A discusión

STORE VS. NATIONAL BRANDS:
A PRODUCT LINE MIX PUZZLE*

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ABSTRACT

This paper examines retailers' strategic decisions about store brand introduction when each retailer can stock a limited number of brands. The different product line mix equilibria depend on demand parameters that measure the cross-effect across national and store brands and the cross-effect within each brand type, thus leading to a simple testable implication. Store brand introduction is determined by the combination of the three effects that result from replacing a national brand by a store brand; the direct effect, the exclusivity effect and the in-store effect. Interestingly enough, we identify conditions under which similar retailers take different decisions concerning their product line mix.

JEL Classification: L13, L23.

Key words: store brands, retail duopoly, product line mix.

1   Introduction

Over the last decades, private labels brands or store brands have become a popular and profitable marketing strategy both in the United States and Europe. According to AC-Nielsen's report *The Power of Private Label 2005* private label sales accounted for 17% of the value sales over the 12 months ending the first quarter of 2005. Europe is the leading private label region with an aggregated private label share in value sales of 23% for the 17 European countries included in the study. Switzerland (45%), Germany (30%), Great Britain (28%), Spain (26%) and Belgium (25%) were the five European countries with the highest private label shares. North America with a penetration rate of 16%, saw a significant growth of private label sales up 7%.[1] There are a number of reasons explaining the substantial development of private label programmes. Among other benefits, private labels add diversity to a retailer's product line in a category; a retailer utilizes them as a measure of exclusivity to differentiate from manufacturers' brands, since by definition private labels can only be sold by the retailer that carries them; and the retailer gross margins on their private labels are higher than those obtained on national brands.

A concentrated retail sector appears to facilitate the growth of private labels (Dobson, 1998, Steiner, 2004). Store brands give retailers a stronger position against brand manufacturers. In particular, they serve as a competitive tool in obtaining price concessions from manufacturers. Private label products are further advantaged by the retailer's ability to delist certain brands and the managing of scarce shelf space.[2] From a consumer perspective, store brands are viewed as reasonable quality products that are typically priced lower than leading manufacturer brands. Despite the rather apparent advantages of private label products, they are not introduced by every store and in every product category. Managerial strategies about private label introduction are not solely driven by the manufacturer-retailer relationship but are also an important dimension of competition in oligopolistic retail markets.[3]

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[1] This recent report includes 38 countries and 80 categories and was released in Septemper 2005.

[2] "The newly national Macy's chain is a prime example of a retailer focused on driving the top and bottom lines through the sales of its own brands. According to a report in the *Chicago Tribune*, the retailer has jettisoned designer labels such as Prada, Miu Miu and Jimmy Choo, while others such as the Yves St. Laurent handbag collection have been seriously cutback." (Extracted from "Store Brands and Exclusive Labels Make or Break the Retailer", by George Anderson, 9/12/06 in www.retailwire.com).

[3] As reported in the AC Nielsen study, within individual product categories, refrigerated foods are now the largest private label category worldwide with a 32 % share, having replaced paper, plastic and wraps in the top spot. Alternatively, private label shares within the areas of personal care, cosmetics and baby food were the smallest at less than 5%.

This paper addresses retailers' strategic decisions about private label introduction when shelf space is limited and shows the relevance of demand interactions. There are two distinguishing features to our model as compared with the existing theoretical literature. Firstly, previous analyses have mainly examined the rivalry among national brands and private labels looking at a single retailer. We consider retailer interaction as it is obvious that retailers compete on a number of dimensions, and product line mix is an important one among them; the development of private label programmes should be regarded as a strategic variable in retail competition. Secondly, and within the context of a retail duopoly, the scarcity of shelf space (and its role to extract better terms of payment) is taken into account; if a retailer decides to introduce a store brand then it needs to decide which national brand to take off the shelf in favour of the store brand.[4] As such the model will be stylized to illustrate that private labels are not always introduced by every store; this occurs despite the fact of not giving any advantage to national brands neither in demand or cost grounds.

Specifically, a multi-stage game is developed where retailers, after knowing the transfer prices, decide whether to introduce their store brands and then compete in the market. The key question is to identify what conditions make it profitable for retailers to replace a national brand with a private label. Our analysis draws attention to demand parameters that measure the cross-effect across brand types, that is, national brands vis a vis store brands, and the cross-effect within each brand type. With one national brand manufacturer we find i) that only one retailer introduces its store brand when the cross effect within brand types exceeds the cross effect across brand types, and ii) that the retailer that does not replace obtains better terms of payment from the manufacturer. These results are shown to be robust under quite a number of changes in the assumptions. For a retailer, store brand introduction implies both a direct effect, because of saving on the transfer price born by the national brand, and an exclusivity effect, because the store brand is uniquely sold by that one retailer. This gives rise to always having one store brand in the market. Whether both retailers take the replacement decision depends on which of the cross effects is higher; retail interaction leads to the market configuration that entails a softer intensity of competition. In case the national brand manufacturer may retain one of the retailers he will do so by lowering the transfer price, a result that also follows from the retailer's threat to drop the national brand.

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[4]Steiner (2004) refers that the provision of favourable shelf space is one of the most potent weapons in the private label's arsenal. The retailer's power to decide on the placement of national brands and its private labels on the store shelves is a distinguish prerogative from competition among national brands.

With two national brands and assuming that retailers are multiproduct sellers, iii) the equilibrium entails no store brands in the market when the cross effect across brand types is large enough, greater than that within brand types, and for a low enough transfer price, and iv) it is shown that only one of the retailers introduces its store brand for a certain range of the demand parameters and the transfer price. The intuition here heavily lies on the importance of the in-store effect. This additional effect stems from retailers' decision about their product line mix in order to internalize competition to their own advantage. The in-store effect is negative and large when the cross effect within brand types is well below the cross effect across brand types. A low transfer price supposes a small direct effect and therefore neither retailer introduces its store brand. But as the transfer price increases, and when the cross effect across brand types is not too large, we show that one retailer finds store brand introduction advantageous in equilibrium. Therefore, similar retailers take different decisions because one prefers internalizing competition between a national brand and its store brand whereas another is best off carrying two national brands. Most interestingly, the assumption that national brands are supplied by a competitive fringe of manufacturers, v) results in either no store brand introduction, or one store brand in the market, or two store brands, depending on the particular relationships between the demand parameters. This finding further emphasizes the relevance of retailer interaction rather than upstream market power.

Despite the increasing growth in private labels as well as their competitive interaction with national brands, little theoretical research exists in this area. To the best of our knowledge, the modeling of *retailer competition* when a retailer's decision to carry a store brand must take into account that *shelf space is limited* has not yet been undertaken.[5]

Mills (1995) and Narasimhan and Wilcox (1998) considered a successive monopoly to examine the strategic role of a private label that is sold along with a national brand product. These authors took an important first step in exploring how private label introduction affects the manufacturer-retailer relationship. They showed that private label marketing allows the retailer to elicit a price concession from the national brand manufacturer, that it improves the performance of the vertical structure and that it reduces the double marginalization problem.[6] Recently, Gabrielsen and Sørgard (2007), using a

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[5]There is an extensive literature devoted to private label analysis, most of which is of an empirical nature. A wide number of issues have been examined such as the variation in store brand penetration by product category (Sethuraman, 1992, Hoch and Banerji, 1993) and by retailer (Dhar and Hoch, 1997) and the competitive pricing interaction between national brands and private labels (Putsis, 1997, Cotterill and Putsis, 2000, Gabrielsen et al., 2001, and Harris et al., 2002).

[6]Bontems et al. (1999) analyze the retailer's choice of quality for the private label product when marginal costs of production are quality dependent and find some contradictory results to Mills (1995) and

vertical differentiation approach with loyal as well as switching consumers, allow the national brand producer to condition the transfer price on whether the retailer introduces a private label or not. Interestingly, private label introduction may lead to a price increase on the national brand, which can be detrimental to welfare thus pointing out a conflict between the private and the social incentives. A relevant feature, which is common to the aforementioned analyses, is worth being noted: the retailer is a single-product seller who becomes a multi-product seller when the private label is introduced.

When the retailer markets several national brands, it must consider the cross-price sensitivity measuring the intensity of inter-brand competition between the national brands. Adding a private label makes the retailer to regard not only the cross-price sensitivity between existing national brands but also the price sensitivity between a national brand and the store brand. A setting with several national brands and a retailer is considered by Raju et al. (1995). These authors provide an analytical framework for examining conditions under which a private label is more likely to increase category profits for the retailer. Sayman et al. (2002) extend their model by allowing the retailer also to decide how the store brand is positioned relative to the two national brands. The optimal strategy for the retailer is to position the store brand as close as possible to the leading national brand, which means reducing the consumers' perceptual distance between the store brand and the national brand. In this way, the retailer can decrease the monopoly power of the leading national brand and increase their own relative bargaining power as lower transfer prices are achieved – the double marginalization problem is mitigated. It is often observed that store brands imitate the category leader. Both these papers analyze the profitability of adding a store brand to an assortment of national brands. On the other hand, Scott Morton and Zettelmeyer (2004) have considered two-part tariff contracts in a setting where a retailer chooses between carrying two national brand and replacing one of them by a private label, in addition to the label's positioning. These authors offer an explanation of why a retailer values store brands instead of existing national brands. The retailer introduces a private label that is a close substitute for the national brand. The retailer's control over private label positioning coupled with limited shelf space[7] make store brands so valuable to him.[8]

Narasimhan and Wilcox (1998).

[7] The relevance of retailers' limited shelf space was firstly analyzed by the pioneering papers by Shaffer (1991a, 1991b), who examined optimal strategies for upstream firms when setting their pricing strategies in the presence of retailers' stocking decisions.

[8] It is commonly argued that private labels serves as a product differentiation device for a retailer. Vertical differentiation is essentially quality-driven and thus the paper by Bontems et al. (1999) can be viewed to treat a positioning strategy by the retailer, that is, by his choice about the perceptual distance between store brands and national brands at the eyes of consumers. The models of store brand positioning

Our analysis can be seen as complementary to the above literature by considering the strategic interaction between retailers when deciding upon private label introduction, obtaining results that are consistent with some of the observed variety of phenomena in the retailing sector.[9] Our modelling leads to a simple testable condition concerning the strategic use of store brands which basically relies on demand parameters, and provides some useful managerial implications. The paper is organized as follows. The next section presents a formal model of retail competition by first describing the basic assumptions and then solving for the equilibrium store brand strategies when there is just one national brand manufacturer. A robustness analysis is undertaken. Section 3 analyzes the case of several national brands and Section 4 concludes the paper.

2 A Simple Model with Retail Competition

2.1 Basic Assumptions

We next present the model assumptions and the general demand structure for two national brands and two store brands (one for each of the two retailers) from which the different settings follow. Two retailers face a large number of consumers. The representative consumer maximizes $V(q_{n1}, q_{n2}, q_{s1}, q_{s2}, y)$ subject to the budget constraint $I = y + \mathbf{q}^T \mathbf{p}$, where I is consumer's income and y is the amount of the composite commodity taken as numéraire, $\mathbf{q}^T = (q_{n1}, q_{n2}, q_{s1}, q_{s2})$ and $\mathbf{p}^T = (p_{n1}, p_{n2}, p_{s1}, p_{s2})$, where $q's$ and $p's$ denote total output and price for each brand, respectively. There are two brand types, national and store, and subscripts denote national brand one, national brand two, store brand from retailer one and store brand from retailer two, respectively.[10] The function V is linear in the composite good and can be written as $V = y + U(q_{n1}, q_{n2}, q_{s1}, q_{s2})$. The function U is assumed quadratic and strictly concave as follows

by Sayman et al. (2002) and by Scott Morton and Zettelmeyer (2004) can be catalogued as models of horizontal and of vertical differentiation, respectively. Recently, Choi and Coughlan (2006) have combined both differentiation diimensions. Furthermore, the manufacturer might employ counterstrategies to lessen the force of private label programmes. Precisely, Mills (1999) has considered that widening the quality gap is one such measure that allows the manufacturer to fight back and limit the diversion of profits to the retailer due to private label introduction.

[9] There are many other features that help characterize the recent change in the retailing landscape such as the substantial quality improvement of private labels, the growth of superstores at the expense of traditional outlets, brand reputation creating loyalty to a particular supermarket chain, advertising expenditure and so on. See Dobson (1998) for a comprehensive survey and a discussion on the economic welfare implications of private label products.

[10] The terms private label and store brand will be used interchangeably throughout the paper. We will however employ subscripts s to keep with the notation in the received literature in this area.

$$U = \boldsymbol{\alpha}^T \mathbf{q} - \frac{1}{2}\mathbf{q}^T \mathbf{Z}\mathbf{q}, \tag{1}$$

where $\boldsymbol{\alpha}^T$ is equal to $(\alpha_n, \alpha_n, \alpha_s, \alpha_s)$ and

$$\mathbf{Z} = \begin{pmatrix} \beta_n & \gamma_n & \varepsilon & \varepsilon \\ \gamma_n & \beta_n & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \beta_s & \gamma_s \\ \varepsilon & \varepsilon & \gamma_s & \beta_s \end{pmatrix}.$$

Concavity is guaranteed for $\mathbf{Z}$ a positive definite matrix. This is the case as long as the eigenvalues are strictly positive, that is, $\beta_h > \gamma_h$, for $h = n, s$, and $\varepsilon^2 < \frac{(\beta_n + \gamma_n)(\beta_s + \gamma_s)}{4}$. The maximization of (1) subject to the budget constraint results in the following inverse linear demand system,

$$p_{n1} = \alpha_n - \beta_n q_{n1} - \gamma_n q_{n2} - \varepsilon q_{s1} - \varepsilon q_{s2}, \tag{2}$$

$$p_{n2} = \alpha_n - \gamma_n q_{n1} - \beta_n q_{n2} - \varepsilon q_{s1} - \varepsilon q_{s2}, \tag{3}$$

$$p_{s1} = \alpha_s - \varepsilon q_{n1} - \varepsilon q_{n2} - \beta_s q_{s1} - \gamma_s q_{s2}, \tag{4}$$

$$p_{s2} = \alpha_s - \varepsilon q_{n1} - \varepsilon q_{n2} - \gamma_s q_{s1} - \beta_s q_{s2}, \tag{5}$$

where β_h measures own effects of brands, γ_h captures the cross-effect within each brand type, and ε measures the cross-effect across brand types. Note that although own effects dominate cross effects within each brand type, the latter may or may not exceed the cross effect across brand types, i.e. $\gamma_h \lesseqgtr \varepsilon$. Although a different utility function could lead to another demand structure, and there is conflicting evidence regarding the fit of linear demand to market data, we employ the above structure for analytical tractability. In fact, we consider the simplest and most interpretable possible case, although stylized, in which we have the complete spectrum of brand type configurations.

Since we are particularly concerned with studying the strategic introduction of a private label, and for the sake of the exposition, we assume symmetry as follows: $\alpha_n = \alpha_s = 1$, $\beta_n = \beta_s = 1$, and $\gamma_n = \gamma_s = \gamma$. Consequently the range for the cross effect parameters is characterized by $0 < \gamma < 1$ and $0 < \varepsilon < \frac{1+\gamma}{2}$, or equivalently $0 < \varepsilon < 1$ and $\max\{2\varepsilon - 1, 0\} < \gamma < 1$. The fact that $\alpha_n = \alpha_s$ means that neither the national brand nor the store brand is given any "ad hoc" advantage that could be traced back to consumers' preferences; this simplification allows us to better focus our studying of the replacement of national brands by store ones for strategic reasons. The received literature assumes that the base level demand for a national brand is larger than for a store brand. We wish to put store brands on equal terms to national brands to show that even so store brands

are not always introduced. The relevant parameter space is therefore a subset of the 1×1 square in the (γ, ε) space as presented in Figure 1.

We assume that retailers are not differentiated in the sense that consumers of a brand receive the same utility no matter which retailer is selling that particular brand to them. Each manufacturer has constant marginal costs c which without loss of generality are assumed to be zero. If a retailer sells a national brand product it pays w_j per unit of output to manufacturer j, for $j = 1, 2$. Each private label is supplied by a competitive fringe of manufacturers at constant marginal costs t, assumed to be zero. By assuming both national and store brands marginal costs to be zero we are ensuring that any difference in transfer prices between both brand types can only be explained as a result of market power on the part of the national brand manufacturing industry. Finally, in order to take into account that shelf space is limited, a retailer is forced to replace a national brand in case a store brand is introduced.

As already indicated, it is our purpose to analyze the introduction of store brands from a strategic point of view. We will then develop a model with a duopolistic retail structure and begin with the existence of one national brand. Since a retailer's shelf space is limited, we assume that if a private label is carried this can only be done by replacing the national brand. An extension to the case of two national brands is taken up in Section 3 below, where retailers are multiproduct sellers.

3 No Competition Upstream: The monopolist manufacturer case.

Suppose a monopolist national brand manufacturer, say of brand one, whose product is sold through two undifferentiated retailers. The contract used by the manufacturer is a linear price contract. We propose the following multi-stage game with observed actions. In the first stage, the manufacturer sets the transfer price w. In the second, the replacement stage, retailer one, R_1, and retailer two, R_2, decide independently and simultaneously whether to introduce its own store brand. If so the store brand replaces the national brand (strategy r), where r stands for replacement, otherwise it sells the national brand (strategy n), where n stands for no replacement. There is quantity competition in the last stage of the game and then consumers purchase the products. We look for the subgame perfect equilibria of the game.

Note that three qualitatively different quantity competition outcomes may arise: a) a homogeneous duopoly when both retailers sell the national brand, where the inverse
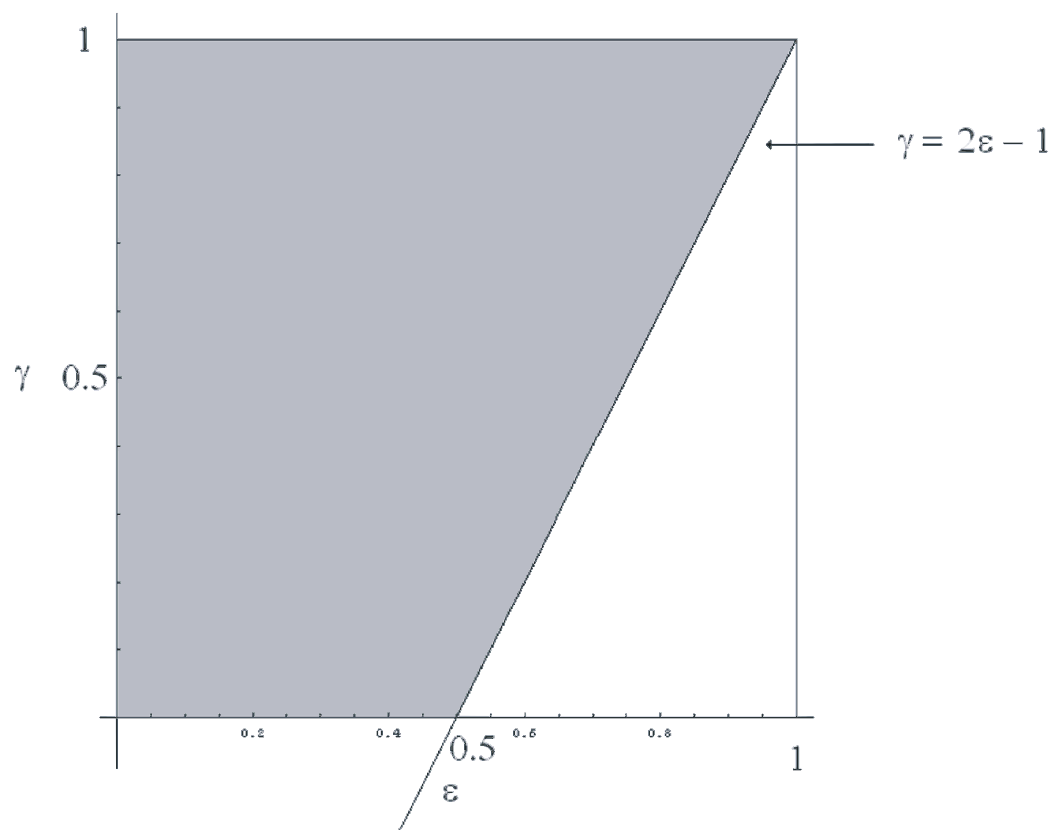
Figure 1

demand functions are obtained by taking $\mathbf{q}^T = (q_{n1}, 0, 0, 0)$; b) a demand symmetric differentiated duopoly where the national brand competes against a store brand, but with different marginal costs for retailers, now considering either $\mathbf{q}^T = (q_{n1}, 0, q_{s1}, 0)$ or $\mathbf{q}^T = (q_{n1}, 0, 0, q_{s2})$; and c) a symmetric differentiated duopoly with two store brands where now $\mathbf{q}^T = (0, 0, q_{s1}, q_{s2})$.

The timing of the game captures that private label introduction is a long-term decision compared with the decision on quantities. Furthermore, as the manufacturer decides first it anticipates the fact that retailers may drop its brand, which is taken into account when deciding on its transfer price, and this leads to retailers attaining better terms of payment from the manufacturer.

For any given w, the second stage decision by retailers gives rise to three different subgames. In case that no retailer replaces the national brand, the (n, n) subgame, retailers' payoffs are $R_1^{nn}(w) = R_2^{nn}(w) = \frac{(1-w)^2}{9}$, with $w \leq 1$ for nonnegative outputs. The pair of superscripts denote the store brand strategy taken by retailer one and retailer two, respectively. In case one of the retailers replaces the national brand whereas the other does not, the (r, n) and (n, r) subgames, the corresponding payoffs are given by $R_1^{rn}(w) = R_2^{nr}(w) = \frac{(2-\varepsilon(1-w))^2}{(4-\varepsilon^2)^2}$ and $R_2^{rn}(w) = R_1^{nr}(w) = \frac{(2-\varepsilon-2w)^2}{(4-\varepsilon^2)^2}$, with $w \leq (1 - \frac{\varepsilon}{2})$ for nonnegative outputs. Finally, when both retailers decide to introduce its store brand, subgame (r, r), the manufacturer is left out of the market and both retailers obtain the following payoffs, $R_1^{rr} = R_2^{rr} = \frac{1}{(2+\gamma)^2}$.

A retailer's best reply, say R_1, in the second stage is as follows: First, R_1 will introduce the store brand when R_2 does not if and only if $R_1^{rn}(w) - R_1^{nn}(w) > 0$. The difference is equal to $\frac{((2-\varepsilon)(1-\varepsilon)+(4-\varepsilon)(1+\varepsilon)w)((2-\varepsilon)(5+\varepsilon)-(4+\varepsilon)(1-\varepsilon)w)}{9(4-\varepsilon^2)^2} \equiv \Phi_r(w)$, and $\Phi_r(w)$ is positive for the relevant range of w. Therefore, one of the retailers always introduces its store brand.

Second, R_1 introduces the store brand when R_2 does so if and only if $R_1^{rr} - R_1^{nr}(w) = \left(\frac{1}{2+\gamma} + \frac{2-\varepsilon-2w}{4-\varepsilon^2}\right)\left(\frac{2(2+\gamma)w-(2-\varepsilon)(\gamma-\varepsilon)}{(2+\gamma)(4-\varepsilon^2)}\right) \equiv \Phi_{rr}(w)$ is positive. It happens that $\Phi_{rr}(w) > 0$ for $w > \frac{(2-\varepsilon)(\gamma-\varepsilon)}{2(2+\gamma)} \equiv \bar{w}$. It follows that $\bar{w} > 0$ if and only if $\gamma > \varepsilon$ and it is smaller than the bound imposed on w to guarantee nonnegative outputs. Otherwise, for $\varepsilon \geq \gamma$, $\bar{w} < 0$ and then $\Phi_{rr}(w) > 0$ for all w. The analysis is similar for retailer two.

In the first stage the manufacturer, M, decides on the value of the transfer price that maximizes its payoffs. Note that depending on the value of w, one of the retailers may

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find it profitable not to introduce its store brand. The manufacturer's payoffs function is:

$$M(w) = \begin{cases} w\left(\frac{2-\varepsilon-2w}{4-\varepsilon^2}\right) & for\ 0 \leq w \leq \bar{w} \\ 0 & for\ \bar{w} < w < \left(1-\frac{\varepsilon}{2}\right) \end{cases} \qquad (6)$$

From the inspection of the critical value $\bar{w}$, it is clear that for $\gamma < \varepsilon$, M is unable to retain one of the retailers in the sense that it will get zero payoffs irrespective of w since both retailers will always decide to introduce the store brand. Firms' payoffs are $M(w) = 0$ and $R_1^{rr} = R_2^{rr} = \frac{1}{(2+\gamma)^2}$. However, for $\gamma > \varepsilon$, the manufacturer has to set a transfer price that maximizes its profits subject to the restriction that $w \in [0, \bar{w}]$. In doing so, it compares the value of w that maximizes $w\left(\frac{2-\varepsilon-2w}{4-\varepsilon^2}\right)$ when w is unrestricted, denoted by w^*, with $\bar{w}$. It turns out that w^* is equal to $\frac{2-\varepsilon}{4}$ and this value is always greater than $\bar{w}$. Therefore, we conclude that for $\varepsilon < \gamma < 1$, M chooses a transfer price equal to $\bar{w}$, one of the retailers decides not to introduce its store brand and the payoffs for the manufacturer are $M(\bar{w}) = \frac{(2-\varepsilon)(\gamma-\varepsilon)}{2(2+\gamma)}$, for the retailer that replaces are $R_1^{rn} = R_2^{nr} = \frac{(2+\gamma-\varepsilon))^2}{4(2+\gamma)^2}$ and for the one that does not replace are $R_2^{rn} = R_1^{nr} = \frac{1}{(2+\gamma)^2}$. The next proposition summarizes the subgame perfect equilibrium of this game.

Proposition 1 *When the cross effect across brand types exceed that within brand types both retailers replace the national brand. However, when the opposite occurs, only one retailer introduces its store brand. The other retailer does not, but it obtains better terms of payment from the manufacturer than in a setting where it had not the option to introduce a store brand.*

Sequentiality and replacement when introducing a store brand in the presence of retail competition make it such that a retailer may find it unprofitable to market a store brand. To see the intuition consider a standard vertical structure with just one retailer. Given the assumptions of the model the monopolist retailer will sell the national brand and not introduce the store brand as long as α_n exceeds α_s; it will of course be indifferent if $\alpha_n = \alpha_s = 1$. In other words, demand parameters related with the intensity of competition are obviously irrelevant. When there is a retail duopoly, retailers choose the combination of brands leading to softer competition intensity. Note that there is just one national brand and that retailers are single-product sellers. The introduction of a store brand by a retailer has, in this context, two effects. On the one hand, and because of limited shelf space, a national brand is replaced by the store brand. This directly affects retailer's profits since each brand type implies a different margin for the retailer. This is a *direct effect*, which is positive as long as the national brand bears a positive transfer price. On the other hand, there exists an *exclusivity effect* because replacement implies leaving the

rival retailer as the unique seller of the national brand. The two effects are in favour of replacement and therefore one store brand in the market is guaranteed. The most competitive outcome is reached when neither retailer introduces its store brand for there is intra-brand competition in the national brand and retailers are always worse under a homogeneous duopoly. Note however that the exclusivity effect is not present when we compare the market configuration with one store brand and another with two. In both cases each retailer is an exclusive seller of a particular brand. We then conclude that whether both retailers or just one of them introduces a store brand depends on the distance between the own-effect and each of the cross-effects and the size of the direct effect. Consider that the cross-effect across brand types, ε, exceeds the cross-effect within each brand type, γ. Then, a duopoly where both store brands are in the market supposes a distance between γ and β (assumed equal to one) that is greater than the distance between ε and β were there a duopoly with the national brand and just one store brand. The former setting entails a lower degree of competition intensity, retailers' payoffs are therefore higher and we find introduction of both the store brands. The contrary happens when γ exceeds ε and then the national brand is replaced by just one retailer since the manufacturer is able to reduce the magnitude of the direct effect, by lowering the transfer price, to convince the retailer not to replace.

This simple modeling provides an easy testable implication to better understand why some outlets introduce store brands in some categories and not in others. Further note that, from Proposition 1 above, a retailer may threat the national brand manufacturer with not carrying his brand to achieve better terms of payment vis a vis a situation where this option was not available. In fact a positive externality takes place since there is already one retailer that replaces the national brand, and then the other retailer obtains a lower transfer price in case of non-replacement.

3.1   Robustness

In this subsection we wish to analyze how changes in a given assumption will affect the result. One wonders whether the foregoing result continues to hold a) assuming away market power on the side of the national brand manufacturer, b) with an alternative timing, c) with price competition among retailers, d) in the case the manufacturer is capable of conditioning the transfer price charged depending on whether the contract is exclusive or not and, e) under other terms of payment in the contract.

A competitive fringe of manufacturers.
Suppose that the national brand is supplied by a competitive fringe of manufacturers.

In such an eventuality the transfer price is zero since there is no market power in the upstream manufacturing industry. The payoffs corresponding to the above multi-stage game, where the first stage is obviously void, are $R_1^{rr}(w=0) = R_2^{rr}(w=0) = \frac{1}{(2+\gamma)^2}$, $R_1^{nr}(w=0) = R_2^{rn}(w=0) = R_2^{nr}(w=0) = R_1^{rn}(w=0) = \frac{1}{(2+\varepsilon)^2}$ and $R_1^{nn}(w=0) = R_2^{nn}(w=0) = \frac{1}{9}$. It is easy to see that Proposition 1 continues to hold for the special case $w=0$. Interestingly, the fact that not all retailers introduce a store brand arises regardless of the existence of manufacturer market power.

Timing

Consider now several changes in the sequence of moves. As suggested by the empirical evidence above, stores have developed private label programmes in sequence. In particular and to account for such sequentiality in the strategic introduction of store brands we assume that one retailer decides whether to replace the national brand in the initial stage. Next, the manufacturer decides on the value of the transfer price in the second stage; note that w will depend now on the previous choice made by retailer. Then the second retailer decides its store brand strategy. Finally, retailers compete in quantities. This change in timing has no effect on the statement on Proposition 1, with the qualification that it is the retailer that moves first the one that always replaces the national brand when the equilibrium results in replacement by only one retailer.

An alternative change in timing is that retailers simultaneously decide on their store brand strategies first and then, knowing the decision made before, the manufacturer decides on the transfer price. In such an eventuality, the manufacturer is unable to influence the retailers' decision, and the national brand is left out of the market.

Price competition.

Assume now that there is price competition in the last stage of the game. In such a case it clearly follows that there is always a unilateral incentive to replace a national brand since otherwise both retailers obtain zero profit due to Bertrand competition in the national brand. Both store brands are in the market as long as one retailer finds replacement profitable when the rival has introduced its store brand, i.e. when $\Phi_{rr}^{price}(w)$ is positive, where

$$\Phi_{rr}^{price}(w) = \frac{(1-\gamma)}{(1+\gamma)(2-\gamma)^2} - \frac{((1-\varepsilon) - (\frac{2-\varepsilon^2}{2+\varepsilon})w)^2}{(1-\varepsilon^2)(2-\varepsilon)^2}.$$

Therefore, we reach the same qualitative conclusion as in Proposition 1. Whenever $0 < \gamma < \varepsilon$ the function $\Phi_{rr}^{price}(w)$ is always positive even for $w = 0$ and thus both retailers replace the national brand. For $\gamma > \varepsilon$, the manufacturer sets the transfer price,

$\bar{w}^{price}$, that solves $\Phi_{rr}^{price}(\bar{w}^{price}) = 0$ and only one retailer replaces the national brand.

Contingent transfer price.

Now suppose that the manufacturer is capable of setting the transfer price charged to the retailers conditional on whether the contract is exclusive or not, that is, in the first stage of the game the manufacturer posts a pair (w^e, w^{ne}). Superscript e indicates that just one retailer sells the manufacturer's brand; superscript ne corresponds to the case where both retailers market the national brand. Accordingly, the retailers payoffs are: $R_1^{rr} = R_2^{rr} = \frac{1}{(2+\gamma)^2}$, $R_1^{rn}(w^e) = R_2^{nr}(w^e) = \frac{(2-\varepsilon(1-w^e))^2}{(4-\varepsilon^2)^2}$, $R_2^{rn}(w^e) = R_1^{nr}(w^e) = \frac{(2-\varepsilon-2w^e)^2}{(4-\varepsilon^2)^2}$ and $R_1^{nn}(w^{ne}) = R_2^{nn}(w^{ne}) = \frac{(1-w^{ne})^2}{9}$. Retailer R_1's unilateral incentive to replace the national brand is given by the difference $R_1^{rn}(w^e) - R_1^{nn}(w^{ne}) = \frac{9(2-\varepsilon(1-w^e))^2 - (4-\varepsilon^2)^2(1-w^{ne})^2}{9(4-\varepsilon^2)^2}$, which is always positive since $w^{ne} \in [0,1]$ for positive outputs in the (n,n) subgame. Then, whatever (w^e, w^{ne}) be each retailer will replace the national brand provided the other does not. Finally, retailer R_1's incentive to replace the national brand provided the rival has done it is given by the expression $R_1^{rr} - R_1^{nr}(w^e) = \frac{(4-\varepsilon^2)^2 - (2+\gamma)^2(2-\varepsilon-2w^e)^2}{(2+\gamma)^2(4-\varepsilon^2)^2}$, which is positive as long as $w^e > \bar{w}$. Therefore, in the case of contingent transfer prices, the manufacturer's payoffs are independent of w^{ne} and coincide with expression (6) where w^e is substituted for w. The manufacturer chooses a pair of transfer prices where $w^e = \bar{w}$ for $\varepsilon < \gamma < 1$, any real number otherwise, and w^{ne} is always any real number. Then, the conclusions of Proposition 1 also follow in this context.

Other terms of payment: fixed fee and two-part tariff contracts.

We assume now that, in the first stage, the manufacturer proposes retailers the following two alternative contracts: a fixed fee contract $(w = 0, F)$ and a two-part tariff contract, (w, F) where w is the transfer price and F is the up-front fixed fee to be paid by a retailer willing to market the national brand. Several conclusions are reached (see the Appendix for the details). First, concerning the fixed fee contract, no qualitatively different results appear since only one retailer can be retained by the national brand manufacturer and this requires that γ be greater than ε. On top of that, the fixed fee contract would never be chosen by the manufacturer since the two-part tariff contract yields higher profits. Second, there are two interesting findings regarding two-part tariff contracts (see Proposition in the Appendix): i) Neither store brand is introduced for both γ and ε close to one, in particular for $2\varepsilon - 1 < \gamma < \min\{\gamma^{n/nn}(\varepsilon), 1\}$ and $\varepsilon > 0.945$ (see the horizontal stripped area in Figure 2); ii) Only one store brand is introduced when $max\{2\varepsilon - 1, \gamma^0(\varepsilon), \gamma^{n/nn}(\varepsilon)\} < \gamma < 1$ and $0 < \varepsilon < 0.980$ (see the light grey and vertical stripped areas in Figure 2);[11] iii)

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[11]Note that $\gamma^{n/nn}(\varepsilon)$ is a threshold for γ that implies that if $0.918 < \varepsilon$ and $\gamma < \gamma^{n/nn}(\varepsilon)$ then the
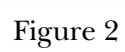
Figure 2

The manufacturer is unable to keep any retailer in the dark grey area in Figure 2.

Finally, if the manufacturer is able to choose between a linear and a two-part tariff contract, then we find that both types of contracts might be chosen by the manufacturer at equilibrium: the linear contract in the vertical stripped area and the two-part tariff contract in both the horizontal stripped and light grey areas in Figure 2. The two-part tariff contract is used mainly to avoid store brand introduction by retailers when the linear contract is unable to do it (the horizontal stripped and light grey areas where $\gamma < \varepsilon$), but the linear contract is preferred to the two-part tariff contract in more cases, for $\gamma^{l/tp}(\varepsilon) < \gamma < 1$ and $0 < \varepsilon < 0.915$. For the cases where $max\{0, 2\varepsilon - 1\} < \gamma < \gamma^{l/tp}(\varepsilon)$ and $0 < \varepsilon < 0.903$, the dark grey area, the manufacturer cannot avoid the introduction of store brands regardless of the contract used.

4   An Extension to Several National Brands

In this section we are going to consider the case where there are two national brands and retailers are assumed to be multi-product sellers. Each retailer may carry the two national brands or *replace one of them* by its own store brand. Note that these assumptions enrich the possible market configurations ranging from a successive duopoly with multi-product retailers carrying two national brands to a configuration with the highest product diversity, i.e. the one with the two national and the two store brands. They also allow us to understand whether retailers, given that store brands are introduced, prefer competition between national brands rather than intra-brand competition in one of the national brands keeping the other one out of the market.

We proceed to analyze the following multi-agent and multi-stage game with observed actions. In the first stage, the manufacturers M_i decide simultaneously and independently on the transfer prices, w_i for $i = 1, 2$. In the second stage, the replacement stage, retailers decide simultaneously and independently whether to replace national brand one by its store brand (strategy r_1), or replace national brand two by its store brand (strategy r_2), or not to introduce its store brand (strategy r_0). A particular subgame will be denoted by a pair (r_i, r_j) for $i, j = 0, 1, 2$, and where r_i denotes retailer one's choice whereas r_j denotes retailer two's choice. There is quantity competition in the last stage of the game and then consumers purchase the products. We will look for the subgame perfect equilibria of the

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manufacturer is better off if the two retailers sell its national brand at equilibrium. Also $\gamma^0(\varepsilon)$ is a threshold for γ that implies that for γ greater than $\gamma^0(\varepsilon)$ the equilibrium manufacturer payoffs when one retailer sells the national brand are positive. Finally, $\gamma^{l/tp}(\varepsilon)$ is a threshold for γ that implies that for γ greater than the threshold, the manufacturer is better off if a linear contract is employed rather than a two-part tariff.

game.[12]

The product line mix decision is clearly influenced by the extent of competition envisaged, as captured by the different demand parameters, by multi-product retail competition and by scarce shelf space. The received literature has ignored the strategic interaction in such a decision which makes interesting in itself to focus on the analysis of the replacement stage. As mentioned in the introduction, it is a stylized fact that not all retailers introduce their store brands, and if they have their own brand it is not present in every product category despite the very many benefits store brands have. We are then particularly interested in finding conditions ensuring that none of the retailers will introduce a store brand as well as for the case where only one retailer replaces one of the national brands while the rival does not. In view of this, we will first examine the *unilateral incentive* to introduce a store brand (or no store brands in the market whenever such an incentive does not exist), and then characterize the equilibrium where only *one retailer introduces its store brand.*

The introduction of a store brand by a retailer has another effect, in this context, in addition to the *direct effect* and *the exclusivity effect,* already mentioned in the case of one national brand. Since retailers are multiproduct sellers replacement also affects the decision that the retailer takes regarding the output of the other national brand. Here it is important which brand types the retailer carries since, for two national brands, the internalization of competition is weighted by a factor γ whereas it is weighted by ε when carrying a national brand and its store brand. This additional effect is related with in-store competition which is the competition among the two brands of a retailer's product line, and will be referred to as the *in-store effect.* Hence for ε smaller than γ in-store competition is softer if there is replacement. Furthermore, the exclusivity effect is now more elaborate as it is linked to the strategic behaviour in both the retailers' choice of outputs and product line mix. Thus, when a retailer decides to replace a national brand it is introducing another brand exclusively marketed by that particular retailer, but at the same time it is granting the rival retailer the role of exclusive seller of the national brand that is replaced by the former retailer. Then, the exclusivity feature for brands sold at any particular choice (r_i, r_j) made by retailers is affecting the power that a retailer has to sustain prices above marginal costs when choosing equilibrium outputs. Note that the exclusivity effect favours replacement when both brand types are symmetric.[13] Finally, it

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[12] Provided the symmetry of national and store brands and the fact that retailers are undifferentiated, four different subgames need to be analyzed out of the possible nine subgames. In particular, the different subgames are: (r_0, r_0), (r_1, r_0), (r_1, r_1) and (r_1, r_2); noting that (r_0, r_1), (r_2, r_0), (r_0, r_2), are symmetric to (r_1, r_0); (r_2, r_2) is alike to (r_1, r_1) and finally (r_2, r_1) is similar to (r_1, r_2). The equilibrium outputs corresponding to each subgame are relegated to the Appendix.

[13] To isolate the exclusivity effect consider that transfer prices are zero (a null direct effect) and that $\gamma = \varepsilon$ to make national brands and store brands symmetric, which implies a zero in-store effect. It happens

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is worth mentioning that the direct and the exclusivity effects do not show up in a setting where the retailer adds a new product to its product line and faces no competition.

4.1 Unilateral incentive to introduce a store brand.

Consider any given pair of transfer prices (w_1, w_2) and that retailer R_2 does not replace any national brand, will R_1 do it? The answer is affirmative if either of the following two inequalities holds: $R_1^{r_1 r_0}(w_1, w_2) > R_1^{r_0 r_0}(w_1, w_2)$ or $R_1^{r_2 r_0}(w_1, w_2) > R_1^{r_0 r_0}(w_1, w_2)$. Before proceeding, we state the following intermediate result.

Consider any given pair of transfer prices (w_1, w_2) and that one of the retailers does not replace any national brand. In case the other retailer introduces its store brand, it will replace the national brand with the greatest transfer price. That is, $R_1^{r_1 r_0} > R_1^{r_2 r_0}$ and $R_2^{r_0 r_1} > R_2^{r_0 r_2}$ iff $w_1 > w_2$.

Next, and in order to simplify the analysis and extract some clear-cut conclusions, we focus on the case where one of the national brands is provided by a competitive fringe of manufacturers, e.g. national brand two, while the other is provided by a monopolist manufacturer. Given that, we assume that $w_1 \geq w_2 = 0$, and by the above intermediate result, retailer one has a unilateral incentive to introduce its store brand will exist as long as $R_1^{r_1 r_0}(w_1, 0) > R_1^{r_0 r_0}(w_1, 0)$ - we keep arguing in terms of retailer one, which is symmetric to retailer two. Besides, it is important to note that when the inequality does not hold for any retailer we have that the subgame (r_0, r_0) is a Nash equilibrium and no store brands are marketed. With some abuse of notation, we will denote q_{nkl} the output sold by retailer l of national brand k when national brands are not sold exclusively.

Retailers' payoffs for these subgames can be written as follows, $R_1^{r_1 r_0} = (1-\varepsilon^2)(q_{s1}^{r_1 r_0})^2 + \frac{1}{9}, R_1^{r_0 r_0} = (1 - \gamma^2)(q_{n11}^{r_0 r_0})^2 + \frac{1}{9}$. Then we find that R_1 will replace national brand one as long as $q_{s1}^{r_1 r_0}$ be large enough with respect to the equilibrium output of the replaced brand, weighted by a function relating the cross effects across and within brand types; in particular for $q_{s1}^{r_1 r_0} > \sqrt{\frac{(1-\gamma^2)}{(1-\varepsilon^2)}} q_{n11}^{r_0 r_0}$. We now define $x_1 = 1 - w_1$, where $q_{s1}^{r_1 r_0}$ is decreasing in x_1 and $q_{n11}^{r_0 r_0}$ is increasing. Let x^u be the value of x_1 that solves $q_{s1}^{r_1 r_0}(x^u) = \sqrt{\frac{(1-\gamma^2)}{(1-\varepsilon^2)}} q_{n11}^{r_0 r_0}(x^u)$, with the following implication:[14] for all $x_1 < x^u$ there is a unilateral incentive to replace national brand one. It is proven in the Appendix that x^u is a decreasing function in ε and also that x^u evaluated at $\varepsilon = \gamma$ is greater than one, while if evaluated at $\varepsilon = \frac{1+\gamma}{2}$ it is

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that: i) both retailers sell a lesser amount of the non exclusive brand and at the same price than before the store brand introduction; ii) both retailers sell a larger amount of the brand they sell exclusively and at a higher price than before. The replacement by one retailer results in higher profits for both retailers, then concluding that the exclusivity effect is in favour of replacement.

[14] The precise expression is $x^u = \frac{A^2\gamma - 9(1+\gamma)(1-\gamma^2)(2-\varepsilon)\varepsilon(1-\varepsilon^2) + 6A(1+\gamma)(1-\varepsilon)\sqrt{(1-\varepsilon^2)(1-\gamma^2)}}{A^2 - 9(1-\varepsilon^2)(1-\gamma^2)\varepsilon^2}$, where $A = 4(1+\gamma) - (5+3\gamma)\varepsilon^2$.

smaller than one. We also define the value for the cross effect across brand types ε such that $x^u = 1$ and denote it by ε^u. Thus, we are ready to state the following results. The proofs are straightforward.

Proposition 2 *There is a unilateral incentive to introduce a store brand for a high enough transfer price w_1. The w_1 required is greater as ε approaches $\frac{1+\gamma}{2}$. Besides, if $0 < \varepsilon < \gamma$, such unilateral incentive exists irrespective of the value of w_1.*

Proposition 3 *No store brands will be in the market (i.e. (r_0, r_0) is a NE) if and only if ε is sufficiently large, in any case greater than γ, and w_1 low enough.*

The consideration both of multiproduct sellers and the strategic behaviour associated with replacement yield different results concerning the univocal relationship between the cross effect across brand types and the cross effect within brand types established in the previous section. It seems natural to find that a unilateral incentive to store brand introduction exists when the cross effect within brand types is the highest as this means that in-store competition is strong; then, a retailer will replace national brand one no matter the value of the transfer price to better internalize in-store competition. But replacement can also arise when the cross effect across brand types is the highest of the two, despite this meaning a negative in-store effect. The shaded area in Figure 3 displays an interval for ε where the negative in-store effect is, for any transfer price, offset by the direct and the exclusivity effects. Finally, a retailer is better off carrying national brand two and its store brand when the transfer price of national brand one as well as ε are sufficiently high. In fact, as the cross effect across brand types increases a higher transfer price is required for this replacement strategy to be profitable. The reference situation is a multiproduct duopoly downstream with intra-brand competition in both the national brands. What Proposition 2 states is when a retailer prefers introducing its store brand though leaving the rival retailer as the exclusive dealer of national brand one. The fact that the transfer price of this brand is rather high (a large direct effect) more than compensates for the exclusivity granted to the rival and a cross effect within brand types below the one across brand types (a negative in-store effect), thereby making the replacement strategy profitable. Certainly, a low enough transfer price turns store brand introduction harmful for retailers and the equilibrium entails no store brands in the market. The analysis with several national brands leads to conclusions that are in contrast with the previous section. With one national brand manufacturer, we found that at least one of the retailers decides to introduce its store brand. Here we find cases where neither retailer opts for store brand introduction.
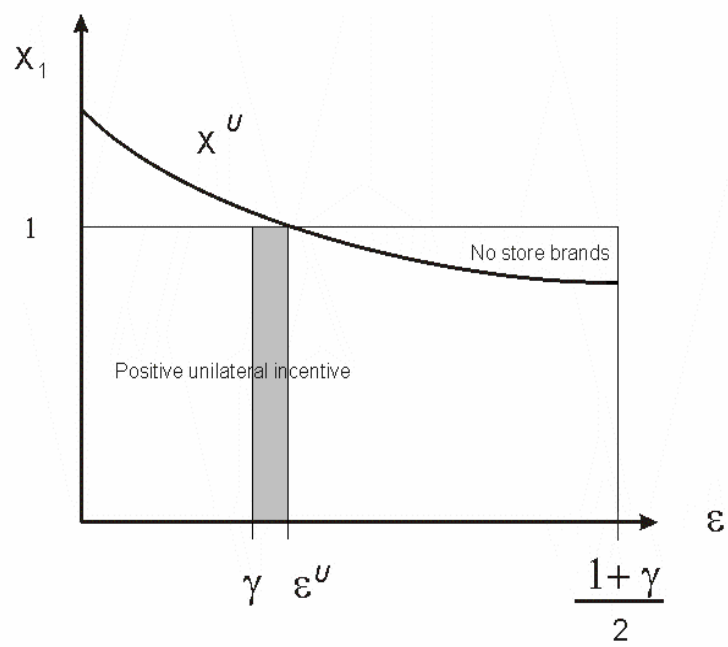
Figure 3

4.2   One store brand in the market.

We have already noted that there is evidence of some category products with extremely low penetration rates of store brands (see footnote 3 above). In fact, it is commonly observed that not all retailers replace national brands in all categories. Our focus in this section is on characterizing conditions to explain asymmetric retailer behaviour in their replacement strategies. We now wish to specify conditions under which (r_1, r_0) is a Nash equilibrium, this meaning that one of the retailers does not replace national brand one although it is supplied at a positive transfer price. This setting entails three brands in the market, retailer one's store brand and both the national brands; there is intra-brand competition in national brand two as it is sold by both retailers and the other is exclusively sold by retailer two. Three conditions must be simultaneously satisfied:

a) retailer one must find it profitable to replace national brand one by its store brand, which corresponds with the unilateral incentive previously analyzed; b) retailer two has to be better off not replacing national brand one; c) nor replacing national brand two, provided that the rival decides to introduce its store brand. *Condition b)* is formally stated as $R_2^{r_1 r_0}(w_1, 0) > R_2^{r_1 r_1}(w_1, 0)$, where $R_2^{r_1 r_0} = (1 - \gamma^2)(q_{n12}^{r_1 r_0})^2 + \frac{1}{9}$ and $R_2^{r_1 r_1} = (1 - \varepsilon^2)(q_{s2}^{r_1 r_1})^2 + \frac{1}{9}$. Therefore, if $q_{n12}^{r_1 r_0} > \sqrt{\frac{(1-\varepsilon^2)}{(1-\gamma^2)}} q_{s2}^{r_1 r_1}$ then it happens that retailer R_2 does not replace national brand one. There exists an x^e defined by the value of x_1 that solves the following equality $q_{n12}^{r_1 r_0}(x^e) = \sqrt{\frac{(1-\varepsilon^2)}{(1-\gamma^2)}} q_{s2}^{r_1 r_1}$. Thus for all $x_1 > x^e$, retailer two prefers not to replace national brand one provided retailer one does.[15]

It is important to underline that if $\varepsilon = 0$ then $x^e > 1$, if $\varepsilon = \gamma$ then $x^e = 1$ and if $\varepsilon = \frac{1+\gamma}{2}$ then $x^e < 1$. Besides $\frac{dx^e}{d\varepsilon} < 0$ when it is evaluated in each of these three values for ε. We conclude that condition b) can only be satisfied for $\gamma < \varepsilon$. To understand the intuition behind this, consider the case where $\varepsilon = \gamma$ so that from the retailer two's point of view, national brand one and its store brand only differ by the transfer price paid. Both brands are exclusive brands for him and they would suppose the same level of in-store competition. Therefore, in case both brand types coincide in the transfer price, i.e. $x_1 = 1$, then the equilibrium outputs $q_{n12}^{r_1 r_0}(x_1 = 1)$ and $q_{s2}^{r_1 r_1}$ are equal and hence the retailer is indifferent between both the national and the store brand. Now it is easy to understand that when γ is smaller than ε at the transfer price $w_1 = 0$, the intensity of in-store competition is weaker when the two national brands are carried. Therefore, retailer two obtains higher profits when non replacement. As a consequence, for $\gamma < \varepsilon$ the transfer price that makes the retailer indifferent is greater than zero (i.e. $x^e < 1$).

As for *condition c)*, formally specified as $R_2^{r_1 r_0}(w_1, 0) > R_2^{r_1 r_2}(w_1, 0)$, note that both these subgames imply that retailer two is an exclusive dealer of national brand one; it

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[15]The specific expression of x^e is $x^e = \frac{(1+\gamma)(2+\gamma-3\varepsilon^2)(2\gamma+(1+\gamma)\varepsilon)+A\sqrt{(1-\varepsilon^2)(1-\gamma^2)}}{2(1+\gamma)(1+\varepsilon)(2+\gamma-3\varepsilon^2)}$.

therefore becomes important to see how intense is competition from the other product carried by retailer two. This certainly depends on the feature of exclusivity inherited by retailer one's store brand introduction. On the one hand, if retailer two does not introduce its store brand then he will hold exclusivity in one of the national brands but will face intra-brand competition in the other - subgame (r_1, r_0). On the other hand, in case he opts for store brand introduction then the four brand types will be in the market and retailer two will be the sole dealer of two of them, its store brand and one of the national brands - subgame (r_1, r_2). Both of the above possibilities are affected by the size of ε. Retailer two's payoffs can be expressed as follows: $R_2^{r_1 r_0} = (1 - \gamma^2)(q_{n12}^{r_1 r_0})^2 + \frac{1}{9}$ while $R_2^{r_1 r_2} = (q_{n1}^{r_1 r_2})^2 + (q_{s2}^{r_1 r_2})^2 + 2\varepsilon q_{n1}^{r_1 r_2} q_{s2}^{r_1 r_2}$, with $q_{s2}^{r_1 r_2} > q_{n1}^{r_1 r_2}$. It is also important to remark that $q_{n1}^{r_1 r_2}$ is increasing in x_1, whereas $q_{s2}^{r_1 r_2}$ is if and only if $\varepsilon < \sqrt{\frac{-4 + 8\gamma - \gamma^2}{3}} < \frac{1 + \gamma}{2}$ and decreasing otherwise.[16] Besides, it can be proven that $R_2^{r_1 r_2}$ is increasing in x_1. We next define by x^{ne+} and x^{ne-} the two roots that solve $R_2^{r_1 r_0}(x_1) - R_2^{r_1 r_2}(x_1) = 0$, a quadratic polynomial in x_1 which can be concave or convex. It occurs that when the polynomial is concave, then condition c) holds for $x^{ne-} < x_1 < x^{ne+}$. Then a sufficiently low x_1 is required so that retailer two would better not introduce its store brand.

In sum, (r_1, r_0) is a Nash equilibrium if and only if *condition a* $(x_1 < x^u)$; *condition b* $(x_1 > x^e)$ and *condition c* $(x^{ne-} < x_1 < x^{ne+})$ are simultaneously satisfied for $\gamma < \varepsilon$. For illustration purposes Figure 4 is drawn for the particular case $\gamma = 0.3$ and ε varies from γ to $\frac{1 + \gamma}{2}$.[17]

The shaded area in Figure 4 identifies, given $\gamma = 0.3$, *parameter conditions for ε and w_1 such that (r_1, r_0) is a Nash equilibrium in the replacement stage.* A necessary condition is that the cross effect across brand types exceeds the cross effect within brand types $(\gamma < \varepsilon)$, a negative in-store effect. Given this, an equilibrium with only one store brand typically arises for intermediate values of ε and for a given interval of the transfer price w_1. First of all, the transfer price must be high enough to induce replacement by one of the retailers, which implies a large enough direct effect. The equilibrium output of the store brand increases in w_1 whereas the equilibrium output of national brand one decreases in w_1; the *right* transfer price level combined with appropriate values of the demand parameters precisely make the replacement strategy profitable. Secondly, the transfer price has to be sufficiently low to discourage replacement of national brand one by the rival retailer. The reason is that if it decided to replace, then national brand one

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[16] The expression $\sqrt{\frac{-4 + 8\gamma - \gamma^2}{3}}$ is positive iff $\gamma > 2(2 - \sqrt{3}) \simeq 0.5358)$.

[17] Figure 4 includes the graphs of the expressions $\underline{x}^a$ and $\underline{x}^b$ which stand for the lower value for x_1 that ensures positive equilibrium outputs for all subgames. In particular, $\underline{x}^a = 1 - \underline{w}_1^a$ is the value defined by $q_{n1}^{r_1 r_2}(\underline{w}_1^a, 0) = \frac{1}{2 + \gamma + 3\varepsilon} - \frac{(2 + \gamma)w_1^a}{2((2 + \gamma)^2 - 9\varepsilon^2)} - \frac{(2 - \gamma)w_1^a}{2((2 - \gamma)^2 - \varepsilon^2)} = 0$; while $\underline{x}^b = 1 - \underline{w}_1^b$ is the value defined by $q_{n21}^{r_2 r_2}(w_1) = \frac{-3\varepsilon + (2 + \gamma)(1 - w_1^b)}{3(2 + \gamma - 3\varepsilon^2)} = 0$. Thus, the lower bound for x_1 is $\min\{\underline{x}^a, \underline{x}^b\}$.
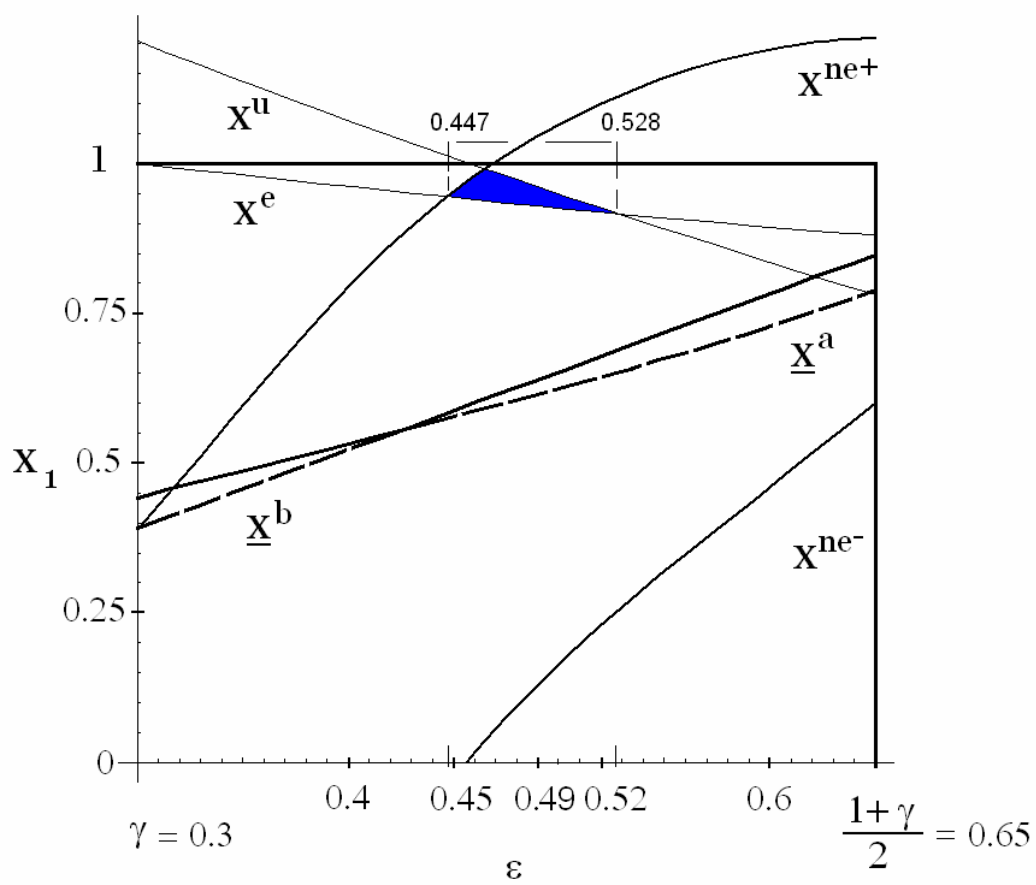
Figure 4

would not be in the market and the corresponding equilibrium quantities would not depend on the transfer price w_1. Thus, since retailer two's payoffs are decreasing in the transfer price when replacement does not occur we have that store brand introduction is precluded for a low enough transfer price. Last but not least, we must also ensure that retailer two would be worse off if replacing national brand two. The two market configurations to be compared suppose that national brand one is carried exclusively by retailer two; the equilibrium output of that brand decreases in the transfer price w_1. It is the other product sold that makes a difference although their equilibrium outputs increase in the transfer price. The fact that the cross effect within brand types γ is smaller than the cross effect across brand types ε is in favour of not replacing. Still, a sufficiently high transfer price is required for no store brand introduction to be the equilibrium choice of retailer two. Overall, retailer two's profits are decreasing in the transfer price in both these subgames. No replacement is explained by the fact that the variation in profits is more marked in subgame (r_1, r_2). Hence, being a common retailer of national brand two is preferred for a sufficiently high transfer price. However, as the cross effect across brand types increases, competition downstream is more intense if both retailers introduce their store brands; the required transfer price need not be too high to have replacement by just one of the retailers (in fact, high values of ε guarantee that national brand two is not replaced by retailer two regardless of the value of the transfer price).

At this point one wonders whether the replacement stage equilibrium (r_1, r_0) is part of the equilibrium of the full game. In other words, we wish to ensure that the range of parameters for which (r_1, r_0) is an equilibrium of the full game is non-empty. We now proceed with the example in Figure 4 to solve for the first stage of the proposed multi-stage game. The next table summarizes the subgame perfect equilibrium for different values of the cross effect across brand types.

ε	0.3	0.4	0.45	0.49	0.52	0.6
w_1^*	0.08857	0.06545	0.05663	0.07099	0.07431	0.10622
replacement stage	(r_1, r_2)	(r_1, r_2)	(r_1, r_0)	(r_1, r_0)	(r_0, r_0)	(r_0, r_0)

It illustrates that the manufacturer, by choosing the *right* transfer price, w_1^*, will induce the replacement stage equilibrium that maximizes its payoffs. How is w_1^* computed? Take the case for $\varepsilon = 0.49$. It happens that w_1^* is obtained from x^e evaluated at $\gamma = 0.3$ and $\varepsilon = 0.49$. If the manufacturer set a higher transfer price then it would induce the replacement equilibrium (r_1, r_1) which is not an optimal choice. Now, a sufficiently low transfer price would induce (r_0, r_0); it is the one obtained from x^u evaluated at $\gamma = 0.3$ and $\varepsilon = 0.49$, $w_1 = 0.03917$. The manufacturer then selects w_1^* since it is better off with one

retailer at a higher transfer price than employing two retailers but a lower transfer price. A similar procedure applies for other values of ε. We conclude that $\{w_1^* = 0.07099, \ (r_1, r_0)\}$ is an equilibrium for the full game. On the other hand, if $\varepsilon = 0.52$ the manufacturer is better off when it implements (r_0, r_0) by setting $w_1^* = 0.07431$.

Finally, note that in case $\varepsilon = 0.45$ the manufacturer must decide whether implementing the replacement equilibrium (r_1, r_0) by setting $w_1^* = 0.05663$ or implementing the replacement equilibrium (r_1, r_2) by setting the transfer price obtained from x^{ne+} evaluated at $\gamma = 0.3$ and $\varepsilon = 0.45$, $w_1 = 0.04764$. The manufacturer is better off with the former option. Thus $\{w_1^* = 0.05663, \ (r_1, r_0)\}$ is an equilibrium for the full game.

It is worth remarking that a) the equilibrium transfer price is lower than the interior solution, and b) there are three equilibria in the replacement stage that appear in the equilibrium for the full game. In particular, there are combinations of ε, γ, and w_1 where both retailers would choose (r_1, r_1) but this is never in the manufacturer's interest.

We are claiming that manufacturer of national brand one, who holds market power, can keep itself in the market with a certain transfer price, and for a given combination of the cross effects parameters. Even more, for some parameter values we find that national brand two, though supplied by a competitive fringe of manufacturers, is sold by both retailers - this national brand is similar to a store brand in costs terms. The analysis suggests that it is retailer interaction and demand conditions that determine the variety of equilibria rather than an issue of market power. To further emphasize this point we briefly discuss the case where both national brands are supplied by a competitive fringe of manufacturers.

4.3   The case of no market power upstream

This subsection is aimed at providing some theoretical basis that the emergence of market configurations where none of the retailers or only one of them introduce its store brand is not to be attributed to the existence of upstream market power, which might be used to retain retailers. Rather it results from retailer strategic interaction and a particular relationship between the demand parameters. In this particular context, only the in-store and the exclusivity effects derived from the store brand introduction are present.

Consider that both national brands are supplied competitively, which amounts to the case when both transfer prices, w_1 and w_2 are equal to zero, i.e. to the case where $x_1 = 1$ above. We may then present the analysis solely in terms of the cross effects γ and ε. Let us now define the values for the cross effect across brand types ε such that $x^e = 1$ and $x^{ne+} = 1$ and denote them by ε^e and ε^{ne}, respectively. Further note that ε^u, ε^e and ε^{ne} are a function of γ, where ε^e is equal to γ, and that both ε^u and ε^{ne} are

greater than γ and smaller than $\frac{1+\gamma}{2}$ as indicated above. It also happens that $\varepsilon^u < \varepsilon^{ne}$ for $\gamma < 0.58677$, the opposite otherwise. How to link these threshold values ε^u, ε^e and ε^{ne} to the previous analysis? The former has to do with the unilateral incentive to store brand introduction (*condition a* above) so that values of the cross effect across brand types below ε^u imply higher profits under the replacement strategy, i.e. $R_1^{r_1 r_0} > R_1^{r_0 r_0}$ and $R_2^{r_0 r_1} > R_2^{r_0 r_0}$. For the interpretation of the other two, consider that one retailer has decided to market its store brand, then the ε^e threshold is related with the replacement of the *same* national brand already replaced by the rival (*condition b* above) and the latter is related with replacing a *different* national brand than the rival retailer (*condition c* above). Therefore, values below ε^e imply higher profits under the former replacement strategy, i.e. $R_1^{r_1 r_1} > R_1^{r_0 r_1}$; and values below ε^{ne} mean that the latter replacement strategy is preferred to no replacement, i.e. $R_2^{r_1 r_2} > R_2^{r_1 r_0}$ and $R_1^{r_2 r_1} > R_1^{r_0 r_1}$.

Finally, using the result stated above if w_1 and w_2 are equal then both national brands become symmetric and therefore, $R_1^{r_1 r_0} = R_1^{r_2 r_0}$ and $R_2^{r_0 r_1} = R_2^{r_0 r_2}$. Also, the following Lemma can be shown (see the Appendix).

Lemma 1 *In case of store brand introduction by both retailers, each retailer always prefers to replace the national brand that is kept by the rival retailer. That is $R_1^{r_j r_i} > R_1^{r_i r_i}$ and $R_2^{r_i r_j} > R_2^{r_i r_i}$ for $i, j = 1, 2$ and $i \neq j$, $\forall \, \gamma \in (0, 1)$ and $\forall \, \varepsilon \in (0, \frac{1+\gamma}{2})$.*

This result means that, provided both retailers introduce its store brand, exclusivity on both national brands is preferred to intra-brand competition in one of them. In other words, replacement of the same national brand by both retailers is not an equilibrium in the replacement stage. The intuition can be cast in terms of the well known dampening-of-competition effect of exclusive dealing but now in a setting with two multi-product retailers.

We proceed by noting that $\varepsilon^e = \gamma$ and that ε^e is the lowest of the thresholds, then we can distinguish two different cases depending on the sign of the in-store effect.

i) Consider a positive in-store effect: $0 < \varepsilon < \gamma$ and $0 < \gamma < 1$. It happens that the following orderings hold for retailer one: $R_1^{r_j r_i} > R_1^{r_i r_i} > R_1^{r_0 r_i}$ $i, j = 1, 2$ and $i \neq j$; also $R_1^{r_2 r_0} = R_1^{r_1 r_0} > R_1^{r_0 r_0}$ (mutatis mutandis for retailer two). Therefore, the following result is obtained:

Proposition 4 *If $\varepsilon < \gamma$ there is a NE in the replacement stage where each retailer replaces one and a different national brand to that of the rival. This is obtained without market power upstream and for all $\gamma \in (0, 1)$.*

ii) We next focus on the case of a negative in-store effect, $\varepsilon \in \left(\gamma, \frac{1+\gamma}{2}\right)$, and distinguish two different subcases: ii.1) for $0 < \gamma < 0.58677$, where the following ranking holds $\gamma < \varepsilon^u$

$< \varepsilon^{ne} < \frac{1+\gamma}{2}$; ii.2) for $0.58677 < \gamma < 1$, where $\gamma < \varepsilon^{ne} < \varepsilon^{u} < \frac{1+\gamma}{2}$. We may characterize the Nash equilibrium in each of the two different subcases and find that only two different equilibria arise for the case where $\gamma < 0.58677$: either both retailers opt for the replacement strategy if $\gamma < \varepsilon < \varepsilon^{u}$, or no store brand is marketed if $\varepsilon^{ne} < \varepsilon < \frac{1+\gamma}{2}$, or finally the two type of equilibria arise when $\varepsilon^{u} < \varepsilon < \varepsilon^{ne}$. Next, for the ii.2) subcase three different equilibria arise as stated in the next Proposition:

Proposition 5 *Consider* $0.58677 < \gamma < 1$, *the following pair of strategies are the NE in the replacement stage:*

a) Both retailers decide to replace one different national brand when $\varepsilon \in (\gamma, \varepsilon^{ne})$

b) Only one retailer introduces its store brand for $\varepsilon \in (\varepsilon^{ne}, \varepsilon^{u})$.

c) Finally, store brands are not introduced when $\varepsilon \in (\varepsilon^{u}, \frac{1+\gamma}{2})$.

This simplified setting offers all the possible equilibria one may find. Let us elaborate on the latter subcase since it is here where competition intensity is stronger because products are less differentiated and this highlights the effects of retail strategic behaviour. We can see that the replacement strategy chosen by retailers can be explained by how distant the cross effect across brand types is from the cross effect within brand types. As the former cross effect increases, retailers are at equilibrium less inclined towards replacement. This occurs because the store brand is perceived by consumers as more similar than the national brand to be replaced relative to the other national brand. Therefore, when ε is very large (part c) in Proposition 5, replacement would increase significantly the negative in-store effect which could not be compensated for by the positive exclusivity effect of selling a store brand, thus leading to a no replacement strategy for both retailers. On the other hand, when ε is close to γ, the exclusivity effect always dominates the in-store effect.

5   Concluding Remarks

Private labels are an important phenomenon both in European and U.S. retailing and increasing in importance. This paper looks at the strategic interaction between national brands and private labels in a retail duopoly when each retailer can stock a limited number of brands. While the model leaves some issues unexplained, nevertheless it does provide a theoretical argument to better understand why we observe a varied pattern of store brand introduction across outlets and across product categories. Multiproduct retailers have to decide their product line mix in order to better internalize in-store competition while also alleviating retail competition, in addition to the direct effect that results from replacement of a national brand by a store brand. We contribute to the received literature by drawing attention to how consumers differently perceive brand types. Our simple approach suffices

to explain different levels of private label penetration once accounting for retailer strategic interaction; we provide a theoretical justification as to why undifferentiated retailers undertake different equilibrium strategies about private label introduction. Differently, a store brand's poor performance can be explained in case a retailer has introduced it when that strategy is not optimal for given demand conditions. The results have been obtained neither assuming demand or cost asymmetries that favour store brand introduction, nor demand or cost uncertainties. Other features also characterize the retail sector and our analysis seems a natural starting point for further research.

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A Robustness

A.1 Timing

We consider the following timing: firstly, R_1 decides whether to replace, secondly, M chooses the transfer price and, finally, R_2 decides whether to replace.

a) R_1 decides to replace the national brand by its store brand.

For any given w, the third stage payoffs to R_2 are obtained as follows.

a.1) If it decides to introduce its store brand, then the manufacturer is left out of the market and payoffs $R_1^{rr} = R_2^{rr} = \frac{1}{(2+\gamma)^2}$.

a.2) If R_2 does not introduce the store brand then the payoffs are $R_1^{rn} = \frac{(2-\varepsilon(1-w))^2}{(4-\varepsilon^2)^2}$ and $R_2^{rn} = \frac{(2-\varepsilon-2w)^2}{(4-\varepsilon^2)^2}$.

Therefore, R_2 introduces the store brand when R_1 does so if and only if $R_2^{rr} - R_2^{rn} > 0$, that is, for $w > \frac{(2-\varepsilon)(\gamma-\varepsilon)}{2(2+\gamma)} \equiv \bar{w}$.

In the second stage the manufacturer decides on the profit maximizing value of the transfer price. Note that w will depend on previous retailer one's choice, that is $w(r)$ or $w(n)$. In this subgame, the manufacturer's payoffs function is given by:

$$M(w(r)) = \begin{cases} w(r)\left(\frac{2-\varepsilon-2w(r)}{4-\varepsilon^2}\right) & for\ 0 \leq w(r) \leq \bar{w} \\ 0 & for\ \bar{w} < w(r) < 1 \end{cases}$$

From the inspection of the critical value $\bar{w}$, it is clear that for $\gamma < \varepsilon$, firms' payoffs are $M^{rr} = 0$ and $R_1^{rr} = R_2^{rr} = \frac{1}{(2+\gamma)^2}$ because M is unable to retain R_2. However, for $\gamma > \varepsilon$, by similar reasoning as in the text, the manufacturer sets the transfer price $w(r) = \frac{(2-\varepsilon)(\gamma-\varepsilon)}{2(2+\gamma)} \equiv \bar{w}$. Firms get the following payoffs:

$M^{rn} = \frac{(2-\varepsilon)(\gamma-\varepsilon)}{2(2+\gamma)}$, $R_1^{rn} = \frac{(2+\gamma-\varepsilon))^2}{4(2+\gamma)^2}$ and $R_2^{rn} = \frac{1}{(2+\gamma)^2}$.

b) R_1 does not introduce its store brand.

In this case, M knows that $R_2's$ decision is to introduce its store brand regardless of w. Therefore, M solves an unrestricted maximization problem on $w(n)$, which yields $w(n) = \frac{2-\varepsilon}{4} = w^*$. The payoffs are $M^{nr} = \frac{(2-\varepsilon)}{8(2+\varepsilon)}$, $R_1^{nr} = \frac{1}{4(2+\varepsilon)^2}$ and $R_2^{nr} = \frac{(4+\varepsilon)^2}{16(2+\varepsilon)^2}$.

In stage one R_1 compares $\frac{1}{(2+\gamma)^2}$ with $\frac{1}{4(2+\varepsilon)^2}$ if $\gamma < \varepsilon$, or $\frac{(2+\gamma-\varepsilon))^2}{4(2+\gamma)^2}$ with $\frac{1}{4(2+\varepsilon)^2}$ if $\varepsilon < \gamma < 1$. It follows that R_1 always introduces its store brand.

A.2 Other terms of payment

We consider two alternative terms of payment to linear contracts: a) the fixed fee contract, a contract with a zero transfer price and a positive fixed up-front fee, $\{w = 0,\ F\}$; and b) the two-part tariff contract, a contract including a variable and a fixed payment, $\{w,\ F\}$.

Finally, we compare the linear contract $\{w, \ F = 0\}$ with the two-part tariff in part c) below.

a) Fixed fee contract.

Retailers' payoffs for each subgame that arises at the end of the second stage are:

$R_1^{nn}(w = 0) = R_2^{nn}(w = 0) = \frac{1}{9} - F$, $R_1^{rn}(w = 0) = R_2^{nr}(w = 0) = \frac{1}{(2+\varepsilon)^2} - F$,

$R_2^{rn}(w = 0) = R_1^{nr}(w = 0) = \frac{1}{(2+\varepsilon)^2}$ and $R_1^{rr} = R_2^{rr} = \frac{1}{(2+\gamma)^2}$.

- A retailer's best reply, say R_1, in the second stage is as follows.

First, R_1 introduces its store brand when R_2 does not if and only if $R_1^{rn}(w = 0) - R_1^{nn}(w = 0) = \Phi_r(w = 0) + F$ is positive. As happens in the linear price contract, this difference is always positive since now a positive constant has been added.

Second, R_1 introduces its store brand when R_2 does if and only if the difference $R_1^{rr} - R_1^{nr}(w = 0) = \Phi_{rr}(w = 0) + F$ is positive. This is so if either $\Phi_{rr}(w = 0) = \frac{(4+\gamma+\varepsilon)(\varepsilon-\gamma)}{(2+\gamma)^2(2+\varepsilon)^2} > 0$ regardless of F, or $F > -\Phi_{rr}(w = 0)$ when $\gamma > \varepsilon$. Since the manufacturer's payoffs are $M(w = 0, F) = F$ if $R_1^{rr} - R_1^{nr}(w = 0) < 0$, or zero otherwise, the manufacturer sets the highest F that makes the retailer not to replace the national brand, that is $F^f = -\Phi_{rr}(w = 0) = \frac{(4+\gamma+\varepsilon)(\gamma-\varepsilon)}{(2+\gamma)^2(2+\varepsilon)^2}$, superscript f stands for fixed fee. Thus, the manufacturer's payoffs are $M^f = F^f$, which are positive for $\varepsilon < \gamma < 1$. Note that this result would have been obtained when a two-part tariff contract had been implemented with the restriction of a nonnegative transfer price.

b) Two-part tariff contract.

Retailers' payoffs for each subgame are now given by:

$R_1^{nn}(w) = R_2^{nn}(w) = \frac{(1-w)^2}{9} - F$, $R_1^{rn}(w) = R_2^{nr}(w) = \frac{(2-\varepsilon(1-w))^2}{(4-\varepsilon^2)^2} - F$,

$R_2^{rn}(w) = R_1^{nr}(w) = \frac{(2-\varepsilon-2w)^2}{(4-\varepsilon^2)^2}$ and $R_1^{rr} = R_2^{rr} = \frac{1}{(2+\gamma)^2}$.

We analyze two possible situations, i) that both retailers sell the national brand; and ii) that only one of them does.

-i) The difference $R_1^{rn}(w) - R_1^{nn}(w) = \Phi_r(w) + F$ is positive regardless of F as long as $\Phi_r(w) > 0$, which holds for $w \in (\frac{-(2-\varepsilon)(1-\varepsilon)}{(4-\varepsilon)(1+\varepsilon)}, 1 - \frac{\varepsilon}{2})$. However, $\Phi_r(w)$ is negative for w smaller than $\frac{-(2-\varepsilon)(1-\varepsilon)}{(4-\varepsilon)(1+\varepsilon)}$. Therefore, $R_1^{rn}(w) - R_1^{nn}(w) > 0$ for $F > -\Phi_r(w)$ and $w < \frac{-(2-\varepsilon)(1-\varepsilon)}{(4-\varepsilon)(1+\varepsilon)}$. Then, if the manufacturer wants to retain a retailer provided that the other retailer does not replace the national brand, it will maximize the following expression:

$$M = 2w\frac{(1-w)}{3} + 2F, \text{ subject to } F \leq -\Phi_r(w) \text{ and } w < \frac{-(2-\varepsilon)(1-\varepsilon)}{(4-\varepsilon)(1+\varepsilon)}.$$

The maximization yields $w_{nn}^{tp} = \frac{16-36\varepsilon+10\varepsilon^2+\varepsilon^3}{2(32-7\varepsilon^2+2\varepsilon^4)}$, which is smaller than $\frac{-(2-\varepsilon)(1-\varepsilon)}{(4-\varepsilon)(1+\varepsilon)}$ for ε greater than 0.6995. Then $F_{nn}^{tp} = \frac{-(16-24\varepsilon+3\varepsilon^3-\varepsilon^4)(48-16\varepsilon^2+3\varepsilon^3+\varepsilon^4)}{4(32-7\varepsilon^2+2\varepsilon^4)^2}$ which is positive for ε greater than 0.6995. Superscript tp stands for two-part tariff. Substituting back, the

manufacturer's profits are equal to $M_{nn}^{tp} = \frac{-16+24\varepsilon-8\varepsilon^2+\varepsilon^4}{2(32-7\varepsilon^2+2\varepsilon^4)}$, where M_{nn}^{tp} is positive for ε greater than 0.9180. Then we conclude that the option for the manufacturer to retain both retailers is only possible for ε greater than 0.9180.

-ii) The difference $R_1^{rr} - R_1^{nr}(w) = \Phi_{rr}(w) + F$ is positive if either $\Phi_{rr}(w) > 0$ regardless of F, or if $F > -\Phi_{rr}(w)$ when $w \leq \bar{w} \equiv \frac{(2-\varepsilon)(\gamma-\varepsilon)}{2(2+\gamma)}$. Manufacturer's profits are $M(w,F) = w\left(\frac{2-\varepsilon-2w}{4-\varepsilon^2}\right) + F$ for $w \leq \bar{w}$ and $F \leq -\Phi_{rr}(w)$. Solving $\frac{\partial M(w,F)}{\partial w} = \frac{-(4(2-\varepsilon^2)w+(2-\varepsilon)\varepsilon^2)}{(4-\varepsilon^2)} = 0$, yields $w_n^{tp} = \frac{-(2-\varepsilon)\varepsilon^2}{4(2-\varepsilon^2)} < 0$. Substituting back the fixed fee is $F_n^{tp} = \frac{\gamma(4+\gamma)(2-\varepsilon)^2-4\varepsilon(4-5\varepsilon+\varepsilon^3)}{4(2+\gamma)^2(2-\varepsilon^2)^2}$ which is positive for $\frac{2\varepsilon(1-\varepsilon)}{2-\varepsilon} < \gamma < 1$, where $\frac{2\varepsilon(1-\varepsilon)}{2-\varepsilon}$ is greater than $2\varepsilon - 1$ for $\varepsilon > \frac{2}{3}$. Finally, manufacturer's profits when only one retailer replaces are $M_n^{tp} = \frac{\gamma(4+\gamma)(2-\varepsilon)^2-4\varepsilon(4-3\varepsilon)}{8(2+\gamma)^2(2-\varepsilon^2)}$, which is positive for $\gamma > \gamma^0(\varepsilon) = \frac{2\sqrt{2(2-\varepsilon^2)}}{2-\varepsilon} - 2$, where $\gamma^0(\varepsilon) > 2\varepsilon-1$ for $0 < \varepsilon < 0.9031$ and $\gamma^0(\varepsilon) < \varepsilon$. Therefore, the option of retaining a retailer is possible for $max\{\gamma^0(\varepsilon), 2\varepsilon-1\} < \gamma < 1$.

The difference $M_n^{tp} - M_{nn}^{tp}$ is a second degree concave polynomial in γ. It is negative as long as $0.918 < \varepsilon$ and $\gamma < \gamma^{n/nn}(\varepsilon) = \frac{2\sqrt{2(2-\varepsilon^2)(32-7\varepsilon^2+2\varepsilon^4)(256-320\varepsilon+4\varepsilon^2+124\varepsilon^3-39\varepsilon^4-8\varepsilon^5+6\varepsilon^6)}}{(256-320\varepsilon+4\varepsilon^2+124\varepsilon^3-39\varepsilon^4-8\varepsilon^5+6\varepsilon^6)} - 2$. Superscript n/nn stands for the comparison between cases i) and ii). It follows that $\gamma^{n/nn}(\varepsilon) < 2\varepsilon-1$ for $0 < \varepsilon < 0.943$ and $\gamma^{n/nn}(\varepsilon) > 1$ for $\varepsilon > 0.980$. Combining the conditions on ε and γ for both M_{nn}^{tp} and M_n^{tp} to be positive along with the condition for $M_n^{tp} - M_{nn}^{tp} < 0$, yields the next result. See Figure 2 in the text.

Proposition 6 *Consider that the manufacturer sets a two part tariff contract.*
a) The manufacturer cannot retain any retailer for $\gamma \in [\max\{0, 2\varepsilon-1\}, \gamma^0(\varepsilon)]$ and $0 < \varepsilon < 0.903$.
b) The manufacturer sets (w_n^{tp}, F_n^{tp}) and retains only one retailer as long as either,
b.1) $\gamma \in [\gamma^0(\varepsilon), 1]$ and $0 < \varepsilon < 0.903$, or
b.2) $\gamma \in [2\varepsilon-1, 1]$ and $0.903 < \varepsilon < 0.943$, or
b.3) $\gamma \in [\gamma^{n/nn}(\varepsilon), 1]$ and $0.943 < \varepsilon < 0.980$.
c) The manufacturer sets $(w_{nn}^{tp}, F_{nn}^{tp})$ and retains both retailers as long as $\gamma \in [2\varepsilon-1, \min\{\gamma^{n/nn}(\varepsilon), 1\}]$ and $0.943 < \varepsilon < 1$.
Furthermore, the variable part of the two part tariff contract is always set below zero at equilibrium.

Finally we compare the manufacturer's profits when either a fixed fee contract or a two-part tariff contract is used. Note that this comparison is only meaningful for $M^f > 0$ i.e. $\gamma > \varepsilon$. The difference $M_n^{tp} - M^f = \frac{\varepsilon^4}{8(2+\varepsilon)^2(2-\varepsilon^2)}$ is always positive, then a fixed fee contract is never endogenously chosen by the manufacturer.

The two-part tariff contract is preferred to the fixed fee contract by the manufacturer.

c) Linear vs. two-part tariff contracts.

Finally, consider that the manufacturer has the option to choose between a linear price and a two part tariff contract. Note that the linear contract option is only possible for $\gamma > \varepsilon$. We then have the following result, $M^l = M(\bar{w}) = \frac{(2-\varepsilon)(\gamma-\varepsilon)}{2(2+\gamma)} > M_n^{tp}$ for $\gamma > \gamma^{l/tp}(\varepsilon) = \frac{2[-(1-\varepsilon)(4-3\varepsilon^2+\varepsilon^3)+\sqrt{(2-\varepsilon)(2-\varepsilon^2)[4+6\varepsilon-(2-\varepsilon)(2+4\varepsilon+\varepsilon^2)\varepsilon^2]]}}{(2-\varepsilon)(6+\varepsilon-4\varepsilon^2)}$. Superscript l/tp stands for the comparison between linear and two-part tariff contracts. It follows that $\gamma^{l/tp}(\varepsilon) > \varepsilon$ for all ε and $0 < \gamma^{l/tp}(\varepsilon) < 1$ for $0 < \varepsilon < 0.915$.

Proposition 7 *The manufacturer is better off with a linear price contract for $\gamma \in [\gamma^{l/tp}(\varepsilon), 1]$ and $0 < \varepsilon < 0.915$.*

The manufacturer is better off with a two-part tariff contract for $\gamma \in [\max\{\gamma^0(\varepsilon), 2\varepsilon - 1\}, \min\{\gamma^{l/tp}(\varepsilon), 1\}]$ and $0 < \varepsilon < 1$.

If both retailers introduce their store brands then the manufacturer is indifferent between either contract, which happens for $\gamma \in [\max\{0, 2\varepsilon - 1\}, \gamma^0(\varepsilon)]$ and $0 < \varepsilon < 0.903$.

B   Several National Brands.

B.1   Third Stage Equilibrium: Several National Brands.

We display in this Appendix the equilibrium outputs of the following subgames.

$(r_0, r_0)-$ *Subgame*

The national brands are sold by both retailers, no store brand is marketed.

$q_{nij}(w_1, w_2)$ denotes the *ith* national brand equilibrium output sold by R_j.

$q_{n11}^{r0r0}(w_1, w_2) = q_{n12}^{r0r0}(w_1, w_2) = \frac{(1-w_1)-\gamma(1-w_2)}{3(1-\gamma^2)}$

$q_{n21}^{r0r0}(w_1, w_2) = q_{n22}^{r0r0}(w_1, w_2) = \frac{(1-w_2)-\gamma(1-w_1)}{3(1-\gamma^2)}$

Equilibrium outputs are positive as long as $\gamma < \frac{1-w_1}{1-w_2} < \frac{1}{\gamma}$.

Equilibrium retailers' margins are,

$p_{n1}^{r0r0}(w_1, w_2) - w_1 = \frac{(1-w_1)}{3}$

$p_{n2}^{r0r0}(w_1, w_2) - w_2 = \frac{(1-w_2)}{3}$

Consider the case where $w_2 = 0$. Note that the first order conditions for retailer R_1's optimization problem are $p_{n1}^{r0r0}(w_1, 0) - w_1 = q_{n11}^{r0r0}(w_1, 0) + \gamma q_{n21}^{r0r0}(w_1, 0)$,

$p_{n2}^{r0r0}(w_1, 0) = q_{n21}^{r0r0}(w_1, 0) + \gamma q_{n11}^{r0r0}(w_1, 0)$ and we know that $p_{n2}^{r0r0}(w_1, 0) = \frac{1}{3}$. Then the following expression follows $q_{n21}^{r0r0} = \frac{1}{3} - \gamma q_{n11}^{r0r0}$. Thus it is easy to write R_1's payoffs as:

$R_1^{r0r0}(w_1, 0) = (1 - \gamma^2)(q_{n11}^{r0r0}(w_1, 0))^2 + \frac{1}{9}$.

Similarly for R_2's payoffs

$R_2^{r0r0}(w_1, 0) = (1 - \gamma^2)(q_{n12}^{r0r0}(w_1, 0))^2 + \frac{1}{9}$.

$(r_1, r_0)-$ *Subgame*

The two national brands are marketed and R_1 also markets its store brand. The equilibrium outputs are,

$$q_{n12}^{r1r0}(w_1, w_2) = \frac{-\varepsilon(1-\gamma)+2(1-\varepsilon^2)(1-w_1)-[2\gamma-(1+\gamma)\varepsilon^2](1-w_2)}{(1-\gamma)[4(1+\gamma)-(5+3\gamma)\varepsilon^2]]}$$

$$q_{n21}^{r1r0}(w_1, w_2) = \frac{-6\varepsilon(1+\gamma)+3\varepsilon^2(1-w_1)+[4(1+\gamma)+\varepsilon^2](1-w_2)}{3[4(1+\gamma)-(5+3\gamma)\varepsilon^2]]}$$

$$q_{n22}^{r1r0}(w_1, w_2) = \frac{3\varepsilon\gamma(1-\gamma)-6\gamma(1-\varepsilon^2)(1-w_1)+[2(2+\gamma^2)-(5+\gamma)\varepsilon^2](1-w_2)}{3(1-\gamma)[4(1+\gamma)-(5+3\gamma)\varepsilon^2]]}$$

$$q_{s1}^{r1r0}(w_1, w_2) = \frac{2(1+\gamma)-\varepsilon(1-w_1)-(2+\gamma)\varepsilon(1-w_2)}{[4(1+\gamma)-(5+3\gamma)\varepsilon^2]]}$$

Equilibrium retailers' margins are

$$p_{n1}^{r1r0}(w_1, w_2) - w_1 = \frac{-3\varepsilon(1-\gamma^2)+6(1+\gamma)(1-\varepsilon^2)(1-w_1)-[2\gamma(1+\gamma)-(3+\gamma)\varepsilon^2](1-w_2)}{3[4(1+\gamma)-(5+3\gamma)\varepsilon^2]]}$$

$$p_{n2}^{r1r0}(w_1, w_2) - w_2 = \frac{(1-w_2)}{3}$$

$$p_{s1}^{r1r0}(w_1, w_2) = \frac{6(1+\gamma)(1-\varepsilon^2)-3\varepsilon(1-\varepsilon^2)(1-w_1)-\varepsilon(2-\gamma-\varepsilon^2)(1-w_2)}{3[4(1+\gamma)-(5+3\gamma)\varepsilon^2]]}$$

Consider the case where $w_2 = 0$. Note that the first order conditions for R_1's problem are

$$p_{n2}^{r1r0}(w_1, 0) = q_{n21}^{r1r0}(w_1, 0) + \varepsilon q_{s1}^{r1r0}(w_1, 0), \; p_{s1}^{r1r0}(w_1, 0) = q_{s1}^{r1r0}(w_1, 0) + \varepsilon q_{n21}^{r1r0}(w_1, 0),$$

and by $p_{n2}^{r1r0}(w_1, 0) = \frac{1}{3}$, the following expression follows $q_{n21}^{r1r0} = \frac{1}{3} - \varepsilon q_{s1}^{r1r0}$. Thus R_1's payoffs read

$$R_1^{r1r0}(w_1, 0) = (1 - \varepsilon^2)(q_{s1}^{r1r0}(w_1, 0))^2 + \frac{1}{9}.$$

For R_2 we have that $p_{n1}^{r1r0}(w_1, 0) - w_1 = q_{n12}^{r1r0}(w_1, 0) + \gamma q_{n22}^{r1r0}(w_1, 0)$,

$$p_{n2}^{r1r0}(w_1, 0) = q_{n22}^{r1r0}(w_1, 0) + \gamma q_{n12}^{r1r0}(w_1, 0).$$ Using $p_{n2}^{r0r0}(w_1, 0) = \frac{1}{3}$ then $q_{n22}^{r1r0} = \frac{1}{3} - \gamma q_{n12}^{r1r0}$. Thus it is easy to write R_2's payoffs as:

$$R_2^{r1r0}(w_1, 0) = (1 - \gamma^2)(q_{n12}^{r1r0}(w_1, 0))^2 + \frac{1}{9}.$$

$(r_1, r_1)-$ *Subgame*

Only national brand two is marketed while both store brands are present.

$$q_{n21}^{r1r1}(w_2) = q_{n22}^{r1r1}(w_2) = \frac{-3\varepsilon+(2+\gamma)(1-w_2)}{3(2+\gamma-3\varepsilon^2)}$$

$$q_{s1}^{r1r1}(w_2) = q_{s2}^{r1r1}(w_2) = \frac{1-\varepsilon(1-w_2)}{(2+\gamma-3\varepsilon^2)}$$

Equilibrium outputs are positive as long as $\frac{3\varepsilon}{2+\gamma} < (1 - w_2) < \frac{1}{\varepsilon}$.

Equilibrium retailers' margins are

$$p_{n21}^{r1r1}(w_2) - w_2 = p_{n22}^{r1r1}(w_2) - w_2 = \frac{(1-w_2)}{3}$$

$$p_{s1}^{r1r1}(w_2) = p_{s2}^{r1r1}(w_2) = \frac{3(1-\varepsilon^2)-(1-\gamma)\varepsilon(1-w_2)}{3(2+\gamma-3\varepsilon^2)}$$

For $w_2 = 0$, note that the first order conditions for retailer R_j are $p_{n2}^{r1r1}(0) = q_{n2i}^{r1r1}(0) + \varepsilon q_{si}^{r1r1}(0) = \frac{1}{3}$,

$$p_{si}^{r1r1}(0) = q_{si}^{r1r1}(0) + \varepsilon q_{n1i}^{r1r1}(0).$$ From the former condition we obtain $q_{n2i}^{r1r1}(0) = \frac{1}{3} - \varepsilon q_{si}^{r1r1}(0).$

Then, R_2's and R_1's payoffs read

$R_i^{r1r1}(0) = (1 - \varepsilon^2)(q_{si}^{r1r1}(0))^2 + \frac{1}{9}$, $i = 1, 2$.

$(r_2, r_2)-$ Subgame

Only national brand one is marketed while both store brands are present. The expressions for the equilibrium outputs $q_{n11}^{r2r2}(w_1) = q_{n12}^{r2r2}(w_1)$ and $q_{s1}^{r2r2}(w_1) = q_{s2}^{r2r2}(w_1)$ are the same as for $q_{n21}^{r1r1}(w_2) = q_{n22}^{r1r1}(w_2)$, and $q_{s1}^{r1r1}(w_2) = q_{s2}^{r1r1}(w_2)$, respectively, writing w_1 instead of w_2 (and viceversa). Similarly for margins.

Equilibrium outputs are positive as long as $\frac{3\varepsilon}{2+\gamma} < (1 - w_1) < \frac{1}{\varepsilon}$.

Note that the first order conditions for R_i are $p_{n1}^{r2r2}(w_1) - w_1 = q_{n1i}^{r2r2}(w_1) + \varepsilon q_{si}^{r2r2}(w_1) = \frac{1-w_1}{3}$, and

$p_{si}^{r2r2}(w_1) = q_{si}^{r2r2}(w_1) + \varepsilon q_{n1i}^{r2r2}(w_1)$. From the former we obtain $q_{n1i}^{r2r2}(w_1) = \frac{1-w_1}{3} - \varepsilon q_{si}^{r2r2}(w_1)$.

Then, retailers' payoffs read
$R_i^{r2r2}(w_1) = (1 - \varepsilon^2)(q_{si}^{r2r2}(w_1))^2 + \frac{(1-w_1)^2}{9}$, $i = 1, 2$.

$(r_1, r_2)-$ Subgame

The two national brands are marketed and both retailers also market their store brand. The equilibrium outputs, and retail margins are,

$$q_{ni}^{r1r2}(w_1, w_2) = \frac{1}{2+\gamma+3\varepsilon} - \frac{(2+\gamma)(w_1+w_2)}{2((2+\gamma)^2-9\varepsilon^2)} - \frac{(2-\gamma)(w_i-w_j)}{2((2-\gamma)^2-\varepsilon^2)}, \ i = 1, 2$$

$$q_{si}^{r1r2}(w_1, w_2) = \frac{1}{2+\gamma+3\varepsilon} + \frac{3\varepsilon(w_1+w_2)}{2((2+\gamma)^2-9\varepsilon^2)} + \frac{\varepsilon(w_i-w_j)}{2((2-\gamma)^2-\varepsilon^2)}, \ i = 1, 2$$

$$p_{ni}^{r1r2}(w_1, w_2) - w_i = q_{ni}^{r1r2}(w_1, w_2) + \varepsilon q_{sj}^{r1r2}(w_1, w_2), \ i, j = 1, 2; \ j \neq i$$

$$p_{si}^{r1r2}(w_1, w_2) = q_{si}^{r1r2}(w_1, w_2) + \varepsilon q_{nj}^{r1r2}(w_1, w_2), \ i, j = 1, 2; \ j \neq i$$

And profits:
$R_1^{r1r2} = (q_{n2}^{r1r2})^2 + (q_{s1}^{r1r2})^2 + 2\varepsilon q_{n2}^{r1r2} q_{s1}^{r1r2}$.
$R_2^{r1r2} = (q_{n1}^{r1r2})^2 + (q_{s2}^{r1r2})^2 + 2\varepsilon q_{n1}^{r1r2} q_{s2}^{r1r2}$.

C Proofs

Proof of Result 1.

We use in the proof the following notation: $x_i = 1 - w_i$, $i = 1, 2$; and $A = 4(1 + \gamma) - (5 + 3\gamma)\varepsilon^2 > 0$. The difference $R_1^{r1r0} - R_1^{r2r0}$ can be written as the sum of two terms, one coming from the difference in payoffs for the national brand and the other for the store brand, as follows:

$$R_1^{r1r0} - R_1^{r2r0} = \frac{(x_2-x_1)[-6(1+\gamma)\varepsilon+(4(1+\gamma)+\varepsilon^2)(x_1+x_2)]}{9A} + \frac{(x_2-x_1)[-2(1+\gamma)(2(1+\gamma)-(1+3\gamma)\varepsilon^2)-(1-\gamma)\varepsilon(1+\gamma+\varepsilon^2)(x_1+x_2)]}{3A^2}$$

It is easy to prove that the first term is always positive while the second is positive for a sufficiently large $(x_1 + x_2)$.

Collecting both terms we get that $R_1^{r1r0} - R_1^{r2r0} > 0$ if and only if

$$(x_2 - x_1)[(16(1+\gamma)^2 - (1+\gamma)(13+15\gamma)\varepsilon^2 - 2(1+3\gamma)\varepsilon^4)(x_1 + x_2)$$
$$-36(1+\gamma)^2\varepsilon(1-\varepsilon^2)] > 0$$

Note that the second factor is positive as long as $(x_1+x_2) > \frac{36(1+\gamma)^2\varepsilon(1-\varepsilon^2)}{(16(1+\gamma)^2-(1+\gamma)(13+15\gamma)\varepsilon^2-2(1+3\gamma)\varepsilon^4)}$ where the denominator is positive for all (ε, γ) in their relevant domain. We prove that the above condition is always satisfied since by adding up the conditions for $q_{n21}^{r1r0}(w_1, w_2)$ and $q_{n11}^{r2r0}(w_1, w_2)$ to be positive yields that $(x_1 + x_2) > \frac{3(1+\gamma)\varepsilon}{(1+\gamma+\varepsilon^2)}$. Finally, $\frac{3(1+\gamma)\varepsilon}{(1+\gamma+\varepsilon^2)}$ is greater than $\frac{36(1+\gamma)^2\varepsilon(1-\varepsilon^2)}{(16(1+\gamma)^2-(1+\gamma)(13+15\gamma)\varepsilon^2-2(1+3\gamma)\varepsilon^4)}$ iff $(1 + \gamma - 2\varepsilon^2)A > 0$ which is true for all (ε, γ) in their relevant domain. Then, we conclude that $R_1^{r1r0} - R_1^{r2r0} > 0$ iff $(x_2 - x_1) > 0$.

Proof that x^u is decreasing in ε

The function $F(x^u, \varepsilon) = (1 - \varepsilon^2)(q_{s1}^{r1r0}(x^u, \varepsilon))^2 - (1 - \gamma^2)(q_{n11}^{r0r0}(x^u, \varepsilon))^2 = 0$ implicitly defines a function that relates x^u with ε. We look for the sign of $\frac{dx^u}{d\varepsilon} = -\frac{\frac{\partial F}{\partial \varepsilon}}{\frac{\partial F}{\partial x^u}}$. Thus, we have the following:

a) $\frac{\partial F}{\partial x^u} = 2q_{s1}^{r1r0}\frac{\partial q_{s1}^{r1r0}}{\partial x^u}(1 - \varepsilon^2) - 2(1-\gamma^2)q_{n11}^{r0r0}\frac{\partial q_{n11}^{r0r0}}{\partial x^u} < 0$ since $\frac{\partial q_{s1}^{r1r0}}{\partial x^u} < 0$ and $\frac{\partial q_{n11}^{r0r0}}{\partial x^u} > 0$.

b) $\frac{\partial F}{\partial \varepsilon} = 2q_{s1}^{r1r0}\frac{\partial q_{s1}^{r1r0}}{\partial \varepsilon}(1 - \varepsilon^2) - 2\varepsilon(q_{s1}^{r1r0})^2 - 2(1-\gamma^2)q_{n11}^{r0r0}\frac{\partial q_{n11}^{r0r0}}{\partial \varepsilon}$, but $\frac{\partial q_{n11}^{r0r0}}{\partial \varepsilon} = 0$, then $\frac{\partial F}{\partial \varepsilon} = 2q_{s1}^{r1r0}(\frac{\partial q_{s1}^{r1r0}}{\partial \varepsilon}(1-\varepsilon^2) - \varepsilon q_{s1}^{r1r0})$. There remains to prove that $(\frac{\partial q_{s1}^{r1r0}}{\partial \varepsilon}(1-\varepsilon^2) - \varepsilon q_{s1}^{r1r0}) < 0$ in order to $\frac{dx^u}{d\varepsilon}$ be negative.

Firstly, by using $q_{s1}^{r1r0} = \frac{2(1+\gamma)-(2+\gamma)\varepsilon-\varepsilon x_1}{[4(1+\gamma)-(5+3\gamma)\varepsilon^2]}$ and $\frac{\partial q_{s1}^{r1r0}}{\partial \varepsilon} = \frac{-(2+\gamma+x_1)[4(1+\gamma)-(5+3\gamma)\varepsilon^2]+4(1+\gamma)(5+3\gamma)\varepsilon}{[4(1+\gamma)-(5+3\gamma)\varepsilon^2]^2}$, we have that $(\frac{\partial q_{s1}^{r1r0}}{\partial \varepsilon}(1 - \varepsilon^2) - \varepsilon q_{s1}^{r1r0}) < 0$ when

$$-(2 + \gamma + x_1)[4(1 + \gamma) - (3 + 5\gamma)\varepsilon^2] + 2(1 + \gamma)\varepsilon(2(3 + \gamma) - (5 + 3\gamma)\varepsilon^2) < 0.$$

Secondly, we know two things, that $4(1 + \gamma) - (3 + 5\gamma)\varepsilon^2 > 0$ for all $\varepsilon \in (0, \frac{1+\gamma}{2})$, and that $x_1 > \gamma$ in order to $q_{n11}^{r0r0} > 0$. Thus, the above expression is negative if $-2(1 + \gamma)[4(1 + \gamma) - (3 + 5\gamma)\varepsilon^2] + 2(1 + \gamma)\varepsilon(2(3 + \gamma) - (5 + 3\gamma)\varepsilon^2) < 0$, or equivalently when $-4(1 + \gamma) + 2(1 - \gamma)\varepsilon + (5 + 3\gamma)\varepsilon^2 < 0$.

Solving for ε, it happens that for $\varepsilon \in [\frac{\gamma-1-\sqrt{21+30\gamma+13\gamma^2}}{5+3\gamma}, \frac{\gamma-1+\sqrt{21+30\gamma+13\gamma^2}}{5+3\gamma}]$ the expression is negative.
Since $\frac{\gamma-1-\sqrt{21+30\gamma+13\gamma^2}}{5+3\gamma} < 0 < \varepsilon < \frac{1+\gamma}{2} < \frac{\gamma-1+\sqrt{21+30\gamma+13\gamma^2}}{5+3\gamma}$.
Then we conclude that $\frac{dx^u}{d\varepsilon} < 0$.

Proof of Lemma 1: The sign of $R_2^{r1r2} - R_2^{r1r1}$ is given by the sign of $(1 - \gamma)K(\gamma, \varepsilon)$ where

$$K(\gamma, \varepsilon) = 20 + 24\gamma + 9\gamma^2 + \gamma^3 - 6(2 - \gamma - \gamma^2)\varepsilon - (69 + 33\gamma + 6\gamma^2)\varepsilon^2 + 18(1 - \gamma)\varepsilon^3 + 54\varepsilon^4.$$

Take the first partial derivative with respect to ε, which is given by

$$\frac{\partial K}{\partial \varepsilon} = -6[2 - \gamma - \gamma^2 + (23 + 11\gamma + 2\gamma^2)\varepsilon - (9 - \gamma)\varepsilon^2 - 36\varepsilon^3]$$

It is negative if and only if $[2 - \gamma - \gamma^2 + (23 + 11\gamma + 2\gamma^2)\varepsilon - (9 - \gamma)\varepsilon^2 - 36\varepsilon^3]$ is positive. Note that $[2 - \gamma - \gamma^2 + (23 + 11\gamma + 2\gamma^2)\varepsilon - (9 - \gamma)\varepsilon^2 - 36\varepsilon^3] > \varepsilon[(23 + 11\gamma + 2\gamma^2) - (9 - \gamma)\varepsilon - 36\varepsilon^2]$. The polynomial $(23 + 11\gamma + 2\gamma^2) - (9 - \gamma)\varepsilon - 36\varepsilon^2$ is positive as long as ε takes on values between the roots. It can be checked that the smallest root is negative and that the upper bound on ε is smaller than the greatest root. Therefore the polynomial is positive for all $\varepsilon \in (0, \frac{1+\gamma}{2})$ and it follows that $\frac{\partial K}{\partial \varepsilon}$ is negative.

Since $\frac{\partial K}{\partial \varepsilon}$ is negative, it suffices to prove that $K(\gamma, \varepsilon)$ is positive when evaluated at $\varepsilon = \frac{1+\gamma}{2}$. It happens that $K(\gamma, \frac{1+\gamma}{2}) = \frac{1}{8}(19 - 30\gamma + 14\gamma^3 - 3\gamma^4)$ which is positive $\forall$ $\gamma \in (0, 1)$.