MARKET EQUILIBRIUM UNDER THE CIRCUMSTANCES OF SELECTABLE ECONOMIC CONDITIONS

Osamu Keida

WP-AD 2006-02

Correspondence: Kumamoto Gakuen University (E-mail:keida@kumagaku.ac.jp)

Editor: Instituto Valenciano de Investigaciones Económicas, S.A.
Primera Edición Febrero 2006
Depósito Legal: V-1109-2006

MARKET EQUILIBRIUM UNDER THE CIRCUMSTANCES OF SELECTABLE ECONOMIC CONDITIONS *

Osamu Keida †

ABSTRACT

This paper presents an analysis of market equilibrium under the circumstances with several discrete economic conditions by using pure exchange economy model. First, as preliminary analysis, it will show the 'temporal' market equilibrium under a given distribution of population over the different circumstances in section 2. Next, in section 3 our study will prove the existence of market equilibrium in the case that economic agents can choose their economic conditions freely for their utility maximization. Finally our research tries to approximate our model to the residential location model through the specified assumptions on initial endowments and agent's preference, and it derives some properties of equilibrium consumptions and prices.

Keywords: equilibrium, local goods, excess utility,
JEL classification: D51, R13, R20

1 Introduction

We shall try to extend the ordinal market equilibrium analysis to a more general case: the market consisted of selectable economic circumstances by using pure exchange economy model, and then to approximate this model to the urban economic model by making assumptions specific.

In the ordinal market model an economic agent determines the optimal behavior to the prices of goods, and the difference between the market demand and the market supply of each good affects its price and other prices, which influences each agent's behavior of demand or supply again. An equilibrium solution is obtained by this repetition if it exists.

By contrast, in urban economic models and regional economic models the prices of some goods like land rent differ depending on the locations of economic activities, and then the consumption quantities of such goods are different at distinct locations. The supplies of such goods also differ at every locations. Each agent of the economy selects a locational point in order to determine his optimal economic behavior for maximal utility or profit. Thus, the eminent feature of urban economic models and regional economic models is the choice of a locational point which means the choice of prices at which an agent faces on his location. The agent performs utility maximization or profit

*The author wishes to thank the anonymous referee of IVIE for helpful comments, and the author also wishes to thank Professor Otani, Y. of Kyushu Sangyo University for useful comments on West Japan Economic Conference(2003 and 2005).

†Kumamoto Gakuen University (E-mail:keida@kumagaku.ac.jp) and University of Alicante (visiting scholar from September 2004 to February 2005)

maximization on his locational point. If the utility or profit is smaller than the one derived in other point, then the agent will change his location for a better level of utility or profit. In the end an equilibrium is attained so that the level of each agent's utility or profit is indifferent in the same group type even if his location is different from other agents.

We shall extend an analysis of the typical market model to show the existence of market equilibrium of an extended model in which the choice of location means the choice of the prices of goods dependent on locations. There has been made a lot of works on the general equilibrium model so far. Among them we find some platform researches with a rigid and strict framework: Arrow and Hahn [2], Debreu [3] and Hildenbrand [3]. On the other hand we encounter some papers of axiomatic approach in urban economics as in Turnbull[8] and [9] which spur us to make an analysis in this direction. Thus we try to apply the results of the general equilibrium theory to the case of 'selectable' economic circumstances by relying on a newer research of Villar [10] who gives us a clear-cut procedure of proof.

To this end we shall introduce a new key concept for equilibrium:'excess utility', which is a distinctive feature of this paper. We can review a lot of splendid works such as Alonso[1], Mills[6] and Wheaton[11] in the area of urban economics. The structure of a city: housing demand, rent, population and city size were well analyzed with comparative analysis of equilibrium. However, since the interests of these researches oriented toward the structure of urban area, the equilibrium mechanism toward equilibrium in an urban area was not specified clearly like the general equilibrium theory. Here we try to reinforce equilibrium mechanism in the urban economic models to employ a new concept for equilibrium:'excess utility' which corresponds to excess demand in the general equilibrium theory and has an important role in equilibrium analysis.

The basics of our model is as follows. An economy consists of different economic circumstances that each agent can choose freely. The economy has two kinds of goods; one type of good of which price changes with the circumstances and the other type of good of which price is constant over the circumstances. These goods which are given to all agents in certain amounts at an initial state are traded among the agents as an pure exchange economy. The market clearing condition of all goods does not necessarily yield a 'permanent' but a 'temporary' equilibrium of the economy. This is because the agent distribution over the circumstances is given when the market condition becomes cleared. If the utilities of agents are different in the circumstances and some agents obtain smaller levels of utility, then those agents will change their location in seeking a better utility level, which breaks the temporary equilibrium. However such repetition seeking for a new 'temporary' equilibrium attains at a 'permanent' equilibrium in the end. The 'permanent' equilibrium is the situation in which agents will not change their location at temporary equilibrium. A new concept of agent's 'excess' utility away from average utility plays a significant role for the permanent equilibrium in Section 3 corresponding to the market clearing condition of goods. Once the economy satisfies the new condition that agent's excess utility becomes zero, the economy attains an equilibrium of agent distribution which results in a permanent equilibrium of the economy.

Another purpose of this paper is to show the property of equilibrium in the economy of selectable economic circumstances. The property is obtained in the model specified with additional assumptions on the agent's initial holdings of goods and preference for goods, which approximate our model to the residential location model. The main feature of the residential location model is that consumption of residential service or land increases with the distance from city center, the consumption of composite goods decreases with the distance, and the price of residential service declines with the distance. This paper derives the same kind of properties of goods and price like the urban economic model through the specification of the model.

The paper has four sections as follows. In Section 2, a basic model setting is made following Villar[10] and Debreu [3]. After model building, we shall make a preliminary analysis of a 'temporary' market equilibrium when the location of each agent is given. Section 3 treats the main task of this paper to prove the market equilibrium when each agent choose its location freely. Here is given a new condition of excess utility over average utility. Section 4 gives further analysis of equilibrium which provides some properties of equilibrium in a specified economy with additional assumptions on initial endowments and preference. The result of this section coincides with the results of the residential location model. Finally the conclusion follows in Section 5.

2 Model setting and 'temporary' market equilibrium

First, we will make a model here to perform a preliminary analysis of 'temporary' equilibrium in selectable economic circumstances.

Economic circumstance and goods

An 'economic circumstance' is defined as the place of economic activity that brings the prices of goods and the income of an agent to change. In this paper it is assumed that economic circumstances are discrete, and that the total number of the circumstances is $\tilde{j}$, indexed by $j = 1, 2, \cdots, \tilde{j}$. Each economic agent is assumed to choose only one from the economic circumstances. Thus the each circumstance is mutually exclusive as a candidate for agent's location.

Two types of goods:

- 'Local goods' dependent on economic circumstances: the each market of local goods is formed in each economic circumstance one by one, and then different markets have different prices depending on the circumstances. For example different values of rent are bided on different points. The number of local goods is assumed to be 1, and Good 1 is the local goods.

- 'General goods' independent of economic circumstances: only one market is constituted with all the circumstances for each gneneral good. Then a common price level of a general good is formed over the economy. Here, the number of the general goods is assumed to be $\tilde{l} - 1$, and the goods are indexed by $l = 2, \cdots, \tilde{l}$.

The prices of goods are denoted by a vector $\boldsymbol{p}$. Since the local goods form different markets based on the economic circumstances, different prices are formed in those circumstances. Then, the price of a local good at economic circumstance j is denoted by p_{1j}, and the price of general good 2 to $\tilde{l}$, by p_l.

$$\boldsymbol{p} = (\underbrace{p_{11}, p_{12}, \cdots, p_{1j}, \cdots, p_{1\tilde{j}}}_{\text{prices of local goods}}, \underbrace{p_2, p_3, \cdots, p_{\tilde{l}}}_{\text{prices of general goods}}) \in P \subset R_+^{\tilde{j}+\tilde{l}-1}$$

Economic agent (consumer) and consumption plan

It is assumed that there exist $\tilde{i}$ types of agents, and that I is the set of agents. Agent type i is the continuous entity which is a point in the interval $I_i \equiv [a_i, b_i]$, which is divided into the intervals I_{ij} that type i consumers locate in j at the initial state. Denote the number of agents of I_i, $n(I_i)$ as the size of the interval $n_i = b_i - a_i > 0$, and the number of agents of I_{ij} as $n(I_{ij}) \equiv n_{ij}$.

$$\sum_{j}^{\tilde{j}} n_{ij} = n_i.$$

Let us denote further the size of the type i agents who change his locational point from j in the initial state to h in the terminal state as n_{ijh}. $\tilde{\boldsymbol{n}}_{ij} \equiv (n_{ij1}, \cdots n_{ij\tilde{j}})$ is the distribution of agent i at a initial state, and $\tilde{\boldsymbol{n}}_{ij}$ is defined on the simplex

$$N^{ij} \equiv \left\{ \tilde{\boldsymbol{n}}_{ij} \in R_+^{\tilde{j}} \mid \sum_{h=1}^{\tilde{j}} n_{ijh} = n_{ij} \right\}.$$

The distribution of all agent types is $\boldsymbol{n} \equiv (\tilde{\boldsymbol{n}}_{11}, \tilde{\boldsymbol{n}}_{12}, \cdots, \tilde{\boldsymbol{n}}_{\tilde{i}\tilde{j}})$. The space N is defined as

$$N \equiv \prod_{i=1}^{\tilde{i}} \prod_{j=1}^{\tilde{j}} N^{ij}.$$

Consumption of goods and its utility

An economic agent selects a consumption plan which maximizes utility under his budget constraint. This consumption plan here includes the selection of one economic circumstance. However, Section 2 supposes that each agent makes consumption decision on an economic circumstance given to him.

The agents of the same type are assumed to be identical in terms of taste in consumption of goods, and then those agents have a identical utility function. The utility function u^{ij} of the type i agent of the initial circumstance j is defined on a consumption set of $R^{\tilde{j}+\tilde{l}-1}$.

Suppose that type i agent of the initial economic circumstance j has moved to the circumstance h. The consumption vector of the agent at the circumstance h is

$$\boldsymbol{x}^{\ ijh} = (0, \cdots, 0, x_{1h}^{ij}, 0, \cdots, 0, x_{2h}^{ij}, \cdots, x_{\tilde{l}h}^{ij}) \in R^{\tilde{j}+\tilde{l}-1}, \tag{1}$$

where x_{1h}^{ij} is the consumption of goods 1 of the agent type i of the initial circumstance j at the economic circumstance h, and $x_{lh}^{ij} (l = 2, \cdots, \tilde{l})$ is the consumption of the general goods of the agent. The parts of local goods and general goods are also expressed as

$$\bar{\boldsymbol{x}}^{ijh} \equiv (0, \cdots, 0, x_{1h}^{ij}, 0, \cdots, 0), \qquad \tilde{\boldsymbol{x}}^{ijh} \equiv (x_{2h}^{ij}, \cdots, x_{\tilde{l}h}^{ij}).$$

The consumptions of local goods at the circumstances other than h are always zero for the agent who locates at the circumstance h. Therefore, the only $(x_{1h}^{ij}, \tilde{\boldsymbol{x}}^{ijh})$ determine the utility of the agent at circumstance h. The utility function $u^{ij}(\boldsymbol{x}^{ijh})$ can be written as

$$u^{ijh}(x_{1h}^{ij}, \tilde{\boldsymbol{x}}^{ijh}),$$

where the consumption element is

$$(x_{1h}^{ij}, \tilde{\boldsymbol{x}}^{ijh}) \in X_h^{ij} \subset R^{\tilde{l}}.$$

Consumption plan and analysis of market equilibrium

We will make an analysis of market equilibrium in two steps in this paper.

Consumption planning in two steps:

Step1: Optimal consumption and market equilibrium under the economic circumstance given to each agent: an optimal consumption is made when each agent's location is given at some economic circumstance. (analysis in Section 2)

Step2: Optimal consumption by selection of economic circumstance: an optimal consumption is made under selection of economic circumstance. (analysis in Section 3)

In Step1 we consider an optimal consumption plan when an economic circumstance is given to each agent. Since an agent consumes the local good only at his own circumstance, the consumption of the good is zero at other circumstances. For example, when a consumer is the type i agent who moves from the initial economic circumstance j to the circumstance h, his consumption takes place at h. Hence his consumption will be $x_{1h}^{ij} \geq 0, x_{lh}^{ij} \geq 0 (l = 2, \cdots, \tilde{l})$ and $x_{1m}^{ij} = 0 (m = 1, 2, \cdots, \tilde{j}, m \neq h)$.

In Step2 each agent will attain his optimal consumption by selection of the most preferable economic circumstance. All circumstances are the candidates for his location, and then each agent tries to maximize utility by reselecting the most preferable circumstance. If the type i agent of initial circumstance j changes his location from h to h', then his consumption vector becomes $(0, \cdots, 0, x_{1h'}^{ij}, 0, \cdots, 0, x_{2h'}^{ij}, \cdots, x_{\tilde{l}h'}^{ij})$ from $(0, \cdots, 0, x_{1h}^{ij}, 0, \cdots, 0, x_{2h}^{ij}, \cdots, x_{\tilde{l}h}^{ij})$.

Initial endowment of goods and budget set

The agents of a same type are assumed to have an identical initial endowments of goods, if their initial circumstances are the same location. Although the agents belong to the same type, if their initial circumstances are different then their initial endowments might be different. As for the type i agent of initial circumstance j, his initial holding of good 1 is denoted as ω_{1j}^{ij}, and the initial holdings of other goods, as ω_l^{ij}. Hence the initial holdings of goods of the agent of type i at initial circumstance j is

$$\boldsymbol{\omega}^{ij} = (0, \cdots, 0, \omega_{1j}^{ij}, 0, \cdots, 0, \omega_2^{ij}, \cdots, \omega_{\tilde{l}}^{ij}),$$

which is decomposed with local goods $\bar{\boldsymbol{\omega}}^{ij} \equiv (0, \cdots, 0, \omega_{1j}^{ij}, 0, \cdots, 0)$ and general goods $\tilde{\boldsymbol{\omega}}^{ij} \equiv (\omega_2^{ij}, \cdots, \omega_{\tilde{l}}^{ij})$.

A consumer is supposed here to decide his optimal consumption with the initial holdings of goods which is sold to make his incime of budget for consumption plan.

Consumption budget and demand correspondence

The type i agent of the initial economic circumstance j makes purchase of his necessary goods by using the initial holdings of goods $\boldsymbol{\omega}^{ij}$. Let us denote the budget set of goods which the consumer of this type can buy with the initial holdings at the circumstance h as

$$\beta^{ijh}(p_{1j}, p_{1h}, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \tilde{\boldsymbol{\omega}}^{ij}) \equiv \left\{ \left(x_{1h}^{ij}, \boldsymbol{x}^{ijh} \right) \in X_h^{ij} \mid p_{1h} x_{1h}^{ij} + \tilde{\boldsymbol{p}} \cdot \boldsymbol{x}^{ijh} \leq p_{1j} \omega_{1j}^{ij} + \tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij} \right\}.$$

The optimal consumption is the consumption that attains a maximal utility under the budget set. Since the budget set of an agent depends on the prices of goods and initial holdings, the optimal consumption, i.e., the consumer's demand of goods is determined by those prices and the initial holdings. Looking at the type i agent of the initial circumstance j who makes consumption plan at the circumstance h, the agent's demand $\boldsymbol{x}^{ijh}$ is determined by the demand correspondence. That is,

$$\boldsymbol{x}^{ijh} \in \xi^{ijh}\left(p_{1j}, p_{1h}, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \tilde{\boldsymbol{\omega}}^{ij}\right),$$

where the local goods and the general goods are

$$\begin{aligned} x_{1h}^{ij} &\in \xi_{1h}^{ijh}\left(p_{1j}, p_{1h}, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \tilde{\boldsymbol{\omega}}^{ij}\right), \quad \text{and} \\ \tilde{\boldsymbol{x}}^{ijh} &\in \left(\xi_{2}^{ijh}\left(p_{1j}, p_{1h}, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \tilde{\boldsymbol{\omega}}^{ij}\right), \cdots, \xi_{\tilde{l}}^{ijh}\left(p_{1j}, p_{1h}, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \tilde{\boldsymbol{\omega}}^{ij}\right)\right). \end{aligned}$$

By denoting the general goods as

$$\tilde{\xi}^{ijh}\left(p_{1j}, p_{1h}, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \tilde{\boldsymbol{\omega}}^{ij}\right) = \left(\xi_{2}^{ijh}\left(p_{1j}, p_{1h}, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \tilde{\boldsymbol{\omega}}^{ij}\right), \cdots, \xi_{\tilde{l}}^{ijh}\left(p_{1j}, p_{1h}, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \tilde{\boldsymbol{\omega}}^{ij}\right)\right),$$

the demand correspondence is

$$\xi^{ijh}\left(p_{j}, p_{h}, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \tilde{\boldsymbol{\omega}}^{ij}\right) \equiv \left(\xi_{1h}^{ijh}(p_{j}, p_{h}, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \tilde{\boldsymbol{\omega}}^{ij}), \tilde{\xi}^{ijh}(p_{j}, p_{h}, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \tilde{\boldsymbol{\omega}}^{ij})\right).$$

The correspondences of all agents are combined into a vector.

$$\xi\left(\boldsymbol{p}, \omega_{1j}^{ij}, \tilde{\boldsymbol{\omega}}^{ij}\right) \equiv \left(\xi^{111}(p_{j}, p_{1}, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \tilde{\boldsymbol{\omega}}^{ij}), \cdots, \xi^{\tilde{i}\tilde{j}\tilde{j}}(p_{j}, p_{\tilde{j}}, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \tilde{\boldsymbol{\omega}}^{ij})\right)$$

Axioms on consumption

It is assumed that the consumption set, the utility function, and the initial endowment of goods satisfy the following conditions.

Axiom 1 As for the type i agent of a initial economic circumstance j who makes a consumption plan at a circumstance h

(i) $X_{h}^{ij} \subset R^{\tilde{l}}$ is a non-empty closed and convex set bounded from below.

(ii) $u^{ijh} : X_{h}^{ij} \to R$ is a continuous, quasi-concave and non-satiable utility function.

(iii) The elements of initial endowments are $\boldsymbol{\omega}^{ij} \in X_{h}^{ij}$, and there exists $\boldsymbol{x}^{ij0} \in X_{h}^{ij}$ such that $\boldsymbol{x}^{ij0} < \boldsymbol{\omega}^{ij}$.

If we take a large enough positive scalar $\alpha > 0$, then the consumption $\boldsymbol{x}^{ijh}$ can be smaller than the vector $\alpha\boldsymbol{e}$, where $\boldsymbol{e} = (1, \cdots, 1)$. Let us denote the set of consumption vectors that satisfy this relation as $X_{h}^{ij}(\alpha)$.

$$X_{h}^{ij}(\alpha) = \{\boldsymbol{x}^{ijh} \in X_{h}^{ij} | \boldsymbol{x}^{ijh} \leq \alpha\boldsymbol{e}\}$$

$X_{h}^{ij}(\alpha)$ is a compact set, and the image which $X_{h}^{ij}(\alpha)$ projects x^{ijh} to $R^{\tilde{l}}$ is also compact. A budget set $\beta^{ijh}(p_{1j}, p_{1h}, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \tilde{\boldsymbol{\omega}}^{ij})$ is non-empty from Axiom (iii). Then the budget set is expressed as a non-empty and continuous correspondence form the price set $P^{jh} \subset$

$R_+^{\tilde{l}+2}$ to the goods set $X_h^{ij}(\alpha)$.[1],where P^{jh} is the set of prices of local goods at j and h and prices of general goods. Since this is a restricted correspondence to goods set $X_h^{ij}(\alpha)$, the budget set is also restricted one. Let us denote this restricted budget set as $\hat{\beta}^{ijh}(p_{1j}, p_{1h}, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \tilde{\boldsymbol{\omega}}^{ij})$. The utility function has a maximum on the budget set from Axiom (ii). Therefore the following theorem is derived from Berge's maximum theorem in Appendix. This is the direct application of the typical general equilibrium theory to this model.

Theorem 1 *Suppose that (i), (ii) and (iii) of consumption axioms hold. Let α be a large enough positive constant such that $X_h^{ij}(\alpha) \subset X_h^{ij}$ is non-empty. Then, for the type i agent of the initial economic circumstance j who makes a consumption plan at circumstance h, its restricted demand correspondence $\hat{\xi}^{ijh}$ is a non-empty, compact and upper-hemicontinunous correspondence.*

Exchange economy

Let us denote the distribution of the type i agents over on the economic circumstances at an initial state as $I_{ij}(i = 1, 2, \cdots, \tilde{i}, j = 1, 2, \cdots, \tilde{j})$. Since each economic agent is characterized by the initial endowments of goods, consumption goods set and utility function, we can define an exchange economy as follows.

$$\mathcal{E} = \{(((n(I_{ij})(X_h^{ij}, u^{ijh}, \boldsymbol{\omega}^{ij}))_{i=1}^{\tilde{i}})_{j=1}^{\tilde{j}})_{h=1}^{\tilde{j}}\}$$

where $n(I_{ij})$ is the number of the elements of the set I_{ij}, that is, $n(I_{ij}) = n_{ij}$.

Allocation and feasibility

The allocation of goods in an exchange economy $\mathcal{E}$ is expressed by an element of product of consumption sets of all economic agents, i.e., a point $(((\boldsymbol{x}^{ijh})_{h=1}^{\tilde{j}})_{j=1}^{\tilde{j}})_{i=1}^{\tilde{i}}$ in $\prod_{i=1}^{\tilde{i}} \prod_{j=1}^{\tilde{j}} \prod_{h=1}^{\tilde{j}} X_h^{ij}$. We say that a point $(((\boldsymbol{x}^{ijh})_{h=1}^{\tilde{j}})_{j=1}^{\tilde{j}})_{i=1}^{\tilde{i}}$ is a feasible allocation if the point satisfies the next two conditions.

(a) $(((\boldsymbol{x}^{ijh})_{h=1}^{\tilde{j}})_{j=1}^{\tilde{j}})_{i=1}^{\tilde{i}}$ is an allocation,

(b) $\sum_{i=1}^{\tilde{i}} \sum_{j=1}^{\tilde{j}} n_{ijh} x_{1h}^{ij} \leq \sum_{i=1}^{\tilde{i}} \sum_{j=1}^{\tilde{j}} n_{ihj} \omega_{1h}^{ij} \quad (h = 1, 2, \cdots, \tilde{j}),$

$\sum_{i=1}^{\tilde{i}} \sum_{j=1}^{\tilde{j}} \sum_{h=1}^{\tilde{j}} n_{ijh} x_l^{ij} \leq \sum_{i=1}^{\tilde{i}} \sum_{j=1}^{\tilde{j}} \sum_{h=1}^{\tilde{j}} n_{ihj} \omega_l^{ij} \quad (l = 2, 3, \cdots, \tilde{l}).$

Walras equilibrium in an exchange economy

Walras equilibrium in an exchange economy is the sate of a feasible allocation with its supporting price system which is $(((\boldsymbol{x}^{ijh*})_{h=1}^{\tilde{j}})_{j=1}^{\tilde{j}})_{i=1}^{\tilde{i}}, \boldsymbol{p}^*)$. This equilibrium is obtained when the agent distribution $\boldsymbol{n} = ((\tilde{\boldsymbol{n}}_{ij})_{i=1}^{\tilde{i}})_{j=1}^{\tilde{j}}$ is given. If the agent distribution changes, then the allocation with its supporting price system is not in equilibrium any longer. Hence the equilibrium $(((\boldsymbol{x}^{ijh*})_{h=1}^{\tilde{j}})_{j=1}^{\tilde{j}})_{i=1}^{\tilde{i}}, \boldsymbol{p}^*)$ is temporary because the agent distribution is given for this equilibrium in the economy.We call this Walras equilibrium as 'temporary market equilibrium'.

[1]See page 106 to 109 of G, Debreu *An Axiomatic Analysis of Economic Equilibrium*(1959).

Excess demand correspondence

An agent decides his optimal consumption or demand based on the initial holdings of goods, and for this purpose the agent makes an exchange of his goods in the markets. Each economic circumstance forms the demand of Good 1 differently. At the circumstance h the demand of Good 1 of the type i agent is $\sum_{i=1}^{\tilde{i}}\sum_{j=1}^{\tilde{j}} n_{ijh}\xi_{1h}^{ijh}(p_j, p_h, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \boldsymbol{\omega}^{ij})$. Since the total endowments of Good 1 at h is $\sum_{i=1}^{\tilde{i}}\sum_{j=1}^{\tilde{j}} n_{ijh}\omega_{1h}^{ij}$, the excess demand of Goods 1 at the economic circumstance h is

$$\zeta_{1h}(\boldsymbol{p}) \equiv \sum_{i=1}^{\tilde{i}}\sum_{j=1}^{\tilde{j}} n_{ijh}\xi_{1h}^{ijh}(p_j, p_h, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \boldsymbol{\omega}^{ij}) \quad - \quad \sum_{i=1}^{\tilde{i}}\sum_{j=1}^{\tilde{j}} n_{ijh}\omega_{1h}^{ij}$$
$$(\quad h = 1, 2, \cdots, \tilde{j}). \tag{2}$$

As for the general goods the excess demand is

$$\zeta_{l}(\boldsymbol{p}) \equiv \sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}}\sum_{h=1}^{\tilde{j}} n_{ijh}\xi_{l}^{ijh}(p_j, p_h, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \boldsymbol{\omega}^{ij} \quad) \quad - \sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ijh}\omega_{l}^{ij}.$$
$$(\quad l = 2, \cdots, \tilde{l}). \tag{3}$$

Combine (2) and (3) in order to express as the excess demand vector

$$\zeta(\boldsymbol{p}) \equiv (\zeta_{11}(\boldsymbol{p}), \cdots, \zeta_{1\tilde{j}}(\boldsymbol{p}), \zeta_2(\boldsymbol{p}), \cdots, \zeta_{\tilde{l}}(\boldsymbol{p})) \tag{4}$$

The excess demand $\zeta(\boldsymbol{p})$ is a mapping $\zeta : P \to Z$ of $R^{\tilde{j}+\tilde{l}-1}$ on $R^{\tilde{j}+\tilde{l}-1}$. A temporary equilibrium is obtained under some agent distribution $\boldsymbol{n}$ being given. What we need for the proof of the temporary equilibrium is the relation between P and Z in the mapping.

The next theorem is derived from the above preparations for the temporary equilibrium.

Theorem 2 ***:Existence of temporary market equilibrium under the economic circumstances being given to all agents***

Let $Z \subset R^{\tilde{j}+\tilde{l}-1}$ be a compact set, and $\zeta : P \to Z$ be an upper hemi-continuous correspondence with non-empty, compact and convex value. Suppose furthermore that for all $\boldsymbol{z} \ \in \zeta(\boldsymbol{p})$, and for all $\hat{\boldsymbol{p}} \in \hat{P}$, $\hat{\boldsymbol{p}}\hat{\boldsymbol{z}} = 0$ under $\boldsymbol{n}$ being given. Then, there exist $\hat{\boldsymbol{p}}^* \in \hat{P}$ and $\hat{\boldsymbol{z}}^* \in \xi(\boldsymbol{p}^*)$ such that $\hat{z}_i^* \leq 0$, with $\hat{p}_i^* = 0$ if $\hat{z}_i^* < 0$.

(Proof)

1. Proof of Feasibility (b): $\sum_{i=1}^{\tilde{i}}\sum_{j=1}^{\tilde{j}} n_{ijh}x_{1h}^{ij} \leq \sum_{i=1}^{\tilde{i}}\sum_{j=1}^{\tilde{j}} n_{ihj}\omega_{1h}^{ih}$ $(h = 1, 2, \cdots, \tilde{j})$ and $\sum_{i=1}^{\tilde{i}}\sum_{j=1}^{\tilde{j}}\sum_{h=1}^{\tilde{j}} n_{ijh}x_{l}^{ij} \leq \sum_{i=1}^{\tilde{i}}\sum_{j=1}^{\tilde{j}}\sum_{h=1}^{\tilde{j}} n_{ihj}\omega_{l}^{ih}$ $(l = 2, 3, \cdots, \tilde{l})$ to be satisfied.

If each agent performs his consumption within the budget, then the following inequality holds. As for the type i agent of the initial circumstance j who make consumption plan at circumstance h, his budget constraint is

$$\boldsymbol{p}\boldsymbol{x}^{ijh} \leq \boldsymbol{p}\boldsymbol{\omega}^{ij},$$

that is,

$$p_{1h}x_{1h}^{ij} + \tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{x}}^{ijh} \leq p_{1j}\omega_{1j}^{ij} + \tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij}.$$

By summing up the inequalities for all the agents of type i of the initial circumstance j and the present circumstance h we get

$$n_{ijh}p_{1h}x_{1h}^{ij} + n_{ijh}\tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{x}}^{ijh} \leq n_{ijh}p_{1j}\omega_{1j}^{ij} + n_{ijh}\tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij}.$$

Further summing the above inequalities for all agents in the whole economy gives

$$\begin{aligned}\sum_{h=1}^{\tilde{j}}\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ijh}p_{1h}x_{1h}^{ij} \quad + &\quad \sum_{h=1}^{\tilde{j}}\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ijh}\tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{x}}^{ijh} \\ \leq &\quad \sum_{h=1}^{\tilde{j}}\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ijh}p_{1j}\omega_{1j}^{ij} + \sum_{h=1}^{\tilde{j}}\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ijh}\tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij}.\end{aligned}$$

The left-hand side is the product of the price vector $(p_{11}, \cdots, p_{1\tilde{j}}, p_2, \cdots, p_{\tilde{l}})$ and the vector of total sum of demands $(\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ij1}x_{11}^{ij}, \cdots, \sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ij\tilde{j}}x_{1\tilde{j}}^{ij}, \sum_{h=1}^{\tilde{j}}\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}}$ $n_{ijh}x_{2h}^{ij}, \cdots, \sum_{h=1}^{\tilde{j}}\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ijh}x_{\tilde{l}h}^{ij})$. The right-hand side is also the product of price vector $(p_{11}, \cdots, p_{1\tilde{j}}, p_2, \cdots, p_{\tilde{l}})$ and vector of total sum of initial endowments $(\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}}$ $n_{i1h}\omega_{11}^{ij}, \cdots, \sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{i\tilde{j}h}\omega_{1\tilde{j}}^{i1}, \sum_{h=1}^{\tilde{j}}\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ijh}\omega_{2}^{ij}, \cdots, \sum_{h=1}^{\tilde{j}}\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ijh}\omega_{\tilde{l}}^{ij})$. Therefore, the feasibility (b) is satisfied.

2. Proof of that the mapping of price adjustment is non-empty,compact and convex;
Define a mapping $\mu : Z \to P$ as follows:

$$\mu(\boldsymbol{z}) = \{\boldsymbol{p} \in P \mid \boldsymbol{p} \cdot \boldsymbol{z} \geq \boldsymbol{p'} \cdot \boldsymbol{z}, \forall \boldsymbol{p'} \in P\}$$

Since P is compact, the continuous function $\boldsymbol{p} \cdot \boldsymbol{z}$ has a maximum on P when z is given. $\mu(\boldsymbol{z})$ is bounded because P is compact. $\mu(\boldsymbol{z})$ is also closed from the continuous function $\boldsymbol{p} \cdot \boldsymbol{z}$. If $\boldsymbol{p}$ and $\boldsymbol{p'} \in \mu(\boldsymbol{z})$ then $\lambda\boldsymbol{p} + (1-\lambda)\boldsymbol{p'} \in \mu(\boldsymbol{z})$, for all $\lambda \in [0, 1]$ Thus the $\mu(\boldsymbol{z})$ is convex.

3. Proof of upper hemi-continuousness of $\mu(\boldsymbol{z})$.
$\boldsymbol{p} \cdot \boldsymbol{z}$ is continuous on $P \times Z$. Take a constant correspondence $\nu(\boldsymbol{z}) = P$ for all $\boldsymbol{z} \in Z$. Then the Maximum theorem concludes that $\mu(\boldsymbol{z})$ is upper hemi-continuous.

4.Proof of existence of fixed points under the restricted demand correspondence.
Since the restricted goods set $X_h^{ij}(\alpha)$ is a compact set, the elements of this set form a compact set. It follows from the theorem 1 that the demand correspondence of each agent, $\hat{\xi}^{ijh}$ is an upper hemicontinunous correspondence with convex values. Then, the market demands summed up through all economic agents have an upper hemicontinunous correspondence with convex values, and the excess demand $\zeta_{1h}(\boldsymbol{p})$ and $\zeta_l(\boldsymbol{p})$ defined by (2) and (3) also has the same property.
Make a new mapping $\phi : Z \times P \to P \times Z$ such that for all $(\boldsymbol{z}, \boldsymbol{p}) \in Z \times P$,

$$\phi(\boldsymbol{z}, \boldsymbol{p}) = \zeta(\boldsymbol{p}) \times \mu(\boldsymbol{z}).$$

$Z \times P$ is a non-empty,compact convex set, and $\phi(\boldsymbol{z}, \boldsymbol{p})$ is an upper hemi-continuous correspondence with non-empty, compact and convex values. Thus Kakutani's fixed point theorem concludes the existence of a fixed point under the restricted demand correspondence.

5. Proof that $\boldsymbol{x}^{ijh*}$ maximize utility of the agent under X_h^{ij}.

Let $\boldsymbol{x}^{ijh*}$ maximize utility under $X_h^{ij}(\alpha)$, that is, $\boldsymbol{x}^{ijh*}$ maximize the utility under

$$\beta^{ijh}(p_{1j}, p_{1h}, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \boldsymbol{\omega}^{ij}) \equiv \left\{ \left(x_{1h}^{ij}, \boldsymbol{x}^{ijh} \right) \in X_h^{ij} \mid p_{1h} x_{1h}^{ij} + \tilde{\boldsymbol{p}} \cdot \boldsymbol{x}^{ijh} \leq p_{1j} \omega_{1j}^{ij} + \tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij} \right\}.$$

Now suppose that a different point $(x_{1h}^{ij'}, \boldsymbol{x}^{ijh'})$ in X_h^{ij} could attain bigger utility than $\boldsymbol{x}^{ijh*}$, that is, $u^{ijh}(x_{1h}^{ij'}, \boldsymbol{x}^{ijh'}) > u^{ijh}(x_{1h}^{ij*}, \boldsymbol{x}^{ijh*})$. Since $(x_{1h}^{ij*}, \boldsymbol{x}^{ijh*})$ is an inner point of $[-\alpha \boldsymbol{e}, \alpha \boldsymbol{e}]$, there exist some $\lambda : 0 < \lambda < 1$ such that

$$(x_{1h}^{ij''}, \boldsymbol{x}^{ijh''}) = \lambda(x_{1h}^{ij*}, \boldsymbol{x}^{ijh*}) + (1-\lambda)(x_{1h}^{ij'}, \boldsymbol{x}^{ijh'}) \in [-\alpha \boldsymbol{e}, \alpha \boldsymbol{e}]$$

Since X_h^{ij} is convex, $(x_{1h}^{ij''}, \boldsymbol{x}^{ijh''}) \in X_h^{ij}(\alpha)$. Thus, it follows from the convexity of preference that $u^{ijh}(x_{1h}^{ij''}, \boldsymbol{x}^{ijh''}) > u^{ijh}(x_{1h}^{ij*}, \boldsymbol{x}^{ijh*})$. This contradicts that $(x_{1h}^{ij*}, \boldsymbol{x}^{ijh*})$ maximizes the utility on the $X_h^{ij}(\alpha)$. Therefore, $\boldsymbol{x}^{ijh*}$ maximizes the utility on the X_h^{ij}.

□

Theorem 2 ensures that a temporary market equilibrium exists when the agent distribution is given.

3 Market equilibrium under selectable economic circumstances

When economic circumstances are given for all economic agents, their maximal utility levels may become different at each circumstance because of the different consumptions of goods at equilibrium. So long as an economic circumstance is given for an agent, this situation will continue. But, if each agent can choose the best economic circumstance for his utility maximization, what will happen? Probably each agent tries to choose such an economic circumstance so as to achieve a highest utility level. For this reason the number of agents in each circumstance will change and then affect the consumptions of agents, which creates an another challenge for equilibrium. Therefore we shall analyze the market equilibrium in the case that each agent can choose an economic circumstance, that is, in the case of the selectable economic circumstances.

According to Theorem 2, when an allocation of economic agents in the economic circumstances, i.e., $\boldsymbol{n} \equiv (\tilde{\boldsymbol{n}}_{11}, \tilde{\boldsymbol{n}}_{12}, \ldots, \tilde{\boldsymbol{n}}_{\tilde{i}\tilde{j}})$ is given, there exist some prices of market equilibriums. If the distribution of agents changes, then some different temporal equilibrium prices will be determined correspondingly. Thus the temporal equilibrium prices correspond to the distribution of the numbers of agents respectively. Let us denote the temporal equilibrium prices as $\hat{\boldsymbol{p}}^*(\boldsymbol{n})$, and here we have the next lemma.

Lemma 1 *$\hat{\boldsymbol{p}}^*(\boldsymbol{n})$ is upper hemi-continuous with respect to $\boldsymbol{n}$.*

(Proof) Suppose that $\hat{\boldsymbol{p}}^{*0}(\boldsymbol{n}^0)$ is a temporal equilibrium price vector at $\boldsymbol{n}^0$, and that $\hat{\boldsymbol{p}}^{*q} \in \hat{\boldsymbol{p}}^*(\boldsymbol{n}^q)$ is a temporal equilibrium price vector corresponding to $\boldsymbol{n}^q$ which converges to the $\boldsymbol{n}^0$. Further suppose that $\lim_{n^q \to n^0} \hat{\boldsymbol{p}}^{*q} = \hat{\boldsymbol{p}}^* \notin \hat{\boldsymbol{p}}^{*0}(\boldsymbol{n}^0)$. Then since $\hat{\boldsymbol{p}}^{*0}(\boldsymbol{n}^0)$ is a temporal equilibrium price, the excess demand is $(\forall \boldsymbol{p}^{*0} \in \hat{\boldsymbol{p}}^{*0}(\boldsymbol{n}^0)) : \zeta(\boldsymbol{p}^{*0}) \leq 0$. We get the next relation.

$$\zeta(\hat{\boldsymbol{p}}^*) > 0 \geq \zeta(\boldsymbol{p}^{*0}) \tag{5}$$

On the other hand when $\boldsymbol{n}^q \to \boldsymbol{n}^0$, the excess demand of local goods and general goods are

$$\begin{aligned}
\lim_{n^q \to n^0} \zeta_{1h}(\hat{\boldsymbol{p}}^{*q}) &= \lim_{n^q \to n^0} \{\sum_{i=1}^{\tilde{i}} \sum_{j=1}^{\tilde{j}} n_{ijh} \xi_{1h}^{ijh}(\boldsymbol{p}^{*q}) - \sum_{i=1}^{\tilde{i}} \sum_{j=1}^{\tilde{j}} n_{ijh} \boldsymbol{\omega}_{1h}^{ih}\} \\
&= \sum_{i=1}^{\tilde{i}} \sum_{j=1}^{\tilde{j}} n_{ijh} \xi_{1h}^{ijh}(\hat{\boldsymbol{p}}^{*}) - \sum_{i=1}^{\tilde{i}} \sum_{j=1}^{\tilde{j}} n_{ijh} \boldsymbol{\omega}_{1h}^{ih} \quad (h = 1, 2, \cdots, \tilde{j}) \qquad (6)
\end{aligned}$$

$$\begin{aligned}
\lim_{n^q \to n^0} \zeta_{l}(\hat{\boldsymbol{p}}^{*q}) &= \lim_{n^q \to n^0} \{\sum_{h=1}^{\tilde{j}} \sum_{j=1}^{\tilde{j}} \sum_{i=1}^{\tilde{i}} n_{ijh} \xi_{l}^{ijh}(\boldsymbol{p}^{*q}) - \sum_{h=1}^{\tilde{j}} \sum_{j=1}^{\tilde{j}} \sum_{i=1}^{\tilde{i}} n^{ijh} \boldsymbol{\omega}_{l}^{ih}\} \\
&= \sum_{h=1}^{\tilde{j}} \sum_{j=1}^{\tilde{j}} \sum_{i=1}^{\tilde{i}} n_{ijh} \xi_{l}^{ijh}(\hat{\boldsymbol{p}}^{*}) - \sum_{h=1}^{\tilde{j}} \sum_{j=1}^{\tilde{j}} \sum_{i=1}^{\tilde{i}} n^{ijh} \boldsymbol{\omega}_{l}^{ih} \quad (l = 2, \cdots, \tilde{l}) \qquad (7)
\end{aligned}$$

The right-hand side of (6) and (7) is positive form (5). Therefore, for $\boldsymbol{n}$ closer to $\boldsymbol{n}^0$ the excess demands of local goods and general goods become

$$\sum_{j=1}^{\tilde{j}} \sum_{i=1}^{\tilde{i}} n_{ijh} \xi_{1h}^{ijh}(\boldsymbol{p}^{*}(\boldsymbol{n})) - \sum_{j=1}^{\tilde{j}} \sum_{i=1}^{\tilde{i}} n^{ijh} \boldsymbol{\omega}_{l}^{ih} \quad (h = 1, 2, \cdots, \tilde{j}) \quad \text{and} \qquad (8)$$

$$\sum_{h=1}^{\tilde{j}} \sum_{j=1}^{\tilde{j}} \sum_{i=1}^{\tilde{i}} n_{ijh} \xi_{l}^{ijh}(\hat{\boldsymbol{p}}^{*}(\boldsymbol{n})) - \sum_{h=1}^{\tilde{j}} \sum_{j=1}^{\tilde{j}} \sum_{i=1}^{\tilde{i}} n^{ijh} \boldsymbol{\omega}_{l}^{ih} \quad (l = 2, \cdots, \tilde{l}) \qquad (9)$$

which are both positive. But since $\hat{\boldsymbol{p}}^*(\boldsymbol{n})$ is a temporal equilibrium price, the above relations are $\zeta(\hat{\boldsymbol{p}}^*(\boldsymbol{n}))$ which must be non-positive in equilibrium. That (8) and (9) are positive contradicts $\zeta(\hat{\boldsymbol{p}}^*(\boldsymbol{n})) \leq 0$. Therefore we get the relation

$$\lim_{n^q \to n^0} \hat{\boldsymbol{p}}^{*q} \in \hat{\boldsymbol{p}}^{*0}(\boldsymbol{n}^0).$$

□

When the type i agent of the initial economic circumstance j makes consumption plan at h his budget set $\beta^{ijh}(\boldsymbol{p})$ is a non-empty and continuous correspondence. The utility function $u^{ijh}(\boldsymbol{x}^{ijh})$ is also a continuous function of $\boldsymbol{x}^{ijh}$ from Axiom 1-(ii). The demand correspondence $\xi^{ijh}(\boldsymbol{p})$ is upper hemi-continuous on P. Therefore the indirect utility function $u^{ijh}(\boldsymbol{p}) = u^{ijh}(\xi^{ijh}(\boldsymbol{p}))$ is continuous on P form Berge's maximum theorem in the appendix. For saving symbols, let us use the same symbol to denote the indirect utility function of type i agent as u^{ijh}. When the type i agents of the initial circumstance j locate over all the circumstances to make their consumption, the utilities of the type i agents are $u^{ij}(\boldsymbol{p}) \equiv (u^{ij1}(\boldsymbol{p}), u^{ij2}(\boldsymbol{p}), \cdots, u^{ij\tilde{j}}(\boldsymbol{p}))$. From Lemma 1, the temporal equilibrium price vector is upper hemi-continuous with respect to the distribution of agents $\boldsymbol{n}$, and the indirect utility function $u^{ij}(\boldsymbol{p})$ is continuous with respect to the price $\boldsymbol{p}$. Then we obtain the next lemma.

Lemma 2 *When a price vector $\boldsymbol{p}$ is a temporal equilibrium price, the utility $u^{ij}(\boldsymbol{p}(\boldsymbol{n}))$ is upper hemi-continuous with respect to $\boldsymbol{n}$.*

The utilities of all agent types are expressed as

$$u(\boldsymbol{p}) \equiv (u^{11}(\boldsymbol{p}), u^{12}(\boldsymbol{p}), \cdots, u^{ij}(\boldsymbol{p}), \cdots, u^{\tilde{i}\tilde{j}}(\boldsymbol{p})),$$

which are upper hemi-continuous with respect to $\boldsymbol{n}$.

'Excess utility' and 'permanent' equilibrium states of economy

When each agent is freely able to choose his location for his consumption of goods, he will try to choose a better circumstance for bigger utility. What will be the choice criterion of economic circumstance by an economic agent? There might be conceived several ways for it. Here we shall try to construct a mechanism based on difference from 'average utility' of agents.

If the utility of an agent is bigger than the average utility of the same group of agents, he will think himself in a better state, and if his utility is smaller than the average utility, he will think that he is in a inferior state and he should take some action for better utility. The average utility can be a standard level when each agent is to take his action for better utility. Hence we first define the average utility of the same type agents as 'average utility', and next we define the difference of each agent's utility from the average utility as 'excess utility'. As long as there exists excess utility, each agent will change his location to get a better circumstance for him, and then the distribution of one type group will continue to fluctuate accordingly.

Once the distribution of population ceases to change and remains constant in the end, an agent will no longer have an incentive to change his location in the economy, that is, the agent will attain the same utility level with the agents in the same group of the economy. This can be viewed as an equilibrium state of agent distribution. In what follows we will formulate this mechanism.

The average utility $\bar{u}^{ij}$ is a weighted average of utilities of the type i agents of the initial circumstance j. That is, the average utility $\bar{u}^{ij}$ is the weighted average utility with weights $n_{ijh}(h = 1, \cdots, \tilde{j})$ when the number of agents who move to $h(h = 1, \cdots, \tilde{j})$ is n_{ijh}.

$$\bar{u}^{ij}(\boldsymbol{n}) \equiv \frac{1}{n_{ij}} \sum_{h=1}^{\tilde{j}} n_{ijh} u^{ijh}(\boldsymbol{p}(\boldsymbol{n}); \omega^{ij}_{1j}, \tilde{\boldsymbol{\omega}}^{ij}) \tag{10}$$

Define the difference of each agent's utility from the average utility of the agent type as 'excess utility'. The excess utility v^{ijh} of the type i agent of the initial circumstance j at the circumstance h is

$$v^{ijh}(\boldsymbol{n}) \equiv u^{ijh}(\boldsymbol{p}(\boldsymbol{n}); \omega^{ij}_{1j}, \tilde{\boldsymbol{\omega}}^{ij}) - \bar{u}^{ij}(\boldsymbol{n}). \tag{11}$$

The excess utilities of the type i at all circumstances are expressed by a vector, $v^{ij}(\boldsymbol{n}) \equiv (v^{ij1}(\boldsymbol{n}), \cdots, v^{ij\tilde{j}}(\boldsymbol{n}))$. The excess utilities of all agent types are

$$v(\boldsymbol{n}) \equiv (v^{11}(\boldsymbol{n}), \cdots, v^{\tilde{i}\tilde{j}}(\boldsymbol{n})).$$

The set V^{ij} of the excess utilities of the type i agents of the initial economic circumstance j can be defined so as to be compact and convex.

$$V^{ij} \subseteq R^{\tilde{j}}$$

We can find the compactness and convexity of the set V^{ij} as follows. First, take any two $\boldsymbol{v}^{ij}$ and $\boldsymbol{v}^{ij'}$ of V^{ij} and make a convex combination $\lambda\boldsymbol{v}^{ij} + (1-\lambda)\boldsymbol{v}^{ij'}, (\lambda \geq 0)$. The excess utilities $\boldsymbol{v}^{ij}$ and $\boldsymbol{v}^{ij'}$ of V^{ij} correspond to the consumptions $((x_{11}^{ij}, \tilde{\boldsymbol{x}}^{ij1}), \cdots, (x_{1\tilde{j}}^{ij}, \tilde{\boldsymbol{x}}^{ij\tilde{j}}))$ and $((x_{11}^{ij'}, \tilde{\boldsymbol{x}}^{ij1'}), \cdots, (x_{1\tilde{j}}^{ij'}, \tilde{\boldsymbol{x}}^{ij\tilde{j}'}))$ respectively. Since the consumption set $X_h^{ij}(h = 1, 2, \cdots, \tilde{j})$ with $(x_{1h}^{ij}, \tilde{\boldsymbol{x}}^{ijh})$ is convex, $\prod_{h=1}^{\tilde{j}} X_h^{ij}$ is also convex. There exits an price $\boldsymbol{p}$ to support a convex combination $\lambda((x_{11}^{ij}, \tilde{\boldsymbol{x}}^{ij1}), \cdots, (x_{1\tilde{j}}^{ij}, \tilde{\boldsymbol{x}}^{ij\tilde{j}})) + (1-\lambda)((x_{11}^{ij'}, \tilde{\boldsymbol{x}}^{ij1'}), \cdots, (x_{1\tilde{j}}^{ij'}, \tilde{\boldsymbol{x}}^{ij\tilde{j}'}))$, $(\lambda \geq 0)$ which belongs to $\prod_{h=1}^{\tilde{j}} X_h^{ij}$. Therefore, $\lambda\boldsymbol{v}^{ij} + (1-\lambda)\boldsymbol{v}^{ij'}, (\lambda \geq 0)$ can be achieved under the price $\boldsymbol{p}$ to be in the V^{ij}, that is V^{ij} is convex.

Next, the consumption set $X_h^{ij}(\alpha)$ is compact, and the utility function u^{ijh} is continuous with respect to consumption. Then the image of consumption set $X_h^{ij}(\alpha)$ by utility function u^{ijh} is also compact, which in turn makes the excess utility set V^{ij} compact. Therefore, the excess utility set V^{ij} is compact and convex.

Define the set of excess utility sets V^{ij} as

$$V \equiv \prod_{i=1}^{\tilde{i}} \prod_{j=1}^{\tilde{j}} V^{ij} \subseteq R^{\tilde{i}\tilde{j}\tilde{j}}.$$

The mapping of excess utility is $v : N \to V$. Since the average utility $u^{ij}(\boldsymbol{n})$ is upper hemi-continuous with respect to $\boldsymbol{n}$, the excess utility $v^{ij}(\boldsymbol{n})$ is also upper hemi-continuous with respect to $\boldsymbol{n}$ from (10) and (11). Therefore the excess utility v is upper hemi-continuous with respect to the agent distribution $\boldsymbol{n}$.

Lemma 3 *A mapping $v(\boldsymbol{n})$ is upper hemi-continunous on the population distribution set N.*

<u>**Equilibrium under the economy with selectable economic circumstances**</u>

In Section 2, the temporal economic equilibrium is defined under the economy with a given distribution of agents. Here we define the equilibrium in the case that each agent can choose his location for consumption freely.

Define the equilibrium of exchange economy as the feasible allocation with a supporting price vector and a feasible distribution of agents $(((\boldsymbol{x}^{ijh*})_{h=1}^{\tilde{j}})_{j=1}^{\tilde{j}})_{i=1}^{\tilde{i}}, \boldsymbol{p}^*, ((\tilde{\boldsymbol{n}}^*)_{j=1}^{\tilde{j}})_{i=1}^{\tilde{i}})$. It is the temporal equilibrium in which each agent will not change his economic circumstance for better utility. Once the equilibrium $(((\boldsymbol{x}^{ijh*})_{h=1}^{\tilde{j}})_{j=1}^{\tilde{j}})_{i=1}^{\tilde{i}}, \boldsymbol{p}^*, ((\tilde{\boldsymbol{n}}^*)_{j=1}^{\tilde{j}})_{i=1}^{\tilde{i}})$ is obtained, the distribution of agents $((\tilde{\boldsymbol{n}}^*)_{j=1}^{\tilde{j}})_{i=1}^{\tilde{i}})$ will not change any more. The corresponding temporal equilibrium $(((\boldsymbol{x}^{kij})_{k\in A^{ij}}^*)_{j=1}^{\tilde{j}})_{i=1}^{\tilde{i}}, \boldsymbol{p}^*)$ will not change either. Therefore, this equilibrium is a 'permanent' equilibrium in the meaning that the equilibrium will not change so long as the distribution of agents remaining in the same state. Here we call this 'permanent' equilibrium as market equilibrium.

Define a mapping $g^{ij}(\boldsymbol{v}^{ij}) : V^{ij} \to N^{ij}$ of the type i agents of the initial circumstance as

$$g^{ij}(\boldsymbol{v}^{ij}) \equiv \{\tilde{\boldsymbol{n}}_{\boldsymbol{ij}} \in N^{ij} | \tilde{\boldsymbol{n}}_{\boldsymbol{ij}} \cdot \boldsymbol{v}^{\boldsymbol{ij}} \geq \tilde{\boldsymbol{n}}' \cdot \boldsymbol{v}^{\boldsymbol{ij}}, \quad \forall \tilde{\boldsymbol{n}}' \in N^{ij}\}$$

Lemma 4 $\tilde{\boldsymbol{n}}_{\boldsymbol{ij}} \cdot \boldsymbol{v}^{\boldsymbol{ij}} = \tilde{\boldsymbol{n}}_{\boldsymbol{ij}} \cdot v^{ij}(\boldsymbol{n}) = 0$ *for* $\boldsymbol{n} = (\tilde{\boldsymbol{n}}_{11}, \cdots, \tilde{\boldsymbol{n}}_{ij}, \cdots, \tilde{\boldsymbol{n}}_{\tilde{i}\tilde{j}}) C\boldsymbol{v}^{ij} = v^{ij}(\boldsymbol{n})$.

(Proof)

$$\begin{aligned}
\tilde{\boldsymbol{n}}_{\boldsymbol{ij}} \cdot \boldsymbol{v}^{\boldsymbol{ij}} &= (n_{ij1}, n_{ij2}, \cdots, n_{ij\tilde{j}})(v^{ij1}(\boldsymbol{n}), v^{ij2}(\boldsymbol{n}), \cdots, v^{ij\tilde{j}}(\boldsymbol{n})) \\
&= (n_{ij1}, n_{ij2}, \cdots, n_{ij\tilde{j}})(u^{ij1}(\cdot) - \bar{u}^{ij}, u^{ij2}(\cdot) - \bar{u}^{ij}, \cdots, u^{ij\tilde{j}}(\cdot) - \bar{u}^{ij}) \\
&= \sum_{h=1}^{\tilde{j}} n_{ijh} u^{ijh}(\cdot) - \bar{u}^{ij} \sum_{h=1}^{\tilde{j}} n_{ijh} \\
&= \sum_{h=1}^{\tilde{j}} n_{ijh} u^{ijh}(\cdot) - \bar{u}^{ij} n_{ij} \\
&= \sum_{h=1}^{\tilde{j}} n_{ijh} u^{ijh}(\cdot) - \left(\frac{1}{n_{ij}} \sum_{h=1}^{\tilde{j}} n_{ijh} u^{ijh}(\cdot) \right) n_{ij} \\
&= 0
\end{aligned}$$

□

Lemma 5 *$g^{ij}(\boldsymbol{v}^{ij})$ is non-empty, compact and convex.*

(Proof) Since N is compact, and $\tilde{\boldsymbol{n}}^{\boldsymbol{ij}} \cdot \boldsymbol{v}^{ij}$ is continuous on $N^{ij} \subseteq N$, $\tilde{\boldsymbol{n}}^{\boldsymbol{ij}} \cdot \boldsymbol{v}^{ij}$ necessarily has its finite value.

The $g^{ij}(\boldsymbol{v}^{ij})$ is bounded because of compactness of N and closed from the definition of $g^{ij}(\cdot)$. Then $g^{ij}(\boldsymbol{v}^{ij})$is compact.

The convexity of $g^{ij}(\boldsymbol{v}^{ij})$ can be shown as follows. Make a convex combination $\lambda\tilde{\boldsymbol{n}}_{ij} + (1-\lambda)\tilde{\boldsymbol{n}}'_{ij}$ of $\tilde{\boldsymbol{n}}_{ij}, \tilde{\boldsymbol{n}}'_{ij} \in g^{ij}(\boldsymbol{v}^{ij})$ for $0 < \lambda < 1$. We conduct the next calculation to the convex combination.

$$\begin{aligned}
(\lambda\tilde{\boldsymbol{n}}_{ij} + (1-\lambda)\tilde{\boldsymbol{n}}'_{ij})\boldsymbol{v}^{ij} &= \lambda\tilde{\boldsymbol{n}}_{ij} \cdot \boldsymbol{v}^{ij} + (1-\lambda)\tilde{\boldsymbol{n}}'_{ij} \cdot \boldsymbol{v}^{ij} \\
&\geq \lambda\tilde{\boldsymbol{n}}''_{ij} \cdot \boldsymbol{v}^{ij} + (1-\lambda)\tilde{\boldsymbol{n}}''_{ij} \cdot \boldsymbol{v}^{ij}, \quad \forall\tilde{\boldsymbol{n}}''_{ij} \in N^{ij} \\
&= \tilde{\boldsymbol{n}}''_{\boldsymbol{ij}} \cdot \boldsymbol{v}^{ij}.
\end{aligned}$$

Thus $\lambda\tilde{\boldsymbol{n}}_{ij} + (1-\lambda)\tilde{\boldsymbol{n}}'_{ij} \in g^{ij}(\boldsymbol{v}^{ij})$.

□

Lemma 6 *The correspondence $g^{ij}(\boldsymbol{v}^{ij})$ is upper hemi-continuous with respect to $\boldsymbol{v}^{ij}$.*

(Proof) The lemma is proved by using Berge's Maximum theorem.

$\boldsymbol{n}_{\boldsymbol{ij}} \cdot \boldsymbol{v}^{ij}$ is continuous on $N^{ij} \times V^{ij}$. Next, we make the correspondence $\delta_{ij} : V^{ij} \to N^{ij}$ so that $\delta_{ij}(\boldsymbol{v}^{ij}) = N^{ij}$ holds for each $\boldsymbol{v}^{ij}$. This constant correspondence δ_i is continuous.

Therefore $\delta_{ij}(\boldsymbol{v}^{ij}) = N^{ij}$ is upper hemi-continuous form Maximum theorem.

□

Let us define the correspondence $g(\boldsymbol{v})$ by use of $g^{ij}(\boldsymbol{v}^{\boldsymbol{ij}})$.

$$g(\boldsymbol{v}) \equiv (g^{11}(\boldsymbol{v}^{11}), g^{12}(\boldsymbol{v}^{12}), \cdots, g^{\tilde{i}\tilde{j}}(\boldsymbol{v}^{\tilde{i}\tilde{j}}))$$

that is, $g : V \to N$.

Lemma 7 *The correspondence $g(\boldsymbol{v})$ is upper hemi-continuous with respect to $\boldsymbol{v}$.*

(Proof) Take any point $\boldsymbol{v}^0$ of V. Next If we take $\boldsymbol{v}$ so that its element $\boldsymbol{v}^{ijq} \rightarrow \boldsymbol{v}^{ij0}(i = 1, \cdots, \tilde{i}, j = 1, \cdots, \tilde{j})$, then $\boldsymbol{v}^q \rightarrow \boldsymbol{v}^0$. Now that $\tilde{\boldsymbol{n}}_{ij}^q \in g^{ij}(\boldsymbol{v}^{ijq})$ from the upper hemi-continuousness of $g^{ij}(\boldsymbol{v}^{ij})$ and $\tilde{\boldsymbol{n}}_{ij}^q \rightarrow \tilde{\boldsymbol{n}}_{ij}^0$, we get $\tilde{\boldsymbol{n}}_{ij}^0 \in g^{ij}(\boldsymbol{v}^{ij0})$ for each $\tilde{\boldsymbol{n}}_{ij}^0$ of $\boldsymbol{n}^0 = (\tilde{\boldsymbol{n}}_{11}^0, \tilde{\boldsymbol{n}}_{12}^0, \cdots, \tilde{\boldsymbol{n}}_{\tilde{i}\tilde{j}}^0)$. Thus we obtain $\boldsymbol{n}^0 \in g(\boldsymbol{v}^0)$ when $\boldsymbol{v}^q \rightarrow \boldsymbol{v}^0$.

□

Theorem 3 *: Existence of market equilibrium*

Suppose that $V \subset R^h$ is compact and convex set, $v : N \rightarrow V$ is upper hemi-continuous correspondence with non-empty compact and convex value, and further that $\boldsymbol{vn} = 0$ for all $\boldsymbol{v} \in v(\boldsymbol{n})$ and all $\boldsymbol{n} \in N$. Then, there exist $\boldsymbol{n}^* \in N$,$\boldsymbol{v}^* \in v(\boldsymbol{n}^*)$ such that $\boldsymbol{v}^* \leq 0$, with $n_h^* = 0$ if $v_h^* < 0$.

(Proof) Define the mapping $\mu : V \times N \rightarrow N \times V$ as follows.

$$\mu(\boldsymbol{v}, \boldsymbol{n}) = g(\boldsymbol{v}) \times v(\boldsymbol{n}) \quad \text{for } \forall(\boldsymbol{v}, \boldsymbol{n}) \in V \times N$$

Then $V \times N$ is a non-empty compact set because V and N are both non-empty compact sets. Since ξ is upper hemi-continuous correspondence with non-empty compact convex value from Theorem 1, v has also the same property. g is also upper hemi-continuous correspondence with non-empty compact convex value from Lemma 5 and 7. Therefore μ has the same property.

Then there exists $(\boldsymbol{v}^*, \boldsymbol{n}^*) \in \mu(\boldsymbol{v}^*, \boldsymbol{n}^*)$ from Kakutani's Fixed Point Theorem. That is,

$$\begin{aligned} \boldsymbol{n}^* \in g(\boldsymbol{v}^*), \qquad & \boldsymbol{v}^* \in \zeta(\boldsymbol{n}^*), \\ 0 = \boldsymbol{n}^*\boldsymbol{v}^* \quad \geq & \quad \boldsymbol{n}\boldsymbol{v}^* \quad \forall \boldsymbol{n} \in N \end{aligned}$$

Since $\boldsymbol{n} \geq 0$ for all $\boldsymbol{n} \in N$ and $0 \notin V$, we get $v^* \leq 0$. If $v_k^* < 0$ then, $n_k^* = 0$.

□

According to Theorem 3 there exist some utility level that agent distribution will not change and remain constant, when each agent is able to choose an economic circumstance for his utility maximization. Such values are the utility and the agent distribution of 'permanent' equilibrium. The corresponding equilibrium values of consumptions and prices which are determined by Theorem 2 are the 'permanent' equilibrium of prices and consumptions. Hence Theorem 3 with Theorem 2 guarantees 'permanent' market equilibrium when each agent is able to choose an economic circumstance. In other words, these theorems have extended the existence of market equilibrium to the case of the selectable economic circumstances. The economic models in which the price of some goods changes in different locations such as residential location and public service consumption location can be unified and studied by our model, which might be contribution to economics.

4 Properties of the equilibrium under selectable economic circumstances

What is the properties of equilibrium solutions? Our interests in the properties awakes the investigation in the study of equilibrium analysis. For this purpose the model needs to be specified in more detail, because it is too general to investigate such properties. Since the model of this paper copes with selectable economic circumstances, the specification of the model is made in the direction of the urban residential model that consumers face to different prices of land rent in a city. Then we will analyze the properties of the equilibrium solutions with an approach to urban residential model.

Urban residential location model

First we review the urban economic model with a monocentric city[2]. The monocentric urban city model has a circular city that has a central business district(CBD) in the city center. The residents commute to the city center to work and to earn same income. They make consumption of goods and service with the disposable incomes after commuting cost. Since the consumers are assumed to be identical in taste, they attain the same level of utility in equilibrium. The given rent at city boundary and the exogenous urban population enables us to obtain the utility level of consumers in the city.

It is clear from the analysis of the residential location model that the quantity of housing service (land) is increasing with the distance from the city center, and that the price of housing service become decreasing with the distance. Generally the interest of the analysis relates to the effects of the change of parameters on economic variables. The parameters are the exogenous population size, land rent at city boundary, consumer's income and commuting cost parameter that influence the quantity and price of housing service, the city size and consumer's utility level.

The assumptions of urban residential location model

The setting of a basic residential location model are:

U1. Each location point is distinguished according to the distance from the city center.

U2. All consumers reside in a city and commute to the CBD to work.

U3. All consumers earn the same nominal income.

U4. The income after the commuting cost, which is spent on the consumption of the goods and service, is different depending on the distance to the city center from the residence.

U5. A consumer has the same preference on goods and service and the utility function is identical.

U6. The consumer's taste to goods and service is unrelated to the residential location.

U7. The goods and service for consumption are housing service (land) and composite goods.

U8. The urban population is given in a fixed size for a closed city model.

[2]For example, see Brueckner,'The Structure of Urban Equilibria: A Unified Treatment of the Muth-Mills Mode'(1987).

U9. The land rent (agricultural land rent) at a city boundary is given exogenously.

Consistency with the assumptions of the urban residential model

To reside in a locational point in the city described in U1 and 2 corresponds to locate a different economic circumstance of this model. However, our model is fundamentally different from the urban residential location model on the assumption of income. While the initial endowments of goods is the source of income in our model, the money income is given in the urban residential model. From U4 the diposable income after commuting cost which is spent on goods and service becomes smaller with the distance from the city center. The problem is how to correspond the initial endowments to the disposable income. The utility function of U5 is basically consistent with our model. The U6 is the typical setting in the urban residential model, and the taste of goods can be also modeled to be indifferent from economic circumstances in a basic model. The housing service or the land rent of which price differs in the distance from the city center in U7, corresponds to the local goods of the model of this paper, and the composite goods correspond to the general goods. The fixed population size in U8 matches the given size of agents in the economy of our model. While the city size is determined by U9, the size of economic environments is given in our model.

Thus the essential points to approximate our model to the urban residential location are :

1. The money income is a common value in the entire city.
2. the disposable income deducted after commutation expense decreases successively with the distance form the city center.

Here we employ the following assumptions on the initial endowments of goods.

Assumption 1 *Each agent of the same type has the same quantities of local goods and general goods as initial holdings, which are irrespective with economic circumstances, that is,*

(i) *Local goods :* $\omega_{11}^{i1} = \omega_{12}^{i2} = \cdots = \omega_{1j}^{ij} = \cdots = \omega_{1\tilde{j}}^{i\tilde{j}} \qquad (i = 1, 2, \cdots, \tilde{i})$

(ii) *General goods :* $\omega_l^{i1} = \omega_l^{i2} = \cdots = \omega_l^{ij} = \cdots = \omega_l^{i\tilde{j}}$
$(i = 1, 2, \cdots, \tilde{i}, \quad l = 2, 3, \cdots, \tilde{l})$

This assumption of initial endowments matches with the constant money income of the residential location model. The income to be spent on goods should be different according to the economic circumstances. It is plausible that the income for consumption changes depending on economic circumstances because of some kind of transportation like the urban economic model. We assume here that while each agent can hold the initial holdings of local good for consumption or income without loss at initial state, he can only use smaller quantities of general goods because of some kind of loss like transportation. Then we set as follows.

Assumption 2 *Local goods and general goods for consumption;*

(i) *Local goods can be used in the initial endowments without loss of quantities.*

(ii) *The quantities of initial endowments of general goods can be used less easily as the number of an economic circumstance is larger.The degree of ease for consumption appears as reduction of the amount of the initial possession of general goods, and the degree is expressed by the residual coefficient of the quantities of initial endowments. The residual coefficient in each economic circumstance* a_j *is* [3]

$$1 = a_1 > a_2 > \cdots > a_{\tilde{j}} > 0.$$

An agent's preference for goods with respect to economic circumstance is assumed in the followings.

Assumption 3 *An agent's preference doesn't change by the difference of the economic circumstance.*

Budget constraint and equilibrium solution

We shall show that the model under Assumption 1, 2 and 3 has market equilibrium. The difference from Section 2 is the budget that is specified in more detail in this section.

When a type i agent of the initial circumstance j makes consumption plan, his budget for consumption of goods changes depending on his economic circumstance. His disposable income will be $p_{1j}\omega_{1j}^{ij} + a_h \tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij}$ if his circumstance is h, and it will be $p_{1j}\omega_{1j}^{ij} + a_{h'} \tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij}$ if his circumstance is h'. When a type i agent of the initial circumstance j makes a consumption plan at h, his budget set becomes

$$\beta^{ijh}(p_{1j}, p_{1h}, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \tilde{\boldsymbol{\omega}}^{ij}) \equiv \left\{ \left(x_{1h}^{ij}, \boldsymbol{x}^{ijh} \right) \in X_h^{ij} \mid p_{1h}x_{1h}^{ij} + \tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{x}}^{ijh} \leq p_{1j}\omega_{1j}^{ij} + a_h \tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij} \right\}.$$

While the feasible condition of market equilibrium is the same as Section 2 as for local goods, the conditions for general goods change into

$$\sum_{i=1}^{\tilde{i}} \sum_{j=1}^{\tilde{j}} \sum_{h=1}^{\tilde{j}} n_{ijh} x_{lh}^{ij} \leq \sum_{i=1}^{\tilde{i}} \sum_{j=1}^{\tilde{j}} \sum_{h=1}^{\tilde{j}} n_{ihj} a_h \omega_l^{ih} \quad (l = 2, 3, \cdots, \tilde{l}).$$

In addition the excess demand correspondence of local goods is the same as Section 2, but the excess demand correspondence of general goods become

$$\zeta_l(\boldsymbol{p}) \equiv \sum_{j=1}^{\tilde{j}} \sum_{i=1}^{\tilde{i}} \sum_{h=1}^{\tilde{j}} n_{ijh} \xi_l^{ijh}(p_j, p_h, \tilde{\boldsymbol{p}}, \omega_{1j}^{ij}, \boldsymbol{\omega}^{ij} \quad) \quad - \sum_{j=1}^{\tilde{j}} \sum_{i=1}^{\tilde{i}} \sum_{h=1}^{\tilde{j}} n_{ihj} a_h \omega_l^{ih}.$$
$$(\quad l = 1, 2, \cdots, \tilde{l}).$$

We can apply the analysis of Section 2 to the local goods, and it is only necessary for us to show that the feasible condition of general goods is satisfied. If the conditions are fulfilled, then the argument on the proof of the theorem 2 in Section 2 will applied to this section as it is, and market equilibrium will exist.

We shall exhibit that the feasible conditions of the general goods are fulfilled. When a type i agent of the initial circumstance j makes a consumption plan at h, the budget constraint is

$$p_{1h}x_{1h}^{ij} + \tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{x}}^{ijh} \leq p_{1j}\omega_{1j}^{ij} + a_h \tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij}.$$

[3] Here we assume the 'iceberg' type transportation cost.

The number of the type i agents of the initial circumstance j who change his circumstance form j to h is n_{ijh}. By summing up the budget constraints of those agents,

$$n_{ijh}p_{1h}x_{1h}^{ij} + n_{ijh}\tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{x}}^{ijh} \leq n_{ijh}p_{1j}\omega_{1j}^{ij} + n_{ijh}a_h\tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij}.$$

Further summing up the LHS and RHS of the above inequalities in the whole economy leads to

$$\begin{aligned}\sum_{h=1}^{\tilde{j}}\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ijh}p_{1h}x_{1h}^{ij} &+ \sum_{h=1}^{\tilde{j}}\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ijh}\tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{x}}^{ijh} \\ &\leq \sum_{h=1}^{\tilde{j}}\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ijh}p_{1j}\omega_{1j}^{ij} + \sum_{h=1}^{\tilde{j}}\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ijh}a_h\tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij}.\end{aligned}$$

The LHS and the RHS are arranged as follows.

$$\begin{aligned} LHS &= (p_{11}, \cdots, p_{1\tilde{j}}, p_2, \cdots, p_{\tilde{l}}) \\ &\times (\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ij1}x_{11}^{ij}, \cdots, \sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ij\tilde{j}}x_{1\tilde{j}}^{ij}, \sum_{h=1}^{\tilde{j}}\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ijh}x_{2h}^{ij}, \cdots, \sum_{h=1}^{\tilde{j}}\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ijh}x_{\tilde{l}h}^{ij}) \\ RHS &= (p_{11}, \cdots, p_{1\tilde{j}}, p_2, \cdots, p_{\tilde{l}}) \\ &\times (\sum_{h=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{i1h}\omega_{11}^{i1}, \cdots, \sum_{h=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{i\tilde{j}h}\omega_{1\tilde{j}}^{1\tilde{j}}, \sum_{h=1}^{\tilde{j}}\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ijh}a_h\omega_2^{ij}, \cdots, \sum_{h=1}^{\tilde{j}}\sum_{j=1}^{\tilde{j}}\sum_{i=1}^{\tilde{i}} n_{ijh}a_h\omega_{\tilde{l}}^{ij})\end{aligned}$$

Comparison of the corresponding each elements of quantity vectors in LHS and RHS brings to show that those are the feasible conditions (b). Therefore the economy of this section has market equilibrium.

Theorem 4 *The model of Section 2 has market equilibrium under Assumption 1, 2 and 3.*

Properties of market equilibrium

We shall analyse the properties of market equilibrium. In the followings, we attach asterisk to denote the quantities in equilibrium. For example, the equilibrium consumptions are x_{1j}^{ih*} and x_{lh}^{ij*} $(h, j = 1, \cdots, \tilde{j}, l = 2, \cdots, \tilde{l})$.

First, we find that the consumptions cannot be the same as the initial holdings, that is, the equilibrium consumptions at $j \neq j'$ cannot be $\omega_{1j}^{ij} = x_{1j}^{ij} = x_{1j'}^{ij'} = \omega_{1j'}^{ij'}, \omega_l^{ij} = x_{lj}^{ij} = x_{lj'}^{ij'} = \omega_l^{ij'}$. This is because the initial holdings for the consumption of general goods will decrease depending on the increase in number of circumstances from Assumption 2, and then the same type agent will consume different general goods, which lead to different to utility levels. Therefore the equilibrium consumption must be different from the initial holdings.

We will clarify the properties of market equilibrium through the analysis of equilibrium quantities of the type i agent of the initial circumstance j.

Lemma 8 *Under Assumption 1,2 and 3 the consumptions of the local goods at economic circumstances h and $h'(h < h')$ by type i agent of the initial circumstance j are $x_1^{ih*} \neq x_1^{ih'*}$.*

the consumptions of the local goods at j and $j'(j > j')$ is $x_1^{1j} \neq x_1^{1j'}$.

(Proof) This is proved by contradiction. Suppose that the consumptions of local goods at both circumstances are equal, that is, $x_{1h}^{ij*} = x_{1h'}^{ij*}$.

From the quasi-concavity of preference we obtain

$$p_{1h}x_{1h}^{ij*} + \sum_{k=2}^{\tilde{l}} p_k x_{kh}^{ij*} \leq p_{1h}x_{1h'}^{ij*} + \sum_{k=2}^{\tilde{l}} p_k x_{kh'}^{ij*},$$

and we also get from the quasi-concavity of preference

$$p_{1h'}x_{1h'}^{ij*} + \sum_{k=2}^{\tilde{l}} p_k x_{kh'}^{ij*} \leq p_{1h'}x_{1h}^{ij*} + \sum_{k=2}^{\tilde{l}} p_k x_{kh}^{ij*}.$$

From these two inequalities we obtain

$$\sum_{k=2}^{\tilde{l}} p_k (x_{kh'}^{ij*} - x_{kh}^{ij*}) \leq 0 \leq \sum_{k=2}^{\tilde{l}} p_k (x_{kh'}^{ij*} - x_{kh}^{ij*}).$$

It is needed for this inequality that $x_{kh}^{ij*} = x_{kh'}^{ij*}, (k = 2, \cdots, \tilde{l})$. This means that the consumptions of local goods and general goods at h and h' are equal respectively. But, as we already have argued, the consumptions at different economic circumstances cannot be equal as for each goods. Therefore the consumptions of local goods at equilibrium is $x_{1h}^{ij*} \neq x_{1h'}^{ij*}$.

□

From Lemma 8 the consumptions of the local goods at any h and h' are limited to the cases of $x_{1h}^{ij*} > x_{1h'}^{ij*}$ or $x_{1h}^{ij*} < x_{1h'}^{ij*}$. The possible combination of general goods and the local goods in the equilibrium would be

$$\sum_{k=2}^{\tilde{l}} p_k x_{kh}^{ij*} \quad < \quad \sum_{k=2}^{\tilde{l}} p_k x_{kh'}^{ij*} \quad \text{in correspondence with} \quad x_{1h}^{ij*} > x_{1h'}^{ij*}, \tag{12}$$

$$\sum_{k=2}^{\tilde{l}} p_k x_{kh}^{ij*} \quad > \quad \sum_{k=2}^{\tilde{l}} p_k x_{kh'}^{ij*} \quad \text{in correspondence with} \quad x_{1h}^{ij*} < x_{1h'}^{ij*}. \tag{13}$$

The reason is as follows. If $x_{1h}^{ij*} > x_{1h'}^{ij*}$ and $\sum_{k=2}^{\tilde{l}} p_k x_{kh}^{ij*} > \sum_{k=2}^{\tilde{l}} p_k x_{kh'}^{ij*}$ then the agent at h can consume more than the agent at h', which makes the utility of agent at h bigger than that of the agent at h'. This case cannot be in equilibrium. The case $x_{1h}^{ij*} < x_{1h'}^{ij*}$ and $\sum_{k=2}^{\tilde{l}} p_k x_{kh}^{ij*} < \sum_{k=2}^{\tilde{l}} p_k x_{kh'}^{ij*}$ cannot be in equilibrium either. Therefore possible combination of consumptions are (12) and (13).

Further analysis shows that the only latter case, (13) holds in an equilibrium.

Property 1 *Under Assumption 1, 2 and 3 the equilibrium consumptions of the local good and the general goods are*

$$x_{1h}^{ij*} < x_{1h'}^{ij*} \quad \textit{and} \quad \sum_{k=2}^{\tilde{l}} p_k x_{kh}^{ij*} > \sum_{k=2}^{\tilde{l}} p_k x_{kh'}^{ij*}.$$

(Proof) We shall show that the case : $x_{1j}^{ij*} > x_{1h'}^{ij*}$ and $\sum_{k=2}^{\tilde{l}} p_k x_{kh}^{ij*} < \sum_{k=2}^{\tilde{l}} p_k x_{kh'}^{ij*}$ contradicts the relation of disposable incomes

$$p_{1j}\omega_{1j}^{ij} + a_h \tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij} > p_{1j}\omega_{1j}^{ij} + a_{h'} \tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij}.$$

The prices corresponding to x_{1h}^{ij*} and $x_{kh}^{ij*}(k = 2, 3, \cdots, \tilde{l})$ are p_{1h} and p_k. The next relation follows from the quasi-concavity of preference.

$$p_{1j}\omega_{1j}^{ij} + a_h \tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij} = p_{1h}x_{1h}^{ij*} + \sum_{k=1}^{\tilde{l}} p_k x_{kh}^{ij*} < p_{1h}x_{1h'}^{ij*} + \sum_{k=1}^{\tilde{l}} p_k x_{kh'}^{ij*} \tag{14}$$

On the other hand, when the consumptions at the circumstance h' meet with the prices $p_{1h'}$ and p_k respectively, we also obtain the next relation from the quasi-concavity of preference.

$$p_{1h'}x_{1h'}^{ij*} + \sum_{k=1}^{\tilde{l}} p_k x_{kh'}^{ij*} < p_{1h'}x_{1h}^{ij*} + \sum_{k=1}^{\tilde{l}} p_k x_{kh}^{ij*}$$

This is transformed into

$$p_{1h'}(x_{1h}^{ij*} - x_{1h'}^{ij*}) > \sum_{k=2}^{\tilde{l}} p_k(x_{kh'}^{ij*} - x_{kh}^{ij*}), \tag{15}$$

and (14) is changed into

$$0 < p_{1h}(x_{1h}^{ij*} - x_{1h'}^{ij*}) < \sum_{k=2}^{\tilde{l}} p_k(x_{kh'}^{ij*} - x_{kh}^{ij*}). \tag{16}$$

It follows from the relations (15) and (16) that

$$p_{1h}(x_{1h}^{ij*} - x_{1h'}^{ij*}) < p_{1h'}(x_{1h}^{ij*} - x_{1h'}^{ij*}).$$

Thus $p_{1h} < p_{1h'}$.

Finally the price relation $p_{1h} < p_{1h'}$ leads us to the properties of this Property 1. We obtain

$$p_{1h}x_{1h'}^{ij*} + \sum_{k=1}^{\tilde{l}} p_k x_{kh'}^{ij*} < p_{1h'}x_{1h'}^{ij*} + \sum_{k=1}^{\tilde{l}} p_k x_{kh'}^{ij*} = p_{1j}\omega_{1j}^{ij} + a_{h'} \tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij} \tag{17}$$

(14) and (17) give us the relation

$$p_{1j}\omega_{1j}^{ij} + a_h \tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij} < p_{1j}\omega_{1j}^{ij} + a_{h'} \tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij}.$$

This contradicts the premise of budgets at economic circumstance h and h'

$$p_{1j}\omega_{1j}^{ij} + a_h \tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij} > p_{1j}\omega_{1j}^{ij} + a_{h'} \tilde{\boldsymbol{p}} \cdot \tilde{\boldsymbol{\omega}}^{ij}.$$

This means that the case: $x_{1h}^{ij*} > x_{1h'}^{ij*}$ and $\sum_{k=2}^{\tilde{l}} p_k x_{kh}^{ij*} < \sum_{k=2}^{\tilde{l}} p_k x_{kh'}^{ij*}$ cannot be in an equilibrium. Thus the consumptions of the local and the general goods are $x_{1h}^{ij*} < x_{1h'}^{ij*}$ and $\sum_{k=2}^{\tilde{l}} p_k x_{kh}^{ij*} > \sum_{k=2}^{\tilde{l}} p_k x_{kh'}^{ij*}$ in an equilibrium.

□

Property 2 *Under Assumption 1, 2 and 3 the equilibrium prices of the local goods at* $h < h'$ *are* $p_{1h} > p_{1h'}$.

(Proof)

The property is proved from the quasi-concavity of preference as in the proof of Property 1.

$$p_{1h}x_{1h}^{ij*} + \sum_{k=2}^{\tilde{l}} p_k x_{kh}^{ij*} < p_{1h}x_{1h'}^{ij*} + \sum_{k=2}^{\tilde{l}} p_k x_{kh'}^{ij*}$$

$$p_{1h'}x_{1h'}^{ij*} + \sum_{k=2}^{\tilde{l}} p_k x_{kh'}^{ij*} < p_{1h'}x_{1h}^{ij*} + \sum_{k=2}^{\tilde{l}} p_k x_{kh}^{ij*}$$

Transformation brings the next relations

$$p_{1h}(x_{1h'}^{ij*} - x_{1h}^{ij*}) > \sum_{k=2}^{\tilde{l}} p_k(x_{kh}^{ij*} - x_{kh'}^{ij*}), \tag{18}$$

$$0 < p_{1h'}(x_{1h'}^{ij*} - x_{1h}^{ij*}) < \sum_{k=2}^{\tilde{l}} p_k(x_{kh}^{ij*} - x_{kh'}^{ij*}). \tag{19}$$

Thus from (18) and (19) we get $p_{1h'}(x_{1h'}^{ij*} - x_{1h}^{ij*}) < p_{1h}(x_{1h'}^{ij*} - x_{1h'}^{ij*})$, which means $p_{1h'} < p_{1h}$.

□

What we have shown is in property 1 and 2; If an agent's initial holdings of goods and the initial general goods are identical but decreases like 'iceberg' when he changes his location, then 1: among the agents who are the same type and the same initial economic circumstance, the agent's consumption of local goods increases with the number of circumstance and the expenditure on the circumstance general goods decreases with the number of circumstance, and 2: the price of the local goods decreases with the number of circumstance. It might be viewed that this results correspond with the residential location model in urban economics and the economic model of this section approximates it well.

However there still exists difference between the two models. Property 1 of this paper insists that the properties of goods consumption in equilibrium holds only among the same type agents who are in the same initial economic circumstance. By contrast, in the residential location model the consumption of residential service increases and the consumption of composite goods decreases with distance in an equilibrium. That is, the consumption of those goods at a location closer to the city center is larger than that at a location farther from the city center. The difference comes from whether a model assumes the initial location of agents or not. The residential model does not assume the initial location of agents and it leads the general property of consumption of goods. But the model of this paper assumes the initial location of agents, and then the property of consumption of goods hold only the same agents who are in the same initial circumstance. So if we restrict the model of this paper to allow only one type of agents who locate in the only one economic circumstance at the initial state, then the results would be same as the residential location model. Therefore from the point of the general equilibrium the residential location model can be viewed that the residents who are in one location at initial time spread over urban areas to locate in equilibrium.

5 Conclusion

We have analyzed market equilibrium of pure exchange economy under selectable economic circumstances in this paper.

The eminent feature of this paper is that there are some goods of which prices change with circumstance, and that an agent is able to choose a circumstance for his consumption of goods. The one of the main purposes of this paper is to prove the existence of market equilibrium in such an economy, and we have obtained Theorem 3. Preceding to Theorem 3, we proved Theorem 2: 'a temporary' equilibrium' which is derived when agent distribution is given. This is the direct application of typical general equilibrium theory to our model.

Finally we obtained Theorem 3: market equilibrium which is viewed as 'permanent' equilibrium. This market equilibrium is derived in the case that each agent is able to choose an economic circumstance for his utility maximization. There exists a common utility level of each agent type and its supported agent distribution over circumstances on this equilibrium. Therefore this paper has extended the existence of market equilibrium to the case of selectable economic circumstances. The economic models in which the price of some goods changes in different locations such as residential location and public service consumption location can be unified and studied by our model, which might be contribution to economics.

In Section 4 we have derived some properties about consumptions and prices of equilibrium in the model specified in Assumption 1, 2 and 3, which approximate our model to the residential location model of urban economics.The assumptions employed in Section 4 are that the initial holdings of goods of agents are identical regardless of economic circumstances and those reduce like 'iceberg' depending on their locations when they consume, and that agent's preference for goods is independent of economic circumstances. By this specification we have derived the same result of goods and prices as the residential location model. In this meaning the model specified in the Section 4 is thought to approximate the residential location model well. At the same time the residential location model can be viewed that the agents who locate at one location point at initial state spread over urban areas to locate in equilibrium from the point of the general equilibrium analysis.

However we have still some limitations in this model. The first point is the assumption on the consumption of local goods. This paper assumes that an agent consumes goods in the circumstance that he choose. An agent could consume goods at more than two circumstances generally and theoretically, which is excluded in this paper. But our assumption may be partially justified because the goods such as local goods is consumed only at the locatinal circumstance of an agent like residence.

The second point is that our analysis is made on based on discrete economic circumstances. If we would pursue more strict correspondence with the typical model of residential location in urban economics, then the continuous economic circumstances should have been assumed. The results upon the continuous assumption of cirucumstance would be applied more generally. However we have employed the assumption of discrete economic circumstances in this paper because of simplicity and tractability of analysis. This assumption should be relaxed for more general analysis in future research.

The third point is that our analysis is based on a pure exchange economy which lacks the production side of a market economy. This is just because of simplicity of analysis. For more complete analysis it is preferable to take the such aspect into consideration. The inclusion of production side will needs some consideration on assumption of production sites, that is, whether production activity take place in every circumstance for more general analysis or production activity take place in some limited circumstances for simpler analysis.

Appendix

Berge's Maximum Theorem

Suppose that $D \subset R^k$ and $X \subset R^l$ are compact. Further suppose that $\beta : D \to X$ is a non-empty continuous correspondence, $u : X \to R$ is a continuous function, and $\xi : D \to X$ is $\xi(\boldsymbol{d}) = \{\boldsymbol{x} \in \beta(\boldsymbol{d}) | u(\boldsymbol{x})$ have maximum.$\}$. Then the correspondence ξ is upper hemi-continuous, and the function $v(\boldsymbol{d}) = u(\boldsymbol{x})$ is continuous.

Kakutani's Fixed Point Theorem

Let K be a non-empty compact and convex subset, and $\Gamma : K \to K$ be a upper hemi-continuous correspondence with non-empty compact and convex value. Then there exists a fixed point $\boldsymbol{x}^* \in \Gamma(\boldsymbol{x}^*)$.

References

[1] Alonso, W., *Location and land use*, Harvard University Press., 1964

[2] Arrow, K.J., and Hahn, F.H., *General Competitive Analysis*, Holden-Day.Inc., 1971

[3] Bruekner, J.K. 'The Structure of Urban Equilibria: A Unified Treatment of the Muth-Mills Mode', *Handbook of Regional and Urban Economics*, vol.2, North-Holland, 1987

[4] Debreu, Gerard, *Theory of Value-An Axiomatic analysis of Economic Equilibrium-*, Yale University Press, 1959

[5] Hildenbrand, W. and A.P. Kirman, *Equilibrium Analysis*, North-Holland,1988

[6] Keida, O., 'Axiomatic approach to housing location model', *Studies in contemporary economics (Western Association of Economics)* ,No.7,1999. (in Japanese)

[7] Mills, E.S., *Studies in the structure of the urban economy*, Johns Hopkins University Press, 1972

[8] Moore,James C. *Mathematical methods for economic theory 2*, Springer,1999

[9] Turnbull,G. E. 'The pure theory of household location: an axiomatic approach', *Journal of Regional Science,* vol. 30, 549-562, 1990.

[10] Turnbull,G. E. 'The substitution theorem in urban consumer theory', *Journal of Urban Economics,* vol. 33, 331-343, 1993.

[11] Villar, Antonio, *Equilibrium and Efficiency in Production Economies*, Springer-Verlag, 2000.

[12] Wheaton, W.C., 'A comparative static analysis of urban spatial structure', *Journal of Economic Theory*, vol.9, 223-237, 1974