote for the former, citizens aged more than 25 can vote for both. The current electoral law allows for regional specific rules for each house. Over 20 regions, in 18 (19 for the congress) the electoral law is based on the proportional principle. For my computations, I used parties number of received votes, in order to disregard local specificities.

In 2006 a centre-left coalition elected Romano Prodi prime minister. The winning coalition officially included eight parties at the Senate; most of them were created *ad hoc* before the election, to profit of some exotic features of the electoral law, only few of them had an own leader and a programme independent from the main party's. I focus on a small independent party: UDEUR. It represented about 1% of citizens on a national basis, it was pivotal and, when in 2008 it left the coalition, the government lost the majority, new elections were called.

Coalition members were aware of the consequences of this party leaving: the majority controlled three more senators than the opposite coalition, including the external support received by some "senatori a vita"²⁴. During the last months of Mr. Prodi's government, UDEUR's senators used their influence on medias and their pivotal positions (threatening to leave) to obtain some major changes in several laws, especially in the "Finanziaria".²⁵ They carried on their own agenda and clearly showed that their real power within the coalition was more than 1%.

²³Election results are public, they can be found on the web page of the *Ministero degli interni*, on the web page of each of the two houses of Parliament, and on many independent web pages.

²⁴"Senatori a vita" are senators who are not elected (e.g., former Republic Presidents). They sit in Parliament for life.

²⁵The "Legge Finanziaria" is one of the most important Italian laws, it determines the forthcoming year public expenditure.

In what follows, I consider the left and the right coalitions as two single parties (named CL and CR) and UDEUR as a third independent party that can form a coalition with either party.²⁶ Table 2 first column summarises the 2006 Senate seats shares under proportional rule. The second column is the “cube rule” estimation of the share of seats under plurality rule.

	Seats - Proportional Rule	Seats - Plurality Rule
CL	49.5%	49.999%
CR	49.5%	49.999%
UDEUR	1%	0.002%

Table 2: Seats Share - 2006 Italian Senate

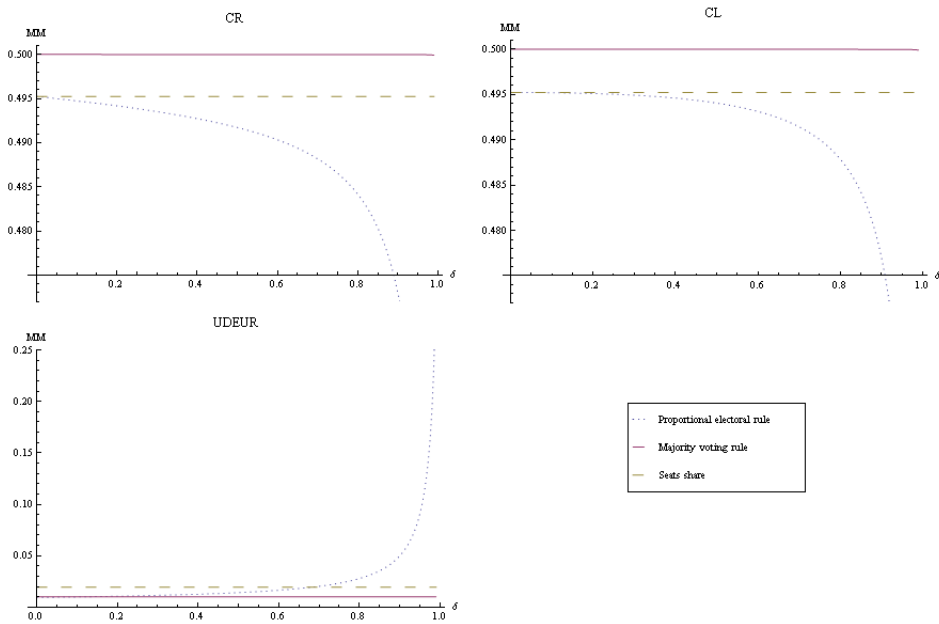


Figure 5: Budget share depending on δ

The budget share depends on the discount factor: Picture 5 depicts the equilibrium share of each party for different levels of δ . The straight line is for the majority voting rule; the dotted one, for the proportional one; the dashed one represents parties’ shares of votes; each chart corresponds to one party: (from top left) CR, CL and UDEUR. Under plurality rule, both CR and CL’s shares increase at the expenses of UDEUR, which share is extremely large under the proportional rule, if the discount factor gets larger (e.g., for $\delta = 0.99$, under proportional representation its expected share is 33%, while under plurality rule it is 0.02%).

²⁶This would have been politically plausible; UDEUR is a centre party, its leader already formed some coalitions with the centre-right party and he run with it at the 2009 European Parliament elections.

Equation 1 measures the Euclidean distance between a party’s average power and the optimal one, given voters preferences; the larger its value, the bigger is the difference between the distribution of power within the government and voters’ preferences. With the 2006 election’s data, the difference in misrepresentation (MM , Equation 6) is drawn in Figure 6 as a function of δ . For $\delta > 66.7\%$, plurality rule ensures a better representation of voters; the opposite is true for $\delta < 66.7\%$.

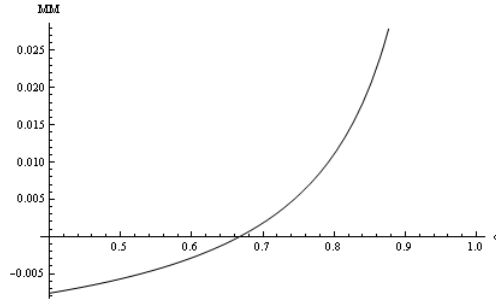


Figure 6: Misrepresentation as a function of δ

After the 2008 elections, due to a change in the political strategy of the two main parties, only four parties are now represented in Parliament: PD-IDV (the centre-left party),²⁷ PDL (the centre-right party), Lega Nord and UDC, the smallest party.²⁸ Table 3 summarises their shares.

	Seats - Proportional Rule	Seats - Plurality Rule
PD-IDV	41.7%	49.8%
PDL	41.5%	49.2%
Lega Nord	10.5%	0.8%
UDC	6.2%	0.2%

Table 3: Seats Share - 2008 Italian Congress

UDC did not obtain enough seats to form a two party coalition and resulted to be a “dummy player”, that is: regardless of the coalition, its contribution is always irrelevant, thus it never belongs to a winning coalition. The successful coalition was the centre-right one (PDL with Lega Nord), and Mr. Berlusconi was elected prime minister. Lega Nord has already proved that it is not willing to accept the coalition’s decisions without negotiating. PDL already withdrew more than once

²⁷PD and IDV, through a pre-electoral agreement, run together and shared both programme and candidate prime minister.

²⁸To be more precise, one more party (SVP) is represented. SVP is a local party from a cross-border region, where the majority of citizens speaks German. Aimed to protect linguistic minorities, a special electoral rule allowed SVP to obtain 2 seats at Parliament (equivalent to 0.3%).

own law proposals not in line with Lega Nord’s program, and promoted others, against the will of most of the parliament (including several PDL leaders).²⁹

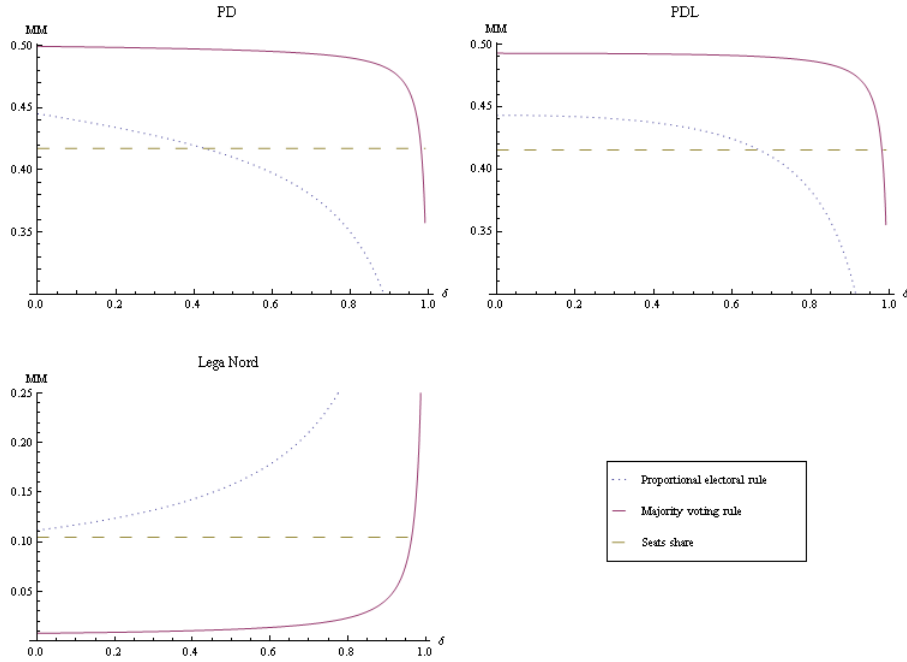


Figure 7: Budget share depending on δ

Figure 7 shows the share of budget of each party, according to the value of δ for PD, PDL and Lega Nord. Under the proportional rule, Lega Nord obtains a share of budget considerably larger than the share of votes received (e.g., with a share of votes of 10.5% and for $\delta = 80\%$ the expected share of budget for Lega Nord is 26.8%, while under majority voting it would be 2.5%). Considering aggregate data and the level of misrepresentation computed by Equation 6, Figure 8 shows that, for $\delta \geq 64.64\%$ the majoritarian rule is preferable to the proportional one.

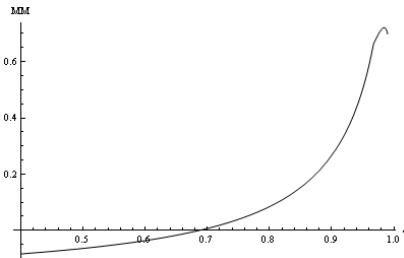


Figure 8: Misrepresentation as a function of δ

²⁹For instance, in April 2009 Lega Nord, by threatening to leave the coalition, obtained from Mr. Berlusconi’s party to change the day of a referendum, at an estimated cost of 400 million euros.

4.1 Comments on the cube rule

I assumed the cube rule to hold with $\tau = 3$. Although empirically tested, the cube rule lacks of theoretical foundations. The real share of seats depends on the distribution of preferences over districts.³⁰ For some countries, $\tau = 3$ may be a poor proxy; different values for τ account for idiosyncratic differences in the electoral system, the distribution of voter preferences, etc.³¹

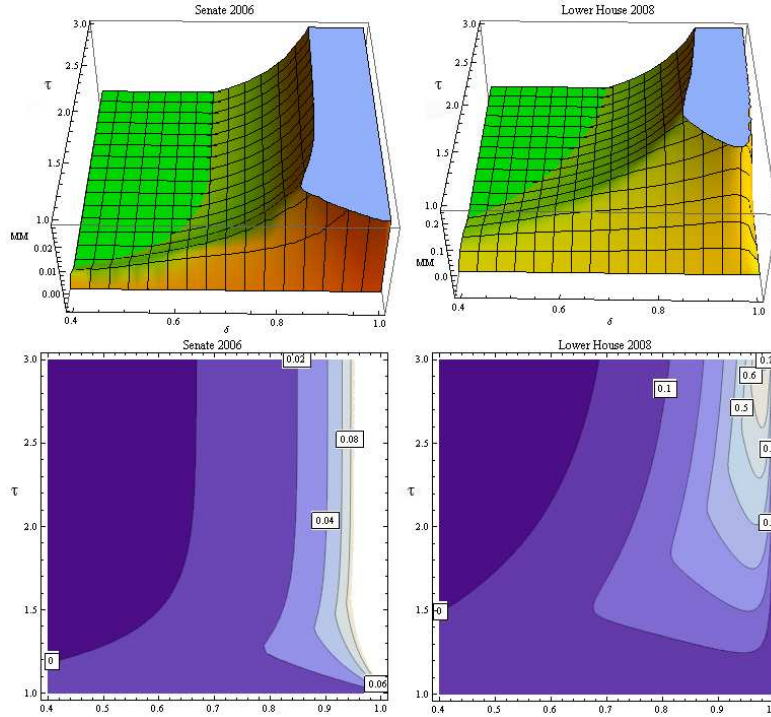


Figure 9: Changes both in δ and τ

Let's see how previous results change in δ and τ . In the 2006 case, we have two big parties and one very small; in the 2008 case, the smallest party is relatively big. Figure 9 shows how the misrepresentation index changes over δ and τ . Whatever the value for τ , non proportional voting systems perform better if parties are patient.

For low levels of δ , non proportional systems perform better only when the value of τ is small. In Figure 10 we can see that the peak is close to one for low levels of δ (each line corresponds to a different level of δ , lower lines are for lower values of δ). When the smallest party is very small, the majoritarian rule might

³⁰For instance, in a country where, in all districts, parties' shares are (40%, 30%, 30%), under plurality rule the first party obtains 100% of seats, while the cube rule predicts a share (54%, 23%, 23%).

³¹As a general rule, we should expect τ to be larger in a country with one big party many small local parties, while τ should be smaller than 3 for countries with heterogeneous districts, strong local parties and no big national parties.

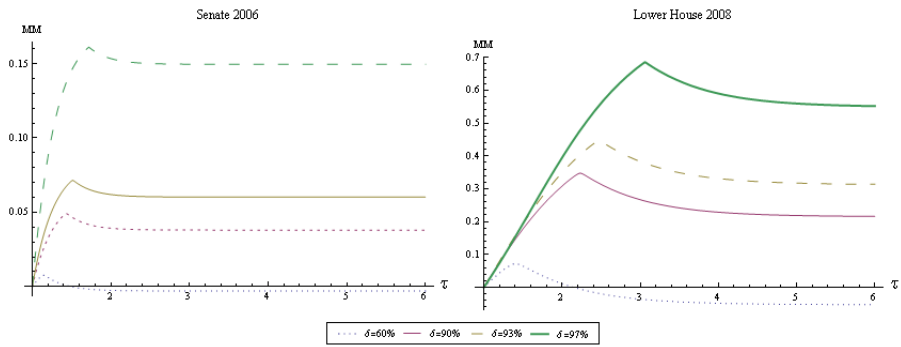


Figure 10: Effect of τ for different levels of δ

distort too much and a small value for τ would be preferable (this can be obtained artificially, with a mixed electoral rule, or it can simply be a consequence of the geographical distribution of preferences). On the opposite, when the smallest party obtains a large share of votes, the majority voting distortion is smaller and it is preferable to have a τ closer to three.

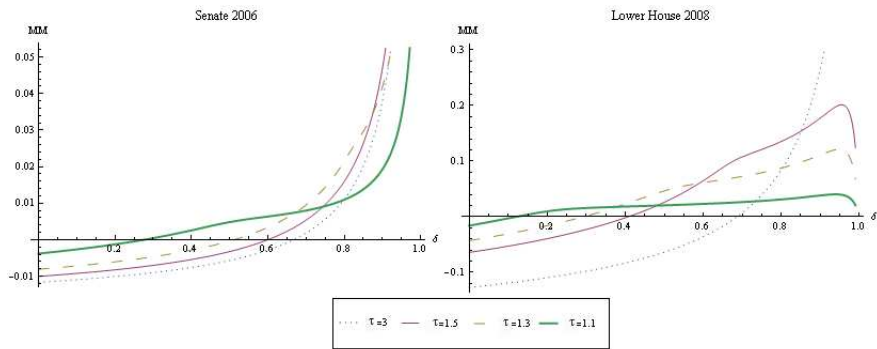


Figure 11: Effect of δ for different levels of τ

Figure 11 shows how the level of misrepresentation changes with δ for different levels of τ . The value of δ for which proportional and majority voting rule are equivalent is an increasing function of τ : the smaller the value of τ , the more it is likely that the introduction of distortions in favour of large parties is beneficial.

5 Conclusions

Electoral systems are a social compromise. Each country chooses its voting rules according to political, cultural, historical and social reasons. Many countries (e.g., Italy), preferred the proportional systems while others (e.g., U.K. or U.S.A.) chose the plurality rule. Most countries adapted their system according to local needs. My work focused on the two basic electoral systems (i.e., purely proportional versus

plurality rule) disregarding local specificities.

Proportional electoral rules are costly in term of governability: the number of represented parties in the winning coalition tends to increase; governments' expected duration falls and the average time to introduce structural changes increases, because of the long negotiation time required. According to proportional systems advocates, on the other hand, decisions reflect citizens' preferences since, by definition, Parliament's composition reflects precisely voters' preferences.

It is generally disregarded that decisions are mainly taken by the government and, within the Parliament, by the majority of members. Coalitions form to support a government and parties' power depends on their role in the coalition, not on their shares of seats in Parliament. Given the distortion due to negotiation and the importance of bargaining during the coalition formation stage, it results pointless to measure the degree of representativeness of Parliament. What matters is the relation between voters' preferences and a party's power within the government.

I showed that, especially when parties are patient at the coalition formation stage, the distortion resulting from the negotiation process (filter 2) increases small parties' power; at the election stage (filter 1), plurality rule distorts Parliament's representativeness; the two distortions have opposite sign. If parties are impatient, filter 2 distortion is negligible, thus a non-distorting electoral system is better; when parties are patient enough, the magnitude of the distortion increases and it is beneficial to have a non-proportional electoral system. My model shows that the idea that, under proportional rules, governments are always more representative is false; plurality rule can be preferable from a representativeness perspective.

The Italian example is instructive: during the 15th legislature, a party representing 1% of voters, could threaten the government, obtained to substantially change part of the 2008 "Finanziaria" law and determined the government's fall the 23rd January, 2008. Similarly, during the first year of the 16th legislature, with about 11% of votes at the 2008 elections, a party severely affected government's behaviour on very discussed laws, such as the law to reform the justice system, the immigration laws and the law on federalism. With a less proportional system (for instance under plurality rule), small parties' role would reduce and more decisions would be taken by parties representing a larger subset of the population.

Plurality rule may reduce by too much the smallest party's power. Preliminary results from section 4.1, in particular the study of misrepresentation for different values of τ , however suggest that proportional rule is never the most representative rule, better results can be achieved by introducing some distortions aimed to increase the power of big parties.

Appendix

A Proof of Proposition 2

The generic share of budget with three parties is resumed in table 4.³²

Shares	Formateur		
	1	2	3
z_1	$1 - z_2^1 - z_3^1$	$\frac{\delta}{1-\delta e_2} (e_1 z_1^1 + e_3 z_1^3)$	$\frac{\delta}{1-\delta e_3} (e_1 z_1^1 + e_2 z_1^2)$
z_2	$\frac{\delta}{1-\delta e_1} (e_2 z_2^2 + e_3 z_2^3)$	$1 - z_1^2 - z_3^2$	$\frac{\delta}{1-\delta e_3} (e_1 z_2^1 + e_2 z_2^2)$
z_3	$\frac{\delta}{1-\delta e_1} (e_2 z_3^2 + e_3 z_3^3)$	$\frac{\delta}{1-\delta e_2} (e_1 z_3^1 + e_3 z_3^3)$	$1 - z_1^3 - z_2^3$

Table 4: Generic shares with three parties

By solving the system of equations, we can (for each of the eight scenarios) compute the continuation value for each party. For the case $\{(1,3), (2,3), (3,2)\}$, results are obtained as follows:

$$z_1^1 = 1 - z_3^1 \quad (7a)$$

$$z_2^2 = 1 - z_3^2 \quad (7b)$$

$$z_3^3 = 1 - z_2^3 \quad (7c)$$

$$z_3^1 = \delta(e_1 z_3^1 + e_2 z_3^2 + e_3 z_3^3) \quad (7d)$$

$$z_3^2 = \delta(e_1 z_3^1 + e_2 z_3^2 + e_3 z_3^3) \quad (7e)$$

$$z_2^3 = \delta(e_2 z_2^2 + e_3 z_2^3). \quad (7f)$$

Noticing that 7d=7e, we obtain that $z_1^1 = z_2^2$ and $z_3^1 = z_3^2$. Combining 7b with 7e and 7f with 7c, we obtain:

$$z_2^2 = 1 - \frac{\delta e_3}{1 - \delta e_1 - \delta e_2} z_3^3 \quad (8a)$$

$$z_3^3 = 1 - \frac{\delta e_2}{1 - \delta e_3} z_2^2. \quad (8b)$$

Solving the system, we obtain that

$$z_2^2 = \frac{1 - \frac{\delta e_3}{1 - \delta e_1 - \delta e_2}}{\frac{1 - \delta + \delta^2 e_1 e_3}{(1 - \delta e_1 - \delta e_2)(1 - \delta e_3)}} \quad (9a)$$

$$z_3^3 = \frac{\frac{1 - \delta e_2 - \delta e_3}{1 - \delta e_3}}{\frac{1 - \delta + \delta^2 e_1 e_3}{(1 - \delta e_1 - \delta e_2)(1 - \delta e_3)}}. \quad (9b)$$

³²Note that, according to which coalition is formed, some of the cells in the table take the value zero.

After some simplifications and using the property that $\delta e_1 + \delta e_2 + \delta e_3 = \delta$, we obtain the results summarised in table 5.

		Formateur		
Shares		1	2	3
z_1		$\frac{(1-\delta)(1-\delta e_3)}{1-\delta+\delta^2 e_1 e_3}$	0	0
z_2		0	$\frac{(1-\delta)(1-\delta e_3)}{1-\delta+\delta^2 e_1 e_3}$	$\frac{(1-\delta)\delta e_2}{1-\delta+\delta^2 e_1 e_3}$
z_3		$\frac{(1-\delta e_2 - \delta e_3)\delta e_3}{1-\delta+\delta^2 e_1 e_3}$	$\frac{(1-\delta e_2 - \delta e_3)\delta e_3}{1-\delta+\delta^2 e_1 e_3}$	$\frac{(1-\delta)(1-\delta e_2) + \delta^2 e_1 e_3}{1-\delta+\delta^2 e_1 e_3}$

Table 5: Shares at equilibrium

The continuation value depends on the coalition and on the identity of the formateur. To have a stationary equilibrium, each party always chooses to form the same coalition when it is the formateur. For stationarity, its choice must be, at every period of time, the best response to others' player behaviour and the strategy has to be always the same. Committing to a given strategy allows parties to modify their continuation value when they are not formateur.

Within the eight scenarios, we look for Nash simultaneous stationary subgame perfect equilibria in pure strategies (SSPPS). Each player has two possible actions (consisting in forming a coalition with either of the remaining parties). Comparing expected payoffs of each party in each situation (through a reduced form game matrix of payoff), we notice that only scenario $\{(1, 3), (2, 3), (3, 2)\}$ is SSPPS. If for example party 2 always form a coalition (2,3) and party 3 a coalition (3,2), then for party 1 it is a best-reply strategy to form a coalition (1,3).

To check that this scenario is really an equilibrium, take the generic recognition probabilities (a, b, c) . By definition of continuation value, $v_j^* = az_j^1 + bz_j^2 + cz_j^3$ thus:

$$v_1 = \frac{a(1-\delta)(1-\delta c)}{1-\delta+\delta^2 ac} + 0 + 0 \quad (10)$$

$$v_2 = 0 + b \frac{(1-\delta)(1-\delta c)}{1-\delta+\delta^2 ac} + c \frac{(1-\delta)\delta b}{1-\delta+\delta^2 ac} = \frac{b(1-\delta)}{1-\delta+\delta^2 ac} \quad (11)$$

$$v_3 = (a+b) \frac{(1-\delta b - \delta c)\delta c}{1-\delta+\delta^2 ac} + c \frac{(1-\delta)(1-\delta b) + \delta^2 ac}{1-\delta+\delta^2 ac} = \frac{(1-\delta c - \delta b)c}{1-\delta+\delta^2 ac}, \quad (12)$$

and thus $v = \left(\frac{a(1-\delta)(1-\delta c)}{1-\delta+\delta^2 ac}, \frac{b(1-\delta)}{1-\delta+\delta^2 ac}, \frac{(1-\delta c - \delta b)c}{1-\delta+\delta^2 ac} \right)$.

For $a = e_1$, $b = e_2$, $c = e_3$, and knowing that $z_j^i = \delta v_j$ for $i \neq j$, we are back to results in table 5.

We check now that no player wants to deviate: we refer to an equilibrium E via the corresponding formed coalition when a party is formateur. Let's call E^* the above proposed equilibrium (that is $\{(1, 3), (2, 3), (3, 2)\}$) and define then E^i the alternative candidate equilibrium if party i deviates.

Since we look for stationary pure strategy equilibria, to show that no player wants to deviate, I show that a) $E^* \succeq_1 E^1 = (\{1, 2\}, \{2, 3\}, \{3, 2\})$, b) $E^* \succeq_2 E^2 = (\{1, 3\}, \{2, 1\}, \{3, 2\})$ and c) $E^* \succeq_3 E^3 = (\{1, 3\}, \{2, 3\}, \{3, 1\})$.

a) $E^* \succeq_1 E^1$ if and only if the continuation value of party 3 when the equilibrium is E^* is smaller than the one of party 2 in the equilibrium E^1 , that is iff $v_3(E^*) < v_2(E^1)$, which means $\frac{(1-\delta e_2 - \delta e_3)e_3}{1-\delta+\delta^2 e_1 e_3} < \frac{(1-\delta e_2 - \delta e_3)e_2}{1-\delta+\delta^2 e_1 e_2}$. Thus $e_3(1-\delta+\delta^2 e_1 e_2) < e_2(1-\delta+\delta^2 e_1 e_3)$. Since $e_3 < e_2$, it is clear that $e_3(1-\delta) < e_2(1-\delta)$.

b) $E^* \succeq_2 E^2$ iff $v_3(E^*) < v_1(E^2)$, which means $\frac{(1-\delta e_2 - \delta e_3)e_3}{1-\delta+\delta^2 e_1 e_3} < \frac{(1-\delta e_1 - \delta e_3)e_1}{1-\delta+\delta^2 e_2 e_3}$. From $0.5 > e_1 > e_2 > e_3$, it is a matter of simple algebra to show that the left hand side is always smaller than the right hand side.

c) $E^* \succeq_3 E^3$ iff $v_2(E^*) < v_1(E^3)$, which means $\frac{(1-\delta)e_2}{1-\delta+\delta^2 e_1 e_3} < \frac{(1-\delta)e_1}{1-\delta+\delta^2 e_2 e_3}$. Result follows directly.

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