LEARNING, LIFE-CYCLE AND ENTREPRENEURIAL INVESTMENT*

Claudio Campanale**

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** Departamento de Fundamentos del Anàlisis Económico, Universidad de Alicante, 03071, Alicante, Spain. Voice: ++34965903614 - Fax: ++34965903898. E-mail: claudio@merlin.fae.ua.es

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ABSTRACT

In this paper I present a calibrated model of life-cycle occupation and investment decisions where households choose between paid work and entrepreneurship and conditional on the latter how much of their savings to invest in their business. The returns to entrepreneurial activity are modeled through Bayesian learning. The model is able to reproduce the main stylized facts of entry in and exit out of self-employment over the life-cycle. It also suggests a partial explanation of the recent finding of Moskowitz and Vissing-Jørgensen (2002) that entrepreneurs seem not to require a premium for the extra risk of their private equity investment.

Keywords: Occupational choice, portfolio choice, entrepreneurship, firm dynamics, learning, private equity premium.

1 Introduction

Understanding entrepreneurial investment is of fundamental importance, given the role that privately held firms play in overall economic activity. This can be summarized by a few facts: according to Evans and Leighton (1989) a little more than one tenth of Americans operate a business on a full time basis and they give employment to about a tenth of all wage workers. The economic importance of entrepreneurial households is even greater though, since these households appear more than proportionately in the higher percentiles of the income and net worth distribution holding 40 percent of total national wealth. Since they hold undiversified portfolios where equity in their own business makes a large part of their asset holdings, this translates into a size of the private equity market that is of the same order of magnitude of the public equity market.

Surprisingly enough not much effort has been made to model and study quantitatively the relationship between entrepreneurial activity and the economic decisions of those who carry it out, that is, actual and potential entrepreneurs. The purpose of this paper is to make a first effort at filling this gap by bringing together different strands of literature to understand the main features of entrepreneurial investment.

To address this issue I develop a life-cycle partial equilibrium model where agents work, retire and then die. During working life agents make an occupational choice: at each point in time they decide between working for pay and becoming entrepreneurs. This choice is always reversible so that it is repeated in every period until retirement. Workers supply their labor in exchange for a wage and choose how much to consume and how much to save in a risky financial asset. Entrepreneurs receive earnings from their business, choose their optimal consumption and saving plan and decide how to allocate their wealth between equity in their business and the financial asset. Three key features characterize the way entrepreneurial activity is modeled. First I assume that the average return to a private firm is fixed in the course of its life and not known at the time the entry decision is made. Agents make the decision to enter self-employment based on a noisy signal of this average return. After that, they learn about it over time by applying Bayes rule to the realized stochastic returns, possibly reverting to paid employment if they find out that the returns to the firm are on average low. Second, exogenous imperfections in financial markets force the entrepreneur to finance the private business out of his own wealth and a minimum equity requirement is assumed. Finally the process for entrepreneurial earnings is taken to be highly correlated with the process for the return to equity invested in the private firm; this is meant to capture the fact that most entrepreneurs work in the same business where they invest their wealth.

The model is calibrated and its quantitative properties are examined. The goal is to check if the joint operation of life-cycle occupation, consumption and investment decisions and the learning mechanism can explain the main features of entrepreneurial investment and more specifically to assess the marginal contribution of the latter. The results are promising along two dimensions. The first concerns the dynamics of the occupational choice implied by entrepreneurship: the model with imperfect information about project quality and learning generates quantitatively reasonable patterns of entry in and exit out of self-employment over time and along the household life-cycle whereas a similarly calibrated model without those features shows counterfactual predictions. The second one concerns the portfolio dimension of entrepreneurial activity. Coupled with the empirically plausible assumption that entrepreneurial income is on average greater than that of paid workers, the model with learning also suggests a partial explanation to the surprising fact recently discovered by Moskowtiz and Vissing-Jørgensen (2002) of the absence of a private equity premium. Since the model is partial equilibrium it does not directly address this asset pricing fact. What it does is to provide an explanation for why despite the lack of an observed private equity premium and the higher risk of this investment many agents choose to become entrepreneurs and put large shares of their wealth in their firm.

The rest of the paper is organized as follows. In the next subsection of the introduction I provide a brief description of the relation between the current paper and the different lines of research that it attempts to bring together. Then in Section 2 the model is described, in Section 3 I present the calibration, in Section 4 the results and finally in Section 5 the conclusions.

1.1 Contribution with Respect to the Literature

The present research relates to a number of papers, both empirical and theoretical from different areas. First of all the life-cycle precautionary savings models like Carroll (1997) or Hubbard, Skinner and Zeldes (1994) and successive extensions that add a portfolio choice to the basic model like for example Campbell et al. (1999) or Cocco, Gomes and Maenhout (2005). With these papers the present model shares its basic structure, that is, the fact of analyzing, in a partial equilibrium setting, the optimal life-cycle choices of an agent that faces uninsurable idiosyncratic earnings risk and borrowing constraints and goes through the stages of working life and retirement. It departs from them in that it adds to the basic consumption and saving decision the choice between a menu of two different earnings processes that are interpreted as different occupations, that is, self-employment and paid work. It also allows the subset of households who choose the former to have access to a wider set of assets that include private equity and studies the portfolio choice in this case.

Second it relates to the literature on the choice to become entrepreneurs. This literature is essentially empirical and provides background motivation for the present research. Among these works the most direct reference is a paper by Evans and Leighton (1989) that estimates a model of self-employment entry and exit using NLS and CPS data. Other related papers are the ones that investigate the role of household wealth in the decision to become entrepreneurs: Evans and Jovanovic (1989), using the NLS find that wealth matters and suggest the existence of binding liquidity constraints, Hurst and Lusardi (2004) using the PSID find the opposite, that is, except for very wealthy individuals, the transition to entrepreneurship seems unaffected by initial wealth, suggesting that liquidity constraints are not an important impediment to starting a business for the large majority of US households. Still in the empirical approach is the work by Hamilton (2000) who estimates self versus paid employment earnings. He uses the SIPP and constructs a number of different measures for self-employment earnings; when the most comprehensive one is used he finds that median earnings are higher for workers while mean earnings are higher for entrepreneurs. He also finds that mean tenure profiles are steeper for entrepreneurs whose earnings start below but then overtake those of paid-workers.

A third line of research that bears important common points with the present paper is the one about industry dynamics of which two notable examples are Jovanovic (1982) and Hopenhayn (1992). Those papers study, in a theoretical framework, the long-run equilibrium of an industry as it results from the entry and exit decisions of firms. In those papers firms' decisions are studied abstracting from the choices of their owners and the decision to enter or exit the industry is based on the value of being in the industry compared to an exogenous outside option. The main difference from the current paper is then that here firm dynamics is dictated by the occupational, saving and portfolio decisions of the owner and the value of the option to quit is the endogenously determined value of switching from self to paid employment. Particularly strong is the relationship with the cited work of Jovanovic, since I borrow from that work one key assumption governing occupational choice, that is, that returns to entrepreneurship are learned by Bayes' rule while running the business.

Finally the present research relates to the cited work by Moskowitz and Vissing-Jørgensen and the few papers that have tried to rationalize the surprising fact they discovered. Briefly stated the two authors report that entrepreneurial risk is quite substantial because returns are very volatile and because entrepreneurs invest large shares of their financial wealth, as well as their human capital in their firm. Given this, one would expect risk averse entrepreneurs to demand a large premium for firm specific risk. The two authors though find that private equity in aggregate provides about the same return as the much less risky public equity. Among the papers that have proposed explanations to this finding is the one by Hopenhayn and Vereshchagina (2005) who propose a theory based on borrowing constraints, the existence of an outside opportunity and endogenous risk choice. The possibility of exit creates a non concavity in the agent's continuation value. Risky projects provide lotteries that eliminate this non concavity and are particularly valuable to low wealth agents that will then undertake them at no premium over the safe project. Others are the paper by Hintermeier and Steinberger (2005) who construct a life-cycle portfolio choice model where entrepreneurship is interpreted as investing in a third asset, private equity, on top of risk-free bonds and risky stock and Polkovnichenko (2003) who presents a static model to asses the excess return that private equity must pay over public equity when only a small portion of the agent's human capital is actually invested in the business. The present work shares the dynamic life-cycle framework with the work of Hintermeier and Steinberger and the two facts that it both studies a full occupational choice model and addresses issues related to firm dynamics with Hopenhayn and Vereshchagina. Its distinctive features are the assumption of unobserved heterogeneity in firm returns and learning on the one hand and the analysis of firm dynamics in the context of life-cycle occupational choices on the other.

2 The Model

The life-cycle problem studied in this model is set in a partial equilibrium context, taking as given the processes for asset returns and earnings. The model is populated by finitely lived agents. Time is discrete and the model period is assumed to be one year. Agents live for a maximum of $T = 80$ years and during the course of their life face an age changing probability of surviving. If they are still alive they work until model age 45 and retire afterwards. Model age is equal to real-life age minus twenty. In what follows agents' age is denoted with t while a will denote the age of the business the agent may run.[1]

Utility is defined over the stream of consumption enjoyed during lifetime and there are no bequests. Period utility is discounted at the rate β. Let p_{t+1} be the probability that an agent survives until age $t+1$ conditional on being alive at age t. Agents maximize

[1] Since I will simulate a cohort of agents all born at the same time, t will be used for time as well as agents' age without ambiguity.

$$E_0 \sum_{t=0}^{T} \beta^t \left[\prod_{j=0}^{t} p_j \right] u\left(c_t\right) \tag{1}$$

where c_t is period consumption, E_0 is the expectation operator at the beginning of working life and $p_0 = 1$.

During working life agents are endowed with a certain amount of human capital that they supply inelastically. This endowment evolves deterministically over time but is equal for all agents of a given age. In other words the endowment of human capital ω_t follows the same path over the lifetime of all agents.

During working life agents have to make an occupational choice: they have to decide between working for pay and becoming entrepreneurs. The decision is made every period and is always reversible.

Workers sell their human capital endowment to the market, get a wage and choose optimally how to split their resources between consumption and savings which occur through the single financial asset available in the economy. I call this asset stock. I denote the amount of public stocks held at time t by S_t and impose a short sale constraint $S_t \geq 0$. This asset pays a stochastic gross return R_t which I assume to be i.i.d. over time. The process for the stock return is uncorrelated with all other stochastic processes in the economy. The wage rate is normalized to 1. Workers' earnings are given by $\omega_t \varepsilon_t$ where ε_t is a stochastic shock which is assumed to be $LN\left(0, \sigma_\varepsilon^2\right)$ and i.i.d. over time. The shock is also independent across agents and uncorrelated with the return to the financial asset. Finally workers get the chance to observe a signal on the quality of a project that they can potentially run if they decide to be self-employed in the next period. I will give more details about the specific description of project quality in what follows when talking about the entrepreneurial problem.

Entrepreneurs have to use their human capital to run their business. They can invest their wealth in the same financial asset as workers, but have also access to a second asset, that is, the equity in their own business. Private equity investment K_t is subject to a minimum requirement $\underline{K} > 0$. As it will become clear later, given that the entrepreneurial technology is assumed to be linearly additive in the agent's endowment of labor and financial investment, this positivity constraint is needed to have private equity investment also among newly entered entrepreneurs. This fits with the empirical evidence that most private equity is held by individuals who are at the same time the managers of the business. At the theoretical level extensive literature starting with Jensen and Meckling (1976) demonstrates how asymmetric information about managerial actions impose the entrepreneurs to commit personal funds to finance their project justifying the assumption made here. The critical element in the model is the way returns to private equity are defined. There are two components that determine the return to equity invested in a private business. A first component is firm specific and fixes the average project return over its entire life. With a slight abuse of notation I denote this component x. In fact in the course of their life agents may enter and exit self-employment more than once and so have different x at different times; however I use x instead of x_t to stress the fact that the average project return is fixed for a given project. It is assumed that there is a population distribution of x described by the random variable $X \sim N\left(0, \sigma_x^2\right)$. While agents are workers they get a draw from X. If they decide to start a business they keep the same value of x while they operate it, if they decide to continue as paid employees they get a new draw from X in the coming period. Successive draws from X are independent over time[2]. The second component, denoted with u, is a noise taken from a random variable $U \sim N\left(0, \sigma_u^2\right)$ and is i.i.d. over time and across firms. In practice an entrepreneur receives a sequence of i.i.d. draws y from a distribution $N\left(x, \sigma_u^2\right)$. Agents know the exact distribution of X and U but they can't observe their own realization x. What they observe is a signal that is a known function of the two components:

$$\rho_t = \lambda + \exp\left(x + u_t\right). \tag{2}$$

Also ρ_t is the actual return to wealth invested in a business with average return $\lambda + \exp\left(x + \sigma_u^2/2\right)$. Since entrepreneurs can in fact see the sum $(x + u_t)$ and know the distribution of the two components they can update their beliefs about the average return of their project by using the initial prior distribution of x with mean $\widehat{x}_\tau = 0$, corresponding to the true population average with the sequence of observations $y_t = x + u_t$, $t = \tau + 1, \ldots\ldots, \tau + A$ where $\tau + 1$ indexes the age when the business was started and A is the number of signals received. Denoting by $\widehat{x}_{t+1}$ the expectation of the posterior distribution of the average return parameter, it can be easily shown that it follows the law of motion:

$$\widehat{x}_{t+1} = (1 - \Lambda_{a+1})\widehat{x}_t + \Lambda_{a+1} y_{t+1} \tag{3}$$

where $1 - \Lambda_{a+1}$ and Λ_{a+1} are the weights assigned to the prior average and new observation respectively, and are determined according to the Bayesian

[2] This assumption implies that the experience acquired by running a firm is entirely lost when the business is closed. One consequence is that the model cannot capture the fact that the probability of entry is higher for former business owners than for the general population as reported in Quadrini (2000). One alternative assumption would be that the draw on the project quality is taken from a better distribution if an agent had previously run a business in her life. This would help matching the fact reported by Quadrini and would increase the value of starting a business possibly leading to an even lower private equity premium. Adding this assumption would further complicate the solution of the model and is not pursued further here.

updating rule. [3] These weights are a deterministic function of the variance of the distribution of X and U and of the number of signals received, that is, the age of the business. The iteration starts with $\widehat{x}_t = 0$, the mean of the initial prior distribution which coincides with the distribution of x in the population. Finally λ is a constant that shifts up and down the expected project return without affecting other moments of its distribution.

Entrepreneurs also receive income from their endowment of human capital ω_t. Entrepreneurial earnings are given by $\omega_t \xi_t$ where ξ_t follows an i.i.d. process and $\xi \sim LN\left(\mu_\xi, \sigma_\xi^2\right)$. The process for ξ is highly correlated with ρ and hence with the noise u, but the correlation is not perfect. The assumption that the correlation between u and ξ is positive but not perfect is meant to capture the fact that according to the SCF, a member of the household is often the manager of the business at the same time that other members work outside the business. This presumably leads to high correlation between households earnings and return to private equity. A second property of ξ is that its mean μ_ξ is increasing with the average return to private equity so that business owners who run a good firm earn both a higher return on the financial capital they optimally choose to invest in and higher income from the fixed human capital they are endowed with.

All agents retire at the same age and those who are entrepreneurs at that age get back the equity in their business and convert it into stocks at no cost. All retired agents receive a fixed pension benefit and simply choose optimal consumption and savings. Post-retirement utility is subsumed in the function $V\left(d_{Ret}\right)$ which results from the optimal consumption/saving plan solved by an agent in the first year of retirement, given that he starts with current resources equal to d_{Ret}. Workers, entrepreneurs and retired agents all have access to the same risky financial asset.

Given the description above I can write the recursive formulation of the household utility maximization problem during working age. At each age t before retirement the household compares the utility of becoming a worker with that of choosing entrepreneurship. The optimal utility of choosing to become a worker in the next period for an agent whose state is described by the triple $(d_t, \widehat{x}_t, a)$, where $\widehat{x}_t$ and a have been described before and d_t are currently available resources, is:

$$V_t^{pe}\left(d_t, \widehat{x}_t, a\right) = \max_{c_t, S_{t+1}} u\left(c_t\right) + \beta p_{t+1} E_t V_{t+1}\left(d_{t+1}, \widehat{x}_{t+1}, 1\right) \quad (4)$$

[3] See Sargent and Ljunqvist (2000) for a derivation of this formula as well as the exact definition of Λ.

$$c_t + S_{t+1} \leq d_t \tag{5}$$

$$d_{t+1} = R_{t+1} S_{t+1} + \varepsilon_{t+1} \omega_{t+1} \tag{6}$$

$$\widehat{x}_{t+1} = (1 - \Lambda_1)\, \widehat{x}_t + \Lambda_1 y_{t+1} \tag{7}$$

$$\widehat{x}_t = 0 \tag{8}$$

$$S_{t+1} \geq 0,\ c_t \geq 0. \tag{9}$$

Notice that since agents receive a first signal about a new project while they are working for pay, the variable a is set to 1 in the right hand side of the Bellman equation; for the same reason in the law of motion for $\widehat{x}_{t+1}$ I have the index on the weight Λ taking the value one.

The utility of becoming an entrepreneur is instead defined as:

$$V_t^{se}(d_t, \widehat{x}_t, a) = \max_{c_t, S_{t+1}, K_{t+1}} u(c_t) + \beta p_{t+1} E_t V_{t+1}(d_{t+1}, \widehat{x}_{t+1}, a+1) \tag{10}$$

$$c_t + S_{t+1} + K_{t+1} \leq d_t \tag{11}$$

$$d_{t+1} = R_{t+1} S_{t+1} + \rho_{t+1} K_{t+1} + \xi_{t+1} \omega_{t+1} \tag{12}$$

$$\widehat{x}_{t+1} = (1 - \Lambda_{a+1})\, \widehat{x}_t + \Lambda_{a+1} y_{t+1} \tag{13}$$

$$\widehat{x}_{t+1} = x \ if \ a + 1 \geq L \tag{14}$$

$$K_{t+1} \geq \underline{K} > 0,\ S_{t+1} \geq 0,\ c_t \geq 0. \tag{15}$$

A few comments are needed about the evolution over time of the estimated average returns. First notice that the state variables referring to the returns to equity in the business are $\widehat{x}$ and a. While the true average return on equity invested in the business is fixed by x, agents do not observe it and use the estimated value $\widehat{x}$ instead. As the law of motion of $\widehat{x}_t$ follows a first order process, $\widehat{x}_t$ must appear as a state variable in the agent's decision problem. Also, the number of years a firm has been operated, a, is a state variable since agents need to know the distribution of estimated private equity returns next period in order to compute $E_t V_{t+1}$. That distribution is normal with a variance that evolves according to a deterministic law and is a function of the number of noisy observations on the parameter, that, is the age of the business. After a finite number of periods, denoted with L, the true value of x is revealed to agents, so that $\widehat{x}_t = x$ for $a \geq L$. I make this assumption for numerical reasons[4]. However, learning takes place very fast so that after a few periods the estimate that agents make about their average return is very precise and changes only marginally as new signals accrue. Since these changes drive the exit of firms, truncating learning after some periods does not affect firm dynamics. For expositional reasons in what follows I will label businesses whose type is already known "mature" and businesses whose type is still being learned "young".

Finally, the value function at time t is obtained as the result of the optimal occupational choice, that is:

$$V_t\left(d_t, \widehat{x}_t, a\right) = \max\left\{V_t^{pe}\left(d_t, \widehat{x}_t, a\right), V_t^{se}\left(d_t, \widehat{x}_t, a\right)\right\}. \tag{16}$$

The model is analytically intractable and therefore it is solved numerically. The decision rules obtained from the agent's dynamic programming problem are then used together with simulated series of shocks to produce individual histories of actual decisions and statistics are computed on the resulting aggregates.

3 Calibration

In this section I describe the choice of parameters in the model. The model period is assumed to be one year. Period utility is assumed to be of the standard CRRA form, that is:

$$u\left(c_t\right) = \frac{c_t^{1-\alpha}}{1-\alpha}. \tag{17}$$

The risk aversion parameter α is set to 2 and the subjective discount rate β is set at 0.96 values consistent with most macro studies like for example Aiyagari (1994). The maximum length of life is deterministic but there is a probability of dying at each age $1-\ p_{t+1}$ which I calibrate using mortality tables for the U.S. male population taken from "The Berkeley Mortality Database" (available at http://www.demog.berkeley.edu). Each agent's endowment of human capital ω_t evolves deterministically over time and is measured using the age profile estimated by Cocco, Gomes and Maenhout (2005) for high school graduates.

[4] An agent at the end of working life may have been in business for up to 45 years. Therefore by not truncating learning I would need 45 state space points along the firm age dimension slowing down the program considerably.

The risky financial asset in the economy, which I interpret as the index of public equity, has an average gross return of 1.08 and takes with equal probability the values [0.92, 1.24] corresponding to a standard deviation of 16 percent in line with the historical standard deviation of the S&P 500 index.

While working as a paid employee, the human capital endowment ω_t is hit by an i.i.d. shock ε_t which is assumed to be $LN\left(\mu_\varepsilon, \sigma_\varepsilon^2\right)$. I set $\mu_\varepsilon = 0$ and $\sigma_\varepsilon^2 = 0.025$. This latter value is taken from Hubbard, Skinner and Zeldes (1994) where it is the estimate of the standard deviation of the i.i.d. innovation to the AR(1) process they use to model wage earnings[5].

Next I describe how I choose the parameters related to the private equity return function:

$$\rho = \lambda + \exp\left(x + u\right). \tag{18}$$

The population distribution of average project quality from which x is taken is $N\left(0, \sigma_x^2\right)$ and the distribution of the noise u is $N\left(0, \sigma_u^2\right)$. In order to choose the two variances I first fix the ratio σ_x^2/σ_u^2 and then set the level of one of the two. The ratio is crucial in determining the statistical properties of the law of motion of the estimated average project return parameter $\widehat{x}_{t+1}$ which in turn are the key elements in fixing the exit rate of firms. Intuitively a lower ratio, is associated with noisier observations on the average project quality, hence learning takes place more slowly and exit rates are larger for a longer period. The criterion I follow to fix the ratio is then to match the ten year survival rate of firms reported by Dunn et al. (1988) for manufacturing firms and is obtained by setting $\sigma_x^2/\sigma_u^2 = 3/4$.

Given the ratio σ_x^2/σ_u^2, I then fix the level of the variances. Lacking an estimate of project risk based on actual data I consider three different levels of project risk. I pick the variance of the shock so that the model generates endogenously some given values of the standard deviations of the returns: 0.28, 0.35 and 0.5. These three values are very close to the ones used in Heaton and Lucas (2002), the highest value is close to the average volatility of individual stocks in the CRSP sample (see Campbell et al. 2001).

The other component of the pay-off to becoming entrepreneurs is the stream of earnings the agents receives when his human capital is used in his business

[5] The reason for this choice is that in this way the standard deviation of wage earnings one period ahead conditional on the current value is the same for the two models. Assuming AR(1) shocks would enrich the predictions of the model along the occupational choice dynamics. For example is this case agents would tend to enter entrepreneurship when they receive lower labor earnings shocks since this would predict a lower value of continuing as paid employees, something that is found in the data (see Evans and Leighton, 1989). On the contrary the findings of the model about the excess return of private equity would be basically unchanged since the return on the private equity index depends mainly on the risk of the financial investment faced by those who choose to become entrepreneurs.

instead of being rented out to the market. Here the calibration strategy is based on two goals. First it targets the ratio of average self-employment earnings to average worker earnings in the population: this is taken to be 1.2 following the value reported in Hamilton (2000). [6] Second it assumes a high correlation between this flow of earnings and the return to the investment the entrepreneurs makes in his business. This concerns both the average of the two and the idiosyncratic shocks that determine their yearly realizations. Unfortunately there is no empirical reference to fix the correlation coefficient so I set it to 0.75. The choice of a high number is meant capture the fact that most private businesses are run by a member of the household that owns them so that a "good business", or "good luck" in any given year of its life, are reflected on both earnings and capital returns. Because of the arbitrary choice of this number some sensitivity analysis is performed.

Another crucial variable in determining both entry and the model generated return on the index of private equity is the minimum capital requirement. In order to calibrate this parameter I try to make it comparable in size with the wage an agent would get as a worker. To do so I take data form Hurst and Lusardi (2002) about the amount of equity needed to start a business and compare it with the wage of beginning high school workers taken from the life-time profiles of earnings reported in Hubbard, Skinner and Zeldes (1994). As a result of this comparison I set the minimum equity requirement equal to one year of 20 year old worker wages.

Finally I describe how I set the constant λ. Notice that once all the other parameters have been fixed λ controls the average return on private equity simply by shifting it up and down. Rather than setting λ and looking at the resulting allocation as is typical in the quantitative literature about portfolio choice, I work the other way round by gauging a given average entrepreneurial portfolio composition and backing out the average private equity return that supports it. In practice I set the constant λ to a level such that entrepreneurs will hold on average 32 percent of their portfolio in business equity, a number taken from Moskowitz and Vissing-Jørgensen (2002).[7]

[6] Hamilton reports three different measures of entrepreneurial earnings. The one I use here is what he calls the EAD which includes what the entrepreneurs draws from the business plus the change in the equity invested net of the opportunity cost of that investment.

The finding that entrepreneurial earnings are higher than those of wage earners is common in the literature. Such result is obtained for example in Rosen and Willen (2002) as well. Moskowitz and Vissing-Jørgensen (2002) themselves find that conditional on a number of characteristics, like age and education the self-reported wages of entrepreneurs are higher than those of workers in the SCF.

[7] This number is obtained by multiplying the average share of private equity for those who have positive amounts of it by the percentage of that private equity invested in one single business. The two figures are 41 and 82 percent respectively. This choice is motivated by the fact that in my model agents can only run one business at a time.

4 Results

In the present section I report the results of the quantitative experiments. The presentation is organized in two subsections. In the first one a benchmark case is considered: a number of entrepreneurial investment features are examined in the models with and without learning and a discussion of the mechanism driving the results is conducted. In the second section I report results from a sensitivity analysis.

4.1 The Benchmark Case

I consider the benchmark case to be the one with intermediate project risk. This is obtained by setting $\sigma_u^2 = 0.048$, which generates an observed standard deviation of returns of 0.354 a value that is very close to the intermediate project risk considered in Heaton and Lucas (2002). The results are organized in two subsections, the first one about the dynamics of entry and exit from entrepreneurship and the second about entrepreneurial investment. They are reported both for the model with learning and the one without it. The latter case is obtained from the former by assuming that agents draw a value of x and not $x + u$ so that they make their entry decision based on the knowledge of the true project return. During the life of the project they then receive the usual sequence of i.i.d. draws from u. Also the constant λ that controls average return is adjusted so that the mean share investment in private equity is kept constant at its 32 percent target.

4.1.1 Entrepreneurial Dynamics

Results are reported in Table 1 and Figures 1 and 2. Table 1 reports data on firm entry and exit. In the model with learning the entry rate is 2.4 percent and the survival rate at a 10 year horizon is 23.6 percent. These two figures match exactly their calibration target, taken respectively from Evans and Leighton (1989) and Moskowitz and Vissing-Jørgensen (2002). In order to evaluate the performance of the model we then have to look at other statistics. The survival rate at a shorter horizon of 5 years in the model is of 30 percent, close to the empirical value of 38.5 percent. The average fraction of households who are in self-employment in the model is 14.1 percent which is close to the 11 percent figure in the SCF according to Moskowitz and Vissing-Jørgensen. Also the average age of firms is 7 years in the model and 10.3 in the SCF and the cumulative exit rate, that is 14.3 percent in the model, is according to Evans and Leighton (1989) equal to 21.6 percent using the CPS. Overall the statistics reported above show that the model does a very good job at reproducing the

Table 1: Entrepreneurial Dynamics, Intermediate Risk

	Data	Learning	No learning
Average entry rate	2.4 %	2.4 %	3.0 %
Survival rate (10 years)	23.6	23.6	87.9
Survival rate (5 years)	38.5	29.8	94.6
Cumulative exit rate	21.6	14.3	4.4
Fraction of entrepreneurs	11.0	14.1	38.3
Average firm age	10.3	7.0	22.2

main features of entry in and exit out of entrepreneurial activity. Learning with i.i.d. signals seems to take place slightly faster than what the data suggest: this shows up in that the survival rate at 5 year horizon is somewhat lower than its empirical counterpart. Another factor that helps explaining the small deviations of the model from the data are the two assumptions that there are no bequests and that retirement age is the same for workers and entrepreneurs, while in the data entrepreneurs tend to retire a few years later. Both effectively shorten the life of successful businesses reducing the average firm age in the model compared to the one in the data. If we look at the third column of Table 1 we see that the model without learning gives rise to very counterfactual results except for the average entry rate which is at 3 percent, a number that is only slightly above its data counterpart. The survival rates at 10 and 5 year horizons are 87.9 percent and 94.6 percent respectively and the corresponding cumulative exit rate is a puny 4.4 percent. Given the entry rate, such a low exit rate implies that the average share of entrepreneurs in the economy is 38.3 percent, more than three times the empirical value and the average firm age is 22.2 years, double the one in the data. The interpretation of these results is the following: in the model with learning the average quality of a project is observed imperfectly with the consequence that many agents who indeed have good entrepreneurial ideas may never try to carry them out, while others whose projects are of poor quality will try anyway. With time and the accrual of further signals about project quality these latter entrepreneurs will be selected out of the industry. This mechanism increases exit rates and keeps average firm age and the share of entrepreneurs in the population at values that are consistent with the empirical evidence. In the model with perfect information about project quality all and only those agents that have good entrepreneurial ideas enter self-employment. Given that the distribution of project quality in the population is the same in both models this implies about the same yearly entry rate but with perfect information there will be no exits except for those determined by retirement and the unlikely event of a string of very bad realizations of the idiosyncratic shock to returns.

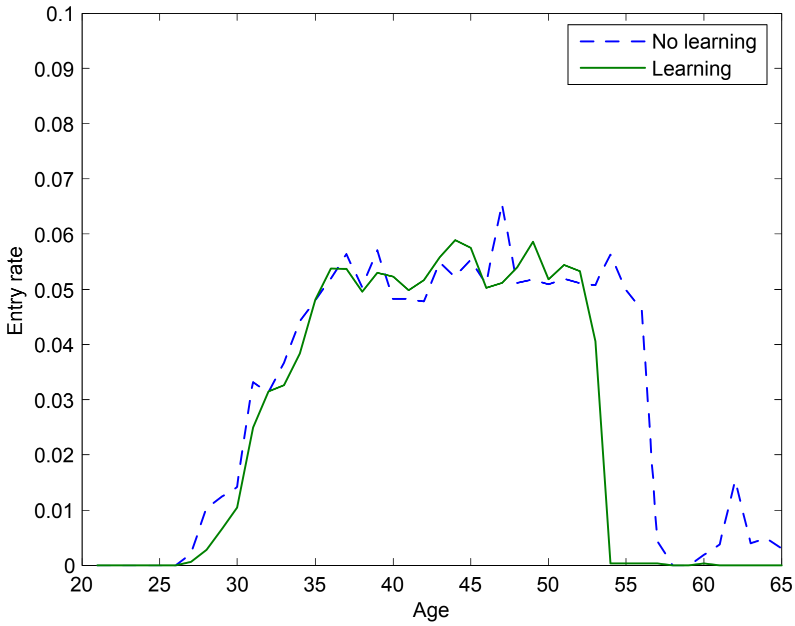

Figure 1: Figure 1: Entry rate by household age

The consequence is an implausible average length of entrepreneurial spells and a share of self-employed in the population that is too large.

Results about the way entry and the fraction of self-employed evolves over the life-cycle are reported in Figures 1 and 2. In Figure 1 I report the entry rate into self-employment by age of the household for both the model with learning and the model without learning. The two lines show a qualitatively similar pattern: entry is zero initially, then around age 27 it picks up quickly and reaches a plateau starting at 35. In the model without learning the rate of entry declines around age 57 but remains positive until retirement, while in the model with learning it becomes 0 around age 54. The interpretation of the pattern observed from the beginning to mid working life relies on the role of the borrowing constraint and the minimum investment scale: agents need to finance their business out of their own funds so they need some initial accumulation in order to exploit their entrepreneurial ideas when they occur. Given the small initial investment required around age 35 virtually all agents have enough wealth to start a business so that the entry rate stabilizes at the rate of arrival of potentially viable projects. The decline of entry rates to zero after age 54 in the model with learning can be explained by observing that, because of the high failure rates, the expected return early on in self-employment are lower than in paid employment, so that agents make the initial investment because this is needed to enjoy the option of the more favorable expected pay-off later on if the business proves to be successful. As retirement gets closer the horizon over which these later favorable returns can be enjoyed shortens making entry not attractive. The pattern of entry over the life-cycle is qualitatively similar to the one in the data except for the late decline in entry rate. As Evans and Leighton (1989) show the entry rate remains constant or even increases close to retirement. Another difference is that entry rates in the data, while increasing over the first part of life are never zero. The explanation for the first discrepancy lies in the assumption that retirement is fixed at age 65 for entrepreneurs in the model while in reality entrepreneurs tend to retire later. [8]

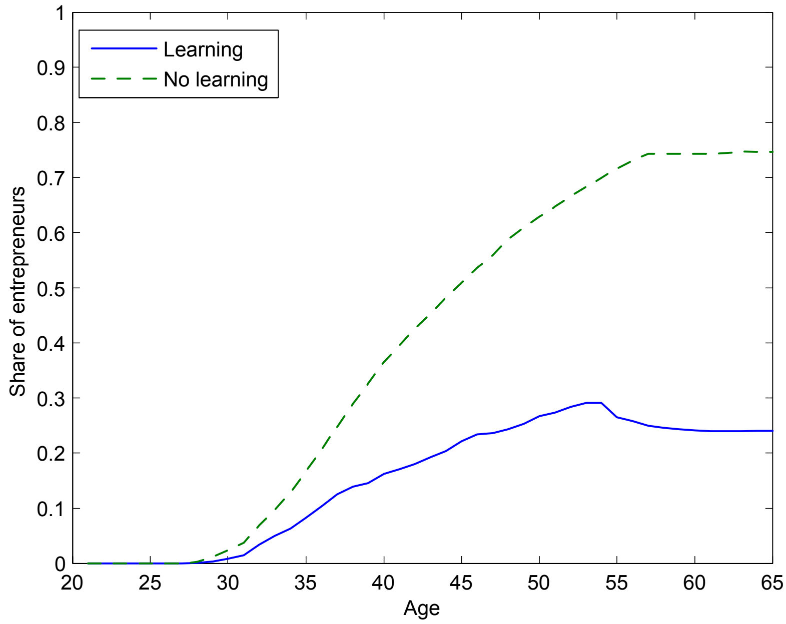

Figure 2: Share of entrepreneurs by household age

Allowing later retirement for the self-employed in the model would increase the relative value of opening a business not only because of the longer horizon left to reap the fruits if the business is successful, but also because later in life the

[8] For example Heaton and Lucas (2000) show that the share in private equity does not decline to 0 after age 65 according to the SCF, suggesting that many entrepreneurs continue their activity beyond that age.

value of the alternative occupation declines as lower pensions substitute wage earnings. As far as the second discrepancy is concerned the missing element in the current model are bequest and inter-vivos transfers that would allow some agents to have enough wealth to start a business even in the very first years of working life. As far as the percentage of entrepreneurs over the life-cycle is concerned Figure 2 shows the counter-factually high values in the model without learning according to which after age 55 about 75 percent of the population would be self-employed. In the model with learning we see that the percentage of the population that is self-employed increases early in life and then remains roughly constant after age 45, a pattern that is consistent with the evidence in Evans and Leighton (1989). Quantitatively the plateau is a little above 20 percent while it is around 13 percent in the data. This excess is in part explained by the fact that the average share of entrepreneurs is itself higher in the model than in the data in part because of the above mentioned absence of entry early in life which concentrates the mass of entrepreneurs towards mid-life.

Overall the results about the evolution of the percentage of self-employed over the life-cycle confirm the ability of the two key mechanisms proposed here, that is the liquidity constraint and learning to explain the patterns found in the data. Moreover the discrepancies found can be attributed to modeling choices that are not essential to those mechanisms, in particular the omission of inter-generational transmission of wealth and the assumption that retirement occurs at a fixed age that is the same for both workers and entrepreneurs.

4.1.2 Entrepreneurial Portfolio and Returns

Next we can look at results concerning entrepreneurial investment. Here the focus is on the portfolio allocation of the self-employed and the premium that private equity needs to pay to obtain that allocation. Given its partial equilibrium formulation, the model cannot directly address the recently emerged private equity premium puzzle. The question that I try to answer is then similar to the one in Heaton and Lucas (2002) and Polkovnichenko (2003) who, for a given excess risk generated by the idiosyncratic nature of private equity, look at the premium that must be paid to induce agents to make that investment. Contrary to those papers, here the size of the investment is not fixed but can be adjusted subject to a minimum equity requirement. Consequently the question itself is slightly altered: I look at the excess return on the index of private equity that generates the observed average cross-sectional share of private capital in the portfolio of entrepreneurs. The return on the index of private equity is defined as the average of individual firm returns — the ρs — weighted by the firms' share in total private equity. Results are reported in Table 2 for both the

Table 2: Entrepreneurial Portfolio and Return, Intermediate Risk

	Learning	No learning
Std. of returns	35.4 %	34.1 %
Private equity index return: all firms	9.2	10.5
Private equity index return: "mature firms"	11.4	–
Public equity return	8.0	8.0
Portfolio share of private equity: "mature firms"	41.0	–
Portfolio share of private equity: "all firms"	31.0	32.0

model with learning and the one without. In the top line we see the standard deviation of returns which determines the risk of the entrepreneurial firm; its value of about 35 percent corresponds to the intermediate case of Heaton and Lucas (2002) and is somewhat more than double the historical volatility of the S&P 500 index. In the last line we see that in both models the average entrepreneur holds about 32 percent of his wealth in a business as does the average American entrepreneur: this was a calibration target. In the second line we can see that the return to private equity that is needed to match that target is 9.2 percent in the model with learning. As can be read in the fourth row the exogenous return on the risky financial asset is 8 percent, corresponding to a 1.2 percent premium. If we look at the model with perfect information about project quality we see that the private equity return is in this case 10.5 percent implying a 2.5 percent premium over public equity. The difference between the private equity premia measures the contribution of learning and selection to the explanation of the low observed return to private equity: in practice under this calibration the premium is cut by 50 percent and left close to a modest 1 percent.

The explanation for this result is the following. In the model with learning at the time the entry decision is made the entrepreneur faces a very low expected return on his project because of the high probability that the true underlying return is low. As time passes and bad firms are selected out, the expected return increases. For firms that have learned their type the premium paid on the financial investment must be high if the entrepreneur is to invest a large fraction of his wealth in his business. This can be seen in the third row where the return to the set of mature firms — those whose type has been revealed — is reported: this value is 11.4 percent, 3.4 percentage points above the return to the financial asset. However the return on the index results from the aggregation of the return to all individual firms and these include the many young firms with low returns that will be selected out. Notice though that the allocation of wealth to private equity results from an optimal portfolio choice so that the premium paid to mature firm just compensates them exactly for the idiosyncratic risk of the project. One question then remains to be answered, that is, why entrepreneurs are willing to make the initial financial investment even though it is more risky and pays a lower expected return than the stock. Here the answer lies in the occupational choice dimension of the decision to invest one's wealth in a business. Choosing self-employment means foregoing wages to substitute them with an alternative earnings process. Consistent with the evidence I assume that on average self-employment earnings are greater than wages, consequently successful entrepreneurship gives a substantially higher return on the agents' human capital. The return to the initial financial investment in the business can then be thought as the sum of two components, the directly measured return and the option value of learning: the agent by investing in his firm buys the option to enjoy the higher expected earnings of a successful business if it proves to be so. We may then think about the negative premium on the initial investment as the price of this option.

Table 2 shows two more results about entrepreneurial investment and returns. First if we look at the table the average return to private equity in the complete set of businesses is 9.2 percent, while the return to mature firms only is 11.4 percent. Clearly the return to learning firms must be even lower than the average: as it can be seen from Figure 3 returns are monotonically increasing in the age of the business from a low -9 percent for newly founded firms to the 11.4 percent for mature firms mentioned above. Second, looking at the last two rows of Table 2, it can be seen that the average share of private equity in the complete set of households is 31 percent while it is 41 percent in the subset of households owning older firms whose type has been revealed. The share of portfolio in private equity of households who are still learning the quality of their project is then even lower than 31 percent implying that over time the portfolio share invested in the business increases for continuing entrepreneurs. This increase is the direct consequence on portfolio allocations of the pattern of increasing returns over firm age induced by learning and selection that was described above. While at present there is no empirical evidence supporting the existence of a pattern of increasing returns over firms' age, there is evidence in favor of the second result. [9] Gentry and Hubbard (2000), using the 1983 and 1986 waves of the SCF — the only two that have a panel dimension — found that among continuing entrepreneurs the share of wealth invested in business

[9] There is a large empirical literature on firm dynamics that explores the relationship between age and different characteristics of firms but it focuses on survival and growth rates rather than returns. Moreover as Evans (1987) points out the sample of firms examined largely under-represents the very small ones that are an important element of a model like the one in this paper that is focused on firms as the result of an occupational choice. For these reasons that literature is not of much help in the present context.

assets increases over time.

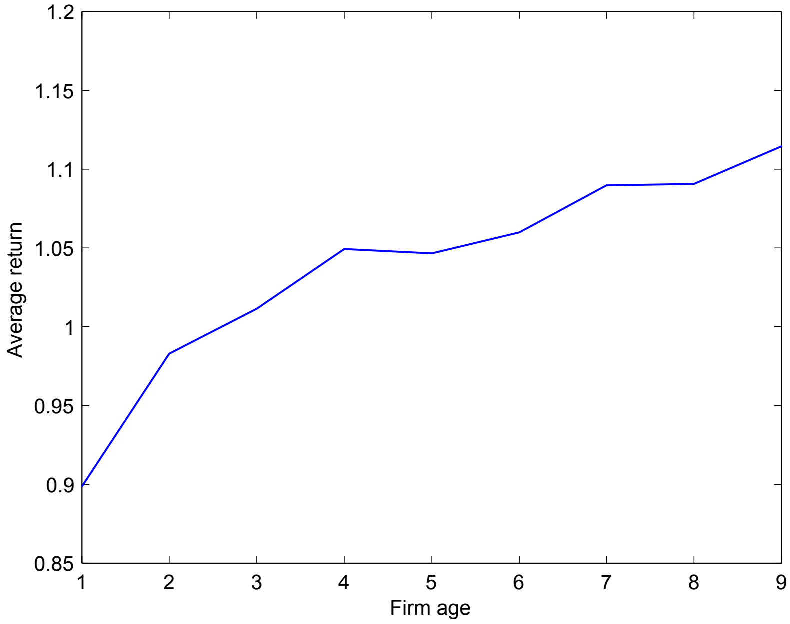

Figure 3: Return by age of firms

A number of caveats have to be made about the results in this section. First it was said before that the initial investment carries an unmeasured return in the form of the value of high entrepreneurial earnings in case of success. This return is unmeasured only as return to the financial investment in the early life of the firm; the earnings that create it are instead perfectly measured monetary earnings. This is important to stress because the existence of unmeasured benefits, like the one of being "your own boss" are often cited as explanations for the choice to become entrepreneurs. These kinds of benefits are not modeled in the present research; if they were they could help further reduce the private equity premium. Second, given the choice of a CRRA utility function and the value of risk aversion the model premium may seem at first sight huge and might suggest a 100 percent allocation to private equity. The reason why it is not is that with the current calibration private equity is really very risky: both the standard deviation of returns and the correlation between self-employment earnings and returns are very high and as Heaton and Lucas (2000) show in a traditional model of portfolio allocation between a stock and a bond a relatively modest positive correlation between earnings and stock returns tilts the demand towards bonds significantly. Whether this calibration is reasonable is difficult to assess since the evidence, like for example the one about the standard deviation of returns cited in Moskowitz and Vissing-Jørgensen, is indirect and based on returns to individual publicly traded stocks, probably because of lack of sufficiently good data on private firms. [10] For this reason in the next section I report the results of a sensitivity analysis on the parameters that control the amount of risk entailed by private equity investment. Third, the present model makes a distinction between the earnings of the entrepreneur and the return on the money invested in the closely held business even though both enter jointly the occupation decision. This distinction is commonplace; for example in the SCF a separate question is asked about the wages and profits entrepreneurial households make out of their firm. Based on this distinction the empirical work, like the cited works of Moskowitz and Vissing-Jørgensen (2002) or Hamilton (2000) look at one of the two components in isolation and suggest that some non-measured benefits of entrepreneurship or some form of overconfidence must exist to justify this occupational choice. This distinction is somewhat arbitrary and may be misleading since in the end what determines the choice to become entrepreneurs is the expected value and volatility of the overall income stream that the business provides, while any of the two components taken separately is not sufficiently informative. The important step forward that the present paper takes is precisely that here the choice to become entrepreneurs is based on the overall pay-off that a household that is making the entry decision expects to receive. In the empirical literature the approach I propose here has been adopted by Rosen and Willen (2002) who estimate an entrepreneurial income process on PSID data and find that the combination of average and volatility is such that all but extremely risk averse individuals would choose entrepreneurship. This result stands in sharp contrast with the one in Moskowitz and Vissing-Jørgensen although the lack of information about the investment of financial resources in the business limits the conclusiveness of their analysis.

4.2 Sensitivity Analysis

In this section I report the results that obtain when the variance of the i.i.d. shocks to entrepreneurial income and returns is changed and summarize briefly results of other sensitivities. In reporting the results I focus both on self-employment dynamics and on the returns and portfolio choice of entrepreneurs.

[10] The available data on private entrepreneurs come from the SCF and the PSID. The SCF has very good quality asset data and allows to correctly represent the wealthy entrepreneurs but lacks the panel dimension that would be needed to estimate the actual risk of private businesses. The PSID has the panel dimension but collects data on assets only every five years.

Table 3: Entrepreneurial Dynamics, Low Risk

	Data	Learning	No learning
Average entry rate	2.4 %	2.4 %	2.9 %
Survival rate (10 years)	23.6	24.0	87.8
Survival rate (5 years)	38.5	29.5	94.7
Cumulative exit rate	21.6	14.1	4.6
Fraction of entrepreneurs	11.0	14.1	38.3
Average firm age	10.3	6.9	21.8

First I consider a low risk scenario where I set $\sigma_u^2 = 0.033$. Those results are reported in Table 3 and 4. This corresponds to the low risk project considered in Heaton and Lucas (2002) as can be seen from the standard deviation of returns equal to 27.3 percent reported in the first line of Table 4. A look at Table 3 shows that the dynamics of self-employment is very similar to the one of the intermediate risk case. In the model with learning the entry rate is 2.4 percent and the 10 year survival rate is 24.0 percent: once again these were calibration target so they correspond to the data counterparts. The survival rate at 5 years is 29.5 percent close to but a little below the 38.5 percent registered in US data. The overall exit rate is 14.1 percent and the fraction of entrepreneurs in the population is 14.1 percent as well. Both figures are close to the data that show a cumulative exit rate of 21.6 percent and an average fraction of entrepreneurs of 11.0 percent. The model then underestimates the first figure and overestimates the second. When the no learning case is considered we get once again very counterfactual predictions except for the entry rate that with a 2.9 percent figure is only slightly above the US data. Exit is very low at a rate of 4.6 percent which is also reflected in the extremely high survival rates: 94.7 percent at a five year horizon and 87.8 percent at a 10 year horizon. The consequences are a share of entrepreneurs of 38.3 percent and an average firm age of 21.8 years, both much higher than what is found in the data. Results concerning entrepreneurial portfolio allocation can be found in Table 4. Here there are some changes compared to the intermediate risk case. The return to the private equity index in the model with learning is 7.9 percent while the exogenous return to the risky financial asset is 8 percent implying in this case that the model can even generate a negative private equity premium. In the model without learning by contrast the return to the index of private equity is 9 percent, 1.1 percentage points above the one of the model with learning. As in the intermediate risk case, in the model with learning, mature firms face a higher expected return, which is now 9.7 percent and the owning household invest a larger share of its portfolio in the business, that is, 40.3 percent.

Table 4: Entrepreneurial Portfolio and Return, Low Risk

	Learning	No learning
Std. of returns	27.3 %	26.2 %
Private equity index return: all firms	7.9	9.0
Private equity index return: "mature firms"	9.7	–
Public equity return	8.0	8.0
Portfolio share of private equity: "mature firms"	40.3	–
Portfolio share of private equity: "all firms"	32.6	32.9

Next we look at the results for the high risk case obtained by setting σ_u^2 equal to 0.072. With this value of the variance of the noise, the model with learning generates a standard deviation of returns of 48 percent in line with the high project risk considered in Heaton and Lucas (2002). This value is also close to the volatility of individual stock returns reported in Campbell et al. (2001). Statistics about entrepreneurial dynamics are reported Table 5. In the model with learning, given an entry rate of 2.3 percent and a survival rate at 10 years of 23.8 percent, both chosen to match the values in the data, we find that the cumulative exit rate is 14.3 percent and the fraction of entrepreneurs is 13.7 percent, the first slightly below the one in the data and the second a little above. The survival rate at a 5 year horizon is 29.6 percent and the average firm age is 7 years. In the model without learning again except for the entry rate of 2.9 percent all other statistics are very far from the data: at a low rate of 4.4 percent exit virtually reflects retirement of the firm's owner and the survival rates at both chosen horizons exceed 90 percent. As a consequence the average share of entrepreneurs in the population is 38.8 percent and the average firm age is 22.8 years. Table 6 reports the results about the private equity premium. Given the extremely high risk of investment in a business in this case the return needed to induce entrepreneurs to hold on average 32 percent of their wealth in private equity in the model without learning is 13.6 percent, 5.6 percentage points above the return to the diversified portfolio of stocks. In the model with imperfect information about project quality, the operation of low productivity firms reduces the private equity return by about 2 percentage points to 11.8 percent, leaving an unexplained premium of 3.8 percent. Looking in more details at the model with learning we see that mature entrepreneurs enjoy a 14.6 percent return but even with such a high return their optimal portfolio choice is to invest only 34 percent of their wealth in private equity. The fact that continuing entrepreneurs invest larger portfolio shares in their firm found by Gentry and Hubbard (2000) is confirmed also in this case even though the magnitude of the effect of the higher expected return is lower than

Table 5: Entrepreneurial Dynamics, High Risk

	Data	Learning	No learning
Average entry rate	2.4 %	2.3 %	2.9 %
Survival rate (10 years)	23.6	23.8	90.3
Survival rate (5 years)	38.5	29.6	96.3
Cumulative exit rate	21.6	14.3	4.4
Fraction of entrepreneurs	11.0	13.7	38.8
Average firm age	10.3	7.0	22.8

with less risky projects.

Other sensitivities with respect to the correlation between the earnings process of the entrepreneurs and the return to his private equity investment were performed. Briefly summarized the results were that reducing this correlation does not affect the dynamics of entry in and exit out of self-employment but reduces the excess return of private equity needed to trigger the targeted portfolio allocation.

Summarizing the results of this section it can be said that changes in the risk of the entrepreneurial project — in the range considered here — does not affect the basic life-cycle patterns of entry into self-employment and the exit dynamics out of it. The intuition is that the former depend mainly from the joint operation of the liquidity constraint and the life-cycle accumulation of wealth that provides the resources to overcome it. The latter depends on the assumption of Bayesian learning of an i.i.d. process and the calibration of the ratio between the uncertainty about project quality and the volatility of the idiosyncratic risk, rather than on the latter only. On the contrary changes in the risk of the return process affect directly the portfolio choice that entrepreneurs make and therefore the required premium for a given investment. The intuition here is straightforward: a higher variance of returns or higher correlation with the earnings process of the self-employed implies more risk and therefore a higher premium to induce the same portfolio allocation.

5 Conclusions

This paper presents a Jovanovic's (1982) style industry dynamics model with learning about the quality of firms but it makes the value of the outside option endogenous by merging that framework with a life-cycle occupational choice model where the alternative to stay in the industry is to work as paid employee. The model is calibrated and its quantitative properties are examined showing how the joint operation of life-cycle accumulation, liquidity constraints

Table 6: Entrepreneurial Portfolio and Return, High Risk

	Learning	No learning
Std. of returns	48.0 %	47.0 %
Private equity index return: all firms	11.8	13.6
Private equity index return: "mature firms"	14.6	–
Public equity return	8.0	8.0
Portfolio share of private equity: "mature firms"	34.0	–
Portfolio share of private equity: "all firms"	31.0	32.3

and learning can explain quite well the main features of the life-cycle entry and exit patterns of entrepreneurs. The model also looks at the decisions of entrepreneurs about how to allocate their wealth between the private firm they own and other assets and addresses the private equity premium puzzle. While its partial equilibrium nature does not allow it to provide a resolution of this puzzle the model still provides a useful explanation that is based on heterogeneity of firms' average returns and the higher earnings that the successful ones provide to their owner: in the model successful firms indeed provide the high returns needed to compensate for idiosyncratic risk but the return on the index is lowered by the many young firms with low pay-offs that are doomed to exit. In turn households are willing to accept this lower pay-off early on because it is a price to pay to enjoy the higher return on human capital if the entrepreneurial idea proves to be good.

The present paper also points to other avenues for research. For example it would be interesting to look at how the prospect of becoming entrepreneurs affects savings and what are the relative patterns of wealth accumulation of entrepreneurs vs. workers: this work was done by Quadrini (2000) and Cagetti and De Nardi (2002) in an infinite horizon context but it would be useful to analyze it in the context of a finite horizon model like the one in this paper. More importantly in addressing entrepreneurial returns the present paper clearly points to the need to reassess the issue of the private equity premium puzzle by explicitly acknowledging its occupational choice dimension. This would require considering the average and the volatility of the whole stream of income that a given investment in a closely held business guarantees his owner, rather than looking separately at the wage and capital return component. This is challenging empirical work because the data are of much lower quality than those about public firms, however it is definitely needed to reach a better understanding of entrepreneurial activity. All this work is beyond the scope of the present project and is left for future research.

References

[1] Cagetti M. and De Nardi M. (2002). "Entrepreneurship, Frictions and Wealth". *Federal Reserve Bank of Minneapolis Working Paper* 620.

[2] Campbell John Y.; Lettau Martin; Malkiel Burton G.; Xu Yexiao. "Have Individual Stocks Become More Volatile? An Empirical Exploaration of Idiosyncratic Risk." *Journal of Finance*, February 2001, 56(1), pp. 1-44.

[3] Campbell John Y., Cocco Joao, Gomes Francisco and Maenhout Pascal. 1999. "Investing Retirement Wealth ? A Life-cycle Model". Working paper, Harvard University.

[4] Cocco Joao; Gomes Francisco and Maenhout Pascal. 2005. "Portfolio Choice over the Life-Cycle". *Review of Financial Studies*, 18(2): 491-533.

[5] Dunne Timothy; Roberts Mark J. and Samuelson Larry. 1988. "Patterns of Firm Entry and Exit in U.S. Manufacturing Industries." *RAND Journal of Economics*, 19(4), 495-515.

[6] Evans David S. 1987. "Test of Alternative Theories of Firm Growth". *Journal of Political Economy*, 95(4): 657-674.

[7] Evans David S. and Leighton Linda S. 1989. "Some Empirical Aspects of Entrepreneurship". *American Economic Review*, 79(3): 519-535.

[8] Gentry, William M. and Hubbard Glenn R. "Entrepreneurship and Household Saving".National Bureau of Economic Research, (Cambridge, MA) Working Paper No. 7894, September 2000.

[9] Gomes, Francisco and Michaelides Alexander. 2002. "Life-Cycle Asset Allocation: A Model with Borrowing Constraints, Uninsurable Labor Income Risk and Stock-Market Participation Costs". Working Paper

[10] Hamilton, Barton H. 2000. "Does Entrepreneurship Pay ? An Empirical Analysis of the Returns to Self-Employment", *The Journal of Political Economy*, 108 (3): 604-631

[11] Heaton John and Lucas Deborah. 2000. "Portfolio Choice in The Presence Of Background Risk", *Economic Journal*. 110: 1-26.

[12] Heaton John and Lucas Deborah. 2000. "Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk". *The Journal of Finance*. 55(3): 1163-1198

[13] Heaton John and Lucas Deborah. 2002. "Capital Structure, Hurdle Rates, and Porfolio Choice- Interactions in an Entrepreneurial Firm", Working Paper.

[14] Hintermaier Thomas and Steinberger Thomas. 2005. "Occupational Choice and the Private Equity Premium Puzzle". *Journal of Economic Dynamics and Control*. 29: 1765-1783

[15] Hopenhayn Hugo A.. and Vereshchagina Galina. 2003. "Risk-Taking by Entrepreneurs". Working Paper.

[16] Hubbard Glenn; Skinner Jonathan and Zeldes Stephen. 1994. "The Importance of Precautionary Motives in Explaining Individual and Aggregate Saving". *Carnegie-Rochester Conference Series in Public Policy*. 40: 59-126.

[17] Hurst Erik and Lusardi Annamaria. 2002. "Liquidity Constraints, Wealth Accumulation and Entrepreneurship". *Journal of Political Economy*, 112 (2): 319-348.

[18] Jensen, Michael C. and Meckling William H. 1976. "Theory of the Firm: Managerial Beahavior, Agency Costs and Ownership Structure". *Journal of Financial Economics*3: 305-360.

[19] Jovanovic, Boyan. 1982. "Selection and The Evolution of Industry". *Econometrica*, 50: 649-70.

[20] Ljungqvist, Lars and Sargent Thomas J. 2000. *Recursive Macroeconomic Theory*, Cambridge, Massachussets, MIT press.

[21] Moskowitz, Tobias and Vissing-Jørgensen Annette. 2002. "The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle ?". *American Economic Review*, 92(4): 745-778.

[22] Polkovnichenko Valery. "Human Capital and the Private Equity Premium". *Review of Economic Dynamics*, October 2003, 6 (4) pp. 831-845.

[23] Quadrini Vincenzo. 2000. "Entrepreneurship, saving and social mobility", *Review of Economic Dynamics*, 3,(1): 1-40.

[24] Rosen Harvey and Willen Paul. 2002. "Risk, Return and Self-Employment". Working Paper.