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POOLING AND REDISTRIBUTION WITH MORAL HAZARD*

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ABSTRACT

We study a model in which risk-averse consumers obtain mutual insurance by participating voluntarily in pools. More precisely, consumers commit to contributing a fraction of their future uncertain endowment to a common pool. In exchange, they gain the right to receive a share of the total return of the pool, in proportion to their promises. Consumers influence the likelihood of the good state of nature by undertaking a hidden action. We therefore provide a model of mutual insurance with moral hazard. We first analyze the equilibrium properties of the model and then illustrate how an aggregate pool of heterogenous consumers Pareto dominates the two segregated pools.

Keywords: moral hazard, pool of promises, heterogeneous consumers.

JEL Classiffication: D5, H23, O16.

1 Introduction

In this paper we study a model in which risk-averse consumers obtain mutual insurance by participating voluntarily in pools. More precisely, consumers commit to contributing a fraction of their future uncertain endowment to a common pool. In exchange, they gain the right to receive a share of the total return of the pool, in proportion to their promise. Consumers influence the likelihood of the good state of nature by undertaking a hidden action. We therefore provide a model of mutual insurance with moral hazard.

In our view, it is helpful to distinguish mutual insurance organizations according to the way in which their members interact. A typical form of mutual insurance is the one in which members interact strategically. Historically, these organizations emerged to cater to the particular needs of specific populations limited in their access to the credit, insurance and goods markets. Economic literature has been trying to understand these organizations by focusing on both their endogenous formation and the mechanisms adopted to overcome the problems arising from information asymmetries (among the most recent contributions are those of Bloch et al. (2005), Cabrales et al. (2003), De Weerdt and Dercon (2006)). Case studies of mutual insurance reveal that it relies heavily on monitoring and enforcement mechanisms that sometimes constrain the capability of the organization to provide full insurance.

A different type of mutual insurance organization arises when members do not interact strategically and the price of insurance is taken as given. This happens when such organizations have many members. Possible examples of big insurance pools are national insurance systems (health, disability, and pension). Historically, these were first created as mutual insurance funds mainly associated with occupations, but they have gradually turned into broad national systems. This trend has also been associated, at least in Europe, with an increase in redistribution in such systems (Pestieau (2006)). Naturally, a consumer takes the *price* of social insurance as given.[3]

Like Dubey and Geanakoplos (2002), in the present paper we study a model of insurance pools with many participants, although our aim is substantially different from theirs. In fact, their main goal is to overcome the problem of the existence of equilibrium in the competitive model with adverse selection, first studied by Rothschild and Stiglitz (1976). We, on the other hand, study big insurance pools with moral hazard, focusing on the possibilities of redistribution that emerge among heterogenous groups of consumers.[4] In particular, we wish to stress the redistribution possibilities of insurance pools, even in the presence of moral hazard.

Several differences emerge in adapting the framework of Dubey and Geanakoplos (2002) to a model encompassing ex-ante moral hazard. The most significant is that the proportion of high risk consumers becomes endogenous, since it depends on the action undertaken, which is consumers' optimal choice.[5] Since consumers take the total return of the pool as given and are unrestricted in the amount of insurance they can obtain, the structure that we have in mind may appear not to be compatible with the incentives consumers need to undertake high effort. At equilibrium consumers may end up choosing high insurance coverage and low effort. This is, indeed, one possible equilibrium configuration, but it is not the only one. In particular, there exists an equilibrium in which some consumers choose high effort, and others low effort.[6]

After studying the case of a homogeneous group of consumers, we consider the case of heterogeneous groups and show how, in an economy of poor and rich consumers, an aggregate pool of promises may represent a Pareto improvement on segregated pools of promises. As will become clear, the rich can, at no cost, redistribute towards the poor because they are wealthier and more able to prevent bad outcomes.

Our paper proceeds as follows. Section 2 introduces the model and Section 3 characterizes its equilibrium. Two examples of the mixed pool equilibrium are provided in Section 4. Section 5 analyzes the possibility of a pool of comitments by both rich and poor and concludes that such a heterogenous pool Pareto dominates the two segregated pools. Section 6 concludes. All proofs are relegated to the Appendix.

2 The model

We consider a pure exchange economy with a single consumption good. The economy is populated by a large number of (ex-ante) identical consumers, and lasts for two periods $t = 0, 1$. Today, at $t = 0$, there is no consumption, and consumers face an idiosyncratic uncertainty regarding their endowments at $t = 1$. Tomorrow each consumer may be in either of two states of nature $s \in \mathcal{S} = \{G, B\}$, where G stands for good and B for bad. Individual endowments are denoted by the vector $w = (w_G, w_B) \in \mathbb{R}^2_+$, with $w_G > w_B \geqslant 0$. It is assumed that at $t = 1$ endowments are verifiable, so that it is possible to determine the idiosyncratic state in which consumers find themselves.

Consumers may influence the likelihood of the states of nature by undertaking an action $a \in \mathcal{A} = \{L, H\}$, corresponding to low effort and high effort, respectively. In what follows, π_a denotes the probability of the good state when action a is chosen, with $1 > \pi_H > \pi_L > 0$. The (dis-)utility of the action is c_a, and $c_H > c_L = 0$. The tradeoff is thus clear: on the one hand undertaking action H increases the likelihood of the good state of nature, but on the other hand it is costly since it requires effort. It is assumed that the action chosen by a consumer is not verifiable, so information is asymmetric.

Preferences are represented by an expected utility function $U(x, a) : \mathbb{R}^2_+ \times \mathcal{A} \to \mathbb{R}$ that depends on state contingent consumption bundle $x = (x_G, x_B) \in \mathbb{R}^2_+$, and action as follows:

$$U(x, a) := \pi_a u(x_G) + (1 - \pi_a) u(x_B) - c_a ,$$

where the (Bernoulli) utility function u is twice differentiable, strictly increasing, and strictly concave.

Since the action set is binary, preferences represented by U are inherently non-convex, as shown in Figure 1, which illustrates preferences for state contingent consumption bundles. The high action indifference curve is steeper than the low action one, and thus when the two indifference curves cross they make a kink. Low and high effort action indifference curves cross at the *switching locus*, a set of consumption bundles defined as follows:

$$\{x \in \mathbb{R}^2_+ \mid U(x,H) = U(x,L)\}.$$

Therefore, the switching locus is the set of consumption bundles that makes consumers indifferent between choosing the low and high actions. Above it, closer to the certainty line, consumers choose the low effort action. Below it, consumers prefer to choose the high effort action.[7] The intuition is the following: If contingent endowments are very different, a consumer would rather strongly try to avoid the bad state of nature by undertaking the high effort action. If, by contrast, contingent endowments are not that different, it is not worth making the effort to avoid the bad state of nature.

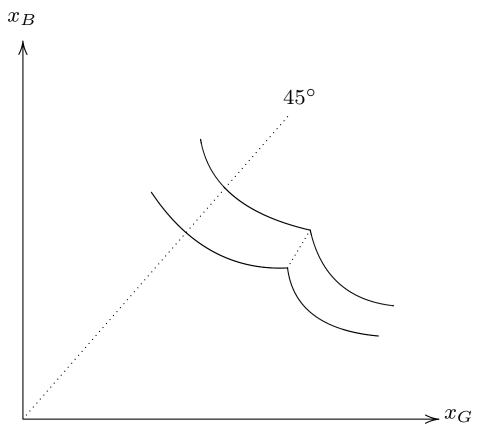

Figure 1: Indifference curves and switching locus.

We assume that each consumer faces uncertainty independently of other consumers. This assumption, added to the fact that there is a large number of consumers, rules out aggregated uncertainty, hence the aggregate (average) endowment is independent of states of nature. We denote the aggregate (average) endowment when action a is chosen by:

$$\bar{w}_a = \pi_a w_G + (1 - \pi_a) w_B.$$

If all consumers undertake action H, aggregate endowment is

$$\bar{w}_H = \pi_H w_G + (1 - \pi_H) w_B.$$

If however they all take action L, aggregate endowment is

$$\bar{w}_L = \pi_L w_G + (1 - \pi_L) w_B.$$

Finally, if a fraction $q \in [0, 1]$ of consumers undertake action H, aggregate endowment is

$$\bar{w}_q = q\bar{w}_H + (1 - q)\bar{w}_L. \tag{1}$$

2.1 The pool of promises

Since consumers are risk-averse, they would like to smooth their consumption across idiosyncratic states. We assume consumers may *voluntarily* constitute a pool of promises so as to effectively insure themselves. The pool works as follows: Today, each consumer makes the voluntary *promise* to deliver to the pool a fraction of tomorrow's endowment . In exchange, tomorrow, each consumer is entitled to receive a *share* of the total deliveries to the pool proportional to the promise made. We will call the latter the *return* of the pool.

Tomorrow, once the state of nature is realized, each consumer engages in a net trade with the pool by transferring the difference between what he/she is entitled to receive and the *delivery* to the pool − what he/she has promised. If this difference is positive (negative), so is the net trade, and his/her final income is higher (lower) than it would have been without the pool.

Since realized endowments are verifiable, we assume that promises are always honored when the realized endowment is positive. If endowment is zero, then consumers are allowed not to deliver. This means that consumers who do not contribute because of lack of endowment are still entitled to receive their share of the pooled deliveries.

It helps to think of more specific scenarios. For the sake of simplicity, we consider a pool of only two consumers whose endowment tomorrow may be either $w_G = 1$ or $w_B = 0$. In Example 1 in Table 1, both consumers promise to deliver to the pool 0.5

	Example 1		Example 2	
	Consumer 1	Consumer 2	Consumer 1	Consumer 2
Promise	0.5	0.5	1.5	0.5
Share	1/2	1/2	3/4	1/4
Endowment	1	0	1	0
(Net) Delivery	0.5	0	0.75	0
Return	0.5(1/2)	0.5(1/2)	0.75(3/4)	0.75(1/4)
Consumption	3/4	1/4	13/16	3/16

Table 1: Examples of the pool of promises

of their endowments of tomorrow. Since they have promised the same proportion, tomorrow they will both be entitled to receive the same share of the pool, even though they may have made different deliveries. Consumer 1, who faces the good state of nature, delivers 0.5 to the pool, while Consumer 2 delivers zero. The pool thus amounts to 0.5, to be equally shared. Consumer 1 ends up consuming 3/4 of his endowment (0.5 not delivered, plus the net trade with the pool, 1/4) while Consumer 2 ends up consuming 1/4. In Example 2, however, because consumers promise differently they are entitled to different shares of the pool. This example also illustrates that promises may be higher than 1, i.e., gross deliveries might be higher than w_G: Consumer 1 promises 1.5 of his tomorrow endowment and Consumer 2 0.5, they are entitled to a share of the pool of 3/4 and 1/4 , respectively.

Since the pool we have in mind incorporates promises from a continuum of consumers, these examples are merely aimed at understanding the mechanism of promise, mainly that *promises, and not deliveries, determine the share of the pool.* Moreover, in our set-up, consumers may undertake different actions that influence the likelihood of states of nature and thus the risk that they face. Promises made to the pool can hence be different depending on the action undertaken. Yet, contingent on the action, consumers will all promise the same since they are ex ante equal.

Therefore, if q, respectively $(1-q)$, consumers make the high, respectively low, effort and commit accordingly θ_H, respectively θ_L, each consumer promising θ_a has the right to the following share of the pool:

$$\frac{\theta_a}{q\theta_H + (1-q)\theta_L}.$$

where, with consistent notation, we let θ_a be the promise of those consumers choosing action a. The deliveries effectively made to the pool depend on the state of nature, although aggregate endowment is certain, as discussed above. Therefore, *return per promise*, defined as

$$\kappa = \frac{q\theta_H \bar{w}_H + (1-q)\theta_L \bar{w}_L}{q\theta_H + (1-q)\theta_L}\,, \tag{2}$$

is itself independent of the state of nature, and by construction $w_B < \bar{w}_L \leqslant \kappa \leqslant \bar{w}_H < w_G$. In what follows, we say that the pool return per promise κ is *consistent* if it satisfies (2).

2.2 Consumers' problem

Consumers take the return of the pool as given, and choose their promises and actions so as to maximize expected utility. A promise is $\theta \geqslant 0$, which implies that negative insurance is ruled out and that consumers are not constrained in their promises. In particular, as stressed above, they may commit a fraction greater than one.

When promising θ, they give up θw_s in the s state of nature. Giving up θw_s allows them to receive $\theta\kappa$, where κ denotes the pool's return per promise, as given by (2). Consumers form expectations about the pool's return per promise in the same spirit as they do about prices in a competitive economy. Given these expectations, they choose their optimal promise and optimal action. In equilibrium, expectations are fulfilled and all optimal plans are realized without resorting to strategic behavior. We then have that a consumer's state contingent consumption is:

$$\begin{aligned} x_G &= w_G - \theta(w_G - \kappa), \\ x_B &= w_B - \theta(w_B - \kappa). \end{aligned}$$

The above contingent budget constraints can be re-written in a way that shows the transfer of consumption from the good state of nature to the bad one. Therefore, let us first eliminate θ , and rewrite the above constraints as follows:

$$x_B \quad = \quad w_B - \frac{\kappa - w_B}{w_G - \kappa}(x_G - w_G), \tag{3}$$

From the above equation, it can be seen that by giving up $(w_G - \kappa)$ units of consumption in the good state a consumer gets $(\kappa - w_B)$ units of consumption in the bad state per unit of promise, since by construction $w_B < \kappa < w_G$. Therefore, through pool consumers can indeed transfer consumption from the good state of nature to the bad one, that is they can buy insurance.

Now let us eliminate κ , and rewrite the contingent budget constraints as follows

$$x_B \quad = \quad x_G - (w_G - w_B)(1 - \theta). \tag{4}$$

The above equations merely state how much is left over for consumption in the bad state of nature for a promise θ . When $\theta = 1$, all the endowment is promised to the pool whatever the state of nature. Consequently consumption is equalized in the two states of nature, i.e. consumers are fully insured. More generally, from the above equation we conclude that:

$$x_B \lesseqqgtr x_G \quad \Leftrightarrow \quad \theta \lesseqqgtr 1.$$

Clearly, a solution must satisfy both constraints. Graphically, it corresponds to the intersection of equations (3) and (4), as shown in Figure 2, borrowed from Dubey and Geanakoplos (2002).

Figure 2 illustrates precisely the transfer of consumption from the good state of nature to the bad. The horizontal axis represents consumption in the good state of nature, and the vertical represents consumption in the bad state. In the figure it is assumed that contingent endowments are $(w_G, w_B) = (w_G, 0)$. The pool returns κ per promise and thus consumption is transferred from the good state to the bad through the price line κ that joins $(w_G, 0)$ and $(0, w_G\kappa/(w_G - \kappa))$. This "κ-price" is the representation of constraint (3) above.

Since consumers commit θw_G of their endowment, their final consumption bundle must lie on the 45° degree "θ-quantity" line starting at $(w_G(1 - \theta), 0)$. This is the representation of constraint (4) above. The final consumption bundle is given by

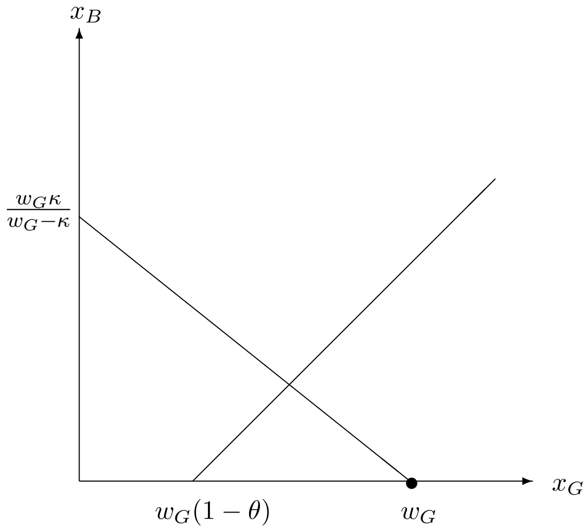

Figure 2: Transferring consumption from the good to the bad state of nature: κ-price line and θ-quantity line.

the intersection of the two lines. It becomes clear now that promising more than 1 is possible as long as the net trade with the pool is positive, i.e., as long as the θ-quantity line crosses the κ-price line for positive x_G.

Summarizing the above discussion, the consumers' problem can be written as follows:

$$\max_{x,\theta,a} \quad \pi_a u(x_G) + (1-\pi_a)u(x_B) - c_a\,,$$

$$\text{s.t.} \quad x_G = w_G - \theta(w_G - \kappa)\,, \tag{5a}$$

$$x_B = w_B - \theta(w_B - \kappa)\,, \tag{5b}$$

$$\theta \geqslant 0\,, \quad x_G \geqslant 0\,, \quad x_B \geqslant 0\,, \tag{5c}$$

$$a \in \mathcal{A}\,. \tag{5d}$$

As a last comment on the consumers' problem, it can be pointed out that non-negative constraints on consumption imply that $\theta \leqslant w_G/(w_G - \kappa)$, from which it can be concluded that $\theta > 1$ is indeed admissible, since $w_G/(w_G - \kappa) > 1$.

In what follows, let (x^*, θ^*_a, a^*) be a solution of the above maximization problem, and $\psi(\kappa, w)$ its solution set.

3 Equilibrium

In equilibrium, consumers maximize their utility by taking as given the return of the pool, which is endogenously determined in a consistent way. Formally, we propose the following:

Definition 3.1. *An equilibrium with a pool of promises is* $(x^*, \theta_a^*, a^*, q^*, \kappa^*)$ *such that:*

(a) $(x^*, \theta_a^*, a^*) \in \psi(\kappa^*, w)$,

(b) κ^* *is consistent*,

(c) q^* *satisfies:*

(1) $q^* = 0$ *if* $(x^*, \theta_a^*, a^*) \in \psi(\kappa^*, w) \Rightarrow a^* = L$ [Low Action Equilibrium] ,

(2) $q^* = 1$ *if* $(x^*, \theta_a^*, a^*) \in \psi(\kappa^*, w) \Rightarrow a^* = H$ [High Action Equilibrium] ,

(3) $q^* \in (0, 1)$ *otherwise* [Mixed Action Equilibrium] .

In the above definition, it is stated that the equilibrium values of q must be properly related to the optimal choices of consumers. In particular, $q = 0$ can only arise at equilibrium if $a = L$ is the optimal choice for every consumer. Similarly, $q = 1$ can only arise if $a = H$ is the optimal choice for every consumer. Finally, for $q \in (0, 1)$ to arise in equilibrium, both $a = H$ and $a = L$ must be optimal choices of consumers.

In what follows, we first consider *uniform pool equilibria*, in which all consumers undertake the same action, and then *mixed pool equilibria*, in which some consumers undertake the high effort action and others the low effort action.

3.1 Uniform pool equilibrium

In this section we first show that a high action equilibrium never arises (Proposition 1), and then we propose conditions for the existence of low action equilibrium (Proposition 2).

Proposition 1. [Impossibility of high action equilibrium]

There cannot be an equilibrium in which all consumers undertake the high action, i.e. , if $(x^*, \theta_a^*, a^*, q, \kappa)$ *is an equilibrium with pool of promises, then* $q \neq 1$.

Proposition 1 states that $q = 1$ leads to a non consistent κ. From the definition of κ in equation (2) it is easily seen that $q = 1$ would imply $\kappa = \bar{w}_H$. Yet, when consumers anticipate such a low price for insurance, their optimal choice is to buy a lot of insurance (above the switching locus) and to choose $a = L$. However, from the definition of equilibrium it follows that if all consumers carry out the low effort action, then $q = 0$, and therefore $\kappa = \bar{w}_H$ cannot be consistent.

In Proposition 2 we state the condition under which a low action equilibrium exists. It is helpful to consider first the following problem:

$$
\begin{aligned}
\max_{x,\theta} \quad & \pi_H u(x_G) + (1 - \pi_H) u(x_B) - c_H \,, \\
\text{s.t.} \quad & x_G = w_G - \theta(w_G - \bar{w}_L) \,, \\
& x_B = w_B - \theta(w_B - \bar{w}_L) \,, \\
& \theta \geqslant 0 \,, \quad x_G \geqslant 0 \,, \quad x_B \geqslant 0 \,.
\end{aligned}
$$

The solution of the above problem is the optimal consumption bundle for a consumer who can buy insurance at the price $\kappa = \bar{w}_L$, conditional on choosing $a = H$. Let $(\hat{x}_H, \hat{\theta}_H)$ represent this solution, and let $(\hat{x}_L, \hat{\theta}_L) = ((\bar{w}_L, \bar{w}_L), 1)$ be the solution of the consumers' problem when facing the same price $\kappa = \bar{w}_L$ but conditional on choosing $a = L$. We can now state Proposition 2.

Proposition 2. [Possibility of low action equilibrium]

If $U(\hat{x}_L, L) > U(\hat{x}_H, H)$ *, then a low action equilibrium exists.*

In a low action equilibrium, $q = 0$ and therefore $\kappa = \bar{w}_L$. Anticipating this price of insurance, conditional on choosing $a = L$, consumers optimal bundle is the full insurance one $\hat{x}_L$, while conditional on choosing $a = H$, their optimal bundle is $\hat{x}_H$. If $(\hat{x}_L, L)$ is preferred to $(\hat{x}_H, H)$, then indeed all consumers undertake the low action, and hence $q = 0$ and $\kappa = \bar{w}_L$ is consistent.

The next section covers the mixed pool equilibrium, which arises when there is no low action equilibrium.

3.2 Mixed pool equilibrium

In a mixed pool equilibrium, some consumers are willing to undertake the high action and some others the low action. Yet, since consumers are ex ante equal, this can only happen if they are all indifferent between undertaking one action or the other. In the following proposition we state the condition under which this happens.

Proposition 3. [POSSIBILITY OF MIXED ACTION EQUILIBRIUM]
If $U(\hat{x}_L, L) < U(\hat{x}_H, H)$, then a mixed action equilibrium exists.

If the condition in Proposition 2 does not hold, then consumers prefer to undertake the high action when faced with price $\kappa = \bar{w}_L$. Moreover, from Proposition 1 it is known that consumers always undertake the low action when faced with price $\kappa = \bar{w}_H$. Since the consumers' problem solution is a closed and upper hemicontinuous correspondence, there exists a price $\hat{\kappa} \in (\bar{w}_L, \bar{w}_H)$ at which consumers are indifferent between high and low action.[8] It follows that at this price they split into high and low action in the proportion $q \in (0, 1)$ that ensures that $\hat{k}$ is consistent.

Figure 3 illustrates an equilibrium with a mixed pool of promises for the case in which $w_B = 0$. The two bundles (x_L, L), and (x_H, H) yield the same utility, and insurance is bough at the price $\hat{\kappa}$.

To help understand the mixed pool equilibrium we propose the following. Firstly, in Section 4 we introduce two numerical examples. Secondly, in Section 5 we show how in an economy of poor and rich consumers an aggregate pool of promises may represent a Pareto improvement on segregated pools of promises. This result is surprising since one would expect that the rich would enhance their welfare if they pool only among themselves. However, as will become clear, the poor may play an important role in ensuring a consistent return per promise.

4 Examples of Mixed Pool of Promises

We follow by presenting two specific examples, so that the question at stake is more clearly understood. In the first example it is assumed that $u(x) = \log(x)$, $w_G > w_B = 0$, and $\pi_H = 1 - \pi_L$.

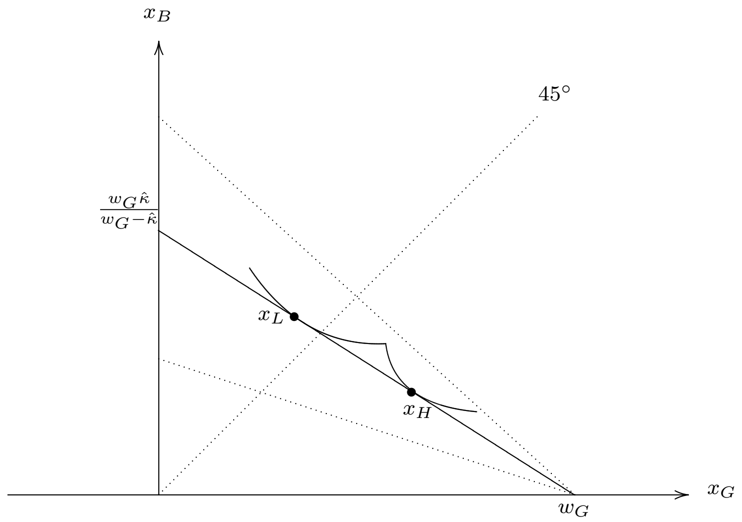

Figure 3: Mixed Pool equilibrium.

As introduced in Section 2.2, given κ, consumers choose a consumption bundle, a promise, and an action so as to maximize expected utility, under the budget constraints described above. Using the first order condition for an interior maximum, one can easily find the following closed form solutions:

$$\theta_a(\kappa) = \frac{(1-\pi_a)w_G}{w_G - \kappa}, \tag{7a}$$

$$x_a(\kappa) = \left(\pi_a w_G, \frac{(1-\pi_a)w_G\kappa}{w_G - \kappa}\right). \tag{7b}$$

Conditional on the action chosen, optimal promises are decreasing both on contingent endowment and the probability of the good state, since if the good state is very likely one does not need to insure too much.

As it is clear from equation (7b), the optimal level of contingent consumptions depends on the pool's return per promise. One can thus find the value $\hat{\kappa}$ above (below) which consumers prefer to take the low (high) effort action. This is given by the solution of the following equation:

$$U(x_H(\hat{\kappa}), H) = U(x_L(\hat{\kappa}), L) \quad \Leftrightarrow \quad \hat{\kappa} = \frac{w_G}{1 + \exp\left\{\frac{c_H}{(\pi_H - \pi_L)}\right\}}. \tag{8}$$

The return per promise $\hat{\kappa}$ is increasing in the endowment of the good state of nature and decreasing in the cost of the high effort action, meaning that the less costly the action the greater $\hat{\kappa}$.

For $\hat{\kappa}$ to be consistent, it must hold that

$$\hat{\kappa} = \frac{q\theta_H(\hat{\kappa})\bar{w}_H + (1-q)\theta_L(\hat{\kappa})\bar{w}_L}{q\theta_H(\hat{\kappa}) + (1-q)\theta_L(\hat{\kappa})}.$$

Substituting the optimal level of promises as given by equation (7a), and solving for q, the proportion of consumers undertaking the high effort action that guarantees $\hat{\kappa}$ consistency can be found:

$$\hat{q} = \frac{\pi_H - \pi_H\pi_L\left(1 + exp\left\{\frac{c_H}{\pi_H - \pi_L}\right\}\right)}{\pi_H - \pi_L}.$$

Figure 4 illustrates a mixed pooling equilibrium. There are three price-lines represented: Those associated with the lower and higher probabilities of the good state, and that associated with the pool return per promise (between the other two). Associated with the $\hat{\kappa}$ that sustains the mixed pooling equilibrium are $\hat{q}$ and two levels of promises, $\theta_H(\hat{\kappa})$ and $\theta_L(\hat{\kappa})$. The promise of consumers undertaking the high effort action is smaller than that of those undertaking the low effort action. In particular $\theta_L(\hat{\kappa}) > 1$, and consumers overinsure. The final consumption bundles are those at the intersection of each of the θ_a-quantity lines with the $\hat{\kappa}$-price line. At these points, optimality requires indifference curves to be tangential to the $\hat{\kappa}$-price line. Finally these two indifferent curves are associated with the same level of utility (they cross at the switching locus).

This mixed pool equilibrium could be an illustration of the following numerical example: $c_H = 0.21$, $(\pi_H, \pi_L) = (2/3, 1/3)$, and $w = (1.5, 0)$. In the mixed pool equilibrium, $\hat{\kappa} = 0.52$ and $\hat{q} = 0.1$. Equilibrium consumption and promises for those consumers choosing high action and low action are, respectively, $(x_H, \theta_H) = ((1, 0.27), 0.51)$ and $(x_L, \theta_L) = ((0.5, 0.53), 1.02)$. The level of utility achieved is $U(x_H, \theta_H) = U(x_L, \theta_L) = -0.65$.

Alternatively, the following numerical example can be represented: $u(x) = x^{\gamma}/\gamma$ with $\gamma = 0.5$, $c_H = 0.163$, $(\pi_H, \pi_L) = (2/3, 1/3)$ and $w = (1, 0)$. An equilibrium with a mixed pool of promises exists in this case, and it is such that $\hat{\kappa} = 0.4$ and $\hat{q} = 0.56$. Equilibrium consumption and promises for those consumers choosing the high action or the low action, are $(x_H, \theta_H) = ((0.85, 0.095), 0.23)$, and $(x_L, \theta_L) = ((0.27, 0.48), 1.21)$, respectively. The level of utility achieved is $U(x_H, H) = U(x_L, L) = 1.27$.

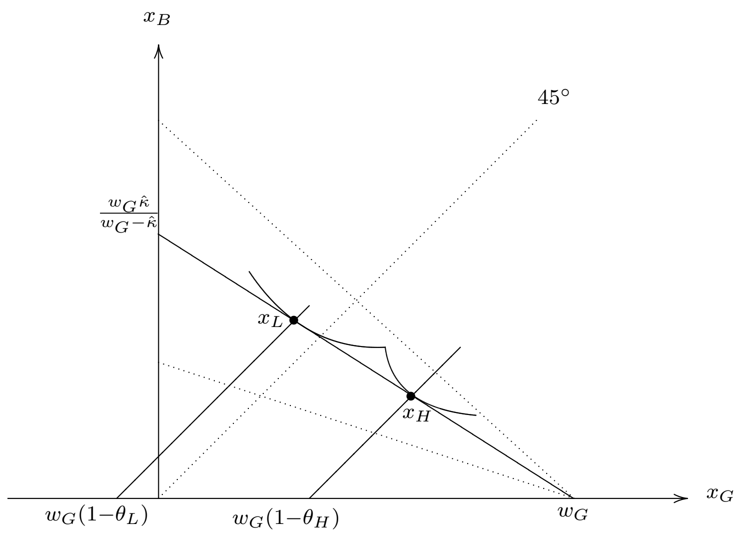

Figure 4: Mixed Pool equilibrium.

5 Pool of Promises with Heterogeneous Consumers

The equilibrium concept just introduced may be extended to the case of heterogeneous groups of consumers. In particular it may be used to explore the possibility of forming an aggregate pool, Pareto-superior to segregated ones, that would never arise in the absence of moral hazard.

For the sake of specificity, we assume there are two large groups of consumers, *rich* ($\mathcal{R}$) and *poor* ($\mathcal{P}$), differing in the distribution of their endowments, so that the former have higher expected endowments than the latter. In the absence of moral hazard, it is easy to understand that consumers from the rich group would never agree to join an aggregate pool with those from the poor group. To see this, let $\lambda^{\mathcal{R}}$ and $\lambda^{\mathcal{P}}$ denote, respectively, the proportion of rich and poor consumers, with $\lambda^{\mathcal{R}} + \lambda^{\mathcal{P}} = 1$. Moreover, let $\bar{w}^{\mathcal{R}}$ and $\bar{w}^{\mathcal{P}}$ be the expected endowments of the former and the latter, with $\bar{w}^{\mathcal{R}} > \bar{w}^{\mathcal{P}}$. In this case, the total expected endowments that an aggregate pool can guarantee to its members is $\lambda^{\mathcal{R}}\bar{w}^{\mathcal{R}} + \lambda^{\mathcal{P}}\bar{w}^{\mathcal{P}}$. This is lower than $\bar{w}^{\mathcal{R}}$, the expected endowment that a segregated pool of the rich alone can guarantee to its members.

In the presence of moral hazard, an aggregate pool of poor and rich may emerge. We assume that one group is still richer than the other in the sense that no matter what action is chosen, rich consumers have higher expected endowments than poor consumers. In this case the proportion of consumers choosing high action is endogenously determined, and in particular a mixed pooling equilibrium may emerge. Therefore, it may happen that a high enough proportion of them doing high effort will offset the reduction of the return of the pool due to the presence of the poor consumers.

5.1 The pool of promises

When groups are segregated each pool of promises has a return per promise given by (2), which we recall here:

$$\kappa^i = \frac{q^i \theta^i_H \bar{w}^i_H + (1-q^i)\theta^i_L \bar{w}^i_L}{q^i \theta^i_H + (1-q^i)\theta^i_L}\,, \tag{9}$$

for $i \in \{\mathcal{P}, \mathcal{R}\}$. When an aggregated pool is formed, its return per promise depends on the deliveries of both types of consumers:

$$\kappa = \frac{\sum_i \lambda^i q^i \theta^i_H \bar{w}^i_H + \sum_i \lambda^i (1-q^i)\theta^i_L \bar{w}^i_L}{\sum_i \lambda^i q^i \theta^i_H + \sum_i \lambda^i (1-q^i)\theta^i_L}\,. \tag{10}$$

Also in this case, the return of an aggregate pool is consistent if it satisfies (10).

5.2 Equilibrium

When groups are segregated, the equilibrium concept is analogous to the one presented in Definition 3.1. Each pool's return per promise is given by (9), and for consumers in group i choices are optimal given κ^i.

When an aggregated pool is considered, the equilibrium definition must be amended to take into account that the pool's return per promise is determined by (10) and that consumers' choices must be optimal given this. These properties are summarized as follows:

Definition 5.1. *An equilibrium with an aggregate pool of promises is* $(x^{i*}, \theta_a^{i*}, a^{i*}, q^i, \kappa)$, *such that, for* $i \in \{\mathcal{P}, \mathcal{R}\}$*:*

(*a*) $(x^{i*}, \theta_a^{i*}, a^{i*}) \in \psi^i(\kappa, w^i)$ *,*

(*b*) κ *is consistent ,*

(*c*) q^i *satisfies:*

 (1) $q^i = 0$ *if* $(x^{i*}, \theta_a^{i*}, a^{i*}) \in \psi^i(\kappa, w^i) \Rightarrow a^{i*} = L, \forall i$ *,*

 (2) $q^i = 1$ *if* $(x^{i*}, \theta_a^{i*}, a^{i*}) \in \psi^i(\kappa, w^i) \Rightarrow a^{i*} = H\ \forall i$ *,*

 (3) $q^i \in (0, 1)$ *otherwise .*

Moreover, let $q := \sum_i \lambda^i q^i$ *be the aggregate proportion of consumers undertaking high effort .*

Consistently with the previous section, an equilibrium such that $q \in (0, 1)$ is referred to here as an equilibrium with a mixed (aggregate) pool of promises.

5.3 Mixed aggregate pool: an example

We provide an explicit example of a mixed aggregate pool. As in Example 1, we assume that $u^i(x) = \log(x)$, $w_G^i > w_B^i = 0$, $\pi_H^{\mathcal{R}} = \pi_H^{\mathcal{P}}$, and $\pi_H^i = 1 - \pi_L^i$. Moreover,we assume that $w_G^{\mathcal{P}} = \alpha w_G^{\mathcal{R}}$, with $0 < \alpha < 1$.

It is straightforward to apply the former formulation to the heterogenous case. Under the assumptions considered, optimal promises and optimal contingent consumption levels are:

$$\theta_a^{\mathcal{R}}(\kappa) = \frac{(1-\pi_a)w_G^{\mathcal{R}}}{w_G^{\mathcal{R}}-\kappa} \tag{11a}$$

$$\theta_a^{\mathcal{P}}(\kappa) = \frac{(1-\pi_a)\alpha w_G^{\mathcal{R}}}{\alpha w_G^{\mathcal{R}}-\kappa} \tag{11b}$$

$$x_a^{\mathcal{R}}(\kappa) = \left(\pi_a w_G^{\mathcal{R}}, \frac{(1-\pi_a)w_G^{\mathcal{R}}\kappa}{w_G^{\mathcal{R}}-\kappa}\right) \tag{11c}$$

$$x_a^{\mathcal{P}}(\kappa) = \left(\pi_a \alpha w_G^{\mathcal{R}}, \frac{(1-\pi_a)\alpha w_G^{\mathcal{R}}\kappa}{\alpha w_G^{\mathcal{R}}-\kappa}\right) \tag{11d}$$

Consistently with our previous statement, the promises of the poor are greater than those of the rich. It also follows from equation (8) that $\hat{\kappa}$ is increasing in endowment, so that the return per promise that makes consumers indifferent as to which action to undertake is higher for the rich. We let $\hat{\kappa}^i$ represent this critical return per promise of group $i \in \{\mathcal{P}, \mathcal{R}\}$. Therefore, considering a candidate equilibrium κ, one of the following configuration may happen:

1. $\hat{\kappa}^{\mathcal{P}} < \hat{\kappa}^{\mathcal{R}} < \kappa$, and both poor and rich choose to take the low effort action. Hence, $q = 0$.

2. $\hat{\kappa}^{\mathcal{P}} < \hat{\kappa}^{\mathcal{R}} = \kappa$, and the poor choose to take the low action while the rich are indifferent. Hence, $q^{\mathcal{P}} = 0$, $q^{\mathcal{R}} \in (0,1)$, and $q \in (0,1)$.

3. $\hat{\kappa}^{\mathcal{P}} < \kappa < \hat{\kappa}^{\mathcal{R}}$, and the poor choose to take the low action while the rich take the high action. Hence, $q^{\mathcal{P}} = 0$, $q^{\mathcal{R}} = 1$, and $q \in (0,1)$.

4. $\hat{\kappa}^{\mathcal{P}} = \kappa < \hat{\kappa}^{\mathcal{R}}$, and the poor are indifferent while the rich take the high action. Hence, $q^{\mathcal{P}} \in (0,1)$, $q^{\mathcal{R}} = 1$, and $q \in (0,1)$.

5. $\kappa < \hat{\kappa}^{\mathcal{P}} < \hat{\kappa}^{\mathcal{R}}$, and both poor and rich take the high action. Hence, $q = 1$.

Three things are worthy of note here. First, since $\hat{\kappa}^{\mathcal{P}} < \hat{\kappa}^{\mathcal{R}}$, poor consumers will never undertake a more costly action than rich ones. Second, incorporating the additional assumption that the poor consumers bear a higher cost when undertaking the high action merely increases the disparity between between $\hat{\kappa}^{\mathcal{P}}$ and $\hat{\kappa}^{\mathcal{R}}$. This is because $\hat{\kappa}$ is decreasing in the cost of effort as can easily be seen from equation (8). Third, some of the above might never arise in equilibrium because κ might be non consistent. In particular, we already know from Proposition 1 that case 5 can never arise in equilibrium.

We now provide a numerical illustration of case 2. We assume equal sized groups and that $(\pi_H, \pi_L) = (2/3, 1/3)$. Poor consumers are just like those in the first example of homogenous consumers in the end of Section 4. When pooling promises only among themselves they generate an equilibrium such as the one described in that example. The first column of Table 2 synthesizes the information concerning the segregated mixed pool of the poor.

Rich consumers, by contrast, have a higher contingent endowment in the good state of nature $w^{\mathcal{R}} = (2, 0)$, and it is less costly for them to take the high action $c_H^{\mathcal{R}} = 0.2$. This assumption is natural when interpreted in terms of the access that wealthier people have to accident-preventing practices. The second column of Table 2 presents information concerning the segregated pool of the rich. When 10% of them take the high action, they can generate a mixed pool with a return per promise of 0.7. In this case, they achieve a utility level of -0.35, clearly superior to what the poor can get by themselves.

Now let the two groups enter a common pool, characterized in the third column of Table 2. Recall that in this example the return per promise of the aggregate pool equals the return per promise of the mixed segregated pool of the rich alone. Therefore, facing the same return per promise, the rich have no reason to commit differently and end up with the same level of utility. The poor are however much better off. They face a higher return per promise, which makes them want to promise more. As anticipated, none of them undertakes the high action.

The message of this example is that by mixing poor and rich in a pool of promises there is room for a Pareto improvement: the poor are better off and the rich are equally well off. This follows since the proportion of the rich undertaking the high action increases, which is possible because they are still indifferent with respect to which action to choose.

The economy therefore gains from two different effects. Firstly, the rich are more

	Pool of Poor	Pool of Rich	Heterogenous Pool	
c_H	0.21	0.20		
q	0.1	0.1	$q^{\mathcal{P}} = 0$	$q^{\mathcal{R}} = 0.8$
κ	0.52	0.7	0.7	
θ_H^*	0.51	0.51		$\theta_H^{\mathcal{R}^*} = 0.51$
x_H^*	$(1, 0.27)$	$(4/3, 0.36)$	$x_H^{\mathcal{P}^*} = (1, 0.44)$	$x_H^{\mathcal{R}^*} = (4/3, 0.36)$
θ_L^*	1.02	1.02	$\theta_L^{\mathcal{P}^*} = 1.25$	$\theta_L^{\mathcal{R}^*} = 1.02$
x_L^*	$(0.5, 0.53)$	$(2/3, 0.72)$	$x_L^{\mathcal{P}^*} = (0.5, 0.875)$	$x_L^{\mathcal{R}^*} = (2/3, 0.72)$
U_{C_H}	-0.65	-0.35		$U_{C_H}^{\mathcal{R}} = -0.35$
U_{C_L}	-0.65	-0.35	$U_{C_L}^{\mathcal{P}} = -0.32$	$U_{C_L}^{\mathcal{R}} = -0.35$

Table 2: Example of a mixed heterogenous pool of promises

active in preventing the bad state of nature. This process increases the aggregate expected endowment since a rich consumer in the good state of nature is endowed with more than a poor one. Secondly, the rich bear a lower cost in preventing the bad state of nature and this reduces the economy's overall cost of preventing accidents. In other words, the rich can, at no cost, redistribute towards the poor because they are wealthier and are more able to prevent bad outcomes.

6 Conclusion

We analyze the pool of promises in a setting with ex-ante moral hazard, in which agents affect the probability distribution of events. This additional freedom allows for the possibility of some consumers making high efforts and others low efforts in equilibrium, even though consumers are ex-ante alike. Consequently, besides the low effort equilibrium, it is also possible that economies end up in a mixed pool equilibrium with q consumers undertaking the high action. When a heterogeneous population is considered, we show how the rich, who are also more able, can redistribute towards the poor at no cost, i.e., the heterogenous pool Pareto dominates the two segregated pools.

A natural question to raise here is whether a mixed equilibrium can be implemented. The pool organizer can be thought of as allocating consumers to promise levels according to the q that guarantees a consistent return per promise. Again it should be stressed that consumers are completely indifferent to this process, since they obtain the same level of utility whatever the action.

In our view this framework is of particular interest in developing countries. As Pauly et al. (2006) suggests, it seems reasonable to think of insurance cooperatives as an adequate form of insurance organization for these countries. In fact, on the one hand, tax systems are often more deficient, which compromises compulsory public insurance schemes. On the other hand, the populations of these countries are poorer and more often excluded from the market. In developing countries, mutual insurance solutions have indeed emerged for smaller communities. For example, Cabrales et al. (2003) analyze a specific mutual fire insurance scheme used in Andorra, De Weerdt and Dercon (2006) find evidence of risk-sharing across networks within a village in Tanzania, and Murgai et al. (2002) study water transfers on two water courses in Pakistan. We additionally argue that a voluntary mutual insurance scheme, such as the pool of promises, could be implemented at national level.

For application to developing economies, it seems reasonable to extend this model so that it encompasses aggregate uncertainty. Another interesting extension is to consider the possibility of limiting promises. This has the same effect as partial insurance has in standard models with moral hazard: it makes a high action enhancing consumers' welfare incentive compatible. In a heterogenous pool, the consequences of limiting promises are however not as straightforward. This is precisely the next point on our research agenda.

7 Appendix

In this Appendix, we prove all the propositions in the text. To that end, we find it useful to modify the consumers' problem as follows. We eliminate θ by substituting (5a)−(5b) for the κ-price line given by (3). Note that (3) can then be written as

$$\frac{\kappa - w_B}{w_G - \kappa}(x_G - w_G) + (x_B - w_B) = 0 \quad \Leftrightarrow \quad (\kappa - w_B)(x_G - w_G) + (w_G - \kappa)(x_B - w_B) = 0\,.$$

Multiplying both sides by $(w_G - \kappa)$ and letting

$$\pi = \frac{\kappa - w_B}{w_G - w_B}\,, \tag{12}$$

the consumers' problem can be rewritten as the following *generalized consumers' problem*:

$$\begin{aligned} \max_{x,a} \quad & \pi_a u(x_G) + (1-\pi_a)u(x_B) - c_a\,, \\ \text{s.t.} \quad & \pi(x_G - w_G) + (1-\pi)(x_B - w_B) = 0\,, && \text{(13a)} \\ & 0 \leqslant x_G \leqslant w_G\,, && \text{(13b)} \\ & a \in \mathcal{A}\,. && \text{(13c)} \end{aligned}$$

With consistent notation, let $\varphi(\pi, w)$ denote its solution set. Since in the above problem the choice set is a compact-valued and continuous correspondence and the objective function is continuous, it follows from the Maximum Theorem that $\varphi(\pi, w)$ is a closed and upper hemicontinuous correspondence.

We now introduce Lemma 7.1 and Lemma 7.2 , which will be useful in characterizing the equilibria of the pool of promises. Lemma 7.1 relates the solutions of the generalized consumers' problem to those of the problem introduced on page 11.

Lemma 7.1. *If (x, a) satisfies (13a)–(13c) for some π , then (x, θ, a) satisfies (5a)–(5d) for $\theta = (w_G - x_G)/(w_G - \kappa)$ and κ given by* (12)*. Moreover, if (x, θ, a) satisfies (5a)–(5d) for some κ , then (x, a) satisfies (13a)–(13c) for π given by* (12)*.*

Lemma 7.2 is due to Hellwig (1983a) and characterizes the consumer's optimal solution.[9]

Lemma 7.2. [Hellwig (1983a), Lemma A.1]*. If $(x, a) \in \varphi(\pi, w)$ and $\pi_a \leqslant \pi$, then $x_G \leqslant x_B$ and $a = L$.*

Proof of Lemma 7.2. Consider the following function:

$$v(a, x_G, \pi) = \pi_a u(x_G) + (1-\pi_a)u\left(w_B + \frac{\pi}{1-\pi}(w_G - x_G)\right) - c_a\,,$$

with $a \in \mathcal{A}$, $x_G \in [0, w_G]$ and $\pi \in [\pi_L, \pi_H]$. A solution of the generalized consumers' problem corresponds to a maximum of $v(\cdot,\cdot,\pi)$. Since v is concave in x_G , and since $\pi_a \leqslant \pi$ implies $\partial v(a, w_G, \pi)/\partial x_G < 0$, the maximum is reached at some x_G such that $x_G = 0$ or $x_G > 0$. In the first case, we have that $x_B = w_B + (\pi w_G/(1-\pi)) > 0 = x_G$, while in the second case $\partial v(a, x_G, \pi)/\partial x_G = 0$ implies $x_B = w_B + (\pi(w_G - x_G)/(1-\pi)) \geqslant x_G$, since $u'(\cdot)$ is decreasing. ∎

Proof of Proposition 1. [Impossibility of high action equilibrium]
If $q = 1$, then $\kappa = \bar{w}_H$ and, by (12) , $\pi = \pi_H$. From the analysis of the generalized consumer's problem it follows that in this case $(x, a) \in \varphi(\pi_H, w)$ implies $a = L$, and, by Lemma 7.1, so does $(x, \theta_a, a) \in \psi(\bar{w}_H, w)$. By definition of equilibrium, $q = 0$ and this is the required contradiction. ∎

Proof of proposition 2. [POSSIBILITY OF LOW ACTION EQUILIBRIUM]
If $U(\hat{x}_L, L) > U(\hat{x}_H, H)$, then $(x, a) \in \varphi(\pi_L, w)$ implies $a = L$, and, by Lemma 7.1, so does $(x, \theta_a, a) \in \psi(\bar{w}_L, w)$. By definition of equilibrium, $q = 0$, hence κ is consistent. ∎

In order to prove Proposition 3, we need the following lemma, due to Hellwig (1983b). Since the original source may be difficult to access, we offer a complete proof, following closely the original one, to facilitate the task of readers.

Lemma 7.3. [HELLWIG (1983B), LEMMA 3.2].
For any given (π_H, π_L, w) such that $1 > \pi_H > \pi_L > 0$ and $w_G > w_B \geqslant 0$ there exists a $c > 0$ such that, for $c > c_H > 0$, $(x, a) \in \varphi(\pi_L, w)$ implies $x_G > x_B$ and $a = H$. In this case, there exists a price of insurance $\hat{\pi}$, an overinsurance consumption bundle x_L and a partial insurance consumption bundle x_H such that $\{(x_H, H), (x_L, L)\} = \varphi(\hat{\pi}, w)$.

Proof of lemma 7.3.
We first prove the first part of the proposition, then the second part. Let

$$\xi(x) \quad := \quad \pi_H u(x_G) + (1 - \pi_H) u(x_B)\,, \tag{14a}$$

$$\chi(\pi_L, w) \quad := \quad \arg\max\left\{\xi(x) \,|\, \pi_L(x_G - w_G) + (1 - \pi_L)(x_B - w_B) = 0\,,\ 0 \leqslant x_G \leqslant 0\right\}. \tag{14b}$$

From the first order conditions of the problem in (14b), we can easily verify that $x \in \chi(\pi_L, w)$ implies $x_G > x_B$, hence $x \neq \hat{x}_L$, where we recall that $\hat{x}_L = (\bar{w}_L, \bar{w}_L)$.[10] Since $\hat{x}_L$ satisfies the constraint set in (14b), $x \in \chi(\pi_L, w)$ also implies $\xi(x) > \xi(\hat{x}_L)$, where the strict inequality follows from the strict concavity of ξ. For $x \in \chi(\pi_L, w)$, we let $\xi^* = \xi(x) - \xi(\hat{x}_L)$. From the above analysis it follows that ξ^* it is strictly positive, and from the Maximum Theorem it follows that it is continuous. Therefore it reaches a minimum on a compact set $\mathcal{D} \subset \mathbb{R}^2_+ \times [0,1]^2$ such that $(w, \pi_H, \pi_L) \in \mathcal{D}$ implies $w_1 > w_2 \geqslant 0$ and $1 > \pi_H > \pi_L > 0$. Let $c > 0$ be this minimum. If follows that there exists $0 < c_H < c$ such that $c_H < \xi^*$, hence that $\xi(\hat{x}_L) < \xi(x) - c_H$ with $x \in \chi(\pi_L, w)$. This concludes the proof of the first part of the lemma.

As for the second part, let Π be the set of insurance prices at which which consumers choose full, or overinsurance, and low effort, which is formally defined as follows:

$$\Pi := \{\pi \,|\, (x, L) \in \varphi(\pi, w)\,,\ x_B \geqslant x_G\}\,.$$

Notice that $\pi_H \in \Pi$, hence $\Pi \neq \varnothing$. Moreover, we assume that $\pi_L \notin \Pi$. The first part of lemma 7.3, which has just been proved, guarantees that this is indeed legitimate. Finally, let

$$\hat{\pi} := \inf\{\pi \,|\, \pi \in \Pi\}\,, \tag{15}$$

One should notice that $\hat{\pi} > \pi_L$. Indeed, if $\hat{\pi} = \pi_L$, there exists a sequence $\pi^n \in \Pi$ such that $\pi^n \to \pi_L$. In this case there exists a sequence $(x^n, L) \in \varphi(\pi^n, w)$ with $x^n_G \geqslant x^n_B$ and $x^n \to x$

for some x. Since φ is a closed correspondence, it follows that $(x, L) \in \varphi(\pi_L, w)$ with $x_B \geqslant x_G$. However, this contradicts $\pi_L \notin \Pi$. Let us now consider the following sequence:

$$\pi_h := \left(1 - \frac{1}{h}\right) \hat{\pi} + \left(\frac{1}{h}\right) \pi_L .$$

By construction, $\hat{\pi} > \pi_h > \pi_L$. Therefore, by definition of $\hat{\pi}$, $(x, a) \in \varphi(\pi_h, w)$ cannot imply $a = L$ and $x_B \geqslant x_G$. Moreover, it cannot imply $a = L$ and $x_B < x_G$ because of lemma 7.2. Hence, one can conclude that $(x, a) \in \varphi(\pi_h, w)$ implies $a = H$ and $x_G > x_B$. Moreover, if $(x_h, H) \in \varphi(\pi_h, w)$, then x_h belongs to the constraint set in the generalized consumer problem, which is compact. Therefore the series x_h converges to some x_H as h tends to infinity. Summing up, we have that $\pi_h \to \hat{\pi}$ and $x_h \to x_H$ as $h \to \infty$, and $(x_h, H) \in \varphi(\pi_h, w)$ for every h. Since φ is upper hemicontinuous, it follows that $(x_H, H) \in \varphi(\hat{\pi}, w)$. Let us now consider the following sequence:

$$\pi_l := \left(1 - \frac{1}{l}\right) \hat{\pi} + \left(\frac{1}{l}\right) \pi_H .$$

From the definition of $\hat{\pi}$, there exists a sequence $\pi_l \in \Pi$ such that $\pi_l \to \pi$. It follows that there exists a sequence $(x^l, L) \in \varphi(\pi^l, w)$ with $x^l_B \geqslant x^l_G$. As a consequence, there exists a subsequence converging to some (x_L, L). Since φ is closed, it follows that $(x_L, L) \in \varphi(\hat{\pi}, w)$. This concludes the proof. ∎

Proof of proposition 3 [Possibility of mixed action equilibrium].
If $U(\hat{x}_L, L) < U(\hat{x}_H, H)$, then $(x, a) \in \varphi(\pi_L, w)$ implies $a = H$ and $x_G > x_B$. In this case, by lemma 7.3 there exists a price of insurance $\hat{\pi}$, an overinsurance consumption bundle x_L and a partial insurance consumption bundle x_H such that $\{(x_H, H), (x_L, L)\} = \varphi(\hat{\pi}, w)$. Therefore, from Lemma 7.1 we know that for $\hat{\kappa}$ given by (12) with $\pi = \hat{\pi}$, there exist $\theta_L > 1 > \theta_H > 0$ such that $\{(x_H, \theta_H, H), (x_L, \theta_L, L)\} = \psi(\hat{\kappa}, w)$. By definition of equilibrium, it must be that $q \in (0,1)$, and $\hat{\kappa}$ is consistent, which implies in particular that q is given by

$$q = \left[1 + \frac{\theta_H\,(\hat{\kappa} - \bar{w}_H)}{\theta_L\,(\bar{w}_L - \hat{\kappa})}\right]^{-1} .$$

Since $\pi_L < \hat{\pi} < \pi_H$, hence $\bar{w}_L < \hat{\kappa} < \bar{w}_H$, we verify that indeed $q \in (0,1)$, and this concludes the proof. ∎

Notes

[1]Universidad de Alicante.

[2]DPTEA - Luiss Guido Carli.

[3]We have particularly in mind heterogeneous consumers participating voluntarily in big pools of health, disability or unemployment insurance, in the presence of moral hazard. While in reality most social insurance schemes are compulsory, there are nevertheless some examples of voluntary social insurance. For instance, in 1981 Chile turned its compulsory social health insurance system

into a voluntary one. Also, recently, the residents of the state of New York have been offered several state health insurance programs. These target currently non-insured individuals who are above the traditional Medicaid income limits or employers not providing health insurance. Individuals may voluntarily qualify for the "Family Health Plus" program or the "Healthy NY", for example. Even in more welfare states with a more European tradition, the growing number of self-employed persons increases the number of individuals relying on voluntary social insurance. Goulão (2005) studies the implications of a voluntary social insurance scheme with adverse selection.

[4]As has been well known since Arrow (1963), moral hazard in health, disability and unemployment insurance is highly significant.

[5]This shows up a major conceptual difference with the setup of Dubey and Geanakoplos (2002), since in their adverse selection model the proportion of high-risk consumers and low-risk consumers is an exogenous parameter.

[6]Since consumers are ex-ante equal, this result requires them to be indifferent to undertaking either of the actions.

[7]Figure 1 only represents the relevant part of each indifference curve, i.e. the high effort indifferent curve below the switching locus, and the low effort indifferent curve above it.

[8]See the Appendix for details.

[9]We thank an anonymous referee for suggesting a useful simplification of the proof of this result.

[10]This also follows from the observation that in the above problem the price of insurance is less than fair.

References

Anderson S. and Baland J.M. "The economics of roscas and intra-household resource allocation". *Quarterly Journal of Economics*, 117(3), 963–965, 2002.

Arnott R. and Stiglitz J. "The basic analytics of moral hazard". *Scandinavian Journal of Economics*, 90(3), 383–413, 1988.

Arrow K.J. "Uncertainty and the welfare economics of medical care". *American Economic Review*, 53(5), 941–973, 1963.

Besley T. and Gouveia M. "Alternative systems of health care provision". *Economic Policy*, 19, 199–258, 1994.

Bloch F., Genicot G., and Ray D. "Informal insurance in social networks", 2005. Working Paper.

Cabrales A., Calvó-Armengol A., and Jackson M.O. “La crema: A case study of mutual fire insurance”. *Journal of Political Economy*, 111, 425–458, 2003.

De Weerdt J. and Dercon S. “Risk-sharing networks and insurance against illness”. *Journal of Development Economics*, 81, 337–356, 2006.

Dubey P. and Geanakoplos J. “Competitive pooling: Rothschild-Stiglitz reconsidered”. *The Quarterly Journal of Economics*, 117, 529–1570, 2002.

Dubey P., Geanakoplos J., and Shubik M. “Default and punishment in general equilibrium”. *Econometrica*, 73, 1–37, 2005.

Goulão C. “Voluntary social insurance”, 2005. Mimeo.

Hellwig M. “Moral hazard and monopolistically competitive insurance marktes”. *The Geneva Papers on Risk and Insurance*, 26, 44–71, 1983a.

Hellwig M. “On moral hazard and non-price equilibria in competitive insurance markets”. *SFB 21 Discussion Paper No. 109, University of Bonn*, 1983b.

Murgai R., Winters P., Sadoulet E., and de Janvry A. “Localized and incomplete mutual insurance”. *Journal of Development Economics*, 67, 245–274, 2002.

Pauly M. “Overinsurance and public provision of insurance: The roles of moral hazard and adverse selection”. *Quarterly Journal of Economics*, 88, 44–54, 1974.

Pauly M.V., Zweifel P., Scheffler R.M., Preker A.S., and Basset M. “Private health insurance in developing countries”. *Health Affairs*, 2, 369–379, 2006.

Pestieau P. *The Welfare State in the European Union. Economic and Social Perspectives.* Oxford University Press, 2006.

Rothschild M. and Stiglitz J. “Equilibrium in competitive insurance markets: an essay on the economics of imperfect information”. *Quarterly Journal of Economics*, 90, 624–649, 1976.