

A discusión

ON THE POSITIVE EFFECTS OF TAXATION ON EDUCATION*

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WP-AD 2007-30

Editor: Instituto Valenciano de Investigaciones Económicas, S.A.

Primera Edición Diciembre 2007

Depósito Legal: V-213-2007

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* I am indebted to my supervisor Iñigo Iturbe-Ormaetxe for his advice, comments and corrections. I thanks the Spanish Ministry of Education for financial support, FPU grant AP2003-0563.

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ABSTRACT

In the economic literature a constant tax rate on labor income has usually a neutral or negative effect on education. The effect is neutral in the absence of non-deductible costs and it is negative in the presence of them. A positive effect is obtained in the presence of non-deductible profits or uncertainty in the returns to education. In this model education is treated as a signalling device for the level of human capital and agents choose freely their labor supply under certainty and perfect financial markets. Within this framework a constant tax rate on labor income has a positive effect on education under certainty and in the absence of non-deductible costs or profits as long as consumption and leisure are complementary and the amount of transfers and family income is low enough.

Keywords: Education, taxes.

JEL classification: I20,H20,H24

1 Introduction

Education has positive effects on production, productivity, growth, technological implementation, technological growth, and it might have positive externalities in a direct economic sense. The social externalities also have positive economic effects: an educated society is claimed to be less aggressive, more healthy, and to have lower crime levels. An educated society is also claimed to be more informed about political issues increasing the efficiency of democracy. Due to all these positive effects on the economy and the society as a whole, the study of the effect of any policy on education is a highly relevant issue.

The literature on education has analyzed the effects of the tax policy on education. In particular, the effect of a constant (proportional) labor income tax on education depends on the presence of non-deductible costs or profits. According to human capital accumulation theories, in the absence of non-deductible costs or profits, a constant labor income tax has no effect on education. This neutrality is due to the fact that both costs and profits are reduced in the same proportion. In the presence of non-deductible costs, which is the most common scenario, a constant tax rate has a negative effect. In the presence of non-deductible profits the effect will be positive.¹ A non-deductible cost can be any monetary cost not subject to fiscal deduction (tuition fees, traveling costs, books, housing) or any direct utility cost different from an opportunity cost, as effort in some studies. The existence of non-deductible profits has been argued by assuming a consumption value of education or any kind of direct utility increase as, for example, social status. In the presence of both costs and profits the effect becomes ambiguous. The screening or signaling literature yields similar results: the presence of non-deductible costs or profits will determine the sign of the effect related to a constant labor income tax. In both cases the presence of a labor income tax reduces the return to education and makes investment in physical capital more attractive and, therefore, reduces the investment in education.

A positive effect of taxes on education has been obtained in the context of un-

¹For a general proof see Eaton and Rosen 1980.

certainty in the returns to education as in Eaton and Rosen (1980) in a human capital model, or Poutvaara (2002) in a signaling model. In both cases the positive effect is obtained relying on a higher uncertainty in the return of education and the risk attitude of agents. If education increases or decreases return uncertainty is still an open debate, so in this model I will use a certain return for both educated and uneducated agents.

I assume that although human capital can be accumulated through education or training, it is hardly observable by firms. Under this assumption a time demanding educational process that includes teaching, tests, exams and the evaluation of students made by professional educators does not only increase the stock of human capital, but obtaining a diploma also proves the achievement of at least some level of human capital that is previously determined. If an educational process has enough reputation, the labor market interprets correctly the signal and pays higher wages to agents with a diploma, in the same way that educational institutions will attach to the stated human capital levels as they are interested in building a good reputation to ensure the enrolment of future students. Following this idea, there exists only a discrete set of human capital levels that can be signaled in the labor market through education.²

In my model the government will set the human capital requirement to obtain a diploma, the amount of monetary investment, and the tuition fees. This implies that agents that choose to educate will only choose the time they invest in education. In reality, this is how agents choose, since the human capital requirements to obtain a diploma are more or less known in advance, the cost in form of fees is posted, and the monetary investment in education (quantity and quality of the staff, facilities, laboratories...) is not a direct choice of the agent, but it is observable. Nevertheless, the time that agents devote studying is widely a free choice. The model can be extended to include privately provided education where educational institutions choose their requirements or to a mixture of privately and publicly provided education.

The model has two periods. In the first period agents choose their education,

²Firms can, through time and experience, infer more information about the human capital of workers. To develop this idea will require a huge expansion of the model.

labor supply, consumption, and savings. In the second period agents choose their consumption and labor supply. Only two different educational levels are allowed, the lowest one corresponds to the human capital stock generated during compulsory schooling and the highest one to the human capital after non-compulsory higher education. As any other level cannot be signaled, there is no interest in acquiring additional human capital. The result is a signaling model where education is a binary decision: to acquire or not higher education. The signal is assumed perfect as it gives perfect information to firms about the achieved human capital level. Firms attach a concrete wage for both type of agents, educated and uneducated ones, so the return, understood as wage level, is perfectly known in advance for both types. The educative system works in two ways, as a human capital production function and as a signaling device. During the first period educated agents invest the required amount of time and money to achieve the necessary human capital level to pass the final exams, obtaining a diploma that is used to signal the reached human capital level.

The present model has two significative differences with previous signaling models:

1. It is a two-period model where agents derive utility in both periods and not just maximize lifetime income or derive utility in a single period.
2. Agents can freely choose their labor supply in both periods. This gives an additional dimension to the model that is usually not present in signaling models that involve education.

Within this framework and in the absence of non-deductible costs or profits, the effect of a constant tax rate is positive on the educational choice as long as consumption and leisure are complement and the amount of transfers and non-labor (family) income is low enough.

This positive effect of taxes is due to the labor supply choice in the second period and to the reduction of the opportunity cost of education in the first period. Obviously a higher tax rate always implies a lower opportunity cost of education in

terms of lost income as also a relatively lower return in terms of second period wage. Due to the endogeneity of the labor supply the relation between consumption and leisure is highly relevant; there is a change in the labor supply decision of agents that implies a change in the derived utility. When leisure and consumption are complements the reduction in wages has a lower impact on utility than when the variables are substitutes, the relative utility loss of agents facing a high wage with respect to those facing a low wage is smaller when they are complements so the utility gain related to a lower opportunity cost prevails and a higher number of agents choose to educate.

The main difference with previous models is that the positive effect is obtained under certainty and in absence of any kind of non-deductible costs or profits.

Transfers have a negative effect on education as expected, but family income has also a negative effect on education. A higher transfer or family income provides higher consumption possibilities making then education less attractive. This negative relation of family income and education goes against most of the empirical evidence that suggest a positive correlation between education and family income.

This apparently counter-intuitive result can be solved inside the model in several ways by changing some assumptions. A commonly used assumption is to allow a positive correlation between family income and ability but also other explanations are possible. The educated wage might be positively correlated with family income; this fact can be explained through matching in the labor market. A higher family income allows agents to reject low-wage job offers and, therefore, it increases the probability of a good match. Another possible explanation for a positive correlation between educated wage and family income can be the access to the "family" network to find a job. Many agents find their work through networks. The higher is family income the better is the network to find a high-wage educated job and, therefore, a higher family income induces a higher probability of finding a high-wage educated job. A fourth explanation to the positive relation between education and family income is through parental guidance. Parents' wish their offsprings to educate and they will encourage education using monetary incentives if they can, high-income parents will pay any cost of education and provide additional resources to their

offsprings if they choose to educate while non of these resources are made available to those choosing not to educate. This can be reflected in the model as a reduction in family income for uneducated agents.

2 Model

I build a model where a society of heterogenous agents lives for two periods of length one. Agents are endowed with one unit of time in each period. In both periods agents choose their labor supply and their consumption. In the first period they also choose the amount they wish to save and the educational level they wish to signal. The educational choice is a binary choice: to acquire or not a previously determined level of human capital. The return to education is a higher wage in the second period, and its cost is the required monetary and time investment that depends on the ability of the agent to accumulate human capital.

2.1 Agents

Agents are heterogeneous with respect to two exogenous characteristics: their ability to accumulate human capital (e) and their family income (y). Ability is distributed in $[e, \bar{e}]$ according to the distribution function $E(e)$, while family income is distributed in $[y, \bar{y}]$ with distribution function $Y(y)$. Since agents differ in two characteristics they are represented in a two-dimensional space that is assumed to be continuous of mass one. For simplicity both characteristics are assumed to be independent and their joint distribution is just $J(e, y) = E(e)Y(y)$. All agents will have the same utility function and the same discount factor δ . As instantaneous utility function for both periods I use the following CES form:

$$U_i(C_i, L_i) = (aC_i^\rho + bL_i^\rho)^{\frac{\phi}{\rho}} \text{ for } i = 1, 2, \quad (1)$$

with parameters $a, b > 0$, $\rho \leq 1$, $\phi \in (0, 1)$, where C_i and L_i are effective consumption and leisure in period i , respectively. Effective consumption is total consumption in each period (c_i) minus the minimum consumption required (m_i).

The parameter ϕ is not present in the normal representation of a CES function. It gives additional curvature to the utility function and it ensures that the CES is homogeneous of degree ϕ . The additional curvature is needed to avoid corner solutions that arise when savings are allowed. In particular, when $\phi = 1$ agents will either save all or borrow all, which is equivalent to derive utility in only one of the two periods.³

Agents' objective function is then:

$$U_1(C_1, L_1) + \delta U_2(C_2, L_2). \quad (2)$$

Agents will consume and supply labor in both periods, in order to maximize their objective function. During the first period agents choose the amount they wish to save (S) that will pay some interest rate (r) in the second period and also their human capital level from the set of discrete levels of human capital that are available. For simplicity I will assume only one possible human capital level that corresponds to \bar{H} . The extension to several education levels is not difficult but messy.

2.2 Education

The educational process acts as a human capital accumulation function that depends on the innate ability of agents to accumulate human capital (e), the time (h) and monetary (f) investments in human capital accumulation:⁴

³The result for such cases are also provided.

⁴The human capital accumulation function is similar to the one used by Ben-Porath (1967).

$$H(h, f, e) = q + (eh)^\alpha f^\beta, \quad (3)$$

where $\alpha, \beta > 0$, $\alpha + \beta < 1$, and $q > 0$. The variable q represents the human capital level that corresponds to compulsory education. This level of human capital is the same for all individuals. The educational process increases human capital but also ensures that agents get at least a fixed level of human capital (\bar{H}) that is required to obtain a diploma.

The government fixes the monetary investment in human capital equal to \bar{f} and it also fixes the amount of human capital that is necessary to obtain a diploma. This determines the amount of human capital that can be signaled in the labor market. It can also subsidize the cost of education so that the effective cost of education is $f \leq \bar{f}$. Once fixed \bar{f} and \bar{H} , the time investment that a particular agent needs to achieve the required level of human capital becomes fixed and corresponds to:

$$\tilde{h}_i = \frac{1}{e_i} \left(\frac{\bar{H} - q}{\bar{f}^\beta} \right)^{1/\alpha}. \quad (4)$$

The relation of the ability and time can also be expressed as:

$$e_i = \frac{1}{\tilde{h}_i} \left(\frac{\bar{H} - q}{\bar{f}^\beta} \right)^{1/\alpha}. \quad (5)$$

As ability is a function of time investment and other exogenously fixed variables, for simplicity I solve the model according to the required time investment.

The screening mechanism is assumed to be perfect: all agents that reach the required level of human capital obtain a diploma. This implies that, since agents have perfect information, no one that chooses to educate will fail. Assuming that the screening mechanism is not perfect, and therefore that there exists a positive probability of failing the exam when the required human capital is achieved, does not alter the main results of the model. It will just reduce the education threshold due to the risk involved in education that is not present for uneducated agents.

2.3 Wages

Firms pay salaries according to the signaled human capital. So there will exist an uneducated wage that corresponds to a human capital of q and an educated wage that corresponds to the human capital \bar{H} . I call them w_1 and w_E , respectively. The government imposes a constant tax rate (t) on labor income, so net wages are $W_1 = (1 - t)w_1$ and $W_E = (1 - t)w_E$, respectively.

Observe that in the first period all agents have a stock of human capital that corresponds to q and the educated agents cannot signal their higher human capital level until the educational process has concluded, what happens at the end of the first period.⁵ So in the first period all agents get the same wage that moreover must be equal to W_1 . In order to allow for an increase in the human capital of uneducated agents through experience or training I will distinguish the first period wage of both types (W_1) from the second period uneducated wage (W_U).⁶ At some points, for simplicity, I will assume that $W_1 = W_U$. The total supply of human capital does not affect wages so they are constant. The inclusion of a production function might be interesting as wages will be endogenous and they will depend on the total amount of human capital supplied in the labor market.

2.4 Budget constraints

In each period agents can receive transfers from the government (g_i) and they have to afford the minimum consumption. In the first period they also are endowed with their family income. For simplicity I collect all this variables into just two:

$$B_1 = y + g_1 - m_1, \tag{6}$$

$$B_2 = g_2 - m_2. \tag{7}$$

⁵For different approach see Swinkels (1999).

⁶This requires that firms infer through experience some information about the human capital of their workers.

An educated agent will face the following budget constraints:

$$C_1 = (1 - h - L_1)W_1 + B_1 - f - S_E \text{ and } C_2 = (1 - L_2)W_E + B_2 + (1 + r)S_E. \quad (8)$$

For an uneducated agent I have:

$$C_1 = (1 - L_1)W_1 + B_1 - S_U \text{ and } C_2 = (1 - L_2)W_U + B_2 + (1 + r)S_U. \quad (9)$$

2.5 Education decision

The problem that agents face consists of deciding whether to acquire education or not, knowing that if they educate they will reach the utility level derived from solving:

$$\begin{aligned} & \underset{C_1, C_2, L_1, L_2, S_E}{Max} (aC_1^\rho + bL_1^\rho)^{\frac{\phi}{\rho}} + \delta (aC_2^\rho + bL_2^\rho)^{\frac{\phi}{\rho}} \\ & \text{s.t } C_1 = (1 - h - L_1)W_1 + B_1 - S_E - f; \\ & C_2 = (1 - L_2)W_E + B_2 + (1 + r)S_E; \\ & L_1 \in (0, h); L_2 \in (0, 1), \end{aligned}$$

and if they do not educate they will reach the utility level derived from solving:

$$\begin{aligned} & \underset{C_1, C_2, L_1, L_2, S_U}{Max} (aC_1^\rho + bL_1^\rho)^{\frac{\phi}{\rho}} + \delta (aC_2^\rho + bL_2^\rho)^{\frac{\phi}{\rho}} \\ & \text{s.t } C_1 = (1 - L_1)W_1 + B_1 - S_U; \\ & C_2 = (1 - L_2)W_U + B_2 + (1 + r)S_U; \\ & L_1 \in (0, 1); L_2 \in (0, 1). \end{aligned}$$

An agent will choose to educate only if the utility level with education is higher or equal than without education.

3 Results

The first-order conditions for an educated agent in a interior solution yield:

$$\left. \begin{aligned} S_E &= \frac{(1-h)W_1 + B_1 - f - V_E(W_E + B_2)}{1 + (1+r)V_E} \\ C_1 &= \frac{b^{1/\rho-1}\Delta}{b^{1/\rho-1} + a^{1/\rho-1}W_1^{\rho/\rho-1}} ; C_2 = \frac{1}{V_E} \frac{b^{1/\rho-1}\Delta}{b^{1/\rho-1} + a^{1/\rho-1}W_E^{\rho/\rho-1}} \\ L_1 &= \frac{a^{1/\rho-1}W_1^{1/\rho-1}\Delta}{b^{1/\rho-1} + a^{1/\rho-1}W_1^{\rho/\rho-1}} ; L_2 = \frac{1}{V_E} \frac{a^{1/\rho-1}W_E^{1/\rho-1}\Delta}{b^{1/\rho-1} + a^{1/\rho-1}W_E^{\rho/\rho-1}} \end{aligned} \right\} \quad (10)$$

The term V_E is a function that depends on net wages in the following way:

$$V_i = \left((1+r)\delta \frac{\theta(W_i)}{\theta(W_1)} \right)^{\frac{1}{\phi-1}}, \quad (11)$$

where again $\theta(\cdot)$ is a function of the net wage represented as:

$$\theta(W_i) = (ab)^{\frac{\phi}{\rho}} \left(b^{1/\rho-1} + a^{1/\rho-1}W_i^{\rho/\rho-1} \right)^{\frac{\phi(1-\rho)}{\rho}}, \quad (12)$$

and the term Δ follows the expression:

$$\Delta = V_E \frac{(1+r) \left((1-h)W_1 + B_1 - f \right) + (W_E + B_2)}{1 + (1+r)V_E}. \quad (13)$$

The interior solution conditions are:

$$\Delta \leq \left(\frac{b}{aW_1} \right)^{1/\rho-1} + W_1, \quad (14)$$

$$\Delta \leq V_E \left(\left(\frac{b}{aW_E} \right)^{1/\rho-1} + W_E \right), \quad (15)$$

for the first and second period, respectively.

Plugging the optimal values into the objective function, I obtain the utility level of an educated agent as:

$$X^\phi \left(\frac{V_E^\phi \theta(W_1) + \delta \theta(W_E)}{(1 + (1+r)V_E)^\phi} \right), \quad (16)$$

where $X = (1+r)((1-h)W_1 + B_1 - f) + (W_E + B_2)$ is the maximum possible lifetime income.

In a similar way the interior solution for an uneducated individual yields:

$$\left. \begin{aligned} S_U &= \frac{W_1 + B_1 - V_U(W_U + B_2)}{1 + (1+r)V_U} \\ C_1 &= \frac{b^{1/\rho-1}\Lambda}{b^{1/\rho-1} + a^{1/\rho-1}W_1^{\rho/\rho-1}} ; C_2 = \frac{1}{V_U} \frac{b^{1/\rho-1}\Lambda}{b^{1/\rho-1} + a^{1/\rho-1}W_U^{\rho/\rho-1}} \\ L_1 &= \frac{a^{1/\rho-1}W_1^{1/\rho-1}\Lambda}{b^{1/\rho-1} + a^{1/\rho-1}W_1^{\rho/\rho-1}} ; L_2 = \frac{1}{V_U} \frac{a^{1/\rho-1}W_U^{1/\rho-1}\Lambda}{b^{1/\rho-1} + a^{1/\rho-1}W_U^{\rho/\rho-1}} \end{aligned} \right\} \quad (17)$$

The term Λ is as follows:

$$\Lambda = V_U \frac{(1+r)(W_1 + B_1) + (W_U + B_2)}{1 + (1+r)V_U}. \quad (18)$$

The interior solution conditions for the first and second period are, respectively:

$$\Lambda \leq \left(\frac{b}{aW_1} \right)^{1/\rho-1} + W_1, \quad (19)$$

$$\text{and } \Lambda \leq V_U \left(\left(\frac{b}{aW_U} \right)^{1/\rho-1} + W_U \right). \quad (20)$$

The level of utility obtained by an uneducated individual is:

$$Y^\phi \left(\frac{V_U^\phi \theta(W_1) + \delta \theta(W_U)}{(1 + (1+r)V_U)^\phi} \right), \quad (21)$$

where again $Y = (1+r)(W_1 + B_1) + (W_U + B_2)$ is the maximum possible lifetime income.

Equations (16) and (21) state that agents wish to maximize their maximum possible lifetime income powered to ϕ and weighted by a function that depends on net wages and other exogenous parameters. In particular, it depends on the tax rate.

Using Equations (16) and (21) a particular agent will educate if:

$$X^\phi \left(\frac{V_E^\phi \theta(W_1) + \delta \theta(W_E)}{(1 + (1 + r)V_E)^\phi} \right) \geq Y^\phi \left(\frac{V_U^\phi \theta(W_1) + \delta \theta(W_U)}{(1 + (1 + r)V_U)^\phi} \right). \quad (22)$$

That above expression can be written as:

$$X \geq \chi Y, \quad (23)$$

where χ is the following expression:

$$\chi(\cdot) = \left(\frac{(1 + (1 + r)V_U)^{1-\phi} \theta(W_U)}{(1 + (1 + r)V_E)^{1-\phi} \theta(W_E)} \right)^{\frac{1}{\phi}}. \quad (24)$$

Substituting X and Y into Equation (23) and simplifying I obtain that any individual will educate if:

$$1 - h \geq \chi + \frac{(\chi - 1)(1 + r)B_1 + B_2}{(1 - t)(1 + r)w_1} + \frac{\chi w_U - w_E}{(1 + r)w_1} + \frac{1}{(1 - t)} \frac{f}{w_1}. \quad (25)$$

The threshold time investment is therefore:

$$\widehat{h} = 1 - \chi - \frac{(\chi - 1)(1 + r)B_1 + B_2}{(1 - t)(1 + r)w_1} - \frac{\chi w_U - w_E}{(1 + r)w_1} - \frac{1}{(1 - t)} \frac{f}{w_1}. \quad (26)$$

Any agent that needs this time or less to achieve the fixed level of human capital will choose to educate ($\widetilde{h}_i < \widehat{h}$), implying that any agent with ability higher or equal than:

$$\widehat{e} = \frac{1}{\widehat{h}} \left(\frac{\overline{H} - q}{f^\beta} \right)^{1/\alpha}, \quad (27)$$

will choose to educate.

Proposition 1 *The function χ has a value greater or equal than one ($\chi \geq 1$). The value is strictly greater if $W_E > W_U$.*

Proof. See the Appendix. ■

Proposition 1 indicates that if an agent chooses to educate then X must be strictly greater than Y , so the maximum lifetime income when educated must be strictly greater than the maximum lifetime income when uneducated for an educated agent. This implies that there is less education than in a similar setup in which agents maximize simply lifetime income. It also says that $(\chi - 1) \geq 0$, which implies that any transfer or family income has a negative effect on education. The negative effect of transfers was as expected, but the negative effect of family income on education is not, since it is usually found that higher family income implies higher levels of education. This apparently counter-intuitive result can be explained inside the model if a positive correlation between y and e is allowed. There exist other possible ways to revert this result as for example allowing a positive correlation between y and w_E . This positive correlation can be supported using matching models or labor networks. A higher family income allows in a matching model to look for a job during more time, increasing the probability of a good match. A higher family income can be related to a network that is more suitable to find a high wage educated vacancy.⁷ If the access to family income depends on parents' choice then parental guidance might also explain the positive relation between high family income and high education, since parents' might restrict the access to family income if agents choose not to educate. If there exists any kind of social rewards to education they might be positively correlated with family income.

If financial markets are not perfect low-income families cannot borrow to finance education. In this case there is a positive relation between family income and edu-

⁷Using a similar intuition a low income family network can be more suitable to find high wage uneducated vacancies.

cation for those families that are financially constrained and a negative relation for those families that are not. This scenario seems more realistic.

The effect of a change in the tax rate on the time investment threshold can be stated as:

$$\frac{d\hat{h}}{dt} = - \left(\frac{(1+r)(W_1+B_1)+W_U+B_2}{(1+r)W_1} \right) \frac{d\chi}{dt} - \frac{\chi-1}{(1-t)^2} \left(\frac{(1+r)B_1+B_2}{(1+r)w_1} \right) - \frac{1}{(1-t)^2} \frac{f}{w_1}. \quad (28)$$

Proposition 2 *When consumption and leisure are complements the derivative of χ with respect to t is negative. This implies that, when the monetary cost of education is zero, or nearly zero, and the sum of transfers and family income is low enough, an increase in the tax level has a positive effect on education.*

Proof. See the Appendix. ■

As the derivative of χ is negative with respect to t when consumption and leisure are complements, it is clear that when education is free ($f = 0$) and the sum of transfers and family income is low enough, or alternatively the minimum consumption requirements are high ($(1+r)B_1 + B_2 \simeq 0$),⁸ the derivative of the time investment threshold is positive so that higher taxes imply more education, since more agents will choose to educate.

A higher tax rate always implies a lower cost of education in terms of lost income as also a lower return in terms of wage in the second period. The relevance of the relation between consumption and leisure is due to the labor supply choice. When leisure and consumption are complements the reduction in wages has a lower impact on utility than when the variables are substitutes, the relative utility loss of agents facing a high wage with respect to those facing a low wage is smaller when they are complements so the utility gain related to a lower opportunity cost prevails and a higher number of agents choose to educate. A more detailed explanation is provided after proposition 5.

⁸The minimum consumption requirements can be quite high in a modern society.

This effect works through the labor supply decision in the second period as a similar result is obtained when agents derive utility only in the second period. In such a model agents supply labor inelastically in the first period and save all their income to be consumed in the second period. Solving that model I obtain as a threshold time investment the following expression:

$$\hat{h} = 1 - \varkappa - \frac{(\varkappa - 1)(1 + r)B_1 + B_2}{(1 - t)(1 + r)w_1} - \varkappa \frac{w_U - w_E}{(1 + r)w_1} - \frac{1}{(1 - t)} \frac{f}{w_1}, \quad (29)$$

where \varkappa follows the expression:

$$\varkappa(\cdot) = \left(\frac{\theta(W_U)}{\theta(W_E)} \right)^{\frac{1}{\phi}}. \quad (30)$$

This expression is identical to (26), except for the term \varkappa .

Proposition 3 *The value of \varkappa is greater than one ($\varkappa \geq 1$). It is strictly greater if $W_E > W_U$.*

Proof. See the Appendix. ■

Similarly to Proposition 1, Proposition 3 reflects the negative effect of transfers and family income on education.

Proposition 4 *The expression χ is bounded above by the expression \varkappa : $\chi < \left(\frac{\theta(W_U)}{\theta(W_E)} \right)^{\frac{1}{\phi}}$.*

Proof. See the Appendix. ■

From Proposition 4 it is clear that equation (29) overestimates the effects of transfers, family income and the uneducated wage. In fact, the time investment threshold is always higher in this model, so less agents educate. This difference is due to the labor supply and to the savings decision in the first period. Agents that choose to educate will borrow (or save less) to finance education, and their labor supply decision in the first period helps also to decrease the cost of education since not all the time devoted to education implies a loss in income. The difference

between \varkappa and χ depends also on the value of ϕ . The closer to zero is ϕ , the closer is χ to \varkappa . For ϕ close to one, χ is close to one.

Proposition 5 *The derivative of \varkappa with respect to t is negative if consumption and leisure are complements: $\frac{d\varkappa}{dt} \leq 0$ if $\rho < 0$ and $W_E > W_U$.*

Proof. See the Appendix. ■

Proposition 5 indicates a similar behavior with respect to taxes and this result shows that the labor supply decision in the second period is the relevant variable to induce the positive reaction to an increase in the tax rate, as long as consumption and labor are complements. When considering this case, the effect of a change in the tax rate can be decomposed in two effect, the first one is related to savings since the change in taxes affects the first period wage and the second one is related to the change in the wage of the second period. This can be mathematically expressed through the derivative of the indirect utility function of the second period ($V(W_i, S_i)$) with respect to taxes:

$$\frac{\partial V(W_i, S_i)}{\partial t} = \frac{\partial V(W_i, S_i)}{\partial S_i} \frac{\partial S_i}{\partial W_1} \frac{\partial W_1}{\partial t} + \frac{\partial V(W_i, S_i)}{\partial W_i} \frac{\partial W_i}{\partial t} \quad \text{for } i = E, U.$$

The first term of the RHS corresponds to the effect on savings and can be expressed as.

$$\frac{\partial V(W_i, S_i)}{\partial S_i} \frac{\partial S_i}{\partial W_1} \frac{\partial W_1}{\partial t} = \begin{cases} -(1+r)hw_1\theta(W_E) & \text{for educated agents.} \\ -(1+r)w_1\theta(W_U) & \text{for uneducated agents.} \end{cases}$$

As $h \in (0, 1)$ and $\theta(W_E) < \theta(W_U)$ the loss of utility due to the increase in taxes is less for educated agents, there is a reduction in the opportunity cost of education and education becomes more attractive, independently of the relation between consumption and leisure.

The second terms of the RHS corresponds to the wage effect in the second period and can be expressed as.

$$\frac{\partial V(W_i, B_i)}{\partial W_i} \frac{\partial W_i}{\partial t} = \frac{-W_i \theta(W_i)}{(1-t)} (1-L_i^*) \text{ for } i = E, U.$$

Here $W_E \theta(W_E) > W_U \theta(W_U)$, independently of the relation between consumption and leisure. The relevance of the relation between consumption and labor is related to the labor supply choices. When consumption and leisure are substitutes then $(1-L_E^*) > (1-L_U^*)$ in any case and the difference between these two terms is always higher than the same difference when consumption and leisure are complements since the income and substitution effect work in opposite directions, in fact it can be the case that $(1-L_E^*) < (1-L_U^*)$. This implies that the relative loss of utility of educated agents with respect to uneducated agents is always lower when consumption and leisure are complements.

Then, for educated agents there is a relative gain of utility due to the reduction in the opportunity cost of education, in terms of savings, and a relative loss of utility due the reduction of the second period wage. This relative loss is always lower when consumption and leisure are complements, in a way that the gain related to the reduction in opportunity cost dominates, making education more attractive. If consumption and leisure are substitutes the dominant effect is the loss due to the reduction in the second period wage, making then education less profitable.

A similar intuition can be applied to the case where individuals extract utility from both periods, the mechanism is much more complicated and it works through changes in the labor supply choices in both periods and the saving choice.

If the same problem is solved assuming that agents care only about first period utility I get:

$$\hat{h} = 1 - \frac{w_U - w_E}{(1+r)w_1} - \frac{1}{(1-t)} \frac{f}{w_1}.$$

There are no negative effects due to transfers or family income and taxes only affect through the last term related to the cost of education, so for costless education taxes do not have any effect. This is similar to the cases where χ is close to one.

Is the complementarity assumption too strong? There exists empirical evidence of complementarity between consumption and leisure in the literature (Stern 1976). Complementarity between consumption and leisure is also a way to explain the backward sloping labor supply function that is empirically observed.

4 Discussion

The main conclusion of this paper is that higher tax rates on labor income can have positive effects on education. The requirements to observe such effect are, perhaps, not too restrictive: Consumption and leisure must be complements, a fact that is empirically observed; the monetary cost of education must be low or zero, something that is observed in many European countries; and the sum of transfers and family income must be low enough to just cover the minimum consumption requirements. This last requirement is perhaps the most difficult to argue. Assume that the reverse is true, then any agent, not working at all, must reach positive levels of effective consumption. This might be true in some countries, but mainly due to positive transfers and governmental help that are offered only in some concrete kind of situations, like tagging policies. Usually any agent that works and/or reaches some minimum level of income has not access to the mentioned transfers. Without these transfers and help is difficult to believe that an agent will reach a positive level of effective consumption without working. Then the sum of transfers and family income must be, in most of the cases, lower or equal than the sum of minimum consumption requirements.

In many European countries higher education is free or nearly free and marginal tax rates are high; the combination of these two facts might act increasing education instead of decreasing it. The high proportion of college students observed in some European countries that cannot be explained by the sole wage differential might be explained by this fact. In the particular case of Norway the population of agents with a college degree and the number of university students has nearly doubled between 1987 and 2002, and interestingly the increase is concentrated on those fields that

have a lower return.⁹ In practice these agents face a constant tax rate since their income is close to the uneducated income. In other Nordic countries the return to education is also quite low. Even when the tax schedule is highly progressive the average tax rate difference between agents with tertiary education (educated) and agents with lower secondary education (uneducated) is much lower than in the U.S.A.¹⁰ In the case of Swedish and Danish women the average tax difference between educated and uneducated is less than 5%.¹¹ In many developed countries the advantages that students have in form of discounts, tax exemptions, easy access to loans, housing help, grants and other forms of subventions can make the monetary cost of education even negative, and then education is even more attractive and the effect of an increase in taxes encourages even more education.

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¹⁰Data from OECD Economic Studies No 34.

¹¹Data from OECD Economic Studies No 34.

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5 Appendix.

5.1 Solving utility levels of educated and uneducated agents.

An educated agent must solve:

$$\begin{aligned}
 & \underset{C_1, C_2, L_1, L_2, S_E}{Max} (aC_1^\rho + bL_1^\rho)^{\frac{\phi}{\rho}} + \delta (aC_2^\rho + bL_2^\rho)^{\frac{\phi}{\rho}} \\
 & s.t \ C_1 = (1 - h - L_1)W_1 + B_1 - S_E - f; \\
 & \quad C_2 = (1 - L_2)W_2 + B_2 + (1 + r)S_E; \\
 & \quad L_1 \in (0, h); L_2 \in (0, 1).
 \end{aligned}$$

This is equivalent to:

$$\underset{S_E}{Max} \left\{ \begin{array}{l} \underset{L_1}{Max} \left\{ (a((1 - h - L_1)W_1 + B_1 - S_E - f)^\rho + bL_1^\rho)^{\frac{\phi}{\rho}} \right\} \\ + \delta \underset{L_2}{Max} \left\{ (a((1 - L_2)W_2 + B_2 + (1 + r)S_E)^\rho + bL_2^\rho)^{\frac{\phi}{\rho}} \right\} \end{array} \right\}$$

Plus interior solution conditions.

So agents maximize choosing leisure in each period taking savings as given and then maximize the expression given above choosing the amount saved.

An educated agent solves in the second period.

$$\underset{L_2}{Max} (a((1 - L_2)W_E + B_2 + (1 + r)S_E)^\rho + bL_2^\rho)^{\frac{\phi}{\rho}} .$$

The first-order condition is:

$$a((1 - L_2)W_E + B_2 + (1 + r)S_E)^{\rho-1} W_E = bL_2^{\rho-1}.$$

Then the optimal level of leisure in the second period is:

$$L_2^* = \frac{a^{1/\rho-1}W_E^{1/\rho-1}(W_E + B_2 + (1 + r)S_E)}{b^{1/\rho-1} + a^{1/\rho-1}W_E^{\rho/\rho-1}},$$

The interior solution condition is then:

$$\frac{a^{1/\rho-1}W_E^{1/\rho-1}(W_E + B_2 + (1 + r)S_E)}{b^{1/\rho-1} + a^{1/\rho-1}W_E^{\rho/\rho-1}} \leq 1.$$

This can we expressed as:

$$(W_E + B_2 + (1 + r)S_E) \leq \frac{b^{1/\rho-1}}{a^{1/\rho-1}W_E^{1/\rho-1}} + W_E.$$

The optimal labor supply and consumption in the second period can be stated as:

$$1 - L_2^* = \frac{b^{1/\rho-1} - a^{1/\rho-1}W_E^{1/\rho-1}(B_2 + (1 + r)S_E)}{b^{1/\rho-1} + a^{1/\rho-1}W_E^{\rho/\rho-1}},$$

$$\text{and } C_2^* = \frac{b^{1/\rho-1}(W_E + B_2 + (1 + r)S_E)}{b^{1/\rho-1} + a^{1/\rho-1}W_E^{\rho/\rho-1}}.$$

Plugging this results in the instantaneous utility function I obtain the indirect instantaneous utility function of the second period as.

$$\left(a \left(\frac{b^{1/\rho-1}(W_E + B_2 + (1 + r)S_E)}{b^{1/\rho-1} + a^{1/\rho-1}W_E^{\rho/\rho-1}} \right)^\rho + b \left(\frac{a^{1/\rho-1}W_E^{1/\rho-1}(W_E + B_2 + (1 + r)S_E)}{b^{1/\rho-1} + a^{1/\rho-1}W_E^{\rho/\rho-1}} \right)^\rho \right)^{\frac{\phi}{\rho}}$$

$$\begin{aligned} \implies (W_E + B_2 + (1+r)S_E)^\phi & \left(ab \frac{b^{1/\rho-1}}{(b^{1/\rho-1} + a^{1/\rho-1}W_E^{\rho/\rho-1})^\rho} + ab \frac{a^{1/\rho-1}W_E^{\rho/\rho-1}}{(b^{1/\rho-1} + a^{1/\rho-1}W_E^{\rho/\rho-1})^\rho} \right)^{\frac{\phi}{\rho}} \\ \implies (W_E + B_2 + (1+r)S_E)^\phi & (ab)^{\frac{\phi}{\rho}} \left(b^{1/\rho-1} + a^{1/\rho-1}W_E^{\rho/\rho-1} \right)^{\frac{\phi(1-\rho)}{\rho}}. \end{aligned}$$

I call

$$\theta(W) = (ab)^{\frac{\phi}{\rho}} \left(b^{1/\rho-1} + a^{1/\rho-1}W^{\rho/\rho-1} \right)^{\frac{\phi(1-\rho)}{\rho}},$$

and then I can write the instantaneous utility of the second period as:

$$(W_E + B_2 + (1+r)S_E)^\phi \theta(W_E).$$

Observe that $\frac{d\theta(W)}{dW} < 0$.

Equivalently in the first period educated agents solve:

$$\text{Max}_{L_1} \left(a \left((1-h-L_1)W_1 + B_1 - f - S_E \right)^\rho + bL_1^\rho \right)^{\frac{\phi}{\rho}}.$$

The first-order condition state that:

$$a \left((1-h-L_1)W_1 + B_1 - f - S_E \right)^{\rho-1} W_1 = bL_1^{\rho-1}.$$

The optimal leisure, labor supply and consumption in the first period are then.

$$L_1^* = \frac{a^{1/\rho-1}W_1^{1/\rho-1} \left((1-h)W_1 + B_1 - f - S_E \right)}{b^{1/\rho-1} + a^{1/\rho-1}W_1^{\rho/\rho-1}},$$

$$1 - h - L_1^* = \frac{(1 - h)b^{1/\rho-1} - a^{1/\rho-1}W_1^{1/\rho-1}(B_1 - f - S_E)}{b^{1/\rho-1} + a^{1/\rho-1}W_1^{\rho/\rho-1}},$$

$$C_1^* = \frac{b^{1/\rho-1}((1 - h)W_1 + B_1 - f - S_E)}{b^{1/\rho-1} + a^{1/\rho-1}W_1^{\rho/\rho-1}},$$

and the interior solution condition is:

$$((1 - h)W_1 + B_1 - f - S_E) \leq \frac{b^{1/\rho-1}}{a^{1/\rho-1}W_1^{\rho/\rho-1}} + W_1.$$

Assuming savings fixed the indirect utility function in the first period yields:

$$((1 - h)W_1 + B_1 - f - S_E)^\phi \theta(W_1).$$

Once I have both instantaneous indirect utility functions I can compute the optimal amount of savings as:

$$\underset{S_E}{Max} ((1 - h)W_1 + B_1 - f - S_E)^\phi \theta(W_1) + \delta(W_E + B_2 + (1 + r)S_E)^\phi \theta(W_E).$$

The first-order condition yields:

$$((1 - h)W_1 + B_1 - f - S_E)^{\phi-1} = (1 + r)\delta \frac{\theta(W_E)}{\theta(W_1)} (W_E + B_2 + (1 + r)S_E)^{\phi-1}.$$

I call

$$V_E = \left((1 + r)\delta \frac{\theta(W_E)}{\theta(W_1)} \right)^{\frac{1}{\phi-1}} > 0.$$

Then the optimal level of savings is:

$$S_E = \frac{(1-h)W_1 + B_1 - f - V_E(W_E + B_2)}{1 + (1+r)V_E},$$

and I obtain that:

$$(1-h)W_1 + B_1 - f - S_E = V_E \frac{(1+r)((1-h)W_1 + B_1 - f) + (W_E + B_2)}{1 + (1+r)V_E},$$

and

$$W_2 + B_2 + (1+r)S_E = \frac{(1+r)((1-h)W_1 + B_1 - f) + (W_E + B_2)}{1 + (1+r)V_E}.$$

Plugging this result in the optimal values of leisure, consumption and labor supply I obtain the solutions stated in (10). Solutions for uneducated are easily obtained in a similar way.

5.2 Proof of Proposition 1

Proof. $\left(\frac{(1 + (1+r)V_U)^{1-\phi}\theta(W_U)}{(1 + (1+r)V_E)^{1-\phi}\theta(W_E)} \right)^{\frac{1}{\phi}} \geq 1$. I develop and simplify the expression as follows.

$$\begin{aligned} & \left(\frac{(1 + (1+r)V_U)^{1-\phi}\theta(W_U)}{(1 + (1+r)V_E)^{1-\phi}\theta(W_E)} \right)^{\frac{1}{\phi}} \\ &= \left(\frac{\theta(W_U)^{\frac{1}{1-\phi}} + (1+r) \left(\delta(1+r) \frac{\theta(W_U)}{\theta(W_1)} \right)^{\frac{1}{\phi-1}} \theta(W_U)^{\frac{1}{1-\phi}}}{\theta(W_E)^{\frac{1}{1-\phi}} + (1+r) \left(\delta(1+r) \frac{\theta(W_E)}{\theta(W_1)} \right)^{\frac{1}{\phi-1}} \theta(W_E)^{\frac{1}{1-\phi}}} \right)^{\frac{1-\phi}{\phi}} \\ &= \left(\frac{\theta(W_U)^{\frac{1}{1-\phi}} + (1+r) \left(\frac{\delta(1+r)}{\theta(W_1)} \right)^{\frac{1}{\phi-1}}}{\theta(W_E)^{\frac{1}{1-\phi}} + (1+r) \left(\frac{\delta(1+r)}{\theta(W_1)} \right)^{\frac{1}{\phi-1}}} \right)^{\frac{1-\phi}{\phi}} \end{aligned}$$

$$= \left(\frac{\theta(W_U)^{\frac{1}{1-\phi}} + (\delta(1+r)^\phi)^{\frac{1}{\phi-1}} \theta(W_1)^{\frac{1}{1-\phi}}}{\theta(W_E)^{\frac{1}{1-\phi}} + (\delta(1+r)^\phi)^{\frac{1}{\phi-1}} \theta(W_1)^{\frac{1}{1-\phi}}} \right)^{\frac{1-\phi}{\phi}}$$

The last expression is greater than one as $(\delta(1+r)^\phi)^{\frac{1}{\phi-1}} \theta(W_1)^{\frac{1}{1-\phi}} > 0$ and $\theta(W_U) > \theta(W_E)$ if $W_U < W_E$. ■

5.3 Proof of Proposition 2

Proof. $\frac{d\chi}{dt} < 0$ if $\rho < 0$. I need to develop the derivative.

$$\begin{aligned} & \frac{d}{dt} \left(\frac{\theta(W_U)^{\frac{1}{1-\phi}} + (\delta(1+r)^\phi)^{\frac{1}{\phi-1}} \theta(W_1)^{\frac{1}{1-\phi}}}{\theta(W_E)^{\frac{1}{1-\phi}} + (\delta(1+r)^\phi)^{\frac{1}{\phi-1}} \theta(W_1)^{\frac{1}{1-\phi}}} \right)^{\frac{1-\phi}{\phi}} \\ &= \frac{1-\phi}{\phi} \left(\frac{\theta(W_U)^{\frac{1}{1-\phi}} + (\delta(1+r)^\phi)^{\frac{1}{\phi-1}} \theta(W_1)^{\frac{1}{1-\phi}}}{\theta(W_E)^{\frac{1}{1-\phi}} + (\delta(1+r)^\phi)^{\frac{1}{\phi-1}} \theta(W_1)^{\frac{1}{1-\phi}}} \right)^{\frac{1-\phi}{\phi}-1} \frac{d}{dt} \frac{\theta(W_U)^{\frac{1}{1-\phi}} + (\delta(1+r)^\phi)^{\frac{1}{\phi-1}} \theta(W_1)^{\frac{1}{1-\phi}}}{\theta(W_E)^{\frac{1}{1-\phi}} + (\delta(1+r)^\phi)^{\frac{1}{\phi-1}} \theta(W_1)^{\frac{1}{1-\phi}}} \\ &= \frac{1-\phi}{\phi} \chi \left(\frac{\theta(W_U)^{\frac{1}{1-\phi}} + (\delta(1+r)^\phi)^{\frac{1}{\phi-1}} \theta(W_1)^{\frac{1}{1-\phi}}}{\theta(W_E)^{\frac{1}{1-\phi}} + (\delta(1+r)^\phi)^{\frac{1}{\phi-1}} \theta(W_1)^{\frac{1}{1-\phi}}} \right)^{-1} \frac{d}{dt} \frac{\theta(W_U)^{\frac{1}{1-\phi}} + (\delta(1+r)^\phi)^{\frac{1}{\phi-1}} \theta(W_1)^{\frac{1}{1-\phi}}}{\theta(W_E)^{\frac{1}{1-\phi}} + (\delta(1+r)^\phi)^{\frac{1}{\phi-1}} \theta(W_1)^{\frac{1}{1-\phi}}} \end{aligned}$$

To make the derivative of the last part I develop the derivative of $\theta(W)^{\frac{1}{1-\phi}}$ with respect to t .

$$\frac{d}{dt} \theta(W)^{\frac{1}{1-\phi}} = \frac{1}{1-\phi} \theta(W)^{\frac{\phi}{1-\phi}} \frac{d}{dt} \theta(W)$$

The derivative of the last part is

$$\begin{aligned} \frac{d}{dt} \theta(W) &= \frac{\phi(1-\rho)}{\rho} (ab)^{\frac{\phi}{\rho}} (b^{1/\rho-1} + a^{1/\rho-1} W^{\rho/\rho-1})^{\frac{\phi(1-\rho)}{\rho}-1} \left(\frac{\rho}{\rho-1} a^{1/\rho-1} W^{\rho/\rho-1-1} \right) (-w) \\ &= \frac{\phi}{1-t} \frac{a^{1/\rho-1} W^{\rho/\rho-1}}{b^{1/\rho-1} + a^{1/\rho-1} W^{\rho/\rho-1}} \theta(W) \end{aligned}$$

Then

$$\frac{d}{dt} \theta(W)^{\frac{1}{1-\phi}} = \frac{\phi}{1-\phi} \frac{a^{1/\rho-1} W^{\rho/\rho-1}}{b^{1/\rho-1} + a^{1/\rho-1} W^{\rho/\rho-1}} \frac{1}{1-t} \theta(W)^{\frac{1}{1-\phi}}$$

Then you can write the derivative of

$$\frac{\theta(W_U)^{\frac{1}{1-\phi}} + (\delta(1+r)^\phi)^{\frac{1}{\phi-1}} \theta(W_1)^{\frac{1}{1-\phi}}}{\theta(W_E)^{\frac{1}{1-\phi}} + (\delta(1+r)^\phi)^{\frac{1}{\phi-1}} \theta(W_1)^{\frac{1}{1-\phi}}}$$

That is too long to state here, but you can develop and observe that it is negative if $W_E > W_U \geq W_1$ and $\rho < 0$.

When $W_U = W_1$ the derivative can be wrote as

$$\frac{\frac{\phi}{1-\phi} a^{1/\rho-1} b^{1/\rho-1} \theta(W_1)^{\frac{1}{1-\phi}} \theta(W_E)^{\frac{1}{1-\phi}} \left(1 + (\delta(1+r)^\phi)^{\frac{1}{\phi-1}}\right)}{\left(\theta(W_E)^{\frac{1}{1-\phi}} + (\delta(1+r)^\phi)^{\frac{1}{\phi-1}} \theta(W_1)^{\frac{1}{1-\phi}}\right)^2} \left(\frac{W_1^{\rho/\rho-1} - W_E^{\rho/\rho-1}}{\left(b^{1/\rho-1} + a^{1/\rho-1} W_1^{\rho/\rho-1}\right) \left(b^{1/\rho-1} + a^{1/\rho-1} W_E^{\rho/\rho-1}\right)} \right)$$

That is negative if $W_1 < W_E$ and $\rho < 0$. ■

5.4 Proof of Proposition 3

Proof. It is straightforward since $\theta(W_U) > \theta(W_E)$. ■

5.5 Proof of Proposition 4

Proof. $\chi < \left(\frac{\theta(W_U)}{\theta(W_E)}\right)^{\frac{1}{\phi}}$ follows easily from the last expression for χ in the proof of proposition one as

$$\frac{\theta(W_U)^{\frac{1}{1-\phi}} + (\delta(1+r)^\phi)^{\frac{1}{\phi-1}} \theta(W_1)^{\frac{1}{1-\phi}}}{\theta(W_E)^{\frac{1}{1-\phi}} + (\delta(1+r)^\phi)^{\frac{1}{\phi-1}} \theta(W_1)^{\frac{1}{1-\phi}}} < \left(\frac{\theta(W_U)}{\theta(W_E)}\right)^{\frac{1}{1-\phi}}. \quad \blacksquare$$

5.6 Proof of Proposition 5

Proof. The derivative of $\left(\frac{\theta(W_U)}{\theta(W_E)}\right)^{\frac{1}{\phi}}$ is negative if $\rho < 0$.

$$\begin{aligned} \frac{d}{dt} \left(\frac{\theta(W_U)}{\theta(W_E)}\right)^{\frac{1}{\phi}} &= \frac{1}{\phi} \left(\frac{\theta(W_U)}{\theta(W_E)}\right)^{\frac{1}{\phi}-1} \left(\frac{\theta(W_E) \frac{d}{dt} \theta(W_U) - \theta(W_U) \frac{d}{dt} \theta(W_E)}{\theta(W_E)^2}\right) \\ &= \frac{1}{\phi} \left(\frac{\theta(W_U)}{\theta(W_E)}\right)^{\frac{1}{\phi}} \frac{1}{\theta(W_U) \theta(W_E)} \left(\theta(W_E) \frac{d}{dt} \theta(W_U) - \theta(W_U) \frac{d}{dt} \theta(W_E)\right) \end{aligned}$$

where $\frac{1}{\phi} \left(\frac{\theta(W_U)}{\theta(W_E)}\right)^{\frac{1}{\phi}} \frac{1}{\theta(W_U) \theta(W_E)} > 0$ and

$$\frac{d}{dt} \theta(W) = \frac{\phi}{1-t} \frac{a^{1/\rho-1} W^{\rho/\rho-1}}{b^{1/\rho-1} + a^{1/\rho-1} W^{\rho/\rho-1}} \theta(W)$$

So $(\theta(W_E) \frac{d}{dt} \theta(W_U) - \theta(W_U) \frac{d}{dt} \theta(W_E))$ is equal to

$$\frac{\phi a^{1/\rho-1} \theta(W_E) \theta(W_U)}{1-t} \left(\frac{W_U^{\rho/\rho-1}}{b^{1/\rho-1} + a^{1/\rho-1} W_U^{\rho/\rho-1}} - \frac{W_E^{\rho/\rho-1}}{b^{1/\rho-1} + a^{1/\rho-1} W_E^{\rho/\rho-1}} \right)$$

where $\frac{\phi a^{1/\rho-1} \theta(W_E) \theta(W_U)}{1-t} > 0$ and the parenthesis is equal to

$$\left(\frac{b^{1/\rho-1}W_U^{\rho/\rho-1} + W_U^{\rho/\rho-1}a^{1/\rho-1}W_E^{\rho/\rho-1} - b^{1/\rho-1}W_E^{\rho/\rho-1} - W_E^{\rho/\rho-1}a^{1/\rho-1}W_U^{\rho/\rho-1}}{(b^{1/\rho-1} + a^{1/\rho-1}W_U^{\rho/\rho-1})(b^{1/\rho-1} + a^{1/\rho-1}W_E^{\rho/\rho-1})} \right)$$

$$= b^{1/\rho-1} \left(\frac{W_U^{\rho/\rho-1} - W_E^{\rho/\rho-1}}{(b^{1/\rho-1} + a^{1/\rho-1}W_U^{\rho/\rho-1})(b^{1/\rho-1} + a^{1/\rho-1}W_E^{\rho/\rho-1})} \right)$$

That is negative is $\rho < 0$ and $W_E > W_U$.

The derivative is

$$\left(\frac{\theta(W_U)}{\theta(W_E)} \right)^{\frac{1}{\phi}} \frac{a^{1/\rho-1}b^{1/\rho-1}}{1-t} \left(\frac{W_U^{\rho/\rho-1} - W_E^{\rho/\rho-1}}{(b^{1/\rho-1} + a^{1/\rho-1}W_U^{\rho/\rho-1})(b^{1/\rho-1} + a^{1/\rho-1}W_E^{\rho/\rho-1})} \right). \blacksquare$$