A discusión

FRAMING EFFECTS IN PUBLIC GOODS: PROSPECT THEORY AND EXPERIMENTAL EVIDENCE*

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ABSTRACT

This paper studies, both theoretically and experimentally, frame effects in the context of a public good game in which players have to make a costly contribution either *i)* to achieve or *ii)* not to lose a non excludable monetary prize. Our protocol leads to public good provision (not deterioration) only if a certain contribution level is achieved. Since both frames differ with respect to the reference point, we use Prospect Theory to derive testable predictions. In particular, Prospect Theory predicts more contribution in the second frame. Our evidence suggests that *a)* subjects' behavior is highly sensitive to frames and *b)* the theoretical prediction is confirmed except when the threshold is low. We also estimate the parameters which better suit our experimental evidence, partly confirming previous results in the literature.

1 Introduction

Free-riding is a pervasive problem in situations where societies have to decide the level of provision of some public good. This is so because public goods have the feature of being non-excludable. In particular, when we talk about pure public goods, it is assumed that excludability is only feasible at an infinite cost. Then governments cannot use rationing by price which implies that a competitive market cannot generate a Pareto efficient level of the public good. This is the reason of the so-called *free rider problem.* Since any individual perceives that she will benefit from the public good irrespective of her contribution to finance it, she will have no incentives to contribute voluntarily. If the public good is to be financed by voluntary contributions, its level will fall short its efficient level. This conclusion is somehow mitigated by the extensive, and extremely robust across a wide variety of treatment conditions, experimental evidence on the classic Voluntary Contribution Mechanism protocol. Here we find that subjects initially set a contribution which is halfway between the Pareto-efficient level and the free-riding level. If the same protocol is repeated for a finite number of times, average contribution declines over time, but stays always above the Nash equilibrium level. More efficient results are usually obtained in experiments in which the mechanism is modified by introducing a threshold in the total contribution, below which the public good is not provided (the lower this threshold, the higher the incentives to free ride).[1] These experimental protocols, usually termed as Voluntary Contribution Threshold Games, have, usually, multiple equilibria. Precisely, all strategy profiles where exactly the threshold is reached are equilibria of the underlying game.

Consider now a slightly different frame, in which there is a set of individuals *who are already enjoying some public good.* However, they realize that at some point in the future the existing public good can deteriorate, or even disappear. To prevent this possibility they need, somehow, to cooperate. We shall refer to this frame as *prevention of public good deterioration* (PPGD), as opposed to the most classical case of *public good provision* (PGP). The crucial difference between the two cases of PGP and PPGD is just whether individuals have initially the public good or not.

Focusing on threshold games, the aim of the paper is to answer, both theoretically and experimentally, to this very simple question:

Do people contribute more in PPGD, rather than in PGP?

Different cognitive biases could induce individuals to contribute more in one setting, rather than in the other. Under the *endowment effect*, individuals

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[1]See, for example, Ledyard (1995).

value more a good that they own rather than the same good when they do not own it [see Thaler (1980)]. If this effect would arise with public goods, it would imply that individuals value the public good more in PPGD than in PGP. Under the *omission bias*, individuals have the tendency to judge harmful actions as worse than equally harmful omissions [see Baron (1988)]. In our set-up, this could imply again more contribution in PPGD, since not contributing can be seen as an harmful action (as it can lead to the destruction of the public good), while in PGP not contributing can be seen as an harmful omission. However, and due to strategic considerations, the predictions of these effects are not clear. For example, it could be the case that individuals contribute less in PPGD if they believe that their mates are prone to suffer from either one of those biases and, thus, are going to contribute more. It seems that we need a careful theoretical analysis in order to draw predictions.

The aim of this paper is to analyze, theoretically and experimentally, a Voluntary Contribution Threshold Game under both frames. As it turns out, under VNM (Von Neumann-Morgenstern) preferences, both frames yield the same equilibrium prediction, which only depends on the contribution threshold, as the (symmetric) Bayesian Nash equilibrium takes the form of a cutoff rule, such that an individual will contribute if and only if her individual cost of contributing is below some threshold value c^* [see Palfrey and Rosenthal (1991)]. However, given that both frames differ in terms of the initial position, it seems natural to use Prospect Theory to derive testable predictions, because this approach takes explicitly into account that individuals' preferences depend on the *reference point* they use to evaluate costs and benefits of different alternatives. In this respect, our paper can be seen as a crossing between Prospect Theory and Bayesian Nash equilibrium in Public Goods provision. To the best of our knowledge, this is the first paper which applies Prospect Theory to strategic uncertainty.

One key element of Prospect Theory is *loss aversion*, that is, the behavioral assumption that postulates that individuals, from their reference viewpoint, value losses more than gains. Again, *this should imply more contribution in case of PPGD.* The starting point of this paper is exactly to check this preliminary conjecture by carefully evaluating the "Prospect Equilibria" of our model, for the widest range of relevant parameters. In this respect, our detailed analysis discloses a complex set of conditions, both on the relevant parameters of Prospect Theory and the thresholds, for which more contribution is expected under one frame, rather than the other. This consideration notwithstanding, for the parameter range usually considered by this literature, *our original conjecture is validated, predicting more contribution in the case of PPGD,* for all thresholds. This is the theoretical conjecture we bring

into the lab, for its empirical validation.

Our evidence partially confirms our theoretical conjecture, as we find that PPGD yields higher contribution when the threshold is sufficiently high. By contrast, when the threshold is set to its minimum, the opposite holds. In this respect, our results contrast very much with previous experiments on classic voluntary contribution mechanisms that find more contribution when the problem is framed as a positive externality (like in our PGP treatment), rather than when it is framed as a negative externality (PPGD).[2] Finally, we also estimate (by maximum likelihood) the basic parameters of Prospect Theory which better adjust to our experimental evidence, getting point estimates which confirm previous studies on similar experimental frameworks.

The remainder of the paper is arranged as follows. In Section 2, we set up the public good problem under the two frames: PGP and PPGD. We show that, under VNM preferences, the cutoff cost value c^* which identifies symmetric Bayesian Nash equilibrium is constant across players and frames. Section 3 looks at our theoretical framework from the point of view of Prospect Theory. We see that different frames yield different equilibria, which we characterize in a sequence of propositions as functions of all reference points and contribution thresholds. In Section 4 we calibrate the model using point estimates borrowed from related articles to provide theoretical predictions for our experiment, whose basic design is described in Section 5. In Section 6 we present our experimental results and we study which factors affect the decision to contribute. We also estimate the parameters of Prospect Theory. Finally, Section 7 concludes.

2 The basic model

There is a group of N individuals. An individual is indexed by $i \in \{1, .., N\}$. Each individual has one unit of input that she can either consume privately or contribute. The public good is provided if and only if at least k individuals contribute, where $1 \leq k \leq N$. The input of individual i has a privately known value c_i that can be interpreted as the cost of contributing. We assume that c_i is uniformly distributed within the interval $[0, 1]$ (to simplify notation we will skip the index whenever possible and we will write only c). We further assume that all individuals value equally the public good and we call this common value $g \leq 1$.

Table 1 describes player i's monetary payoffs when the number of individuals other than i that are contributing is n and the contribution threshold is k. We denote by C (NC) the action of (non-) contributing,

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[2]See Andreoni (1995), Sonnemans *et al.* (1998) and Dufwenberg *et al.* (2006).

States of the world	$n > k - 1$	$n = k - 1$	$n < k - 1$
Probabilities	p	q	r
C	g	g	0
NC	$g + c$	c	c

$$(1)$$

Table 1: Voluntary Contribution Threshold Game

where $p = \Pr(n > k - 1)$, $q = \Pr(n = k - 1)$, and $r = \Pr(n < k - 1)$. State probabilities are determined in equilibrium. A symmetric Bayesian Nash equilibrium (BNE) has the form of a cutoff rule: Individual i contributes if and only if her cost c is below some threshold value c^*, common to all individuals. To solve for c^*, we note that it is that value that makes an individual to be indifferent between C and NC. Then, it must satisfy:

$$g(p + q) = p(g + c^*) + qc^* + rc^*. \tag{2}$$

From (2) we get:

$$c^* = qg = \Pr(n = k - 1)g. \tag{3}$$

In a BNE of the game of Table 1, a given player contributes whenever $c < c^*$, and does not contribute whenever $c > c^*$. Then, c^* is defined implicitly by the following condition:

$$c^* = \binom{N - 1}{k - 1} \left\{ (c^*)^{k-1} (1 - c^*)^{N-k} \right\} g. \tag{4}$$

By analogy with our experimental conditions, consider that $N = 3$ and $g = 10/11$. If $k = 1$ the unique BNE is $c^* = 0.32$. If $k = 2$ there are two equilibria, one with $c^* = 0$ and another one with $c^* = 0.45$. Finally, if $k = 3$ the only equilibrium is $c^* = 0$. Notice that the above BNE are valid not only for the case of PGP, but also for the case of PPGD.[3]

3   Prospect theory

A substantial body of evidence shows the failure of expected utility theory to predict actual behavior in simple choice problems under uncertainty. Starmer (2000) reviews this evidence as well as many of the theories that have been proposed to account for it.

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[3] The monetary payoffs in the case of PPGD can be obtained by subtracting g from every cell of Table 1.

Among these theories, the best known is Prospect Theory proposed by Kahneman and Tversky (1979) and Tversky and Kahneman (1992).[4] Prospect Theory distinguishes two phases in the decision process: *editing* and *evaluation*. In the editing phase a number of operations may be applied to organize and reformulate the options in order to facilitate subsequent evaluation and choice. In particular, the various options are formulated as distributions of *gains* and *losses* with respect to some *reference point*. In the evaluation phase the decision maker evaluates each option and chooses the one with the highest value.

The overall value of an edited prospect, denoted V, is expressed in terms of two functions: a probability weighting function w and a subjective value function v applied to gains and losses.

Let $(x, \pi; y, 1-\pi)$ denote a prospect that gives a π chance at x and a $1-\pi$ chance at y, where x and y are gains or losses with respect to some reference point taken by the decision maker (usually her current asset position). If the prospect involves only gains or only losses, $x > y > 0$ or $x < y < 0$, then it can be represented as involving a sure gain (loss) y and an additional gain (loss) with probability π, and its value is

$$V(x, \pi; y, 1 - \pi) = v(y) + w(\pi)\left[v(x) - v(y)\right]. \qquad (5)$$

If $xy \leq 0$, the prospect is evaluated simply as

$$V(x, \pi; y, 1 - \pi) = w(\pi)v(x) + w(1 - \pi)v(y). \qquad (6)$$

Note that both expressions are identical if $w(\pi) + w(1 - \pi) = 1$, which is not assumed in Prospect Theory.

An essential feature of Prospect Theory is that the carriers of value are gains and losses rather than final states. Capturing *loss aversion*—"losses loom larger than corresponding gains"—the value function v is assumed to be steeper in losses than in gains, $v'(-x) > v'(x) > 0$, for $x > 0$. Reflecting the principle of *diminishing sensitivity* also observed in psychology —"the impact of a change diminishes with the distance from the reference point"— it is assumed that the value function v is concave in gains and convex in losses, $v''(x) \leq 0 \leq v''(-x)$, for $x > 0$. For $x > 0$ we define the coefficient of loss aversion as $\lambda(x) := -v(-x)/v(x)$. In many applications it is assumed a constant coefficient. Furthermore, existing empirical evidence suggests a value of around 2. No loss aversion corresponds to the case $\lambda = 1$.

The *weighting function* w is assumed to be increasing in probabilities, with $w(0) = 0$ and $w(1) = 1$. However, departing from Expected Utility

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[4]Camerer (2000) reviews the ability of Prospect Theory to explain empirical evidence.

Theory, the weighting function is generally nonlinear in probabilities. While the natural reference point for outcomes is the current asset position $x = 0$, for probabilities there are two natural reference points: certainty $\pi = 1$ and impossibility $\pi = 0$. The principle of diminishing sensitivity applied to the weighting of probabilities gives rise to a probability weighting function that is concave for small probabilities and then convex for larger ones, resulting in an (inverted) *S-shaped* function. Kahneman and Tversky (1979) provide evidence that for small probabilities π the weighting function w is *subadditive*, $w(r\pi) > rw(\pi)$ for $0 < r < 1$ and *overweights* probabilities $w(\pi) > \pi$. However, there is evidence that suggests that for all $0 < \pi < 1$, $w(\pi) + w(1 - \pi) < 1$, a property called *subcertainty* by Kahneman and Tversky (1979).

In addition, the evidence also suggests that the weighting function is *regressive*, intersecting the diagonal from above, *asymmetric*, with fixed point at about $1/3$, and *reflective*, assigning equal weights to an equal probability of a gain or a loss (Prelec, 1998).

3.1 Frames

If players evaluate risky prospects in terms of gains and losses with respect to a reference point, the game of provision of the public good can be framed into four natural forms. The successful provision of the public good can be seen as the realization of a gain (PGP) or, instead, as the elimination of a loss (PPGD). Because of loss aversion, the value of the provision of the public good will be larger if it is put in the latter framing. Similarly, the cost of contributing can be put as a loss or, instead, as a lack of realizing a gain. Because of loss aversion, the former will tend to be more discouraging of contributing. These two dimensions can be combined into four configurations that can be seen as the game perceived from four natural reference points: $x_0 = 0$, $x_0 = c$, $x_0 = g$, and $x_0 = g + c$, where g is the common value of the public good and c is the individual cost of contributing. As it will be seen below, the effect of the change in the reference point will not be due only to loss aversion but also to the nonlinearity of the probability weighting function as well as to the curvature of the value function in gains and losses.

In the sequel we focus only on two reference points, namely $x_0 = c$ and $x_0 = g$. The reason is that with reference point $x_0 = 0$ (respectively, $x_0 = g + c$), the individual faces a choice between two prospects that involve only gains (respectively, losses) and, thus, loss aversion plays no role.[5]

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[5]The interested reader can find an analysis with all four reference points in a previous version of the paper published as University of Ferrara Working paper 15/2008. It is available at:
http://www.unife.it/dipartimento/economia/pubblicazioni/collana-quaderni-

Reference point $x_0 = c$ **(Game G_c).** In G_c the provision of the public good is seen as a gain. However, contributing to the public good involves a loss. This is the standard way of presenting a problem of public good provision. In Table 2, the payoff matrix of Table 1 is modified by subtracting c from every cell.

	$n > k-1$	$n = k-1$	$n < k-1$
Probabilities	p	q	r
C	$g-c$	$g-c$	$-c$
NC	g	0	0

(7)

Table 2: Game G_c

Player i has to choose between two prospects:

$$C = (g-c, p+q; -c, r) \text{ or } NC = (g, p). \tag{8}$$

Notice that the payoff $g - c$, corresponding to contributing when sufficient number of others also contribute, may be a gain or a loss, since it may be the case that $c > g$. In this latter case, C is strictly dominated.

Reference point $x_0 = g$ **(Game G_g).** If $x_0 = g$, the provision of the public good is not seen as a gain, but instead, as eluding a loss. On the other hand, the cost of contributing is not seen as a loss but, instead, it is seen as not realizing a gain. As a consequence, payoffs in this situation are obtained by subtracting from Table 1 the value of the public good g, as Table 3 shows.

	$n > k-1$	$n = k-1$	$n < k-1$
	p	q	r
C	0	0	$-g$
NC	c	$-(g-c)$	$-(g-c)$

(9)

Table 3: Game G_g

Here player i has to choose between two prospects,

$$C = (-g, r) \text{ or } NC = (c, p; -(g-c), q+r), \tag{10}$$

where C involves a risky loss, while NC may result in a gain or a loss, in the non-trivial case where $g > c$.

We now turn to analyzing how equilibria are affected by framing. In the next section, we study public goods whose provision requires only a single individual contribution, $k = 1$. In subsequent sections we turn to consider first the case $k = N$ and then the case $1 < k < N$.

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3.2 One contribution is enough (Γ_1)

In this section we consider the polar case where the public good is provided whenever there is at least one contribution, that is, $k = 1$. As we shall see, this is a simple case where it is clear both the relevance of the reference point and the contrast with the predicted behavior under Expected Utility.

We focus on symmetric pure strategy equilibria with $g = 1$, and $c \in [0, 1]$. Let s be a cutoff strategy profile where each individual i contributes if her cost $c_i < c = c^*$ and she does not contribute if her cost $c_i > c$. Let $q(c)$ be the probability that no individual other than i contributes, that is,[6]

$$q(c) = (1 - c)^{N-1}. \tag{11}$$

Note that q is decreasing in c, going from 1 at $c = 0$ to 0 at $c = 1$.

We analyze equilibria for the reference points $x_0 = c$ and $x_0 = 1$.

(a) **Reference Point** $x_0 = c$: The equilibrium condition is:

$$v(1 - c) = w(1 - q(c))v(1),$$

or:

$$\frac{v(1 - c)}{v(1)} = w(1 - q(c)). \tag{12}$$

Since the left-hand side is decreasing from $\frac{v(1)}{v(1)} = 1$ to $\frac{v(0)}{v(1)} = 0$, and the right-hand side is increasing from 0 to 1, there is a unique symmetric equilibrium $c_c^* \in (0, 1)$.

(b) **Reference Point** $x_0 = 1$: The equilibrium condition is:

$$0 = w(1 - q(c))v(c) + w(q(c))v(c - 1). \tag{13}$$

Since the right-hand side is an increasing function of c that goes from $v(-1) < 0$ to $v(1) > 0$, it follows that there is a unique symmetric equilibrium $c_1^* \in (0, 1)$.

We summarize previous results in the following:

Proposition 1 *Suppose the public good is provided as long as at least one individual contributes, i.e. $k = 1$. Under Prospect Theory, for both reference points $x_0 = c$ and $x_0 = 1$, there exists a unique symmetric equilibrium. These equilibria are interior to $(0, 1)$ and are the unique solution to Equations (12) and (13), respectively.*

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[6]Note that when $k = 1$, the third column in Tables 1-3 plays no role.

3.2.1 Ranking the probability of contribution by reference point

In this section, we shall rank the probabilities of contribution under both reference points (all proofs can be found in the Appendix). Note first that, if we take $g = 1$, all prospects at choice with reference point $x_0 = c$ are nonnegative and, thus, loss aversion plays no role. Also note that if the value of the public good is relatively small, as it is likely to be in an experimental setting, the value function can be taken as linear in gains and linear in losses.

Proposition 2 *Suppose $k = 1$. Under Prospect Theory, at the symmetric equilibrium, there is more contribution with reference point $x_0 = 1$ than with reference point $x_0 = c$ if and only if loss aversion is high enough. That is:*

$$c_1^* > c_c^* \text{ if and only if } \lambda(1 - c_c^*) > \overline{\lambda}_c, \tag{14}$$

where the threshold $\overline{\lambda}_c$ is defined by (37).

  When the value function v is linear in gains, the threshold $\overline{\lambda}_c$ can be written as:

$$\overline{\lambda}_c = \frac{c_c^*}{w(q)}, \tag{15}$$

where $q = q(c_c^*)$.

  If in addition, the probability weighting function is linear, then $\overline{\lambda}_c = 1$, so that $c_1^* > c_c^*$ as long as there is loss aversion, i.e., $\lambda > 1$.

3.2.2 Comparison with expected utility

Let c_{eu} be the symmetric equilibrium probability of individual contribution for linear VNM utility in the game where a single contribution is enough for the public good to be provided. Clearly, c_{eu} is the unique solution to the equilibrium condition:

$$c_{eu} = q(c_{eu}) = (1 - c_{eu})^{N-1}. \tag{16}$$

Since q is decreasing both in c and N, Condition (16) implies that c_{eu} is decreasing in N: the larger the group, the smaller the equilibrium probability of contribution for each individual. In particular, by analogy with our experimental setting where $N = 3$, we have $c_{eu} = .382$.

  Let c_f be the interior fixed point of the probability weighting function, i.e., $w(c_f) = c_f$, $0 < c_f < 1$. Prelec (1998) reports estimates of the fixed point c_f that range from .30 to .39. In the next proposition we compare c_c^* and c_{eu}^* for the case in which, as all empirical estimates suggest, $c_f < 1/2$.

Proposition 3 *Suppose $k = 1$. Under Prospect Theory, at the symmetric equilibrium, the probability of contribution with reference point $x_0 = c$ is greater than according to Expected Utility Theory if the fixed point of the weighting function c_f is less than $1/2$. That is:*

$$c_c^* > c_{eu} \ if \ c_f < 1/2. \tag{17}$$

This result also holds if the probability weighting function w is linear but the value function v is strictly concave in gains.

Now we compare c_1^* with c_{eu}.

Proposition 4 *Suppose $k = 1$. Under Prospect Theory, at the symmetric equilibrium the probability of contribution with reference point $x_0 = 1$ is greater than according to Expected Utility Theory if and only if loss aversion is high enough. That is:*

$$c_1^* > c_{eu} \ if \ and \ only \ if \ \lambda(1 - c_{eu}) > \overline{\lambda}_{eu}, \tag{18}$$

where the threshold $\overline{\lambda}_{eu}$ is defined as:

$$\overline{\lambda}_{eu} = \left(\frac{w(1 - c_{eu})}{v(1 - c_{eu})} \right) \left(\frac{v(c_{eu})}{w(c_{eu})} \right). \tag{19}$$

In particular, for $N = 2$, $\overline{\lambda}_{eu} = 1$. For $N > 2$, $\overline{\lambda}_{eu} < 1$, if v is linear in gains and $c_{eu} \leq c_f$.

3.3 All contributions required (Γ_N)

Now we turn to the other polar case, where the provision of the public good requires that all players contribute.

So far, we have considered that the value of the public good is $g = 1$, the same as the supremum of the support of the distribution of private costs. As it will be apparent later, it is convenient to allow g to take values below 1. When $g < 1$ the strategy of contributing regardless of the cost is clearly dominated. In this way, we get a sharper contrast between EUT and PT that can be tested experimentally. We focus our analysis in the limit of the equilibrium as g goes to 1.

As in the previous section, let $q(c)$ be the probability that a player is decisive (i.e., pivotal), given that all other players contribute if and only if their cost is less than c. Then, $q(c) = c^{N-1}$. Clearly, q is an increasing function in c going from 0 at $c = 0$ to 1 at $c = 1$, and decreasing in N.

We start with the equilibrium analysis in the reign of Expected Utility Theory with linear VNM utility function. According to Expected Utility Theory, the equilibrium c_{eu} is characterized by $g(c_{eu})^{N-1} = c_{eu}$. Then, if $g < 1$ the unique equilibrium is $c_{eu} = 0$. If $g = 1$ and $N = 2$, there is a continuum of equilibria $[0,1]$. If $g = 1$ and $N > 2$, there are only two equilibria, one at $c_{eu} = 0$ and one at $c_{eu} = 1$.

We now turn to analyzing the equilibrium under Prospect Theory for our two reference points.

(a) Reference point $x_0 = g$

The equilibrium condition at the reference point $x_0 = g$ is:[7]

$$w(1 - q)v(-g) = v(c - g). \tag{20}$$

Clearly, $c = 0$ is always an equilibrium and $c = 1$ is also an equilibrium if and only if $g = 1$. However, these equilibria are not stable in the sense that the best reaction to a small deviation results in a further deviation. Besides, the equilibrium $c = 1$ is not robust in the sense that it is not the limit of any equilibria as g goes to 1. That is, even though it is an equilibrium when $g = 1$, there is no equilibrium close to it when g is slightly smaller than 1. Proposition 5 builds upon these facts to establish a number of properties for the symmetric equilibria at reference point $x_0 = g$.

Proposition 5 *Suppose the public good is provided as long as all players contribute, i.e. $k = N$. Under Prospect Theory with linear value function in losses, the set of symmetric equilibria for reference point $x_0 = g$ includes $c = 0$, and $c = 1$ if and only if $g = 1$. In addition:*

(i) For $N = 2$, at $g = 1$ and in the limit as $g \to 1$, the only interior equilibrium is:

$$c_1^* = 1 - c_f. \tag{21}$$

(ii) For $N > 2$, at $g = 1$ and in the limit as $g \to 1$, there is no interior equilibrium greater than or equal to $(1 - c_f)$.

(iii) More generally, for $g = 1$ and in the limit as $g \to 1$, if $c_{1,m}$ is the maximum interior equilibrium for $N = m > 1$, then the maximum interior equilibrium for $N = m + 1$, if it exists, satisfies:

$$c_{1,m+1} < c_{1,m}. \tag{22}$$

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[7]When $k = N$ the first column in Tables 1-3 plays no role.

(iv) For $N > 2$ there can be a continuum of equilibria in $[0, 1 - c_f)$, no interior equilibrium, or any number of them in that interval.

(v) For $g = 1$ and in the limit as $g \to 1$, if for $N = m > 2$ there is no interior equilibrium, then there is no interior equilibrium for any $N > m$.

(b) Reference point $x_0 = c$

The equilibrium condition at the reference point $x_0 = c$ is:

$$w(q)v(g - c) + w(1 - q)v(-c) = 0. \tag{23}$$

Clearly, $c = 0$ is always an equilibrium and $c = 1$ is also an equilibrium if and only if $g = 1$.

Proposition 6 *Suppose $k = N$. Under Prospect Theory the set of symmetric equilibria for reference point $x_0 = c$ includes $c = 0$, and $c = 1$ if and only if $g = 1$. In addition:*

(i) With loss aversion but linear value functions in gains and losses, and linear weighting function, there is no additional equilibrium.

(ii) With loss aversion but linear value functions in gains and losses, there is no interior equilibrium greater than or equal to $(1 - c_f)$.

(iii) For $g = 1$ and in the limit as $g \to 1$, if $c_{c,m}$ is the maximum interior equilibrium for $N = m > 1$, then the maximum interior equilibrium for $N = m + 1$, if it exists, satisfies:

$$c_{c,m+1} < c_{c,m}. \tag{24}$$

It follows from Proposition 6 that for $N = 2$ and g close to 1, the most efficient equilibrium involves more contribution for reference point $x_0 = g$ than for reference point $x_0 = c$. In particular, $c_c^* < c_g^* \leq 1 - c_f$. When $N > 2$, and provided that $c_g^* \geq c_f$, we also have that $c_c^* < c_g^*$, that is, there is more contribution with $x_0 = g$ rather than with $x_0 = c$.

3.4 Intermediate contribution requirements: $1 < k < N$

Finally, we consider the case when the number of contributions required for the provision of the public good is greater than 1 and less than N.

Three probabilities are important to determine the symmetric equilibria. Suppose every player $j \neq i$ is following the strategy of contributing whenever her cost is less than a threshold c. As above, let $q(c)$ be the probability that i is decisive for the provision of the public good, i.e., $q(c)$ is the probability that exactly $k - 1$ players other than i have a cost less than c. Similarly, let $p(c)$ be the probability that the public good is provided regardless of what i does, i.e., $p(c)$ is the probability that at least k players other than i contribute. Finally, let $r(c)$ be the probability that the public good is not provided regardless of i's choice. We have:

$$q(c) = \binom{N-1}{k-1} c^{k-1}(1-c)^{N-k}, \tag{25}$$

$$p(c) = \sum_{j=k}^{N-1} \binom{N-1}{j} c^j (1-c)^{N-1-j}, \tag{26}$$

and:

$$r(c) = 1 - p(c) - q(c). \tag{27}$$

These facts will be useful in the following

Lemma 7 *(i) p and $p + q$ are increasing in c.*
(ii) r and $q + r$ are decreasing in c.
(iii) q is increasing in c for $c < \frac{k-1}{N-1}$, and decreasing thereafter. Moreover, $q(0) = q(1) = 0$.

Under Expected Utility Theory, the symmetric equilibrium condition is:

$$q(c_{eu}) = c_{eu}. \tag{28}$$

>From (25) it follows that $c = 0$ is an equilibrium and $c = 1$ is not an equilibrium. Any other equilibrium has to satisfy:

$$\binom{N-1}{k-1} c^{k-2}(1-c)^{N-k} - 1 = 0. \tag{29}$$

For $k = 2 < N$ the only equilibrium other than 0 is $c_{eu} = 1 - 1/(N-1)^{1/(N-2)}$. Since the left-hand side of (29), $L(c)$, is an increasing affine transformation of $q(c)$ it follows from Lemma 11 that $L(c)$ is increasing in c if $c < (k-2)/(N-2)$ and it is decreasing thereafter. It is then clear that for $2 < k < N$ there are at most two positive equilibria, one of them greater than $(k-2)/(N-2)$ and the other one smaller than $(k-2)/(N-2)$.

It can also be shown that for any $k > 2$, if N is large enough there are two positive equilibria, and that if N is small enough there is no positive equilibrium.

In particular, for $N = 4$ and $N = 5$ there is no positive equilibrium with $N > k > 2$. For $N = 6$ there are positive equilibria only if $k = 2$ and $k = 3$.

In the next proposition we compare the prediction of Expected Utility Theory with that of Prospect Theory for both reference points.

Proposition 8 *Suppose the public good is provided as long as k players contribute. Under Prospect Theory with loss aversion but linear value functions in gains and losses, and linear weighting function, the maximum equilibria satisfy:*

$$c_c^* < c_{eu}^* < c_1^*, \tag{30}$$

with weak inequality whenever the maximum is zero.

Note that the maximum equilibrium is the most efficient. Note also that if the weighting function is not linear, for sufficient loss aversion then the efficient equilibrium is lowest at reference point $x_0 = c$ and largest at reference point $x_0 = 1$.

It can also be seen that for $k = 2$ and $N > k$ but small, given a degree of loss aversion $\lambda \geq 1$, if the probability weighting function is sufficiently regressive, then the equilibrium under Expected Utility Theory is larger than any of the equilibria under Prospect Theory, whatever the reference point.

4   Calibration

Table 4 summarizes the theoretical findings of Section 3, as far as contribution cost thresholds across frames are concerned, in a more compact way, comparing equilibrium vales for the Bayesian Equilibria (which, as we know, are constant across frames) and for Prospect Theory. In line with our experiment, we focus on the case $N = 3$.

As Table 4 shows, in general we have:

$$c_c^* < c_1^*, \tag{31}$$

that is, the highest equilibrium contribution results at reference point $x_0 = g = 1$. Except in the case when $k = 3$, for the result to hold λ must be sufficiently high. As for the comparison with contribution levels predicted by

	Equilibrium threshold	Requirements
$k = 1$	$c_{eu}^* < c_c^* < c_g^*$	Loss aversion high enough
$k = 2$	$c_c^* < c_g^* < c_{eu}^*$	Loss aversion high enough
		Weighting function sufficiently regressive
$k = 3$	$0 = c_{eu}^* \leq c_c^* < c_g^*$	$c_g^* \geq c_f$

Table 4: Summary of theoretical results

	$\lambda = 1.25$		$\lambda = 1.5$		$\lambda = 2$		EUT
k	$x_0 = c$	$x_0 = g$	$x_0 = c$	$x_0 = g$	$x_0 = c$	$x_0 = g$	
1	.458	.419	.458	.444	.458	.484	.366
2	.204	.315	.176	.325	.132	.338	.450
3	0	0.269	0	.269	0	.269	0

Table 5: Framing Effects on the Equilibrium Probability of Contribution Public good value $g = .91$. Weighting function parameter $\delta = .56$

	$\delta = 0.56$		$\delta = 0.61$		$\delta = 0.71$		EUT
k	$x_0 = c$	$x_0 = g$	$x_0 = c$	$x_0 = g$	$x_0 = c$	$x_0 = g$	
1	.458	.444	.435	.439	.403	.429	.366
2	.176	.325	.190	.335	.217	.362	.450
3	0	.269	0	0	0	0	0

Table 6: Framing Effects on the Equilibrium Probability of Contribution Public good value $g = .91$. Loss aversion parameter $\lambda = 1.5$

		$k = 1$	$k = 2$	$k = 3$	Obs
Frequency of	T_c	406 (**35.68**)	452 (**39.48**)	351 (**30.76**)	3,424
contribution	T_g	319 (**27.69**)	528 (**45.83**)	664 (**57.64**)	3,456
Frequency of	T_c	846 (**74.34**)	399 (**34.85**)	45 (**3.94**)	3,424
provision	T_g	726 (**63.02**)	510 (**44.27**)	243 (**21.09**)	3,456
Observations		2,290	2,297	2,293	6,880

Table 8: Frequency of contribution and provision across treatments

the Bayesian equilibrium, c_{eu}, this is the highest when $k = 2$, and the lowest otherwise.

We now proceed in the analysis by reporting calibrations for the equilibria under Prospect Theory from the two reference points and under Expected Utility Theory. We assume that the value function (5-6) is linear both in gains and losses, with constant coefficient $\lambda(x) \equiv -v(-x)/v(x) = 2 > v'(x) = 1$, for $x > 0$. For the weighting function $w(\pi)$, we shall employ the specification proposed by Tversky and Kahneman (1992):

$$w(\pi) = \frac{\pi^\delta}{\left(\pi^\delta + (1 - \pi)^\delta\right)^{1/\delta}}. \tag{32}$$

Table 5 presents equilibrium probabilities of contribution with, by analogy with the experimental conditions, $N = 3$ and $g = .91$, and for all possible levels of contribution requirements, k, from 1 to 3. Using a maximum likelihood estimation procedure, Camerer and Ho (1994) get an estimate for δ of .56, which we use for our calibration exercise, using different values for λ, 1.25, 1.5 and 2, respectively. When there are multiple equilibria, the *efficient* interior equilibrium is reported. The number of interior equilibria is reported in brackets when it is greater than 1.

When $k = 1$, the numbers in Table 5 confirm our theoretical analysis: the equilibrium probabilities when $x_0 = g$ are greater than those when $x_0 = c$ if and only if the degree of loss aversion is high enough. It should be noticed that the reason why the equilibrium proportion corresponding to $x_0 = c$ is not affected by λ is because, since players experience only gains with respect to their reference point, loss aversion plays no role. When $k = 2$, we obtain that the probability of contribution is always higher when $x_0 = g$. Finally, when $k = 3$ we also obtain a higher proportion of contribution when $x_0 = g$. Again, we must remark that when $k = 3$ and $x_0 = g$, given our linear specification for $v(.)$, loss aversion plays no role. Comparing the predictions of Prospect Theory with those of Expected Utility we find more contribution with Prospect Theory when $k = 1$ and $k = 3$, and the opposite when $k = 2$. Finally, we obtain a clear prediction with Prospect Theory for the effect of increasing k. In all cases we find that the probability of contribution reduces with k.

In Table 6 we fix $\lambda = 1.5$, letting δ in the probability weighting function (32) take values which have been found by the relevant literature: $\delta = .56$ estimated by Camerer and Ho (1994), $\delta = .61$ estimated by Tversky and

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	Equilibrium threshold	Requirements
$k=1$	$c_{eu}^* < c_c^* < c_g^*$	Loss aversion high enough
$k=2$	$c_c^* < c_g^* < c_{eu}^*$	Loss aversion high enough
		Weighting function sufficiently regressive
$k=3$	$0 = c_{eu}^* \leq c_c^* < c_g^*$	$c_g^* \geq c_f$

Table 4: Summary of theoretical results

	$\lambda = 1.25$		$\lambda = 1.5$		$\lambda = 2$		EUT
k	$x_0 = c$	$x_0 = g$	$x_0 = c$	$x_0 = g$	$x_0 = c$	$x_0 = g$	
1	.458	.419	.458	.444	.458	.484	.366
2	.204	.315	.176	.325	.132	.338	.450
3	0	0.269	0	.269	0	.269	0

Table 5: Framing Effects on the Equilibrium Probability of Contribution
Public good value $g = .91$. Weighting function parameter $\delta = .56$

	$\delta = 0.56$		$\delta = 0.61$		$\delta = 0.71$		EUT
k	$x_0 = c$	$x_0 = g$	$x_0 = c$	$x_0 = g$	$x_0 = c$	$x_0 = g$	
1	.458	.444	.435	.439	.403	.429	.366
2	.176	.325	.190	.335	.217	.362	.450
3	0	.269	0	0	0	0	0

Table 6: Framing Effects on the Equilibrium Probability of Contribution
Public good value $g = .91$. Loss aversion parameter $\lambda = 1.5$

		$k=1$	$k=2$	$k=3$	Obs
Frequency of	T_c	406 (**35.68**)	452 (**39.48**)	351 (**30.76**)	3,424
contribution	T_g	319 (**27.69**)	528 (**45.83**)	664 (**57.64**)	3,456
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Observations		2,290	2,297	2,293	6,880

Table 8: Frequency of contribution and provision across treatments

Kahneman (1992), and $\delta = .71$ estimated by Wu and Gonzalez (1996), respectively.

From Table 6 we see that the case in which the prediction seems more robust to changes in the relevant parameters is when $k = 2$, where we always find more contribution in $x_0 = g$. As in Table 5, when $k = 1$, which frame yields more contribution appears to be highly sensitive to the level of δ : when δ is low, we find more contribution in $x_0 = c$, but the result reverses when δ is high. Finally, when $k = 3$, we see that for sufficiently high values of δ, contribution is 0 under either frame.

5   Experimental design

In what follows, we describe the features of the experiment in detail.

5.1   Subjects

The 6 experimental sessions were run at the Laboratory for Theoretical and Experimental Economics (LaTEx) of the Universidad de Alicante. A total of 144 students (24 per session) were recruited among the undergraduate student population of the Universidad de Alicante -mainly, undergraduate students from the Economics Department with no (or very little) prior exposure to game theory. In all sessions, subjects were divided into two *matching groups* of 12. Subjects from different matching groups never interact with each other throughout the session. The sessions lasted approximately 60' each.

5.2   Treatments

All the experimental sessions were run in a computer lab.[8] Instructions were provided by a self-paced, interactive computer program that introduced and described the experiment. Subjects were also provided with a written copy of the experimental instructions, identical to what they were reading on the screen. In each session, subjects played 24 rounds of two treatments, for a total of 48 rounds. As explained in Section 2, a treatment is uniquely

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[8]The experiment was programmed and conducted with the software *z-Tree* (Fischbacher, 2007).

defined by a reference point. We focus only in two cases: $x_0 = c$ and $x_0 = g$, the reason being that for the two remaining reference points $x_0 = 0$ and $x_0 = 1 + c$, all prospects are either nonnegative or non-positive, and loss aversion plays no role.

Let call T_c and T_g the contribution game in which the reference point is equal to c and g respectively and D_1 (D_2) be the design in which treatment T_c (T_g) is played first (see Table 7).

	D_1	D_2
	Sessions 1 to 3	Sessions 4 to 6
Rounds 1-24	T_c	T_g
Rounds 25-48	T_g	T_c

Table 7: Experimental sessions

In each session, the 24 subjects were divided into 2 *cohorts* of 12, with subjects from different cohorts never interacting with each other throughout the session. We shall therefore read our experimental data under the assumption that the history of each individual cohort (4 for each design, D_1 and D_2) corresponds to an independent observation of our experimental environment. Within each round $t = 1, ..., 48$, in each cohort, 4 groups of 3 subjects were randomly determined.

All monetary payoffs in the experiments were expressed in Spanish Pesetas (1 euro worth approximately 166 Pesetas).[9] The value of the prize g was fixed to 50 pesetas at all times. Consistently with our theoretical framework, the cost for contributing was, for all subjects and rounds, an independent draw $c \sim U[0, \bar{c}]$, with $\bar{c} = 55$ pesetas. Let *time interval* $\tau_i = \{3(i-1) < t \leq 3(i)\}$, $i = 1, ..., 8$, be the subsequence of the $i - th$ 3 rounds of each treatment. Within each time interval τ_i, subjects experienced each and every possible $k \in \{1, 2, 3\}$, with the order being randomly determined within each τ_i. We did so to keep under control the time distance between two rounds characterized by the same value of k. After being told the current level of k and c, each subject had to:

1. Choose whether to contribute or not for that round;

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[9]It is standard practice, for all experiments run in Alicante, to use Spanish Pesetas as experimental currency. The reason for this design choice is twofold. First, it mitigates integer problems, compared with other currencies (USD or Euros, for example). On the other hand, although Spanish Pesetas are no longer in use (substituted by the Euro in the year 2002), Spanish people still use Pesetas to express monetary values in their every-day life. In this respect, by using a "real" (as a opposed to an artificial) currency, we avoid the problem of framing the incentive structure of the experiment using a scale (e.g. "Experimental Currency") with no cognitive content.

2. Elicit their belief on the number of contributors in their group (excluding herself). Every correct guess would be paid 5 pesetas at the end of that round.[10]

After each round each agent was informed of the contribution decision of the other group members (i.e. the outcome for that round), together with her payoff (on both dimensions: belief and contribution game) and the average payoff of her group members (only as for the contribution decision was concerned). The same information was also given in the form of a *History table*, so that subjects could easily review the results of all the rounds that had been played so far.

At the beginning of each treatment, subjects received 1.000 pesetas (1 euro is approximately 166 pesetas) as initial endowment. A particular care was devoted in explaining the two different treatments (i.e. the two frames). As for T_c, subjects would gain $g = 50$ pesetas if the number of contributors in their group reached the target k (with c being subtracted from their initial endowment; in T_g subjects would loose g from their initial endowment if the numbers of contributors did not reach target, gaining c in case of non contribution. Subjects received, on average, 15 euros for a 45' session. At the end of the sessions, subjects were asked to answer a detailed questionnaire on their socio-demographic characteristics, together with standard questions to estimate their pro-social behavior.[11]

6 Experimental Results

In what follows, we shall report our experimental results in detail. In Section 6.1 we present some descriptive statistics; while in Section 6.2 we estimate some (panel) logit regressions which take more carefully into account the impact of all our experimental conditions on outcome and behavior distributions. In reading the experimental evidence, our first concern will be to test the theoretical conjectures of Section 2, which have been calibrated, by analogy with our experimental conditions, in Tables 5 and 6. Let p_x^k denoting the equilibrium probability under Prospect Theory when $N = 3$, the reference point is x and the contribution threshold is equal to k (with p_{eu}^k denoting the corresponding BNE probability). The results from our simulations provide us with the following testable hypotheses.

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[10]We borrow this design feature from Nyarko and Schotter (2002). See also Gachter and Renner (2006).

[11]The complete set of instructions and a copy of the questionnaire can be downloaded from http://merlin.fae.ua.es/iturbe/experiments.html.

1 $H_0 : p_c^1 = p_g^1$ ($H_1 : p_c^1 \neq p_g^1$). When $k = 1$, we know from Tables 5 and 6 that which frame dominates in term of overall contribution is highly sensitive to the level of the relevant parameters, so that, it is not clear how to formulate the alternative. Also notice that, in this case, $p_{eu}^1 = .366$.

2 $H_0 : p_c^2 = p_g^2$ ($H_1 : p_c^2 < p_g^2$). As we already noticed, $k = 2$ is the case in which prediction seems more robust to changes in the parameters, with higher contribution for $x_0 = g$. In this case, we also have $p_{eu}^2 = .450$.

3 $H_0 : p_c^3 = p_g^3$ ($H_1 : p_c^3 \neq p_g^3$). When $k = 3$, prediction seems highly sensitive to the choice of δ, predicting 0 contribution (like in the Expected Utility model) under either frame when the latter is high enough.

6.1 Descriptive Statistics

In the upper part of Table 8 we show the number of subjects who contribute across different treatments, while in the lower part we show the number of cases in which the public good is provided (or its deterioration is prevented). We also report percentages in bold face.

As we explained earlier, interpreting our results, we shall consider treatment T_c (T_g) as the standard case of PGP (PPGD). In this respect, we observe that, for T_c, the frequency of contribution is higher for the intermediate level of $k = 2$, than for the low and high levels. By contrast, in T_g the frequency of contribution rises with k. If we compare both treatments for a given value of k, we observe a higher frequency of contribution in T_c when $k = 1$, while the result reverses when $k = 2$ or $k = 3$.

When $k = 1$, we observe $p_c^1 = .36$ and $p_g^1 = .28$. Looking at the calibrations in Table 5 we see that Prospect Theory could predict this pattern if the coefficient of loss aversion is low. When $k = 2$, we observe $p_c^2 = .39$ and $p_g^2 = .46$, with both values significantly higher than the prediction of Prospect Theory. Nevertheless, Prospect Theory predicts correctly that contribution is higher in treatment T_g. We also notice that, when $k = 2$, the probability of contribution predicted by Expected Utility (.45) is remarkably close to actual behavior.

When $k = 3$, we get $p_c^3 = .31$ and $p_g^3 = .58$. This is the case in which the difference is largest. Also notice that the frequency of contribution is much

		$k = 1$	$k = 2$	$k = 3$	Obs
Frequency of	T_c	406 (**35.68**)	452 (**39.48**)	351 (**30.76**)	3,424
contribution	T_g	319 (**27.69**)	528 (**45.83**)	664 (**57.64**)	3,456
Frequency of	T_c	846 (**74.34**)	399 (**34.85**)	45 (**3.94**)	3,424
provision	T_g	726 (**63.02**)	510 (**44.27**)	243 (**21.09**)	3,456
Observations		2,290	2,297	2,293	6,880

Table 8: Frequency of contribution and provision across treatments

higher than what was predicted by both Prospect Theory and Expected Utility. This is particularly true in treatment T_c, where the prediction is no contribution at all.

As we already noticed, our results contrast very much with other previous experiments (such as Andreoni (1995) and Sonnemans et al. (1998)) that find more contribution when the problem is framed as a positive externality (T_c) than when it is framed as a negative externality (T_g). By contrast, we obtain such a result only when $k = 1$.

In the bottom part of Table 8 we report the frequency of public good provision (not deterioration). Here we see that these frequencies decline with k, with a much stronger effect in T_c, where public good provision is basically null when $k = 3$.

In Figure 1 we refine this evidence, by disaggregating contribution frequencies for treatment, contribution thresholds k and cost levels, c.

The two diagrams report, one for each treatment (T_c and T_g), the relative frequency of contribution for each possible threshold level k. We partition the cost levels into 11 subintervals of size 5 in the x axis, averaging out contribution frequencies for each subinterval. Not surprisingly, average frequency of contribution is decreasing in the cost level, and this effect is much more pronounced in T_c. On the other hand, while in T_g average contribution increases with k, the same does not happen for T_c. We also notice that, for any given k, contribution schedules display very similar patterns between T_c and T_g (except for the case $k = 3$, in which subjects contribute uniformly more for all cost intervals).

Does contribution follow any time trend? In Figure 2 we plot relative frequencies of contribution across the 8 time intervals (see section 5.2).

As Figure 2 shows, relative frequencies basically stay constant over time. We observe some "endgame effects" only when $k = 3$. In T_c, we observe a moderately decreasing time trend in contribution, while in treatment T_g the trend displays an inverted U-shape, as contribution frequency rises until the middle of the session, declining later on. The overall impression we get from Figures 1 and 2 is that contribution is highly sensitive to c in both treatments, while in T_g it is also sensitive to k, with only marginal changes over time for both cases.

We also check the evolution of group behavior in Figure 3), where we track the relative frequencies with which a) the public good was successfully
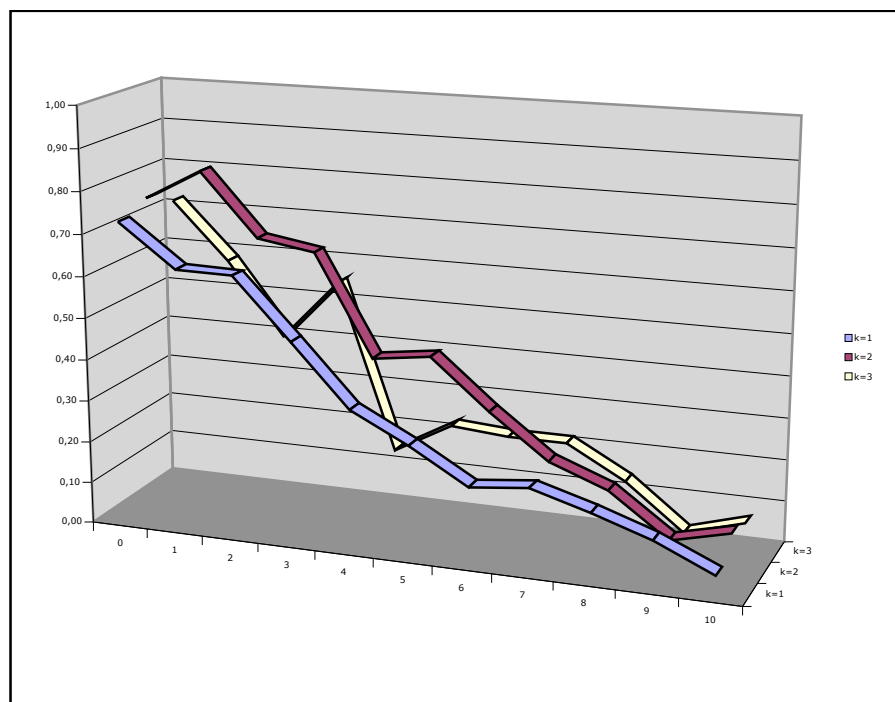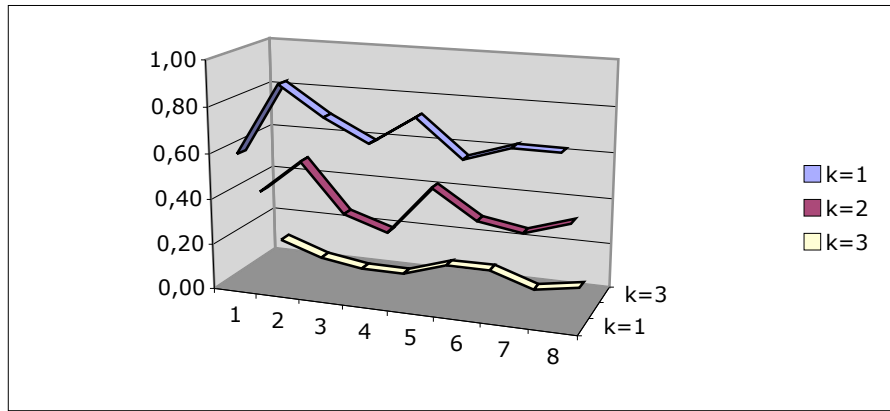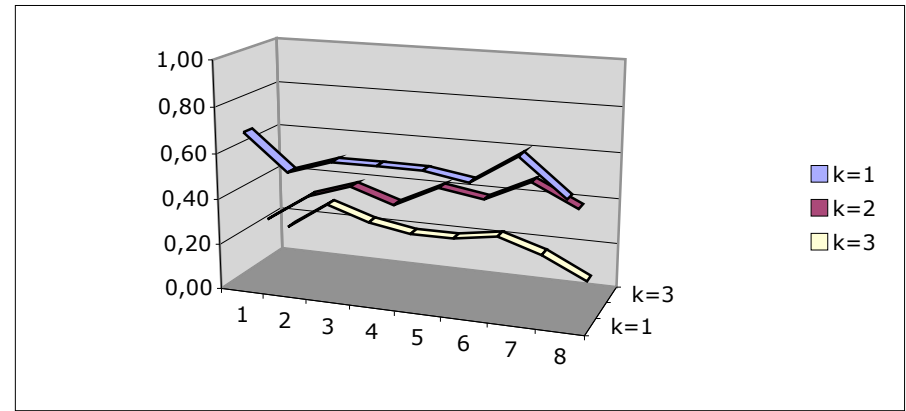
a)
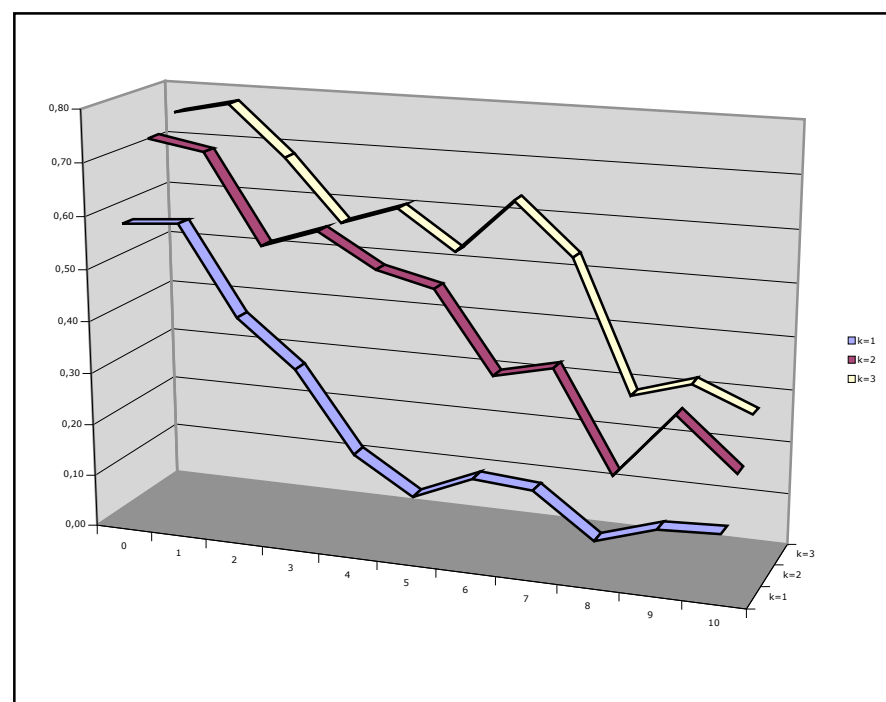
b)

Fig. 1
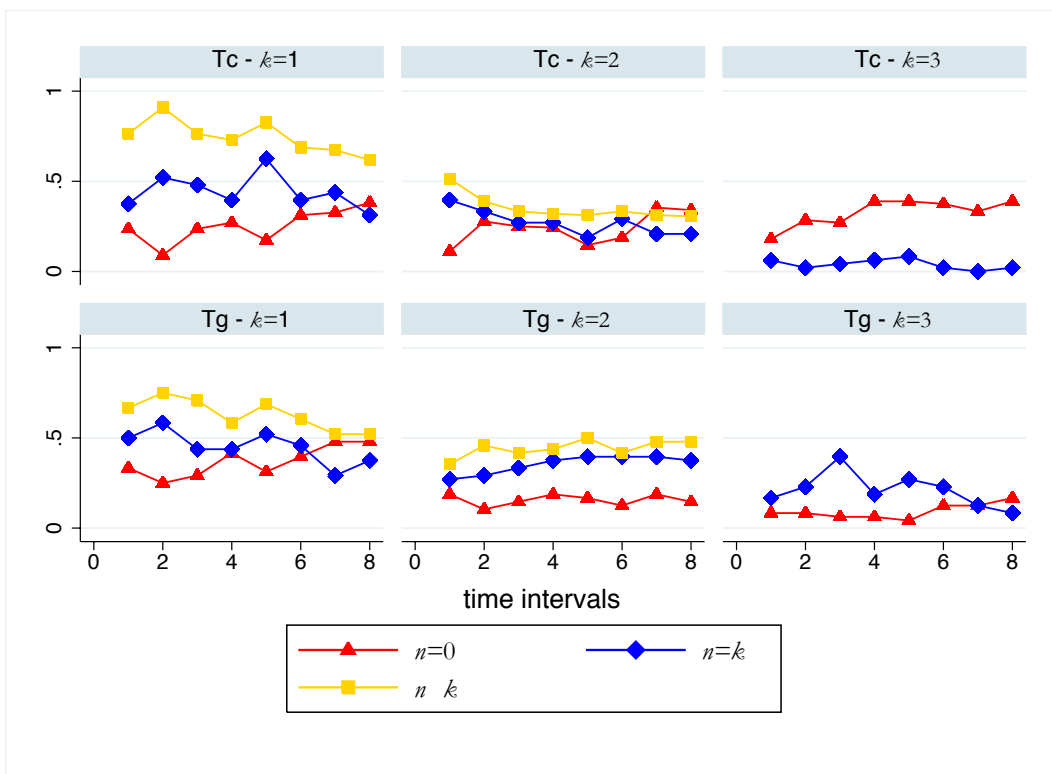
a)

b)

Fig. 2

Fig. 3

achieved, and b) it was achieved efficiently. Figure 3) also reports, conditional on the frame and contribution threshold, the frequency of plays in which nobody contributed.

First notice that, for $k = 1$, higher contribution in T_c is mainly due to "inefficient overprovision", since the frequency of outcomes where only one group member contributes is basically constant across frames (around 50% of total observations). As for $k > 1$, we also see that in T_c the number of no-contribution outcomes is substantially higher, even when as time proceeds, neither frame seems capable to sustain unanimous cooperation.

We now turn our attention to the extent to which contributing is *individually rational*, that is, whether it corresponds to a best-reply to the current strategic situation. We can look at this question from two complementary viewpoints: an *ex-ante* or an *ex-post* perspective, that is, consistency of contribution decision with the elicited belief of Stage 2, or consistency of contribution decision with the *actual* opponents' behavior, respectively.

As for the former interpretation, Table 9 looks at the extent to which elicited beliefs in Stage 2 depend on k.

Each row (column) in Table 9 corresponds to one particular k (point belief). We also report the value of mean beliefs. In both treatments, when $k = 1$ and $k = 2$ the modal belief is 1. However, when $k = 3$ the modal belief is 0 in T_c and it is 2 in T_g. We also find that the mean belief in treatment T_g is higher than in T_c for $k = 2$ and (especially) $k = 3$, and it is lower for $k = 1$. More precisely, when $k = 3$, there is a striking difference between the frequency of subjects forecasting that 2 group members are contributing (28.73% in T_c *vs.* 55.12% in T_g) rather than 0 (41.15% and 20.75%, respectively).

When the belief is equal to $k - 1$, we say that the subject believes that she is *pivotal*, or decisive. Overall, the frequency with which subjects feel that they are pivotal is much higher in T_g. This is in clear contrast with the VNM prediction, as the unique Bayesian Nash Equilibrium should imply beliefs concentrated at 0 in both treatments. In general, we find a positive and highly significant correlation between the probability of cooperating and the belief of being pivotal (.2987 and .4418 in T_c and T_g respectively, *p*-value of 0 in both cases). This result coincides with the one obtained in similar

	Beliefs in treatment T_c				Beliefs in treatment T_g			
	0	1	2	Mean	0	1	2	Mean
$k=1$	268	681	203	0.94	304	688	160	0.87
	23.26	**59.11**	**17.62**		**26.39**	**59.72**	**13.89**	
$k=2$	234	667	251	1.01	166	651	335	1.15
	20.31	**57.90**	**21.79**		**14.41**	**56.51**	**29.08**	
$k=3$	474	347	331	0.87	239	278	635	1.34
	41.15	**30.12**	**28.73**		**20.75**	**24.13**	**55.12**	
Total	976	1,695	785	0.94	709	1,617	1,130	1.12
	28.24	**49.05**	**22.71**		**20.52**	**46.79**	**32.70**	

Table 9: Elicited beliefs in Stage 2

Others	Beliefs in treatment T_c				Beliefs in treatment T_g			
	0	1	2	Total	0	1	2	Total
0	449	689	306	1,444	283	565	306	1,154
	31.09	**47.71**	**21.19**		**24.52**	**48.96**	**26.52**	
1	425	745	372	1,542	297	771	514	1,582
	27.56	**48.31**	**24.12**		**18.77**	**48.74**	**32.49**	
2	102	234	102	438	129	281	310	720
	23.29	**53.42**	**23.29**		**17.92**	**39.03**	**43.06**	
Total	976	1,668	780	3,424	709	1,617	1,130	3,456
	28.50	**48.71**	**22.78**		**20.52**	**46.79**	**32.70**	

Table 10: Contribution in Stage 1 and elicited beliefs in Stage 2

experiments by Offerman *et al.* (1996). However, it should be noticed that this evidence is not constant across contribution thresholds, k. In particular, when $k = 1$, in treatment T_c these two variables are negatively correlated (-0.0502 with $p = 0.0905$), while correlation is positive and significant in all other cases.

Table 10 analyses whether there is consistency between elicited beliefs and actual behavior in both treatments. Each row (column) of Table 10 corresponds to a particular contribution level of i's teammates, n, (i's point belief).

The cells in the main diagonal correspond to those situations in which beliefs turn out to be correct. We observe that subjects tend to be over-optimistic in both treatments. As an example, consider treatment T_c, the table on the left. The total number of cases in which subjects believe that 0, 1, and 2 of their group members are cooperating are 976, 1,668 and 780, respectively. However, the true numbers are 1,444, 1,542 and 438, respectively. Subjects believe that 0 is less likely than what it really is, and they believe that 1 and 2 are more likely than what they really are. This result is in line with those obtained by Palfrey and Rosenthal (1991). We also find that in treatment T_c, for any contribution level n, modal (point) belief is always 1.

Do elicited beliefs change over time? In Figure 4 we look at the evolution of subjects mean beliefs along the experimental time line.
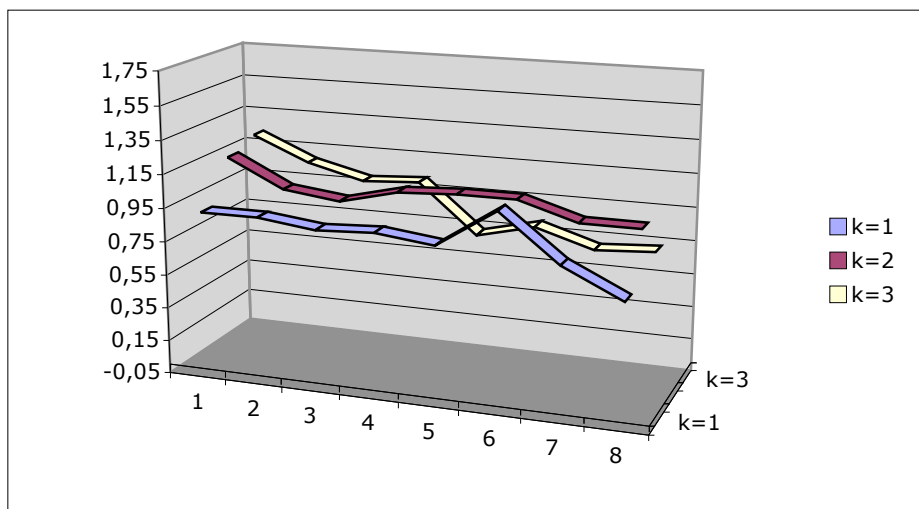
In Figure 4 we observe that in T_g mean beliefs increase with k, confirming the evidence shown in Table 9. We also see that beliefs remain basically constant over time. In T_c we see that beliefs are less dispersed over k, and only in the case $k = 3$ we find a decreasing trend.

We conclude this section by tracking best-reply dynamics in Figure 5, disaggregated by treatment and threshold contribution, both taking into account the ex-ante and the ex-post interpretation.
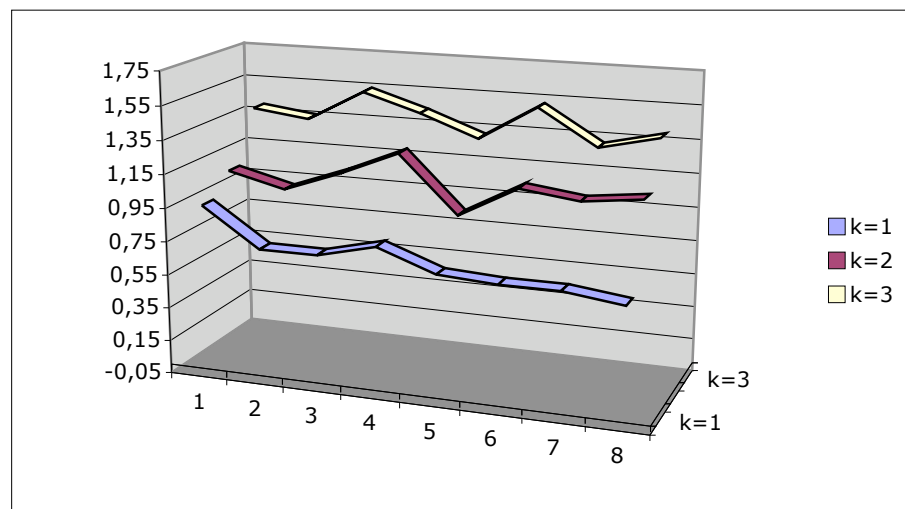
Each diagram of Figure 5 reports the relative frequency of best-replies across time. The top two diagrams refer to our ex-ante interpretation (i.e. best-reply to elicited beliefs), the two bottom diagrams refer to our ex-post perspective (i.e. best-reply to the current opponents' strategy profile). As

Others	Beliefs in treatment T_c				Beliefs in treatment T_g			
	0	1	2	Total	0	1	2	Total
0	449	689	306	1,444	283	565	306	1,154
	31.09	**47.71**	**21.19**		**24.52**	**48.96**	**26.52**	
1	425	745	372	1,542	297	771	514	1,582
	27.56	**48.31**	**24.12**		**18.77**	**48.74**	**32.49**	
2	102	234	102	438	129	281	310	720
	23.29	**53.42**	**23.29**		**17.92**	**39.03**	**43.06**	
Total	976	1,668	780	3,424	709	1,617	1,130	3,456
	28.50	**48.71**	**22.78**		**20.52**	**46.79**	**32.70**	

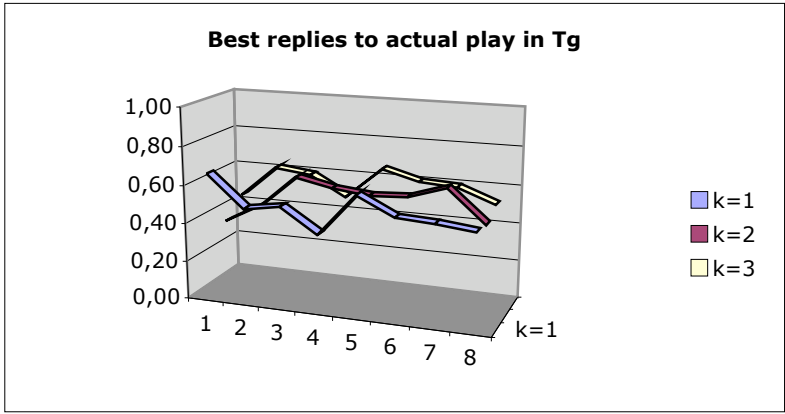**Table 10:** Contribution in Stage 1 and elicited beliefs in Stage 2
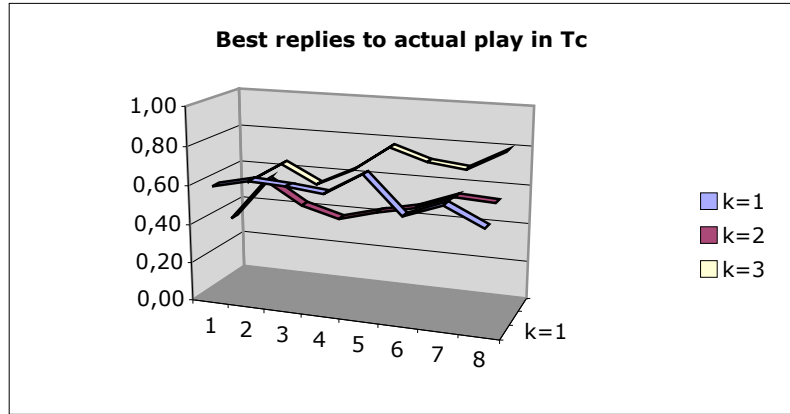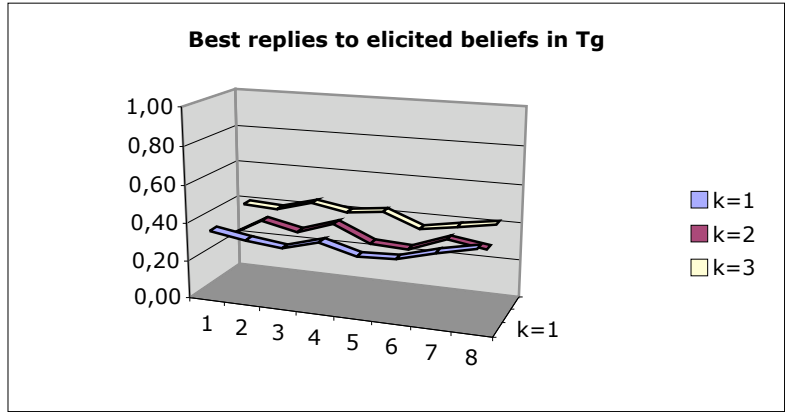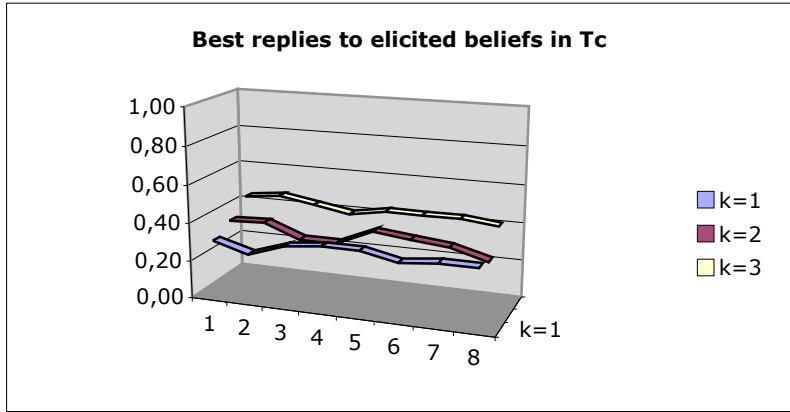
a)

b)

Fig. 4

Fig. 5

Figure 5 shows, subjects' average frequency of best-responses to their beliefs is higher than with respect to the actual behavior of the others. Also notice that learning effects seem negligible, as the average frequency of best-replies stays basically constant through time (the only noticeable exception is T_c when $k = 3$).

6.2   Panel regressions

To fully exploit the panel structure of our data set, in this section we shall run some regressions in which individual heterogeneity is controlled for. Our aim is to study with more detail which are the main factors that affect subjects' decision to contribute. In particular, we estimate a logit model in which the dependent variable is the probability of contribution. Here we present some of the explanatory variables that we will consider:

- **belief pivotal:** This dummy variable takes value 1 if the subject believes she is pivotal, and 0 otherwise;

- **forecast precision:** This variable takes value 1 (-1) [0] if the difference between elicited beliefs and actual contribution of the other group members was positive (negative) [zero]. That is, this variable equals 1 (-1) in case of over-optimistic (over-pessimistic) beliefs;

- **cost:** This is individual cost. It is a discrete variable that takes values (uniformly distributed) between 1 and 55;

- **treatment:** This is a binary variable that takes value 0 (1) in T_c (T_g). To capture any possible interaction between cost and treatment we construct the variable **cost×treat**;

- **k1×treat1, k2×treat1, k2×treat2, k3×treat1, k3×treat2** are dummy variables to control possible interactions between treatments and thresholds. For example **k1×treat2** is equal to 1 when $k = 1$ and the treatment is T_g, and 0 otherwise. The default case is, therefore, treatment T_c and $k = 1$;

- **sequence:** Again a binary variable that is 0 (1) if the observation is taken from a treatment played first (second) in the sequence;

- **period:** This variable refers to the number of the round within each treatment. It goes from 1 to 24;

- **forecast previous:** We construct this variable as the difference between elicited beliefs and the actual behavior of the others one time interval behind. It can take values $-2, -1, 0, +1$, and $+2$. For example, if one subject reports a belief of 2, but no one of the other two subjects have contributed, then this variable takes the value $2 - 0 = +2$. Positive values represent over-optimistic beliefs and negative values over-pessimistic beliefs.

- **outcome previous:** This variable tells us whether the public good was provided or not in the previous period. It takes value 1 if the public good was (not) provided/not deteriorated and 0 otherwise;

- **contribution previous:** A binary variable equal to 1 if the subject has contributed in the previous period and 0 otherwise;

- **forecast previous k:** This variable is constructed like **forecast previous**, but it refers to what happened the last time played with the same k;

- **contribution previous k**: Similar to **contribution previous**, but it refers to whether the subject decided to contribute in the last period in which k was the same than in the current period.

We also include as explanatory variables the information of individual characteristics that we obtained from the questionnaire. We present summary statistics of a selection of our variables in Table 11.

Mean contribution is .395. That is, subjects contribute in almost 40% of the cases. The positive values of forecast precision, forecast previous, and forecast previous k reflect the fact that, on average, subjects believe than the others are contributing more than what they are actually doing.

In Table 12 we present the main results from our regression on the probability of contribution. The second and third columns show the estimated coefficients and the standard errors, respectively. Since the logit model is nonlinear, from this information we can interpret only the sign and the significance of the coefficients. To give an idea of the size of the effects of the different regressors, in Column 4 we report the odds ratios. Odds are ratios of two probabilities, the probability of contribution and the probability of not contribution. While probabilities vary with the value of the regressors, odds ratios remain constant. They are calculated by exponentiating the regression coefficients. That is, for the case of the first regressor (belief pivotal),

Variable	Mean	St dev	Min	Max	Obs
forecast precision	.190	.761	-1	1	6,912
contribution	.395	.489	0	1	6,880
belief pivotal	.413	.492	0	1	6,912
cost	27.60	16.041	0	55	6,912
forecast previous	.242	.961	-2	2	6,738
outcome previous	.402	.490	0	1	6,738
contribution previous	.394	.489	0	1	6,738
forecast previous k	.242	.959	-2	2	6,737
outcome previous k	.409	.492	0	1	6,737
cost previous k	.396	.489	0	1	6,737
gender	.403	.490	0	1	6,912
year course	1.9	1.294	1	9	6,720
education	2.937	.995	1	5	6,912
family size	4.049	1.030	1	7	6,912
rooms	6.771	1.499	1	9	6,912
weekly budget	36.049	34.956	0	300	6,912
risk tolerance	.1667	.373	0	1	6,912
work	3.285	.620	1	4	6,912
family	3.833	.408	1	4	6,912
politics	2.326	.744	1	4	6,912
religion	1.937	.813	1	4	6,864
trust	.180	.385	0	1	6,912
inequality	.875	.331	0	1	6,912

Table 11: Summary Statistics of selected variables

Variable	Coefficient	St error	Odds	Prob change	
				$0 \rightarrow 1$	Min$\rightarrow$max
belief pivotal***	1.457	.116	4.29	.331	
belief pivotal×treat	.293	.202	1.34	.068	
cost***	−.075	.006	.93	−.012	−.745
cost×treat***	.028	.007	1.03	.006	.363
k1×treat2***	−1.153	.229	.32	−.225	
k2×treat1	−.211	.145	.81	−.047	
k2×treat2***	−.738	.184	.48	−.154	
k3×treat1	−.274	.199	.76	−.061	
k3×treat2	.007	.231	1.01	−.002	
sequence**	−.270	.109	.76	−.062	
period***	−.017	.005	.98	−.004	−.092
forecast previous k**	.109	.051	1.12	.025	.099
contribution previous k**	.183	.077	1.20	.042	
gender	.078	.081	1.08	.018	
rooms	−.028	.037	.97	−.007	−.051
weekly budget	.001	.001	1.00	.000	.102
risk tolerance	−.069	.151	.93	−.016	
work	.032	.110	1.03	.007	.015
family	.135	.129	1.15	.026	.060
politics	.066	.049	1.07	.015	.046
religion	.047	.053	1.05	.011	.033
inequality**	.285	.141	1.33	.063	
constant	.102	.751			

Level of significance: *** : 1%; ** : 5%. Number of observations 5,858.
Pseudo R^2 = .2474

Table 12: Logit regression on the decision to contribute

we have that $4.29 = \exp(1.457)$, and so on. Odds ratios are equal to 1 if the regressor has no effect on the dependent variable, are larger than 1 if the effect is positive and are smaller than 1 if the effect is negative. In Column 5 we compute the effect on the absolute value of the probability of contribution when the regressor increases 1 unit. This is particularly interesting for those regressors that are dummy variables. For those that are not, we also report in Column 6 the change in the probability of contribution when the regressor moves from its minimum to its maximum value.

To highlight a couple of interesting results from Table 12, we observe the huge impact that the belief of being pivotal has on the probability of contribution. When a subject believes she is pivotal, compared to when she believes she is not, her odds ratio multiplies by more than 4. In absolute value, the probability of contribution rises in 1/3, going from .1 to .43. The positive effect of the belief of being pivotal seems to be larger in T_g, as implied by the fact that the coefficient of belief pivotal$\times$treatment is positive, although this effect is not significantly different from zero (p value .147). As we should expect, the individual cost has a negative effect, but this effect is somehow mitigated in treatment T_g, since the interaction between cost and treatment is positive and significant. To see the different effect of a change in cost in each treatment, we calculate that an increase in c of one standard deviation (approximately 16) changes the odds of contributing by a factor of 0.3006 in T_c, while the change in T_g is by a factor of 0.4722. We also see that our dummy variables for treatment and k depress the probability of contribution. Finally order effects seem to matter, as the probability of contribution is lower in treatments played last in the sequence. We also observe a decreasing time trend in contribution. From the first to the last period, the probability of contribution reduces by as much as -.1.

To summarize we see that even after controlling for all our experimental conditions, the basic message we get from Section 6.1 remains. Prevention is better than cure when k is high (i.e. when is relatively difficult for the public good to be achieved/not deteriorated). When public good provision is relatively easier the opposite holds.

6.3   Estimating Prospect Theory

In this section we use the data we have obtained in our experiment to estimate the parameters of Prospect Theory. We will outline briefly our empirical strategy in which we use a similar approach to that of Harrison and Rutström

(2006) and Harrison (2007).

We use a simple stochastic specification to specify likelihoods conditional on our model. Every time that an agent has to choose between contributing and not contributing, we assume that the subject uses Prospect Theory to evaluate the two alternatives. Call $V(C)$ and $V(NC)$ the values that the subject assigns to the two alternatives under Prospect Theory, and call $\Delta V = V(C) - V(NC)$, the difference between these two values. For each individual decision we calculate this difference. Using ΔV we define the cumulative probability of the choice that we observe using the logistic function $\Lambda(\Delta V)$ as:

$$\Lambda(\Delta V) = \frac{\exp(\Delta V)}{1 + \exp(\Delta V)}. \tag{33}$$

Now the likelihood, given Prospect Theory, depends on the estimates of the parameters of the model and the observed choices. We will restrict ourselves to the simplest version of Prospect Theory where the value function is linear for gains and losses. Then, we need to estimate just two parameters, the parameter λ that captures the degree of loss aversion and the parameter δ that determines the shape of the probability weighting function. The conditional log-likelihood is, therefore:

$$\ln L(\lambda, \delta; y, X) = \sum_i \left[(\ln \Lambda(\Delta V) \mid y_i = 1) + (\ln(1 - \Lambda(\Delta V)) \mid y_i = 0) \right], \tag{34}$$

where $y_i = 1$ (respectively, $y_i = 0$) means that the agent decides to contribute (not to contribute), and X are individual characteristics.

However, it remains to describe how we compute the values $V(C)$ and $V(NC)$ for each individual decision. Suppose, for example, that a given subject faces the problem described in Table 1. To compute $V(C)$ and $V(NC)$, we need not only the values assigned to the different payoffs and the reference point from which to compute gains and losses, *but also the probabilities that the subject assigns to how many of the other subjects she believes are contributing.* That is to say, we treat each decision within the frame of individual choice under uncertainty, where *uncertainty is only strategic.* In the notation of Table 1, we need to assign values to p and q. Unfortunately, we do not have that information. The only information we have is what we call the "elicited beliefs" on the number of other group members contributing for that round. Here we explain how we derive the values of p and q using these beliefs.

Consider the viewpoint of individual i and call π the (iid) probability that she assigns to the fact that anyone of the others will contribute. Given that group size is 3 in our experimental setup, the probabilities that i assigns

to the events that 0, 1, or 2 subjects are contributing are then $(1 - \pi)^2$, $2\pi(1 - \pi)$, and π^2, respectively. When we ask about beliefs, subjects can only answer 0, 1, or 2. If the belief is 0, this means that $(1 - \pi)^2$ is higher than both $2\pi(1-\pi)$ and π^2. This implies that $\pi < 1/3$. If her beliefs are that exactly 1 subject will contribute, then $1/3 < \pi < 2/3$, and if her beliefs are that 2 subjects will contribute, then $\pi > 2/3$.

Given these restrictions imposed on π for the different beliefs elicited from subjects, we need to go a step further in order to estimate our model. In particular, we need to fix the values of π for the different stated beliefs. Among the various possibilities, we shall assume that π takes the values $0, 1/2$, and 1 when the stated beliefs are 0, 1, and 2, respectively.[12] This implies that when an subject declares a belief 0, she believes that the events that 0, 1, or 2 subjects are contributing have probabilities 1, 0, and 0, respectively. If her belief is 1, these probabilities are $\frac{1}{4}, \frac{1}{2}$, and $\frac{1}{4}$, respectively. When her belief is 2, probabilities are 0, 0, and 1.[13] Table 13 reports estimates for λ and δ given our estimation strategy. The reported estimated standard errors of the parameters of λ_i^k and θ_i take also into account matching group clustering.

	Coefficient	Std. error	95% conf. interval
$\widehat{\lambda}$	1.29	.2103	$[.878, 1.702]$
$\widehat{\delta}$	1.30	.0783	$[1.151, 1.458]$

Number of observations 6,880

Table 13: Estimated Prospect Theory parameters

The result that we get for the shape of the probability weighting function is not as usually assumed, since we get an estimation of δ higher than 1. This is similar to the result in Harrison (2007) where he gets a value of 1.27. Our estimation of λ suggest that the degree of loss aversion is quite low, something we already inferred from our descriptive statistics.

In Table 14, we report (equilibrium) probabilities of contribution (as in Tables 5 and 6), given our estimated parameters.

We get similar results to those of Tables 5 and 6. Given the large estimate for δ, we find no contribution at all when $k = 3$. This exercise should be taken with a lot of caution, given the simple procedure we use to construct the probabilities that individuals assign to the fact that others are contributing.

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[12]This corresponds to the value of π that maximizes the likelihood function associated to each case.

[13]Another possibility would be to use, as proxies of the value of π induced by each stated belief, the midpoint of each interval, $\frac{1}{6}, \frac{1}{2}$ and $\frac{5}{6}$, respectively.

	$x_0 = c$	$x_0 = g$	EUT
$k = 1$	.366	.378	.366
$k = 2$	.531	.574	.450
$k = 3$	0	0	0

Table 14: Framing Effects on the Equilibrium Probability of Contribution
Public good value $g = .91$; $\lambda = 1.29$; $\delta = 1.30$.

7   Conclusions

Inspired by the seminal works of Kahnemann and Tversky (dated more than 30 years from now), economists have learned that *frames matter* since they affect the way in which people understand problems and plan to solve them. In our paper, we study frame effects in the classic problem of public good provision, a problem which has important policy implications. To this aim, we applied Prospect Theory to get different equilibrium distributions in the four possible different problems that differ with respect to the reference point. Our basic theoretical conjecture would call for: a) Different contribution probabilities in the two frames T_c and T_g tested in the lab with b) more contribution in T_g (basically, because of loss aversion). In this respect, our experimental evidence backs definitely up the first working hypothesis. In particular, we find that the biggest difference happens when $k = 3$; as for the second, this is true when k, the threshold below which public good is not provided/not maintained, is high.

One lesson from our experimental evidence is that, if unanimity is needed, it is better to frame the problem as prevention of a bad than as provision of a public good. On the contrary, when the threshold is low and the temptation to free ride is highest, it is better to frame the problem as public good provision.

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8   Appendix

8.1   Proof of Proposition 2

To save notation we call $b = c_c^*$. Evaluating the right-hand side of (13) at $b = c_c^*$ and using (12):

$$R(b) = w(1 - q(b))v(b) + w(q(b))v(b - 1) = \frac{v(1 - b)v(b)}{v(1)} + w(q(b))v(b - 1).$$
(35)

Since $\lambda(1 - b) = -v(b - 1)/v(1 - b)$,

$$R(b) = \frac{v(1 - b)v(b)}{v(1)} - \lambda(1 - b)w(q(b))v(1 - b).$$
(36)

Since the expression above is linear and decreasing in $\lambda(1 - b)$ and $R(b)$ is increasing in b, it follows that for $\lambda(1 - b)$ greater than the threshold:

$$\overline{\lambda}_c = \frac{v(c_c^*)}{v(1)w(q(c_c^*))},$$
(37)

we have $R(b) < 0$ so that $c_1^* > b = c_c^*$, while the inequality is reversed if $\lambda(1 - b)$ is less than $\overline{\lambda}_c$.∎

8.2   Proof of Proposition 3

To compare c_c^* with c_{eu}, we evaluate the left- and right-hand sides of the equilibrium condition of the former at the latter:

$$L(c_{eu}) = \frac{v(1 - c_{eu})}{v(1)} \geq 1 - c_{eu},$$
(38)

by concavity of v in gains, and with equality if v is linear in gains:

$$R(c_{eu}) = w(1 - c_{eu}).$$
(39)

If $c_f < 1/2$, then $c_f < 1 - c_{eu}$ for all $N > 1$, so that $1 - c_{eu} > w(1 - c_{eu})$. Thus, $L(c_{eu}) > R(c_{eu})$, which implies $c_c^* > c_{eu}$.∎

8.3   Proof of Proposition 4

At c_{eu}, the right-hand side of the equilibrium condition for c_1^*, Expression (13) becomes:

$$R(c_{eu}) = w(1 - c_{eu})v(c_{eu}) + w(c_{eu})v(c_{eu} - 1).$$
(40)

Then (18) follows since R is increasing.

For $N = 2$, since $c_{eu} = 1/2 = 1 - c_{eu}$, clearly $\overline{\lambda}_{eu} = 1$.

For $N > 2$, if v is linear in gains,

$$\overline{\lambda}_{eu} = \left(\frac{w(1 - c_{eu})}{v(1 - c_{eu})}\right) \left(\frac{v(c_{eu})}{w(c_{eu})})\right) = \left(\frac{w(1 - c_{eu})}{1 - c_{eu}}\right) \left(\frac{c_{eu}}{w(c_{eu})})\right), \qquad (41)$$

is less than $c_{eu}/w(c_{eu})$ since $1 - c_{eu} > 1/2 \geq c_f$, by regressiveness and sub-certainty of w at c_f. Thus, if $c_{eu} \leq c_f$ then $\overline{\lambda}_{eu} < 1$ since $c_{eu} \geq w(c_{eu})$.∎

8.4 Proof of Proposition 5

To prove (i) we observe that when $N = 2$, the equilibrium condition becomes:

$$w(1 - c) = 1 - c, \qquad (42)$$

which is satisfied only at $c = 1 - c_f$.

When $N > 2$, we see that $w(1 - c^{N-1}) > w(1 - c)$ since $w(1 - c^{N-1})$ is an increasing function of N. So, we have that $w(1 - (1 - c_f)^{N-1}) > w(1 - (1 - c_f)) = 1 - (1 - c_f) = c_f$. Any $c \in (0, 1)$ can be an equilibrium only if $c < 1 - c_f$, as stated in (ii).

Note that (ii) and (i) imply (iii) for the case $m = 2$.

To show (iii) for $m > 2$, let $c_n \in (0, 1)$ be the supremum interior equilibrium for $N = n > 3$. Let $x_n = 1 - c_n$ and $G_N(z) = f_N(z) - w^{-1}(z)$ be the left-hand side minus the right-hand side of the equilibrium condition (42). Let m be an integer such that $n > m > 2$. Note that $G_m(1 - c_f) = f_m(1 - c_f) - c_f > 0$. Note also that because f_N, and consequently G_N, are increasing in N for any $x \in (0, 1)$, $G_m(x_n) < G_n(x_n) = 0$. Then, by continuity of $G_m(x)$ in $x \in (0, 1)$, it follows that there exists $x_m \in (1 - c_f, x_n)$ such that $G_m(x_m) = 0$. Thus $c_m = 1 - x_m$ is an interior equilibrium for $N = m$, satisfying $c_m < c_n$. This proves (iv).

To show (v) note that if there is no interior equilibrium for $N = m > 2$, then $G_m(x) > 0$ for all $x \in (1 - c_f, 1)$ and $G_m(1 - c_f) > 0$. Now since G_N is increasing in N, the same inequality holds for any larger $N > m$, and no interior equilibrium exists either.

To show (iv), for any given integer $n > 2$, take a weighting function such as for $1 > p > \underline{p}_n$ is defined by:

$$w_n(p) := f_n^{-1}(p) = 1 - (1 - p)^{1/(n-1)} \text{ if } p \in \left[\underline{p}_n, 1\right], \qquad (43)$$

where $\underline{p}_n > f_n(c_f)$. For $p < \underline{p}_n$ $w_n(p)$ can be defined to have a fixed point $p = c_f$ and have all the properties assumed in Prospect Theory. With such

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a weighting function, it is clear from the equilibrium condition (**??**) that for $N = n$, the set of equilibria is $[\underline{p}, 1] \cup \{0\}$, while for $N > n$ there is no interior equilibrium. Similarly it can be shown that for any subset $S \subset (c_f, 1]$ with S being a union of closed intervals included in $(c_f, 1]$, there is a weighting function satisfying the properties stated in Prospect Theory and with fixed point c_f, such that S is the set of equilibria for some $N > 2$.■

8.5 Proof of Proposition 6

(i) With linear value functions in gains and losses, and linear weighting function, the equilibrium condition becomes:

$$q(g - c) - \lambda c(1 - q) = 0. \tag{44}$$

Since $q = c^{N-1}$:

$$\lambda = \frac{g - c}{c} \frac{c^{N-1}}{1 - c^{N-1}}. \tag{45}$$

The term on the right is an increasing function of g and it is a decreasing function of N, then we have:

$$\lambda = \frac{g - c}{c} \frac{c^{N-1}}{1 - c^{N-1}} \leq \frac{1 - c}{c} \frac{c}{1 - c} = 1, \tag{46}$$

which is a contradiction with the fact that there is loss aversion.

(ii) With linear value functions in gains and losses the equilibrium condition becomes:

$$\lambda = \frac{g - c}{c} \frac{w(c^{N-1})}{w(1 - c^{N-1})}. \tag{47}$$

Since again the term on the right is increasing with g and decreasing with N, we have that:

$$\lambda \leq \frac{w(c)}{c} \frac{1 - c}{w(1 - c)}. \tag{48}$$

Then, if the term in the right is less than one, we get a contradiction with loss aversion. We see that this will be the case if $c \geq 1 - c_f$. First, for any $c \geq 1 - c_f$, $1 - c \leq c_f$ and $w(1 - c) \geq 1 - c$, which implies $\frac{w(1-c)}{1-c} \geq 1$. Secondly, as $c_f < 1/2$ we know that $c \geq 1 - c_f > c_f$ and, therefore, $w(c) < c$ and $\frac{w(c)}{c} < 1$. To sum up, we get:

$$\frac{w(c)}{c} < \frac{w(1 - c)}{1 - c}, \tag{49}$$

which implies $\lambda < 1$.

(iii) This is immediate since the term $\frac{w(c^{N-1})}{w(1-c^{N-1})}$ is a decreasing function of N.

8.6   Proof of Proposition 8

Recall that the equilibrium condition under Expected Utility (28) is:

$$q(c) = c. \tag{50}$$

(i) We show the first inequality of the proposition. For reference point $x_0 = c$ the equilibrium condition making indifferent the prospects at choice (8), becomes:

$$w(p+q)v(1-c) + w(r)v(-c) = w(p)v(1). \tag{51}$$

With linear value function both in gains and in losses and linear probability weighting function, the equilibrium condition (51) becomes:

$$(p+q)(1-c) - \lambda r c = p, \tag{52}$$

so:

$$q(c) = [1 + (\lambda - 1)r(c)]\, c. \tag{53}$$

Suppose that there exists a solution $c_c \in (0, 1)$ to (53). Then, since for $k > 1$, $r(c_c) > 0$, and by loss aversion $\lambda > 1$:

$$q(c_c) = [1 + (\lambda - 1)r(c_c)]\, c_c > c_c. \tag{54}$$

So that at c_c the left-hand side of the equilibrium condition (50) is greater than the right-hand side $L_0(c_c) > R_0(c_c)$. Since at $c = 1$, for that equilibrium condition the inequality is the opposite, $L_0(1) = 0 < R_0(1) = 1$, and both sides are continuous, it follows that there exists an equilibrium $c_{eu} > c_c$.

   (ii) We finally show the second inequality of the proposition. For reference point $x_0 = 1$ the equilibrium condition making indifferent the prospects at choice (10), becomes:

$$w(r)v(-1) = w(p)v(c) + w(q+r)v(-1+c). \tag{55}$$

With linear value function both in gains and in losses and linear probability weighting function, the equilibrium condition (55) becomes:

$$-\lambda r = pc - \lambda(q+r)(1-c). \tag{56}$$

So:

$$q(c) = c\left[1 - \left(1 - \frac{1}{\lambda}\right)p(c)\right]. \tag{57}$$

Suppose that there exists a solution $c_0 \in (0, 1)$ to (50). Then, since for $k < N$, $p(c_{eu}) > 0$, and by loss aversion $\lambda > 1$:

$$q(c_{eu}) = c_{eu} > c_{eu}\left[1 - \left(1 - \frac{1}{\lambda}\right)p(c_{eu})\right]. \tag{58}$$

So that at c_{eu} the left-hand-side of the equilibrium condition (57) is greater than the right-hand side $L_1(c_{eu}) > R_1(c_{eu})$. Since at $c = 1$, for that equilibrium condition the inequality is the opposite, $L_1(1) = 0 < R_1(1)$, and both sides are continuous, it follows that there exists an equilibrium $c_1 > c_{eu}$.■