A discusión

PEER GROUP EFFECTS AND OPTIMAL EDUCATION SYSTEM*

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PEER GROUP EFFECTS AND OPTIMAL EDUCATION SYSTEM

Marisa Hidalgo

ABSTRACT

The belief that peers' characteristics influence the behavior and outcomes of students in school has been important in shaping public policy. How peers affect individuals depends on the educational system prevailing. I analyze two different systems: tracking and mixing, and I propose several criteria to compare them. I find that at compulsory level, average human capital across the population is maximized under tracking, although tracking does not dominates mixing according to first order stochastic dominance. The education system that maximizes college attendance depends on the income level in the population and on the opportunity cost of college attendance.

Keywords: Peer Effects, Tracking, Mixing, Income Premium

JEL Classification: D63, I28, J24.

1 Introduction

The belief that peers' characteristics in school influence the behavior and outcomes of students has been important in shaping public policy. But, what are we exactly referring to when talking about "peer effects"? Arnott and Rowse (1987) used this term to refer to the effect on an individual's academic performance of the ability distribution of their peers. In general there has been limited attention given to the mechanisms through which peers effects work. The most common perspective is that peers, like families, are sources of motivation, aspirations, and direct interactions in learning. Moreover, peers may affect the classroom process (aiding learning through questions and answers, contributing to the pace of instruction, or hindering learning through disruptive behavior à la Lazear (2001)).

Interest in social interactions, neighborhood effects, and social dynamics has seen recently a revival. A small literature has emerged that studies the generation of persistent inequality among a population due to neighborhood effects of various kinds. All these effects have the consequence of inducing sub-optimal levels of education for some groups of the population. These neighborhood effects consist mainly of three types: investment, role-model and peer-group influences. Investment refers to local public good provision. It occurs when the poor are segregated in a community: due to the low tax base, funding of local education is low, and hence children receive less education than in richer communities. Under the role-model effect, the behavior of one individual in a group is influenced by the characteristics of and earlier behavior of older members of the group. Peer group influences refer to contemporaneous influences and so may be reciprocal.[1]

Peer group effects have played an important role in a number of policy debates including: ability tracking, anti-poverty programs in both rural areas and urban ghettos, and school desegregation. The peer group composition of schools is, therefore, undeniably important in the minds of parents as well as policy makers at the local and state level. If peer effects exist, the government should, therefore, be able to take them into account in order to better achieve policy objectives. An example of this is the choice between streaming (or tracking) and mixing students of different abilities

[1] See Roemer and Wets (1994), Maski (1993) and Durlauf (2002). Roemer and Wets (1994) and Streufert (2000) show how economic segregation can lead to inaccurate assessments of the economic payoff to education. The basic idea in this type of analysis is that by depriving children in poor neighborhoods of successful role models (which is a necessary consequence of economic segregation), they make inferences on the benefits to education that are biased downward.

in public schools.

There is a great deal of controversy regarding the practice of ability grouping or tracking (to group students in classrooms according to their ability level). The main argument is that by narrowing the range of students' abilities within the classroom, teachers can target instruction to a level more closely aligned with students' needs than in more heterogeneous environments. The critics of ability grouping argue that when students are segregated, disadvantaged students lose any positive peer effects that might be gained from being with more able students. In keeping with this view, there has been considerable movement in the US towards eliminating the practice of grouping students according to ability.[2] In Europe there is currently an intense debate in response to the publication of the PISA 2000 and 2003 Reports and the quite different results achieved by countries with different education systems. In fact, simple cross-country comparisons show that there is no statistically significant correlation between the level of stratification in the education system and country mean performance. However, the more differentiated and selective education systems tend to show larger performance differences between students from more and less advantaged family backgrounds.[3]

The influence of peers ability on own educational achievement is well documented but still controversial. Most of works focus on the average innate ability within the class as the main characteristic of the student's classmates which can affect achievement. On the one hand, for example, Evans, Oates and Schwab (1992), and more recently Arcidiciano and Nicholson (2002) find a significant peer group effect that vanishes when they control for endogeneity. On the other hand, Henderson, Mieszkowski, and Sauvageau (1978), Summers and Wolfe (1977) and more recently Zimmer and Toma (2000) report significant positive influences of higher achieving peers on achievement.

The existence of peer effects and its relation with different policies of grouping students have been studied theoretically as well as empirically. The very first empir-

[2]For example, data from the Schools and Staffing Survey suggest than 20% of school with programs for gifted children in 1990 had eliminated the programs by 1993 (Figlio and Page (2000)).

[3]The Programme for International Student Assessment (PISA) tested 15 year-old students in the subjects mathematics, science and reading proficiency in the first half of 2000 and 2003, in all OECD countries. For example, while Finland and The Netherlands achieved the top ranks, Germany and Spain were placed below or just above the OECD average in both reports. These countries are suited for a comparison of a streamed or tracked system (Germany and The Netherlands) to a single type schooling system (Finland and Spain). See PISA 2003 Report.

ical works on peer effects focused on ethnic and racial groups. Following this line, Schofield (1995) made a review on the impact of desegregation (or detracking) on students academic achievement. More recently, and focusing on the effects of grouping students by ability the majority of works conclude that relative to outcomes in mixed groups, students placed in the low track are hurt while those allocated in the high track gain. This result is consistent with our model. Therefore, the remaining question is whether the losses of the former compensate or not the gains of the latter. Argys et al.(1996) conclude by saying that on net terms, if all students in their sample were placed in heterogeneous classes (mixing), average test scores could be expected to decline. However, Betts and Shkolnik (2000) find little or no differential effects of grouping students. Finally, Figlio and Page (2000) find no evidence that tracking hurts low ability children. Theoretical contributions are more scarce. Among others we find the works by de Bartolome (1990) and Epple, Newlon and Romano (2002). Arnott and Rowse (1987) studied the optimal allocation of students and educational expenditures over classrooms when peer group effects are present. They concluded that the optimal allocation, when the objective is to maximize the sum of students results, depends on the properties of the educational production function.

The aim of this paper is to study public intervention in education when the government, taking into account the process of human capital accumulation and, in particular, the peer effects on students' achievement, has to decide the optimal education system. I analyze two different education systems. The first one, tracking, consists on grouping students based on innate ability. The second one, mixing, implies that the ability distribution is the same in all classrooms. Both education systems must be understood as polar cases.

Our model is an economy in which individuals live for two periods. Individuals differ in two aspects: innate ability, and family background. In the first period individuals attend compulsory education where they accumulate human capital. The acquisition of human capital reflects the influence of family and peers factors. As I said above, I consider two different educational systems at compulsory level: mixing and tracking. At some point of the first period, students must also decide whether to attend college or not. If they do, they spend the second part of this first period at college. If they do not, they enter immediately in the labour market working as unskilled workers. By attending college they become skilled workers. During the second period all individuals work. Those who went to college as skilled workers and those who did not go as unskilled.

My goal is to evaluate these two systems, using several criteria that have not been used previously in this literature. Note that I propose a normative theory, so I need to define the objectives of the government. Although there are several views about what these objectives should be, till now most of the debates focus on which system leads to higher achievement. I will consider this objective but also some others, that can be classified according to the education level analyzed, compulsory or post-compulsory (college).

This paper contributes to the literature in two directions. First, at compulsory level the paper advances the existing literature on evaluating the consequences on the distribution of human capital under both educational systems, by using the "Veil of Ignorance" assumption, widely used in modern Welfare Economics. Under reasonable assumptions about the human capital production function, I find that average human capital at compulsory level is always maximized under tracking. However, tracking does not dominate mixing according to the criteria of first and second order stochastic dominance. This is equivalent to say that, given the choice between both educational systems to any individual in the population, in complete ignorance of what his/her relative position would be within each system, there would be no unanimously preferred system.

The second contribution of my paper lies on highlighting the importance of analyzing the possible effects of the education system at compulsory level on individuals' outcomes later in life, such as college choice and occupational attainment. Yet, schooling decisions may depend in important ways on the amount of schooling acquired by other individuals. The idea is that an individual's self image may be enhanced when his or her actions are in line with the behavior in the peer group.[4]

The first aim when analyzing college level is to maximize college attendance. It is found that the system that provides the maximum number of college students depends on the opportunity cost of college attendance and on how wealthy society is. In particular if the opportunity cost is low enough, mixing always maximizes college attendance, and the reverse occurs when the opportunity cost is sufficiently high, for any wealth level in the population. For intermediate values of the opportunity cost I find that the education system that maximizes college attendance depends on how rich society is. In particular, for poor societies the optimal education system is

[4]Other reasons for social interaction in schooling (although they are not the focus of this paper) include social learning (Manski (2004)), and strategic consideration, i.e. that it pays off to acquire more schooling if other students acquire more schooling due to labor market competition.

tracking whereas for richer societies the optimal system is mixing.

The second criterion is equality of opportunities which in my model means to guarantee that the individual decision of whether to attend college or not is taken independently of parents' income. I obtain that tracking is the most equitable system in most of the cases except in situations where there is a high level of inequality in the population, and when the minimum level of human capital required to attend college is sufficiently low.

Finally the last criterion consists on maximizing the utility of the worst-off individuals in the society. I found that tracking (mixing) maximizes the probability of college attendance of the worst-off individuals in the economy when the opportunity cost of college attendance is high (low) enough.

The paper is organized as follows. In Section 2 I describe the model and the main features of the human capital distribution under both education systems. In Section 3 I compare the performance of tracking and mixing system at compulsory level, by analyzing the average human capital attained under each system and its effects on the distribution of human capital. Section 4 analyzes the individuals' decision on college attendance. Two criteria are proposed to compare both education systems at this education level: maximum college attendance and equality of opportunities. Finally, Section 5 concludes.

2 Model

2.1 Individuals

I consider an economy in which individuals live for two periods. Population has constant size equal to 1. Individuals in each generation differ in two aspects: their innate ability, θ_0, and their family background denoted by z.[5] To make the model tractable I will assume that θ_0 is uniformly distributed on the interval $[0, 1]$ and that family background z takes only two values, 1 and $x > 1$ with probabilities $1 - \lambda$ and λ, respectively. I assume that both characteristics are independently distributed.

In the first period of their lives, individuals accumulate human capital. At the beginning of this period they attend compulsory education, which is free of charge, and they are not allowed to work. At some point of this first period, they also have to decide whether to attend college or not. I call γ, where $\gamma \in [0, 1]$, the fraction of the

[5] Therefore z could be either the parents' level of income or the parents' human capital.

first period that it is left after attending compulsory education. Those who attend college spend the fraction γ of the first period at college, while the rest of individuals work that fraction γ as unskilled workers. By attending college they become skilled workers.[6]

During the second period of their lives all individuals have one unit of time and work. Those who went to college as skilled workers and those who did not go as unskilled. The wage they receive is proportional to their own level of human capital.

In this model mean income is $\lambda x + (1 - \lambda)$ and income inequality, measured by the variance of income in the population, is $(x - 1)^2\lambda(1 - \lambda)$. Both are increasing with x. Below we analyze the effect of mean income on the distribution of human capital under both education systems.

2.2 Production of Human Capital

At this educational level, individuals are separated into different groups. To simplify matters, I will assume that there are only two groups or classrooms. The production of human capital at compulsory level depends on three factors. The first one is the individual innate ability, θ_0. The second one is the "formal schooling" or "peer group" effect. It will depend on the characteristics of the group in which the individual is placed. These characteristics can be summed up by the mean ability of the group j or "peer" effect, denoted by $\overline{\theta_0}^j$. The third one is "informal schooling"and refers to family background effects, that are captured by z.[7] After attending compulsory education an individual with innate ability θ_0 ends up with a level of human capital θ_1:

$$\theta_1(\theta_0, \overline{\theta_0}^j, z) = \theta_0(1 + r(\overline{\theta_0}^j, z)), \tag{1}$$

where r is the individual rate of return (see below). I propose:

$$r(\overline{\theta_0}^j, z) = (\overline{\theta_0}^j)^\alpha (z)^{1-\alpha}. \tag{2}$$

[6]Note that the parameter γ can be interpreted as the cost of investment in human capital, or the fraction of earnings that would have been received in the absence of the investment.

[7]Galor and Tsiddon (1997) call this factor *home environment externality* and distinguish it from *global technological externality*, by which the aggregate level of human capital of the parents' generation is transferred to the children. The last term has been used by several other studies among others see Benabou (1996).

The acquisition of human capital reflects the influence of family and peers factors, with respective weights $1-\alpha$ and α, where $\alpha \in [0,1]$.[8]

The main properties of r are as follows. First, regarding family background, the individuals' level of human capital is an increasing function of the parental level of human capital but at a decreasing rate, $r_2 > 0$ and $r_{22} < 0$. In addition, note that regarding the peer group effect we have $r_1 > 0$, $r_{11} < 0$ and $r_{12} > 0$.[9]

The empirical evidence establish that the peer group effect is non-linear: the achievement of individual students rises with an improvement in the average quality of their classroom, but this positive effect has decreasing returns.[10]

From Equations (1) and (2) we can observe that the peer effect becomes more effective in the production of human capital as the level of innate ability or parents' income increases, that is $\frac{\partial^2 \theta_1}{\partial \overline{\theta_0}^j \partial \theta_0} > 0$ and $\frac{\partial^2 \theta_1}{\partial \overline{\theta_0}^j \partial z} > 0$.[11]

In the second part of the first period, every individual decides whether to attend college or not. After attending college they will enjoy a further increase in their level of human capital acquired during compulsory level. I denote such an increment by δ and, thus, those individuals who decide to attend college will end up with the following level of human capital:

$$\theta_2 = \theta_1(1+\delta). \tag{3}$$

The findings of the recent empirical literature show that factors that take place in early stages of life are crucial determinants of children's later success.[12] Therefore we

[8] This technology of production of human capital is commonly used in this literature. See for example Benabou (1996) or Epple and Romano (1998 and 2002).

[9] The importance of the parental education in the acquisition of human capital of the individual has been explored theoretically as well as empirically. Among other see Coleman et al. (1966), Becker and Tomes (1986). More recently, among others, Feinstein and Symons (1999) found that parent interest is the principal via by which the attainments of each generation are passed to the next. They also suggested the complementarity between parental interest and peer effect.

[10] See Summers and Wolfe (1977), Henderson, Mieszkowski, and Sauvageau (1978), de Bartolome (1990), and more recently Zimmer and Toma (1997).

[11] The empirical evidence regarding these properties is still mixed. Henderson et al. (1978) find no interaction between own ability and the benefits of an improved peer group, i.e. $\frac{\partial^2 \theta_1}{\partial \overline{\theta_0}^j \partial \theta_0} = 0$. Argys et al. (1996) suggest $\frac{\partial^2 \theta_1}{\partial \overline{\theta_0}^j \partial \theta_0} > 0$. Summers and Wolfe (1977) find some support for higher peer group benefits to lower ability students, that is, $\frac{\partial^2 \theta_1}{\partial \overline{\theta_0}^j \partial \theta_0} < 0$.

[12] In particular, Neal and Johnson (1996) find that differences in educational achievements by the time of high-school completion account for almost all the observed black-white wage gap. See also Keane and Wolpin (1997).

assume that the acquisition of human capital at college is not directly affected by the family nor peers factors, although it is indirectly affected by them since it depends on the level of human capital previously acquired, $\delta = \delta(\theta_1)$. We assume that this increase, that reflects the efficacy of higher education, is an increasing function of the human capital acquired at compulsory level, but at a decreasing rate ($\delta_1 > 0, \delta_{11} < 0$).

It is important to note that the characteristics of the group in which the individual is placed affect her final level of human capital θ_2 through two different channels. First, there is a direct effect since peers affect the human capital acquired at compulsory education. Second, there is also an indirect effect since this level of human capital determines the efficacy of higher education and, thus, as we will see below, the decision of the individual of whether to undertake college education or not.

It is important, therefore, to analyze the different composition of the groups at school, which are determined by the educational system prevailing. This composition is going to be crucial in determining the distribution of human capital across the population and, as we will see below, in the individuals' decision of whether to attend college or not. In the next section I will study the two different educational systems.

2.3 Educational Systems at Compulsory Level

As I said in the Introduction, grouping students based on ability measures (tracking) is very common in the USA and in Europe.[13] In this paper I will consider two different educational systems at compulsory level: mixing and tracking. In this section I will describe them and I will also analyze the distribution of human capital at the end of compulsory school under both systems.

2.3.1 Mixing

Under mixing the ability distribution is the same in both classrooms. We denote the average ability in each classroom by $\overline{\theta_0}^m$. It is the same in both classrooms and coincides with the average ability in the population, m. That is, $\overline{\theta_0}^m = 1/2$.

However, as individuals differ in their parents' level of human capital, there will be two income groups within each classroom: the rich and the poor. I will study now the distribution of θ_1, the human capital at the end of compulsory education. Note

[13] See Oakes et al. (1992) and PISA 2003 Report for US and Europe respectively. For the US case, public schools teachers reported that only 14.4% and 10.8% of tenth-grade students were in heterogeneous (untracked) math classes in 1988 and 1990 respectively, see Rees et al. (1996).

that, with probability equal to λ, θ_1 follows an uniform distribution on $(0, b')$, and with probability $(1-\lambda)$, θ_1 follows a uniform distribution on $(0, a')$, where b' and a' denote the human capital θ_1 acquired by the "best" individual (more skilled) in the rich and the poor income group, respectively:

$$a' = 1 + (1/2)^{\alpha} \tag{4}$$

$$b' = 1 + (1/2)^{\alpha} x^{1-\alpha}. \tag{5}$$

Under mixing, therefore, the C.D.F. (cumulative distribution function) of human capital at the end of compulsory education, denoted by $F_M(\theta_1)$, is:

$$F_M(\theta_1) = \begin{cases} 0 & if \quad \theta_1 \leq 0 \\ \left(\frac{\lambda}{b'} + \frac{(1-\lambda)}{a'}\right)\theta_1 & if \quad 0 \leq \theta_1 \leq a' \\ (1-\lambda) + \frac{\lambda}{b'}\theta_1 & if \quad a' \leq \theta_1 \leq b' \\ 1 & if \quad \theta_1 > b'. \end{cases} \tag{6}$$

It can be easily checked that after any increase in mean income level, due either to an increase in λ or x, the whole function $F_M(\theta_1)$ shifts down, which means that after that change the resulting function dominates the initial one in the sense of first order stochastic dominance.[14]

I denote by $E_M(\theta_1)$ the expected value of θ_1 under mixing, where:

$$E_M(\theta_1) = (1-\lambda)\frac{a'}{2} + \lambda\frac{b'}{2} = \frac{1}{2}\left(a' + \lambda(b'-a')\right),$$

or, using Equations (4) and (5):

$$E_M(\theta_1) = \frac{1}{2}\left[(1 + (1/2)^{\alpha}) + \lambda(1/2)^{\alpha}(x^{1-\alpha} - 1)\right]. \tag{7}$$

Thus $E_M(\theta_1)$ is an average of the mean values of θ_1 in the two income groups, with respective weights $(1-\lambda)$ and λ. From the previous equation we observe that $E_M(\theta_1)$ is an increasing function of both x and λ: in a richer economy in which there are either more individuals with high income level, or the same proportion of individuals but with a higher level of income, there will be a higher aggregate level of human capital than in a poor economy.[15]

[14] Recall that, given two distribution functions $F(\cdot)$ and $G(\cdot)$, $F(\cdot)$ first order stochastically dominates $G(\cdot)$, $F(\cdot) \succeq_{FOSD} G(\cdot)$, if (i)$F(z) \leq G(z)$ for all $z \in R$ and (ii) there exists $\widetilde{z}$ such that $F(\widetilde{z}) < G(\widetilde{z})$.

[15] Note that it can also be easily concluded from Equation (6) and the definition of expected value of a random variable.

2.3.2 Tracking

Under this system students are grouped based on innate ability. For simplicity, I permit at most two tracks. Thus, the median level of innate ability m, is used as a threshold ability to group students. Students are assigned to the high (low) track as long as their ability θ_0 is above (below) the median.

The distribution of human capital within each track is uniform but with different parameters. I denote by $\overline{\theta_0}^h$ and $\overline{\theta_0}^l$ the average ability in the high and low track respectively. Thus, given the distributional assumption on θ_0, I have that $\overline{\theta_0}^h = 3/4$, whereas $\overline{\theta_0}^l = 1/4$. It is clear that for any distribution function of θ_0 the following condition is satisfied:

$$\overline{\theta_0}^l < \overline{\theta_0}^m < \overline{\theta_0}^h. \tag{8}$$

Again, there will be two income groups within each track. In the low track θ_1 follows a uniform distribution on $(0, c)$ with probability λ, and it follows a uniform distribution on $(0, a)$ with probability $(1 - \lambda)$, where a and c denote the human capital acquired by the "best" individual (more skilled) within the poor and the rich, respectively, that is:

$$a = \frac{1}{2}\left(1 + (\frac{1}{4})^{\alpha}\right) \tag{9}$$

$$c = \frac{1}{2}\left(1 + (\frac{1}{4})^{\alpha} x^{1-\alpha}\right). \tag{10}$$

In the same way, in the high track θ_1 follows a uniform distribution on (b, e) with probability λ, and it follows a uniform distribution on (d, f) with probability $(1-\lambda)$. We denote by b and d the human capital θ_1 acquired by the "worst" individual (less skilled) within the poor and the rich, respectively. We denote by e and f the human capital θ_1 acquired by the "best" individual (more skilled) within the poor and the rich, respectively, i.e.:

$$b = \frac{1}{2}\left(1 + (\frac{3}{4})^{\alpha}\right) \tag{11}$$

$$e = \left(1 + (\frac{3}{4})^{\alpha}\right) \tag{12}$$

$$d = \frac{1}{2}\left(1 + (\frac{3}{4})^{\alpha} x^{1-\alpha}\right) \tag{13}$$

$$f = \left(1 + (\frac{3}{4})^{\alpha} x^{1-\alpha}\right). \tag{14}$$

From previous Equations (9) to (14) we have that the two following conditions apply. First we have that $a < c, b < d$ and $e < f$. That is, independently of the ability

group in which the individual is placed, given two individuals with the same level of innate ability, the one whose parents have higher income level will always attain a higher level of human capital. Second, we have that $a > 0, c > 0, e > b$ and $f > d$. This means that, independently of the ability group in which the individual is placed, given two individuals whose parents have the same income level, the one with a higher level of innate ability will always attain a higher level of human capital.

Now I need to introduce some assumptions to ensure that the support of θ_1 is a connected set under tracking, i.e. the density function under tracking denoted by $f_T(\theta_1)$ is strictly positive for all θ_1 in the interval $[0, f]$.

Assumption 1 (A.1): $c > b$.

This assumption ensures that the support of θ_1 in the low track overlaps the support of θ_1 in the high track. In other words, the "best" (the richest and most skilled) individual in the low track obtains more human capital than the "worst" individual in the high track (the poorest and least skilled). This assumption implies a restriction on both x and α. For a fixed α this implies that x has to be above a threshold level : $x > \underline{x}(\alpha) = 3^{\frac{\alpha}{1-\alpha}}$. That is, x must be high enough to compensate the disadvantage of being in the low track.

Assumption 2 (A.2): $a' > d$.

This assumption implies that the "best" individual among the poor obtains under mixing a higher level of human capital than the "worst" individual among the rich in the high track. As in previous Assumption 1, it implies a restriction on both x and α. Fora fixed α it requires that x must be below a threshold level: $x < \overline{x}(\alpha) = ((4/3)^{\alpha}(1 + 2^{1-\alpha}))^{\frac{1}{1-\alpha}}$.[16]

In addition Assumption 2 implies that the two intervals within the high track overlap, as in the low track case. That is, the "best" individual in the low income group has more human capital than the "worst" individual in the high income group.

From Assumptions 1, 2 and Equations (4), (5), (12) and (14), I have that the

[16] One could think that the mixing system represents the public education system whereas tracking represents a private system where only individuals with high levels of innate ability (and rich) are accepted. Thus, A.2 implies that the best student in the public school can achieve a higher level of human capital than the worst student in the private school.

following two conditions hold:

$$e < b' < f, \tag{15}$$

$$d < a' < e. \tag{16}$$

These conditions refer to the relationship between the intervals of θ_1 under tracking and under mixing. In particular we have that, independently of the income group where the individual is placed, an individual with the highest level of innate ability will always achieve a higher level of human capital under tracking than under mixing, that is, $a' < e$ and $b' < f$. Also, for a fixed level of ability, a rich individual under mixing will get more human capital than a poor individual under tracking, i.e., $e < f$.

From the previous two assumptions I have that for any $\alpha \in (0,1)$ the income level of the rich must belong to the following interval :

$$\underline{x}(\alpha) < x < \overline{x}(\alpha). \tag{17}$$

Inversely, one could think of a restriction on α for any x. In Figure 1 I illustrate the different intervals for θ_1 and the relation among them, for both educational systems. Now, under Assumptions A.1 and A.2, the C.D.F. of θ_1 under tracking, denoted by $F_T(\theta_1)$ is as follows:

$$F_T(\theta_1) = \begin{cases} 0 & if \quad \theta_1 \leq 0 \\ \left(\frac{\lambda}{2c} + \frac{(1-\lambda)}{2a}\right)\theta_1 & if \quad 0 \leq \theta_1 \leq a \\ \frac{(1-\lambda)}{2} + \frac{\lambda}{2c}\theta_1 & if \quad a \leq \theta_1 \leq b \\ \left(\frac{\lambda}{2c} + \frac{(1-\lambda)}{e}\right)\theta_1 & if \quad b \leq \theta_1 \leq c \\ \frac{\lambda}{2} + \frac{(1-\lambda)}{e}\theta_1 & if \quad c \leq \theta_1 \leq d \\ \left(\frac{\lambda}{f} + \frac{(1-\lambda)}{e}\right)\theta_1 & if \quad d \leq \theta_1 \leq e \\ (1-\lambda) + \frac{\lambda}{f}\theta_1 & if \quad e \leq \theta_1 \leq f \\ 1 & if \quad \theta_1 > f. \end{cases} \tag{18}$$

As in the case of mixing, an improvement in the mean income level implies that the resulting $F_T(\theta_1)$ will improve in the sense of first order stochastic dominance.

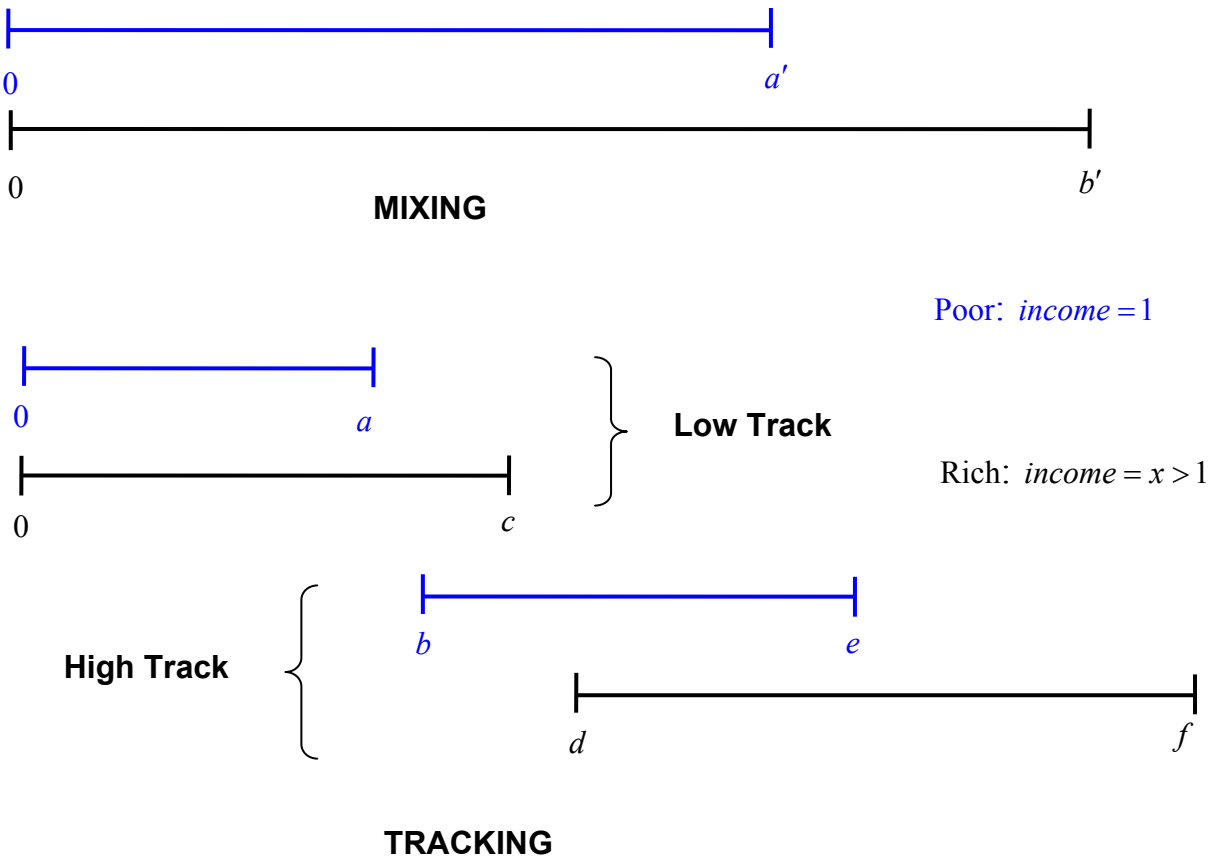

Figure 1. EDUCATIONAL SYSTEMS

When $\alpha = 0$ we have that $F_T(\theta_1) = F_M(\theta_1)$, since the peer effect plays no role on human capital accumulation.

The expected value of θ_1 under tracking is:

$$E_T(\theta_1) = (1-\lambda)\frac{a}{4} + (1-\lambda)\frac{3b}{4} + \lambda\frac{c}{4} + \lambda\frac{3d}{4}, \tag{19}$$

and using Equations (11) to (14):

$$E_T(\theta_1) = \frac{1}{8}\left[(4 + (1/4)^{\alpha}\left(1 + 3^{\alpha+1}\right)) + \lambda(1/4)^{\alpha}\left(1 + 3^{\alpha+1}\right)(x^{1-\alpha} - 1)\right]. \tag{20}$$

As in the case of mixing, the expected value of θ_1 is a weighted average of the mean value of θ_1 in the four income groups previously analyzed. It is increasing both in x and λ (see Footnote 15).

3 Comparing Educational Systems at the Compulsory Level

Most works dealing with the effects of tracking focus just on comparing average test scores in tracked groups relative to mixed groups. Here I adopt the "Veil of Ignorance" approach widely used in modern Welfare Economics (see for example the seminal works of Harsanyi (1953 and 1955) and Rawls (1971)). Under this approach, to evaluate alternative systems individuals must put themselves behind a hypothetical "veil of ignorance", such they ignore their own characteristics. In our case this requires to ignore the value of both θ_0 and z. At this point it is crucial the assumptions made regarding the information available to the individual when taking the decision. I will apply different concepts depending on that information.[17]

The task of individuals behind the "veil of ignorance" is to compare tracking and mixing, assuming that individuals ignore θ_0 and z. To start with, assume that they also know that all of them vlaue positively θ_1, that is, all of them would like to have as much of θ_1 as possible.

This amounts to say that a particular system will be better if it induces an aggregate distribution that dominates the other in the sense of first order stochastic

[17] I will not discuss which are the most appropriate assumptions regarding what individuals should know behind the veil of ignorance, since it is not the focus of the paper. See for example Roemer (1996) for a detailed discussion on that matter.

dominance. However, in my setting no system dominates the other. To see this, note that from $F_T(\theta_1)$ in Equation (18) and $F_M(\theta_1)$ in Equation (6) we have that for any $\theta_1 \in (0, a]$, $(F_T(\theta_1) - F_M(\theta_1)) > 0$ for every λ, α and x. One can also check, using Equations (18) and (6) that for any $\theta_1 \in [d, f]$, $(F_T(\theta_1) - F_M(\theta_1)) < 0$.[18]

Next I assume that individuals are risk neutral. Thus, they will choose that system that maximizes average human capital. [19]

In the next proposition I show that average human capital is always maximized under tracking. While the result is an almost immediate consequence of the model, it is worth stating formally since it facilitates considerably the analysis of the rest of the paper.

Proposition 1 *Let $\alpha > 0$. Then, $E_T(\theta_1) - E_M(\theta_1) > 0$ for all x and λ.*

Proof. From Equations (7) and (20), and rearranging terms we have that the difference between the average human capital under tracking and mixing, $E_T(\theta_1) - E_M(\theta_1)$ is positive if and only if the following condition holds:

$$\frac{1}{8}\left(\lambda x^{1-\alpha} + (1-\lambda)\right)\left(\left(\frac{1}{4}\right)^{\alpha} - \left(\frac{1}{2}\right)^{\alpha}\right) + \frac{3}{8}\left(\lambda x^{1-\alpha} + (1-\lambda)\right)\left(\left(\frac{3}{4}\right)^{\alpha} - \left(\frac{1}{2}\right)^{\alpha}\right) > 0.$$

This expression represents the change in the mean ability when we move from mixing to tracking. The first term represents the losses for those individuals that go to the low track, and the second term represents the gains of those individuals who join the high track. This is equivalent to the following expression:

$$\frac{1}{8}\left(\frac{1}{2}\right)^{\alpha}\left(\lambda x^{1-\alpha} + (1-\lambda)\right)\left(\left(\frac{1}{2}\right)^{\alpha} + 3\left(\frac{3}{2}\right)^{\alpha} - 4\right) > 0.$$

This expression is positive if and only if $\left(\left(\frac{1}{2}\right)^{\alpha} + 3\left(\frac{3}{2}\right)^{\alpha} - 4\right) > 0$ for all α. But this expression is positive and strictly increasing when $\alpha > 0$ and is equal to zero when $\alpha = 0$. This proves the claim. ■

If the goal of government is to maximize average human capital across the population at the end of the compulsory level, it should choose the educational system that groups students according to ability.

[18] Clearly this implies that very risk-averse individuals will always prefer mixing while very risk-loving individuals always prefer tracking.

[19] Since it is implicitly assumed that individuals derive utility from lifetime income, which is supposed to be a linear function of the level of human capital (see Section 4 below), the maximization of the average human capital can be interpreted also as an utilitarian criteria.

An implication of Proposition 1 is that $F_M(\theta_1)$ cuts $F_T(\theta_1)$ from below for all λ, α and x.This means that individuals with low ability will prefer mixing whereas individuals with high ability will prefer tracking. We illustrate this point for some values of the parameters in Figure 2.

We conclude that although average human capital is higher under tracking, it is not true that the distribution of human capital under tracking dominates the distribution under mixing in the sense of first order stochastic dominance. In other words, given the choice between both educational systems, some individuals will prefer tracking and some others will prefer mixing, meaning that when going from one system to another there will always be winners and losers.[20]

Finally assume that all individuals behind the "veil of ignorance" are risk averse. Therefore, individuals will prefer the less risky distribution of human capital. This criteria leads to the concept of second order stochastic dominance.[21]

In the next Proposition I show that there is no system preferred to the other under this criteria.

Proposition 2 *$F_r(\theta_1) \not\succeq_{SOSD} F_s(\theta_1)$ for $r, s = M, T$ and $r \neq s$.*

Proof. See Appendix. ■

We can conclude given the choice between tracking and mixing under the "veil of ignorance", i.e. before they know which will be their own innate ability level, there will be no unanimity in the society on the choice of the educational system.

It is important to stress the fact that the previous result applies to the compulsory level of education. When analyzing average human capital at college level it has to be

[20] Brunello and Giannini (2001) also conclude that neither a mixed nor a tracked system unambiguously dominates the other in terms of efficiency. However, the concept of both mixed and tracked groups differs from ours, since they do not consider the existence of peer group effects. In their model, tracking implies that the individuals are allocated to vocational or academic schools based on measures of their academic talent. Under mixing the students are not streamed and receive both technical and general education.

[21] Recall that, given two distribution functions $F(\cdot)$ and $G(\cdot)$, $F(\cdot)$ second order stochastically dominates $G(\cdot)$, $F(\cdot) \succeq_{SOSD} G(\cdot)$, if (i) $\int_{-\infty}^{y} F(z)dz \leq \int_{-\infty}^{y} G(z)dz$ for all $y \in R$ and (ii) there exists $\widetilde{y}$ such that $\int_{-\infty}^{\widetilde{y}} F(z)dz < \int_{-\infty}^{\widetilde{y}} G(z)dz$.

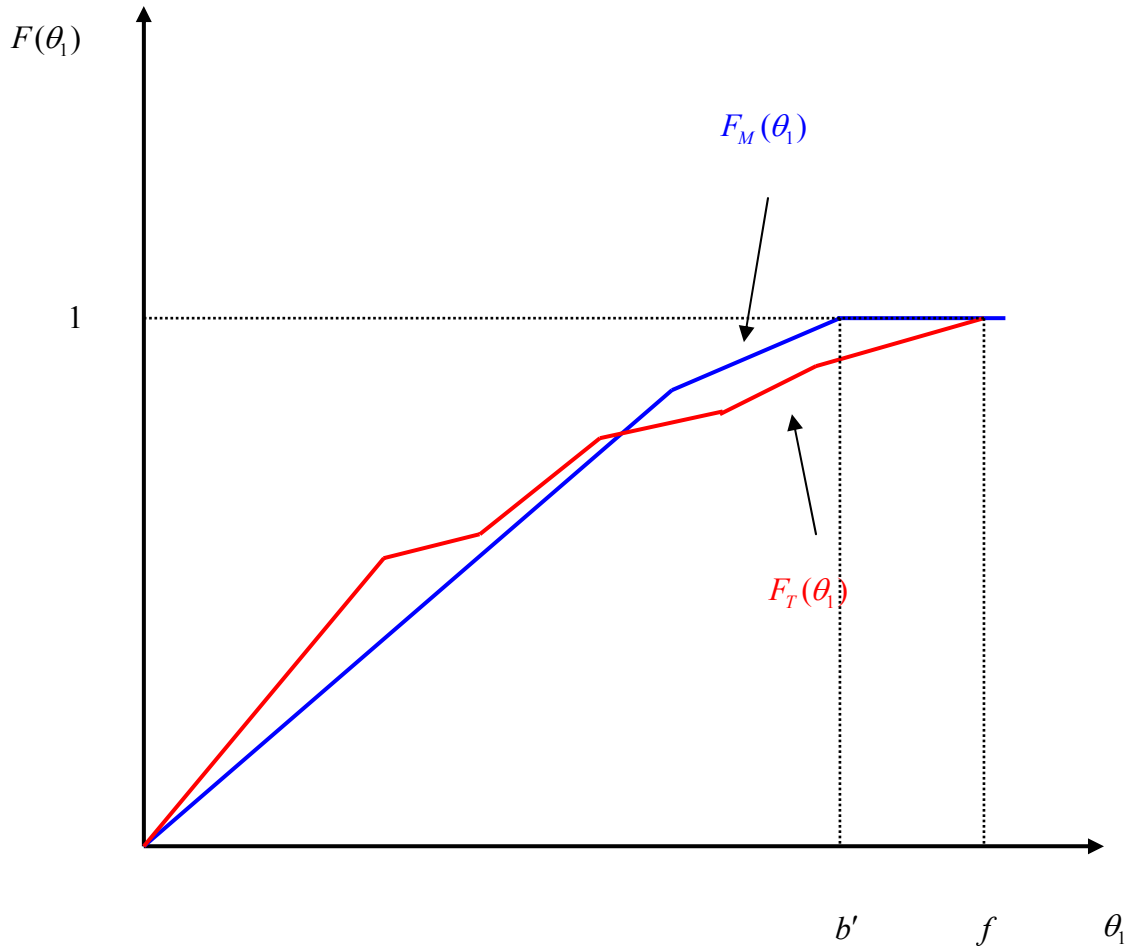

Figure 2. HUMAN CAPITAL DISTRIBUTION

taken into account that some individuals have dropped off the educational system after compulsory education and thus, average human capital is not that straightforward to calculate.

4 Comparing Educational Systems at the College Attendance Level

Now I turn to study how individuals decide whether to attend college or not. I am interested in how the system chosen at compulsory level, tracking or mixing, could affect this decision. I assume that individuals want to maximize their consumption that is equal to their lifetime income, which is supposed to be a linear function of the level of human capital. First, if an individual does not go to college, she will work as an unskilled worker a fraction γ of the first period and the whole second period. Her lifetime income will be $\theta_1(1+\gamma)$.

Second, the lifetime income of those individuals who decide to go to college is the skilled wage, i.e. the increased level of human capital after attending college, $\theta_2 = \theta_1(1+\delta(\theta_1))$.

Finally, individuals take as given the educational system, either tracking or mixing. Then, for all individuals who decide to attend college, the following condition must hold:

$$\theta_1(1+\delta(\theta_1)) \geq \theta_1(1+\gamma),$$

or,

$$\delta(\theta_1) \geq \gamma. \tag{21}$$

This condition determines a minimum level of human capital accumulated through compulsory education, θ_1^*, such that only individuals above that threshold will attend college.[22] That is, $\theta_1^* \in (0, f)$ is the value that satisfies the previous equation with equality.

It is crucial to see in which interval the threshold level θ_1^* is placed, since this determines the composition of the students' body under tracking. Thus, we can distinguish the following three cases:

(i) If $\theta_1^* \in (0, b)$ some individuals from the low track attend college and all individuals from the high track attend college.

[22] To ensure that θ_1^* is interior we will assume that $\delta(\theta_1 = 0) < \gamma < \delta(\theta_1 = f)$.

(ii) If $\theta_1^* \in (b, c)$ some individuals from both tracks attend college. This case is only possible because of Assumption 1.

(iii) If $\theta_1^* \in (c, f)$ only individuals from the high track attend college.

The interval where θ_1^* is placed depends on both the efficacy of higher education described by $\delta(\theta_1)$, and the opportunity cost of attending college measured by γ. In particular, for a given function $\delta(\theta_1)$, an increase in γ will move θ_1^* to the right and a lower proportion of individuals will attend college. In addition, for a fixed γ an upward shift of $\delta(\theta_1)$ implies that a higher proportion of individuals will attend college.

Now I turn to compare both systems after individuals have decided on college attendance. First, I will analyze which system maximizes college attendance. Next I will propose a criteria of equality of opportunities that consists of minimizing the income premium, where this income premium is defined as the difference in the probability of attending college between the rich and the poor under each system.

4.1 Proportion of college students

In Section 3 I concluded that tracking is the system that maximizes average human capital at compulsory level. However, this result should not be overemphasized since tracking may affect decisions on college attendance.[23]

One could think of higher education as positive for the individual himself and her well-being, but one could also think of the positive externalities generated by more highly educated people for the entire society. On this respect, Moretti (2004) find empirical evidence suggesting that an increase in the supply of college graduates not only increases less educated wages, but also high school wages. In addition recent research has pointed to the significance of human capital accumulation for economic growth (see Barro and Sala-i-Martin (1995)) and as a result, there is much policy focus on promoting human capital formation (see, for example, PISA 2003 Report).

An important aspect in this regard is the extent to which the demand for higher education is affected by the education system prevailing at compulsory level through the existence of peer effects. Then, we want to analyze which system, tracking or mixing, provides higher education to the highest number of individuals.

I denote by π_s the proportion of individuals attending college under educational

[23] See Schofield (1995) for a discussion of the possible determinants of the impact of tracking on college attendance from a sociological point of view.

system s, for $s = M, T$, that is $\pi_s = 1 - F_s(\theta_1^*)$. I also define the probability of attending college conditional on individual background. I denote by $\pi_{1,s}$ the probability of attending college for an individual with poor parents ($x = 1$), that is $\pi_{1,s} = 1 - F_s(\theta_1^* \mid x = 1)$, and by $\pi_{x,s}$ the probability of attending college for an individual with rich parents, that is, $\pi_{x,s} = 1 - F_s(\theta_1^* \mid x > 1)$. Both probabilities depend on the minimum level of human capital required to attend college, θ_1^*.

I compare total college attendance under both educational systems. I need first to analyze the value of θ_1 for which both cumulative distributions functions cross each other, since the relation between this value and θ_1^* is crucial to determine the system that maximizes college attendance. I also see that the crucial parameter to determine the value of θ_1 where they cross is λ.

To see this, Propositions 4 and 5 below show that the system that maximizes college attendance depends on the particular income level in the population.

In Section 3 we saw that F_M cuts F_T from below for any value of λ, α and x. The next Proposition shows that the crossing point can be only in one of the two intervals of θ_1, depending on the proportion of rich individuals in the population, λ.

Proposition 3 *There is a unique* $\widetilde{\theta_1} \neq 0$ *such that* $F_T(\widetilde{\theta_1}) = F_M(\widetilde{\theta_1})$. *If* $\lambda < \widehat{\lambda}$, *then* $\widetilde{\theta_1} \in (a, b)$. *If* $\lambda > \widehat{\lambda}$, *then* $\widetilde{\theta_1} \in (c, d)$.

Proof. See Appendix. ■

Moreover I see that $\widetilde{\theta_1}$ is an increasing function of the proportion of rich individuals in the economy λ, independently of the interval where $\widetilde{\theta_1}$ is located.[24]

Thus, we have that if $\theta_1 \in (0, \widetilde{\theta_1})$ then $F_T(\theta_1) - F_M(\theta_1) > 0$, and if $\theta_1 \in (\widetilde{\theta_1}, f)$ then $F_T(\theta_1) - F_M(\theta_1) < 0$. In other words, the density function of θ_1 under tracking accumulates more probability in the tails than under mixing.

When there are few rich people (λ is low), F_M surpasses F_T for a low value of θ_1. The intuition could be that family background cannot offset the peer effect which is stronger under tracking than under mixing. As the society gets richer, average human capital increases as we showed in Section 2.3, and the crossing point $\widetilde{\theta_1}$ moves to the right. In other words, the C.D.F. under mixing will be below the C.D.F. under tracking for a larger interval of values of θ_1.

[24] Note from Equation (18) that if $\widetilde{\theta_1} \in (a, b)$ then, $\widetilde{\theta_1} = \frac{(1-\lambda)a'2cb'}{2(a'\lambda(2c-b')+(1-\lambda)2cb')}$. If $\widetilde{\theta_1} \in (c, d)$ then, $\widetilde{\theta_1} = \frac{\lambda a'eb'}{2(\lambda a'e+(1-\lambda)b'(e-a'))}$.

Now I consider how the income of the rich x affects the relationship between $\widetilde{\theta_1}$ and θ_1^* since this relation, as we will see below, determines the system that maximizes the proportion of college students.

First of all, remember from Equation (21) that the minimum level of human capital required to attend college is increasing with the opportunity cost of college attendance, γ. Now I define two particular values of this opportunity cost that correspond to two different compositions of the college student body under tracking. I denote by $\underline{\gamma}$ the opportunity cost such $\delta(a) = \underline{\gamma}$. That is, when the opportunity cost is $\underline{\gamma}$, we get $\theta_1^* = a$. This value $\underline{\gamma}$ is such that when $\gamma > \underline{\gamma}$, under tracking only poor in the high track attend college. See Figure 1. I denote by $\overline{\gamma}$ the opportunity cost such that $\delta(d) = \overline{\gamma}$. That is, when the opportunity cost is $\overline{\gamma}$, we get $\theta_1^* = d$. In other words, if $\gamma < \overline{\gamma}$ all rich individuals in the high track go to college.

The next Proposition shows that when λ is above some threshold value, that is for rich enough societies, the system that maximizes college attendance is mixing, whereas if λ is below that threshold value, that is for poor societies, the optimal educational system is tracking. I show also that this threshold level is increasing with the opportunity cost of attending college.

Proposition 4 *If $\gamma \in (\underline{\gamma}, \overline{\gamma})$ the education system that maximizes college attendance changes from tracking (poor societies) to mixing (rich societies) as the income level in the population rises. For any income level in the population the following two statements hold:*

(i) If $\gamma \leq \underline{\gamma}$ then mixing maximizes college attendance.

(ii) If $\gamma \geq \overline{\gamma}$ then tracking maximizes college attendance.

Proof. See Appendix. ■

In Figure 3 I illustrate this result. First note that for both extreme values of θ_1^*, and in particular for $\theta_1^* < a$ and $\theta_1^* > d$, the proportion of rich individuals in the population plays no role in the choice of the educational system that maximizes the proportion of college students. If the minimum level of human capital required to attend college is very low, the proportion of college students is maximized under mixing. The intuition could be that, in general, individuals who cannot reach the minimum level of human capital are those with low levels of innate ability. Under mixing, a higher proportion of those individuals will attain the minimum required, since the peer variable is stronger than under tracking.[25]

[25] The case of Spain during the eighties could be suitable to illustrate this result. During those

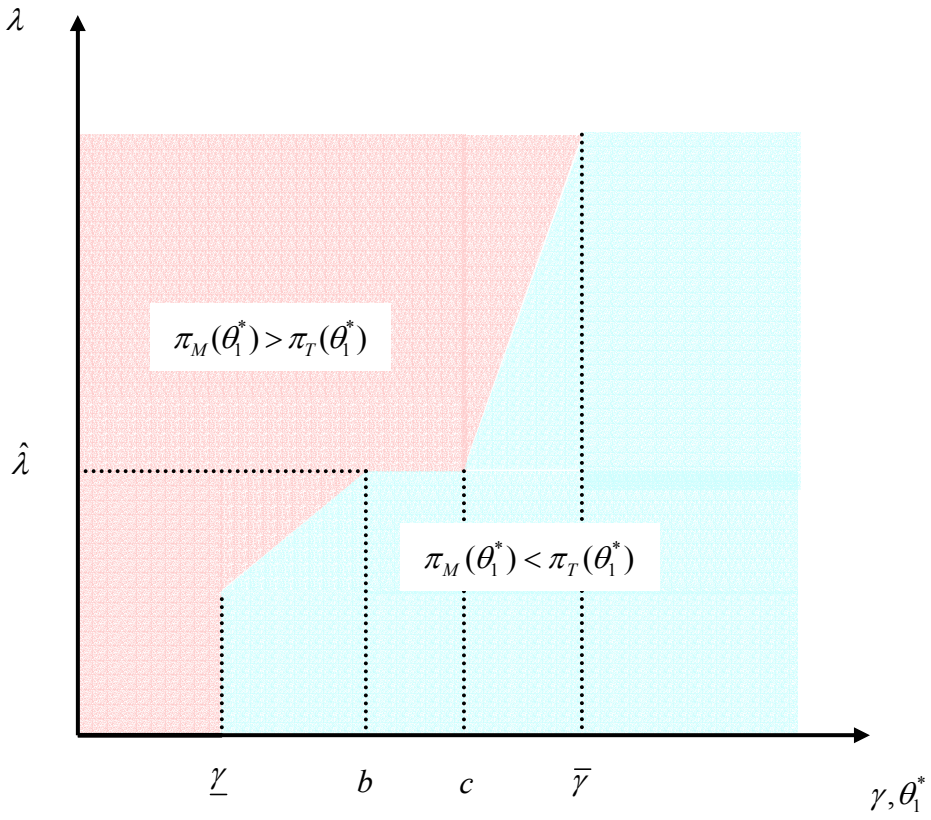

Figure 3. PROPORTION OF COLLEGE STUDENTS

For the same reason, if the minimum level of income required to attend college is high, the proportion of college students is maximized under tracking. In that case the proportion of individuals with larger levels of θ_1 is higher under tracking than under mixing.

For intermediate values of θ_1^* the educational system that maximizes college attendance depends on the income level in the population. In particular, as the proportion of rich individuals rises, the optimal educational system changes from tracking to mixing. The intuition could be as follows. Take as given a minimum level of human capital required to attend college, θ_1^*. In poor societies we ensure that the probability of attending college will be as high as possible under tracking. In these case the minimum human capital required is such that the poor students attending college under tracking belongs to the high track, and for those individuals θ_1 is higher under tracking (since the peer variable is stronger). In rich societies, in order to maximize college attendance we have to ensure that the probability of attending college for rich individuals is as high as possible. Under tracking, the rich individuals (for this interval of θ_1^*) belongs to the low track. But we already know that the human capital of those individuals is higher under mixing (since the peer variable for them is stronger). Thus, the educational system that maximizes college attendance is mixing.

In addition, the previous Proposition implies that as the minimum level of human capital required to attend college rises, the proportion of rich required to maximize college attendance under mixing increases as well. That is, it is needed a higher proportion of rich individuals benefiting from being in mixed classrooms rather than in low tracked classrooms.

4.1.1 Rawlsian Criteria

Throughout the paper it is implicitly assumed that individuals derive utility from income, which they get in the labor market, either as skilled or unskilled workers. Individuals maximize utility when deciding whether to attend college or not.

Suppose now that the government wants to maximize the utility of the worst-off individuals in the society. To do this we have to define first who are the worst-off. If,

years, and given the low rates of college attendance prevailing, the priority of the government was to maximize the proportion of college students. The low opportunity cost of college attendance at that moment, together with a mixing education system at compulsory level yielded an extraordinary increase in the number of college students (from 744,115 in 1983/84 to 1,508,842 in 1995/96. See Estadística Universitaria (2003)).

for example, we take as the worst-off those with innate ability below the median level and with poor parents, the result is quite trivial. Mixing is always better. This comes directly from the properties of the human capital production function (Equations (1) and (2)), since maximizing the utility of these individuals will imply to maximize their human capital at compulsory level θ_1, which in turn will increase their probability of college attendance.[26]

Therefore, I propose to widen the concept of worse-off individuals by considering as the worst-off all individuals with poor parents. Thus, I assume that the government chooses the educational system that maximizes the utility of this group by maximizing their probability of attending college. I define two particular value of the opportunity cost of college attendance. I denote by γ_0 the opportunity cost such that $\delta(a'/2) = \gamma_0$. That is, if the opportunity cost is γ_0 , we get that $\theta_1^* = a'/2$ which is exactly the level of human capital acquired under mixing by poor individuals with median innate ability ($\theta_0 = 1/2$). I denote by γ_1 the opportunity cost such that $\delta(e) = \gamma_1$. That is, if the opportunity cost is γ_1 , we get that $\theta_1^* = e$. In other words, γ_1 is the minimum opportunity cost required to ensure that independently of the education system only rich individuals will attend college

In the next Proposition I show that when the minimum level of human capital required to attend college is low, mixing maximizes college attendance and the reverse occurs for high levels of θ_1^*.

Proposition 5 *Let $\gamma < \gamma_1$. Then if $\gamma < (>)\ \gamma_0$ then mixing (tracking) maximizes college attendance of the worst-off.*

Proof. See Appendix. ■

I illustrate this result in Figure 4. From the human capital production function we have that, for those individuals with innate ability below the median level, their final level of human capital is higher under mixing than under tracking. In particular, under mixing is equal to $(1/2)a'$. Under tracking some of them will be placed in the high track and their final human capital will be b. However, some of those individuals will be placed in the low track, and their final human capital will be a.

Then, when the minimum level of human capital required to attend college θ_1^* is below $(1/2)a'$, the proportion of individuals above θ_1^* is higher under tracking than

[26] Note that this applies to all the individuals with $\theta_0 < m$, except for that individual with $\theta_0 = 0$. It applies to all the individuals for any other uniform distribution with lower bound strictly positive.

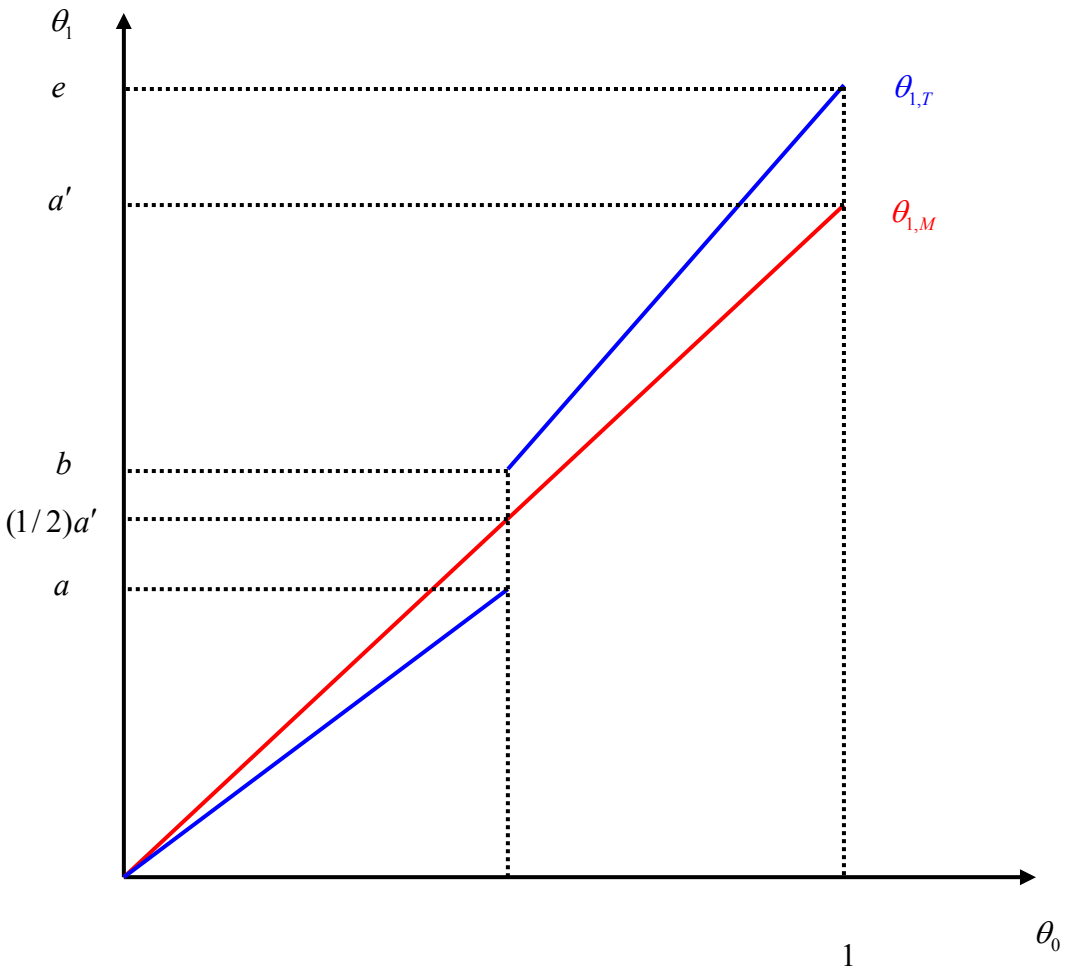

Figure 4. RAWLSIAN CRITERIA

under mixing. The reverse occurs when $\theta_1^* \in ((1/2)a', e)$ and college attendance of the worst-off is maximized with tracking. Finally, when $\theta_1^* > e$, the probability of attending college is zero for the poor, independently of the educational system prevailing.

In the following table I summarize the main results in Propositions 4 and 5:

Table 1.*Maximize College Attendance*

$\lambda \setminus \gamma$	$\gamma < \underline{\gamma}$	$\underline{\gamma} < \gamma < \overline{\gamma}$		$\gamma < \overline{\gamma}$
		$\gamma < \gamma_0$	$\gamma > \gamma_0$	
Rich Societies| $\frac{\text{Total}}{\text{Poors}}$	$\frac{\text{M}}{\text{M}}$	$\frac{\text{M}}{\text{M}}$	$\frac{\text{M}}{\text{T}}$	$\frac{\text{T}}{\text{T}}$
Poor Societies| $\frac{\text{Total}}{\text{Poors}}$	$\frac{\text{M}}{\text{M}}$	$\frac{\text{T}}{\text{M}}$	$\frac{\text{T}}{\text{T}}$	$\frac{\text{T}}{\text{T}}$

Note: *Total=Total College Attendance;Poors=Poors College Attendance;M=Mixing;T=Tracking*

Observe that for poor societies tracking maximizes college attendance both for the poor and total population, when the opportunity cost of attending college is high enough. For rich societies, and provided that the opportunity cost of attending college is low enough, mixing maximizes college attendance both for the poor and total population.

In the following section I use a different criteria to compare both systems: equality of opportunities. In particular I analyze which system minimizes the income gap by trying to bring into line the probability of college attendance between individuals with poor and rich parents.

4.2 Equality of Opportunities

In this section I study which system better guarantees that individuals' decision of whether or not to attend college or not, is taken independently of parents' income.

For both systems I define the income premium $p_s(\theta_1^*)$ or income gap, as the difference in the probability of attending college between the rich and the poor under education system s, for $s = M, T$:

$$p_s(\theta_1^*) = \pi_{x,s} - \pi_{1,s}. \tag{22}$$

If the government wants to guarantee equality of opportunities, it should choose the system s that makes $p_s(\theta_1^*) = 0$. Since this is not possible because $p_s(\theta_1^*)$ is always strictly positive, an educational system is called "equitable" if it minimizes the income premium $p_s(\theta_1^*)$. As we will see below, which system is the most equitable depends on the minimum level of human capital required to attend college.

Observe also that the income premium is defined only for strictly positive values of both $\pi_{x,s}$ and $\pi_{1,s}$, that is, when the minimum level of human capital required to attend college for both systems is such that there are individuals with poor and rich parents attending college. This is equivalent to say that the income premium is defined for every $\theta_1^* \leq a'$.

Since we are comparing the probability of attending college between individuals with different family backgrounds, the result will be driven by the level of income inequality in the population. Remember from Section 2 that the variance of income is $(x-1)^2\lambda(1-\lambda)$, thus it is increasing with x. Therefore, if we take the proportion of rich individuals λ as given, income inequality will be characterized by x.

In the following proposition I state which system is more equitable. I find that this depends both on the minimum level of human capital required to attend college and on the level of income inequality in the population. In particular, income inequality plays a crucial role when the minimum level of human capital required to attend college is low. If this is not the case, tracking is the most equitable system independently of the level of income inequality in the population.

Proposition 6 *Define* $\widetilde{x}(\alpha) = 2^{3\left(\frac{\alpha}{1-\alpha}\right)}$. *There is a minimum level of human capital required to attend college* $\eta_1 \in (a, b)$ *such that:*

(i) When $\theta_1^* \leq \eta_1$ *the following two statements hold:*

(i.1) If $x \leq \widetilde{x}(\alpha)$ *tracking is the most equitable system.*

(i.2) If $x \geq \widetilde{x}(\alpha)$ *mixing is the most equitable system.*

(ii) When $\theta_1^* > \eta_1$ *tracking is the most equitable system independently of the level of income inequality.*

Proof. See Appendix. ■

I illustrate this result in Figure 5. First of all note that when $\theta_1^* \leq \eta_1$ every individual placed in the high track will attend college, and thus both individuals from rich and poor families share the same probability of attending college, which is 1/2. This implies that the income premium under tracking is capturing just the income gap for those individuals placed in the low track.

The proposition above shows that when income inequality is low, tracking is the most equitable system, independently of the level of human capital required to attend college. If this is not the case and income inequality is high, as θ_1^* rises, tracking becomes the most equitable system.

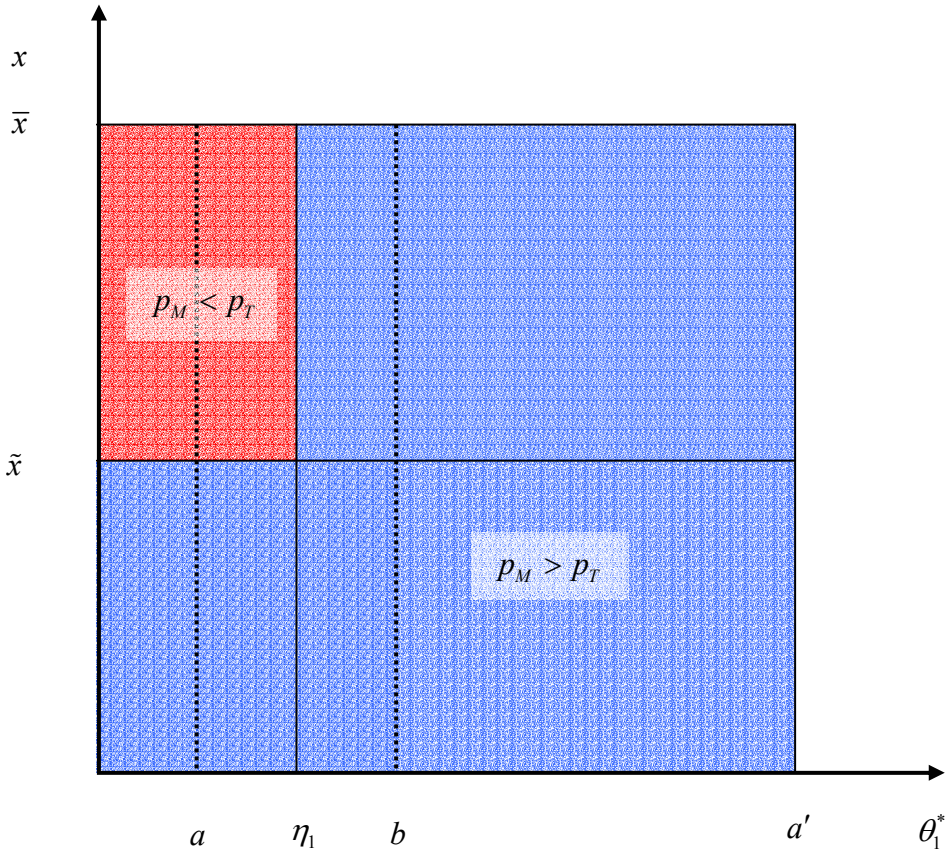

Figure 5. EQUALITY OF OPPORTUNITIES

In addition, the previous proposition shows that when the minimum level of human capital required to attend college is high enough (above b), then tracking is the most equitable system. The intuition could be that when $\theta_1^* > \eta_1$ there are no poor individuals from the low track attending college. Since the positive effect of the peer variable is higher for the most skilled individuals under tracking and, in particular, for those students placed in the high track, their family background is not a crucial factor to determine their final level of human capital. However, under mixing, and for those individuals with the same level of innate ability, it is the case that their backgrounds have a higher relative weight, and as a result the income premium is crucially affected by this variable.

5 Conclusions

In this paper I have analyzed public intervention in education when the government, taking into account the process of human capital production and in particular the peer effect on students' achievement, has to decide how to group students. I analyze two different educational systems. The first one, tracking, consists on grouping students according to their innate ability. The second one, mixing, groups students into completely homogeneous groups. The objective of this paper was to evaluate both educational systems using several criteria. These criteria are classified depending on the educational level analyzed, compulsory or college education.

Some previous works studied the optimal education system at compulsory level by focusing on mean achievement. In particular, Arnott and Rowse (1987) studied the optimal allocation of students and educational expenditures over classrooms when peer effects are present. They concluded that the optimal allocation, when the objective is to maximize the sum of students results, depends on the properties of the educational production function. My paper contributes to this line of research in two directions.

The first contribution of the paper is to introduce the Veil of Ignorance approach in the evaluation of both systems at compulsory level. We saw that under quite reasonable assumptions based on the existing empirical evidence regarding human capital production factors, if we are only interested in maximizing average human capital, the optimal educational system is tracking. However it was also checked that there is no system that dominates the other in the first or second order stochastic sense.

The second contribution of the paper is to highlight the importance of peer effects on college choices. Regarding college attendance it is shown that the system that provides maximum college attendance depends on the opportunity cost of attending college and on the income level in the society. In particular, when the opportunity cost is low, mixing maximizes college attendance. When the opportunity cost is high, tracking maximizes college attendance. With respect to equality of opportunities I found the surprising result that tracking minimizes the income premium in most cases except in situations where there is a high level of inequality in the society, and the level of human capital required to attend college is low. As far as I know there is no previous literature on peer effects on college attendance decisions.

I believe these results are relevant for several issues in the Economics of Education literature. Studies that link persistent levels of inequality in the population to neighborhood effects is an interesting example. Empirical investigation of college choices and the impact of different financial schemes on college attendance decisions could also benefit from our analysis.

In this paper I assumed that the achievement of individual students rises with an improvement in the average quality of their classroom, but at a decreasing rate. Another important issue regarding the non-linearities of peer group effects is the importance of "distance". There is empirical evidence that suggests that peers' effects are stronger when the distance between the individual's innate ability and the average innate ability in the classroom is small, and that as this distance increases, peers' effects become almost negligible. Although I did not model such effect it can be checked that it will only reinforce my main results without adding additional insights. For example, regarding Proposition 1 observe that under tracking, students in the high track gain more because the peer effect is stronger and also because the distance to the average ability in the group is lower. Individuals in the low track first lose because of a lower average ability in the group, but now there is a new positive effect under tracking, since the distance to the average ability in the group is also lower. [27]

The paper allows for some extensions. On the one hand it could be interesting to check the robustness of the main results of the paper to particular features of the model. It could be important to relax some assumptions of the model, in particular

[27] See for example Manski and Wise (1983) where they conclude that students "preferred to enroll in colleges where the average academic ability of the enrolled students was slightly higher than their own. Schools where the average SAT scores of entering freshmen were either too low or too high were relatively disfavored."

some properties of the human capital production function. For example, I could introduce different measures of the so called "peer effect", like some measure of the level of heterogeneity in the group. Moreover, it could be interesting to consider other distributions of innate ability. Other possible ways of modelling the tracking system could be considered. For example, introducing the possibility that students are placed in tracked classes for only a subset of subjects as in Epple, Newlon and Romano (2002). In addition to adding realism, incorporating this possibility would be helpful for determining the design of an optimal educational system. On the other hand it could be also interesting to compare both education systems in a dynamic set up.

6 Appendix

Proof of Proposition 3: (i) $F_T(\theta_1) \not\succeq_{SOSD} F_M(\theta_1)$. Using $F_T(\theta_1)$ from (18) and $F_M(\theta_1)$ from (6) we can check that, $\int_0^b (F_T(\theta_1) - F_M(\theta_1))d\theta_1 > 0$, for every λ, α and x. (ii) $F_M(\theta_1) \not\succeq_{SOSD} F_T(\theta_1)$. Recall that the expected value of a random variable defined on $[0, \overline{z}]$ can be written as: $E[z] = \overline{z} - \int_0^{\overline{z}} F(z)dz$. But then, if $F_M(\theta_1) \succeq_{SOSD} F_T(\theta_1)$ then the following inequality should hold: $f - E_M(\theta_1) \leq f - E_T(\theta_1)$. The final result is immediate from Proposition 1. ■

Proof of Proposition 4: I denote by $\widehat{\lambda}$ the proportion of rich individuals in the economy such that $\frac{\widehat{\lambda}}{1-\widehat{\lambda}} = \Lambda$, where Λ is defined as $\Lambda = \dfrac{2cb'(e-a')}{(b'-2c)a'e}$. In Section 3 it was shown that $F_T(\theta_1) - F_M(\theta_1) > 0$ for all $\theta_1 \in (0, a)$, whereas $F_T(\theta_1) - F_M(\theta_1) < 0$ for all $\theta_1 \in (d, f)$. It is easy to verify also that $F_T(b) - F_M(b) < (>)0$ if and only if $F_T(c) - F_M(c) < (>)0$. If we evaluate the two C.D.F., under mixing and tracking for $\theta_1 = c$, we can check that, $F_M(c) = \dfrac{c}{b'}\lambda + \dfrac{c}{a'}(1-\lambda)$ and $F_T(c) = \dfrac{c}{e}(1-\lambda) + \dfrac{\lambda}{2}$. Thus, from Equation (18) $F_T(c) > F_M(c)$ if and only if $\dfrac{\lambda}{1-\lambda} > \Lambda$. The final result follows immediately from the definition of $\widehat{\lambda}$. ■

Proof of Proposition 5: From Proposition 4 and the fact that F_M always cut F_T from below, a necessary and sufficient condition to ensure that $\pi_M(\theta_1^*) > (<)\ \pi_T(\theta_1^*)$ is that $\widetilde{\theta_1} > (<)\ \theta_1^*$. If $\gamma < \underline{\gamma}$ then, from Proposition 4 we have that $\widetilde{\theta_1} > \theta_1^*$ for all λ. Now assume $\gamma \in (\underline{\gamma}, \overline{\gamma})$. If γ is such that $\theta_1^* \in (b, c)$ then, from Proposition 4 we have that if $\lambda < \widehat{\lambda}$ then, $\widetilde{\theta_1} < \theta_1^*$ and if $\lambda > \widehat{\lambda}$ then, $\widetilde{\theta_1} > \theta_1^*$. Now let γ be such that $\theta_1^* \in (a, b)$ or $\theta_1^* \in (c, d)$. Then, for each θ_1^* there is one λ, denoted by $\widetilde{\lambda}$, such that $\widetilde{\theta_1}(\widetilde{\lambda}) = \theta_1^*$. Thus, since $\widetilde{\theta_1}$ is increasing with λ we have that, if $\lambda < \widetilde{\lambda}$ then $\widetilde{\theta_1} < \theta_1^*$ and if $\lambda > \widetilde{\lambda}$ then, $\widetilde{\theta_1} > \theta_1^*$. From Proposition 4 we have that $\widetilde{\lambda} < \widehat{\lambda}$. Finally, if $\gamma > \overline{\gamma}$ then, from Proposition 4 we have that $\widetilde{\theta_1} < \theta_1^*$ for all λ. ■

Proof of Proposition 6: First I define the threshold level of income $\widetilde{x}(\alpha) = 2^{3\left(\frac{\alpha}{1-\alpha}\right)}$. From Assumptions 1 and 2 $\widetilde{x}(\alpha) \in [\underline{x}(\alpha), \overline{x}(\alpha)]$. From Equations (6) and (22) we have that the income premium under mixing when $\theta_1^* \in (0, a')$ is $p_M(\theta_1^*) = \theta_1^*\left(\frac{b'-a'}{a'b'}\right)$. Take first any $\theta_1^* \in (0, a)$ and the resulting income premium under tracking from Equations (18) and (22) is $p_T(\theta_1^*) = \theta_1^*\left(\frac{c-a}{2ac}\right)$. By A.2 we have that $a < a'$. Thus,

just comparing both income premium and from Equations (4), (5) for $p_M(\theta_1^*)$ and Equations (9) and (10) for $p_T(\theta_1^*)$ we can check that $p_T(\theta_1^*) \geq (\leq)\ p_M(\theta_1^*)$ if and only if $x \geq (\leq)\ \widetilde{x}(\alpha)$. Now take any $\theta_1^* \in (a,b)$. We check that the income premium under tracking is $p_T(\theta_1^*) = \frac{1}{2}\left(1 - \frac{\theta_1^*}{c}\right)$. Let $\eta_1 = \frac{a'b'c}{2c(b'-a')+a'b'}$ be a level of human capital strictly lower than b. Then, if we compare again the income premium under tracking and mixing we find that a sufficient condition to ensure that $p_T(\theta_1^*) \leq p_M(\theta_1^*)$ is $x \leq \widetilde{x}(\alpha)$. If $x > \widetilde{x}(\alpha)$ then, just by comparing both income premiums it can be checked that $p_T(\theta_1^*) \geq (\leq)\ p_M(\theta_1^*)$ if and only if $\theta_1^* \leq (\geq)\ \eta_1$. Take any θ_1^* that belongs to (b,c). From Equations (18) and (22) we obtain $p_T(\theta_1^*) = \theta_1^*\left(\frac{c-b}{2bc}\right)$. As $b < a'$ we find that the income premium under mixing is $p_M(\theta_1^*) = \theta_1^*\left(\frac{b'-a'}{a'b'}\right)$. From Equations (4) and (5) for $p_M(\theta_1^*)$ and Equations (10) and (11) we can check that $p_M(\theta_1^*) > p_T(\theta_1^*)$ is equivalent to the following expression:

$$\left(x^{1-\alpha}-1\right)\left(1+(\tfrac{3}{4})^{\alpha}\right)\left(1+(\tfrac{1}{4})^{\alpha}x^{1-\alpha}\right) > (\tfrac{1}{2})^{\alpha}\left(x^{1-\alpha}-3^{\alpha}\right)\left(1+(\tfrac{1}{2})^{\alpha}\right)\left(1+(\tfrac{1}{2})^{\alpha}x^{1-\alpha}\right).$$

This inequality holds for every x and α. Now take any θ_1^* that belongs to (c,d). From Equations (18) and (22) we obtain $p_T(\theta_1^*) = \frac{1}{2}\left(\frac{\theta_1^*}{b} - 1\right)$. From A.2 we have that $d < a'$, and thus, $p_M(\theta_1^*) > p_T(\theta_1^*)$ implies that the following inequality must hold: $\theta_1^* < \frac{a'b'b}{a'b'-2b(b'-a')}$. A sufficient condition to ensure it is $\frac{a'b'b}{a'b'-2b(b'-a')} \leq d$. The last inequality is equivalent to: $\frac{b'-a'}{a'b'} \geq \frac{d-b}{2bd}$. From Equations (4) and (5) for $p_M(\theta_1^*)$ and Equations (11) and (13) for $p_T(\theta_1^*)$ we check that the last inequality holds if and only if $x \geq (2^{\alpha}(4/3)^{\alpha})^{\frac{1}{1-\alpha}}$. But this is always true since $\underline{x}(\alpha) > (2^{\alpha}(4/3)^{\alpha})^{\frac{1}{1-\alpha}}$. Now, if θ_1^* belongs to (d,a') from Equations (18) and (22) we have that $p_T(\theta_1^*) = \theta_1^*\left(\frac{d-b}{2bd}\right)$. But we have just seen that $\frac{b'-a'}{a'b'} \geq \frac{d-b}{2bd}$, and thus $p_M(\theta_1^*) > p_T(\theta_1^*)$ in this interval. ■

Proof of Proposition 7: In a previous section we have shown that, under both educational systems θ_1 is uniformly distributed on the different intervals determined by each income and ability group. Thus under mixing, if $\theta_1^* \in (0,a')$ then $\pi_{1,M} = 1 - \frac{\theta_1^*}{a'}$ whereas if $\theta_1^* \in (a',b')$ then $\pi_{1,M} = 0$. These probabilities under tracking are as follows. If $\theta_1^* \in (0,a)$ then $\pi_{1,T} = \frac{2a-\theta_1^*}{2a}$, for any $\theta_1^* \in (a,b)$ we have that $\pi_{1,T} = \frac{1}{2}$. Following the same reasoning, if $\theta_1^* \in (b,e)$ then $\pi_{1,T} = (\frac{1}{2}) - \frac{1}{2}\frac{(\theta_1^*-b)}{(e-b)}$. The proof follows just by comparing $\pi_{1,M}$ and $\pi_{1,T}$ for the different intervals. ■

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