IS THE SPEED OF CONVERGENCE A GOOD PROXY FOR THE TRANSITIONAL GROWTH PATH?*

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WP-AD 2005-33

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Editor: Instituto Valenciano de Investigaciones Económicas, S.A.
Primera Edición Noviembre 2005
Depósito Legal: V-4677-2005

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* We thank Craig Burnside, John Duffy, Jordi Caballe, Theo Eicher, Lutz Hendricks, Robert King, Sharon Kozicki, Peter Klenow, KazuoMino, Pietro Peretto, Jordan Rappaport, Tomoya Sakagami, Robert Solow, Jerry Thursby, Steve Turnovsky, Chang Yonsung, John Williams, and seminar participants in many universities and conferences for their comments and suggestions. All remaining errors are our own.
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ABSTRACT

This paper compares transitional dynamics in two alternative R&D non-scale growth models, one includes endogenous human capital, whereas the other does not. We show that focusing on the speed of convergence to discriminate between the two models can be misleading. Our analysis suggest that a better alternative to discriminate between different growth theories is studying the whole adjustment path predicted by them. In addition, we find that the introduction of human capital makes the speed of convergence predicted by the model much less sensitive to exogenous shocks. This last result offers theoretical support to the similar convergence speeds estimated by the literature in different samples.

JEL Classification: O33, O41, O47

Keywords: Convergence, R&D, human capital, asymptotic speed, transitional dynamics

1 Introduction

The growth literature has devoted considerable time and effort in analyzing the speed of convergence predicted by alternative growth models.[1] The speed of convergence is the rate at which a country's output approaches its balanced growth path. One reason for this analysis is its importance to establish the stability of the model's long-run equilibrium. The other crucial reason is that, as pointed out by Ortigueira and Santos (1997) and Eicher and Turnovsky (1999b, 2001), among many others, a desirable property for growth models is to deliver a speed of convergence around 2 percent, figure that is consistent with most cross-country empirical studies.[2] In this paper, however, we show that the speed of convergence in itself maybe a misleading representation of the transitional growth path and that, therefore, we need to study the whole adjustment path predicted by transitional dynamics if we want to discriminate among alternative growth theories.

More specifically, we study convergence speeds in two versions of the type of *hybrid* non-scale R&D-based growth framework studied by Eicher and Turnovsky (1999a, 1999b, 2001), one without human capital and another one that includes endogenous human capital.[3] We follow the standard approach and focus on the asymptotic speed predicted by the system of equations that characterize the model's equilibrium dynamics. We obtain several interesting findings. First, the introduction of human capital decreases the convergence speed. More important, both the non-scale R&D-based growth model with human capital and the one without human capital predict empirically-supported speeds.

We discover, however, that this result alone is not be very informative about the overall capacity of these two models for reproducing convergence episodes. The reason is that small variations in the asymptotic speed can be related to substantial changes in the initial periods of the adjustment path. More specifically, even though both frameworks deliver similar speeds, the transitional dynamics of the model with human capital are able to reproduce important output-convergence experiences such as those of Japan and South Korea much more accurately than the model without human capital. In this sense, the model with human capital arises as a better growth theory.

We also show that the introduction of human capital makes the asymptotic speed of convergence

[1] For example, recent contributions have focused on the effect on the convergence speed of income inequality (Zhang (2005)), government financing (Gokan (2003)), or international labor mobility (Rappaport (2005)).

[2] Barro and Sala-i-Martin (1995) report convergence speeds that vary from 0.4%–3% in Japan, 0.4%–6% in the U.S. and 0.7%–3.4% in Europe. Temple (1998) reports estimates for OECD nations between 1.5% and 3.6%. Authors such as Caselli *et al.* (1996), however, have estimated larger convergence speeds, as high as 10%.

[3] Other papers such as Keller (1996), Eicher (1996), Funke and Strulik (2000), Lloyd-Ellis and Roberts (2002), and Papageorgiou and Perez-Sebastian (2004, forthcoming) present growth models in which both human capital and technological innovation are endogenous. They do not study the speed of convergence.

much less sensitive to changes in the underlying parameter values. This finding can offer theoretical support to Barro and Sala-i-Martin's (1995) result that convergence-speed estimates do not vary substantially across different countries or regions.

The remainder of the paper is organized as follows. Section 2 presents the model with endogenous technical change and human capital, and studies steady-state predictions. Section 3 states the equations that will be employed to analyze transitional dynamics in the two models. Section 4 obtains numerical results for the asymptotic speed of convergence and the adjustment paths of the alternative models. Section 4 concludes.

2 The R&D Model with Human Capital

The models studied in this paper are an extension of the type of non-scale R&D-based framework studied in Eicher and Turnovsky (1999a, 1999b, 2001). As shown by Jones (1995), this type of framework succeeds in reconciling important properties of the data such as increasing R&D intensity with constant output growth rates. We incorporate two modifications: first, we allow for human capital stock to accumulate endogenously over time, and second, technology imitation in our model is costly. These modifications suggested, for example, by Bils and Klenow (2000) make the model more appropriate to analyze countries at different levels of development.

In this section, we first outline the economic environment under which households and firms operate when human capital accumulation is possible. Then we solve the socially optimal problem. Our exposition is focused on aggregate technologies. The main reason is that the human capital technology incorporated in this paper can not be easily derived from a decentralized setup due to aggregation problems.[4]

2.1 Economic environment

The economy consists of identical infinitely-lived agents, and population grows exogenously at rate n. Agents have preferences only over consumption, and choose to allocate their time endowment in three types of activities: consumption-good production, R&D effort, and human capital attainment.

Our model economy is characterized by the following three equations: First, at period t, output (Y_t) is produced using labor (L_{Yt}) and physical capital (K_t) according to the following aggregate

[4] See footnote 6 for a discussion on the aggregation problem of this approach.

Cobb-Douglas technology:

$$Y_t = A_t^{\xi} \, (h_t \, L_{Yt})^{1-\alpha} \, K_t^{\alpha} \quad , \quad 0 < \alpha < 1 \, , \;\; \xi > 0, \tag{1}$$

where h_t represents the effectiveness of average human capital level on labor; α is the share of capital; ξ is a technology externality; and A_t is the economy's technical level.

Second, the R&D equation that determines technological progress is given by

$$A_{t+1} - A_t = \mu A_t^{\phi} \, (h_t L_{At})^{\lambda} \left(\frac{A_t^*}{A_t} \right)^{\psi} , \quad \phi < 1, \;\; 0 < \lambda \leq 1, \;\; \mu, \psi \geq 0, \;\; A_t^* \geq A_t, \tag{2}$$

where L_{At} is the portion of labor employed in the R&D sector at time t; A_t^* is the worldwide stock of existing technology at t, which grows exogenously at rate g_{A^*}; ϕ is an externality due to the stock of existing technology; and λ captures the existence of decreasing returns to R&D effort. The above R&D equation is the one proposed by Jones (1995, 2002) plus a *catch-up* term $\left(\frac{A_t^*}{A_t} \right)^{\psi}$, where ψ is a technology-gap parameter. The catch-up term is also consistent with the "relative backwardness" hypothesis of Findlay (1978) that the rate of technological progress in a relatively backward country is an increasing function of the gap between its own level of technology and that of the advanced country.[5]

Third, we have the schooling equation that determines the way by which human capital accumulates. The human capital technology follows Bils and Klenow (2000), who suggest that the Mincerian specification of human capital (Mincer (1974)) is the appropriate way to incorporate years of schooling in the aggregate production function. Following their approach, human capital per capita is given by

$$h_t = e^{f(S_t)} \, , \tag{3}$$

where $f(S_t) = \eta S_t^{\beta}$, $\eta > 0$, $\beta > 0$; and S_t is the labor force average years of schooling at date t. The derivative $f'(S_t)$ represents the return to schooling estimated in a Mincerian wage regression: an additional year of schooling raises a worker's efficiency by $f'(S_t)$.[6]

[5]Nelson and Phelps (1966) are the first to construct a formal model based on the catch-up term. Parente and Prescott (1994) notice that this formulation implies that development rates increase over time (with A_t^*), and provide empirical evidence that is consistent with this implication. Benhabib and Spiegel (1994) find evidence in favor of an R&D equation with imitation in a large sample of countries.

[6]To be fully consistent with the Mincerian interpretation, $H_{jt} = \sum_{i=1}^{L_{jt}} e^{f(s_{it})}$; where s_{it} is the educational attainment of worker i at date t. The mapping between this expression and equation (3) is not straightforward, and has not been addressed by the literature, with the exception of Lloyd-Ellis and Roberts (2002) who perform only balanced-growth path analysis in a finitely-lived agent framework. The difficulty arises because different cohorts can possess different schooling levels. To make both expressions consistent, we could assume that the first generation of agents pins down the workers' educational attainment, and that posterior cohorts are forced to stay in school until they accumulate this educational level. In this way, all workers would have the same years of education (i.e., $s_{it} = S_t$

We assume that, each period, agents allocate time to human capital formation only after output production has taken place.[7] Let L_{Ht} be the total amount of labor invested in schooling in the economy at date t. Assume that at some point in time, say period 1, the average educational attainment equals *zero*. Next period, given that consumers live for ever, the average years of schooling will be $S_2 = \frac{L_{H1}}{L_2}$, where L_t is the labor size at date t. In period 3, $S_3 = \frac{L_{H1}+L_{H2}}{L_3}$, and so on. Hence, the average educational attainment can be written as

$$S_t = \frac{\sum_{j=1}^{t-1} L_{Hj}}{L_t}. \tag{4}$$

From equation (4), we can write

$$S_{t+1} = \frac{S_t\, L_t + L_{Ht}}{L_{t+1}}, \tag{5}$$

which in turn implies

$$S_{t+1} - S_t = \left(\frac{1}{1+n}\right)\left(\frac{L_{Ht}}{L_t} - n\, S_t\right). \tag{6}$$

2.2 Social planner's problem

Let C_t be the amount of aggregate consumption at date t. A central planner would choose the sequences $\{C_t, S_t, A_t, K_t, L_{Yt}, L_{At}, L_{Ht}\}_{t=0}^{\infty}$ so as to maximize the lifetime utility of the representative consumer subject to the feasibility constraints of the economy, and the initial values L_0, K_0, S_0, and A_0. The problem is stated as follows:

$$\max_{\{C_t, S_t, A_t, K_t, S_t, L_{Yt}, L_{At}, L_{Ht}\}} \sum_{t=0}^{\infty} \rho^t \left[\frac{\left(\frac{C_t}{L_t}\right)^{1-\theta} - 1}{1-\theta}\right], \tag{7}$$

subject to,

$$Y_t = A_t^{\xi}\left(e^{f(S_t)} L_{Yt}\right)^{1-\alpha} K_t^{\alpha}, \tag{8}$$

$$I_t = K_{t+1} - (1-\delta)\, K_t = Y_t - C_t, \tag{9}$$

$$A_{t+1} - A_t = \mu A_t^{\phi}\left(e^{f(S_t)} L_{At}\right)^{\lambda}\left(\frac{A_t^*}{A_t}\right)^{\psi}, \tag{10}$$

for all i) and then $\sum_{i=1}^{L_{jt}} e^{f(s_{it})} = L_{jt}\, e^{f(S_t)}$. However, introducing this into the model would force us to keep track of the different cohorts' years of education across time, thus making the transitional dynamics analysis much more cumbersome, if not impossible. We leave this important issue to future research.

[7] The primary reason for the particular timing of events is mathematical tractability. In particular, this timing allows writing the motion equation of S_{t+1} as a function of S_t and L_{Ht} (see equation (5)). If the timing were the opposite, we would obtain the state variable S_{t+1} as a function of S_t and $L_{H,t+1}$ that could make the optimal control problem significantly more difficult to solve.

$$S_{t+1} - S_t = \left(\frac{1}{1+n}\right)\left(\frac{L_{Ht}}{L_t} - n\,S_t\right), \tag{11}$$

$$L_t = L_{Yt} + L_{At} + L_{Ht}, \tag{12}$$

$$\frac{L_{t+1}}{L_t} = 1 + n, \quad \text{for all } t, \tag{13}$$

$$\frac{A^*_{t+1}}{A^*_t} = 1 + g_{A^*}, \tag{14}$$

$$L_0,\; S_0,\; K_0,\; A_0 \text{ given,}$$

where θ is the inverse of the intertemporal elasticity of substitution; ρ is the discount factor; and δ is the depreciation rate of physical capital. Equation (9) is a feasibility constraint as well as the law of motion of the stock of physical capital; it states that, at the aggregate level, domestic output must equal consumption plus physical capital investment, I_t. Equation (12) is the labor constraint; the labor force – that is, the number of people employed in the output and the R&D sectors – plus the number of people in school must be equal to population.

Solving this dynamic optimization problem obtains the Euler equations that characterize the optimal allocation of labor in human capital investment, in R&D investment, and in consumption/physical capital investment respectively as follows:

$$\left(\frac{C_t}{L_t}\right)^{-\theta}\frac{(1-\alpha)Y_t}{L_{Yt}} = \frac{\rho}{1+n}\left(\frac{C_{t+1}}{L_{t+1}}\right)^{-\theta}\frac{(1-\alpha)Y_{t+1}}{L_{Y,t+1}}\left[1 + f\,'(S_{t+1})\left(\frac{L_{Y,t+1}}{L_{t+1}} + \frac{L_{A,t+1}}{L_{t+1}}\right)\right], \tag{15}$$

$$\begin{aligned}\left(\frac{C_t}{L_t}\right)^{-\theta}\frac{(1-\alpha)Y_t}{L_{Yt}} &= \frac{\rho}{1+n}\left(\frac{C_{t+1}}{L_{t+1}}\right)^{-\theta}\frac{\lambda\,(A_{t+1}-A_t)}{L_{At}} \,* \\ &\quad * \left\{\frac{\xi Y_{t+1}}{A_{t+1}} + \left[1 + (\phi - \psi)\left(\frac{A_{t+2}-A_{t+1}}{A_{t+1}}\right)\right]\left[\frac{\frac{(1-\alpha)Y_{t+1}}{L_{Y,t+1}}}{\frac{\lambda(A_{t+2}-A_{t+1})}{L_{A,t+1}}}\right]\right\},\end{aligned} \tag{16}$$

$$\left(\frac{C_t}{L_t}\right)^{-\theta} = \frac{\rho}{1+n}\left(\frac{C_{t+1}}{L_{t+1}}\right)^{-\theta}\left[\frac{\alpha Y_{t+1}}{K_{t+1}} + (1-\delta)\right]. \tag{17}$$

At the optimum, the planner must be indifferent between investing one additional unit of labor in schooling, R&D, and final output production. The LHS of equations (15) and (16) represent the return from allocating one additional unit of labor to output production. The RHS of equation (15) is the discounted marginal return to schooling, taking into account labor growth. The RHS term in brackets arises because human capital determines the effectiveness of labor employed in output production as well as in R&D. The RHS of equation (16) is the return to R&D investment. An additional unit of R&D labor generates $\frac{\lambda(A_{t+1}-A_t)}{L_{At}}$ new ideas for new types of producer durables. Every new design increases next period's output by $\frac{\xi Y_{t+1}}{A_{t+1}}$ and R&D production by $\frac{dA_{t+2}}{dA_{t+1}}$ *times* $\frac{(1-\alpha)Y_{t+1}}{L_{Y,t+1}}\left[\frac{\lambda(A_{t+2}-A_{t+1})}{L_{A,t+1}}\right]^{-1}$, where the term $\frac{(1-\alpha)Y_{t+1}}{L_{Y,t+1}}\left[\frac{\lambda(A_{t+2}-A_{t+1})}{L_{A,t+1}}\right]^{-1}$ gives the value of one additional design that equalizes labor wages across sectors. Euler equation (17) is standard and states that the planner is indifferent between consuming one additional unit of output today and converting it into capital (thus consuming the proceeds tomorrow).

2.3 Steady-state growth

We now derive the model's balanced-growth path. Solving for the interior solution, equation (12) implies that in order for the labor allocations to grow at constant rates, L_{Ht}, L_{Yt} and L_{At} must all increase at the same rate as L_t. This means that the ratio $\frac{L_{Ht}}{L_t}$ is invariant along the balanced-growth path. Hence, equation (11) implies that, at steady-state (ss), S_{ss} is constant and equals

$$S_{ss}=\frac{u_{H,ss}}{n}, \tag{18}$$

where $u_{H,ss}=\left.\frac{L_H}{L}\right|_{ss}$. Equation (18) shows that along the balanced-growth path, the economy invests in human capital just to provide new generations with the steady-state level of schooling.

Let lower case letters denote per capita variables, and $g_x=G_x-1$ denote the growth rate of x. The aggregate production function, given by equation (8), combined with the steady-state condition $g_{Y,ss}=g_{K,ss}$ delivers the gross growth rate of output as a function of the gross growth rate of technology as

$$G_{Y,ss}=\left(G_{A,ss}\right)^{\frac{\xi}{1-\alpha}}(1+n). \tag{19}$$

Since $G_{A,ss}$ is a constant, it follows from equation (2) that

$$G_{A,ss}=\left[(1+n)^{\lambda}\left(G_{A^*,ss}\right)^{\psi}\right]^{\frac{1}{1+\psi-\phi}}. \tag{20}$$

Equation (20) shows the relationship between the technology frontier growth rate and the technology growth rate of the model economy. Since $\frac{\psi}{1+\psi-\phi}<1$, it is easy to show that there is a unique point at which

$$G_{A,ss}=G_{A^*,ss}=(1+n)^{\frac{\lambda}{1-\phi}}. \tag{21}$$

We focus on a special case: we suppose that $G_{A^*,ss}$ is given by expression (21) and, therefore, so is $G_{A,ss}$.[8] This in turn implies that

$$G_{Y,ss}=G_{C,ss}=G_{K,ss}=(1+n)^{\frac{\lambda\xi}{(1-\alpha)(1-\phi)}}. \tag{22}$$

[8]We could assume that a technology leader shifts outward the world technological frontier according to equation (2) which now reduces to

$$A^*_{t+1}-A^*_t=\mu A_t^{*\phi}\left(h^*_{At}L^*_{At}\right)^{\lambda},$$

Consistent with Jones (1995, 2002) our balanced-growth path is free of *scale effects*. The reason why the model's long-run growth is equivalent to that of Jones even in the presence of a schooling sector, is that at steady state the mean years of education, S_t, reaches a constant level S_{ss}.

3 Transitional Dynamics

We now turn to studying the transitional-dynamics predictions of the model. Our main goal is to compare these predictions to the ones of an identical model but without human capital. As a theory, the model presented above (from now on, *model with H*) emphasizes the important complementary role that human capital accumulation has on final output production and R&D. A non-scale R&D-based growth model without human capital, on the other hand, considers that the role of human capital is less important. This second type of framework (from now on, *model w/o H*) can be obtained by simply removing human capital from the above setup, and corresponds to the class of two-sector non-scale growth model studied by Jones (1995), Eicher and Turnovsky (1999a) and Perez-Sebastian (2000), among others.

In order to generate the system of equations that can help to study transitional dynamics, we need to redefine variables so that their values remain constant at steady state. In particular, the aggregate production function, equation (8), suggests that we normalize variables by the term $A_t^{\frac{\xi}{1-\alpha}} L_t$. We can then rewrite consumption, physical capital and output as $\hat{c}_t = \frac{C_t}{A_t^{\frac{\xi}{1-\alpha}} L_t}$, $\hat{k}_t = \frac{K_t}{A_t^{\frac{\xi}{1-\alpha}} L_t}$ and $\hat{y}_t = \frac{Y_t}{A_t^{\frac{\xi}{1-\alpha}} L_t}$, respectively.

Next, we present the normalized system for the *model with H* and for the *model w/o H*. We also derive the equation that obtains the asymptotic speed of convergence.

3.1 The normalized systems for the model with H

Using equation (15) gives

$$\left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)^{\theta} \left(\frac{u_{Y,t+1}}{u_{Yt}}\right) (G_{At})^{\frac{(\theta-1)\xi}{1-\alpha}} \left(\frac{\hat{y}_t}{\hat{y}_{t+1}}\right) = \left(\frac{\rho}{1+n}\right) \left[f'(S_{t+1})\,(u_{Y,t+1} + u_{A,t+1}) + 1\right]. \tag{23}$$

From the R&D equation (2), we derive G_{At} as

$$G_{At} = \frac{A_{t+1}}{A_t} = 1 - \delta_A + \upsilon \left[e^{f(S_t)} u_{At}\right]^{\lambda} T^{(1+\psi-\phi)}, \tag{24}$$

where now $\frac{A_t^*}{A_t} = 1$ as imitation is not possible at the frontier; and $*$ denotes the value which variables take in the leading country. In such case $G_A^* = 1 + g_A^* = (1+n^*)^{\frac{\lambda}{1-\phi}}$ as in Jones (1995, 2002). Assuming that $n = n^*$, and substituting G_A^* into equation (20) delivers equation (21).

where $T = \frac{A_t^*}{A_t}$; and $\upsilon = \mu \left(A_t^*\right)^{\phi-1} L_t^{\lambda}$, which is a constant.[9] From equation (16) we obtain

$$\left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)^{\theta}\left(\frac{\hat{y}_t}{\hat{y}_{t+1}}\right)\left(\frac{u_{Y,t+1}}{u_{Yt}}\right) = \frac{\rho\left(g_{At}+\delta_A\right)}{G_{At}^{\frac{\xi}{1-\alpha}(\theta-1)+1}}\left(\frac{u_{A,t+1}}{u_{At}}\right) * \\ * \left[\left(\frac{\lambda\xi}{1-\alpha}\right)\left(\frac{u_{Y,t+1}}{u_{A,t+1}}\right)+\left(\frac{1-\delta_A}{\left(g_{A,t+1}+\delta_A\right)}\right)+(\phi-\psi)\right]. \quad (25)$$

Finally, from equation (17) we obtain

$$\frac{1+n}{\rho}\left[\left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)\left(G_{At}\right)^{\frac{\xi}{1-\alpha}}\right]^{\theta} = \alpha\frac{\hat{y}_{t+1}}{\hat{k}_{t+1}} + (1-\delta_K). \quad (26)$$

The system that determines the dynamic equilibrium normalized allocations is formed by the conditions associated with three control and three state variables as follows:

Control Variables:

1. Euler equation for population share in schooling, u_{ht}: Eq. (23).
2. Euler equation for population share in R&D, u_{At}: Eq. (25).
3. Euler equation for normalized consumption, $\hat{c}_t$: Eq. (26).

Subject to the population constraint $u_{Yt} = 1 - u_{At} - u_{ht}$.

State Variables:

1. Law of motion of human capital, S_t: Eq. (6).
2. Law of motion of the technology gap, T_t:

$$T_{t+1} = T_t\left(\frac{G_{A^*t}}{G_{At}}\right). \quad (27)$$

3. Law of motion of normalized physical capital, $\hat{k}_t$:

$$(1+n)\hat{k}_{t+1}\left(G_{At}\right)^{\frac{\xi}{1-\alpha}} = (1-\delta_K)\hat{k}_t + \hat{y}_t - \hat{c}_t, \quad (28)$$

where G_{At} is given by expression (24), $G_{A^*t} = G_{A,ss}$ for all t, and

$$\hat{y}_t = \hat{k}_t^{\alpha}\left[e^{f_Y(S_t)}u_{Yt}\right]^{1-\alpha}. \quad (29)$$

[9]To show that υ is constant requires some algebra. Rewriting the equality in its gross growth form, $\frac{\upsilon_{t+1}}{\upsilon_t} = G_{A^*t}^{\phi-1}(1+n)^{\lambda}$, and given that $G_{A^*t} = G_{A,ss} = (1+n)^{\frac{\lambda}{1-\phi}}$, it follows that $\frac{\upsilon_{t+1}}{\upsilon_t} = 1$. Notice that if A_t^* did not grow according to equation (21), υ could not be constant, making the simulation exercise more tedious.

3.2 The normalized system for the model w/o H

The model economy is now characterized by two control variables (consumption and R&D-labor) and two state variables (physical capital and technology gap). It is straightforward to show that the system of equations that determines the dynamics in the economy without schooling sector consists of Euler conditions (25) and (26), and motion equations (27) and (28), subject to $f(S) = 0$, the population constraint $u_{Yt} = 1 - u_{At}$, $G_{A^*t} = G_{A,ss}$, and equations (24) and (29).

3.3 Asymptotic speed of convergence

To compute the asymptotic speed of convergence, we need to linearize the normalized system of Euler and motion equations around the steady state, and express the resulting system as follows:

$$\vec{x}_{t+1} = D\,\vec{x}_t;$$

where $\vec{x}$ is the vector consisting of the state and control variables; and D is the matrix of first derivatives $(\partial x_{i,t+1}/\partial x_{jt})\ \forall i,j$ evaluated at the steady state, with x_i being the i^{th} component of vector $\vec{x}$. In the *model with H*, the transpose of this vector is $\vec{x}'_t = (\hat{c}_t, u_{At}, u_{Ht}, \hat{k}_t, T_t, S_t)$, whereas for the *model w/o H*, $\vec{x}'_t = (\hat{c}_t, u_{At}, \hat{k}_t, T_t)$.

Second, we compute the eigenvalues associated with the matrix D. Convergence speed is obtained by the largest eigenvalue (denoted as *eigen*) among those contained in the unit circle. In particular, the asymptotic speed of convergence (denoted as *asc* hereon) of normalized variable $\hat{y}$ can be written as

$$asc(\hat{y}) = -\frac{(\hat{y}_{t+1} - \hat{y}_t) - (\hat{y}_{t+1,ss} - \hat{y}_{t,ss})}{\hat{y}_t - \hat{y}_{t,ss}} = 1 - eigen.$$

Given that we are primarily interested in output *per worker*, $\frac{Y}{L_A+L_Y} = \hat{y}A^{\frac{\xi}{1-\alpha}}(u_A + u_Y)^{-1}$ (call it y^w), it is easy to show that its *asc* equals

$$asc\,(y^w) = (1 - eigen)G_{y,ss} - g_{y,ss}. \tag{30}$$

4 Numerical Results

To highlight the changes brought by the introduction of human capital into the model, we first present results for the *model w/o H*. Closed-form solutions neither for general analysis of the model's transitional dynamics nor for matrix D exist. As a consequence, we resort to numerical methods.

Table 1: Benchmark parameter values for the *model w/o H*

α	0.36	ξ	0.1	ρ	0.96	ψ	0.16
δ_K	0.06	λ	0.5	θ	1	T_{ss}	1
δ_A	0.01	ϕ	0.931	n	0.015		

4.1 The asymptotic speed in the model w/o H

Table 1 describes our benchmark economy for the *model w/o H*. For the sake of comparability, the parameter values are those chosen by Eicher and Turnovsky (1999b, 2001). The only exceptions are the parameters ϕ and ψ related to the R&D technology. In particular, these authors consider an economy without a schooling sector and without imitation, assigning a value of 0.5 to ϕ, and of *zero* to ψ. They show that in this environment the stable manifold is two dimensional. Hence, the adjustment path is asymptotically stable and unique. Furthermore, growth rates and convergence speeds can, as a consequence, vary across time and variables. For this parameterization but with $\phi = 0.5$, and $\psi = 0$, our numerical methods obtain an $asc(y^w)$ of 0.0179.[10] This is, actually, the major finding of Eicher and Turnovsky (1999b, 2001) that going from the neoclassical one-sector growth model to a two sector non-scale growth model reduces the asymptotic speed of convergence from about 7 percent to more reasonable values.[11]

As Table 1 says, we instead choose $\phi = 0.931$ and $\psi = 0.16$. That is, we consider a *model w/o H* with an R&D sector that exhibits increasing returns in knowledge and labor, and imitation. The reason to assign a larger value to ϕ is that we want to generate reasonable values for the steady-state growth rate of output per capita. Taking $g_{y,ss} = 1.6\%$, the average g_y in Bils and Klenow's (2000) 91-country sample, implies that $\phi = 0.931$ through equation (22), for given values of λ, n, ξ, and α.[12]

A value of ψ greater than *zero*, in turn, allows reconciling a reasonable $g_{y,ss}$ with fast development experiences, as Perez-Sebastian (2000) shows. Otherwise, the two-sector *hybrid* non-scale R&D-based growth model delivers implausibly low converge speeds, with half lives in the hundred of years.[13] A value of 0.16 for ψ is within the calibrated values that we obtain later in section 4.3.

[10] All numerical results were obtained using MATHEMATICA. Programs are available by the authors upon request.

[11] These authors employ a continuous-time version of the model that provides slightly larger speeds than our discrete-time approach. In particular, for the benchmark economy, the continuous-time analog would imply $asc(y^w) = 0.0184$. The slightly larger speed implied by continuous-time holds across all the models considered in our paper.

[12] It is well known that the empirical literature does not offer much guidance about the value of ϕ.

[13] This result was originally shown by Jones(1995). For example, with the benchmark parameterization but taking

Table 2: Asymptotic speed of convergence for different parameterizations

	Benchmark	$\delta_A = 0.1$	$\lambda = 0.75$	$\psi = 0.25$
Model w/o H	0.0196	0.0410	0.0340	0.0330
Model with H	0.0132	0.0172	0.0145	0.0168

In the benchmark economy given by Table 1, $asc(y^w)$ is 0.0196, which exactly equals the 2% suggested by previous literature. Obviously, this prediction is sensitive to changes in the parameters. Table 2 presents results for different parameterizations. If δ_A increases from 0.01 to 0.1, another empirical-supported value (see Caballero and Jaffe (1993)), the *model w/o H* predicts a relatively large increase from 0.0196 to 0.041. Another example. When λ increases from 0.5 to the 0.75 estimated by Jones and Williams (2000) then $asc(y^w)$ rises to 0.034. Finally, let us think about policy actions that affect the technology-gap parameter ψ. This could occur, for example, because of changes in the degree of barriers to technology adoption along the lines of Parente and Prescott (1994). Suppose that a successful policy to enhance technological adoption causes ψ to increase from 0.16 to 0.25. The consequence is that $asc(y^w)$ becomes 0.033.

4.2 The asymptotic speeds in the model with H

Next, we analyze the speed of convergence in the model with schooling and imitation. To do this, we need to calibrate the human capital technology. Following Bils and Klenow (2000), we assume that

$$f(S) = \eta S^{\beta}, \;\; \eta > 0, \;\; \beta > 0. \tag{31}$$

Then using Psacharopoulos' (1994) cross-country sample on average educational attainment and Mincerian coefficients we estimate η and β. Given equation (31), we can construct the loglinear regression equation

$$\ln\left(Mincer_i\right) = a + b\,\ln S_i + \varepsilon_i, \tag{32}$$

where $Mincer_i = f'(S_i)$ is the estimated Mincerian coefficient for country i; a and b equal $ln(\eta\beta)$ and $(\beta - 1)$, respectively; and ε_i is a random disturbance term. We obtain estimates of $\eta = 0.69$ and $\beta = 0.43$, both significantly different from zero at the 1 percent level, that are very similar to those obtained by Bils and Klenow (2000).

$\phi = 0.5$ and $\psi = 0$, the *model w/o H* generates $g_{y,ss} = 0.0023$. Taking $\phi = 0.913$, $g_{y,ss}$ becomes 0.016, but the implied asc falls to -0.0042, clearly an implausibly low value.

Table 3: Parameter values for the *model with H*

α	0.36	ξ	0.1	ρ	0.96	ψ	0.16	T_{ss}	1
δ_K	0.06	λ	0.5	θ	1	η	0.69	S_{ss}	12.03
δ_A	0.01	ϕ	0.931	n	0.015	β	0.43	$g_{y,ss}$	0.016

Table 3 presents the parameter values used in our numerical exercise. It includes the parameters used in the benchmark economy (Table 1) and the human capital technology parameters ($\eta = 0.69$, $\beta = 0.43$). Given the above values, equations (15), (18) and (22) imply that the steady-state average educational attainment is 12.03 years, close to the 2000 U.S. figure of 12.05 obtained by Barro and Lee (2001). For this economy, the stable manifold is pinned down by three eigenvalues that are contained within the unit circle. That is, the transition is characterized by a three-dimensional stable saddle-path which in turn implies that the adjustment path is asymptotically stable and unique.[14]

Moreover, this economy with human capital predicts an *asc* for output per worker equal to 0.0132. Note that even though this convergence speed is lower than the 0.0196 provided by the *model w/o H*, it is still well within empirical estimates. The reduction in the convergence speed occurs because of the additional schooling sector present in our model. A new sector implies that the same amount of available labor must now be allocated among three (rather than two) sectors, which makes state variables move more slowly towards the balanced-growth path.

Table 2 offers another result worth noting. The *asc* becomes much less responsive to changes in parameter values when we introduce human capital in the R&D-based growth model. When δ_A increases from 0.01 to 0.1, the *model with H* predicts a small increase in $asc(y^w)$ from 0.0132 to 0.0172, much smaller than the change produced in the *model w/o H*. The same is true if λ goes up from 0.5 to 0.75. Now, the *asc* increases with λ from 0.0132 to 0.0145, that is, hardly a 0.1% change. Finally, a much lower sensitivity of the *asc* is also obtained if a policy action varies the technology-gap parameter ψ from 0.16 to 0.25. In particular, in this last case, $asc(y^w)$ increases from 0.0132 to 0.0168. The reason for the different response of the asymptotic speed in the two models is again the one given above. The presence of the additional sector implies a lower allocation of resources to each of the different activities, thus reducing the impact of external shocks.

This low sensitivity of the convergence speed to changes in the parameter values is consistent

[14]This result is robust to reasonable changes in the parameter values.

Table 4: Output, Capital and Schooling in Japan and S. Korea

Country		1960	1963	1990
Japan	Y per worker (%)**	20.6		60.3
	K per worker (%)**	16.9		104.6
	S (years)	10.2		11.0*
S. Korea	Y per worker (%)**		11.0	42.2
	K per worker (%)**		11.6	50.2
	S (years)		3.2	7.7*

* 1987 figures.

** Levels relative to their U.S. counterparts.

with Barro and Sala-i-Martin's (1995) finding that estimated convergence speeds do not vary much across different countries or regions. However, our result does not necessarily imply that policy actions have a small impact on the transition process, as Barro and Sala-i-Martin's result has been interpreted. Far away from the balanced-growth path, policy may have a larger effect on the speed of convergence over subsequent periods because the model allows the convergence speed to vary across time. It is also important to notice that Barro and Sala-i-Martin's finding is obtained for a fairly homogenous group of wealthy regions – namely, U.S. states, European regions, and Japanese prefectures – which are probably close to their steady states.

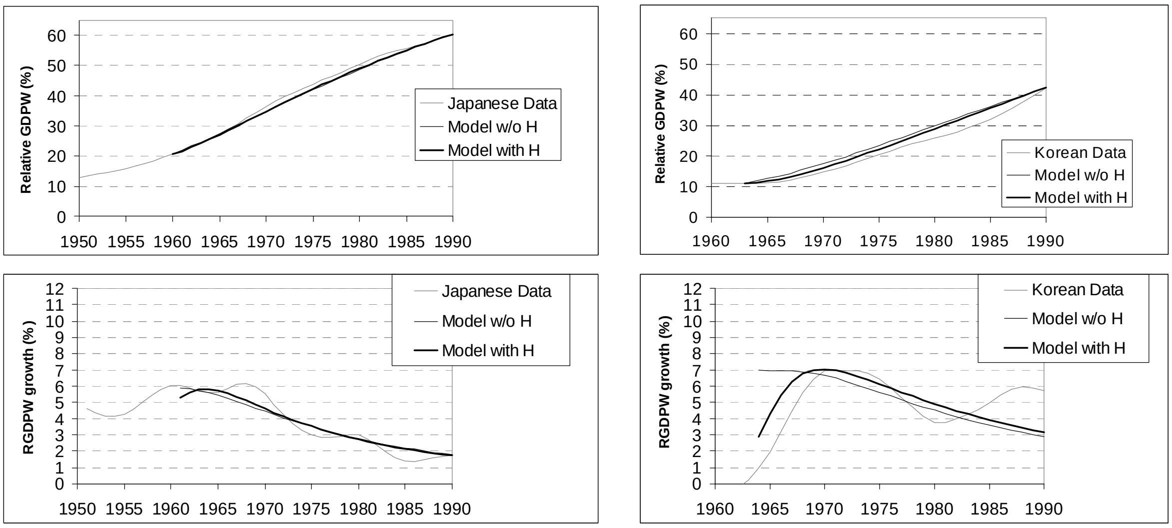
In summary, we find that the non-scale R&D growth model with human capital obtains a convergence speed consistent with the evidence. More importantly, the model implies a convergence speed which is much less responsive to policy actions compared to existing models in the literature.

4.3 Adjustment paths

At least since the seminal work of Lucas (1993), it has been recognized that a desirable property of growth models is to be able to reproduce miraculous experiences. In terms of transitional dynamics analysis, this amounts at least to being able to reproduce the average speed of convergence, and country-specific changes in the output growth trend. The empirical literature has provided estimates of the first one, that is, the average speed, and growth theory has tried to see whether alternative growth models can reproduce these estimates by computing the *asc*.

In the previous section, we have found that the two models that we compare are able to reproduce fairly well the average speed of convergence. In this sense, we could conclude that the above results suggest that the *model with H* does not represent an improvement over the *model w/o H*

Figure 1: Adjustment paths for Japan and S. Korea



to reproduce convergence experiences. Not only that. If we take into account that some estimates of the speed of convergence such as Caselli *et al.* (1996) obtain relatively large convergence-speed estimates, as high as 10%, the lower speed predicted by the *model with H* could be even interpreted as a bad outcome for the model.

Next, we show that the information given by the *asc* is misleading because it does not give any information about the second aspect; namely, the capacity to reproduce country-specific changes in the output growth trend. More specifically, in this section, we compare the capacity of the two models to reproduce country-specific changes in the output growth trend in two important examples: the S. Korean and the Japanese output paths.[15]

Taking the model to the data requires assigning a value to ψ. Here, we follow Parente and Prescott (1994), and assume that countries may differ in their degrees of technology adoption barriers. For simplicity, we suppose that these barriers affect the value of the parameter ψ. To obtain its economy-specific value, we calibrate the parameter ψ to each country's output data. Because we focus on two nations, Japan and South Korea, the value on which the parameter ψ takes will be the one that makes transitional dynamics be able to reproduce the output per worker evolution between 1960 and 1990 in Japan, and between 1963 and 1990 in S. Korea – i.e., their

[15]Once again, we resort to numerical approximation techniques to simulate transitional dynamics. The method used and measures of its accuracy are provided in the Appendix.

average speed of convergence.[16]

The initial values of the stock variables and output data used to calibrate ψ are presented in table 4.[17] The *model with H* requires $\psi = 0.131$ to induce Japan's average speed of convergence, and $\psi = 0.162$ to produce the S. Korean output numbers. The *model w/o H* requires $\psi = 0.10$ for the Japanese development experience, and $\psi = 0.074$ for the S. Korean development experience.

The adjustment paths predicted by both models for the level and growth rates of relative GDP per worker (RGDPW) are depicted in figure 1. The two panels at the top imply that, although the *model with H* does slightly better, both frameworks generate output paths that replicate fairly well the Japanese and the S. Korean data. Something expected knowing that both models generate a similar *asc* for output. However, the message from the bottom charts is different. The *model with H* does a much better job because it predicts that output per worker growth rates do not pick at the beginning of the adjustment path but later on. This is an important feature that characterizes the output-convergence phenomenon as Easterly and Levine (1997), among others, show. Because of this, it can be argued that the *model with H* represents a better theory to explain convergence experiences than the *model w/o H*.

5 Conclusion

In this paper, we have compared transitional dynamics of two alternative non-scale R&D-based models of economic growth. One model incorporates human capital accumulation, whereas the other does not. We have shown that the asymptotic speed of convergence of per-worker output predicted by the model with human capital is consistent with the evidence, although closer to the lower bound suggested by empirical estimates than the one predicted by the model without human capital. This might have led us to believe that the theory in which human capital and technology have an important complementary role does not represent an improvement over the theory that does not emphasize that role.

Interestingly, we have shown that this information given by the asymptotic speed of convergence is misleading. The reason is that transitional dynamics of the model with human capital offers much better predictions regarding the evolution of growth rates for important development experiences. This has led us to conclude that a model that delivers a speed of convergence that complies better

[16] Japan's rapid convergence toward U.S. income levels actually started right after WWII. Unfortunately, the Japanese Education Department does not possess estimates of the average educational attainment before 1960. We are grateful to Tomoya Sakagami who has attempted to obtain this data for us.

[17] All relative measures in the paper are with respect to U.S. levels. Additionally, we follow Parente and Prescott (1994) and smooth all data series using the Hodrick-Prescott filter with the smoothing parameter equal to 25.

with empirical estimates does not necessarily provide a better description of the convergence process. A careful study of the adjustment paths predicted by alternative growth theories starting far away from the balanced growth path is required if we hope to discriminate among them.

The paper has offered another interesting result. We have shown that the introduction of human capital makes the asymptotic speed of convergence much less sensitive to external shocks such as policy actions. This is consistent with Barro and Sala-i-Martin's (1995) result that estimated convergence speeds do not vary much across different region groups that belong to developed nations. Our intuition for this result is that, as we increase the number of state variables, labor must be allocated among more sectors, thus reducing the speed at which they can converge towards the steady-state. But unlike the interpretation that the literature has assigned to Barro and Sala-i-Martin's finding, we can not conclude that policy actions have a small effect on the convergence speed, because non-scale growth frameworks deliver speeds of convergence that can vary over time.

A Data Appendix

The data and programs used in this paper are available by the authors upon request.

- *Income (GDP)* [Source: PWT 5.6]

Cross-country real GDP per worker (chain index, 1985 international prices) is taken from the Penn World Tables, Version 5.6 (PWT 5.6) as described in Summer and Heston (1991). This data set is available on-line at: http://datacentre.chass.utoronto.ca/pwt/index.html.

- *Physical capital stocks* [Source: STARS (World Bank), and PWT 5.6]

Physical capital comes from PWT 5.6. However, this data set reports physical capital starting in 1965. To obtain stocks from 1963 for S. Korea, and from 1960 for Japan, we used the growth rates implied by the STARS physical capital data to deflate the 1965 PWT 5.6 numbers.

- *Education* [Source: STARS (World Bank)]

Annual data on educational attainment are the sum of the average number of years of primary, secondary and tertiary education in labor force. These series were constructed from enrollment data using the perpetual inventory method, and they were adjusted for mortality, drop-out rates and grade repetition. For a detailed discussion on the sources and methodology used to build this data set see Nehru, Swanson, and Dubey (1995).

B Transitional Dynamics Methodology

What follows is a brief explanation of the methodology used in analyzing transitional dynamics. Because there is no analytical solution to our system of Euler and motion equations, we resort to numerical approximation techniques. In our analysis we follow Judd (1992) to solve the dynamic equation system, approximating the policy functions employing high-degree polynomials in the state variables.

In particular, the parameters of the approximated decision rules are chosen to (approximately) satisfy the Euler equations over a number of points in the state space, using a nonlinear equation solver. A Chebyshev polynomial basis is used to construct the policy functions, and the zeros of the basis form the points at which the system is solved; that is, we use the method of orthogonal

Table 5: Accuracy measures in different models

Country	*Model**	ψ	Average Error (%) C	u_H	u_A	Max. Error (%) C	u_H	u_A
Japan	*model with H*	0.131	0.01	0.02	0.01	0.04	0.07	0.04
Japan	*model w/o H*	0.10	0.00	−.−	0.00	0.01	−.−	0.02
S. Korea	*model with H*	0.162	0.06	0.17	0.06	0.27	0.78	0.24
S. Korea	*model w/o H*	0.074	0.01	−.−	0.01	0.02	−.−	0.05

**model with H* refers to the per worker three-sector non-scale growth model with schooling sector. *model w/o H* refers to the two-sector non-scale growth model without schooling sector.

collocation to choose these points. Finally, tensor products of the state variables are employed in the polynomial representations.

This method has proven to be highly efficient in similar contexts. For example, in the one-sector growth model, Judd (1992) finds that the approximated values of the control variables disagree with the values delivered by the true policy functions by no more than one part in 10,000. All programs were written in GAUSS and are available by the authors upon request.

For the cases considered in this paper, Table 5 gives accuracy measures. In particular, we assess the Euler equation residuals over 10,000 state-space points using the approximated rules. For each variable, the measures give the average and maximum current-value decision error that agents using the approximated rules make, assuming that the (true) optimal decisions were made in the previous period. Santos (2000) shows that the residuals are of the same order of magnitude as the policy function approximation error.

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