A discusión

LIFE-CYCLE PORTFOLIO CHOICE: THE ROLE OF HETEROGENEITY AND UNDER-DIVERSIFICATION*

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ABSTRACT

The empirical work on household portfolio choice documents two facts. One, the stock market participation rate is low and hump-shaped over the life-cycle, two, the conditional share of stocks is also low but does not appear to change much during the life-cycle. In contrast the standard life-cycle portfolio choice model predicts high and monotonically increasing participation rates and conditional stock shares that are low and exhibit dramatic changes with age. In this paper I consider a number of extensions to this basic framework. I find that a small per period participation cost is needed to generate a hump shaped life-cycle profile of participation rates. Under a realistic calibration the quantitative effect is minor. Progressive social security and the assumption that the risk of stock portfolios is declining in household wealth as a consequence of better diversification opportunities —an assumption that has some empirical support— though provide substantial amplification and significantly improve the ability of the model to match the data. Under-diversification also reduces the average portfolio share of stocks conditional on participation and, together with the intergenerational transmission of wealth makes it insensitive to age, consistent with the empirical evidence.

Keywords: Portfolio choice, life-cycle, bequests, diversification, social security.

1 Introduction

In this paper I construct and solve the optimal portfolio choice problem of a life cycle household with altruistic bequests. The decision model is then used to simulate an Overlapping Generation, partial equilibrium economy to produce cross sectional profiles of stock market participation rates and portfolio shares of stocks conditional on participation that are the direct counterpart to the profiles of household decisions described by previous empirical studies. In order to perform this comparison a key point of the model is to introduce a substantial degree of household heterogeneity as we observe in the US economy. We build on standard models like the ones in Campbell et al. (1999), Gomes and Michaelides (2005) in that we consider the life cycle joint savings and portfolio decision of households facing idiosyncratic earnings risk and borrowing constraints. In this framework we introduce a richer structure of stock market participation costs by considering both a fixed one time entry cost and a per period participation cost. We pursue the goal of introducing a substantial degree of heterogeneity in three ways. First we assume that beside receiving idiosyncratic earnings shocks agents are ex-ante differentiated by a permanent component of earnings ability and choose the parameters of the labor process so that we can match the earnings inequality observed in the PSID data. Second, in order to take care of the large observed differences in wealth accumulation we introduce a progressive social security system that mimics the one in the US and we allow for a bequest motive with actual transmission of wealth across the generations of a same dynasty. A progressive social security has been showed to induce higher saving rates among high income households thus inducing different wealth-earnings profiles.[1] Intergenerational transmission of wealth also affects heterogeneity in wealth profiles especially at young or very old ages. Finally we also take into account the possibility that because of small fixed costs of trading in each stock, wealthier agents will have better diversified stock portfolios and therefore face a lower variance of returns.

The main results can be summarized as follows. First, a per period cost of participating in the stock market is needed to obtain the characteristic hump shaped life cycle profile of participation rates. Second, the introduction of a progressive formula for social security benefits has an important effect on conditional shares. In particular under such assumption it is optimal for lower income households to invest a larger share of their wealth in the stock market than for high income households. To our knowledge this result is new and has not yet been incorporated in the investment strategies proposed by financial advisors. At a positive level the result seems to run against the empirical evidence

[1]See Huggett and Ventura (2000) for the US and Domeij and Klein (2002) for Sweden.

although it is soon to state this as a new asset allocation puzzle. The progressive social security formula also magnifies the effect of the fixed per period cost in reducing participation rates, although when both are realistically calibrated their joint effect is too weak to bring average participation rates close to their empirical values. Third, as Curcuru et al. (2004) pointed out there may be an important problem of lack of diversification in household stock portfolios; when this is explicitly taken into account in the model it shows great potential in bringing model predictions closer to the data. In particular it can reduce both participation rates and the conditional share by a large amount. Finally the introduction of a bequest motive and actual intergenerational transmission of wealth helps making the life-cycle profile of the conditional stock share virtually independent of age as it seems to be the case in the data. Beside the results obtained we believe the Overlapping Generation approach used in simulating the model is of independent interest. To my knowledge this approach is new to the study of models of life-cycle portfolio choice although it has been frequently used in other areas like the study of social security and wealth distribution issues.[2] This approach seems very promising along two dimensions. First it makes it natural to consider models that exhibit a degree of heterogeneity among agents that is comparable to the one observed in the real economy. Second, with some limitations due to its partial equilibrium nature, it allows to study the effects of changes in policies or other institutional settings on the observed life-cycle portfolio choices. This appears to be an interesting possibility given the large changes in the observed stock market decisions of American households observed in the last 20 years. Exploring all the potentials of this approach is beyond the scope of the present paper so we have focused on the first of the two only.

The rest of the paper is organized as follows. In the next subsection I briefly describe the literature on household asset allocation, both theoretical and empirical. In section 2 I present the description of the model and the choice of parameters. In section 3 I describe the results of the quantitative analysis and finally in section 4 I present some brief conclusions and directions for future work.

1.1 Related Literature

Starting from the nineties and possibly in response to the important changes in financial choices of American families a large literature has developed to study

[2]The number of papers written in both areas is large so that giving an exhaustive list of references would be difficult. More recently an asset allocation problem has also been introduced in life-cycle OLG models in general equilibrium that try to explain the equity premium. See Storesletten, Telmer and Yaron (2007).

the issue of portfolio choice both empirically and theoretically.[3] In the empirical field, works by Poterba and Samwick (1997), Heaton and Lucas (2000), Ameriks and Zeldes (2001) and Bertaut and Starr-McCluer (2000), beside documenting the rise in stock market participation rates that occurred since the early nineties, have described a number of stylized facts about household portfolios in the US. These can be summarized as follows. First, despite the size of the equity premium and even after the recent surge, the participation rate is still only about 50 percent. Second, the participation rate is increasing in wealth and hump-shaped in age.[4] Finally the share of stocks conditional on participation is roughly constant in both age and wealth. These findings for the US economy extend to a number of other industrialized countries like the U.K., Italy, Germany and the Netherlands as reported in the country studies presented in the volume edited by Guiso et al. (2001). At the theoretical level the seminal work by Samuelson (1969) and Merton (1971) pointed to some key properties of portfolio decisions. Samuelson (1969) considered the problem of an agent with no labor income, power utility and facing i.i.d. returns and found out that the optimal share of risky asset is independent of wealth and the horizon. Merton (1971) extended this result to the possibility of a constant labor income stream and concluded that in this case the share of risky assets is constant in total — human plus financial — wealth implying that as the agent ages and his residual human wealth declines he should reduce his exposition to stocks as popular financial advisors suggest. In more recent times the advances in computational methods and computing power allowed scholars to solve models with realistic labor income risks and borrowing and short sale constraints thus merging the portfolio choice and precautionary saving framework. Among the many works produced in this framework are those of Heaton and Lucas (1997 and 2000) and Haliassos and Michaelides (2003) who consider infinite horizon problems and of Campbell, Cocco, Gomes and Michaelides (1999), Cocco, Gomes and Maenhout (2005), Gomes and Michaelides (2005) who look at finite horizon problems. These papers have delivered a number of interesting predictions. First, with an empirically plausible low correlation between labor earnings shocks and stock returns households would enter the stock market first and then diversify towards bonds only as their wealth grows. Second, as a consequence non participation can be justified only by adding some frictions in the form of fixed participation costs. Third they predict that the share of stocks should be declining with age until retirement and then increasing again. The present model is most closely

[3]Two useful surveys of work done and open questions in this area are Guiso, Haliassos and Jappelli (2001) and Curcuru, Heaton, Lucas and Moore (2004)

[4]The relationship between participation and age is somewhat sensitive to the estimation procedure used. In particular when cohort effects are included it tends to be increasing. See Ameriks and Zeldes, (2001).

related to the finite horizon models mentioned above from which it departs in that it gives a thorough consideration to the implications that the large degree of heterogeneity in economic and demographic conditions of households have in shaping the cross section of stock market decisions. Other papers have departed in different directions: Lynch and Tan (2004) and Benzoni, Collin-Dufresne and Goldstein (2004) considered a cyclical and long-term correlation between stock returns and labor earnings, Gomes, Michaelides and Polkovnichenko (2003) and Dammon, Spatt and Zhang (2003) studied the impact of differential tax treatment between assets; Davis, Kubler and Willen (2002) studied the impact on stock investment of the possibility of borrowing at a rate higher than the lending rate; Cocco (2004) and Yao and Zhang (2005) considered a more complex model where housing is added to bonds and stocks in the menu of assets available to the investor; finally a number of authors among whom Campbell and Viceira (1999) studied the impact of return predictability on stock demand.

2 The Model

2.1 Demographic Structure

Time is discrete and the model period is assumed to be one year. The model is populated by finitely lived agents that are linked through altruistic bequests to form infinitely lived dynasties. I let t denote the time period and a the number of periods an agent has spent in the model. Agents enter the model as workers at age 21 so that real-life age is equal to model age plus 20. Every agent can live up to a maximum of $A = 69$ periods, corresponding to age 89. I allow for uncertain life-span by assuming that in every period there is a positive probability $1 - p_{a+1}$ that the agent dies. All agents generate a single descendant after N periods and retire after G periods of life in the model provided they have not died before. The values of N and G are chosen so that agents have a descendant at real age 35 and retire at age 65. The particular demographic structure assumed here allows only two consecutive members of a given dynasty to overlap at each time, thus reducing the number of state variables. The population is kept constant over time by introducing a certain number of new dynasties to replace those that die out because the parent household dies before it is replaced by a descendant one.

2.2 Preferences

Agents value consumption but not leisure. Period utility is defined by a standard utility index $u(c_{i,t})$ and discounted at the rate β. Agents also receive utility from the estate they leave to their descendant upon death. They are truly altruistic so that they value the indirect utility the descendant's will receive from enjoying the bequest. It is assumed that this is discounted at a further rate $\gamma \in [0,1]$ allowing the model to nest life-cycle and fully altruistic households as two polar extremes. Since an agent's utility is defined only starting with its first period of working life, this implies that the bequest motive becomes operational only when the descendant enters the labor force. The assumption of altruistic bequests complicates the solution to the model. The alternative assumption is the so called "warm glow" altruism where an agent values the fact of giving in itself and the utility is defined over the amount of the bequest. Under this assumption though, the curvature of the bequest function would have a fundamental impact on the portfolio choice of the agent. Since there is not at present a consensus on what this parameter should be, assuming true altruism avoids the arbitrariness of this choice.

2.3 Labor and Retirement Income

Investor's i labor income after a periods of life in the model is given by:

$$log(y_{i,t}) = \theta_i + f(a) + z_{i,t} \tag{1}$$

for $a < G$. This formulation implies that there are three components that determine individual earnings. A first component denoted with θ is specific to the individual and fixed for the entire life time; it can be thought as representing his ability as determined by genetics and education and it is assumed to be fully persistent across consecutive members of a dynasty. The second component $f(a)$ is a deterministic function of age that is common to all individuals and is meant to capture the hump in life-cycle earnings that is observed in the data. Finally there is an idiosyncratic component $z_{i,t}$ which is assumed to follow an autoregressive process given by:

$$z_{i,t} = \rho z_{i,t-1} + \upsilon_{i,t} \tag{2}$$

and $\upsilon_{i,t} \sim N(0, \sigma_\upsilon^2)$ and independent over time.

After retirement the agent receives a pension benefit $b(\theta_i, z_{i,G})$ that depends on his permanent earning type and the earnings shock in his last period of working life. This choice allows the model to capture some elements of the progressive US pension system without adding a further state variable.

The general notation for household income will be $Y_{i,a,t}$ where:

$$Y_{i,a,t} = \begin{cases} e^{ln(y_{i,t})} & \text{if } a \leq G \\ b(\theta_i, z_{i,G}) & \text{if } a > G \end{cases} \tag{3}$$

2.4 Financial Assets

In the economy there are two assets in which the agent can invest. First a one period risk-free bond with price q and return $R_f = 1/q$. Second a risky asset called "stock portfolio" with return denoted $R_t(w)$ and defined by the equation:

$$R_{t+1}(w_{i,t}) - R_f = \mu + g(w_{i,t})\varepsilon_{t+1} \tag{4}$$

where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ is an i.i.d. innovation and μ is the expected excess return of the stock investment. Here $g(.)$ is a function that satisfies the following two properties: first $g'(w_{i,t}) \leq 0$ and $\lim_{w_{i,t}\to\infty} g(w_{i,t}) = 1$ where $w_{i,t}$ is the wealth of agent i at time t. Its effect is that the variance of the return to the stock portfolio will be $g(w_{i,t})^2\sigma_\varepsilon^2$ which as it can be seen is potentially wealth dependent. If we assume that $g'(.) = 0$ over the whole range of possible wealth levels then we are back in the standard case in which all agents face the same return process on their stock portfolio, otherwise the model allows the variance of the stock investment to be declining in the agent's wealth. This assumption is supported by the empirical evidence reported for example in the Investment Company Institute's publications "Equity Ownership in America" (1999 and 2002) that shows that while households with moderate levels of financial wealth normally invest in one or two stocks only, wealthier households tend to invest in well diversified portfolios that may contain twenty or more stocks. At the theoretical level this assumption can be justified by the assumption of fixed costs of investing in each single stock. [5] Two more comments are needed about this assumption. First, notice that the average excess return of the stock portfolio is not wealth dependent. While this may seem very strong since it implies that agents who are choosing different portfolios of stocks all receive the same expected return, it is sufficient to capture the effects of under-diversification since both a lower expected return and a higher variance will reduce stock demand. Second, as reported in Curcuru et al. (2004) lack of diversification may take different forms, from ownership of only few stocks in a brokerage account to ownership of equity in the company where a household member works, possibly through retirement plans. The two have somewhat different effects although both will generate a reduction in the exposition to stock market risk of the undiversified household. The choice made here is done mainly to simplify model computation and can be considered as a first analysis susceptible of further refinements.

The amount of bonds and stocks that household i holds at time t is denoted

[5] Explicit modeling of these costs would make the present model almost unsolvable. A theoretical and quantitative analysis in a static, mean variance type of model can be found in Brennan (1975).

with $B_{i,t}$ and $S_{i,t}$ respectively and it is assumed that

$$B_{i,t} \geq 0 \tag{5}$$

$$S_{i,t} \geq 0 \tag{6}$$

meaning that the investor is prevented from borrowing against future labor income or retirement wealth and from selling short stocks.

Participation in the stock market may entail payment of some costs. We allow for the possibility of two different costs. First as many authors have previously done we introduce an initial entry cost F_I that must be paid the first time one invests in a stock portfolio. This cost can be thought of as the cost needed to gather the initial information about the stock market in general; given its nature it creates the need for a new state variable in the problem.[6] We denote this new state variable as $I_{F,i,t}$ where $I_{F,i,t} \in \{0, 1\}$. A value of the index equal to one means that the cost was paid before and a value of zero means that the cost was not paid before. Equation 7 formalizes the fact that this initial information cost is paid only once. Second we allow for the possibility of a per-period participation cost, denoted F_p that must be paid in any subsequent period if the agent decides to invest in the stock market. This cost does not introduce state dependence and may be interpreted as extra time cost of filling tax forms or the monetary cost of brokerage fees. [7] The index $I_{P,i,t}$ is used to denote payment of this cost if it takes the value of one or not payment and therefore not participation in the stock market if it takes the value of zero. With the notation for participation in the stock market given above we can write the following law of motion

$$I_{F,i,t+1} = I_{F,i,t} + (1 - I_{F,i,t})I_{P,i,t} \tag{7}$$

that describes the evolution of the state variable used for payment of the initial entry cost.

2.5 The Household Optimization problem

Given the chosen demographic structure we can divide a household life into two qualitatively different periods. Since an agent has a kid at age 35 that will then enter the labor force at age 21 he won't have an active bequest motive until reaching 55 years of age. At the same time before that age his parent can be alive so the agent is the potential recipient of an inheritance. We can then split an agent's life into a first period up to age 54 when he can receive a bequest but

[6]See for example Campbell et al. (1999) and Gomes and Michaelides (2005)

[7]See Vissing-Jørgensen (2001).

does not value leaving one: we call "early life" this first part. Afterwards and until death the agent cannot receive any more a bequest but values leaving one: we call "late life" this second part. In the next two subsections we describe in turn the dynamic programming problem solved by an agent in the two different stages of life. In order to simplify the notation we will omit the index i that denotes the household. For the same reason I will omit the index that denotes the agent's permanent earning type.

2.5.1 Early Life Problem

Given the description of the model in the previous sections the value function problem in the early stage of life is:

$$V^a(d_t, z_t, I_{F,t}, I_{s,t}) = \max_{c_t, B_{t+1}, S_{t+1}, I_{P,t}} \Big\{ u(c_t) + \beta p_{a+1} \big\{ I_{s,t} \big[p_{a+36} E_t V^{a+1}(d_{t+1}, z_{t+1}, I_{F,t+1}, 1)$$
$$+ (1 - p_{a+36}) E_t V^{a+1}(d_{t+1} + \bar{W}^{a+36}, z_{t+1}, I_{F,t+1}, 0) \big] + (1 - I_{s,t}) E_t V^{a+1}(d_{t+1}, z_{t+1}, I_{F,t+1}, 0) \big\} \Big\} \quad (8)$$

In the above equation the index I_s takes the value of one if the agent's parent is alive at time t and zero otherwise. The interpretation of the equation is the following: on the left-hand side we have the value function of an agent who is a model periods old and whose states are given by his resources d_t, his labor earnings shock z_t and the two indexes that say if the agent had previously paid the initial entry cost and if the agent's parent is alive. This value is the maximized value of the sum of the utility flow from current consumption $u(c_t)$ and future discounted utility where the maximization is performed with respect to consumption, the amount of bonds and stocks to carry to the next period and payment of the entry cost. In turn the continuation value can be either of the two following possibilities. If the agent's parent had died before then the index $I_{s,t}$ takes the value of zero so that continuation utility is the last term of the Bellmann equation $E_t V^{a+1}(d_{t+1}, z_{t+1}, I_{F,t+1}, 0)$, that is, the indirect utility of an agent that has grown one year older and whose parent is dead, given his resources and labor income shock. The other alternative is that the agent's parent is alive at time t so that $I_{s,t} = 1$; in this case with probability p_{a+36} the agent's parent survives so that the household's continuation utility will be the utility of a one year older household whose parent is still alive given resources d_{t+1} and the labor income shock: this is the term $E_t V^{a+1}(d_{t+1}, z_{t+1}, I_{F,t+1}, 1)$ in the Bellmann equation above. With probability $1 - p_{a+36}$ the parent will die next period in which case the continuation utility will be $E_t V^{a+1}(d_{t+1} + \bar{W}^{a+36}, z_{t+1}, I_{F,t+1}, 0)$, that is, the utility of a household that has grown one year older, with no alive parent and with resources that on top of its own personal funds include some "expectation" about the size of the bequest. The notation $\bar{W}^{a+36}$ points to the fact that the assumption made here is that the agent does not know his parent's wealth and uses the average value of wealth in the population of households of the same age and earning type. In general we may assume that heirs have some information about parental wealth but that this is not perfect. The assumption made here that they assume parental wealth is average among their cohort and earning type corresponds to a case of limited information and it is made to reduce the already high computational burden imposed by the program structure. Some discussion is needed to justify this assumption. The way we model how an agent forms his expectation about how much he will inherit affects his decisions since if he expects to receive a larger bequest he will save less. Consequently the modeling choice made here implies that some agents will over-save and some will under-save compared to the case where they had more precise information about parental wealth. The goal of the paper though is to study average life-cycle profiles so that it is reasonable to think that these deviations from a more detailed and perhaps more realistic informational assumption will compensate each other and therefore will be minor.[8] Finally notice that in principle a descendant household can die before its parent does but we rule out altruism between one member of a dynasty and the previous one. The description of the early life problem is completed by the resource constraint and the law of motion of the household's financial resources. The resource constraint reads:

$$c_t + qB_{t+1} + S_{t+1} \leq d_t + Y_{a,t} - I_{P,t}F_P - (1 - I_{F,t})I_{P,t}F_I \tag{9}$$

and the law of motion of household resources is:

$$d_{t+1} = B_{t+1} + R(d_t)S_{t+1} \tag{10}$$

The resource constraint states that the expenditure in consumption, bonds and stocks cannot exceed the sum of financial wealth and income from labor net of payment of the costs of participating in the stock market if the agent decides to do so. In turn these costs are equal to $F_I + F_P$ if participation occurs for the first time in the agent's life and it is F_P if the agent had participated before. The law of motion of financial resources simply states that they are equal to the sum of the realized return on bonds and stock portfolios.

[8] An alternative choice would have been to assume that the agent knows his parent's current wealth and labor earnings shock and uses his decision rules to forecasts the bequest he will receive. This assumption though would imply the addition of two more state variables.

2.6 Late Life Problem

In the second stage of life, when an agent has an active bequest motive but cannot inherit any more the value function problem takes the following form.

$$V^a(d_t, z_t, I_{F,t}, I_{s,t}) = \max_{c_t, B_{t+1}, S_{t+1}, I_{P,t}} \Big\{ u(c_t) + \beta I_{s,t} \big\{ p_{a+1} \times$$
$$\big[p_{a-34} E_t V^{a+1}(d_{t+1}, z_{t+1}, I_{F,t+1}, 1)) + (1 - p_{a-34}) E_t V^{a+1}(d_{t+1}, z_{t+1}, I_{F,t+1}, 0) \big]$$
$$+ (1 - p_{a+1}) p_{a-34} \gamma E_t V^{a-34}(d_{t+1} + \bar{W}^{a-34}, z_m, 0, 0) \big\} +$$
$$\beta (1 - I_{s,t}) p_{a+1} E_t V^{a+1}(d_{t+1}, z_{t+1}, I_{F,t+1}, 0) \Big\} \quad (11)$$

The state variables of this problem are formally the same as in early life but the interpretation of the index $I_{s,t}$ is now different because it refers to the son's living status, with a value of one meaning that he is alive. As usual the value function of an age a agent is the maximized value of the sum of the utility from the flow of current consumption plus continuation utility with the maximization performed with respect to consumption, the amount of financial assets carried to the next period and the stock market participation decision. In turn the continuation utility can be either of the following two possibilities. First if the index $I_{s,t}$ takes the value of zero the agent has no living descendant, so with probability p_{a+1} he survives and enjoys utility $E_t V^{a+1}(d_{t+1}, z_{t+1}, I_{F,t+1}, 0)$, that is, the value of an agent who has grown one year older and does not have a living descendant, given his financial resources and labor efficiency units. Alternatively the agent may have a living descendant. In this case with probability $p_{a+1} \times p_{a-34}$ both parent and descendant survive to the next period so continuation utility will be given by $E_t V^{a+1}(d_{t+1}, z_{t+1}, I_{F,t+1}, 1)$, that is, the value to an $a+1$ year old agent whose son is still alive, with probability $p_{a+1} \times (1 - p_{a-34})$ the parent survives but the son dies so the continuation utility will be the one of an $a+1$ year old agent whose descendant is dead described by the term $E_t V^{a+1}(d_{t+1}, z_{t+1}, I_{F,t+1}, 0)$. Finally and more interestingly with probability $(1 - p_{a+1}) \times p_{a-34}$ the agent himself dies but his son survives so that the transmission of a bequest occurs. Given the altruistic assumption in this case the value to the parent household will be given by $\gamma E_t V^{a-34}(d_{t+1} + \bar{W}^{a-34}, z_m, 0, 0)$, that is, the parent uses the value function of an agent who is 34 years younger than himself next period which corresponds to the age of his son. The parent household needs to form some estimate of the state his son is in and again we make the assumption that the parent does not have such information and simply takes his son to be "average". This means that if he leaves resources d_{t+1} then he expects his son will have resources $d_{t+1} + \bar{W}^{a-34}$ where $\bar{W}^{a-34}$ is the average wealth of agents in the cohorts and earning type cell the son belongs to. The parent also assumes that the descendant received the median labor earnings shock z_m and that he has not paid the fixed entry cost. Also a further discount factor γ is applied to the descendant's utility to capture the possibility of imperfect intergenerational altruism or in the extreme case of no altruism. The justification of this choice is similar to the one given when describing the early life problem: on the one hand it is computationally convenient, on the other hand a more sophisticated choice would not have a mayor impact on average life-cycle profiles that are the object of this study. The description of the late life problem is completed by the resource constraint and the law of motion of financial resources that are the same as the ones reported above when describing the early life problem.

2.7 Parameter Calibration

In this section we describe the choice of the model parameters used in the simulations. Most of the parameters are taken from other studies while a few are chosen so as to match some key target taken from US data.

2.7.1 Preference Parameters

Preferences in our model are defined by three parameters. First the period utility index is of the standard iso-elastic form $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and the coefficient of relative risk aversion σ is set to 7 a value in the range normally chosen in this literature. The other two parameters are β and γ, the discount factor on own and descendant utility. As far as the parameter γ that measures the strength of the bequest motive there is little consensus on what it should be: some authors like Hurd (1989) suggest that this is basically zero and all bequests are accidental, others like Dynan, Skinner and Zeldes (2002) and Cagetti (2002) point to the difficulty of disentangling a bequest motive since agents already accumulate a substantial amount of precautionary wealth late in life that can be passed to heirs when not consumed. Given the focus of the present paper on life-cycle profiles of stock market decisions and their dependence on the evolution of the financial-to-human wealth ratio we fix γ so that the ratio between the flow of bequeathed wealth to total wealth in any period is close to the estimate of 1.4 presented in Gale and Scholz (1994).[9] Unfortunately even if one sets $\gamma = 0$ the model ratio exceeds the target. The likely reason is that in real life a large part of bequests are left by the last surviving spouse to the descendants. Often this is the female in a couple who usually dies later than the male. Here the structure of the household is not modeled and survival probabilities are taken from the male mortality tables, so that bequests are left somewhat earlier in the life

[9] The ratio targeted here corresponds to the sum of bequests and inter-vivos transfers reported by Gale and Scholz. We believe this is more appropriate than targeting the bequest to wealth ratio because in our model all intergenerational transfers occur through bequests.

cycle, thus tend to be larger than in reality. For this reason we run experiments with γ of 0.1 and 0 to consider the cases with and without intergenerational altruism. The first case generates a bequeathed to total wealth ratio of about 1.8 percent, reasonably close to the estimates. The value of β is instead determined endogenously so that once the all the other parameters are given, the average wealth earnings ratio in the population is 5, a value taken from the estimates in Budría-Rodríguez et al. (2001) and Díaz-Giménez et al. (1997). Targeting the wealth-to-earnings and bequest-to-wealth ratios is meant to insure that the profile of financial to human wealth over the life-cycle is consistent with the data especially at the beginning and end of life when the bequest motive and the receipt of an inheritance may have an important impact.

2.7.2 Labor Income Process and Pensions

In order to fully characterize the labor income process we need to specify two different sets of parameters. First we fix the function $f(a)$ that describes the deterministic life-cycle profile of earnings. This is taken form the profile estimated by Cocco, Gomes and Michaelides (2005) for high-school graduates. We think this choice is not restrictive for our economy where agents do not differ by educational attainment since when we aggregate over five year periods the profile is also consistent with the one estimated by Hansen (1993) for the general population. Second we need to specify permanent earnings differences and the stochastic process that determines the yearly evolution of household earnings. To do that we follow the procedure used by Hugget and Ventura (2000). This implies first fixing the standard deviation of the innovation $\upsilon_{i,t}$: we take the value of 0.025 which is consistent with the different estimates available for AR(1) process of earnings.[10] Then we fix the permanent component of individual earnings θ_i so that the two jointly allow the model to match the Gini index of earnings for first year workers. Finally we set ρ, the autocorrelation coefficient of the AR(1) process of earnings to 0.97 so that we can match the Gini index of earnings in the general population.

An important issue is the calibration of the social security system. This is because in the US economy replacement ratios used to compute retirement benefits are progressive. For this reason agents with high earnings will need to accumulate more wealth relative to their earnings to finance retirement consumption when compared to low earners. In order to perform the calibration we proceed in two steps. First we compute the average life-time earnings conditional on an agent's type and last year of work earnings. This forms the base used to compute the pension benefit the agent receives during retirement.

[10]See Hubbard, Skinner and Zeldes (1994) and Hugget and Ventura (2000).

Second the formula used in the US economy is applied to this average lifetime earnings. [11] This formula fixes two bend points at 0.20 and 1.24 times average earnings [12] and attributes a benefit that is 90 percent of earnings up to the first bend point, 32 percent from the first to the second and 15 above that. Retired households also receive social security payments in the form of medical and hospitalization benefits that are independent of their earnings history, so that we also add a fixed component to the benefit and set it approximately equal to 19 percent of average earnings, a value consistent with the one reported in Huggett (2000). To understand the implications of the progressive formula of social security benefit we also consider the case of constant replacement ratio fixed at the average level of the US economy.

2.7.3 Asset Returns and Transaction Costs

We assume that the constant return to bonds R_f is 2% and that the average equity premium is 4% a value that is somewhat below the historical one but is the one commonly used in this literature (e.g. Campbell et al. (1999), Cocco (2001) or Gomes and Michaelides (2005)). As far as the standard deviation of the risky return is concerned we fix its base value at 16% a value consistent with the historical evidence about the volatility of the stock market index. While this value will be used in many of the simulations in others we will explicitly recognize the fact, reported for example in Investment Company Institute (1999 and 2002) that households typically invest in a limited number of stocks and that this number is an increasing function of the household wealth. Given the evidence reported in Campbell et al. (2001) that individual stock returns are substantially more volatile than the stock market index this will presumably lead higher wealth households to have stock portfolios with lower variance. To capture this pattern in a reduced form we postulate the following function:

$$g(w) = 1 + \frac{1}{1 + e^{\delta(w - \bar{w})}} \tag{12}$$

and set $\bar{w} = 30$ and $\delta = 0.2$ implying that agents close to the borrowing constraint face twice the volatility of the stock market index and that this volatility is reached around four times average wealth. We believe this to be a conservative estimate of the difference in stock portfolio volatility between high and low wealth individuals. This is because many low wealth agents do invest in just

[11] This calibration method is not perfect since in general two different agents of the same type that receive the same earnings shock in the last period of working life will have different past earnings histories and therefore also different average lifetime earnings. This method though is the best that can be done without adding a further state variable to keep track of average past earnings.

[12] See Huggett and Ventura (2000) or Social Security Online (2004).

one stock and according to Campbell et al. (2001) the volatility of individual stocks is about 3 times the one on the market portfolio.

Next we have to calibrate the two different costs that agents face to participate in the stock market. There is no empirical estimate of the initial entry cost F_I so we set it to 0.075 which is equivalent to about 3 percent of the yearly average wage, near the value used for example by Gomes and Michaelides (2005). There have been instead efforts to estimate the per period participation cost. Work by Vissing-Jørgensen (2001) and by Paiella (2001) have found values between 50 and 200$ so that we fix F_P to 0.01 a value that makes the model ratio of fixed per period cost to average earnings consistent with the upper bound of the interval just described.

3 Results

In this section I report results for a sequence of models with increasingly rich structure. For convenience of exposition I organize results into two subsections. In the first one I start with a benchmark case that is very similar to the base cases considered in Cocco, Gomes and Maenahout (2005) or Gomes and Michaelides (2005). I then add to the basic model a realistically calibrated social security system and fixed per-period participation cost. In the following subsection I consider the effects of introducing a bequest motive with actual transmission of wealth through the generations and the role of under-diversification. In all cases the focus will be on overall participation rates and on the life-cycle profiles of the participation rate and the portfolio share of stock conditional on participation. Each economy is obtained from the previous one by adding the relevant marginal feature and re-calibrating the subjective discount factor so as to keep the aggregate wealth-to-earnings ratio constant across experiments.

3.1 Fixed per-period participation cost and social security

The first model I present is a benchmark case where each agent receives a pension benefit that is a constant fraction of average past earnings conditional on his earning type and last year of work earning shock. The replacement ratio is fixed at the average replacement ratio implied by the calibrated social security system of the later experiments. This replacement ratio turns out to be 0.502. No fixed per-period cost is assumed. The value of β is 0.89; with this value the wealth-earnings ratio in the economy is 5.02. Results are reported in Figures 1 and 2; relevant variables are reported by 10 year age groups. The first of the two figures reports participation rates by age. The thick continuous line represents the average participation rate by age groups in the economy. As it may be seen

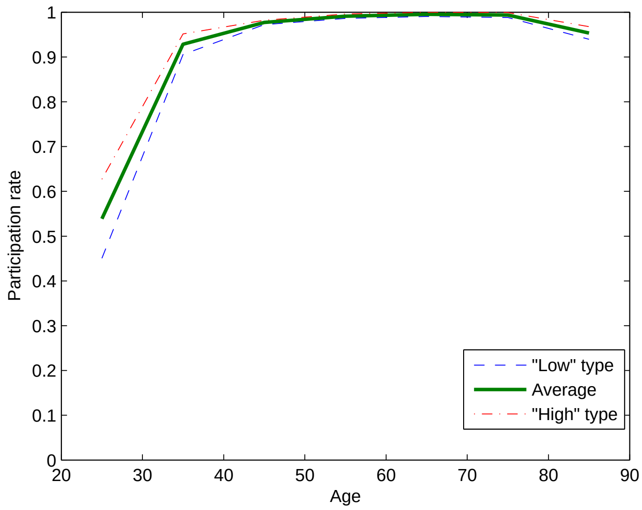

Figure 1: Life-cycle participation rates

this rate is low at young ages since agents have not yet accumulated enough wealth to make it convenient to pay the initial entry cost. As households move into mid age and wealth accumulation to finance retirement consumption picks up the participation rate jumps up to reach a plateau of almost 100 percent; later in life it stabilizes basically reflecting the fact that in the absence of any further cost of staying in the stock market all agents with positive wealth will hold at least some stock in any period of life. This result is counterfactual since a number of studies found that participation rates tend to be hump-shaped over the life-cycle. The graph reports two more lines: the dashed line represents the participation rate for low earning ability households and the dash-dot line does the same for the high ability households. As expected the participation rate for high ability types is higher than the average and that of low earning types is lower. This reflects the fact that the latter have on average lower earning thus less wealth than the former so that a lower fraction will accumulate enough to pay the initial entry cost. It is worth noticing though that this difference is not large. Figure 2 reports the life-cycle profiles of the conditional stock share. These profiles reproduce the well known result that upon paying the entry cost agents would invest 100 percent of their wealth in stocks; after that the portfolio share of stocks declines monotonically and substantially until retirement and then increases somewhat towards the end of life. This profile is in contrast to the empirical evidence that suggests that the conditional share is always well

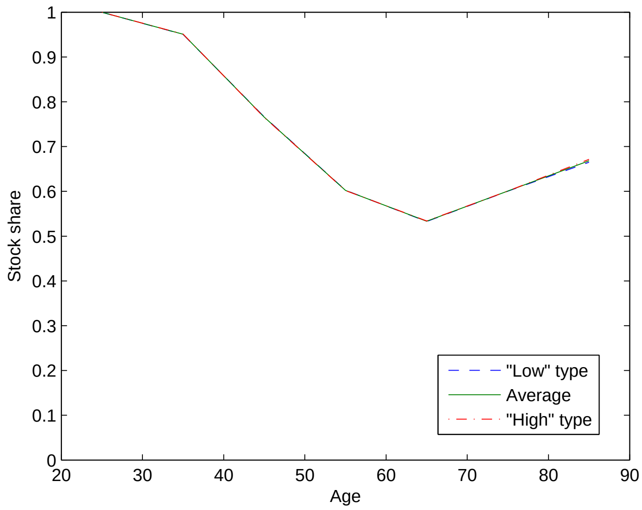

Figure 2: Life-cycle conditional stock share

below 100 percent and is roughly flat or slightly increasing in age. A second feature of the model generated profiles is that the average conditional share is the same for both earning ability types over all of the life-cycle. The intuition for these results is well known and will be explained by way of Figure 3. The figure reports for each age, the ratio between average financial wealth and the average present discounted value of the remaining stream of earnings and pensions until death. [13] The intuition for the portfolio result is that earnings, even though uncertain, are a better substitute for the risk-free bond. Agents with CRRA utility facing i.i.d. stock returns want to keep a constant share of their total wealth in the risky asset so that when financial wealth is low relative to human wealth they would like to invest all of their financial portfolio in stocks while as financial wealth becomes larger they would diversify more and more towards bonds. A comparison between Figure 2 and 3 clearly shows this. The ratio of financial to residual human wealth starts from 0 at age 21, since with no bequests all agents enter working age with no wealth at all. It then picks up quickly as agents start to accumulate for precautionary reasons first and to finance retirement consumption then, while at the same time the shortening of the

[13]Here average wealth is simply the simulated average wealth for each earning type, age group. The same is true for the present discounted value of earnings; the results are obtained when future earnings are discounted at a 3 percent rate, but they would not change significantly when the discount rate is fixed at 2 or 4 percent.

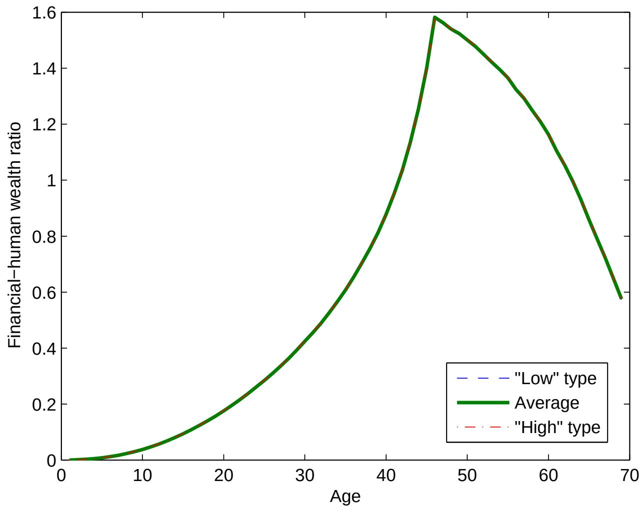

Figure 3: Life-cycle financial to human wealth ratio

remaining horizon reduces human wealth. After retirement then consumption of the accumulated wealth reduces the ratio once more. The inverted V-shape that results mirrors the V-shaped pattern of conditional stock share. While this is no new result the point that is worth stressing is that under proportional replacement ratios the pattern of wealth accumulation of high earning types is simply a scaled up version of that of low types with the scaling factor being the same as the one of earnings, thus the ratio of the financial to residual human wealth is the same for both types of agents. This is reflected in a life cycle profile of the conditional stock share that is the same for both types of agents.

The next step is to introduce a fixed per period cost of participating in the stock market. Since the pattern of wealth accumulation is only marginally affected by this change there is no need to re-calibrate the value of β which is then left at 0.89 as before, generating an average wealth-earnings ratio of 5.03 almost identical to the one in the previous experiment. Results are reported in Figures 4 and 5. The first of the two figures reports the life cycle participation rates. As in the previous case the participation rate is relatively low in the age group 20 to 30 and then increases rapidly as households accumulate wealth to reach a peak of almost 100 percent in mid-life. The novel element here is that late in life there is a non-negligible decline in participation rates down to 70 percent. This result brings the model closer to the data since a hump in participation is empirically observed but cannot be rationalized with the

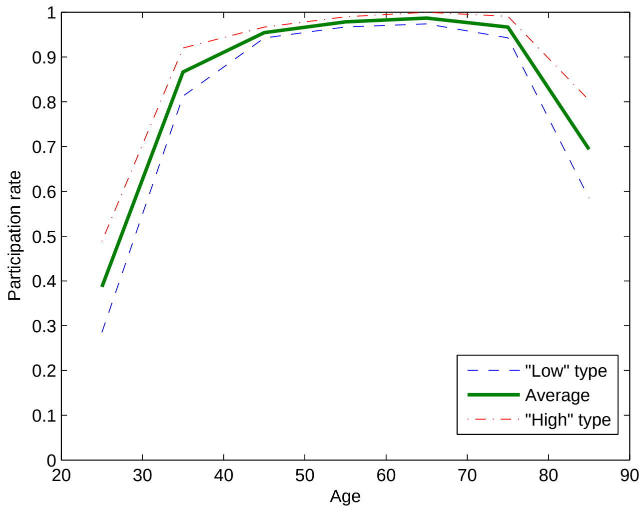

Figure 4: Life-cycle participation rate

usual assumption of a fixed one-time entry cost. The results for the conditional stock share, reported in Figure 5, do not show significant changes compared to the benchmark case: the profile still starts from 100 percent in the youngest age group, declines until around retirement age and then picks up slightly as agents approach the maximum allowed age. This is not surprising since they are driven by the evolution of the financial to human wealth ratio and this is not affected by the introduction of the fixed per period participation cost. The only minor differences are that the increase in conditional stock shares late in life is less pronounced and that late in life the stock share of the high earning ability group is slightly higher. Both are the consequence of the fact that the small participation cost truncates the wealth distribution to the left: while in the general population the financial to human wealth ratio is unchanged compared to the benchmark case, if we confine our attention to the subset of stock market participants this is somewhat higher — and it is more so for the low earnings types — since very low wealth agents won't find it attractive to pay the participation cost.[14]

[14]In principle this mechanism should apply also with the entry cost earlier in life. However very early in life human wealth is very high and the financial to human wealth so low that every agent would like to invest 100 percent in stocks when it becomes convenient to pay the entry cost. In mid-life instead the accumulated level of assets is such that virtually everybody would pay the cost so that this has no bite in truncating the wealth distribution of either type of agent.

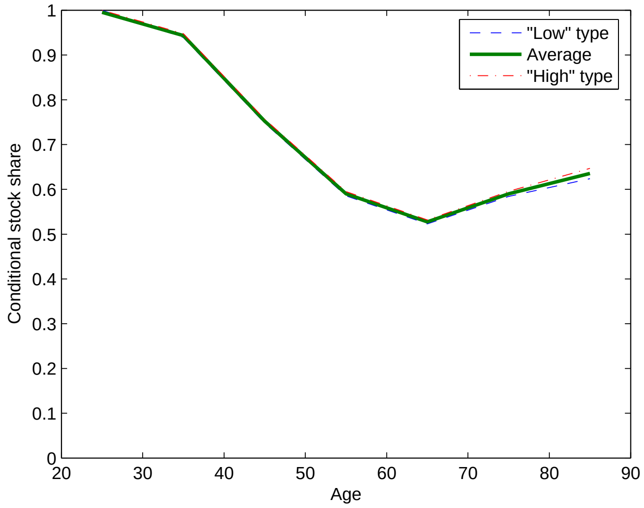

Figure 5: Life-cycle participation rate

The next step is to introduce a progressive social security system with the benefit formula defined in the calibration section and modeled according to the rules of the US social security. In this case the discount factor β needs to be increased to 0.92 in order to keep the wealth to earnings ratio at the target ratio of 5. While this implies that the average life-cycle profile of wealth and the financial to human wealth ratio is not substantially altered from the previous cases, when we look at the two earning ability groups separately the picture is different. With progressive social security, households that have higher earnings face a lower replacement ratio, so that they need to accumulate more assets to smooth their consumption past retirement age, compared to lower earnings households. Even though a high type household may have a lower expected replacement ratio than a low type household if it experiences a sufficiently worse earnings shock, on average high type households will have higher earnings, thus lower replacement ratios.[15] This can be seen in Figure 6 where the curve representing the financial to human wealth ratio for the high earning types lies above the one for low types. The difference is minor in the first decades of working life — when saving occurs mainly for precautionary reasons — but becomes more pronounced in the decade before retirement and even more after that. The effects that this has on households' stock market decisions are reported in

[15]Recall here that the benefit formula is applied to average earnings conditional on both the household earning type and its last year of life earnings shock.

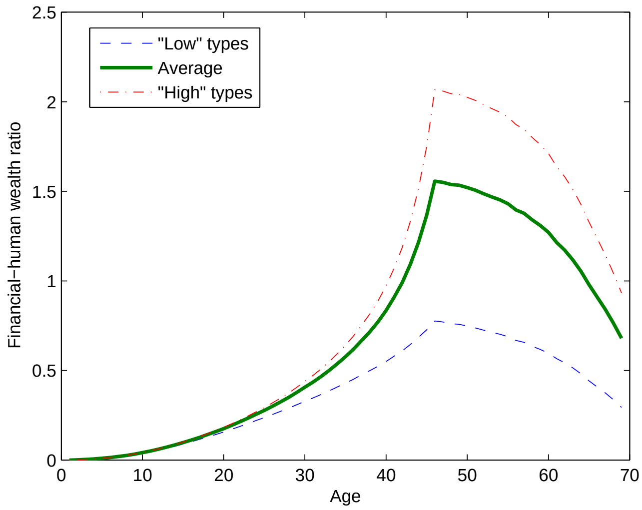

Figure 6: Life-cycle financial-to-human wealth ratio

Figures 4 and 8. The first of the two figures shows that there is a reduction in participation rates for both types of agents especially very late in life. Under progressive social security, those agents facing bad shocks late in their career have very high expected replacement ratios, hence they will choose to enter retirement with very little or no wealth at all and rely entirely on pension benefits to finance their consumption. In both cases the effect is not to participate in the stock market, either because they don't have any asset or because they don't have enough to pay the per period cost. [16] If we look at the conditional portfolio share of stocks in Figure 8 we see that once again the profiles start from a 100 percent share in the first decade of life and then substantially decrease until retirement age, after which they stabilize. The other important feature that emerges from the graph is that early in life the conditional share is about the same for the two types of agents; as retirement approaches high earning types start to choose on average a reduced exposition to stock risk compared to low earning types and the difference becomes substantial after retirement. This is not surprising in light of the well known intuition behind the behavior of conditional stock shares given above and the result in Figure 6 that shows

[16] Indeed an important chunk of the extra non-participation induced by the US formula of pension benefits is the consequence of agents with 0 wealth late in life. This is shown in plots of the fraction of agents with positive wealth against age that are not reported to economize on space.

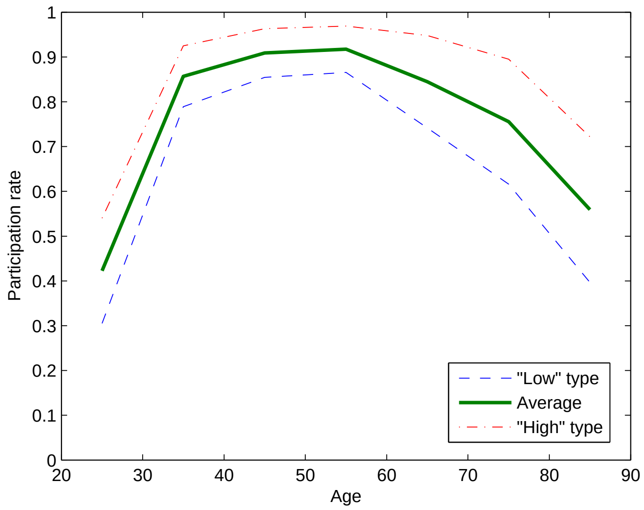

Figure 7: Life-cycle participation rates

substantially higher financial-to-human wealth ratios for high types during retirement. A few comments are needed about this result. It is common among financial advisor to suggest investment strategies that relate the share of portfolio to be invested in the stock market to age. A popular advice is that the share of risky assets should decline with age. [17] The result obtained here is that the suggested strategies omit a key factor, that is, income. Because of the progressive nature of the pension benefit formula, low income households implicitly hold a larger position in the risk-free asset and would benefit the most from exploiting the equity premium, while high income households — who hold a relatively smaller position in risk-free human capital — should try to diversify more to bonds to avoid excessive exposure to stock market risk. At a positive level the available evidence (see for example Vissing-Jørgensen (2001) and Kennickell et al, 2000) seems to point to a positive relation between earnings and the share of stocks adding a new fact that is puzzling to portfolio choice theory, however great caution should be exerted before drawing conclusions from those studies since neither is a perfect empirical counterpart to the plot showed above. [18]

[17] Cocco, Gomes and Meanhout (2005) and Jagannathan and Kocherlakota (1996) report this kind of advice and examine how sound it is from the view point of economic theory.

[18] The positive link between nonfinancial income and the stock share found by Vissing-Jørgensen (2001) is conditional on a complete set of variables that may affect this choice; the one found in Kennickell et al. (2000) is unconditional. The one in this paper is something in

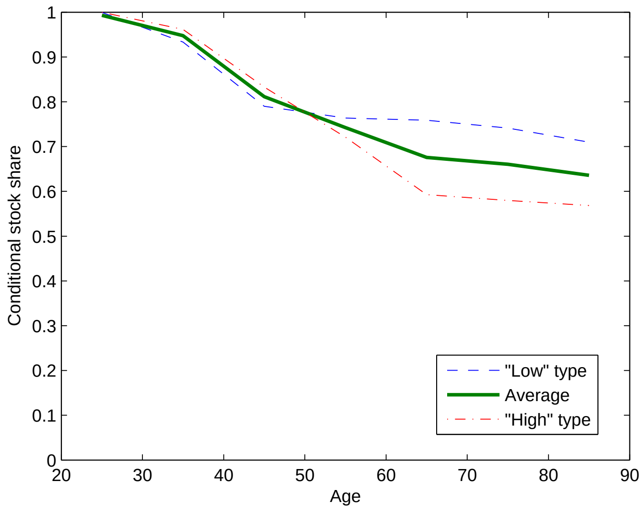

Figure 8: Life-cycle conditional stock share

Table 1: Average participation rates

Benchmark	0.89
Fpc, prop. s.s.	0.83
Nfpc, prog. s.s.	0.856
Fpc, prog. s.s	0.771

Before turning to the role of bequests and under-diversification I will present, with the help of Table 1 summary results about the average stock market participation rate and how this is affected by the fixed per period cost and progressive benefit formula. The participation rate in the benchmark case is 0.89, a value that is very high compared to what is observed in reality. We saw that both a small fixed per period participation cost and social security improve the performance of the model enabling it to obtain a hump-shape life-cycle profile of participation rates. When we try to quantify this effect though, it seems rather small. Adding a fixed participation cost of the size suggested by the studies of Paiella (2001) and Vissing-Jørgensen (2001) only reduces this participation rate to 0.831, just 6 percentage points below the benchmark case. When this is combined with the more spread out wealth accumulation profiles that result from a progressive benefit formula a further decrease of 6 percentage points in participation rates results. Overall then the reduction compared to the benchmark

between since it reports the share of stock by income, conditioning on age.

case is of only 12 percentage points.

3.2 Bequests and under-diversification

In the last section we saw that the benchmark life-cycle portfolio choice model produces a very high participation rate that quickly increases in the first part of life and then stabilizes at 100 percent. It also generates conditional stock shares that start at 100 percent in the first decade and then follow a V-shaped pattern. Both facts seem at odds with the empirical findings. Motivated by these failures I introduced a fixed per-period participation cost and a progressive pension benefit formula. These new features allow the model to produce realistic inverted U shaped pattern of participation rates. However, quantitatively the reduction in the average participation rate falls short of the one needed to match the empirical evidence. Moreover the conditional stock share is still ad odds with the evidence since it is very high, monotonically declining and higher for low earning ability types. For these reasons in this section I consider two more extensions of the model that capture relevant features of the heterogeneity of households in the economy. First I consider the existence of a bequest motive with actual transmission of the parent household's estate to the descendant. Second I introduce the possibility that agents don't actually purchase the stock market index, they buy instead some stock portfolio and that higher wealth households have better opportunities for diversification so that the variance of this stock portfolio declines with the total amount of assets held. The two features are first introduced separately: it is shown that while each of them improves the performance of the model along some dimension they worsen it on others. However when taken together they give satisfactory results.

First I will present the results that are obtained when an active bequest motive and intergenerational transmission of wealth are taken into account. The extension, as in the remaining cases of this section, is done using the last model of the previous section as a starting point. This means that in all models presented in this section there is a fixed per-period participation cost and the formula for social security benefit calculation is progressive. The introduction of bequests increases savings late in life, hence also the average wealth to earnings ratio. The discount factor β is then lowered to 0.885 so as to keep that ratio constant. Results for the life-cycle profile of the participation rate are shown in Figure 9. The figure shows no big difference with the previous case. Participation rates are still hump shaped in age and higher for high earning types than for low earning types. For a given average wealth income ratio the addition of bequests redistributes wealth away from mid-life to the two extremes. This suggests that we should observe higher participation rates early and late in life

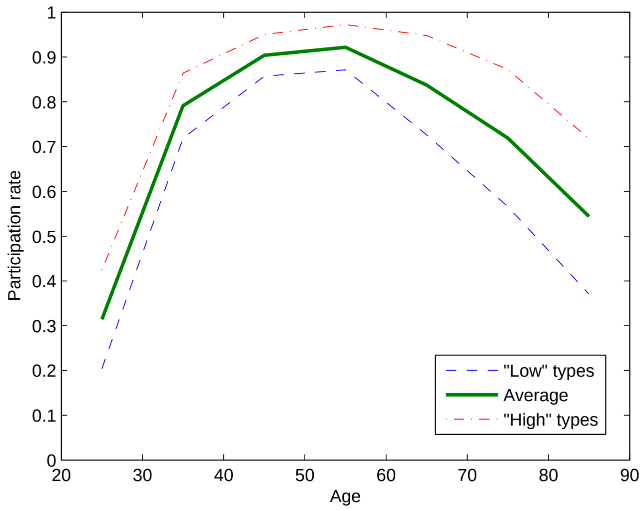

Figure 9: Life-cycle participation rates

and lower participation in the middle. This does not seem to be the case here. Late in life agents tend to have larger wealth, however the fact that the fixed per period cost is small does not translate into large differences in participation rates. Early in life the participation rate is even lower than in the no bequest case, since the direct effect of the fact that a few agents receive bequests is more than compensated by the lower wealth accumulation of the majority of agents brought about by the reduction of the discount factor. The results concerning the conditional stock share are reported in Figure 10. In this case the introduction of bequests seems to generate some improvements since now the conditional share is somewhat lower than 100 percent in the first decade of life. Unfortunately though at a quantitative level the effect is minor since it is still around 95 percent. The intuition for this result is that once intergenerational transfers are allowed, some agents will have substantial amounts of wealth early in life which reduces their optimal stock share. However since young agents have young parents who face low mortality rates, the fraction of them who inherits is small and not sufficient to cause a big decline in the observed average share. Among the other features of the life cycle profiles of the conditional stock share are the the fact that they are declining in age and that starting from mid life the profile of the high earning group is below the one of the low earning group. Both features are shared with the previous model and especially the first one seems to be at odds with the empirical evidence. This justifies the claim made at the

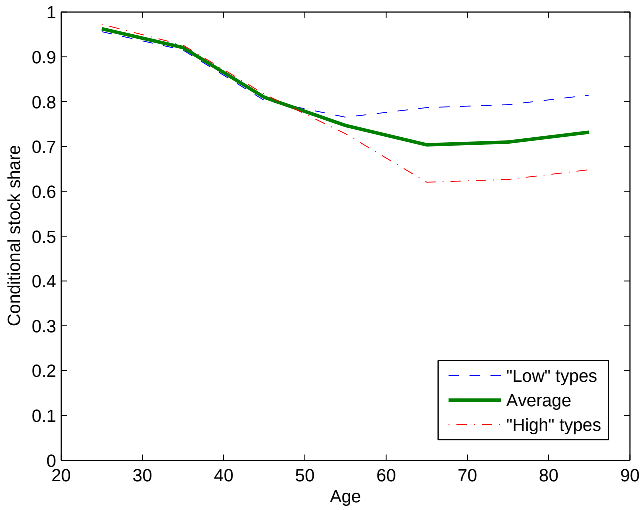

Figure 10: Life-cycle conditional stock share

beginning of this section that the introduction of bequests improves somewhat the performance of the model along some dimensions only, so that it is not a complete solution to the problem of matching model generated profiles with the ones from the data.

Next I turn to the model where there is no intergenerational transmission of wealth but agents face a variance of their stock portfolio return that is declining with their wealth holdings. As it will be shown below the fact that now the risk of holding stocks is greater implies that households will participate less frequently and hold a smaller share conditional on participation. In turn, since portfolios become more heavily tilted towards the low return and safe bond wealth accumulation will proceed at a lower pace. This forces an increase of the discount factor to 0.94 in order to keep the wealth to earnings ratio close to its target. Results for this case are shown in Figure 11 and 12 for the participation and conditional life-cycle profiles respectively. A look at Figure 11 reveals that participation rates are substantially reduced: this reduction occurs among all agents except mid-life high earning types and it is particularly strong for low earning type whose participation rate now barely reaches a peak of 50 percent among the 60 to 70 year old group. The reason for this result is that the increased variance of returns associated with small holdings of wealth substantially reduces the benefit of the equity premium inducing a large number of agents either not to pay the initial cost and never enter the stock market

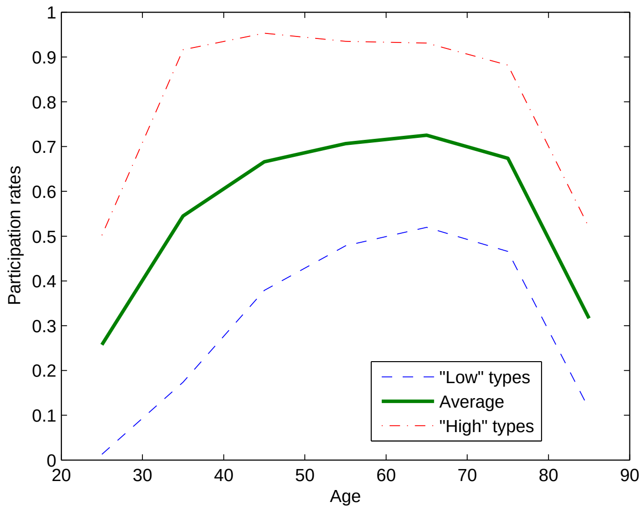

Figure 11: Life-cycle participation rates

or to stop paying the cost earlier and quit it sooner after entering for the first time. The impact of the increased variance of stock returns for small amounts of holdings is also strong when we look at the conditional stock shares. A look at Figure 12 shows that the average conditional share is greatly reduced except in the first decade of working life. For most of the life-cycle it is only about 30 percent while it was about 70 percent in the previous models. The share held by high earning types now lies well above the ones of low earning types. The intuition behind this result is straightforward: now agents face a variance of returns that is higher than the one on the stock market index so they all want to reduce their exposure to stocks. However because this variance declines as the agent becomes wealthier and thus has better opportunities to construct a well diversified stock portfolio, the reduction in stock shares is smaller for high earning and wealthier agents than for the rest of the population. It turns out that this effect is opposite in sign and stronger than the increase in their financial to human wealth ratio induced by the progressive formula for social security benefits. Despite the success of this formulation in reducing the average conditional stock share, it is still true that its life-cycle profile is declining with age especially in the first decades of life.

I next turn to the results that are obtained when both bequests and a variance of the stock portfolio that is declining in wealth are considered at the same time. As before, the introduction of bequests increases wealth accumulation late

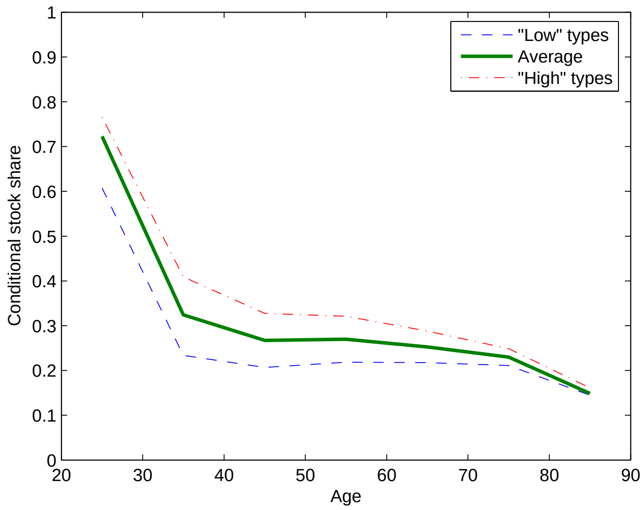

Figure 12: Life-cycle conditional stock share

in life, so that more impatience is needed to keep the average wealth earnings ratio constant. A value of 4.95 for this ratio is obtained by lowering β to 0.90. The life cycle profile of participation rates is depicted in Figure 13; the figure shows no important changes compared to the model with perfect diversification but no bequests. Again the introduction of intergenerational transmission of wealth seems to reduce somewhat the participation rate, especially at young ages. This is because while a few agents inherit early in life so that they can pay the entry cost and start to invest in the stock market, all of them are more impatient which reduces wealth accumulation in the first part of life reducing participation in the stock market as well. The results for the share invested in stocks conditional on participation are reported in Figure 14 and also do not show great changes compared to the previous case. The most notable difference is that now the profile for the average conditional stock share in the population is virtually constant from the 30 to 40 year old group until the oldest group and the decline in the share observed between the first two decades of life is reduced. When moving from the 20 to 30 year old group to the next one the conditional stock share declines from slightly below 50 percent to about 30 percent. In the model without bequests the decline was from 70 to 30 percent. This difference is the consequence of the fact that when intergenerational transmission of wealth is allowed some agents may receive substantial bequests; this increases their financial to human wealth ratio reducing their optimal stock share and

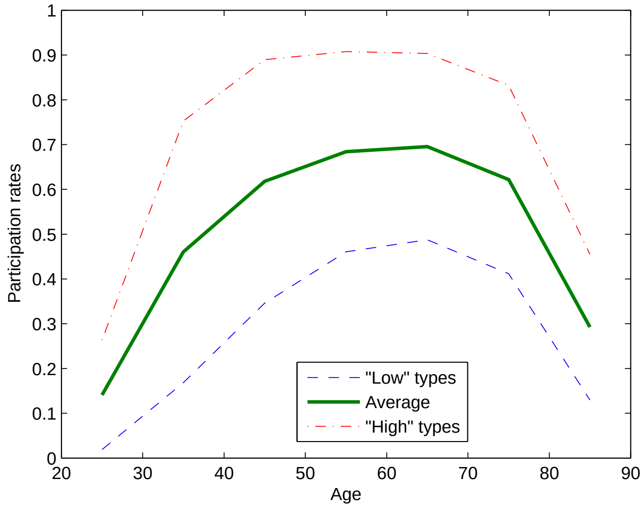

Figure 13: Life-cycle participation rates

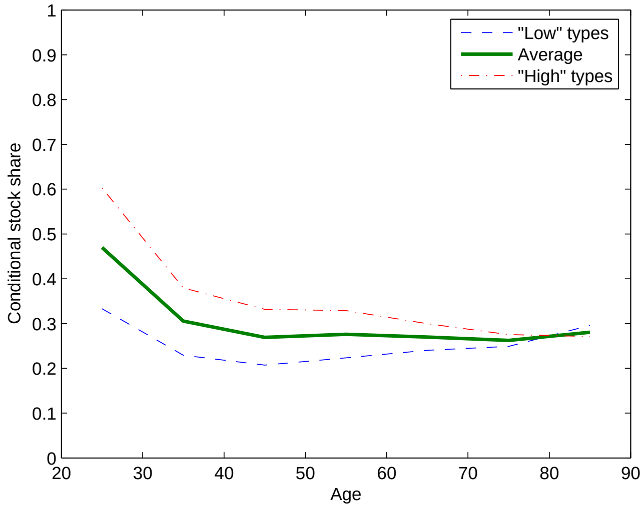

Figure 14: Life-cycle participation rates

Table 2: Average participation rates

Nb, cvr	0.771
Nb, vvr	0.568
B, cvr	0.732
B, vvr	0.509

consequently the average of their age group.

To close this section I report once again figures for the average participation rate. This is done in Table 2. The first line of the table corresponds to the last line of Table 1, that is, a model where social security is progressive, there is a fixed per period participation cost, but there is neither intergenerational transmission of wealth nor wealth related diversification opportunities. As said in the previous section the average participation rate is 0.771 in this case. When bequest are added — third line of Table 2 — the average participation rate goes down to 73.2 percent, a reduction explained by the lower discount factor needed to keep a constant wealth earnings ratio, which delays the initial accumulation of wealth hence postpones the age at which the entry cost is first incurred. Considering the possibility that under-diversification acts more strongly on less wealthy agents reduces the participation rates in a quite substantial way. In the model with no bequests the participation rate is reduced to 56.8 percent — down from 77.1 percent —, in the model with bequests the participation rate goes down to 50.9 percent compared to 73.2 percent in the case with constant volatility of the stock portfolio return. This participation rate is indeed only slightly above the one observed in the most recent issues of the Survey of Consumer Finances.

4 Conclusions

In the present paper I have considered a number of extensions of a widely used model of life-cycle asset allocation and compared its predictions for the cross-sectional profiles of participation rates and conditional stock shares to the ones in the data. The main results that emerged are summarized here for convenience. First a fixed per period participation cost must be added to the model to generate a hump shaped profile of life-cycle participation rates. Second the introduction of a progressive formula to determine social security benefits makes it optimal for low income agents to invest larger shares of wealth in the stock market than high income agents. Moreover by generating more agents with low or no wealth at all it magnifies the effect of participation costs in reducing participation rates, although the two mechanisms jointly are not sufficient to bring the model close to the data. Third, recognizing the lack of diversification in stock portfolios has a strong impact in reducing both the participation rate and the conditional stock share. Finally intergenerational transmission of wealth helps make profiles of the conditional stock share independent of age as the empirical evidence suggests.

An important goal of this paper, as stated before, was to compare model life cycle profiles with the data. Since both participation rates and conditional shares are strongly affected by wealth levels, in pursuing that goal, I tried to constrain the model to generate appropriate values of the average wealth to earnings ratio. This was done by calibrating the value of the discount factor. It is important to recognize that at this stage this approach is more meant to provide sounder qualitative results than to make exact quantitative statements. Among the reasons why this is so, two point to interesting directions for future work. First the wealth earnings ratio targeted in the calibration includes all wealth but it is well known that a substantial part of this wealth is held in the form of housing which is omitted from in the present model. Introducing housing in the model would then be an important extension.[19] Second the model pointed out that the problem of lack of diversification in households' stock portfolios, recognized empirically by Curcuru et al. (2005) may be a force that strongly affect both the decision to participate and the share to invest in the stock market. In the present paper I have chosen a reduced form arbitrary function to model this phenomenon, however in order to make a precise quantitative statement it would be useful to investigate further the problem so as to reach a more accurate formulation.

Finally one of the predictions of the model, that is, the fact that in the presence of progressive social security — and absent the diversification problem — high income households should invest less in stock than low income households conditional on age points to useful further empirical work.

[19]Cocco(2005) and Yao and Zhang (2005) have considered housing in their model. In their model though, housing introduces one more state and choice variable, which given the complexity of the current model is not very desirable. A simpler way to consider housing is the one in Gomes and Michaelides (2005).

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