A discusión

CAPACITY RESTRICTION BY RETAILERS*

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ABSTRACT

A monopolist retailer facing two suppliers producing two symmetric and independent goods improves its bargaining position by commiting to sell only one good. We analyze if this advantage extends to the case where there are two undifferentiated retailers competing in the same market. With linear supply contracts, we have partial capacity restriction in the sense that only one retailer commits to sell only one good. Then, we have that if retailers were to merge, welfare would decrease because the merger reduces the variety of goods available to consumers.

Keywords: retailing, mergers, variety

JEL classification: L13; L41; L42

1 Introduction

In the last few years, concentration in the retailing sector has increased very much. This has been the result of a huge number of both national and cross-border mergers. This trend has raised the concern of antitrust authorities for their possible negative effect on welfare. Apart from the well-known effect they can have on final prices, retail mergers are investigated because they may increase the bargaining power of retailers vis-à-vis suppliers.

One of the worries is that this higher bargaining power may be achieved by delisting some of the goods firms were selling before merger. This will harm welfare, because it will reduce the variety of goods consumers can buy.

Delisting can be a rational strategy, because retailers committing to limit the number of goods they will sell can obtain price discounts from suppliers. In other words, retailers can obtain price discounts by forcing suppliers to compete for the scarce shelf space. Those incentives can be illustrated in the following simple example taken from Gabrielsen and Sorgard (1999). Suppose we have two symmetric upstream firms producing two independent goods. They sell the good through a monopolist retailer. If the retailer takes both goods, each supplier will set wholesale prices above marginal cost. However if the retailer commits to take only one good, competition between suppliers will drive the wholesale price to marginal cost. In a linear example, this latter strategy gives more profits to the retailer.

However, delisting will only occur after merger if those incentives to limit the number of goods increase as competition decreases. This is indeed what we obtain in this paper. When we add a competitor in the retailing sector in the example above, we obtain that only one retailer limit the number of goods she wants to sell. Therefore, both goods are sold before merger: reducing competition in the retailing sector reduces the variety of goods offered to consumers.

Although the merger reduces wholesale prices the effect on variety dominates and the merger reduces welfare.

The most closely related paper to ours is Gabrielsen and Sorgard (1999). They analyze the case where there is one retailer and two suppliers that use linear supply contracts. Goods are substitutes and one of the goods has a larger demand. The retailer chooses to take one good (require exclusivity) if goods are sufficiently symmetric. My paper extends Gabrielsen and Sorgard (1999) to allow for a competitor retailer while simplifying the demand side.

Inderst and Shaffer (2007) also analyze the effect of mergers of retailers on variety. The reduction of variety is explained by the same mechanism as in our paper. The monopolist increases its profits by committing to buy only one of the goods offered by suppliers. The difference is that they consider mergers of non-competing retailers while we focus on the case where retailers that merge compete in the same market. Dana (2006) shows that buyer groups obtain better terms of trade if they are composed by buyers with heterogeneous preferences and they commit to be supplied by a single seller. Here, buyers are final consumers and therefore they do not compete in the market.

In the next section, we consider the basic model where suppliers set linear contracts. We consider two cases depending on whether price discrimination is allowed. In the third section, we consider the same model but with two-part tariff supply contracts. Final conclusions put the paper to an end.

2 Linear contracts

Assume we have two producers (1 and 2). Producer 1 (2) produces good 1 (2). Goods 1 and 2 are independent. Demand of good i (i=1,2) is given by $P_i = a - Q_i$, where P_i and Q_i are respectively the price and the quantity sold of good i. Goods should be sold through two retailers

(1 and 2). To simplify things, we assume that there are no costs of production and retailing.

The crucial ingredient of our paper is that through shop design or other related mechanism the retailers can commit to the number of goods they will sell (0,1 or 2), before suppliers specify the supply contracts. The idea is that by restricting the selling capacity, retailers may indeed induce a reduction in wholesale prices. This is what we want to check next.

In this section, we assume that supply contracts only include a wholesale price that can be different for each retailer. Section 3 solves the model for the case with two-part tariff supply contracts. Furthermore, we assume that retailers compete à la Cournot.

Then we study the following four stage game:

In the first stage, retailers decide how many goods to carry. In the second stage, suppliers decide on the wholesale price to offer to each retailer. In the third stage, retailers that have chosen to carry only one good decide which good to sell. In the last stage, retailers decide how many units to buy from suppliers and how many units to sell in the market.

Next we analyze the game after retailers have decided how many goods to carry. We summarize the decision of the first stage with a vector, where the first component is the number of goods that retailer 1 decides to carry and the second component the goods that retailer 2 decides to carry.

If the decision in the first stage amounts to (2,2), i.e. retailers decide not to restrict the selling capacity, the results of the game are well-known. In this case, the markets are completely separated. If w_{ik} represents the wholesale price that producer i sets for retailer k, we have that producer i maximizes:

$$w_{i1}\left(\frac{a-2w_{i1}+w_{i2}}{3}\right)+w_{i2}\left(\frac{a-2w_{i2}+w_{i1}}{3}\right)$$

The optimal wholesale price is $w_{i1}^{22}=w_{i2}^{22}=\frac{a}{2}$. Each upstream firm obtains $\Pi_U^{22}=\frac{a^2}{6}$ and each retailer, taken into account that they sell two goods, $\Pi_R^{22} = \dfrac{a^2}{18}$.

If only one retailer decides to carry one good (1,0), we have that the competition between producers to be selected by this retailer drives the wholesale prices to zero. Therefore, we have that producers obtain zero profit and the retailer that sells the good $\Pi_R^{10} = \left(\frac{a}{2}\right)^2$.

Next we analyze the two cases that are left (1,2) and (1,1). In both cases we have that both retailers participate but we have restriction in the selling capacity.

In the case[1] (1,2), we have that by restricting capacity, retailer 1 obtains a reduction in the wholesale price. In equilibrium we have that $w_{11} = w_{21} = \frac{a}{4}$ and $w_{21} = w_{22} = \frac{a}{2}$ (Appendix 1). This reduction in the wholesale price translates into higher profits. The retailer that sells only one good obtains $\Pi_R^1 = \left(\frac{a}{3}\right)^2$ and the retailer that sells two goods $\Pi_R^2 = \left(\frac{a}{4}\right)^2 + \left(\frac{a}{12}\right)^2$. Given that $\Pi_R^1 > \Pi_R^2$, we have that specialized shops are more profitable than generalists shops. The reason is that variety is obtained at the cost of higher wholesale prices.

Then the last subgame to analyze is when both retailers have restricted the selling capacity (1,1). In this case, we have that in equilibrium each retailer sells a different good and they pay a wholesale price of $\dfrac{a}{2}$ (Appendix 2). It is like if both markets are separated and producers distribute the goods through only one retailer. Each retailer obtains $\Pi_R^{11} = \left(\frac{a}{4}\right)^2$.

The following payoff matrix describes the game in the first stage. Although retailers can choose not to carry any good, this option has not been included in the matrix, because it is a strictly weakly dominated strategy.

[1] A symmetric argument works for the case (2,1).

		Retailer 2	
		2	1
Retailer 1	2	$\frac{a^2}{18}, \frac{a^2}{18}$	$\left(\frac{a}{4}\right)^2 + \left(\frac{a}{12}\right)^2, \left(\frac{a}{3}\right)^2$
	1	$\left(\frac{a}{3}\right)^2, \left(\frac{a}{4}\right)^2 + \left(\frac{a}{12}\right)^2$	$\left(\frac{a}{4}\right)^2, \left(\frac{a}{4}\right)^2$

Proposition 1 *In equilibrium, we have partial capacity restriction: one retailer decides to offer one good while the competitor offers two.*

Then in equilibrium we have that only one retailer decides to restrict selling capacity. Therefore, we have the coexistence of specialized and generalist retailers. The asymmetric equilibrium prevails because the generalist shop would not obtain a reduction in the wholesale price if she was to reduce further selling capacity. It would only obtain a reduction in variety. We have that welfare is maximized in (2,2). Therefore, the restriction of selling capacity is detrimental for society.

The main object of the paper is to analyze the effect of the merger of both retailers. If retailers merge, they will decide to commit to take only one good. Then the competition between suppliers will drive the wholesale price to zero. The merged firm will obtain $\left(\dfrac{a}{2}\right)^2$ that is higher than the joint profits the merged firms obtained before the merger i.e. the merger is profitable. As far as welfare is concerned, we have that the merger has the positive effect of reducing wholesale prices and the negative effect of both reducing competition and variety. We obtain the negative effect dominates and the merger reduces welfare. This result is stated in the next proposition:

Proposition 2 *The merger of the two retailers reduces variety and welfare.*

2.1 Without price discrimination

In the previous Section, producers were allowed to charge different wholesale prices to retailers. However, this may turn to be illegal. For example the Robert-Patman Act, forbids price discrimination in the input market in order to protect small businesses. In this section, we re do the calculations of the previous section for the case where price discrimination is not allowed. Observe that in this case, a retailer may be less interested in reducing selling capacity, because she knows that any reduction in the wholesale price has to be shared with the competing retailer.

In the subgame where no retailer has reduced the selling capacity (2,2), the equilibrium of the previous section involved no price discrimination. Therefore, forbidding price discrimination will not change the equilibrium.

In the subgame, where both retailers have reduced the selling capacity (1,1), in the previous section we had an equilibrium without price discrimination $w_{11} = w_{21} = w_{12} = w_{22} = \frac{a}{2}$. It should also be an equilibrium now, because forbidding price discrimination can not increase the profits of deviations. A producer to serve both retailers should reduce the wholesale price to $\frac{a}{4}$, but this deviation do not increase the payoff. This equilibrium implies the same payoffs as the case where price discrimination was not forbidden.

Things change radically when only one retailer, say 1, has reduced its selling capacity (1,2). In this case, producers have to compete for the capacity of retailer 1, without the ability to grant her particular discounts. The situation is similar to Varian (1980) where firms face both captive and informed consumers. As in this case, no pure strategy equilibrium exists in the pricing stage. Next lemma describes the symmetric mixed strategy equilibrium.

Lemma 1 *When only one retailer has reduced its selling capacity, producers randomize in wholesale prices, in equilibrium according to the following distribution function:*

$$F(w)=\begin{cases} 0 & \text{if } w<\frac{a}{4} \\ 4+\frac{3a^2}{4w^2-4aw} & \text{if } \frac{a}{4}\le w<\frac{a}{2} \\ 1 & \text{if } w\ge\frac{a}{2} \end{cases}$$

Focusing on retailer 2, a producer can obtain $\frac{a^2}{8}$ by setting $w_i=\frac{a}{2}$. Those are the expected profits she obtains in equilibrium: the additional profits that can be obtained by serving retailer 1 are dissipated by competition. Observe that $\frac{a}{4}$ is the lowest wholesale price such that, serving both retailers, profits of the producer are not lower than $\frac{a^2}{8}$.

We assume that prices are realized at the beginning of stage 3. Then retailer1 will choose to offer the good whose wholesale price is lower. To solve the first stage we have tho calculate the expected profits obtained by retailers when only retailer 1 reduces its selling capacity (1,2).

Retailer 1 obtains:

$$\int_{a/4}^{a/2}\frac{\partial(1-(1-F(w))^2)}{\partial w}\left(\frac{a-w}{3}\right)^2dw=\frac{a^2}{8}(\log[256/3]-4)>\frac{a^2}{18}$$

Retailer 2 obtains:

$$\int_{a/4}^{a/2}\frac{\partial(1-(1-F(w))^2)}{\partial w}\left(\frac{a-w}{3}\right)^2dw+\int_{a/4}^{a/2}\frac{\partial(F(w))^2)}{\partial w}\left(\frac{a-w}{2}\right)^2dw=\frac{a^2}{32}(44-\log[243/4^{32}])>\frac{a^2}{16}$$

Then we still have that the equilibrium is (1,2) (or (2,1)). The difference is that now, because price discrimination is not allowed, the specialized firm earns less profits than the generalist retailer. The reduction obtained by restricting capacity is so important that is the optimal strategy, although price cuts have to be shared with the competitor.

3 Two-part tariff

We analyze the same four stage game as in the previous Section but we consider that supply contracts take the form of two-part tariffs including a wholesale price and a nonnegative fixed fee. The important difference between the two cases is that with linear contracts, producers value retailers competition, because it reduces the double marginalization problem. This is not the case with two part-tariff contracts. A producer obtains the same profits with one or two retailers.

This explains that solving the game is simple when there is one retailer that carries two goods. Upstream firms abandon the other downstream firms (setting an infinite fixed fee) and extract the monopoly profits setting appropriate two part tariffs to the downstream firms that has chosen to sell two goods. Producers have enough with one retailer that sells two goods. The existence of the other retailer does not increase their profitability.

Perhaps more surprisingly, we obtain (see Appendix 3) the same result when each retailer offers only one good. Producers distribute the good through a different retailer, but they are able to extract the full monopoly profits.

Given that, whenever both retailers participate, they obtain zero profits, not participation is not any more a strictly weakly dominated strategy and therefore it can be chosen in equilibrium. Therefore, we have to study the cases where this option is chosen. If one retailer offers one good and the other none (1,0), producers compete for the only slot available and this competitions drives the wholesale price and the fixed fee to zero. Therefore, the upstream firms obtain zero profits and the downstream firm the monopoly profits in one market[2].

Given that retailers only obtain positive profits in (1,0), (0,1), we have multiplicity of equilibria in the first stage. The equilibria follow, (1,0), (0,1),(1,1),(2,1),(1,2),(2,2) where the first component represents the number of goods chosen by downstream firm 1 and the second component the number of goods chosen by downstream firm 2. In the two first equilibria the upstream firms obtain zero profits. In the remaining equilibria, each upstream obtain the monopoly prof-

[2]This is the same result obtained with linear contracts.

its in their market. However, if we introduce a positive but small cost to carry a good, only (1,0) and (0,1) survive. Therefore, the incentive to reduce selling capacity are more intense with two-part tariff, because retailers are less valuable to producers. Proposition 2 summarizes:

Proposition 3 *With two-part tariff and a small but positive cost of capacity, we have strong capacity restriction: one retailer offers to sell a good and the other one quits the market.*

4 Conclusions

We have studied the incentives of retailers to restrict the number of goods they sell in order to obtain bargaining advantages over suppliers. We have obtained that these incentives increase when competition in the retailing sector diminishes. Therefore one important consequence of mergers is that they reduce the variety of goods available to consumers. This effect should be taken into account when evaluating the impact of mergers on welfare.

5 References

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6 Appendix

6.1 Appendix 1

The case (1,2)

Each upstream may always obtain $\dfrac{a^2}{8}$ by choosing $w_{i1} = a$ and $w_{i2} = \dfrac{a}{2}$. Then if, in equilibrium, upstream i is not chosen by 1, then $w_{i2} = \dfrac{a}{2}$. Then we obtain the optimal wholesale prices of upstream j if he sells to 1, given w_{i1} and $w_{i2} = \dfrac{a}{2}$.

$$\underset{w_{ji},w_{j2}}{Max} w_{j1}\left(\frac{a-2w_{j1}+w_{j2}}{3}\right)+w_{j2}\left(\frac{a-2w_{j2}+w_{j1}}{3}\right)$$

$$s.t. \left(\frac{a-2w_{j1}+w_{j2}}{3}\right)^2 \geq \left(\frac{a-2w_{i1}+\frac{a}{2}}{3}\right)^2$$

If $w_{i1} \geq \frac{a}{2}$, the constraint is not binding and $w^*_{j1} = w^*_{j2} = \frac{a}{2}$. If $w_{i1} < \frac{a}{2}$, then the solution is $w^*_{j1} = w_{i1}$ and $w^*_{j2} = \frac{a}{2}$. To check that these are the optimal wholesale prices, we have to check that upstream j obtains more profits than by setting $w_{j1} = a$ and $w_{j2} = \dfrac{a}{2}$. This is the case if $w_{i1} \geq \frac{a}{4}$. To close the construction of the equilibrium we have to check that upstream i choosing w_{i1} and $w_{i2} = \dfrac{a}{2}$ maximizes its payoff given $w^*_{j1} = w_{i1}$ and $w^*_{j2} = \frac{a}{2}$. The only thing to check is whether he can obtain more profits by being chosen by retailer1. In order to do so, the best thing is to set $w'_{i1} = w_{i1} - \varepsilon$ and $w_{i2} = \dfrac{a}{2}$. For what we have just seen, it will obtain less profits if $w_{i1} \leq \dfrac{a}{4}$. Then, in equilibrium we have that $w_{11} = w_{21} = \frac{a}{4}$ and $w_{21} = w_{22} = \frac{a}{2}$.

6.2 Appendix 2

The case (1,1)

In equilibrium, it should be the case that each retailer chooses a different upstream firm and makes positive sales.

Assume that given a vector of wholesale prices, retailer k (l)chooses upstream i (j) and $w_{ik} \leq w_{jl} < \frac{a}{2}$.

If $a - 2w_{jk} + w_{jl} > 0$, the upstream i increases its payoff with $w'_{ik} = a$ and $w'_{il} = w_{jl} + \varepsilon$, where $0 < \varepsilon < \min\{\frac{a}{2} - w_{jl}, \dfrac{a + w_{jl} - 2w_{jk}}{3}\}$. In this case, retailer k chooses upstream j and retailer l chooses upstream i, because $\left(\dfrac{a - 2w_{jl} + w_{jk}}{3}\right)^2 < \left(\dfrac{a - w'_{il}}{2}\right)^2$. Upstream i increases its payoff, because $w'_{il}\left(\dfrac{a - w'_{il}}{2}\right) > w_{ik}\left(\dfrac{a - w_{ik}}{2}\right)$.

If $a - 2w_{jk} + w_{jl} \leq 0$, the upstream increases its payoff with $w'_{ik} = \dfrac{a}{2}$ and $w'_{il} = a$. In this case, retailer l chooses upstream j and retailer k chooses upstream i because it would obtain zero choosing j. Upstream i increases its payoff because $\dfrac{a^2}{8} > w_{ik}\left(\dfrac{a - w_{ik}}{2}\right)$.

Assume that given a vector of wholesale prices, retailer k (l)chooses upstream i (j) and $w_{ik} > \frac{a}{2}$. This implies that

$$\left(\frac{a - w_{ik}}{2}\right)^2 \geq \left(\frac{a - 2w_{jk} + w_{jl}}{3}\right)^2$$

The upstream i increases its payoff with $w'_{ik} = \frac{a}{2}$ and $w'_{il} = a$. Then the retailer l chooses upstream j and retailer k upstream i, because

$$\frac{a^2}{16} > \left(\frac{a - w_{ik}}{2}\right)^2 \geq \left(\frac{a - 2w_{jk} + w_{jl}}{3}\right)^2$$

Then, in equilibrium if retailer k (l)chooses upstream i (j), we must have that $w_{ik} = w_{jl} = \frac{a}{2}$. Then we obtain the values of $w_{il} = w_{jk}$ so that we have indeed an equilibrium. We must have that $w_{il} \geq \frac{3a}{8}$ ($w_{jk} \geq \frac{3a}{8}$) in order that retailer l (k) does not want to choose upstream i (j). Furthermore we must have that $w_{jk} \leq \frac{a}{2}$($w_{il} \leq \frac{a}{2}$) such that upstream i (j) does not want to deviate and sell to both retailers.

6.3 Appendix 3

We analyze the case (1,1) with two-part tariff. Recall that the duopoly output is given by

$$q(w_1, w_2) = \begin{cases} \dfrac{a - w_1}{2} & \text{if } w_1 < -a + 2w_2 \\ \dfrac{a - 2w_1 + w_2}{3} & \text{if } -a + 2w_2 \leq w_1 \leq \dfrac{a + w_2}{2} \\ 0 & \text{otherwise} \end{cases}$$

and the profits in equilibrium by $(q(w_1, w_2))^2$.

Proposition 4 *In an equilibrium in which $D_{k(l)}$ chooses $U_{i(j)}$ we must have that $w_{ik} = 0$ and $F_{ik} = \left(\frac{a}{2}\right)^2 - \max\{0, (q(w_{jk}, w_{jl}))^2 - F_{jk}\}$.*

Proof. If the proposition does not hold, U_i obtains less than $\left(\frac{a}{2}\right)^2$ and it has a profitable deviation.

If $(q(w_{jl}, w_{jk}))^2 \leq F_{jl}$ and $(q(w_{jk}, w_{jl}))^2 > F_{jk}$, if U_i sets $w'_{ik} = \infty, F'_{ik} = \infty, w'_{il} = 0, F'_{il} = \left(\frac{a}{2}\right)^2 - \varepsilon$. Then in the unique equilibrium $D_{k(l)}$ chooses $U_{j(i)}$ and U_i increases its payoff for ε positive but low enough.

If $(q(w_{jl}, w_{jk}))^2 \leq F_{jl}$ and $(q(w_{jk}, w_{jl}))^2 \leq F_{jk}$, if U_i sets $w'_{ik} = 0, F'_{ik} = \left(\frac{a}{2}\right)^2 - \varepsilon, w'_{il} = 0, F'_{il} = \left(\frac{a}{2}\right)^2 - \varepsilon$. Then U_i is chosen by one retailer and increases its payoff for ε positive but low enough.

If $(q(w_{jl}, w_{jk}))^2 > F_{jl}$ and $w_{ik} > 0$, then if U_i deviates to $w'_{ik} = 0, F'_{ik} = F_{ik} + \dfrac{aw_{ik}}{2} - \dfrac{3w_{ik}^2}{8}$, $w'_{il} = \infty, F'_{il} = \infty$, the only equilibrium is that $D_{k(l)}$ chooses $U_{i(j)}$. Observe that for D_l is a strictly dominant strategy to choose U_j. Then D_k chooses U_i because $\left(\dfrac{a}{2}\right)^2 - F'_{ik} > \left(\dfrac{a - w_{ik}}{2}\right)^2 - F_{ik} \geq \max\{0, (q(w_{jk}, w_{jl}))^2 - F_{jk}\}$, where the last inequality comes from the fact that D_k was choosing U_i in the candidate equilibrium. It is possible to check that U_i increases its payoff: $F_{ik} + \dfrac{aw_{ik}}{2} - \dfrac{3w_{ik}^2}{8} - w_{ik}\left(\dfrac{a - w_{ik}}{2}\right) - F_{ik} = \dfrac{w_{ik}^2}{8} > 0$.

If $(q(w_{jl}, w_{jk}))^2 > F_{jl}$ and $w_{ik} = 0$ but $F_{ik} < \left(\frac{a}{2}\right)^2 - \max\{0, (q(w_{jk}, w_{jl}))^2 - F_{jk}\}$, U_i could deviate to $w'_{ik} = 0, F'_{ik} = F_{ik} + \varepsilon$, $w'_{il} = \infty, F'_{il} = \infty$. If ε is positive but small enough we have that the only equilibrium is $D_{k(l)}$ chooses $U_{i(j)}$. Observe that for D_l is a strictly dominant strategy to choose U_j. Then D_k chooses U_i because $\left(\frac{a}{2}\right)^2 - F'_{ik} > \max\{0, (q(w_{jk}, w_{jl}))^2 - F_{jk}\}$. U_i increases its payoff. ■

Proposition 5 *In equilibrium, every upstream firm obtains* $\left(\frac{a}{2}\right)^2$.

Proof. According to Proposition 4, in an equilibrium where $D_{k(l)}$ chooses $U_{i(j)}$ we must have $w_{ik} = w_{jl} = 0$, $F_{ik} = \left(\frac{a}{2}\right)^2 - \max\{0, (q(w_{jk}, 0))^2 - F_{jk}\}$ and $F_{jl} = \left(\frac{a}{2}\right)^2 - \max\{0, (q(w_{il}, 0))^2 - F_{il}\}$. Upstream firms obtain $\left(\frac{a}{2}\right)^2$except when either $(q(w_{jk}, 0))^2 - F_{jk} > 0$ or $(q(w_{il}, 0))^2 - F_{il} > 0$ hold. We are going to see that in those cases there is a profitable deviation. Without loss of generality, assume that $(q(w_{jk}, 0))^2 - F_{jk} > 0$ and $(q(w_{jk}, 0))^2 - F_{jk} \geq \max\{0, (q(w_{il}, 0))^2 - F_{il}\}$. Observe that this implies that $(q(0, w_{jk}))^2 < \left(\frac{a}{2}\right)^2$. Suppose that U_i sets $w'_{ik} = \infty$, $F'_{ik} = \infty$, $w'_{il} = 0$ and $F'_{il} = 2\left(\frac{a}{2}\right)^2 - (q(0, w_{jk}))^2 - \max\{0, (q(w_{il}, 0))^2 - F_{il}\} - \varepsilon$, where

$$\left(\frac{a}{2}\right)^2 - (q(0, w_{jk}))^2 - \max\{0, (q(w_{il}, 0))^2 - F_{il}\} + (q(w_{jk}, 0))^2 - F_{jk} > \varepsilon$$

$$> \max\{0, \left(\frac{a}{2}\right)^2 - (q(0, w_{jk}))^2 - \max\{0, (q(w_{il}, 0))^2 - F_{il}\}\}.$$

It is possible to check that the only equilibrium is that $D_{l(k)}$ chooses $U_{i(j)}$. For D_k is a strictly dominant strategy to choose U_j. Then D_l chooses U_i, because:

$$\begin{aligned}\left(\frac{a}{2}\right)^2 - F'_{il} &= -\left(\frac{a}{2}\right)^2 + (q(0, w_{jk}))^2 + \max\{0, (q(w_{il}, 0))^2 - F_{il}\} + \varepsilon > (q(0, w_{jk}))^2 - F_{jl} = \\ &= (q(0, w_{jk}))^2 - \left(\frac{a}{2}\right)^2 + \max\{0, (q(w_{il}, 0))^2 - F_{il}\}\end{aligned}$$

and

$$\left(\frac{a}{2}\right)^2 - F'_{il} > 0$$

Then it is possible to check that U_i increases the payoff, because $F'_{il} > F_{ik}$. ■

The last thing to show is that an equilibrium indeed exists. $w_{11} = 0, F_{11} = \left(\frac{a}{2}\right)^2, w_{12} = \infty, F_{12} = \infty, w_{21} = \infty, F_{21} = \infty, w_{22} = 0, F_{22} = \left(\frac{a}{2}\right)^2$.