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Equality of opportunity and optimal effort decision under uncertainty*

Aitor Calo-Blanco**

Abstract

We analyze a society that cares about inequality of opportunity. We propose a dynamic setting in which effort is a decision variable that individuals adopt as a solution of an explicit utility maximization program. Effort determines the monetary outcome and it depends on the individual's preferences and circumstances. The planner designs an incentive scheme so as to foster higher incomes, reducing the opportunity cost of effort and productivity for the less favoured agents. Income is assumed to be random, and contrary to the general neutral assumption, we obtain that luck does have a biased and persistent effect on income distribution that may be regarded as unfair. We also study the planner's optimal policy when she cannot infer perfectly the individuals' responsibility feature.

Keywords: Equality of opportunity, effort decision, policy design, luck.

JEL classification: D31, D63, D82.

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** Aitor Calo-Blanco: University of Alicante. E-mail: aitor@merlin.fae.ua.es

1 Introduction

The purpose of the present paper is to study the equality of opportunity approach in a dynamic context in which today's outcome is a random variable (a function of initial endowment, ability and luck, so to speak) that may affect tomorrow's achievements.

The model involves a social planner, a finite set of heterogeneous agents, and a finite time span. The planner aims at implementing an equality of opportunity policy by means of monetary transfers. Contrary to the usual equality of opportunity models, we consider that every individual adopts her effort decision as a solution of an explicit intertemporal maximization problem depending on an environment that includes the policy. Her decisions result in certain outcomes that we associate with personal income. First, we determine the factors that characterize the individual's optimal choice of effort. Second, we study the planner's policy that aims at providing equal opportunity within the society.

Here we introduce into the traditional equality of opportunity framework two features that have been barely studied. First, we consider a non-deterministic set-up in which investment today implies higher expected future gains. Second, we address the problem as a repeated game. To the best of my knowledge, the dynamic approach is something that is completely missing within equality of opportunity literature.

Equality of opportunity policies aim at reducing inequality between individuals with respect to the access to some basic goods or services: income, education, health, etc. The implementation of these policies requires defining properly "equality of what" and finding ways of measuring it. There has been a long and fruitful debate on those aspects in the last years, involving different kinds of social scientists (see Rawls (1971), Dworkin (1981a, 1981b), Sen (1985), Cohen (1989) and Arneson (1989) or Roemer (1993, 1998) among others).¹ A central conclusion is that differences in individual outcomes cannot be the only reference for an egalitarian policy.

¹See also Roemer (1996), Fleurbaey (1996), Peragine (2000, 2002), Fleurbaey and Maniquet (2001), Ruiz Castillo (2003), Villar (2005), Calo-Blanco & Villar (2008).

The reason is that some of those differences are morally acceptable whereas some others are not, depending on the conditioning variables that originate the outcomes. The notion of equality of opportunity tries to capture this distinction by taking into account how individual outcomes depend on both agents' personal decisions (responsibility) and agents' external circumstances (opportunity). Following John Roemer's terminology we call effort to the set of variables related to the agents' responsibility and circumstances to that related to opportunity. The key ethical element is that individuals cannot be held accountable for that part of their outcomes derived from their circumstances. Therefore, in a fair society similar efforts should yield similar outcomes, no matter the agents' circumstances.

The design of an egalitarian policy of this kind calls for a social consensus on the allocation of individuals' traits into the "effort" and "circumstances" categories, and involves particular value judgements that clearly condition the meaning of the results one obtains. The standard approach is to select a specific set of variables that define "opportunity" and treat "effort" as a residual variable that encompasses all those aspects that are not part of the external circumstances. This strategy raises two difficult subjects: The measurement of effort and the treatment of other variables that affect the agents' outcomes (luck in particular). Roemer proposes a way to deal with the measurement difficulty, provided effort is a one-dimensional variable and outcomes are monotonically related to it. If the society is partitioned into types (sets of individuals with similar external circumstances), it can be said that two individuals exert a comparable degree of effort whenever they belong to the same quantile in the outcome distribution of their corresponding types.

Regarding the second difficulty, how should luck be dealt with from an equality of opportunity viewpoint?² Luck is typically subsumed within the effort variable so that outcomes are treated as deterministic and the equality of opportunity policy does not compensate for randomness. What is usually argued to defend this idea is that individuals who exert a comparable degree of effort are facing the same prospects of success (ex-ante lotteries), regardless of their external circumstances.

²We refer implicitly here to what Dworkin calls brute luck (lotteries that an individual cannot escape from), as we place in the realm of circumstances all those random factors that determine each individual's family background, genetic endowment, etc.

Therefore, outcome differences due to luck are ethically acceptable.³ A key point is that the introduction of luck makes the use of quantiles impossible to compare effort degrees, since the monotone relationship assumption does not hold any longer. Therefore, the planner has incomplete information about individuals' level of responsibility, and the planner faces a type of principal-agent situation when designing the equality of opportunity policy.

The previous interpretation of the effect of the luck factor on outcome seems much less compelling in a dynamic setting. In this context the particular realization of the random variable on the initial stages may have a relevant impact on agents that may be regarded as unfair. In order to motivate the previous idea let us put forward the following example. Let us suppose a Ph.D. student that is run over by an irresponsible driver. Because of the severity of the injures she cannot attend the job market meetings, and hence she cannot apply for a job in the best institutions in order to develop her research skills. Therefore, her future outcome (publications here) is clearly affected in the long run for the external shock (the irresponsible driver). Gladwell (2008) collects many similar real examples to this one proposed by us. In one of them it is argued that something as fortuitous as the month in which a hockey player is born may affect seriously his future sports career. Since adolescents born earlier in the year are bigger and more developmentally advanced than the others, they are often identified as better athletes, leading to extra coaching and a higher likelihood of being selected for elite hockey leagues.⁴

The paper is organized as follows. Section 2 contains the basic ingredients of the model. We characterize the individual's intertemporal effort decision under a nonexistent (or constant) public funding policy. We shall assume that higher effort levels are associated with higher expected labour incomes. We obtain that luck appears as an element that explains the resulting income distribution, as the previous realizations of the random variable affects the present transformation of

³Lefranc et al (2007) provide a model in which outcomes are explicitly generated by three different determinants: effort, circumstances and luck. What those authors propose is that equality of opportunity "arises when the outcome distributions conditional on effort cannot be ranked using first and second order stochastic dominance criteria". See also Peragine and Serlenga (2007).

⁴This example is due to Barnsley et al. (1985).

effort into income. We also relate our result to the standard ones in the literature.

Section 3 introduces a social planner that is concerned about inequality of opportunity. Specifically, this planner's designs a system of monetary transfers aims at equalizing the monetary outcomes of those agents who exert a comparable degree of effort (the so-called effort group). Such a policy affects the individuals' optimal effort decision and hence the overall income distribution. As usual, the optimal opportunity policy implies compensations just within effort groups. Moreover, in our setting it provides some compensation for the luck factor, as the planner tries to smooth down its effect on income distribution.

Last section summarizes the main conclusions. All proofs are in the appendix.

2 The basic model

To facilitate the discussion we analyze first the behaviour of individual agents in the absence of a social planner.

2.1 Preliminaries

Consider a society with a finite number of individual agents, $\mathcal{M} = \{1, 2, \dots, m, \dots, M\}$. Individuals are heterogeneous concerning their preferences (responsibility) and their types (circumstances). They make decisions in an finite horizon dynamic setting that involve a utility cost (effort) and result in a monetary outcome that is a random variable.

Individual agents are classed into *types* according to their initial circumstances. Here we consider that any individual's initial endowment consists of a given amount of monetary resources (present wealth), $w \in W = \{\underline{w}, \dots, \bar{w}\}$. We denote by $\mathcal{M}(w)$ the set of agents of type w , for $w = 1, 2, \dots, j, \dots, J$. The effort level chosen by agent m is a real number \tilde{e} in the closed interval $[\tilde{e}_w^L, \tilde{e}_w^H]$, that is type dependent.

We assume that the outcome space for all individuals is a set X with finite support. We write $X = \{x_1, x_2, \dots, x_i, \dots, x_n\}$, for $x_i \in \mathbb{R}_+$, in the understanding

that those outcomes are ranked in an increasing order: $x_1 < x_2 < \dots < x_i < \dots < x_n$. Each point of the outcome space is a stochastic function of individual effort, environmental variables and personal characteristics. We assume that all agents have the same *a priori* distribution over that random variable.

The economic process that transforms effort into income is summarized here by a probability distribution on the monetary outcomes, conditional on the agent's effort and characteristics. The underlying idea is that effort is chosen by the individuals according to their preferences whereas the initial wealth (the type) determines the way in which effort translates into outcomes in terms of a particular probability distribution.

We can write the conditional probability of obtaining income x_i by an agent m of type w as follows:

$$\Pr [x = x_i \mid \tilde{e}, w] = p_i(\tilde{e}, w), \quad \forall i \in \{1, \dots, n\} \quad (1)$$

with $\sum_{i=1}^n p_i(\tilde{e}, w) = 1$. This conditional probability is required to satisfy the following properties:

(i) *Monotone likelihood ratio condition*: For each given w , the functions $\{p^i(\tilde{e}, w)\}_{i=1}^n$ are such that $\tilde{e} \leq \tilde{e}'$ implies that $p^i(\tilde{e}, w) / p^i(\tilde{e}', w)$ is non-increasing in i .

(ii) *Convexity of the distribution function condition (cdfc)*: The functions $\{p_j(\tilde{e}, w)\}_{j=1}^N$ are such that $F_j''(\tilde{e}, w)$ is nonnegative for every $j \in \{1, \dots, N\}$ all \tilde{e}, w , where $F_j(\tilde{e}, w) = \sum_{i=1}^j p_i(\tilde{e}, w)$.

Monotonicity is telling us that the higher the level of effort the higher the probability of obtaining a higher income. Such a condition implies that $\{p^i(\tilde{e}', w)\}$ first order stochastically dominates $\{p^i(\tilde{e}, w)\}$. Finally, Property (ii) requires that the function decreases at a decreasing rate, that is, the cdfc is a form of stochastic diminishing returns to scale.

Let $p_i^H(w) := p_i(\tilde{e}_w^H, w)$, $p_i^L(w) := p_i(\tilde{e}_w^L, w)$ denote the probability of obtaining income x_i by an agent of type w when she exerts the maximum and the minimum level of effort, respectively. In view of the monotone likelihood ratio condition, the

conditional probability of obtaining income x_i by agent m can be rewritten as:

$$p_i(\tilde{e}, w) = ep_i^H(w) + (1 - e)p_i^L(w) \quad (2)$$

where $e \in [0, 1]$, with $e = 1$ iff $\tilde{e} = e_w^H$ and $e = 0$ iff $\tilde{e} = e_w^L$. The monotonicity of the likelihood ratio ensures a one to one relationship between \tilde{e}_m and e , so that e can be understood as the agent's degree of effort (the percentage of the maximal effort, conditional on her type).

The expected income of an agent m that chooses an effort e_m at time t is given by:

$$\bar{x}_m(e_m(t), w_m(t)) = \sum_{i=1}^n [e_m(t)p_i^H(w_m(t)) + (1 - e_m(t))p_i^L(w_m(t))] x_i \quad (3)$$

Note that, under the conditions established, expected income is an increasing function of effort for all types. The extent to which effort translates into expected income depends on the agent's current initial wealth and is reflected in the agent's probability distribution.

The next step is to model how the agent decides her optimal effort.

2.2 Optimal effort decision

Following the approach in Calo-Blanco & Villar (2008) we take effort as a decision variable that each individual chooses as the solution of an explicit utility maximization program.

We assume that the utility function of agent m depends positively on the total current income and negatively on the effort in a separable way. This can be expressed as follows: agent m of type w that obtains an income x_i at point t , with a degree of effort $e_m(t)$, gets a utility:

$$U_m(w(t), e_m(t)) = u_m(x_i) - c_m(e_m(t)) \quad (4)$$

$\forall t \in [0, T]$ and for some end-point T , with the usual derivative signs: $u'(\cdot) > 0$; $u''(\cdot) \leq 0$; $c'(\cdot) > 0$; $c''(\cdot) > 0$. That is, utility is increasing and concave whilst the

cost of effort is increasing and convex. Additionally, we assume that to make the maximum level of effort is extremely costly for the agent, that is $c'(0) = 0, c'(1) = +\infty$.

For the sake of simplicity in exposition, we take a utility function that is linear in income and a strictly convex in effort. More specifically we shall use the following formulation:

$$\begin{aligned} u_m(x_i) &= x_i \\ c_m(e_m(t)) &= (1 - e_m(t))^{-\sigma_m} - 1 \end{aligned} \tag{5}$$

where $\sigma_m \geq 0$ is a parameter that measures the personal disutility of effort (the personal responsibility feature). The higher the value of σ_m , the higher the disutility of making a certain level of effort. The utility function is chosen so as to make computations operational. The specific form of the effort cost function ensures that the optimal effort decision is between 0 and 1.

Each individual has a working life of T periods, and she has to decide the intertemporal choice of effort that maximizes her present and future earnings. The individual derives utility from the consumption of a fraction $(1 - \alpha)$ of her total present income, whereas a fraction α is saved as initial wealth for the next period. The individual takes into account that her current wealth $w(t)$ determines her expected present income.

The problem that any individual faces at the time of making her intertemporal choice is that she is adopting the decision according to expected values. Therefore, at the beginning of every period the individual has to redesign her optimal intertemporal choice for the remaining periods, incorporating the new available pieces of information. At any period $t \in [0, T]$ the individual decides her optimal current effort choice $e_m(t)$ from the solution of the following program:

$$\left. \begin{aligned} \max_{\{e_m(s)\}} \sum_{s=t}^T \beta^{s-t} & \left((1 - \alpha) [\bar{x}_m(e_m(s), w_m(s))] - (1 - e_m(s))^{-\sigma_m} - 1 \right) \\ s.t. : e_m(s) & \in [0, 1] \\ \bar{w}_m(s+1) & = \alpha [\bar{x}_m(e_m(s), w_m(s))] \\ w_m(0) & = w_0^m; \forall m \in \mathcal{M} \end{aligned} \right\} \tag{6}$$

Proposition 1 *The individual's optimal effort decision at any period $t \in [0, T]$ is given by the following expression:*

$$e_m^*(t) = 1 - \left[\frac{(1-\alpha)}{\sigma_m} \sum_{i=1}^n k_i(w_m(t)) x_i (1 + \Delta_t) \right]^{\frac{-1}{1+\sigma_m}} \quad (7)$$

where $k_i(w_m(t)) = p_i^H(w_m(t)) - p_i^L(w_m(t))$, $\Delta_t = \sum_{j=t+1}^T (\beta\rho)^{j-t}$ and $\rho = \alpha \left[\frac{\partial \bar{x}_m(\cdot, t+1)}{\partial w_m(t+1)} \right]$.

Proof. The proof is in appendix A. ■

According to expression 7 we find that at any period $t \in [0, T]$, the higher the expected present or the discounted future gains (measured here through parameter Δ_t), the higher the level of effort made by the agent. It is also easy to check that there is a direct relationship between the individual's optimal choice of effort and her level of responsibility (i.e., e_m^* decreases with σ_m).⁵ Therefore, part of the observed income distribution derives from the differences in the individuals' disutility of effort (a responsibility feature) and can be regarded as socially fair. Implicit in the solution is the role of the initial wealth $w_m(t)$. Higher levels of wealth increase the probability of obtaining higher income because they affect to the agents opportunity cost of effort and productivity through the income probability distribution. The third element that explains the resulting income distribution is luck: the realization of the random variables that determine the transformation of effort into income, and hence luck has an influence on income distribution. This effect comes from the transformation of previous incomes in present wealth. In summary: the income an agent gets depends on circumstances, personal choices, and luck.

As long as the sort of luck experienced in the past affects the individual's optimal behaviour in the long run through the endogenous determination of effort, we argue that the luck factor does call for social compensation, as agents cannot be held accountable for that factor.

One could point out that given that individuals are facing a sort of problem of uncertainty in income, they could completely avoid bad luck just by taking fully insurance about the final income. This would work out in the case in which there

⁵See appendix A.

is no connection between luck and future income. Actually, the insurance can be understood as an explanation of the usual neutral effect of uncertainty assumption in the previous literature. However, our line of reasoning here is that the introduction of an insurance market will not solve the problem. The idea is that if any individual is suffering an initial series of bad luck, the insurance firm would understand the individual's bad results as a signal of a low level of responsibility. Therefore, the insurance firm would charge the individual with a higher premium, even though she is behaving appropriately.

Remark 2.1 *If the individual's optimal effort decision depends on the previous realizations of the monetary outcome, the luck factor will have then a biased and persistent effect on income. Therefore, such a factor does call for social compensation in order to ensure equality of opportunity.*

Finally, our result is closely related to previous features obtained in the literature. Let us consider a very simplified version of the model in which there is neither a link between periods nor a discount factor (i.e., $\beta = 1$). For instance, we can think that the initial set of external circumstances is given by variables like race, gender, family social background and natural skills, among others, and therefore it remains fixed for all periods. If this is the case, the optimal choice of effort turns into the following expression:

$$e_m^* = 1 - \left[\frac{1}{\sigma_m} \sum_{i=1}^n k_i (w_0^m) x_i \right]^{\frac{-1}{(\sigma_m+1)}} \quad (8)$$

which is constant for all periods, independently of the previous realizations of the random variable. Note that this is equivalent to assume that individuals perform a constant level of effort that is exogenously given. It is a well-known result in the literature that if there is no uncertainty in the model and the level of effort is constant, it is possible to infer perfectly the individual's responsibility feature after just one period. What we propose here is an extension of this usual result to the case in which the final income is random. Specifically, we propose the following:

Corollary 2.1 *Under incomplete information, if individuals are making a constant*

level of effort, the planner can infer exactly (after a finite number of periods) the individuals' level of responsibility by means of a simple updating of beliefs process.

Proof. The proof is in appendix B. ■

Roughly speaking, this result can be understood as a sort of extension of the Roemer's solution to the case in which the effort decision cannot be either observed or perfectly inferred.

3 The Planner

In the previous section we have analyzed the individual's optimal choice of effort in the absence of a funding policy. Now we proceed to analyze such optimal behaviour when a social planner that cares about inequality of opportunity is introduced in the model. Such a planner designs a system of monetary transfers that aims at equalizing the monetary outcomes of those agents who exert a comparable degree of effort. As effort is not observable and incomes are random, the planner has to infer the individuals' level of responsibility updating her beliefs according to the observed monetary outcomes and the information on agents' characteristics. Money transfers vary, therefore, according to the messages received by the planner concerning the effort exerted by the agents.

We assume that the planner can identify the agents' current initial wealth and also that she observes the individual incomes and knows their probability distributions. She will use that information to estimate the agents' effort levels and implement the equality of opportunity policy.

At the end of every period the planner adjusts her beliefs about the individuals' level of responsibility according to the available pieces of information. Although the monetary outcome is random and the planner cannot observe the responsibility feature, it is only natural to think that the planner expects from any individual a level of income that should be in accordance with her initial circumstances and the level of effort that she is making.

Because of the uncertainty of income, the planner realizes that sometimes the agent does not achieve the income that was expected from her although she was behaving appropriately. Therefore, in order to mitigate the effect of luck on subsequent incomes the planner aims at smoothing down the effect of initial results on subsequent outcomes.

We define $g_m(t)$ as a function that summarizes the principal's beliefs about all her past experiences with agent m up to period t , to be understood as assessments of individual m 's disutility of effort. Those beliefs can be used to rank all individuals belonging to any type in connection with the responsibility feature, that is, σ_m . Once this classification has been made for all types, the planner would have a tool to make effort comparisons between individuals.⁶

Since the beliefs function summarizes all interactions between the planner and the individual, the realization of previous incomes (that includes the sort of luck experienced in the past) will affect the current beliefs as well. Consequently, every agent chooses an intertemporal effort decision that varies according to her current environment and the planner's funding policy, which is a function of the individual's reputation.

According to the previous description every individual $m \in \mathcal{M}$ makes at time $t \in [0, T]$ a level of effort that maximizes her present and future gains, taking into account that her decision will affect the future beliefs about her, and hence her subsequent incomes, that is:

$$\left. \begin{aligned} \max_{e_m(s)} \sum_{s=t}^T \beta^s & \left((1 - \alpha) [\bar{x}_m(\cdot, s) + g_m(s) \gamma^{w_m(s)}] - (1 - e_m(s))^{-\sigma_m} - 1 \right) \\ \text{s.t. : } & e_m(s) \in [0, 1] \\ & \bar{w}_m(s+1) = \alpha \bar{x}(\cdot, s) \\ & w_m(0) = w_0^m; \forall m \in \mathcal{M} \\ & g_m(s+1) - g_m(s) = f(\bar{x}(\cdot, s)) \\ & g_m(0) = g_m^0 \in \mathbb{R}_+ \end{aligned} \right\} \quad (9)$$

where $f(\cdot)$ is a function that defines how the planner updates her beliefs. The

⁶Needless to say, when the policy is based on the planner's beliefs the result will differ from that one obtained under complete information. However, as the lack of information cannot be completely solved the present solution has to be understood as a second best solution.

individual's present utility function is given by the utility of the expected income plus the monetary incentive received from the planner in that specific period according to her present wealth, $g_m(s) \gamma^{w_m(s)}$. Such a subsidy is scaled down by the planner's beliefs about the individual's level of responsibility. Moreover, the individual takes into account that the effect of effort has an additional feature, as the current level of effort affects the individual's future reputation, and hence it affects her future utility as well. This problem has the following solution:

Proposition 2 *The individual's optimal effort decision at any period $t \in [0, T]$ in system 9 is given by the following expression:*

$$e_m^*(t) = 1 - \left[\frac{(1-\alpha)}{\sigma_m} \left(\sum_{i=1}^n k_i(w_m(t)) x_i [1 + \Delta_t + \Psi_t] + \Gamma_t \right) \right]^{\frac{-1}{1+\sigma_m}} \quad (10)$$

where:

$$\begin{aligned} \Delta_t &= \sum_{j=t+1}^T (\rho\beta)^{j-t} \\ \Psi_t &= \frac{1}{\alpha} f_{\bar{x}} \left(\frac{\partial \bar{x}(\cdot, t+1)}{\partial w_m(t+1)} \right) \sum_{j=t+1}^T \rho^{j-t} \sum_{j=t+2}^T \beta^{j-t} \gamma^{w_m(j)} \\ \Gamma_t &= f_{\bar{x}} \left(\sum_{i=1}^n k_i(w_m(t)) x_i \right) \sum_{j=t+1}^T \beta^{j-t} \gamma^{w_m(j)} \end{aligned}$$

Proof. The proof is in appendix A. ■

Likewise in the previous section, we obtain that the optimal effort decision is positively related with both the level of responsibility and the initial wealth. The difference between this effort decision and the optimal choice of effort in equation 7 is the last terms of the denominator. Such terms indicates the future earnings of making a higher effort due to the incentive mechanism. These gains are twofold. On the one hand, a higher level of effort today will imply a better reputation in the future. On the other hand, a present higher level of effort implies a higher future initial wealth, and therefore indirect future improvements in the individual's reputation. In other words, these values are the future gains that the individual can get if she is investing today in building up a better reputation. Note that such an expression is decreasing in time, that is, the fewer the number of remaining periods to take advantage of the own reputation, the lower the expected gains of

investing in it, and hence the lower the incentive to exert oneself. Again, the individual adapts her optimal choice to the current environment including the luck that she has experienced up to that period.

We also observe that the effort decision is increasing in the monetary incentive as it reduces the opportunity cost of effort and productivity. The value of this variable is actually chosen by the planner, so that she can use it to have an influence on the individual's final income. This can be understood as the way in which the planner makes social compensations.

Remark 3.1 *The social planner can implement an incentive scheme so as to have an influence on the determination of the agents' effort choices. That influence can be used as an instrument to implement an egalitarian policy.*

Finally, we proceed to characterize the funding policy of an egalitarian planner that every period fixes the current incentive scheme.⁷ As we said before, this planner hopes to equalize the current final income of all of those individuals that she "believes" that present a similar degree of responsibility, regardless their initial wealth, that is:

Definition 3.1 *There is equality of opportunity in the society if and only if $\forall m, m' \in \mathcal{M} : g_m(t) = g_{m'}(t)$ the following condition holds:*

$$\bar{x}_m((e_m(t))^*) = \bar{x}_{m'}((e_{m'}(t))^*)$$

Here we assume that the planner considers that two individuals belong to the same type j if both have the same level of current wealth $w_j(t)$. Note that this level of wealth is a function of both opportunity and responsibility factors. However, we know that in a fair society two individuals with the same level of responsibility must end up with the same level of wealth, therefore the planner must compensate individuals for any income difference between both. It can be asserted that there is equality of opportunity in the society if and only if:

⁷In this case individuals must adopt their decisions according to some current expectations about the future incentive scheme.

Proposition 3 *There is equality of opportunity in the society if and only if $\forall m, m' \in \mathcal{M} : g_m(t) = g_{m'}(t)$ the following condition holds:*

$$\begin{aligned} & \sum_{i=1}^n \left[\left(1 - \left(\frac{\Upsilon_m}{\widehat{\sigma}_{t,m}} \right)^{\frac{-1}{(\widehat{\sigma}_{t,m}+1)}} \right) k_i(w_m(t)) + p_i^L(w_m(t)) \right] x_i \\ &= \sum_{i=1}^n \left[\left(1 - \left(\frac{\Upsilon_{m'}}{\widehat{\sigma}_{t,m'}} \right)^{\frac{-1}{(\widehat{\sigma}_{t,m'}+1)}} \right) k_i(w_{m'}(t)) + p_i^L(w_{m'}(t)) \right] x_i \end{aligned} \quad (11)$$

where $\Upsilon_m = (1 - \alpha) \left(\sum_{i=1}^n k_i(w_m(t)) x_i [1 + \Delta_t + \Psi_t] + \Gamma_t \right)$ and $\widehat{\sigma}_{t,m}$ is the level of disutility of effort that in period t the planner guesses from agent m .

Proof. The proof is in appendix A. ■

Since the planner thinks that both agents are similar with respect to their level of disutility, it must be the case that $\widehat{\sigma}_{t,m} = \widehat{\sigma}_{t,m'} = \widehat{\sigma}_t$.⁸ We have previously shown that the final income is directly related to both the external circumstances and the public incentive. If there is no public policy we have that for any pair of levels of wealth $w_m(t) > w_{m'}(t)$, the left hand side of expression 11 is bigger because of the higher probability of obtaining larger incomes. As those individuals are supposed to be identical with respect to the responsibility feature, their income differences must be considered socially unfair. Therefore, in order to equalize the expected income of both individuals at period t the principal must provide the "poorest" individual with a higher fraction of public funds. That is, the planner has to make monetary transfers within effort groups, compensations that will depend on the initial wealth, the agents' level of disutility of effort, and the sort of luck experienced in the past.

Note that we are considering that $\gamma^{w_j(t)}$ is equal for all individuals with current initial wealth j . Therefore, all individuals with the same external circumstances are facing the same maximum incentive, that is, $\forall m(j), m'(j) \in \mathcal{M}(j) \implies \gamma_m^{w_j(t)} = \gamma_{m'}^{w_j(t)} = \gamma^{w_j(t)}$. However, as the planner can use her current beliefs to treat individuals differently, the actual incentive received by each one of them will be different. More precisely, the money received by each one of them would be $g_m(t) \gamma^{w_j(t)}$ and $g_{m'}(t) \gamma^{w_j(t)}$, respectively. Consequently, the planner finds fairest to provide the most diligent workers with a higher fraction of public monetary resources.

⁸Assuming that σ is equally distributed between types.

Remark 3.2 *The equal opportunity feature is concerned about income inequalities within effort groups, whereas income differences between these groups only represent diverse rewards of people's autonomous choices and will not be considered unfair. Moreover, the planner reveals a certain concern for the aggregate welfare as a secondary objective.*

The planner that implements an equality of opportunity policy that maximizes the overall welfare looks for a schedule of monetary transfers that solves the following program:

$$\left. \begin{aligned} \max_{\{\gamma^{w_j(t)}\}_j} & \sum_{m=1}^M \left((1 - \alpha) \left[\bar{x}_m(\cdot, t) + g_m(t) \gamma_m^{w_j(t)} \right] - \left[(1 - \hat{e}_m^*(t))^{-\hat{\sigma}_{t,m}} - 1 \right] \right) \\ \text{s.t.} & \sum_{m=1}^M g_m(t) \gamma_m^{w_j(t)} = \kappa \\ & \hat{e}_m^*(t) = 1 - \left[\frac{\mathbf{r}_m}{\hat{\sigma}_{t,m}} \right]^{\frac{-1}{1 + \hat{\sigma}_{t,m}}}; \forall m \in M \\ & \bar{x}_m(\hat{e}_m^*(t)) = \bar{x}_{m'}(\hat{e}_{m'}^*(t)); \forall m, m' \in \mathcal{M} : g_m(t) = g_{m'}(t) \end{aligned} \right\} \quad (12)$$

where $\kappa \in R_+$ is the planner's budget constraint. If this value is strictly greater than 0 it means that the planner possesses a certain amount of exogenously given monetary resources to implement the equality of opportunity policy. If on the contrary we require that the equality of opportunity policy is budget balanced (i.e., $\kappa = 0$) all the necessary resources to implement the policy must come from the very agents. Then, some of the incentives must be negative, that is, some individuals are being taxed so as to subsidize the rest of the citizens.⁹

4 Concluding remarks

Nowadays equality of opportunity is considered as one of the fairest principle at the time of evaluating outcome and opportunity, and hence it becomes extremely important to define properly the aspects that call for compensation. We have presented a

⁹In that case the individual's incentive scheme should be defined in the following way: $g_m^{tax}(t) \gamma_m^{w_j(t)} = (1 - g_m(t)) \left(-\gamma_m^{w_j(t)} \right)$.

model in which, like in Calo-Blanco & Villar (2008), the individual choice of effort is considered as a decision variable that individuals adopt as a solution of an explicit utility maximization program. This decision determines the individual's monetary outcome, which is assumed to be random. Such an optimal choice depends on different factors like the individuals' environment (initial wealth and the distribution of probabilities), their preferences (singularized here by the disutility of making a level of effort) and the public funding policy (considered as monetary incentives to foster higher incomes).

The introduction of luck through the uncertainty of income exhibits here very interesting features. The standard models in the literature assume that effect of luck on income is even-handed across types, and hence it does not call for social compensation.¹⁰ Nevertheless, if we consider that the realization of previous outcomes may have some influence on subsequent outcomes, it is not possible to assume this neutral effect of luck on income any longer.

We have presented a dynamic setting in which the individual's optimal effort decision, and hence her final income is a function of her responsibility and her circumstances, as such variables reduce the opportunity cost of making a certain level of effort. We have found that the individual's final income is also a function of the sort of luck that she has experienced in the past, as it affects the individual's transformation of effort into income. The key point is that a negative series of bad luck in the initial stages can negatively determine the individual's future circumstances, and hence her future incomes, to a large extent, regardless her personal traits. From our viewpoint, any equality of opportunity policy should also aim to compensate individuals for outcome differences that come from the different kind of luck experienced in the past.

Next, we have introduced a social planner that is concerned about inequality in opportunity terms. She can alter the agents' optimal effort decision, and hence the income distribution, in order to equalize opportunity among individuals. In the case

¹⁰From our viewpoint, this line of argument it is quite debatable. For instance, the possibility of having a road crash is, in principle, independent of the individual's external circumstances. However, the final outcome (the severity of the injures) is clearly dependent on the individual's circumstances, as expensive cars are usually safer than cheap ones.

in which the planner does not have complete information about individuals' level of responsibility, she can use an updating of beliefs mechanism so as to implement a second best policy. Moreover, the planner aims at smoothing down the effect of luck on income distribution, especially for the initial periods, which strongly determines the subsequent behaviours.

The characterization of the optimal funding policy states that the planner must make compensations just within effort groups. On the contrary, any difference between such groups is exclusively given to responsibility features, and hence does not call for social compensation. Additionally, such an optimal policy yields a certain concern for the aggregate welfare of the society.

In the introduction of the present paper we have put forward two examples of how luck can affect the individual's outcome in a repeated game. Once the problem have been analyzed in depth, we can argue now that both the Ph.D. student and the hockey player should be compensated in order to help them to develop their own abilities. For the former, the ministry of education can provide the student with a post doctoral fellowship to visit the institution that best fits her research interests. For the latter, a possible solution is to have a rotating eligible date, something that the Canadian Hockey Association has been discussing about.

Finally, we want to stress the importance of the dynamic approach in terms of efficiency. We have obtained that the sooner the public compensation, the higher the effect of the funding policy and the cheaper the monetary cost for the society. This result is extremely important for outcomes like education or health. For instance, it is very well known that the investment in education has a deeper impact on the students' future success if it is done in the earlier stages of education. Likewise, a patient that is suffering any serious illness will have more chances to fully recover from it if she receives an early treatment.

A Appendix: Proofs

A.1 Proposition 1

Proof. At any period $t \in [0, T]$ the individual decides her choice of effort as the solution of the intertemporal program for the remaining periods $T - t$. This is equivalent to maximize the following intertemporal program :

$$\left. \begin{aligned} \max_{e_m(s)} \sum_{s=0}^{T-t} \beta^s & \left((1 - \alpha) \bar{x}(\cdot, s) - (1 - e_m(s))^{-\sigma_m} - 1 \right) \\ \text{s.t. : } e_m(s) & \in [0, 1] \\ \bar{w}_m(s+1) & = \alpha \bar{x}(\cdot, s) \\ w_m(0) & = w_0^m; \forall m \in \mathcal{M} \end{aligned} \right\}$$

The previous problem has the following Hamiltonian function:

$$\begin{aligned} H & = \beta^s \left((1 - \alpha) \bar{x}(\cdot, s) - (1 - e_m(s))^{-\sigma_m} - 1 \right) \\ & + \delta(s+1) \alpha \bar{x}(\cdot, s) \end{aligned}$$

with the following first order conditions:

$$\begin{aligned} \frac{\partial H}{\partial e_m(s)} & = 0 = \beta^s \left((1 - \alpha) \sum_{i=1}^n k_i(w_m(s)) x_i - \frac{\sigma_m}{(1 - e_m(s))^{\sigma_m+1}} \right) + \delta(s+1) \alpha \sum_{i=1}^n k_i(w_m(s)) x_i \\ \forall s & = 0, \dots, T - t. \end{aligned}$$

$$\frac{\partial H}{\partial w_m(s)} = \delta(s) = \beta^s (1 - \alpha) \left[\frac{\partial \bar{x}(\cdot, s)}{\partial w_m(s)} \right] + \delta(s+1) \alpha \left[\frac{\partial \bar{x}(\cdot, s)}{\partial w_m(s)} \right]$$

\Downarrow

$$\delta(s+1) = \frac{\delta(s)}{\rho} - \beta^s \frac{(1 - \alpha)}{\alpha}; s = 0, \dots, T - t.$$

$$\text{where } \rho = \alpha \left[\frac{\partial \bar{x}(\cdot, s)}{\partial w_m(s)} \right]$$

$$\frac{\partial H}{\partial \delta(s+1)} = w(s+1) = \alpha \bar{x}(\cdot, s)$$

where $k_i(w_m(s)) = p_i^H(w_m(s)) - p_i^L(w_m(s))$. In the second order condition we can solve for the optimal value of the costate variable $\delta(s)$. This equation of differences

has the following solution:

$$\begin{aligned}\delta(s) &= \left(\frac{1}{\rho}\right)^s \delta_0 - \sum_{j=0}^{s-1} \left(\frac{1}{\rho}\right)^{s-j-1} \beta^j \frac{(1-\alpha)}{\alpha} \\ &= \left(\frac{1}{\rho}\right)^s \delta_0 - \sum_{j=0}^{s-1} \left(\frac{1}{\rho}\right)^{s-j-1} b_j; \quad s = 1, \dots, T-t+1.\end{aligned}$$

where δ_0 is a certain constant and $b_j = \beta^j \frac{(1-\alpha)}{\alpha}$. As the value function ends at $T-t$, an increase in $w_m(T-t)$ cannot affect the payoff, so that the transversality condition has a free end point, that is $\delta(T-t+1) = 0$. Thus, $\delta(T-t+1) = \left(\frac{1}{\rho}\right)^{T-t+1} \delta_0 - \sum_{j=0}^{T-t} \left(\frac{1}{\rho}\right)^{T-t-j} b_j = 0 \rightarrow \delta_0 = (\rho)^{T-t+1} \sum_{j=0}^{T-t} \left(\frac{1}{\rho}\right)^{T-t-j} b_j$, and hence:

$$\delta^*(s) = \sum_{j=s}^{T-t} \rho^{j-(s-1)} b_j$$

Substituting this value into the first order condition to eliminate the costate variable:

$$\begin{aligned}\frac{\beta^s \sigma_m}{(1 - e_m(s))^{\sigma_m+1}} &= \beta^s (1 - \alpha) \sum_{i=1}^n k_i(w_m(s)) x_i + \alpha \sum_{i=1}^n k_i(w_m(s)) x_i \delta^*(s+1) \\ \frac{\beta^s \sigma_m}{(1 - e_m(s))^{\sigma_m+1}} &= \beta^s (1 - \alpha) \sum_{i=1}^n k_i(w_m(s)) x_i + \alpha \sum_{i=1}^n k_i(w_m(s)) x_i \sum_{j=s+1}^{T-t} \rho^{j-s} b_j \\ \frac{\beta^s \sigma_m}{(1 - e_m(s))^{\sigma_m+1}} &= \sum_{i=1}^n k_i(w_m(s)) x_i \left(\beta^s (1 - \alpha) + \alpha \sum_{j=s+1}^{T-t} \rho^{j-s} b_j \right) \\ e_m^*(s) &= 1 - \left[\frac{1}{\beta^s \sigma_m} \sum_{i=1}^n k_i(w_m(s)) x_i \left(\beta^s (1 - \alpha) + \alpha \sum_{j=s+1}^{T-t} \rho^{j-s} b_j \right) \right]^{\frac{-1}{(\sigma_m+1)}} \\ e_m^*(s) &= 1 - \left[\frac{1}{\sigma_m} \sum_{i=1}^n k_i(w_m(s)) x_i \left((1 - \alpha) + \frac{\alpha}{\beta^s} \sum_{j=s+1}^{T-t} \rho^{j-s} \beta^j \frac{(1-\alpha)}{\alpha} \right) \right]^{\frac{-1}{(\sigma_m+1)}} \\ e_m^*(s) &= 1 - \left[\frac{(1-\alpha)}{\sigma_m} \sum_{i=1}^n k_i(w_m(s)) x_i \left(1 + \sum_{j=s+1}^{T-t} \rho^{j-s} \beta^{j-s} \right) \right]^{\frac{-1}{(\sigma_m+1)}}\end{aligned}$$

Note that the previous solution is defined for a time span $[0, T-t]$. Therefore we need to scale it adding t periods and fixing $s = 0$, that is, the optimal solution for a particular $t = 0, \dots, T$ would become:

$$e_m^*(t) = 1 - \left[\frac{(1-\alpha)}{\sigma_m} \sum_{i=1}^n k_i(w_m(t)) x_i \left(1 + \sum_{j=s+1}^{T-t} \rho^{j-s} \beta^{j-s} \right) \right]^{\frac{-1}{(\sigma_m+1)}}$$

This expression would give us the individual's optimal effort decision at any period $t \in [0, T]$. Therefore, we would have that $\forall t = 0, \dots, T$ the optimal effort decision chosen by the agent would be:

$$e_m^*(t) = 1 - \left[\frac{(1-\alpha)}{\sigma_m} \sum_{i=1}^n k_i(w_m(t)) x_i \left(1 + \sum_{j=t+1}^T (\rho\beta)^{j-t} \right) \right]^{\frac{-1}{(\sigma_m+1)}}$$

Additionally, we have that $w_m(t)$ can be written as a function all previous income realizations from $t = 0$. More precisely, it can be rewritten as $w_m(t) = \alpha x(t-1)$, where $x(t-1)$ is the realization of income in period $t-1$, which is determined as a stochastic function of the individual's traits at period $t-1$, that is, $w_m(t) = \alpha \phi_{t-1}[w_m(t-1), \sigma_m]$, where ϕ_{t-1} is that stochastic function. Likewise, we can rewrite $w_m(t)$ as $w_m(t) = \alpha \phi_{t-1}[\alpha x(t-2), \sigma_m] = \alpha \phi_{t-1}[\alpha \phi_{t-2}[w_m(t-2), \sigma_m], \sigma_m]$. We can continue such a decomposition up to $t = 0$, and hence for the sake of simplicity we can rewrite the previous expression as: $w_m(t) = \Phi(w_0^m, \sigma_m, \alpha)$. That is, the individual's present wealth is a stochastic function of both her initial endowment and her level of responsibility. Therefore, the presence of uncertainty in the model has a biased and persistent effect on the individual's present labour income. Finally, if we consider a natural model in which individuals have always incentives to work, we have to assume that the disutility of effort is not arbitrarily large, more precisely, it must be the case that $(1-\alpha) \sum_{i=1}^n k_i(w_m(t)) x_i \left(1 + \sum_{j=t+1}^T \rho^{j-t} \beta^{j-t} \right) \geq \sigma_m$, that is, at least up to some extent it must be profitable to make a non-negative level of effort. ■

A.2 Relationship between $e_m^*(t)$ and σ_m

Proof. The relationship between the two variables is given by the following derivative:

$$\frac{\partial e_m^*(t)}{\partial \sigma_m} = - \left(\frac{\sigma_m}{\Upsilon} \right)^{\frac{1}{\sigma_m+1}} \frac{(\sigma_m + 1 - \sigma_m \ln \frac{\sigma_m}{\Upsilon})}{(\sigma_m + 1)^2 \sigma_m}$$

where $\Upsilon = (1-\alpha) \sum_{i=1}^n k_i(w_m(t)) x_i \left(1 + \sum_{j=t+1}^T \rho^{j-t} \beta^{j-t} \right)$. As we are assuming that $\Upsilon \geq \sigma_m$ the the sign of the derivative is inevitably negative, which implies our previous statement between optimal effort and responsibility. ■

A.3 Proposition 2

Proof. The optimal problem in expression 9 has the following Hamiltonian function:

$$H = \beta^s \left((1 - \alpha) [\bar{x}(\cdot, s) + g(s) \gamma^{w_m(s)}] - (1 - e_m(s))^{-\sigma_m} - 1 \right) \\ + \delta(s+1) \alpha \bar{x}(\cdot, s) + \mu(s+1) (g_m(s) + f(\bar{x}(\cdot, s)))$$

with the following first order conditions:

$$\frac{\partial H}{\partial e_m(s)} = 0 = \beta^s \left((1 - \alpha) \sum_{i=1}^n k_i(w_m(s)) x_i - \frac{\sigma_m}{(1 - e_m(s))^{\sigma_m+1}} \right) \\ + \delta(s+1) \alpha \sum_{i=1}^n k_i(w_m(s)) x_i + \mu(s+1) f_{\bar{x}} \left(\sum_{i=1}^n k_i(w_m(s)) x_i \right) \\ \forall s = 0, \dots, T-t.$$

$$\frac{\partial H}{\partial w_m(s)} = \delta(s) = \beta^s (1 - \alpha) \left[\frac{\partial \bar{x}(\cdot, s)}{\partial w_m(s)} \right] + \delta(s+1) \alpha \left[\frac{\partial \bar{x}(\cdot, s)}{\partial w_m(s)} \right] \\ + \mu(s+1) f_{\bar{x}} \left(\frac{\partial \bar{x}(\cdot, s)}{\partial w_m(s)} \right) \\ \Updownarrow \\ \delta(s+1) = \frac{\delta(t)}{\rho} - \beta^t \frac{(1 - \alpha)}{\alpha} + \mu(s+1) f_{\bar{x}} \left(\frac{\partial \bar{x}(\cdot, s)}{\partial w_m(s)} \right) \\ \forall t = 0, \dots, T-t; \text{ where } \rho = \alpha \left[\frac{\partial \bar{x}(\cdot, s)}{\partial w_m(s)} \right]$$

$$\frac{\partial H}{\partial g_m(s)} = \mu(s) = \beta^s (1 - \alpha) \gamma^{w_m(s)} + \mu(s+1)$$

$$\frac{\partial H}{\partial \delta(s+1)} = w(s+1) = \alpha [w_m(s) + \bar{x}(\cdot, s)]$$

$$\frac{\partial H}{\partial \mu(s+1)} = g(s+1) = g_m(s) + f(\bar{x}(\cdot, s))$$

where $k_i(\cdot) = p_i^H(\cdot) - p_i^L(\cdot)$. In the third order condition we can solve for the optimal value of the costate variable $\mu(t)$. This equation of differences has the following solution:

$$\mu(s) = C_1 - (1 - \alpha) \sum_{j=0}^{s-1} \beta^j \gamma^{w_m(j)}; \quad s = 1, \dots, T-t+1.$$

where C_1 is a certain constant. As the value function ends at $T-t$, an increase in $g_m(T-t)$ cannot affect the payoff, so that the transversality condition has a free end

point, that is $\mu(T-t+1) = 0$. Thus, $\mu(T-t+1) = C_1 - (1-\alpha) \sum_{j=0}^{T-t} \beta^j \gamma^{w_m(j)} = 0 \rightarrow C_1 = (1-\alpha) \sum_{i=0}^{T-t} \beta^i \gamma^{w_m(i)}$, and hence:

$$\mu^*(s) = (1-\alpha) \sum_{j=s}^{T-t} \beta^j \gamma^{w_m(j)}; \quad s = 1, \dots, T-t+1.$$

Now we can substitute this result in the second *foc*, and we would get:

$$\begin{aligned} \delta(s+1) &= \frac{\delta(s)}{\rho} - \frac{1}{\rho} \left[\beta^s \rho \frac{(1-\alpha)}{\alpha} + f_{\bar{x}} \left(\frac{\partial \bar{x}(\cdot, s)}{\partial w_m(s)} \right) (1-\alpha) \sum_{j=s+1}^{T-t} \beta^j \gamma^{w_m(j)} \right] \\ &\Downarrow \\ \delta(s+1) &= \frac{\delta(s)}{\rho} - b_s; \quad s = 0, \dots, T-t \end{aligned}$$

This first order condition can be solved for the optimal value of the costate variable $\delta(s)$. This equation of differences has the following solution:

$$\delta(s) = \left(\frac{1}{\rho} \right)^s \delta_0 - \sum_{j=0}^{s-1} \left(\frac{1}{\rho} \right)^{s-j-1} b_j; \quad s = 1, \dots, T-t+1.$$

where δ_0 is a certain constant. As the value function ends at $T-t$, an increase in $w_m(T-t)$ cannot affect the payoff, so that the transversality condition has a free end point, that is $\delta(T-t+1) = 0$. Thus, $\delta(T-t+1) = \left(\frac{1}{\rho} \right)^{T-t+1} \delta_0 - \sum_{j=0}^{T-t} \left(\frac{1}{\rho} \right)^{T-t-j} b_j = 0 \rightarrow \delta_0 = (\rho)^{T-t+1} \sum_{j=0}^{T-t} \left(\frac{1}{\rho} \right)^{T-t-j} b_j$, and hence:

$$\begin{aligned} \delta^*(s) &= \sum_{j=s}^{T-t} \rho^{j-(s-1)} b_j \\ &= \sum_{j=s}^{T-t} \rho^{j-s-1} \left(\frac{1}{\rho} \right) \left[\beta^s \rho \frac{(1-\alpha)}{\alpha} + f_{\bar{x}} \left(\frac{\partial \bar{x}(\cdot, s)}{\partial w_m(s)} \right) (1-\alpha) \sum_{j=s+1}^{T-t} \beta^j \gamma^{w_m(j)} \right] \\ &= \frac{(1-\alpha)}{\alpha} \sum_{j=s}^{T-t} \rho^{j-s-1} \beta^j + f_{\bar{x}} \left(\frac{\partial \bar{x}(\cdot, s)}{\partial w_m(s)} \right) (1-\alpha) \sum_{j=s}^{T-t} \rho^{j-s-1} \sum_{j=s+1}^{T-t} \beta^j \gamma^{w_m(j)} \end{aligned}$$

Substituting $\delta^*(s)$ into the first *foc* to eliminate the costate variable:

$$\begin{aligned} \frac{\beta^s \sigma_m}{(1-e_m(s))^{\sigma_m+1}} &= \beta^s (1-\alpha) \sum_{i=1}^n k_i(w_m(s)) x_i + \alpha \sum_{i=1}^n k_i(w_m(s)) x_i \delta(s+1) \\ &\quad + \mu(s+1) f_{\bar{x}} \left(\sum_{i=1}^n k_i(w_m(s)) x_i \right) \\ \frac{\sigma_m}{(1-e_m(s))^{\sigma_m+1}} &= (1-\alpha) \left[\sum_{i=1}^n k_i(w_m(s)) x_i (1 + \Delta_s + \Psi_s) + \Gamma_s \right] \\ e_m^*(s) &= 1 - \left[\frac{(1-\alpha)}{\sigma_m} \left(\sum_{i=1}^n k_i(w_m(s)) x_i [1 + \Delta_s + \Psi_s] + \Gamma_s \right) \right]^{\frac{-1}{(\sigma_m+1)}} \end{aligned}$$

Note that the previous solution is defined $\forall s = 0, \dots, T - t$. Therefore we need to scale it adding t periods and fixing $s = 0$, that is:

$$e_m^*(t) = 1 - \left[\frac{(1 - \alpha)}{\sigma_m} \left(\sum_{i=1}^n k_i(w_m(t)) x_i [1 + \Delta_t + \Psi_t] + \Gamma_t \right) \right]^{\frac{-1}{(\sigma_m+1)}}; t = 0, \dots, T.$$

where:

$$\begin{aligned} \Delta_t &= \sum_{j=t+1}^T (\rho\beta)^{j-t} \\ \Psi_t &= \frac{1}{\alpha} f_{\bar{x}} \left(\frac{\partial \bar{x}(\cdot, t+1)}{\partial w_m(t+1)} \right) \sum_{j=t+1}^T \rho^{j-t} \sum_{j=t+2}^T \beta^{j-t} \gamma^{w_m(j)} \\ \Gamma_t &= f_{\bar{x}} \left(\sum_{i=1}^n k_i(w_m(t)) x_i \right) \sum_{j=t+1}^T \beta^{j-t} \gamma^{w_m(j)} \end{aligned}$$

This expression would give us the individual's optimal effort decision at any period $t \in [0, T]$. If we consider a natural model in which individuals have always incentives to work, we have to assume that the disutility of effort is not arbitrarily large, more precisely, it must be the case that $(1 - \alpha) \left(\sum_{i=1}^n k_i(w_m(t)) x_i [1 + \Delta_t + \Psi_t] + \Gamma_t \right) \geq \sigma_m$, that is, at least up to some extent it must be profitable to make a non-negative level of effort. ■

A.4 Proposition 3

Proof. By definition, there is EOp if and only if $\forall m, m' \in \mathcal{M} : g_m(t) = g_{m'}(t)$ it holds that $\bar{x}_m((e_m(t))^*) = \bar{x}_{m'}((e_{m'}(t))^*)$. If we substitute the planner's beliefs about the level of effort that individuals are currently making in the previous equation, we obtain expression 11. ■

B Appendix: Proof of corollary 2.1

For any type $w \in W$ let us consider a model with just two feasible levels of income $x \in X = \{A, B\}$, where $A > B$, and two different disutility of effort $J = \{\sigma_h, \sigma_l\}$, with $\sigma_h < \sigma_l$.¹¹ Therefore, we can write the probability of obtaining the high income

¹¹For instance, this can be understood as if we split all feasible incomes in two, the high ones and the low ones. The same can be done for the responsibility feature.

conditional to the disutility of effort as: $p_t^w(A | \sigma_h) = \alpha > \beta = p_t^w(A | \sigma_l)$. Using Bayes' rule it is straightforward to check that the planner's ratio of beliefs about any individual's responsibility feature after T periods turns out to be:

$$\frac{p_T^w(\sigma_h)}{p_T^w(\sigma_l)} = \left(\frac{1-\alpha}{1-\beta}\right)^{\#B} \left(\frac{\alpha}{\beta}\right)^{\#A} \left(\frac{p_0^w(\sigma_h)}{p_0^w(\sigma_l)}\right) \quad (13)$$

where $\#A$ and $\#B$ are the number of times that income A and B comes up respectively after T periods, and $\{p_0^w(\sigma_h); p_0^w(\sigma_l)\}$ are the planner's priors. The higher the value of the ratio, the higher the planner's beliefs that such an agent is "hard-working". The right hand side of equation 13 includes the ratio of planner's priors times a product of probability ratios. We know that the following must be true: $\left(\frac{1-\alpha}{1-\beta}\right) < 1$ and $\left(\frac{\alpha}{\beta}\right) > 1$, and hence the total value of the ratio is between 0 and $+\infty$. On the one hand, we have that the higher the number of times that personal income A appears, the higher the value of the ratio. On the other hand, the lower the number of times that such an income comes up, the lower the value of the ratio of beliefs. If the number of periods is sufficiently large we have that because of the monotone likelihood ratio condition an individual that is "lazy" will obtain a higher fraction of low income, whereas a "hard working" worker will get a higher fraction of high incomes. Therefore, equation 13 goes to 0 for "lazy" agents, whereas for hard-working ones goes to $+\infty$.

Now we can extend the previous result to a more general case with J different disutility of effort and n feasible incomes. For any type $w \in W$ we can split the population in two according to their level of responsibility, with $J_l = \{\sigma_1, \dots, \sigma_q\}$ being the "lazy" workers and $J_h = \{\sigma_{q+1}, \dots, \sigma_J\}$ the "hard-working" ones. Likewise, we can distinguish between high levels of income $x_h = \{x_{p+1}, \dots, x_n\}$ and low ones $x_l = \{x_1, \dots, x_p\}$. Then, applying the previous result we know that after T periods the planner can perfectly class individuals according to the previous partitions. Once individuals are classed as either hard-working or lazy workers the planner can make a similar analysis within each subgroup. For instance, if we focus on the h group we can also split these individuals between high hard-working individuals $J_{hh} = \{\sigma_{q_1+1}, \dots, \sigma_J\}$ and low hard-working ones $J_{lh} = \{\sigma_1, \dots, \sigma_{q_1}\}$. Moreover, we can make a new partition of the monetary outcomes between high and low incomes, $x_{h1} = \{x_{p_1+1}, \dots, x_n\}$ and $x_{l1} = \{x_1, \dots, x_{p_1}\}$ respectively. In

such a case, due to the monotone likelihood ratio condition, it is easy to check that $p(x_{h1} | hh) > p(x_{h1} | lh)$, so that taking up again the results up to period T we can class those individuals according to groups hh and lh . The same can be done for the subset of people belonging to the lazy group. Next, we can repeat the same analysis for the four different subgroups. If we focus on the hh subgroup, we can differentiate people in the following way: $J_{hhh} = \{\sigma_{q_2+1}, \dots, \sigma_J\}$ and $J_{lhh} = \{\sigma_1, \dots, \sigma_{q_2}\}$ and a new classification of incomes $x_{h2} = \{x_{p_2+1}, \dots, x_n\}$ and $x_{l2} = \{x_1, \dots, x_{p_2}\}$. Again, it can be shown that $p(x_{h2} | hhh) > p(x_{h2} | lhh)$, and therefore using again the previous results the planner can use the realizations of incomes up to period T so as to propose a thinner classification for the individuals. This could be done as many times as the number of effort groups in which we would wish to partition the population.

We would like to stress an usual difficulty that arises at the time of defining the number of cells.¹² The fewer the number of cells in which the population is partitioned, the easier to identify the cell to which individual belong. But on the other hand, the higher the number of cells, the finer the welfare evaluation of opportunity. Therefore, there seems to be a trade-off in the number of cells in which the population is partitioned.

¹²See Peragine (2002).

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Ivie

Guardia Civil, 22 - Esc. 2, 1º
46020 Valencia - Spain
Phone: +34 963 190 050
Fax: +34 963 190 055

**Department of Economics
University of Alicante**

Campus San Vicente del Raspeig
03071 Alicante - Spain
Phone: +34 965 903 563
Fax: +34 965 903 898

Website: <http://www.ivie.es>
E-mail: publicaciones@ivie.es