A discusión

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ABSTRACT

We analyze the roles of the government and the incumbent in preventing piracy, and the reasons and incentives why a pirate would want to be a leader in prices. The framework of analysis used is a duopoly model of vertical product differentiation with price competition, where both incumbent and pirate are committed to keep their prices. We find that both government and incumbent have a key role in avoiding the entry of the pirate. We show that the government will not help the incumbent to become a monopolist, even if he installs an antipiracy system, because a monopoly provides the lowest social welfare. However, he will let the pirate enters as a follower or as a leader, or encourage the incumbent to deter the entry of the pirate, which depends on the technology of the government for monitoring piracy. The pirate decides to become a leader to avoid being brought down by the incumbent and the government, although the leader's profit is lower than the follower's profit. Finally, we find that high-income countries with cheaper monitoring technology have lower piracy rates.

JEL classification: K42; L13; L86

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1 Introduction

Over the past few years, most digital products have frequently been illegally copied and sold, to the point where it is possible to find a new product pirated before it is officially launched on the market. We point out two news items taken from Spanish newspaper *EL PAIS*: "New García Márquez Novel Pirated In Colombia Before Its Presentation"(2004b) and, "Pirated Version Of Xbox's Star Game For Christmas Appears On Internet"(2004a).

The importance of piracy is reflected in the studies of piracy, which show that levels of piracy rates are high but vary across regions. In particular, the 2005 Global Software Piracy Study reveals that piracy rates in 2004 were 22% in North America, 35% in the European Union, 61% in the Eastern Europe and 66% in Latin America. The differences between individual countries are even bigger. The piracy rate in 2004 was 21% in the United States, 23% in New Zealand, 91% in Ukraine and 92% in Vietnam. These studies also show the industry losses from piracy by region, highlighting the large losses in North America and European Union because markets there are so large, even though these regions had relatively low piracy rates. According to the 2005 Global Software Piracy Study, losses in 2004 (expressed in millions of dollars) totalled 1546 in Latin America, 2313 in Eastern Europe, 7549 in North America and 12151 in the EU.

In this study and others, the industry losses are represented by the value of pirated software at original prices. This approach implicitly assumes that legal and pirated copies are perfect substitutes, which would imply that, if the purchase of an illegal copy were not available, this would imply the purchase of an original product, i.e. piracy has a negative impact on sales of digital products.

Moreover, there are contradictory verdicts from different courts in this regard. In particular, one of the reasons why the courts ruled that Napster harmed the music industry was the loss of sales of CDs (see Peitz and Waelbroeck (2005)). However, a judge in Alicante (Spain) ruled that commercial piracy harmed neither intellectual property nor the record company's sales of CDs and DVDs (see ruling JUR 2005\240478).

There are many empirical academic studies that seek the relationship between software piracy and consumer income. For example, Marron and Steel (2000) and Rodríguez-Andrés (2006) find that high-income countries have lower piracy rates. Bezmen and Depken II (2006), finds a negative relationship between software piracy and income for various states in the United States.

Piracy has previously been analyzed from a theoretical viewpoint. Some papers have focused on the case of copies made exclusively by end consumers (Johnson (1985), Shy and Thisse (1999) and Bae and Choi (2006)),[1] and others on cases of copies made by a single firm that sells them on the market (Banerjee (2003) and Poddar (2003)). Johnson (1985) supports traditional common wisdom, i.e. that consumers' surplus and social surplus can fall as revenue losses induced by copying reduce the number of creative works, and that this is more likely the greater the supply elasticity and the greater the value consumers

[1] See Peitz and Waelbroeck (2006) for a survey of piracy in which copies are made exclusively by end consumers.

place on product variety. Nevertheless, recent works question such traditional wisdom and maintain that making creators extremely wealthy is a consequence that is welcome to them, but unnecessary to society (Boldrin and Levine (2006)).

Our work is close to that of Banerjee (2003), who considers a monopolist selling original software and a government monitoring and penalizing the pirate. However, that paper commits errors in solving the model which we avoid here (see López-Cuñat and Martínez-Sánchez (2006)). In Banerjee (2003) the incumbent does not consider correctly the consequences of the anticipated pirate's decisions about entry, in particular he does not take into account the setting *deterred entry*, according to the taxonomy of Bain (1956).

In this paper, we develop a duopoly model of vertical product differentiation with price competition to analyze the problem of entry of a pirate in the market, where it is possible that the pirate becomes a leader in prices. We consider that the timing of the game is exogenous,[2] where the pirate has the advantage of deciding first whether to enter or not, because the illegality of the pirate lets him a higher flexibility of movement than the incumbent. Thus, it is necessary that the leader (the incumbent or the pirate) is committed to keep his price.[3] The incumbent's commitment is due to he want to keep his reputation, but the pirate's commitment is due to his illegal feature, which clear him of any responsibility.

We consider that the firms do not compete on quality because we focus on a short run analysis. Thus, we assume that the cost incurred by the incumbent in developing a product is a sunk cost and the production cost of both the incumbent and the pirate is zero, like Wauthy (1996) and Banerjee (2003), but unlike Ronnen (1991), Motta (1993) and Crampes and Hollander (1995). Ronnen (1991) considers firms face quality-dependent fixed cost and compete on quality and price. He finds that even in the absence of externalities an appropriately chosen standard improves social welfare. Nevertheless, Crampes and Hollander (1995) shows that, when the quality-dependent cost is variable, consumers' welfare increases if the firm producing the higher quality does not increase its quality significantly in response to increase in quality by its rival. Motta (1993) considers both kinds of cost and shows that firms differentiate more under Bertrand than under Cournot, and that social welfare is greater when firms compete on prices and also when firms bear fixed costs independently of the kind of competition.

Our analysis shows that monopoly provides the lowest social welfare level, so in equilibrium the government will not make the efforts necessary to blockade the entry of the pirate, even if the incumbent installs an antipiracy system. Nevertheless, in equilibrium, the government will decide to let the pirate enter as a follower (f-outcome), to let the pirate become the leader in prices (l-outcome) or to encourage the incumbent to set a low enough price to successfully deter the entry of the pirate (deterred entry or ne-outcome), which depends on the technology of the government to monitor the piracy, which is represented by the monitoring cost of piracy. These results are according to the taxonomy of Bain (1956).

[2] Hamilton and Slutsky (1990) studies duopoly games where the timing of the game is endogenous.

[3] From a theoretical viewpoint the leader (the incumbent or the pirate) is committed to keep his price because the leader's profit is higher than Bertrand's profit, where Bertrand is the outcome when there is no commitment.

Thus we show that the great variation in piracy rates from one country to another is a consequence of different technologies for monitoring piracy by governments. And we also find that high-income countries with cheaper monitoring technology have lower piracy rates.

An interesting result is that the government will never protect the incumbent fully but only partially. This is because when there is no piracy, the incumbent abuses his dominant position since he sets high prices, and the fact is that although the pirate is engaged in an illegal activity, he helps to moderate the unfair behavior of the incumbent. In short, copyright owner's efforts to reduce piracy should focus not only on the government enforcement but also on pricing strategies. This statement is empirically supported by Papadopoulos (2003).

Finally, according this paper, the decrease of the sales of CDs is not a result of piracy, so industry losses from piracy are lower that claimed by the industry itself, by most studies and by some court rulings.

The rest of the paper is organized as follows. Section 2 describes the model formally. Section 3 considers market equilibrium. Section 4 considers the optimal policy. Section 5 analyzes and compares the results of the model with those of the current theoretical and empirical literature. Section 6 extends the model to the case where the incumbent has the opportunity to decide first whether to enter or not. Section 7 considers the possibility of the incumbent's installing an antipiracy system. Finally, Section 8 concludes.

2 The model

We consider four types of agent: consumers, the developer of an original product (incumbent), a pirate who illegally reproduces and sells it, and the government which is responsible for monitoring and penalizing the pirate.

There is a continuum of consumers indexed by θ, $\theta \in \left[0, \overline{\theta}\right]$.[4] θ is assumed to follow an uniform distribution, and represents the consumers' tastes for the quality of the product. Each consumer is assumed to buy only one unit of the good or not to buy.

The utility of a type θ consumer, following Mussa and Rosen (1978), is,

$$U(\theta) = \begin{cases} \theta q_i - p_i & \text{if the consumer buys the original product} \\ \theta q_p - p_p & \text{if the consumer buys the pirated product} \\ 0 & \text{if the consumer does not buy} \end{cases} \tag{1}$$

where p_i, q_i, p_p and q_p are the price and quality of the original and the pirated product, respectively. We assume $q_i > q_p > 0$.

Let $x_i = p_i/q_i$ and $x_p = p_p/q_p$ be the incumbent's and pirate's hedonic prices, respectively. Since qualities are common knowledge, decisions on prices are equivalent to decisions on hedonic prices. Let $r = q_i/q_p > 1$ be the ratio of qualities. Without loss of generality, we also assume that $x_i, x_p \in \left[0, \overline{\theta}\right]$.

[4] We consider that the market is uncovered, ie. there is always at least one consumer who does not buy at all.

Firms' demand functions are obtained as follows. Let θ_o be a consumer who is indifferent between buying the original and the pirated product. From (1), $\theta_o = (rx_i - x_p) / (r-1)$. Let θ_i be a consumer indifferent between buying from the incumbent and not buying at all, that is, $\theta_i = x_i$. Let θ_p be a consumer indifferent between buying from the pirate and not buying at all, that is, $\theta_p = x_p$.

The demands faced by the incumbent and the pirate are

$$D_i(x_i, x_p) = \begin{cases} \overline{\theta} - \theta_i & \text{if } x_i \leq x_p \\ \overline{\theta} - \min\{\theta_o, \overline{\theta}\} & \text{if } x_i \geq x_p \end{cases} \tag{2}$$

$$D_p(x_i, x_p) = \begin{cases} 0 & \text{if } x_i \leq x_p \\ \min\{\theta_o, \overline{\theta}\} - \theta_p & \text{if } x_i \geq x_p \end{cases} \tag{3}$$

We assume that consumers do not face the risk of prosecution for the use of copies because they did not illegally copy and sell the original product.[5] The government is responsible for monitoring and penalizing the pirate. Let α and G be the monitoring rate and the penalty. Thus, α is the probability of detecting the pirate. We assume $0 \leq G \leq \overline{G}$, where $\overline{G}$ is the maximum legal penalty. Let $C(\alpha)$ be the cost of monitoring piracy. We assume $C(0) = 0, C'(\alpha) > 0$.

We assume that a firm remains in the market if and only if it is making positive profit. If the pirate's illegal operations are detected, which occurs with probability α, he must pay the penalty G and he loses his income. So the expected profits of the incumbent and the pirate, taking into account that detection takes place after sale, are

$$\pi_i(.) = q_i x_i D_i(x_i, x_p), \quad \pi_p(.) = (1-\alpha)\, q_p x_p D_p(x_i, x_p) - \alpha G. \tag{4}$$

We consider that the cost incurred by the incumbent in developing an original product is a sunk cost and the production costs of both the incumbent and the pirate are zero (as in Wauthy (1996) and Banerjee (2003), but unlike Ronnen (1991), Motta (1993) and Crampes and Hollander (1995)).

Let $\alpha G + \alpha\delta I_p(x_i, x_p) - C(\alpha)$ be the net expected revenue of the government, where $I_p(x_i, x_p) = q_p x_p D_p(x_i, x_p)$ represents the pirate's revenue and $\delta \in [0, 1]$ represents the government's ability to reuse the revenue seized from the pirate. The government chooses α and G to maximize the social welfare, which is the sum of the profits both the incumbent and the pirate, the consumer surplus and the net expected revenue of the government.

As can be seen in Figure 1, the complete information game is the following. The government announces α and G to maximize social welfare, and both firms observe the policy variables. Then the pirate decides whether to price first or not. If he decides to price first he becomes the leader on prices, so the incumbent prices the original product taking into account the pirate's price (l-subgame). But if the pirate decides to wait, he becomes a follower on prices that decides whether to enter or not after the incumbent sets the price of the original product (f-subgame).[6] Finally, the consumers decide to buy the original product, the pirated product or neither after they have observed firms' prices.

[5] Which is true for the penal codes of most countries (e.g., see articles 270 to 272 of the Spanish penal code).

[6] We do not consider simultaneous decisions as a pirate's strategy because this strategy is ruled out by the possibility of being the leader.

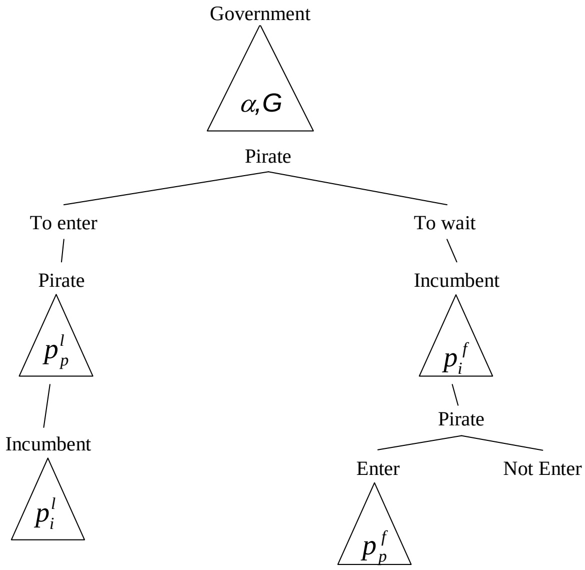

Figure 1: The timing of the game

In the two next sections, we seek to find the subgame perfect equilibrium (SPE) of the game by backward induction. In the following section, we solve the f-subgame and the l-subgame. Next, we look for the decision of the pirate on whether to be the leader. In Section 4, we look for the government's optimal policy (α, G) anticipating the equilibrium of the continuation game.

3 Market Equilibrium

3.1 F-subgame

In this subgame the pirate decides to wait, so he becomes a follower on prices. The pirate's optimal hedonic price, given the incumbent's choice, is obtained by maximizing the pirate's profit. It is similar to the one computed by Ronnen (1991):

$$x_p^{BR}(x_i) = \begin{cases} x_i/2 & \text{if } 0 \leq x_i \leq \frac{2\overline{\theta}(r-1)}{2r-1} \\ rx_i - (r-1)\overline{\theta} & \text{if } \frac{2\overline{\theta}(r-1)}{2r-1} \leq x_i \leq \frac{\overline{\theta}(2r-1)}{2r} \\ \overline{\theta}/2 & \text{if } \frac{\overline{\theta}(2r-1)}{2r} \leq x_i \leq \overline{\theta} \end{cases} \tag{5}$$

By substituting (5) in the pirate's profit, we obtain the pirate's maximum profit $\pi_p^c(x_i) = (1-\alpha)\, q_i \gamma(x_i) - \alpha G$, where

$$\gamma(x_i) = \begin{cases} \frac{x_i^2}{4(r-1)} & \text{if } 0 \leq x_i \leq \frac{2\overline{\theta}(r-1)}{2r-1} \\ \left(rx_i - (r-1)\overline{\theta}\right)\left(\overline{\theta} - x_i\right) & \text{if } \frac{2\overline{\theta}(r-1)}{2r-1} \leq x_i \leq \frac{\overline{\theta}(2r-1)}{2r} \\ \frac{\overline{\theta}^2}{4r} & \text{if } \frac{\overline{\theta}(2r-1)}{2r} \leq x_i \leq \overline{\theta} \end{cases} \tag{6}$$

The pirate decides to enter the market when $\pi_p^c(x_i) > 0$, i.e. when $\gamma(x_i) > g$, where

$$g = \frac{\alpha G}{q_i(1-\alpha)}, \tag{7}$$

which is increasing in α and G, and indicates the government's effort to avoid piracy. Notice that $\gamma(x_i) > g$ is equivalent to $x_i > x_i^{ne}$, where x_i^{ne} is the no-entry hedonic price, which is[7]

$$x_i^{ne} = \begin{cases} \sqrt{4(r-1)g} & \text{if } 0 \leq g \leq \frac{\overline{\theta}^2(r-1)}{(2r-1)^2} \\ \frac{\overline{\theta}(2r-1)-\sqrt{\overline{\theta}^2-4rg}}{2r} & \text{if } \frac{\overline{\theta}^2(r-1)}{(2r-1)^2} \leq g \leq \frac{\overline{\theta}^2}{4r} \\ +\infty & \text{if } \frac{\overline{\theta}^2}{4r} < g \end{cases} \tag{8}$$

Therefore, the pirate's optimal decision is to enter and price $x_p^{BR}(x_i)$ if $x_i > x_i^{ne}$; and to not enter if $x_i \leq x_i^{ne}$. According to the pirate's optimal decision the incumbent anticipates profits

$$\pi_i^c(x_i) = \begin{cases} q_i x_i\left(\overline{\theta} - x_i\right) & \text{if } 0 \leq x_i \leq x_i^{ne} \\ q_i x_i D_i\left(x_i, x_p^{BR}(x_i)\right) & \text{if } x_i^{ne} < x_i \leq \overline{\theta} \end{cases} \tag{9}$$

From maximizing the incumbent's profit in equation (9), we obtain the following values that are relevant for the analysis:

$$\begin{array}{lllll} x_i^f = \frac{\overline{\theta}(r-1)}{2r-1}, & \pi_i^f = \frac{\overline{\theta}^2 q_i(r-1)}{2(2r-1)}, & x_p^f = \frac{\overline{\theta}(r-1)}{2(2r-1)}, & I_p^f = \frac{\overline{\theta}^2 q_i(r-1)}{4(2r-1)^2}, & \theta_o^f = \frac{\overline{\theta}}{2}, \\ x_i^m = \frac{\overline{\theta}}{2}, & \pi_i^m = \frac{\overline{\theta}^2 q_i}{4}, & & & \end{array} \tag{10}$$

where I_p^f is the pirate's revenue when he is the follower. We find that when the government makes little effort to combat piracy (g very low), the pirate enters as a follower and price x_p^f, and when the government makes a major effort (g very high), the entry of the pirate is blockaded, so the incumbent becomes a monopolist that prices at a monopoly price of x_i^m. However, for intermediate levels of government effort, the incumbent finds optimal to set a low enough price (x_i^{ne}) to avoid piracy. These results are summarized in the following proposition.

Proposition 1 *In any SPE, the optimal strategies of the incumbent and the pirate are:*

(a) The pirate will enter the market only if $x_i > x_i^{ne}$ and will price according to (5).

(b) The incumbent will price $x_i^* = x_i^f$ and the pirate will price $x_p^* = x_p^f$ if $g < g_l$, where

$$g_l = \frac{\overline{\theta}^2\left(r - \sqrt{2r-1}\right)}{8(r-1)(2r-1)}. \tag{11}$$

(c) The incumbent will price $x_i^* = x_i^{ne}$ if $g_l \leq g < g_m$, where

$$g_m = \begin{cases} \frac{\overline{\theta}^2(2-r)}{4} & \text{if } 1 < r \leq 3/2 \\ \frac{\overline{\theta}^2}{16(r-1)} & \text{if } 3/2 \leq r. \end{cases} \tag{12}$$

[7] We assume x_i^{ne} is equal to $+\infty$ when $\overline{\theta}/4r < g$ for convenience of analysis only. This means that the pirate is deterred from entering for any price when the government's effort is very high.

(d) The incumbent will price $x_i^* = x_i^m$ if $g_m \leq g$.

Proof: see Appendix A.

Notice that there is no piracy when $g_l \leq g$. When $g_m \leq g$, piracy is only eliminated because of high expenditure by the government in preventing it, so the incumbent can set a monopoly price. However, when $g_l \leq g < g_m$, government intervention must be accompanied by the incumbent setting a low enough price, so the incumbent shares with the government the cost of eliminating piracy.

From equation (12) we get that the effort by the government that leads to monopoly is decreasing in r, which means that blockading the entry of the pirate is easier when the differentiation is higher.

3.2 L-subgame

The l-subgame is reached when the pirate prices first and thus becomes the leader on prices. The incumbent's optimal hedonic price, given the pirate's choice, is obtained by maximizing the incumbent's profit. It is similar to the one computed by Ronnen (1991):

$$x_i^{BR}(x_p) = \begin{cases} \left(\overline{\theta}\,(r-1) + x_p\right)/2r & \text{if } 0 \leq x_p \leq \frac{\overline{\theta}(r-1)}{2r-1} \\ x_p & \text{if } \frac{\overline{\theta}(r-1)}{2r-1} \leq x_p \leq \overline{\theta}/2 \\ \overline{\theta}/2 & \text{if } \overline{\theta}/2 \leq x_p \leq \overline{\theta} \end{cases} \tag{13}$$

The pirate incorporates the incumbent's reaction function into his profit function and chooses the price that maximizes his profit, yielding the following hedonic prices, indifferent consumers and profits:

$$x_i^l = \tfrac{\overline{\theta}(r-1)(4r-1)}{4r(2r-1)}, \quad \pi_i^l = \tfrac{\overline{\theta}^2 q_i(r-1)(4r-1)^2}{16r(2r-1)^2}, \quad x_p^l = \tfrac{\overline{\theta}(r-1)}{2(2r-1)}, \quad I_p^l = \tfrac{\overline{\theta}^2 q_i(r-1)}{8r(2r-1)}, \quad \theta_o^l = \tfrac{\overline{\theta}(4r-3)}{4(2r-1)}, \tag{14}$$

where I_p^l is the pirate's revenue when he is the leader. Since the incumbent's profit is not negative he always enters the market. Notice that the pirate's profit as leader ($\pi_p^l = (1-\alpha)\,I_p^l - \alpha G$) is positive if and only if $g < I_p^l/q_i = g_0$.

3.3 Pirate: leader or follower

In this subsection we analyze the pirate's optimal decision about when to enter the market. From results obtained in each subgame, we get that if the pirate waits he anticipates profit of $\pi_p^F = (1-\alpha)I_p^f - \alpha G > 0$ when $g < g_l$, and $\pi_p^F = 0$ when $g_l \leq g$. But if the pirate prices first he anticipates profit of $\pi_p^L = (1-\alpha)I_p^l - \alpha G$, which is positive if and only if $g < g_0$.

Since $g_l < g_0$, to obtain the pirate's optimal decision we have to compare π_p^F with π_p^L in the three regions given by $g < g_l$, $g_l \leq g < g_0$, and $g_0 \leq g$.

For $g < g_l$, we have $\pi_p^F = (1-\alpha)I_p^f - \alpha G$, and $\pi_p^L = (1-\alpha)I_p^l - \alpha G$. Since $I_p^f > I_p^l$, the pirate decides to wait to price the copy until after the incumbent prices the original product.

For $g_l \leq g < g_0$, we have $\pi_p^F = 0$ and $\pi_p^L > 0$. Since $\pi_p^L > \pi_p^F$, the pirate prices the copy before the incumbent prices the original product.

For $g = g_0$, we have $\pi_p^F = 0$ and $\pi_p^L = 0$. To ensure the existence of an equilibrium it is necessary the pirate becomes a follower that will later not enter.

For $g_0 < g$, we have $\pi_p^F = 0$ and $\pi_p^L < 0$. So the pirate decides to wait and becomes a follower that will not enter the market.

As we can see, the pirate's optimal decision, like the incumbent's optimal decision, depends on the level of expenditure by the government on avoiding piracy. When $g < g_l$, the pirate waits until the incumbent prices the original product since his profit is higher as a follower. However, when $g_l \leq g < g_0$, he prices first because he anticipates a profit of zero as a follower, since the incumbent deters him from entering the market through prices, and a positive profit as a leader, since when he prices first he restricts himself to force the incumbent to not deter him. The following proposition shows the outcomes that arise as a result of pirate's optimal decision:

Proposition 2 *In any SPE,*

(a) The pirate will wait and price the pirated product as a follower $x_p^* = x_p^f$, when $g < g_l$. So the incumbent becomes the leader and prices $x_i^* = x_i^f$.

(b) The pirate will become the leader and price $x_p^* = x_p^l$, when $g_l \leq g < g_0$. So the incumbent becomes the follower and prices $x_i^* = x_i^l$.

(c) The pirate becomes a follower that will later not enter, when $g_0 \leq g$. So the incumbent becomes a monopolist that prices $x_i^* = x_i^{ne}$ when $g_0 \leq g < g_m$, and $x_i^* = x_i^m$ when $g_m \leq g$.

The government's optimal policy is analyzed in the next section.

4 Optimal Policy: analysis of social welfare

The government chooses the optimal policy that maximizes social welfare anticipating the equilibrium of the continuation game. Social welfare is the sum of the profits of the incumbent and the pirate, the consumer surplus and the net expected revenue of the government. Thus, the social welfare of continuation in SPE is:

$$W = \begin{cases} CS^f + \pi_i^f + I_p^f - \alpha\,(1-\delta)\,I_p^f - C\,(\alpha) & \text{if } 0 \leq g < g_l, \\ CS^l + \pi_i^l + I_p^l - \alpha\,(1-\delta)\,I_p^l - C\,(\alpha) & \text{if } g_l \leq g < g_0, \\ CS^{ne} + \pi_i^{ne} - C\,(\alpha) & \text{if } g_0 \leq g < g_m, \\ CS^m + \pi_i^m - C\,(\alpha) & \text{if } g_m \leq g \end{cases} \tag{15}$$

where CS^k is the consumer's surplus in the four outcomes $k \in \{f, l, ne, m\}$ listed in the previous section. The expression of consumer's surplus and his value in the previous outcomes is

$$CS = \int_{\theta_p}^{\theta_o} (\theta q_p - p_p)\, d\theta + \int_{\theta_o}^{\overline{\theta}} (\theta q_i - p_i)\, d\theta \tag{16}$$

$$CS^f = \frac{\overline{\theta}^2 q_i\left(4r^2+r-1\right)}{8(2r-1)^2}; \quad CS^l = \frac{q_i\overline{\theta}^2\left(16r^3+12r^2-15r+3\right)}{32r(2r-1)^2}; \quad CS^{ne} = \frac{q_i\left(\overline{\theta}-x_i^{ne}\right)^2}{2}; \quad CS^m = \frac{\overline{\theta}^2 q_i}{8}. \tag{17}$$

Given that a higher monitoring rate entails a higher cost, $\alpha \equiv \frac{q_i g_l}{q_i g_l + G}$ is decreasing in G and a higher penalty does not entail a higher cost, the government will choose the maximum penalty, which is $\overline{G}$. Notice that social welfare is decreasing in α on the intervals $[0, g_l), [g_l, g_0), [g_0, g_m)$ and $[g_m, +\infty)$, since (i) the values CS^k,$\pi_i^k, I_p^k, k \in \{f, l, m\}$ are independent of α; (ii) the sum $CS^{ne}+\pi_i^{ne} = q_i\left(\overline{\theta}^2 - \left(x_i^{ne}\right)^2\right)/2$ is decreasing in g because x_i^{ne} is increasing in g and $g \equiv \alpha G/q_i\left(1-\alpha\right)$ is increasing in α; and (iii) the monitoring cost of piracy is increasing, $C'\left(\alpha\right) > 0$. So in order to maximize social welfare the government will choose the minimum monitoring rate that lead to different outcomes, which is $\alpha \in \{\alpha_f, \alpha_l, \alpha_{ne}, \alpha_m\}$, where $\alpha_f = 0, \alpha_l = \frac{q_i g_l}{q_i g_l + \overline{G}}, \alpha_{ne} = \frac{q_i g_0}{q_i g_0 + \overline{G}}$ and $\alpha_m = \frac{q_i g_m}{q_i g_m + \overline{G}}$. As a result, since g is increasing in α, social welfare is decreasing in g, so the value of g in the social maximum is reached in $\{0, g_l, g_0, g_m\}$. Since $0 < g_l < g_0 < g_m$, we have $\alpha_f < \alpha_l < \alpha_{ne} < \alpha_m$.

The maximum social welfare is obtained from comparing the following values:

$$\begin{aligned}
&W^f = CS^f + \pi_i^f + I_p^f, \\
&W^l = CS^l + \pi_i^l + I_p^l - \alpha_l\left(1-\delta\right) I_p^l - C\left(\alpha_l\right), \\
&W_0^{ne} = CS_0^{ne} + \pi_{i0}^{ne} - C\left(\alpha_{ne}\right), \\
&W^m = CS^m + \pi_i^m - C\left(\alpha_m\right).
\end{aligned} \tag{18}$$

where $x_{i0}^{ne}, \pi_{i0}^{ne}$ and CS_0^{ne} are the hedonic price, the incumbent's profit and the consumer's surplus at $g = g_0$, respectively, which are:

$$x_{i0}^{ne} = \sqrt{\frac{\overline{\theta}^2(r-1)^2}{2r(2r-1)}}; \quad \pi_{i0}^{ne} = \sqrt{\frac{\overline{\theta}^4 q_i^2(r-1)^2}{2r(2r-1)}} - \frac{\overline{\theta}^2 q_i(r-1)^2}{2r(2r-1)}; \quad CS_0^{ne} + \pi_{i0}^{ne} = \frac{\overline{\theta}^2 q_i\left(3r^2-1\right)}{4r(2r-1)}. \tag{19}$$

Let $\widehat{W}^k = CS^k + \pi_i^k + I_p^k$ be the gross social welfare in outcomes $k \in \{f, l, ne, m\}$. Let $\Delta\widehat{W}_y^x = \widehat{W}^x - \widehat{W}^y$ be the gain in gross social welfare that outcome x generates as regards outcome y. Let $C_l = C\left(\alpha_l\right) + \left(1-\delta\right)\alpha_l I_p^l$ be the social cost that the government supports when the pirate is a leader (l-outcome), where the first term is the cost of monitoring piracy and the second is the expected money loss of the revenue seized from the pirate. For simplicity we call $C_{ne} = C\left(\alpha_{ne}\right)$. The value of the gross social welfare in each outcome and the relationship between them is as follows:

$$\widehat{W}^f = \frac{\overline{\theta}^2 q_i\left(12r^2-9r+1\right)}{8(2r-1)^2}; \quad \widehat{W}^l = \frac{\overline{\theta}^2 q_i\left(48r^3-28r^2-9r+5\right)}{32r(2r-1)^2}; \quad \widehat{W}_0^{ne} = \frac{\overline{\theta}^2 q_i\left(3r^2-1\right)}{4r(2r-1)}; \quad \widehat{W}^m = \frac{3\overline{\theta}^2 q_i}{8} \tag{20}$$

$$\widehat{W}^m < \widehat{W}^f < \widehat{W}^l < \widehat{W}_0^{ne} \tag{21}$$

From (21) and given that $C'\left(\alpha\right) > 0$, we deduce that the monopoly (with no restriction in prices) provides the lowest social welfare due to excessive power of the incumbent in the market. Thus, the government never chooses α_m, and the monopoly is not part of the equilibrium.

As can be seen from Figure 2 and the following proposition, the outcome that maximizes social welfare depends on the relationship between gross social welfare and the cost of monitoring piracy at

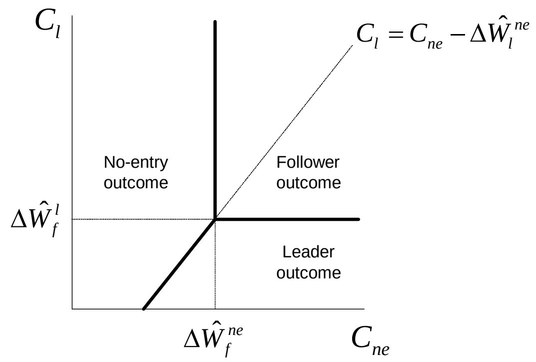

Figure 2: The Government's optimal policy

every outcome. In particular: encouraging the incumbent to set a low enough price for the pirate not to enter the market maximizes the social welfare if the cost of monitoring piracy at this outcome is low enough as regards the sum of the gain of gross social welfare that it generates and the cost of monitoring piracy at the other outcomes (f- and l-outcomes); letting the pirate be the leader maximizes the social welfare if the cost of monitoring piracy at this outcome is low enough as regards the sum of the gain of gross social welfare that it generates and the cost of monitoring piracy at the other outcomes (f- and ne-outcome); and otherwise, letting the pirate enter as a follower maximizes the social welfare, i.e. when the cost of monitoring piracy at l- and ne-outcomes is high.

Proposition 3 *In any SPE, the optimal strategy of the government is:*

(a) $g = 0$, *if* $C_{ne} > \Delta \widehat{W}_f^{ne}$ *and* $C_l > \Delta \widehat{W}_f^l$.

(b) $g = g_l$, *if* $C_{ne} > C_l + \Delta \widehat{W}_l^{ne}$ *and* $C_l < \Delta \widehat{W}_f^l$.

(c) $g = g_0$, *if* $C_{ne} < \Delta \widehat{W}_f^{ne}$ *and* $C_{ne} < C_l + \Delta \widehat{W}_l^{ne}$.

To obtain that the l-outcome is a social maximum it is necessary that the government's ability to reuse the revenue seized from the pirate is high enough (i.e. δ is high enough). This is because the social welfare when the pirate is a follower can be greater than when he is the leader on prices, independently of the value of cost of monitoring piracy. Which happens when $\widehat{W}^l - \alpha_l (1-\delta) I_p^l < \widehat{W}^f$, because the government can reuse little of the revenue seized from the pirate. In this case the maximum social welfare will be letting the pirate enter as a follower or deterring him from entering, which depends on the monitoring cost C_{ne}: for a low enough C_{ne}, deterring entry by the pirate is socially optimal, otherwise the social optimum choice is to let the pirate enter as a follower.

As a numerical example, we illustrate the government's optimal policy when $q_i = 2$, $q_p = 1$, $\overline{\theta} = 1$ and $\delta = 1$. In this case, we have $r = 2$, $\widehat{W}^f = 0.8611$, $\widehat{W}^l = 0.8993$, $\widehat{W}^{ne} = 0.9167$ and $\widehat{W}^m = 0.75$. When $C(\alpha_l) = 0.069$ and $C(\alpha_{ne}) = 0.07$, social welfare in each outcome is $W^f = 0.8611$, $W^l = 0.8303$

and $W^{ne} = 0.8467$, so the government decides $g = 0$. But when $C(\alpha_l) = 0.01$ and $C(\alpha_{ne}) = 0.05$, social welfare in each outcome is $W^f = 0.8611$, $W^l = 0.8893$ and $W^{ne} = 0.8667$, so the government decides $g = g_l$. And when $C(\alpha_l) = 0.049$ and $C(\alpha_{ne}) = 0.05$, social welfare in each outcome is $W^f = 0.8611$, $W^l = 0.8503$ and $W^{ne} = 0.8667$, so the government decides $g = g_0$.

We can check $x_{i0}^{ne} < x_i^l < x_i^f < x_i^m$,[8] which means that the incumbent sets lowest price in *ne*-outcome. This fact and the fact that gross social welfare in *ne*-outcome is highest (see relationship (21)) help to explain why, when the monitoring cost is low enough, the government would rather have only the incumbent producing and selling the product at its lowest price than have the product sold in different qualities and at different prices by the incumbent and the pirate.

Notice that the government incurs a budgetary deficit to reach the outcome where the incumbent deters the entry of the pirate through prices. This deficit comes about because the government incurs a monitoring cost and does not obtain revenue because the pirate does not enter the market. This result suggests it is sometimes necessary incurs a budgetary deficit to reach the socially optimum outcome, which is not took into account by Banerjee (2003) because he assumes the budget is balanced.

So far we have included the pirate's profit in social welfare because he is an agent that generates revenue and helps to moderate the incumbent's abuse of his dominant position.[9] However, we may decide not to include the pirate on ethical and moral grounds. In that case, the results obtained in this section are held if the marginal monitoring cost of piracy is high enough as regards the maximum penalty and the revenue used from the revenue seized from the pirate. See Appendix C for more details.

5 Analysis of equilibrium

5.1 Analysis of demands

The value of demands in every outcome and the relationship between them are the following:

$$D_i^f = \tfrac{\overline{\theta}}{2}; \quad D_p^f = \tfrac{\overline{\theta} r}{2(2r-1)}; \quad D_i^l = \tfrac{\overline{\theta}(4r-1)}{4(2r-1)}; \quad D_p^l = \tfrac{\overline{\theta}}{4}; \quad D_i^{ne} = \overline{\theta} - x_{i0}^{ne}; \quad D_i^m = \tfrac{\overline{\theta}}{2}. \tag{22}$$

$$D_p^l < D_p^f < D_i^m = D_i^f < D_i^l < D_i^{ne} \tag{23}$$

From (23), it can be observed that original product sales do not decrease due to the threat of piracy, as per Shy and Thisse (1999), Banerjee (2003) and Bae and Choi (2006). But, unlike Shy and Thisse (1999), we do not assume network effects between the original and the pirated product. Thus, according to the model developed in this paper, if the purchase of an illegal copy at a lower price were not available,

[8] This result is compatible with those of Mussa and Rosen (1978), where the monopoly price is higher than the competitive price.

[9] We consider that the incumbent abuses his dominant position because he produces a specific product and sets high prices.

this would not imply the purchase of an original product at a higher price. This is because of the effect of the threat of piracy on the incumbent's pricing behavior, the fact that the copy is lower in quality than the original product and the market is uncovered (i.e. there is always at least one consumer who does not buy at all). This result contrasts with most studies about piracy and some court rulings, which assume that the copies sold by the pirate are equivalent to units of products that the incumbent does not sell. But, this one is compatible with the results in Johnson (1985), where copying reduces sales of the original product in spite of the incumbent's pricing behavior. This is because Johnson (1985) assumes that the quality of the copy is the same as the original product and the market is covered (i.e. all consumers buy either the original and the pirated product).

Notice that the demand of the leader, whether it is the incumbent or the pirate, does not depend on the quality of the products, and the total amount of products sold by both on the market when the pirate enters ($D_i + D_p$) is the same, both when he is a leader and when he is a follower. We obtain that both the incumbent and the pirate prefer to be the follower rather than the leader on prices, as in the models of price competition with imperfect substituted products (2004). Moreover, like Shaked and Sutton (1982), we obtain that the top quality firm (incumbent) enjoys greater revenue than its rival (pirate). We also obtain that the pirate establishes the same price whether he is the leader or the follower.

5.2 Measure of competition

A measure of price competition is obtained by taking the ratio of prices, as in Moraga-González and Viaene (2005). From (10-14-24) we deduce that an increase in the quality ratio relaxes price competition and leads to price rises.

$$\frac{p_i^f}{p_p^f} = 2r; \quad \frac{p_i^l}{p_p^l} = \frac{4r-1}{2}; \quad \frac{x_i^f}{x_p^f} = 2; \quad \frac{x_i^l}{x_p^l} = \frac{4r-1}{2r}. \tag{24}$$

It is clear from (24) that the ratio of hedonic prices is constant in the f-subgame and increasing in the l-subgame. This is because the pirate, as a follower, always prices at half the incumbent's price, and as a leader he cannot do this because the final decision lies with the incumbent.

5.3 Comparative static

We now analyze the effects of an increase in product quality taking into account that the government's optimal policy does not change. The results obtained is summarizes in the following proposition.

Proposition 4 *In any SPE, we have that:*

(a) An increase in q_i leads to higher prices and revenue for both incumbent and pirate, although it leads to lower demand to the follower, whether it is the incumbent or the pirate.

(b) The effect of q_p is opposite to q_i, although the effect on the price and revenue of the pirate depend on the initial level of differentiation, independently whether he is the leader or the follower. In particular, it is positive for an initial high differentiation, otherwise it is negative.

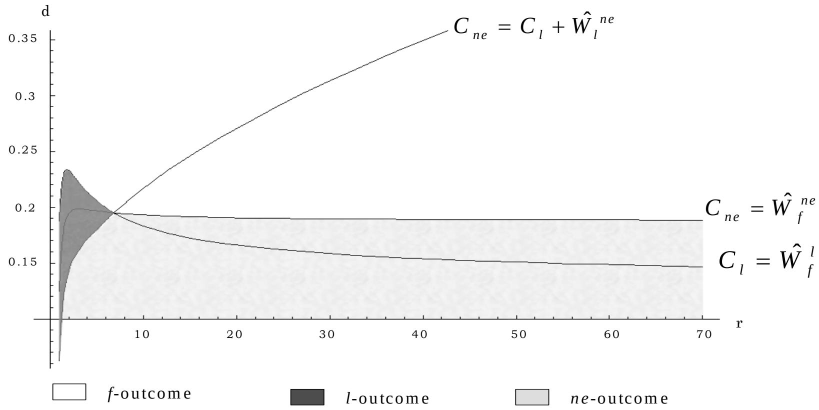

Figure 3: The social optimum

(c) SW^f increases in q_i and q_p.

Proof: see Appendix B.

A bigger differentiation leads to greater market power for both the incumbent and the pirate, independently who is the leader, enabling them to set higher prices and obtain more revenue.[10] An increase in the quality of the pirate induces the incumbent in the outcomes f, l and ne to reduce the price of the original product, and the pirate, as a follower or leader, to rise the price of the pirated product for a high initial differentiation. This is because a rise in q_p implies an increase in the value of the pirated product and a decrease in market power for both firms, since it also implies a lower differentiation.

To give some intuition about the effect of an increase in the ratio of qualities that can imply a change in the government's optimal policy we construct the figure 3, which shows the social optimum according to the value of the marginal monitoring cost of piracy and the ratio of qualities. We have assumed that $\overline{\theta} = q_i = \delta = 1$, $\overline{G} = 1/8$ and the cost of monitoring is lineal ($C(\alpha) = d\alpha$), so the marginal monitoring cost is constant ($C'(\alpha) = d$). As can be seen from this figure, l-outcome is only socially optimal when the marginal monitoring cost level of piracy is intermediate and the quality ratio is small. However, ne-outcome (f-outcome) is socially optimal when the marginal monitoring cost of piracy is low (high) enough, independently of the quality ratio.

5.4 Income and Piracy

We can make another interpretation of θ as a measure of the consumer's income. Thus, a bigger θ is associated with a richer consumer. In this case, the utility of a consumer is $q - (1/\theta)\,p$ if he buys a

[10] These results are the same as those in vertical differentiation literature, where firms are said to prefer to differentiate their products from those of their competitors to restore some monopolistic power (Shaked and Sutton (1982), Champsaur and Rochet (1989)).

product of quality q at price p, and zero if he does not buy any product (see Tirole (1988, p. 96) for more details).

We wish to analyze the relationship between per capita income and piracy, so, we define per capita income as the mean of θ, which is uniformly distributed on $\left[0, \overline{\theta}\right]$. Let $E\left[\theta\right] = \overline{\theta}/2$ be the per capita income and let $R_p = D_p/\left(D_i + D_p\right)$ be the piracy rate. The piracy rates when the pirate enters as follower and as leader and the relationship between them are the following:

$$R_p^f = \frac{r}{3r-1} > R_p^l = \frac{2r-1}{2\left(3r-1\right)} \tag{25}$$

From (22) and (25), we obtain that the demand of both the incumbent and the pirate depends positively on per capita income in every outcome, while the piracy rate is independent. Thus, for every possible outcome $k \in \{f, l, ne, m\}$, we obtain that a bigger per capita income that does not imply a change of outcome does not affect the piracy rate and leads to higher demand and profit (both the incumbent and the pirate), and more effort by the government ($\partial g_k/\partial E\left[\theta\right] > 0 \; \forall k \in \{l, 0, m\}$), which means that in a country with a higher income the government must do a bigger effort to let the pirate become a leader, to encourage the incumbent to deter the entry of the pirate and to blockade the entry of the pirate, because the pirate's revenue is higher.

Moreover, from (23) and (25), we can see that increases in the level of effort by the government that imply a change in outcome cause a decrease in demand for the pirated product and in the piracy rate. This conclusion coincides with that obtained in the empirical work by Rodríguez-Andrés (2006). Moreover, if we take into account that intellectual property receives greater protection in developed economies, which is supported by the empirical work Marron and Steel (2000), and we assume that this is due to cheaper monitoring technology,[11] we get that high-income countries with cheaper monitoring technology have lower piracy rates, as in empirical works by Marron and Steel (2000) and Rodríguez-Andrés (2006).

The empirical work of Bezmen and Depken II (2006) obtains a negative relationship between software piracy and income for various states in the United States. This result is supported by our theoretical model if the monitoring cost of piracy is different between one state and another in the US.

6 Should the incumbent give the pirate the opportunity to be leader?

So far we have assumed that the pirate is the only one that can decide to when enter in the market, so he can be the leader on prices. In this section, we extend the model to the case where the incumbent has the advantage of deciding first whether to enter or not. This raises the question of whether the incumbent gives the pirate the opportunity to be leader.

The extended game is the following. The government announces α and G to maximize social welfare, and both firms observe the policy variables. Then the incumbent decides to price the original product

[11] The cultural factors and the strong institutions, that enforce contracts and protect property, of the developed economies can do the monitoring technology of piracy cheaper.

first or to wait. If he prices first, the pirate observes the price and becomes a follower on prices, so the game continues like the f-subgame. But if he waits, the pirate has the opportunity to be leader: if the pirate price first, the game continues like the l-subgame, but if he waits the game continues like the f-subgame. Finally, the consumers observe the price of the products and then decide to buy the original product, the pirated product or neither.

As in proposition 2, the decisions of both the incumbent and the pirate depend on the level of government spending on avoiding piracy, and the incumbent allows the pirate to be leader on prices, although for a lower interval of values of g. The market equilibrium is summarized in the following proposition.

Proposition 5 *In any SPE of the extended game, the hedonic prices given g are as follows: $x_i = x_i^f$ and $x_p = x_p^f$ if $g \in [0, g_l)$; $x_i = x_i^l$ and $x_p = x_p^l$ if $g \in [g_l, g_i)$; $x_i = x_i^{ne}$ if $g \in [g_i, g_m)$; $x_i = x_i^m$ if $g \in [g_m, +\infty)$; where*

$$g_i = \frac{\overline{\theta}^2 q_i}{64\,(r-1)} \left(2 - \sqrt{\frac{8r^2 - 5r + 1}{r\,(2r-1)^2}}\right)^2 \in (g_l, g_0)\,. \tag{26}$$

The incumbent is indifferent between waiting and acting as leader on prices for any g except on the intervals $[g_l, g_i)$ and $[g_i, g_0)$. When $g \in [g_l, g_i)$ he prefers to wait so as to allow the pirate to act first as the leader, but when $g \in [g_i, g_0)$ he prefers to act first to avoid the entry of the pirate.

The social welfare of continuation in SPE is:

$$W = \begin{cases} CS^f + \pi_i^f + I_p^f - \alpha\,(1-\delta)\,I_p^f - C\,(\alpha) & \text{if } 0 \le g < g_l, \\ CS^l + \pi_i^l + I_p^l - \alpha\,(1-\delta)\,I_p^l - C\,(\alpha) & \text{if } g_l \le g < g_i, \\ CS^{ne} + \pi_i^{ne} - C\,(\alpha) & \text{if } g_i \le g < g_m, \\ CS^m + \pi_i^m - C\,(\alpha) & \text{if } g_m \le g \end{cases} \tag{27}$$

As in Section 4, we have that the government will choose the maximum penalty $\overline{G}$, and social welfare function (27) is decreasing in α on the intervals $[0, g_l)\,, [g_l, g_i)\,, [g_i, g_m)$ and $[g_m, +\infty)$. So in order to maximize social welfare the government will choose the minimum monitoring rate that lead to different outcomes, which is $\alpha \in \{\alpha_f, \alpha_l, \alpha_i, \alpha_m\}$, where $\alpha_i = \frac{q_i g_i}{q_i g_i + \overline{G}}$. As a result, since g is increasing in α, social welfare is decreasing in g, so the value of g in the social maximum is reached in $\{0, g_l, g_i, g_m\}$, which is the same as in Section 4 except g_i. This is because of g_i is the value that lead to ne-outcome in this extended game. Thus, the maximum social welfare in each outcome is the same as in (18) except W_i^{ne}, which is

$$W_i^{ne} = CS_i^{ne} + \pi_{ii}^{ne} - C\,(\alpha_i)\,,$$

where $CS_i^{ne} + \pi_{ii}^{ne}$ is the gross social welfare at x_i^{ne} for $g = g_i$. Since $CS^{ne} + \pi^{ne}$ is decreasing in g and $g_i \in (g_l, g_0)$, we have $CS_0^{ne} + \pi_0^{ne} < CS_i^{ne} + \pi_{ii}^{ne}$. Therefore, monopoly continues providing the lowest social welfare and the outcomes f, l and ne continue being candidates to social optimum. This implies that the optimal policy is similar to the obtained in Section 4, which only difference is that the government chooses $g_i \in (g_l, g_0)$ to reach the outcome where the incumbent deters the pirate from entering the market. This optimal policy is summarized in the following proposition.

Proposition 6 *In any SPE, the optimal strategy of the government is:*

(a) $g = 0$, *if* $C(\alpha_i) > \Delta\widehat{W}_f^{ne_i}$ *and* $C_l > \Delta\widehat{W}_f^l$.

(b) $g = g_l$, *if* $C(\alpha_i) > C_l + \Delta\widehat{W}_l^{ne_i}$ *and* $C_l < \Delta\widehat{W}_f^l$.

(c) $g = g_i$, *if* $C(\alpha_i) < \Delta\widehat{W}_f^{ne_i}$ *and* $C(\alpha_i) < C_l + \Delta\widehat{W}_l^{ne_i}$.

7 Antipiracy System

Since the incumbent wishes to retain his monopoly, we now consider the possibility of the incumbent installing a software protection system (antipiracy system) to prevent his product from being pirated. Following Banerjee (2003), we assume that the original's quality is not damaged by the antipiracy system and the installation cost is fixed at F.

The timing of the new game is as follows: the government decides whether to allow the incumbent to install the antipiracy system or not. If it denies permission to install it, the game continues as in Section 4. But, if it grants permission it announces α and G that maximize the social welfare. Once both firms observe the policy variables, the incumbent decides whether or not to install an antipiracy system. If he does, he becomes a monopolist. But if he does not the game continues as in Section 4. Finally, the consumers observe the price of the products and then decide to buy the original product, the pirated product or neither.

To obtain the SPE we solve the game by backward induction. In the subgame where the government does not allow the antipiracy system to be installed, the optimal policy is that we summarized in proposition 3.

We now solve the subgame where the government allows the system to be installed. If the incumbent installs it he becomes a monopolist and obtains the profit $\pi_i^m - F$. But, if he does not install it, the optimal strategies of both the incumbent and the pirate are that we summarized in proposition 2.

Let $\pi_i^m - \pi_i^f$, $\pi_i^m - \pi_i^l$ and $\pi_i^m - \pi_i^{ne}(g)$ be the incumbent's incentives to install an antipiracy system when the pirate enters (f- and l-outcome) and when the pirate is deterred from entering market (ne-outcome), respectively. Notice that the incumbent's incentives to install the antipiracy system decreases as government spending on monitoring piracy increases, since

$$\pi_i^m - \pi_i^f > \pi_i^m - \pi_i^l > \pi_i^m - \pi_i^{ne}(g) \ \forall g \in [g_0, g_m]$$

As can be seen in the following proposition, the incumbent's optimal decision about whether the antipiracy system is installed depends on the government's policy and on the cost of installation.

Proposition 7 *In any SPE,*

(a) If $F \in [0, \pi_i^m - \pi_{i0}^{ne}]$*, the incumbent installs an antipiracy system if and only if* $g \in [0, g_m)$*.*

(b) If $F \in \left(\pi_i^m - \pi_{i0}^{ne}, \pi_i^m - \pi_i^l\right]$*, he installs an antipiracy system if and only if* $g \in [0, g_0)$*.*

(c) If $F \in \left(\pi_i^m - \pi_i^l, \pi_i^m - \pi_i^f\right]$*, he installs an antipiracy system if and only if* $g \in [0, g_l)$*.*

(d) If $F \in \left(\pi_i^m - \pi_i^f, +\infty\right]$*, he does not install an antipiracy system.*

Notice that the incumbent does not install an antipiracy system when $g \in [g_m, +\infty)$ because of the entry of the pirate is blockaded by the government, and when $F \in \left(\pi_i^m - \pi_i^f, +\infty\right]$ because of the cost of installation is too high.

The maximum social welfare in each outcome when the incumbent does not install an antipiracy system is the same as that obtained in Section 4, which is showed in (18), and social welfare when he installs it is $W_{ApS}^m = \widehat{W}^m - F$.

Taking into account the incumbent's optimal decision about whether the antipiracy system is installed, we obtain the government's optimal policy when he allows installation, which is showed in the following proposition. We observe that for a low installation cost the government decides not to monitor and not to fine piracy, so the incumbent is induced to install an antipiracy system.

Proposition 8 *In any SPE, when the government allows installation, we have:*

(a) If $F \in [0, \pi_i^m - \pi_{i0}^{ne}]$*, the govenment decides* $g = 0$ *when* $F \leq C(\alpha_m)$*, otherwise it decides* $g = g_m$*.*

(b) If $F \in \left(\pi_i^m - \pi_{i0}^{ne}, \pi_i^m - \pi_i^l\right]$*, the govenment decides* $g = 0$ *when* $F \leq C_{ne} - \Delta\widehat{W}_m^{ne}$*, otherwise it decides* $g = g_0$*.*

(c) If $F \in \left(\pi_i^m - \pi_i^l, \pi_i^m - \pi_i^f\right]$*, the govenment decides* $g = 0$ *when* $F \leq \min\left\{C_{ne} - \Delta\widehat{W}_m^{ne}, C_l - \Delta\widehat{W}_m^l\right\}$*, otherwise it decides* $g = g_k$*, where* $k \in (l, ne)$ *is the outcome that maximizes social welfare.*

(d) If $F \in \left(\pi_i^m - \pi_i^f, +\infty\right]$*, the government's policy in proposition 3 is held.*

Finally, by comparing the government's optimal policy when it does not allow the incumbent to install the antipiracy system (proposition 3) with the optimal policy when it does allow the system be installed (proposition 8), we find that the outcome where the incumbent installs an antipiracy system is not an equilibrium because the social welfare in that outcome is lower than social welfare when the pirate enters as follower, i.e. allowing the incumbent installs an antipiracy system is ruled out by letting the pirate enters as a follower, according to the relationship between the gross social welfare in each outcome, which is showed in (21). Therefore, the equilibriums are the same than in Section 4.

8 Conclusions

We analyze the roles of the government and the incumbent in preventing piracy, and the reasons and incentives why a pirate would want to be a leader in prices. The framework of analysis used is a duopoly model of vertical product differentiation with price competition, where both the incumbent and the pirate are committed to keep their prices.

Our analysis shows that the government will not help the incumbent to become a monopolist, even if he installs an antipiracy system, because a monopoly provides the lowest social welfare. However, we obtain that the government, in equilibrium, will decide to let the pirate enters as a follower or as a leader or to encourage the incumbent to set a low enough price to successfully deter the entry of the pirate, which depends on the monitoring cost of piracy. In particular, he lets the pirate enter as a leader when the monitoring cost of piracy in this outcome is low enough as regards the gain in gross social welfare and the monitoring cost at the other outcomes (f- and ne-outcomes); he encourages the incumbent to deter the entry of the pirate when the monitoring cost in this outcome is low enough as regards the gain in gross social welfare and the monitoring cost at the other outcomes, (f- and l-outcomes); and, otherwise, he lets the pirate enter as a follower, i.e. when the monitoring cost of piracy at l- and ne-outcomes is high.

The results in this paper suggest that when the monitoring cost is relatively low and piracy exists, the government must try hard, but not too much to avoid monopoly, or the incumbent must reduce the price of the original product to prevent the entry of the pirate, so the threat of piracy is latent. In this case not just government intervention but also the participation of the incumbent are needed. In short, efforts by the copyright owner to reduce piracy should focus not only on government enforcement but also on pricing strategies. This statement is empirically supported by Papadopoulos (2003).

An interesting result obtained in this paper is that the threat of piracy does not lead to lower sales of the original product, which contrasts with most studies about piracy and some court rulings because they assume that the copies sold by the pirate are equivalent to units of products that the incumbent does not sell. This result means that if the purchase of a pirated product at a lower price were not available, this would not imply the purchase of an original product at a higher price. Therefore, according this paper, the decrease of the sales of CDs is not a result of piracy, so industry losses from piracy are lower than claimed by the industry itself, by most studies and by some court rulings.

We find that government effort has a significant negative effect on piracy rates and that high-income countries with cheaper monitoring technology of piracy have lower piracy rates. These results are supported by empirical studies by Marron and Steel (2000) and Rodríguez-Andrés (2006). We also find that in a country with a higher per capita income the government must do a bigger effort to let the pirate become a leader, to encourage the incumbent to deter the entry of the pirate and to blockade the entry of the pirate, because the pirate's revenue in each outcome is increasing in per capita income.

We assume that both the incumbent and the pirate are in the same economy. If we consider the incumbent as a foreign firm, then the government does not take into account the incumbent's profit on social welfare. In this case, ne-outcome, like a monopoly, not maximizes social welfare because $CS^m < CS^{ne} < CS^f + I_p^f < CS^l + I_p^l$ and $C'(\alpha) > 0$. Thus, the government will let the pirate enter as a follower or as a leader, according to the cost of monitoring piracy. In particular, letting the pirate becomes a leader is socially optimal when the cost of monitoring piracy at this outcome is low enough, otherwise letting the pirate enters as a follower is socially optimal. Moreover, it is also possible that the only social optimum would be that the pirate enters as a follower. This happens when $CS^l + (1 - \alpha_l(1 - \delta)) I_p^l < CS^f + I_p^f$, which is because of the government can reuse little of the revenue seized from the pirate.

However, when the pirate is a foreign firm, the government does not take into account him on social welfare. In this case, the optimal policy obtained when both the incumbent and the pirate belong to the same economy is held if the marginal monitoring cost of piracy is high enough as regards the maximum penalty and the revenue used from the revenue seized from the pirate. See Appendix C for more details.

Thus we show that the great variation in piracy rates from one country to another is a consequence of different technologies for monitoring piracy by governments and the nationality of firms.

Of course, the scope of our results is limited by our assumptions. It would be interesting, for instance, to extend our analysis to the case of multi-product firms, network externalities and switching cost.

Appendix A

Proof of Proposition 1. To maximize (9), we must first obtain the possible local maximum restricted to the intervals $I_1 = [0, x_i^{ne}]$ and $I_2 = [x_i^{ne}, \bar{\theta}]$. Let x_{ki} and π_{ki} be the incumbent's hedonic price and profit in the maximum of $\pi_i^c(\cdot)$ on the interval I_k for $k = 1, 2$, respectively.

First, consider the maximization on I_1. Since the monopoly hedonic price is $x_i^m = \bar{\theta}/2$, we have $x_{1i} = x_i^m$ if $x_i^m \le x_i^{ne}$ and $x_{1i} = x_i^{ne}$ otherwise. In consequence, we have:

$$x_{1i} = \begin{cases} x_i^m & \text{if } g^m \le g, \\ x_i^{ne} & \text{otherwise,} \end{cases} \tag{28}$$

where $g^m = \gamma(\bar{\theta}/2)$, and the maximal value is $\pi_{1i} = \phi_1(x_{1i})$, where $\phi_1(x_i) = q_i x_i(\bar{\theta} - x_i)$ is the monopoly profit function.

Second, consider the maximization on I_2. This case is feasible only when $g \le \bar{\theta}^2/4r$ holds. From (5), it is easy to see that $x_i \ge x_p^{BR}(x_i)$ holds for any $x_i \in [0, \bar{\theta}]$. Therefore, from (2) and (5), maximization on I_2 is equivalent to maximizing

$$\hat{\pi}_i^c(x_i) = \begin{cases} \phi_2(x_i) & \text{if } x_i \le \frac{2(r-1)}{2r-1}\bar{\theta},\ x_i \in I_2 \\ 0 & \text{if } \frac{2(r-1)}{2r-1}\bar{\theta} \le x_i \le \bar{\theta}, \end{cases} \tag{29}$$

where $\phi_2(x_i) = q_i x_i(\bar{\theta} - \frac{2r-1}{2(r-1)}x_i)$ is the incumbent's profit function when the pirate enters the market and reacts optimally.

Let $x_i^f = (r-1)\bar{\theta}/(2r-1)$ be the the maximum of $\phi_2(\cdot)$ over $[0,\,\bar{\theta}]$. Therefore x_i^f is the incumbent's optimal hedonic price when the pirate enters the market and chooses his hedonic price optimally. Assume $g \leq \frac{(r-1)\bar{\theta}^2}{(2r-1)^2}$. In this case, $x_i^{ne} \leq 2(r-1)\bar{\theta}/(2r-1)$ holds, and the maximum of $\pi_i^c(\cdot)$ on I_2 is reached at $x_{2i} = \max(x_i^{ne}, x_i^f)$, with a maximum value of $\pi_{2i} = \phi_2(x_{2i})$. Assume $\frac{(r-1)\bar{\theta}^2}{(2r-1)^2} \leq g \leq \frac{\bar{\theta}^2}{4r}$ or, equivalently, $2(r-1)\bar{\theta}/(2r-1) \leq x_i^{ne}$. Since (29) becomes $\hat{\pi}_i^c(x_i) = 0,\ \forall x_i \in [x_i^{ne}, \bar{\theta}]$, the maximum of $\pi_i^c(\cdot)$ on I_2 is reached at any point in I_2 with a maximum value equal to $\pi_{2i} = 0$.

To summarize the arguments, the maximization of $\pi_i^c(\cdot)$ in (9) leads us to compare

$$\pi_{1i} = \begin{cases} \phi_1(x_i^{ne}) & \text{if } 0 \leq g \leq g^m, \\ \phi_1(x^m) & \text{if } g^m \leq g, \end{cases} \tag{30}$$

with

$$\pi_{2i} = \begin{cases} \phi_2(x_i^f) & \text{if } 0 \leq g \leq \frac{(r-1)\bar{\theta}^2}{4(2r-1)^2}, \\ \phi_2(x_i^{ne}) & \text{if } \frac{(r-1)\bar{\theta}^2}{4(2r-1)^2} \leq g \leq \frac{(r-1)\bar{\theta}^2}{(2r-1)^2}, \\ 0 & \text{if } \frac{(r-1)\bar{\theta}^2}{(2r-1)^2} \leq g \leq \frac{\bar{\theta}^2}{4r}, \\ +\infty & \text{if } \frac{\bar{\theta}^2}{4r} \leq g. \end{cases} \tag{31}$$

Note that the parabola ϕ_1 is always above the parabola ϕ_2 on $[0,\,\bar{\theta}]$. The maximum of ϕ_1 is reached at $x_i = x_i^m = \bar{\theta}/2$, whereas the maximum of ϕ_2 is reached at $x_i = x_i^f$.

The situation of $\gamma(\cdot)$, relative to the points $\frac{(r-1)\bar{\theta}^2}{4(2r-1)^2}$, $\frac{(r-1)\bar{\theta}^2}{(2r-1)^2}$ and $\frac{\bar{\theta}^2}{4r}$, can easily be obtained. From the definition of $\gamma(\cdot)$ in (6), we have $g_m = \frac{\bar{\theta}^2}{16(r-1)}$ if $r \geq 3/2$, and $g_m = \frac{\bar{\theta}^2(2-r)}{4}$ if $1 < r \leq 3/2$. This implies the inequalities $\frac{(r-1)\bar{\theta}^2}{4(2r-1)^2} \leq g_m \leq \frac{\bar{\theta}^2}{4r}$, and we have that $\frac{(r-1)\bar{\theta}^2}{(2r-1)^2} <$ (resp. $>$) g_m if and only if $1 < r < 3/2$ (resp. $3/2 < r$).

In the rest of the proof, we compare π_{1i} with π_{2i}, by separating the arguments into several cases, corresponding to different intervals for the values of g.

(Case 1) Consider first any value $g \in [0,\,\frac{(r-1)\bar{\theta}^2}{4(2r-1)^2})$. Expressions (30-31) imply $\pi_{1i} = \phi_1(x_i^{ne})$ and $\pi_{2i} = \phi_2(x_i^f) = \pi_i^f = \frac{q_i(r-1)\bar{\theta}^2}{2(2r-1)}$ and, from (8), we have $x_i^{ne} < x_i^f$. By solving the corresponding second-degree polynomial equation, we can find a unique point $x_i^0 = \bar{\theta}(1-(2r-1)^{-1/2})/2 < x_i^f$ such that $\phi_1(x_i^0) = \pi_i^f$. Since π_i^f is the maximum value of $\phi_2(\cdot)$, we can see that $\phi_1(x_i^{ne})$ is lower (higher) than $\phi_2(x_i^f) = \pi_i^f$ if and only if x_i^{ne} is lower (higher) than x_i^0. Additionally, from (7) we can show that x_i^{ne} is lower (higher) than x_i^0 if and only if g is lower (higher) than $g_l \in \left(0,\,\frac{(r-1)\bar{\theta}^2}{4(2r-1)^2}\right)$, which is defined in (11).

For the present case, we conclude that the incumbent's equilibrium hedonic price is $x_i^* = x_i^f$ (and the pirate will enter) if $g < g_l$, because then $\pi_i^f > \phi_1(x_i^{ne})$. When $g = g_l$, it may occur that $x_i^* = x_i^{ne}$ (and the pirate will not enter) or, indifferently, $x_i^* = x_i^f$ (and the pirate will enter), because $\phi_1(x_i^{ne}) = \pi_i^f$. If $g_l < g \leq \frac{(r-1)\bar{\theta}^2}{4(2r-1)^2}$, we have $x_i^* = x_i^{ne}$ (and the pirate will not enter), because $\phi_1(x_i^{ne}) > \pi_i^f$.

Note that although the pirate becomes indifferent between entering and not entering when the incumbent prices x_i^{ne}, for SPE to exist it is necessary for the pirate to choose not to enter for that hedonic price.

Whenever $g = \frac{(r-1)\bar{\theta}^2}{4(2r-1)^2}$, the incumbent's optimal hedonic price must be $x_i^* = x_i^{ne}$ and the pirate must not enter. Here, we have $x_i^{ne} = x_i^f$, but $\phi_1(x_i^{ne}) > \pi_i^f$ holds.

(Case 2) Consider $g \in \left(\frac{(r-1)\bar{\theta}^2}{4(2r-1)^2}, \min(g_m, \frac{(r-1)\bar{\theta}^2}{(2r-1)^2})\right)$. Expressions (30-31) imply $\pi_{1i} = \phi_1(x_i^{ne}) > \phi_2(x_i^{ne}) = \pi_{i2}$. In this case, the incumbent will price $x_i^* = x_i^{ne}$ and the pirate will not enter in SPE.

(Case 3, feasible for $3/2 \leq r$) Consider $g \in \left[g_m, \frac{(r-1)\bar{\theta}^2}{(2r-1)^2}\right]$. Here expressions (30-31) lead us to $\pi_{1i} = \phi_1(x^m)$ and $\pi_{2i} = \phi_2(x_i^{ne})$. Evidently, the incumbent will choose $x_i^* = x_i^m$ and the pirate will not enter because $x_i^m < x_i^{ne}$.

(Case 4, feasible for $1 < r \leq 3/2$) Consider $g \in \left[\frac{(r-1)\bar{\theta}^2}{(2r-1)^2}, g_m\right]$. From (30-31), here we obtain $\pi_{1i} = \phi_1(x_i^{ne}) > 0 = \pi_{2i}$. Therefore, the incumbent will price $x_i^* = x_i^{ne}$, and the pirate will not enter.

(Case 5) Consider $g \in \left(\max(g_m, \frac{(r-1)\bar{\theta}^2}{(2r-1)^2}), \frac{\bar{\theta}^2}{4r}\right]$. From (30-31), here we obtain $\pi_{1i} = \phi_1(x_i^m) > 0 = \pi_{2i}$. Therefore, the incumbent will price $x_i^* = x_i^m$, and the pirate will not enter.

(Case 6) Consider $\frac{\bar{\theta}^2}{4r} < g$. In this case, it is optimal for the pirate not to enter given any hedonic price of the incumbent. The incumbent will price $x_i^* = x_i^m$.

Finally, note that the critical values of g that determine the incumbent's optimal hedonic price in $\{x_i^f, x_i^{ne}, x_i^m\}$ are g_l and g_m. ■

Appendix B

Proof of Proposition 4. From (10-14-19) we take partial derivatives with respect to q_i and q_p

$$\frac{\partial p_{i0}^{ne}}{\partial q_i} = \sqrt{\frac{\bar{\theta}^2(4r^2-3r+1)^2}{8r(2r-1)^3}} > 0 \qquad \frac{\partial p_{i0}^{ne}}{\partial q_p} = -\sqrt{\frac{\bar{\theta}^2 r(3r-1)^2}{8(2r-1)^3}} < 0$$

$$\frac{\partial \theta_{o0}^{ne}}{\partial q_i} = \sqrt{\frac{\bar{\theta}^2(3r-1)^2}{8q_i^2 r(2r-1)^3}} > 0 \qquad \frac{\partial \theta_{oo}^{ne}}{\partial q_p} = -\sqrt{\frac{\bar{\theta}^2 r(3r-1)^2}{8q_i^2(2r-1)^3}} < 0$$

$$\frac{\partial \pi_{i0}^{ne}}{\partial q_i} = \sqrt{\frac{\bar{\theta}^4(4r^2-3r+1)^2}{8r(2r-1)^3}} - \frac{\bar{\theta}^2 r(r-1)}{(2r-1)^2} > 0 \qquad \frac{\partial \pi_{i0}^{ne}}{\partial q_p} = -\sqrt{\frac{\bar{\theta}^4 q_i(3q_i-q_p)^2}{8(2q_i-q_p)^3}} + \frac{\bar{\theta}^2(3r-1)(r-1)}{2(2r-1)^2} < 0$$

$$\frac{\partial p_i^l}{\partial q_i} = \frac{\bar{\theta}\left(8r^2-8r+3\right)}{4(2r-1)^2} > 0 \qquad \frac{\partial p_p^l}{\partial q_i} = \frac{\bar{\theta}}{2(2r-1)^2} > 0$$

$$\frac{\partial p_i^l}{\partial q_p} = -\frac{\bar{\theta}(2r(3r-2)+1)}{4(2r-1)^2} < 0 \qquad \frac{\partial p_p^l}{\partial q_p} = \frac{\bar{\theta}\left(2r^2-4r+1\right)}{2(2r-1)^2} \lesseqgtr 0$$

$$\frac{\partial \theta_o^l}{\partial q_i} = \frac{\bar{\theta}}{2q_p(2r-1)^2} > 0 \qquad \frac{\partial \theta_p^l}{\partial q_i} = \frac{\bar{\theta}}{2q_p(2r-1)^2} > 0$$

$$\frac{\partial \theta_o^l}{\partial q_p} = -\frac{\bar{\theta}r}{2q_p(2r-1)^2} < 0 \qquad \frac{\partial \theta_p^l}{\partial q_p} = -\frac{\bar{\theta}r}{2q_p(2r-1)^2} < 0$$

$$\frac{\partial \pi_i^l}{\partial q_i} = \frac{\bar{\theta}^2(4r-1)((r-1)(8r-2)+3)}{16(2r-1)^3} > 0 \qquad \frac{\partial \pi_i^l}{\partial q_p} = -\frac{\bar{\theta}^2(4r-1)(2r(2r-1)+1)}{16(2r-1)^3} < 0$$

$$\frac{\partial I_p^l}{\partial q_i} = \frac{\bar{\theta}^2}{8(2r-1)^2} > 0 \qquad \frac{\partial I_p^l}{\partial q_p} = \frac{\bar{\theta}^2\left(2r^2-4r+1\right)}{8(2r-1)^2} \lesseqgtr 0$$

$$\frac{\partial p_i^f}{\partial q_i} = \frac{\bar{\theta}(2r(r-1)+1)}{(2r-1)^2} > 0 \qquad \frac{\partial p_p^f}{\partial q_i} = \frac{\bar{\theta}}{2(2r-1)^2} > 0$$

$$\frac{\partial p_i^f}{\partial q_p} = -\frac{\bar{\theta}r^2}{(2r-1)^2} < 0 \qquad \frac{\partial p_p^f}{\partial q_p} = \frac{\bar{\theta}\left(2r^2-4r+1\right)}{2(2r-1)^2} \lesseqgtr 0$$

$$\frac{\partial \theta_p^f}{\partial q_i} = \frac{\bar{\theta}}{2q_p(2r-1)^2} > 0 \qquad \frac{\partial \pi_i^f}{\partial q_i} = \frac{\bar{\theta}^2(2r(r-1)+1)}{2(2r-1)^2} > 0$$

$$\frac{\partial \theta_p^f}{\partial q_p} = -\frac{\bar{\theta}r}{2q_p(2r-1)^2} < 0 \qquad \frac{\partial \pi_i^f}{\partial q_p} = -\frac{\bar{\theta}^2 r^2}{2(2r-1)^2} < 0$$

$$\frac{\partial CS^f}{\partial q_i} = \frac{\partial CS^f}{\partial r} = \frac{\bar{\theta}^2\left(8r^3-12r^2+1\right)}{8(2r-1)^3} \gtreqless 0 \qquad \frac{\partial I_p^f}{\partial q_i} = \frac{\bar{\theta}^2}{4(2r-1)^3} > 0$$

$$\frac{\partial CS^f}{\partial q_p} = \frac{\bar{\theta}^2 r^2(10r-3)}{8(2r-1)^3} > 0 \qquad \frac{\partial I_p^f}{\partial q_p} = \frac{\bar{\theta}^2 r^2(2r-3)}{4(2r-1)^3} \gtreqless 0$$

$$\frac{\partial SW^f}{\partial q_i} = \frac{\bar{\theta}^2(12r(r-1)(2r-1)+4r-1)}{8(2r-1)^3} > 0 \qquad \frac{\partial SW^f}{\partial q_p} = \frac{\bar{\theta}^2 r^2(6r-5)}{8(2r-1)^3} > 0$$

■

Appendix C

If we do not include the pirate, the social welfare of continuation in SPE is:

$$SW=\begin{cases} CS^{f}+\pi_i^{f}+\alpha G+\alpha\delta I_p^{f}-C(\alpha) & \text{if } 0\le g<g_l, \\ CS^{l}+\pi_i^{l}+\alpha G+\alpha\delta I_p^{l}-C(\alpha) & \text{if } g_l\le g<g_0, \\ CS^{ne}+\pi_i^{ne}-C(\alpha) & \text{if } g_0\le g<g_m, \\ CS^{m}+\pi_i^{m}-C(\alpha) & \text{if } g_m\le g \end{cases} \tag{32}$$

Given that the social welfare functions (15) and (33) are equal when $g\in[g_0,+\infty)$, we obtain that the government will choose the minimum g that lead to ne- and m-outcomes, which are g_0 and g_m respectively, as in Section 4. Moreover, notice that the conditions $g\in[0,g_l)$ and $g\in[g_l,g_0)$ are equivalent to the conditions $(\alpha,G)\in Z_f$ and $(\alpha,G)\in Z_l$ respectively, where

$$\begin{aligned} Z_f &= \left\{(\alpha,G)\mid \alpha\in\left[0,\tfrac{q_ig_l}{q_ig_l+G}\right) \text{ and } G\in\left[0,\overline{G}\right]\right\}\subset[0,1]\,\text{x}\left[0,\overline{G}\right] \\ Z_l &= \left\{(\alpha,G)\mid \alpha\in\left[\tfrac{q_ig_l}{q_ig_l+G},\tfrac{q_ig_0}{q_ig_0+G}\right) \text{ and } G\in\left[0,\overline{G}\right]\right\}\subset[0,1]\,\text{x}\left[0,\overline{G}\right]. \end{aligned}$$

When the pirate is excluded in social welfare, we have that social welfare positively depends on G. Thus, the maximum social welfare must be reached at $G=\overline{G}$. To maintain the same results that in Section 4 it is necessary that social welfare will be decreasing in α on the intervals $[0,g_l)$ and $[g_l,g_0)$, which is true if $\overline{G}+\delta I_p^f<C'(\alpha)$. Therefore, if $\overline{G}+\delta I_p^f<C'(\alpha)$, the government will choose the minimum monitoring rate that lead to the different outcomes, so the value of g is $\{0,g_l,g_0,g_m\}$ when social welfare is maximum. Therefore, the maximum social welfare is obtained from comparing the following values:

$$\begin{aligned} SW^f &= CS^f+\pi_i^f, \\ SW^l &= CS^l+\pi_i^l+\alpha_l\overline{G}+\alpha_l\delta I_p^l-C(\alpha_l), \\ SW_0^{ne} &= CS_0^{ne}+\pi_{i0}^{ne}-C(\alpha_{ne}), \\ SW^m &= CS^m+\pi_i^m-C(\alpha_m). \end{aligned} \tag{33}$$

The relationship between the sum of consumer surplus and the incumbent's profit among different outcomes is as follows

$$CS^m+\pi_i^m<CS^f+\pi_i^f<CS^l+\pi_i^l<CS^{ne}+\pi_{i0}^{ne} \tag{34}$$

Given $C'(\alpha)>0$ and relationship (34), we deduce that monopoly provides the lowest social welfare and that the outcomes that maximizes social welfare are f, l or ne, according to the relationship between gross social welfare and the cost of monitoring piracy like Section 4. Therefore, the optimal policy is qualitatively similar to that obtained when the pirate is included in social welfare.

Finally, notice that the outcomes where the pirate enters are still possible social optimum, though they provide less social welfare.

However, if the condition $\overline{G}+\delta I_p^f < C'(\alpha)$ is not satisfied the value of g that maximizes social welfare on $[0, g_l)$ and $[g_l, g_0)$ will be one or several values on these intervals according to the value of the marginal cost of monitoring piracy.

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