A discusión

ALTRUISM, EGOISM AND GROUP COHESION IN A LOCAL INTERACTION MODEL*

José A. García Martínez**

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Corresponding author: José A. García Martínez: Universidad de Alicante. Departamento de Fundamentos del Análisis Económico. Apto de Correos 99. 03690 San Vicente del Raspeig. España. Tel: +34 965 903 614 Fax: + 34 965 903 898 E-mail: gama@merlin.fae.ua.es

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ABSTRACT

In this paper we have introduced and parameterized the concept of "group cohesion" in a model of local interaction with a population divided into groups. This allows us to control the level of "isolation" of these groups: We thus analyze if the degree of group cohesion is relevant to achieve an efficient behaviour and which level would be the best one for this purpose. We are interested in situations where there is a trade off between efficiency and individual incentives. This trade off is stronger when the efficient strategy or norm is strictly dominated, as in the Prisoner's Dilemma or in some cases of Altruism. In our model we have considered that agents could choose to be Altruist of Egoist, in fact, they behave as in Eshel, Samuelson and Shaked (1998) model.

1　Introduction

Efficient norms are often applied in certain social contexts. However, we might well ask ourselves why efficient norms are chosen over others in processes of cultural transmission.

In the words of Boyd and Richerson (2002):

> "The fact that group efficient norms can persist does not explain why such norms are widely observed. While punishment and reward can stabilize group efficient norms, they can also stabilize virtually any behavior (Fundenberg & Maskin,1986; Boyd &Richerson,1992a). We can be punished if we lie or steal, but we can also be punished if we fail to wear a tie or refuse to eat the brains of dead relatives. Thus, we need an explanation for why populations should be more likely to wind up at a group efficient equilibrium than one of the vastly greater number of stable but non-group efficient equilibria. Put another way, if social diversity results from many stable social equilibria, then social evolution must involve shifting among alternative stable equilibria. Group efficient equilibria will be common only if the process of equilibrium selection tends to pick out group efficient equilibria."

There are two different kinds of models that focus on these questions: i.e., the local-interaction and the group-interaction models.

From the many group-interaction models proposed in the literature, we consider the Group Selection model as the most interesting and commonly used. It was originally developed by biologists (see Sober and Wilson (1999)) who supposed that the population is segmented into diverse and isolated groups, each with its own independent dynamic evolution. Being able to consider diverse groups within the same population allows us to consider parallel but different forms evolutions at the same time. This increases the probability that some groups achieve efficiency. One of the problems with these models, however, is the mechanism employed for explaining the way in which an efficient group succeeds in spreading its efficiency throughout the entire population. The most widely accepted idea is that there is a sort of "competition" among groups. Thus, the most efficient groups have the best performance and as such, the most common mechanism considered is the extinction of the least-efficient groups. These concepts have already been applied to certain economic issues, (see e.g., Vega-Redondo, F., (1993) and Sjostrom, T., M. Weitzman (1996)). However, the extinction seems to be a too strong mechanism in many contexts. There must be any other way to spread an efficient behavior from a certain group to the rest of the population.

On the other hand, the literature on local interaction, which is also concerned with the selection of equilibria, suggests a natural mechanism that could spread efficiency from a local context towards a global efficiency, by exploiting the overlaps that exist among individuals' neighborhoods. So far, however, this

branch of the literature has not yet considered the concept of a group being able to spread efficient behavior around in the same sense as Group Selection does. Nonetheless, they do consider some vague forms of clustering or grouping, i.e., there is some kind of structure to the population.

One of the most interesting papers which deal with this topic using a local-interaction model is the work of Eshel, Samuelson and Shaked (1998). They use a local-interaction model to exploit the following idea:

If the positive externalities of co-operation (e.g., in a prisoners' dilemma game) or altruistic behavior are locally restricted, the local-interaction model reduces the possibility for other (more distant) players to benefit from co-operation. Consequently, co-operators who are surrounded predominantly by other co-operators, will earn higher payoffs than non-co-operators who are surrounded predominantly by other non-co-operators. Reinforced by imitation co-operation has a better chance of survival. The structure of the population, however, (which is as presented in Ellison (1993)), has a homogeneous and constant neighborhood overlap, which precludes the idea of a group with any meaningful structure.

In Boyd and Richerson (2002) we find a preliminary, though just partial attempt at merging local interaction and group selection. The authors' goal is to find a mechanism that could spread efficient norms, or "group-beneficial" norms, as they refer to them. They consider a population that is made-up of isolated groups in a ring, as in Ellison (1993), except that Ellison considers individuals, not groups. The individual members of a given group interact among themselves in a uniform way and there is no internal structure to the group. Those individuals, who generally learn from the other members of their own group, also have the probability of learning from their neighboring groups. The agents play a co-ordination game and the efficient strategy eventually spreads throughout the entire population, under given values of probability of imitation and depending on the prevailing payoffs. The authors go as far as considering both inter-group learning and intra-group interaction, (i.e., they consider different structures of interaction and observation-imitation), yet they stop short of considering inter-group interaction as well.

Our model, as the model of Boyd and Richerson (2002), wants to link the local interaction and the group selection models. However, unlike them, we consider not only inter-group learning, but also inter-group interaction. In fact, we identify the structure of interaction and the structure of observation-imitation in the framework of a population that is divided into diverse groups.

Thus, in our model we consider groups in a local interaction model and parameterize the degree of isolation of these groups[1]. To achieve this, we introduce the concept of group cohesion in a local interaction model and parameterize it. Indeed, we parameterize the fraction of neighbors that an agent has inside and outside of his group. This allows us to have two extreme and opposite situations (and all situations between them) in the same model: i.e., on the one

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[1] The isolation of a group increases when the number of neighbors that group members have outside of such group decreases.

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hand, if group-cohesion is maximal, the group remains isolated, which is not good news, as there would be no way of spreading efficiency either to or from another group. This is the structure in models of group selection, and this is the reason why these models must consider group extinction. On the other hand, if group-cohesion is minimal, the very concept of a group is meaningless, groups are inessential, i.e., a case in which neighborhoods are unaffected by groups, as in the classical local-interaction model presented in Ellison (1993) and other papers.

Therefore, our model has in one extreme a structure of group selection models and in the other one a conventional local interaction model similar to Eshel et al. (1998) , and , of course, all the intermediate situations.

Thus, we are able to introduce restrictions to the spread of any behavior in such population. The higher is the cohesion of the groups, i.e. the stronger is the role of the boundaries between groups, the more difficult is the spreading of any behavior between groups.

Hence, we are thus able to study the relationship between group-cohesion and the persistence and spread of efficient but dominated strategies, such as altruism. With such a model, we can now answer questions like:

- Is the spread and persistence of altruism benefited (or damaged) by grouping or clustering individuals in a local-interaction model?

- What is the relationship between group-cohesion and the spread of altruism?

We will see that this concept of group plays an important role even in a local-interaction model. A certain level of fragmentation in the population favours the persistence and spread of altruism. Specifically, an intermediate level of group-cohesion is the best scenario for altruism. Indeed, it is better than the classical local-interaction scenario, where groups are inessential, and it is better than a case of almost-isolated groups (i.e., excessive group cohesion).

In the following section, we introduce the model and review the Eshel et al (1998) model, after which, Section 3 describes the analysis and Section 4 presents some simulations.

2   Model

This section is divided into three sub-sections, in the first one we describe the kind of trade-off between individual incentives and efficiency that we consider in our model. We then describe, in the second sub-section, the structure of the population and the characteristics of the cohesion function. In the following sub-section, we specify the payoffs, the learning rule and the dynamics. Finally, we focus on the special case of a linear cohesion function.

2.1 Altruists and Egoists

We consider the same sort of altruistic behavior as the one described in Eshel, Samuelson and Shaked (1998), but within a different population structure. As our study is closest to the work done by Eshel et al., (1998), we first review their model and then explain how it differs from ours.

The model of Eshel, Samuelson and Shaked (1998)

That behavioral model has two key properties. First, people are not rational agents who choose utility-maximizing actions. Rather, they learn which actions work well by imitation. The second property is that interactions between individual agents are all local.

An individual may be either an Altruist or an Egoist. An Altruist provides a public good that supplies one unit of utility to those who receive its benefits. The net cost of providing the public good for the Altruists is c>0, so that the combination of enjoying the benefits of his own public good and bearing the cost of its provision reduces the Altruist's utility by c. Egoists provide no public goods and support no costs. They simply enjoy the benefits of the public goods provided by others.

Time is divided into discrete periods. At the end of each period, after consuming a public good that is available, and bearing the provision costs (in the case of Altruists), each agent then decides, according to a learning rule, whether he wishes to be an Altruist or Egoist during the next period. Instead of choosing the best responses, players tend to imitate the strategies of other players who they have seen earning payoffs. When people are faced with a complex situation, they tend to imitate the behavior of others who have earned high payoffs.

Imitation alone, however, does not seem to guarantee the survival of altruism. Egoists enjoy the same public goods as Altruists do while they bear none of the costs. As such, Egoists obtain higher payoffs than Altruists and imitation only leads players to become Egoists. The above argument, however, only holds if the positive externality of the public good is extended to every agent in the population. The chance of altruism spreading improves if the public good is a local public good. This reduces the possibility of other (more distant) players exploiting co-operation. As a result, Altruists who are surrounded by other Altruists can earn higher payoffs than Egoists who are surrounded primarily by other Egoists. This local interaction, reinforced by imitation, affords altruism a better chance of survival.

To be more precise, they introduce a neighbor structure taken from Bergstrom and Stark(1993) and Ellison (1993). Agents in the model are located around a circle. Agents interact with his two

immediate neighbors, i.e. with one agent to his right and one to his left. When agent i is Egoist, his pay-off is the number of Altruists in his neighborhood, however, when agent i is Altruist his pay-off is the number of Altruists in his neighborhood minus the cost c. The learning rule is as follows: the agent who learns observes his own pay-off and the pay-off and strategy of each agent in his neighborhood. He then chooses to be an Egoist if the average pay-off of the Egoists in his sample exceeds that of Altruists, and choose to be an Altruist in the opposite case. If an agent and his neighbors all play the same strategy, be it Altruists or Egoists, then the agent will continue to play that strategy.

Regarding the *structure of the population*, our model is quite different from the ones outlined above. We consider groups in a local-interaction model and we parameterize the cohesion of these groups. Thus, we parameterize the fraction of neighbors that any group member has outside and inside of his group. We are able to introduce restriction to the spread of any behavior in such population. The higher is the cohesion of the groups, i.e. the stronger is the role of the boundaries between groups, the more difficult is the spreading of any behavior between groups. As such, we are able to increase or decrease the positive externality of the public good produced by clusters or groups of Altruists, i.e., the extension of that externality. We can therefore study the interaction that exists between altruism and egoism in different scenarios, all with neighborhood overlaps that allow the natural spread of such behaviors. In a sense, we can control the degree of the population's segmentation or fragmentation into groups. The model thus permits us to study the influence that the permeability of the boundary between groups has on the spread of a given pattern of behavior. Another difference in our model is that it considers a continuous, rather than a discrete population.

Regarding the *learning rule*, if an agent compares average payoffs in his neighborhood, as in Eshel et al. (1998), the weight that this mechanism of decision place on his own payoffs is the same as that placed on the payoffs of the other agents in his neighborhood.

We could increase the weight of the individual agent's own pay-off, e.g., by letting him compare it with the average of the payoffs obtained by the other members of his neighborhood, and obviously, by those who happened to play the other possible strategy. If this concept were included in the Eshel et al. model, it would be easy to prove that the altruism has no chance of survival[2].

The best response is an extreme case in which the own-pay-off is the only one considered. It seems that the more weight the own-payoff has in the decision-making mechanism, the worst it is for altruism. This seems logical, as the individual payoffs play a more important role in the agent's decision than the payoffs of the others do. Thus, it is more difficult to learn the positive effects

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[2] We have only proven this statement for the main case studied by Eshel el al., (1998), i.e., each agent has two neighborhoods, this is straightforward and immediate to prove. When we consider larger neighborhoods, however, it seems that Altruism can indeed persist .

of following an efficient but dominated strategy like altruism. Since we want to study the relationship between group-cohesion and the spread of altruism, we consider the best learning rule for the spreading of altruism, i.e., the Eshel et al., model. However, to check the robustness of our results, we also consider the learning rule that compares the average pay-off with one's own pay-off.

2.2 Population structure

Individuals are located on the real line. Let $N = \{i \ / \ i \in \mathbb{R}\}$ be the population, a continuum of agents. Thus, each point of the real line represents an agent. We label the set of agents who are situated between two successive or immediate integers as a group, so the size of each group is equal to one. As such, each group has two neighboring groups, one to its left and another to its right. Consequently, integer numbers on the real line can be considered as boundary points between two successive groups. We denominate each group according to its lower integer-boundary, e.g., the group $z \in \mathbb{Z}$ is $G_z = [z, z+1)$.

Each individual i in this population has an interaction and a learning neighborhood. The interaction and the learning neighborhood, in this model, are in fact the same neighborhood. Thus, individual i's neighborhood is a segment of the real line which contains the individual i. We consider the length of that segment to be equal to one, i.e., equal to the size of the group. However, it does not mean that the group of agent i and the neighborhood of this agent are the same segment on the real line. We parameterize the position of that neighborhood with the function $f(i) : \mathbb{R} \longrightarrow [0, \frac{1}{2}]$ which is called cohesion function, see Figure 1. For each i, the cohesion function $f(i)$ gives the size of the segment-neighborhood outside his group, i.e., the fraction of i's neighbors who are outside i's group. Next, we define i's neighborhood, where $\lfloor i \rfloor$ gives the greatest integer less than or equal to i.
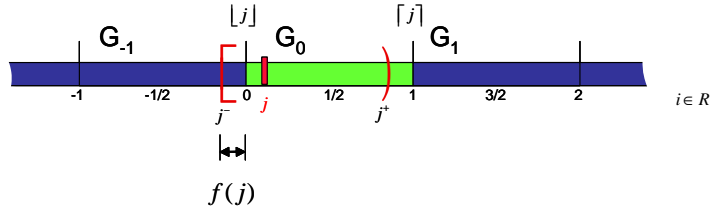

Figure 1: This graph represents the real line. The interval between 0 and 1 is the group $G_0 = [0, 1)$, it is depicted in light grey. The agent j belongs to the group G_0, thus $j \in [0, 1)$. The interaction and learning neighborhood of the agent j is the interval $[j^-, j^+)$.

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Definition 1 *The neighborhood V_i of individual $i \in \mathbb{R}$ is:*

$$V_i = \begin{cases} \{j \in \mathbb{R} \ / \ \lfloor i \rfloor - f(i) \leq j < \lfloor i \rfloor - f(i) + 1\} & \text{if} \ \ i \in [\lfloor i \rfloor, \lfloor i \rfloor + \frac{1}{2}) \\ \{j \in \mathbb{R} \ / \ \lfloor i \rfloor + f(i) < j \leq \lfloor i \rfloor + f(i) + 1\} & \text{if} \ \ i \in [\lfloor i \rfloor + \frac{1}{2}, \lfloor i \rfloor + 1) \end{cases}$$

That V_i is agent i's *interaction and learning neighborhood.*

Even though our one-dimensional model is quite simple, by using different cohesion functions $f(i)$ we can consider broad and diverse population structures. We can change the group's cohesion in many ways. Note that the concept of group-cohesion within a broader population can be a bit fuzzy (see Wasserman and Faust (1994) for several definitions of cohesion). We now present the characteristics of the cohesion function.

2.2.1 Characteristic of the cohesion function.

As we consider that all of our groups have the same structure, the function must be **periodic** with a period equal to one. Let $FractionalPart[i] = |i - \lfloor i \rfloor|$ if $h = FractionalPart[i]$ then $f(i) = f(h)$ for all $i \in \mathbb{R}$, i.e. $f(i) = f(i + 1)$. Furthemore, since we consider that the group structure is **symmetric** around the middle point of the group, the function will be symmetric as well, i.e. $f(h) = f(1 - h)$ for all $h \in [0, \frac{1}{2}]$. Consequently, the function $f(i)$ will be fully characterized by knowing the function $f(h)$ in the interval $h \in [0, \frac{1}{2}]$, because the function is symmetric and periodic. Thus, we can concentrate on the interval $[0, \frac{1}{2}]$.

We also consider that $f(\frac{1}{2}) = 0$, i.e. the cohesion function does not modify the neighborhood of the individual who is in the middle of the group, for this agent, his group and his neighborhood are in fact always the same one. Moreover, we consider $f'(h) \leq 0$ for all $h \in (0, \frac{1}{2})$, i.e. the closer to the center of the group the individual is, the smaller the part of his neighborhood outside the group is. Hence, the function $f(h)$ has the maximum value in $h = 0$ (boundary of the group), and the minimum value in $h = \frac{1}{2}$ (middle point of the group).

Let $x = f(0)$. This parameter, x, is the maximum distance that an external neighbor of a group-member can be from the boundary of the group. We name the parameter x the *external depth* of the group. On the other hand, let $d \in (0, \frac{1}{2}]$ be the argument of f closest to zero, such that $f(d) = 0$, i.e., the value $2d$ gives the number of group members who have neighbors outside the group. We name the parameter d the *internal depth* of the group. See Figure 2.

In such a context, we can see the **cohesion** of the group as the degree of interrelatedness among the group's members.

In general, if we have two cohesion functions $f_1(h)$ and $f_2(h)$ with $f_1(h) \geq f_2(h)$ for all $h \in [0, \frac{1}{2}]$ and $f_1(h) > f_2(h)$ for some $h \in [0, \frac{1}{2}]$ then $f_2(h)$ makes the group's structure more cohesive than $f_1(h)$ does.

With f **lineal**, i.e. $f(h) = x(1 - \frac{h}{d})$ for all $h \in [0, \frac{1}{2}]$, a decrease in either of the parameters d or x obviously causes an increase in the group's cohesion, see Figure 2. On the one hand, if $x = 0$ then we have isolated groups and maximum group-cohesion. On the other hand, if $x = \frac{1}{2}$ and $d = \frac{1}{2}$ then we have

a conventional local interaction model with the same constant overlaps among the neighborhoods, as in Eshel et al[1998]. In this second case, an individual has the same number of neighbors on the right side as on the left side of his neighborhood. He is therefore at the mid-point of his neighborhood.

2.3 Payoffs, learning rule and dynamic.

As we have explained in the previous section, given a particular cohesion function $f(i)$ we have a specific population structure on the real line. In the present section we define the individual payoffs, learning rule and dynamic in a population structure given by a cohesion function $f(i)$.

Time is divided into discrete periods and, as we have already mentioned, we consider altruistic behavior just as Eshel et al., (1998) do. To be more precise, in our population structure, we need to define a state of the system. Let $g_t(i) : \mathbb{R} \longrightarrow \{0,1\}$ be the state of the system in period $t = 0, 1, 2...$ If $g_t(i) = 1$ (0) the agent i is Altruist (Egoist) during period t.

The position of boundary of i's neighborhood are i^- on the left and i^+ on the right (see Figure 1), where:

$$i^- = \begin{cases} \lfloor i \rfloor - f(i) & \text{if } i \in [\lfloor i \rfloor, \lfloor i \rfloor + \frac{1}{2}) \\ \lfloor i \rfloor + f(i) & \text{if } i \in [\lfloor i \rfloor + \frac{1}{2}, \lfloor i \rfloor + 1) \end{cases} \quad \text{and} \quad i^+ = i^- + 1 \qquad (1)$$

The number of i's Altruists neighbors in period t will be[3]:

$$\left| V_t^A(i) \right| = \int\limits_{i^-}^{i^+} g_t(w) \ dw \qquad (2)$$

Thus, i's payoff in period t is:

$$\Pi_t(i) = \left| V_t^A(i) \right| - g_t(i) \ c \qquad (3)$$

Where c is the cost of altruism and $c > 0$.

Remember that an Altruist provides a public good that supplies one unit of utility to those who receive its benefits. Moreover, at the end of each period, after consuming a public good that is available, and bearing the provision costs (in the case of Altruists), each agent then decides, according to a learning rule, whether he wishes to be an Altruist or Egoist during the next period.

We first define the following average payoffs, after that we will specify the learning rule:

The average payoffs of Altruists in i's neighborhood in period t is:

$$\bar{\Pi}_t^A(i) = \begin{cases} 0 & \text{if } \left| V_t^A(i) \right| = 0 \\ \frac{1}{\left| V_t^A(i) \right|} \int_{i^-}^{i^+} g_t(w) \ \Pi_t(w) \ dw & \text{if } \left| V_t^A(i) \right| > 0 \end{cases} \qquad (4)$$

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[3]Obviously, the function $g_t(w)$ is no continuous in general. Nevertheless, as it is bounded, if we assume that it has a finite number of discontinuity points in any finite interval, then the definite integral is possible to calculate.

The average payoffs of Egoists in i's neighborhood in period t is:

$$\bar{\Pi}_t^E(i) = \begin{cases} 0 & \text{if } \left|V_t^A(i)\right| = 1 \\ \frac{1}{1-\left|V_t^A(i)\right|} \int_{i^-}^{i^+} (1 - g_t(w)) \, \Pi_t(w) \, dw & \text{if } \left|V_t^A(i)\right| < 1 \end{cases} \quad (5)$$

We consider the following functions to make the learning rule that determines the population's dynamics more precise:

- If we consider $LR_t^M(i)$, then the agents follow the learning rule in which they compares average payoffs.

$$LR_t^M(i) = \begin{cases} 1 & \text{if } \bar{\Pi}_t^A(i) - \bar{\Pi}_t^E(i) > 0 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

- If we consider $LR_t^P(i)$, then the agents follow the learning rule in which they compares average payoffs with their own.

$$LR_t^P(i) = \begin{cases} 1 & \text{if } \left(\Pi_t(i) - \bar{\Pi}_t^E(i)\right) g_t(i) + (\bar{\Pi}_t^A(i) - \Pi_t(i))(1 - g_t(i)) > 0 \\ 0 & \text{otherwise} \end{cases}$$
$$(7)$$

The following equation represents the dynamic of the system.

If we assume the agents are following the learning rule LR_t^M then the equation is:

$$g_{t+1}(i) = LR_t^M(i)$$

If we assume the agents are following the learning rule LR_t^P then the equation is:

$$g_{t+1}(i) = LR_t^P(i)$$

As we have already mentioned, we first do the analysis assuming that the agents follow the learning rule LR_t^M. We then repeat the analysis, but assuming that they follow the learning rule LR_t^P.

In the following section, we analyse a specific cohesion function, i.e., the linear cohesion function and the parameter that we will use to measure the group cohesion.

2.4   Linear cohesion function and the measure of group cohesion.

Although considering a linear cohesion function might be rather simple, nevertheless it permits us to examine a very diverse population structure. This linear cohesion function is:

$$f(i) = f(h) = \left\{ \begin{array}{ll} x(1 - \frac{h}{d}) & \text{if } h \in [0, \frac{1}{2}) \\ x(1 - \frac{1-h}{d}) & \text{if } h \in [\frac{1}{2}, 1) \end{array} \right. \qquad \text{where } h = FrationalPart[i]$$
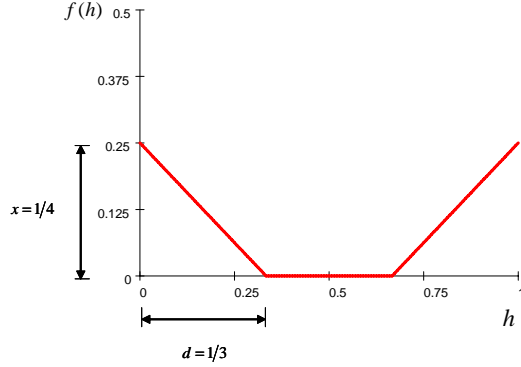
(8)

Figure 2: Linear cohesion function $f(h)$ with external depth $x = \frac{1}{4}$, internal depth $d = \frac{1}{3}$. The vertical axis represents the value of the cohesion function and the horizontal axis represents (without loss of generality) the individuals of group G_0, i.e., the interval $[0, 1)$.

We now briefly describe the relationship between the payoffs of the members of an Altruist group[4] and the different linear cohesion functions (a linear cohesion function is determined by the parameters x and d).

We first focus on the role of the parameter x. It is straightforward to show that, for any strategy profile of the two neighboring groups of an Altruist group: the smaller x (the external depth) is, the greater the payoffs of the members of the Altruist group will be (they might even be equal, but never lower). Conversely, as x increases, the Altruist's payoffs either reduce or stay unchanged.

When we consider an Egoist group, the relationship between the payoffs and the parameter x will be, obviously, the opposite case. Thus, a smaller external depth x will be more beneficial to the Altruist groups and a high x will be more beneficial to the Egoist groups.

Furthermore, the relationship between the degree of cohesion and the parameter x is quite clear. The greater the external depth x is, the weaker the group-cohesion will be. We can therefore state that in the framework of homogeneous groups, high levels of cohesion are good for Altruist groups and negative for Egoist groups.

With regard to the internal depth, d, we have a similar outcome as in the case of the external depth, x, i.e., a similar relationship between the payoffs of

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[4]By the term "Altruist group" we denote a group in which all its members are altruists.

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and Altruist or Egoist group and the parameter d. One interesting characteristic of the parameter d is the following; if d increases, more agents will be able to observe activities that take place outside their own group. Thus, the internal depth (d) determines how many people are able to observe activities outside their group, and the external depth (x) determines how many people from outside the group can be observed by those in the group who can observe. From an individual agent's point of view, a small portion of his neighborhood that lying outside his group is (in general) not very important for his interaction and his payoffs, although it might be very important for the learning process.

When we consider $d = \frac{1}{2}$, we note that every individual agent is able to observe someone from the closest adjacent group. This seems to be the best scenario for the spread of strategies throughout the population. As such, for the sake of simplicity, we assume that $d = \frac{1}{2}$ in the following section. We could certainly do an analysis for any d, but that would only make the analysis more complex, and the outcome would be quite the same as that for the simpler analysis.

From now on we consider $d = \frac{1}{2}$, therefore, *the parameter x will be the measure of group cohesion*. Note that if $x \simeq 0$ then the groups are almost isolated, in fact if $x = 0$ then there is not local interaction and the group are independent, in that case the group cohesion is maximal. If we consider greater x, the group cohesion decreases, thus the group members have more neighbors outside of his group to interact with and learn from. If x takes the maximum value, $x = 1/2$, then the group cohesion is minimal and the group has no significance, in that case we have a classical local interaction model in one dimension, like the ring of the Ellison(1993) model and Eshel[5] et al. (1998), where the agents has the same number of neighbors on the right side that on the left side. See the Figure 3.
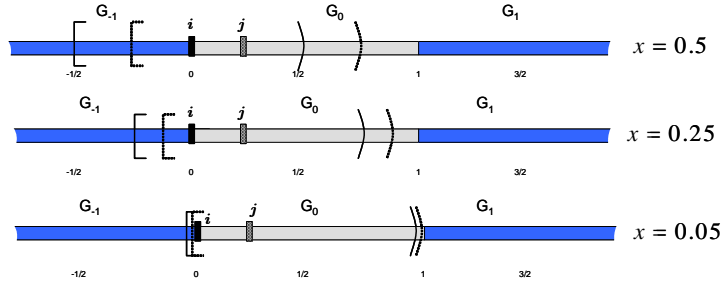

Figure 3: We represent the interaction-learning neighbourhood of agent i (normal brackets) and agent j (dot brackets) for three different values of x, i.e., three levels of group cohesion. The agents i and j belong to group G_0 in light grey. The smaller x is, the greater the level of group cohesion is.

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[5] The only difference is that we consider a continuum population whereas they consider a discrete population.

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Therefore *we identify the parameter x with the level of group cohesion.*

On the one hand, high levels of group cohesion prove to be beneficial for the persistence of an Altruist group. On the other hand, however, low levels of group-cohesion prove to be beneficial for the spreading of a given sort of behavior within a population, and whilst this might be conducive to the spreading of altruism, it would also be suitable for the spreading of egoism as well. It is therefore not clear, as yet, what precise scenario would prove to be the most beneficial setting for the persistence and spread of altruism throughout a population.

In the following section, we do a detailed analysis to identify the best conditions for the persistence and spread of altruism, (i.e., the level of group-cohesion that favours altruism in a population).

3 Analysis

As we assumed in the previous section, we consider (from no on) only *lineal* cohesion functions with $d = 1/2$. Therefore, the group cohesion is give by the parameter $x \in (0, \frac{1}{2}]$. In addition, we consider populations with *homogeneous groups* as initial states. As such, all of the agents in a given group follow the same strategy at $t = 0$. We can thus talk about Altruist and/or Egoist groups at $t = 0$. Such an assumption simplifies matters and is not too artificial either[6]. We can consider that our groups have undergone some previous evolution before joining the global population. Generally, the evolution of an isolated group tends to homogenize its behavior, if we consider a uniform interaction. It seems reasonable to suppose that most groups will be egoistic, although a few of them (or maybe just one) might be altruistic. We can then ask if one (or a few) altruistic groups could persist in a global population (of mainly Egoists groups) and if it (of they) might even be able to spread their altruism throughout the entire population. Moreover, if the Altruism was able to persist and even spread, then we would want to know which level of group cohesion is the better for this.

In an effort to answer such questions, we do the following analysis: We first want to know the values of the parameters c and x (i.e., the cost of altruism and the degree of group-cohesion[7]) that permits the persistence, spread and/or predominance of altruism.

To this aim, we study the long-run behavior of the population in three different initial scenarios:

- *Scenario A*: A predominantly egoistic population with just one Altruist group, from which we determine the conditions (values of c and x) for

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[6] In the next section, using simulations, we relax this assumption and increase the set of initial states.

[7] As we have already mention the parameter x is a good measure of the level of group-cohesion. With $x \simeq 0$, we have an almost-isolated group and the greatest interaction of any agent is with the members of his own group. With $x = \frac{1}{2}$, we make the groups inessential and we have a classical local-interaction model in one dimension population. Therefore, we can identify the parameter x with the level of group-cohesion.

altruism to persist and spread throughout the egoistic population. The relationship between the persistence and spread of altruism and the cost altruism is quite clear. The greater the cost the more difficult it is for altruism to persist and spread. What is not so clear, however, is the relationship between group-cohesion and the persistence and spread of altruism.

- *Scenario B*: A predominantly altruistic population with just one Egoist group, from which we determine the conditions (values of c and x) for the sole Egoist group to be driven to extinction.

- *Scenario C*: A population of Altruists with two Egoist groups from which we determine the conditions for the Egoist groups to be driven to extinction.

Therefore, for the values of c (cost of altruism) and x (level of group-cohesion) that support all of the above conditions, we can verify the following statement: For such values of x and c, if we consider any initial state with only homogeneous groups and with at least one Altruist group, the entire population will follow the altruistic strategy in the long run.

3.1 The learning rule LR^M.

We now determine the values of c and x that support the above conditions, provided these values exist. We assume that all agents use the learning rule LR^M, as defined in (6).

3.1.1 Scenario A: An Altruist group in a world of Egoists.

In period $t = 0$ there is the following initial state: Without loss of generality, we consider that Group $G_0 = [0, 1)$ to be the Altruist group and the remaining groups are egoistic, see Figure 4.

The individual payoffs are zero for the whole population, apart from the Altruists in G_0 and the Egoists closest to the boundary of G_0 and who have altruistic neighbors. Obviously, the smaller the cohesion of the group is, the greater the Altruists' payoffs and the smaller the Egoists payoffs will be. In such a context, cohesion only changes with the parameter x. As we next prove with Condition $A1$, the greater the cost of altruism (c) is, the smaller x must be for altruism to persist in the next period, (i.e., the greater the group-cohesion must be).

We next calculate conditions (values of c and x) under which the group of Altruist persists (Condition $A1$). After that, we calculate the values of c and x under which the Altruism of this group could spread to the entire population (Condition $A2$ an $A3$).
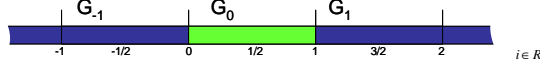
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Figure 4: The group $G_0 = [0, 1)$ is the Altruist group (light grey), the remainder groups are egoistic.

Condition $A1$: At $t = 1$, Altruists continue being Altruists. Here, we calculate the values of c and x that make Altruists continue being Altruists in the following period.

Since we are in a symmetric scenario, we only need to consider the Altruists in $(0, \frac{1}{2})$, see Figure 4. The average payoffs observed by the Altruist i who is located in $(0, \frac{1}{2})$ will be, see equations (4) and (5):

$$\bar{\Pi}_{t=0}^A(i) = \frac{1}{1-f(i)} \int_0^{1-f(i)} (1 - f(w) - c) \, dw$$

$$\bar{\Pi}_{t=0}^E(i) = \frac{1}{f(i)} \int_0^{f(i)} f(w) \, dw$$

The condition required for these altruists to continue being altruists is:

$$\bar{\Pi}_{t=0}^A(i) - \bar{\Pi}_{t=0}^E(i) = 1 - c - \frac{1}{1-f(i)} \int_0^{1-f(i)} f(w) \, dw - \frac{1}{f(i)} \int_0^{f(i)} f(w) \, dw > 0 \quad \text{for all} \quad i \in (0, \tfrac{1}{2}) \tag{A1}$$

Therefore, if $A1$ holds the altruist group continues being altruistic.

Using the linear cohesion function given by (8), condition $A1$ is equivalent to, see Figure 5:

$$c < 1 - \frac{3x}{2}$$

For the values of c and x that hold this expression a group of Altruists surrounded by Egoists continue being Altruist.

The above expression describes an inverse relationship between the cost of altruism (c) and the parameter (x), which is the parameter that measures group-cohesion. Thus, the lower the cohesion of the group is (or the higher x is), the lower the cost of altruism must be to guarantee the persistence of an altruistic group.
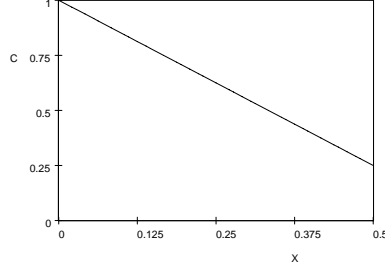
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Figure 5: We represent the function $c = 1 - \frac{3x}{2}$

Condition $A2$: At $t = 1$, an egoistic observer of altruism will want to become an Altruist. Now, we must only concentrate on the egoistic observers of altruism in group G_{-1} (left of G_0), the other group G_1 (right of G_0) is in the same situation, see Figure 4. Since $d = \frac{1}{2}$, the Egoists in the interval $(-\frac{1}{2}, 0)$ will observe Altruists in Group G_0. Note that these Egoists have neighbors in group G_0, therefore, have altruistic neighbors.

We calculate the values of c and x that would encourage such Egoists become Altruists in the following period.

The average payoffs observed by the Egoists of $(-\frac{1}{2}, 0)$ will be:

$$\bar{\Pi}_{t=0}^A(i) \quad = \quad \frac{1}{f(i)} \int\limits_0^{f(i)} (1 - f(w) - c) \; dw$$

$$\bar{\Pi}_{t=0}^E(i) \quad = \quad \frac{1}{1-f(i)} \int\limits_{-\frac{1}{2}}^0 f(w) \; dw = \frac{1}{1-f(i)} \int\limits_0^{\frac{1}{2}} f(w) \; dw$$

The condition for these egoistic observer of altruism will want to become Altruists[8] is:

$$\bar{\Pi}_{t=0}^A(i) - \bar{\Pi}_{t=0}^E(i) = 1 - c - \frac{1}{1-f(i)} \int\limits_0^{\frac{1}{2}} f(w) \; dw - \frac{1}{f(i)} \int\limits_0^{f(i)} f(w) \; dw > 0 \quad \text{for all} \quad i \in (0, \tfrac{1}{2})$$
$$\text{(A2)}$$

It is straightforward to prove that if $(A1)$ holds then $(A2)$ holds, since $\frac{1}{2} \leq 1 - f(i)$.

---

[8]Note that we can always relocate or transfer this condition to the interval $(0, \frac{1}{2})$, because of the characteristics of the function $f(i)$.

Let $A1'$ be the same condition as $A1$, but with a "smaller than" sign in the expression of the condition, i.e. $\bar{\Pi}_t^A(i) - \bar{\Pi}_t^E(i) < 0$ for all $i \in (0, \frac{1}{2})$, and analogously $A2'$.

Remark 2 *Note that, if $A2'$ holds then $A1'$ also holds. Therefore, if $A2'$ holds, altruism will then vanishes completely from the population.*

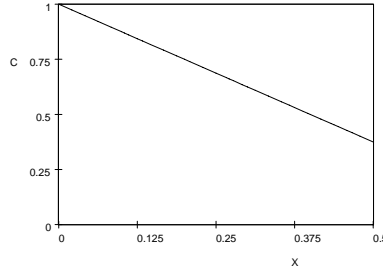Using the linear cohesion function given by (8), the condition $A2$ is equivalent to, see Figure 6:

$$c < 1 - \frac{5x}{4}$$

Figure 6: We represent the function $c = 1 - \frac{5x}{4}$

Therefore, on the one hand, if $c < 1 - \frac{5x}{4}$ then an egoistic observer of altruism will become Altruist. On the other hand, if $c > 1 - \frac{5x}{4}$ then the altruism vanishes completely from the population

Condition $A3$: At $t = 2$, the agents in the half-altruistic and half-egoistic group will want to be Altruists. Thus, in period $t = 1$, if $A1$ holds, then the whole group, G_0, continues to be altruistic and half of the members of the adjacent groups, G_{-1} and G_1, change to altruism, see Figure 7. Since both adjacent groups are under the same situation, we arbitrarily choose Group G_{-1} for our study. We first study the altruistic interval $\left(-\frac{1}{2}, 0\right)$ and the conditions for the Altruists to continue being so, and then the conditions required for the Egoists in the interval $\left(-1, -\frac{1}{2}\right)$ to change to altruism.

Now, the group-cohesion seems to work against the Altruists of G_{-1}. The greater the group-cohesion is, the smaller the payoffs of such Altruists and the greater the payoffs of the Egoists in G_{-1} are. However, in this framework, if the level of cohesion increases, the Altruists' neighborhoods penetrate further into the Egoists' zone, (i.e., they have more Egoists and less Altruists as neighbors). Thus, they observe more Egoists. This could encourage altruism, since the average payoffs of Egoists could decrease. The reason for this, the decrease in the Egoists' payoffs as they move away from altruism.
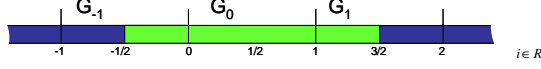
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Figure 7: At $t = 1$, if $A1$ holds, the whole group G_0 continues being Altruist and half of the adjacent groups G_{-1} and G_1 change to altruism.

To clarify this latter point we focus on the opposite case, i.e., the group-cohesion decreases (i.e. x increases), in that case, there might be two different reactions:

- On the one hand, the payoffs for Altruists in G_{-1} increase and the payoffs for Egoists in G_{-1} decrease, which works in favour of altruism.

- On the other hand, an altruist in G_{-1} observes fewer Egoists in his neighborhood but notices that those Egoists obtain high payoffs because they are very close to Altruists, thus, the average payoffs of the Egoists might increase. It could be that this increase of the average Egoist's pay-off was higher than that of the average altruist in that neighborhood. An agent would have therefore incentives to change to egoism if group-cohesion decreases, which works in favour of egoism.

That trade-off makes the upper bound of the next sufficient condition concave and with a maximum.

Altruists in Group G_{-1}. The average payoffs observed by the Altruists who belong to $(-\frac{1}{2}, 0)$, see Figure 7, will be:

$$\bar{\Pi}^A_{t=1}(i) = \frac{1}{\frac{1}{2} + f(i)} \left(\int_{-\frac{1}{2}}^{0} \left(\frac{1}{2} + f(w) - c \right) dw + \int_{0}^{f(i)} (1 - c) \, dw \right)$$

$$\bar{\Pi}^E_{t=1}(i) = \frac{1}{\frac{1}{2} - f(i)} \int_{-(1-f(i))}^{-\frac{1}{2}} \left(\frac{1}{2} - f(w) \right) dw$$

The condition required for these altruists to continue being altruists is:

$$\bar{\Pi}^A_{t=1}(i) - \bar{\Pi}^E_{t=1}(i) = -c + \frac{f(i)}{1+2f(i)} + \frac{1}{\frac{1}{2}+f(i)} \int_{0}^{\frac{1}{2}} f(w) \, dw + \frac{1}{\frac{1}{2}-f(i)} \int_{f(i)}^{\frac{1}{2}} f(w) \, dw > 0 \quad \text{for all} \;\; i \in (0, \tfrac{1}{2})$$

$$(A3_a)$$

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Egoists in Group G_{-1}. The average payoffs observed by the Egoists who belong to $(-1, -\frac{1}{2})$, see Figure 7, will be:

$$\bar{\Pi}_{t=1}^A(i) = \frac{1}{\frac{1}{2} - f(i)} \int_{-\frac{1}{2}}^{-f(i)} (\frac{1}{2} + f(w) - c)\, dw$$

$$\bar{\Pi}_{t=1}^E(i) = \frac{1}{\frac{1}{2} + f(i)} \int_{-1}^{-\frac{1}{2}} \left(\frac{1}{2} - f(w)\right)\, dw$$

This condition for changing from Egoist to Altruist is the same as the one in $A3_a$:

$$\bar{\Pi}_{t=1}^A(i) - \bar{\Pi}_{t=1}^E(i) = -c + \frac{f(i)}{1+2f(i)} + \frac{1}{\frac{1}{2}+f(i)} \int_0^{\frac{1}{2}} f(w)\, dw + \frac{1}{\frac{1}{2}-f(i)} \int_{f(i)}^{\frac{1}{2}} f(w)\, dw > 0 \quad \text{for all} \ \ i \in (0, \tfrac{1}{2})$$

$$(A3_e)$$

Therefore $A3_a \equiv A3_e = A3$.

Using the linear cohesion function given by (8), condition $A3$ is equivalent to, see Figure 8:

$$c < \min\{x, \frac{2x(1 - x^2)}{1 + 2x}\}$$

For the values of c and x that hold this expression the agents in the half-altruistic and half-egoistic group will want to be Altruists.
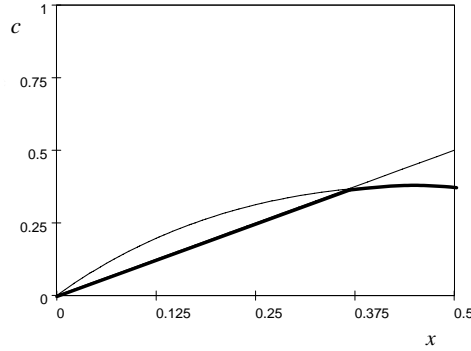

Figure 8: We represent the function $c = \min\{x, \frac{2x(1-x^2)}{1+2x}\}$ with a thick line, it has a maximum at the point $\tilde{x} \cong 0.455$

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The expression $\min\{x, \frac{2x(1-x^2)}{1+2x}\}$ is concave and has a maximum at the point $\tilde{x} \simeq 0.455$. Therefore, the highest cost, c, that is compatible with the spreading of altruism is achieved with an intermediate level of cohesion, see Figure 8.

Note that, if $A1$ and $A3$ hold, at $t = 2$ the groups G_0 G_{-1} G_1 will be Altruist, at $t = 3$ half of the members of groups G_{-2} and G_2 will also be Altruist, at $t = 4$ groups G_0 G_{-1} G_1 G_{-2} G_2 will be Altruist, the process continues spreading Altruism throughout the entire population.

Now, we can state the following proposition.

Proposition 3 *Let $f(i)$ be a cohesion function, LR^M be the learning rule and $d = \frac{1}{2}$ in an initial Egoist population with a group of Altruists. If $A1$, and $A3$ hold, then altruism will spread throughout the entire population.*

Corollary 4 *Let $f(i)$ be a linear cohesion function, LR^M be the learning rule and $d = \frac{1}{2}$ in an initial Egoist population with a group of Altruists. If $c < \min\{1 - \frac{3x}{2}, x, \frac{2x(1-x^2)}{1+2x}\}$, altruism will spread throughout the entire population.*

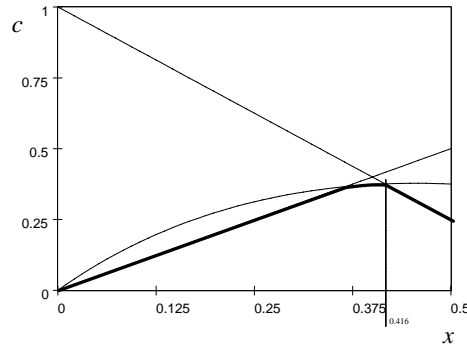In Figure 9 we plot the previous expression.

Figure 9: Conditions A1 and A3 combined, which are equivalent to the expression $c < \min\{1-\frac{3x}{2}, x, \frac{2x(1-x^2)}{1+2x}\}$. We represent the function $c = \min\{1-\frac{3x}{2}, x, \frac{2x(1-x^2)}{1+2x}\}$ with a thick line, below this line the altruism spreads throughout the entire population. This line has a maximum at the point $\tilde{x} \cong 0.416$. Therefore, the highest cost, c, that is compatible with the spreading of Altruism is achieved with an intermediate level of cohesion.

3.1.2 Scenario B: One Egoist group in a world of Altruists.

We now consider a different initial state and work just as before, except that now we calculate the conditions to drive an Egoist group into extinction. In period $t = 0$ there is the following state: Without loss of generality, we consider

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that Group $G_0 = [0, 1)$ is an Egoist group in a population in which all the other groups are altruistic, see Figure 10. The individual payoffs are $1 - c$ for the whole population, apart from the Egoists in G_0 and the Altruists who are closest to the boundary of G_0 and who have egoistic neighbors.

Obviously, the weaker the group-cohesion is, the greater the Egoists' payoffs will be and the smaller those of the Altruists[9] will be. Thus, we can expect that, the greater the cost of altruism (c) is, the smaller x must be to make egoism vanish, (i.e., the greater the group-cohesion must be).
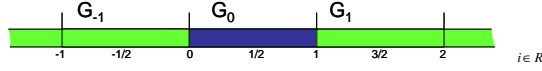
Figure 10: At $t = 0$, Group $G_0 = [0, 1)$ is the Egoist group (dark grey) , the rest of the groups are Altruists (light grey)

Condition B1: Egoists want to be Altruists. We now focus Egoists and calculate the values of c and x that are required to make them become Altruists in the following period.

The average payoffs observed by the Egoists who belong to $(0, \frac{1}{2})$ will be:

$$\bar{\Pi}^A_{t=0}(i) = \frac{1}{f(i)} \int_{-f(i)}^{0} (1 - f(w) - c) \, dw$$

$$\bar{\Pi}^E_{t=0}(i) = \frac{1}{1 - f(i)} \int_{0}^{1-f(i)} f(w) \, dw$$

The condition required for them to change to altruism is:

$$\bar{\Pi}^A_{t=0}(i) - \bar{\Pi}^E_{t=0}(i) = 1 - c - \frac{1}{1-f(i)} \int_{0}^{1-f(i)} f(w) \, dw - \frac{1}{f(i)} \int_{0}^{f(i)} f(w) \, dw > 0 \quad \text{for all} \quad i \in (0, \tfrac{1}{2})$$

$$(B1)$$

As we can clearly see, this condition is equal to $A1$, thus $A1 \equiv B1$.

Condition B2: Altruistic observer of egoism want to continue being Altruists. We now concentrate on the altruistic observer of egoism in Group G_{-1} (left to G_0). The other group, G_1 (right to G_0), is in the same situation, see Figure 10. Since $d = \frac{1}{2}$, the Altruists in the interval $(-\frac{1}{2}, 0)$ will observe Egoists in Group G_0.

---

[9] Evidently, of those who have Egoistic neighbors.

The average payoffs observed by the Altruists who belong to $(-\frac{1}{2}, 0)$ will be:

$$\bar{\Pi}^A_{t=0}(i) = \frac{1}{1-f(i)} \left(\int\limits_{-(1-f(i))}^{-\frac{1}{2}} (1-c)\, dw + \int\limits_{-\frac{1}{2}}^{0} (1-f(w)-c)\, dw \right)$$

$$\bar{\Pi}^E_{t=0}(i) = \frac{1}{f(i)} \int\limits_{0}^{f(i)} f(w)\, dw = \frac{1}{f(i)} \int\limits_{0}^{f(i)} f(w)\, dw$$

The condition required for them to continue being Altruists will be:

$$\bar{\Pi}^A_{t=0}(i) - \bar{\Pi}^E_{t=0}(i) = 1-c-\frac{1}{1-f(i)} \int\limits_{0}^{\frac{1}{2}} f(w)\, dw - \frac{1}{f(i)} \int\limits_{0}^{f(i)} f(w)\, dw > 0 \quad \text{for all} \ \ i \in (0, \tfrac{1}{2})$$

$$(B2)$$

As we can see, that condition is equal to $A2$, thus $A2 \equiv B2$

Therefore, since $A1 \Rightarrow A2$, *if A1 holds, a single group of Egoists surrounded by Altruist groups will become altruistic in just one period.*

3.1.3 Scenario C: Two neighboring Egoist groups in a world of Altruists.

We also need to contemplate this scenario for the following reason; If Conditions $A1$ and $A3$ hold, a string of egoistic groups surrounded by Altruist groups drops to a single group of Egoists, (provided that the string contains an even number of egoistic groups), which later disappears. If the string contains an odd number of Egoist groups, however, it then drops to two adjacent Egoist groups. In the following sub-section, we study the condition under which these two remaining Egoist groups also disappear.

For this purpose, we now consider the following initial state: In period $t = 0$ there is the following state: Without loss of generality, we consider that Groups $G_0 = [0, 1)$ and $G_1 = [1, 2)$ are Egoist groups and the remaining groups are all altruistic, see Figure 11. The payoffs of the individual members of the population are $1 - c$ each, except for the Egoists in G_0, and G_1 and the Altruists that are closest to the left boundary of G_0 and those closest to the right boundary of G_1. Obviously, the weaker the group-cohesion is, the smaller the Altruists' payoffs[10] and the higher the Egoists' payoffs will be.

In period $t = 0$ the groups are in a scenario that is equivalent to the Scenario A. Thus, if $A1$ holds, we can then state that in period $t = 1$ the whole population is altruistic, apart from the agents in $(\frac{1}{2}, \frac{3}{4})$ who continue being Egoists, since they have not observed any Altruists in period $t = 0$, see Figure 11.

Since both groups (G_0 and G_1) are under the same situation in period $t = 1$, we study Group $G_0 = [0, 1)$. That group has a half $[0, \frac{1}{2})$ being Altruists and

---

[10] Evidently, of those who have Egoistic neighbors.

the other half $[\frac{1}{2}, 1)$ being Egoists. Next, we study the condition to change, in period $t = 2$, to the whole group being altruistic.
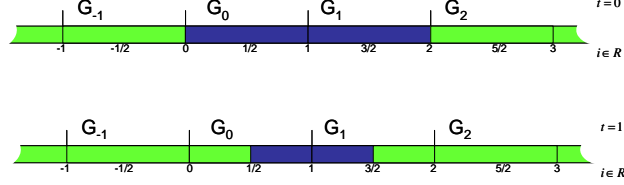

Figure 11: In period $t = 0$, Groups $G_0 = [0, 1)$ and $G_1 = [1, 2)$ are Egoists groups (dark grey), the remainder groups are Altruists group (light grey). In period $t = 1$, if conditions $A1$ holds, both groups (G_0, G_1) will be in the same situation, e.g. ,the group $G_0 = [0, 1)$ has a half $[0, \frac{1}{2})$ being Altruists and the other half $[\frac{1}{2}, 1)$ being Egoists

Condition $C1$: Altruists want to continue being Altruists. The average payoffs observed by the Altruists who belong to $(0, \frac{1}{2})$ at $t = 1$,see Figure 11, will be:

$$
\bar{\Pi}_{t=1}^A(i) = \frac{1}{\frac{1}{2} + f(i)} \left(\int_{-f(i)}^{0} (1 - c) \, dw + \int_{0}^{\frac{1}{2}} (\frac{1}{2} - f(w) - c) \, dw \right)
$$

$$
\bar{\Pi}_{t=1}^E(i) = \frac{1}{\frac{1}{2} - f(i)} \int_{\frac{1}{2}}^{1-f(i)} (\frac{1}{2} - f(w)) \, dw
$$

The condition required for them to continue being Altruists will be:

$$
\bar{\Pi}_{t=1}^A(i) - \bar{\Pi}_{t=1}^E(i) = -c + \frac{f(i)}{1+2f(i)} + \frac{1}{\frac{1}{2}+f(i)} \int_{0}^{\frac{1}{2}} f(w) \, dw + \frac{1}{\frac{1}{2}-f(i)} \int_{f(i)}^{\frac{1}{2}} f(w) \, dw > 0 \quad \text{for all} \ \ i \in (0, \frac{1}{2})
$$

$$(C1)$$

As we can see, that condition is equal to $A3$, $C1 \equiv A3$

Condition $C2$: Observer Egoist of altruism want to change to altruism.
The average payoffs observed by the Egoists who belong to $(\frac{1}{2}, 1)$ will be:

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$$\bar{\Pi}_{t=1}^{A}(i) \quad = \quad \frac{1}{\frac{1}{2} - f(i)} \int\limits_{f(i)}^{\frac{1}{2}} (\frac{1}{2} + f(w) - c) \ dw$$

$$\bar{\Pi}_{t=1}^{E}(i) \quad = \quad \frac{1}{\frac{1}{2} + f(i)} \int\limits_{\frac{1}{2}}^{1+f(i)} (\frac{1}{2} - f(w)) \ dw$$

The condition required for them to change to altruism is:

$$\bar{\Pi}_{t=1}^{A}(i) - \bar{\Pi}_{t=1}^{E}(i) = -c + \frac{1}{\frac{1}{2} - f(i)} \int\limits_{f(i)}^{\frac{1}{2}} f(w) \ dw + \frac{1}{\frac{1}{2} + f(i)} \int\limits_{\frac{1}{2}}^{1+f(i)} f(w) \ dw > 0 \quad \text{for all} \ \ i \in (0, \tfrac{1}{2})$$

(C2)

Using the linear cohesion function given by (8), the condition $C2$ is equivalent to:

$$c < \frac{x + 2x^2 - 4x^3}{1 + 2x}$$

The expression $\frac{x+2x^2-4x^3}{1+2x}$ is concave and has a maximum at the point $\tilde{x} \cong 0.427$, so we have a situation that is similar to Condition $A3$ when we consider a linear cohesion function.

In fact, the previous expression is more restrictive that the condition $A3$ and $A1$ when we consider a linear cohesion function. It is straightforward to prove that if $c < \frac{x+2x^2-4x^3}{1+2x}$, then $c < \min\{1 - \frac{3x}{2}, x, \frac{2x(1-x^2)}{1+2x}\}$ we can see this by plotting those expressions, see Figure 12.

Therefore, if we have any initial population with homogeneous groups, the scenarios A, B and C that we have considered reflect any situation the individuals of those groups could face during the first period. We can therefore state the following proposition:

Proposition 5 *Let $f(i)$ be a cohesion function, LR^M the learning rule and $d = \frac{1}{2}$ in any initial population of homogeneous groups of which at least one group is Altruistic. If $A1$, $A3$ and $C2$ hold, then altruism will spread throughout the entire population.*

Corollary 6 *Let $f(i)$ be a linear cohesion function, LR^M the learning rule and $d = \frac{1}{2}$ in any initial population of homogeneous groups of which at least one group is Altruistic. If $c < \frac{x+2x^2-4x^3}{1+2x}$, then altruism will spread throughout the entire population.*

For any group homogeneous initial state with at least one group of Altruists, we know what will happen, in the long run, in two regions of Figure 13 (linear case):
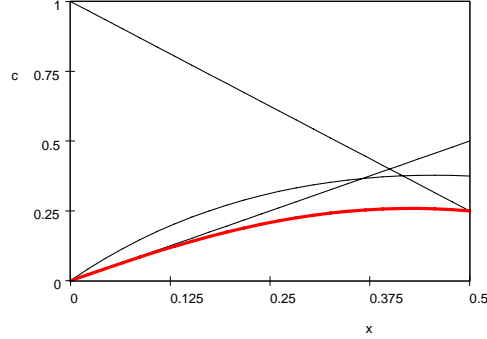
Figure 12: The expression $\min\left\{\frac{x+2x^2-4x^3}{1+2x}, 1-\frac{3x}{2}, x, \frac{2x(1-x^2)}{1+2x}\right\} = \frac{x+2x^2-4x^3}{1+2x}$ has a maximum at the point $\tilde{x} \cong 0.427$, so that the highest cost c that is compatible with the spreading of altruism is only achievable with an intermediate level of cohesion.

- In the region where $C2$ holds (i.e., in the linear case when $c < \frac{x+2x^2-4x^3}{1+2x}$), altruism eventually spreads throughout the population. Furthermore, an intermediate level of group cohesion is the best scenario for the spread of altruism.

- In the region where $A2'$ holds (i.e., in the linear case when $c > 1 - \frac{5}{4}x$) egoism eventually spreads throughout the population.

In the other region (Region III in Figure 13), however, we have no idea of what will happen in the long run, so we shall perform some simulations to verify whether our sufficient conditions are also necessary conditions. Before doing the simulations, however, we first repeat the previous analysis for the other learning rule.
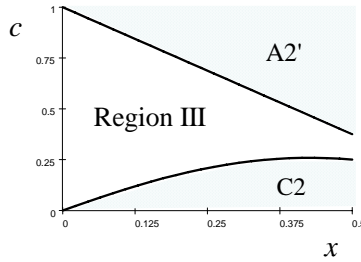

Figure 13: We already know what happens in the regions where either $C2$ or $A2'$ hold, but regarding the other region (Region III), however, we have no idea of what happens in the long run.

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3.2 The learning rule LR^P.

We now assume the agents uses the learning rule LR^P, as defined in 7.

In repeating the previous analysis, we obtain the analogous sufficient conditions when the learning rule compares average pay-off with one's own. The relationships among those conditions[11] are, under the learning rule LR^P the following:

$$A1^P \equiv A2^P \equiv B1^P \qquad and \qquad A3_a^P \equiv A3_e^P \equiv B2^P \equiv C1^P \equiv C2^P$$

Where the conditions $A1^P$ and $A3^P = A3_a^P$ are:

$$\bar{\bar{\Pi}}_t^A(i) - \bar{\bar{\Pi}}_t^E(i) = 1 - c - f(i) - \frac{1}{f(i)} \int_0^{f(i)} f(w)\, dw > 0 \quad \text{for all} \ \ i \in (0, \tfrac{1}{2}) \ \ (\text{A1}^{\text{P}})$$

$$\bar{\bar{\Pi}}_t^A(i) - \bar{\bar{\Pi}}_t^E(i) = -c + f(i) + \frac{1}{\frac{1}{2} - f(i)} \int_{f(i)}^{\frac{1}{2}} f(w)\, dw > 0 \quad \text{for all} \ \ i \in (0, \tfrac{1}{2}) \ \ (\text{A3}^{\text{P}})$$

Using the linear cohesion function given by (8), the condition $A1^P$ and $A3^P$ are respectively equivalent to:

$$c < 1 - 2x + x^2$$

$$c < \frac{x}{2}$$

Where the second one is more restrictive than the first one.

Therefore, with this learning rule we obtain the following propositions:

Proposition 7 *Let $f(i)$ be a cohesion function, LR^P the learning rule, $d = \frac{1}{2}$ and any initial group homogeneous population with at least one Altruist group. If $A1^P$ and $A3^P$ hold , then altruism will spread throughout the entire population.*

Corollary 8 *Let $f(i)$ be a linear cohesion function, LR^P the learning rule, $d = \frac{1}{2}$ and any initial group homogeneous population with at least one Altruist group. If $c < \frac{x}{2}$ then the altruism spreads to the whole population.*

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[11]Where, e.g. $A1^P$ is equivalent to Condition $A1$, but we consider the learning rule LR^P instead of LR^M.

4 Simulation

The purpose of simulations is to check the robustness of the analytical results, first we examine if our sufficient conditions for spreading Altruism are also necessary conditions. After that, we give up the assumption of considering populations of homogeneous groups as initial states. Thus, we consider a broader set of initial states, where there could be Altruists and Egoists in the same group at $t = 0$.

To make simulations we use a discrete version of the theoretical model. We consider a ring in which there are K groups with N individuals each, giving us a total of KN agents in the ring. Moreover, there are no agents at the boundaries and N is an even number. The distance between two adjacent agents is $e = \frac{1}{N}$.

4.1 Are our sufficient conditions also necessary conditions?

In the previous sections, we have identified sufficient conditions for the persistence and spread of altruism. We could ask ourselves, however, whether such conditions are also necessary. In other words, what happen in Region III of Figure 13.

As initial state, we consider a population with just one group of Altruists among other groups of Egoists, and a linear cohesion function with $d = \frac{1}{2}$.

Numerical analysis was conducted for a sizable subset of $(c, x) \in \{\frac{1}{50}, \frac{2}{50}, ..., \frac{50}{50}\} \times \{\frac{1}{50}, \frac{2}{50}, ..., \frac{25}{50}\}$, if $x = \frac{25}{50} = \frac{1}{2}$ the groups then have minimal cohesion and if $x = \frac{1}{50}$ they then have maximal cohesion. After specifying values for (c, x), we then ran the system until convergence was achieved. In the long run, there are three possible outcomes: the population may become either totally altruistic or entirely egoistic, or both types of behavior may persist.

In the following two sections, we present the results of this simulation for the two different learning rules.

4.1.1 Linear cohesion function, $d = \frac{1}{2}$ and the learning rule LR^M

As we can see in Figure 14 we obtained results that were quite similar to the ones previously obtained in the analytic approach, see Figure 9.

As we consider an even number of groups, the sufficient conditions for the spreading of altruism will be given by the expression $c < \min\{1 - \frac{3x}{2}, x, \frac{2x(1-x^2)}{1+2x}\}$, see Figure 9. Theoretical analysis predicts that in the region given by this expression the Altruism spread throughout the entire population[12].

In the results of the simulation, we can see what happens in the region that we did not study in the analytical approach. In the long run, we have both kinds of behavior, altruism and egoism, in this region, see Figure 14. Altruism is able to persist but it never dominates. Therefore, *we verify that our sufficient conditions are also necessary conditions for the spreading of altruism throughout the entire population*, in this case.

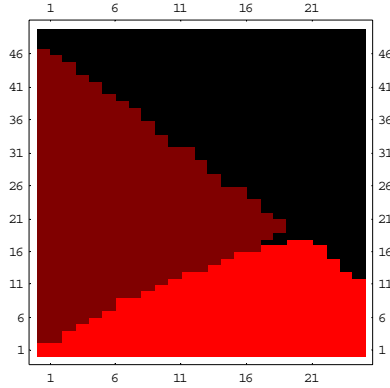[12] We also now that the population becomes egoistic in $c < 1 - \frac{5x}{2}$.

Figure 14: With $K = 4$ and $N = 40$. The vertical axis represent the Altruist's cost $c \in \{\frac{1}{50}, \frac{2}{50}, ..., \frac{50}{50}\}$ and the horizontal axis represents the level of cohesion $x \in \{\frac{1}{50}, \frac{2}{50}, ..., \frac{25}{50}\}$, when $x = \frac{1}{2}$ minimal cohesion and $x = \frac{1}{50}$ maximal cohesion. In the axes only appear the numerator of the value, e.g. if $x = \frac{1}{50}$ then in the axis appears the number 1. Black color represents that the whole population under this conditions (c and x values) becomes egoistic in the long run, light grey altruistic, the other color (dark grey) both behaviors persist.

4.1.2 Linear cohesion function, $d = \frac{1}{2}$ and the learning rule LR^P

Regarding the previous section we only changed the learning rule, instead of LR^M we used LR^P, see Figure 15.

In that case, we also obtained results that were quite similar to the ones previously obtained in the analytic approach. Moreover, the interpretation of the previous simulation is equally applicable to this one.

4.2 A broader set of initial states.

In the previous sections we have studied the long-run behavior for a subset of initial states, i.e., we have only considered populations made up of homogeneous groups as initial states. We could verify the robustness of our previous results by considering any initial state, i.e., a population that can have heterogeneous groups (with both Altruists and Egoists in each group), into this population we can then plant just one homogeneous group of Altruists like a seed.

Thus, when considering a heterogeneous population, if a group becomes altruistic or if a homogeneous group of Altruists is added to such a population, we might well ask ourselves:

Will altruism persist as it did in the previous analysis? Would it spread throughout the entire population? And if the answer is yes, what is the level of group cohesion better for that? In short, we will know if the previous analytical results hold in a broader initial setting.
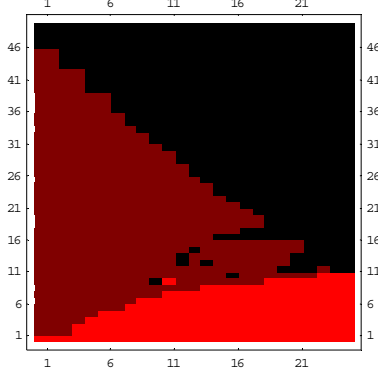
Figure 15: This figure has the same interpretation as Figure14, however, we now consider the learning rule LR^P instead of LR^M.

To this aim, we consider the initial state of a heterogeneous population with just one group of Altruists and with the behavior of the remaining individuals in the others groups being randomly chosen (at $t = 0$). Thus, there will be groups with agents following different behaviors, heterogeneous groups, in the initial state.

Numerical analysis was conducted for a sizable subset of $(c, x) \in \{\frac{1}{25}, \frac{2}{25}, ..., \frac{25}{25}\} \times \{\frac{1}{40}, \frac{2}{40}, ..., \frac{20}{40}\}$, if $x = \frac{20}{40} = \frac{1}{2}$ the groups then have minimal cohesion and if $x = \frac{1}{40}$ they then have maximal cohesion. After specifying values for (c, x), we then ran the system until convergence was achieved. In the long run, there are three possible outcomes: the population may become either totally altruistic or entirely egoistic, or both types of behavior may persist. We repeat the experiment 100 times for each pair of $\{c, x\}$, with different randomly chosen initial states. We then compute the rate of the three different possible situations in the long run. We obtain three values (v_a, v_m, v_e) for each pair $\{c, x\}$, with $v_a + v_m + v_e = 1$ and $v_i \in (0, 1)$ for $i = a, e, m$. These ratios can be taken as an estimation of the size of the three different basins of attraction, e.g. $(v_a, v_m, v_e) = (0.8, 0.03, 0.17)$ the system becames entirely altruistic 80 times in the simulation, entirely egoistic 17 times and mixed 3 times.

In the following two sections, we present the results of this simulation for the two different learning rules.

4.2.1 Linear cohesion function, $d = \frac{1}{2}$ and the learning rule LR^M

In this section we consider a linear cohesion function with $d = \frac{1}{2}$, the learning rule LR^M, $K = 3$ and $N = 20$.

As there are three values linked to each pair (c, x), reflecting the results of the simulation, we show one figure for each value. Thus, we have three figures. The first one represents v_a, i.e., the ratio of times the system became entirely altruistic, see Figure 16. The second one represents v_m, i.e., the ratio of the

times the system maintains a mixed behavior, see Figure 17. The third one represents v_e, i.e., the ratio of the times the system became entirely egoistic, see Figure 18.

As we can clearly see, an intermediate level of cohesion is the scenario that is most conducive to Altruism. Thus, the highest cost, c, that is compatible with the spreading of altruism is achieved with an intermediate level of cohesion. Therefore, the results are quite similar to those of the analytical approach, which means that our results are robust enough.
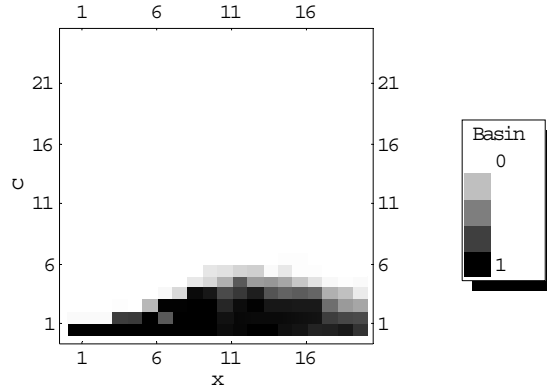
Figure 16: The vertical axis represents the Altruist's cost $c \in \left\{\frac{1}{25}, \frac{2}{25}, ..., \frac{25}{25}\right\}$ and the horizontal axis represents the level of cohesion $x \in \left\{\frac{1}{40}, \frac{2}{40}, ..., \frac{20}{40}\right\}$, when $x = \frac{20}{40} = \frac{1}{2}$ minimal cohesion and when $x = \frac{1}{40}$ maximal cohesion. In the axes only appear the numerator of the value, e.g. if $x = \frac{1}{40}$ then in the axis appears the number 1. The figure represents v_a the number of times the system became entirely altruistic in the long run. The black zones in the graph, $v_a = 1$, mean that under these conditions (those c and x values) the system always became entirely altruistic, i.e. the 100 times that we ran the system. The white zones, $v_a = 0$, mean that it never became entirely altruistic under these conditions (c, x). The greater v_a is, therefore, the darker the shade of grey and the system became entirely altruistic more frequently in our simulations. As we can clearly see, an intermediate level of cohesion is the scenario that is most conducive to Altruism. Thus, the highest cost, c, that is compatible with the spreading of altruism is achieved with an intermediate level of cohesion.
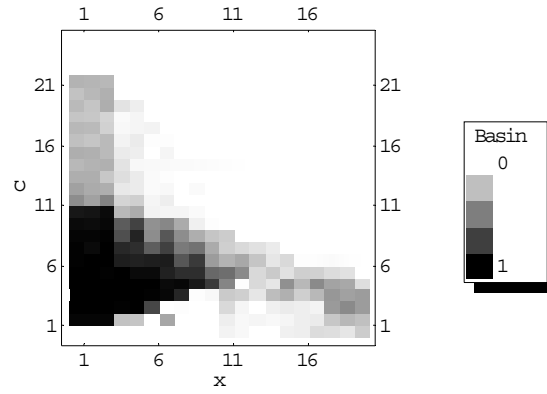
Figure 17: It represents v_m the number of times the system permits both types of behaviour, altruistic and egoistic, to persist in the long run. The black zones in the graph, $v_m = 1$, mean that under these conditions (the c and x values) the system always converged to an heterogeneous state with altruist and egoists, i.e., the 100 times that we ran the system. The white zones, $v_a = 0$, mean that it never converged to a mixed or heterogeneous state in the long run under these conditions (c, x), in that case, the system became entirely altruistic or egoistic. The greater v_a is, therefore, the darker the shade of grey and the system converge to a mixed or heterogeneous state more frequently in our simulations.
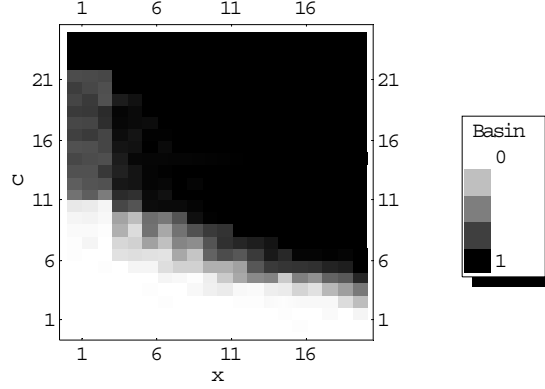
Figure 18: It represents v_e the number of times the system became entirely egoistic in the long run. The black zones in the graph, $v_e = 1$, mean that under these conditions (the c and x values) the system always became entirely egoistic, i.e., the 100 times that we ran the system. The white zones, $v_e = 0$, mean that it never became entirely egoistic under these conditions (c, x). The greater v_e is, therefore, the darker the shade of grey and the system became entirely egoistic more frequently in our simulations.

4.2.2 Linear cohesion function, $d = \frac{1}{2}$ and the learning rule LR^P

We shall now repeat the exercise, but with a change of learning rule, we consider LR^P instead of LR^M. See Figures 19, 20, 21.

In conclusion, with previous simulations, we have therefore verified that our results are robust in broader settings, that an intermediate level of cohesion seems to be the best scenario for the fostering of altruism, i.e., the concept of "the group" or a certain division in the population fosters altruism, although such division or fragmentation should be not too great.

On the other hand, we observe that the learning rule LR^P is somewhat less favorable for altruism, since the weight that an individual's own payoff has in this rule is greater than in LR^M. Note that under LR^M, the individual's own payoff has the same weight in the learning rule as any other payoff in the neighborhood has.
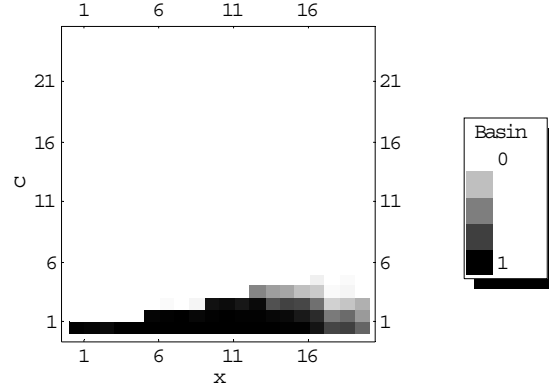
Figure 19: It represents v_a the number of times the system became entirely altruistic in the long run. This figure has the same interpretation as the figure 16. As we can clearly see, an intermediate level of cohesion is the scenario that is most conducive to Altruism.
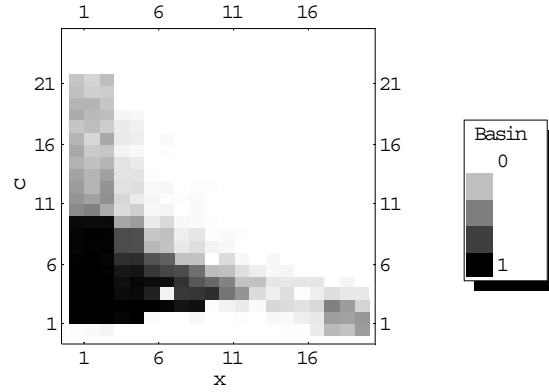

Figure 20: It represents v_m the number of times the system permitted both types of behaviour, altruistic and egoistic, to persist in the long run. This figure has the same interpretation as the figure 17.
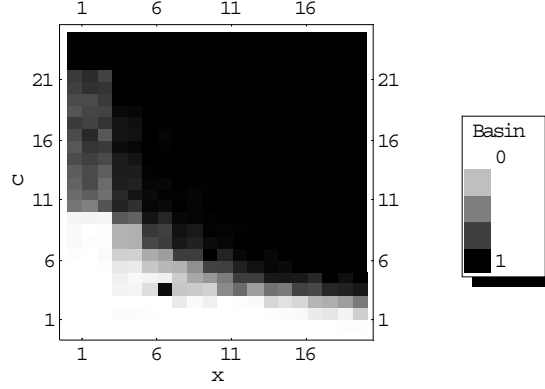
Figure 21: It represents v_e the number of times the system became entirely egoistic in the long run. This figure has the same interpretation as the figure 18.

4.3 Non-Linear cohesion function with $x = \frac{1}{2}$, $d = \frac{1}{2}$ and the learning rule LR^M.

In addition, we also examine what happens when we consider a non-linear cohesion function.

For example, the function $f(h) = x(1 - \frac{h^n}{d^n})$ with $n \in (0,1)$. If $n = 1$ then we have a linear cohesion function. The smaller n is the more convex $f(i)$ will be. As such, if n decreases, the cohesion increases, see Figure 22. In order to do the simulations, we take $x = \frac{1}{2}$ and $d = \frac{1}{2}$ and use the parameter n as a measure of group cohesion. The smaller the parameter n is, the greater the group cohesion is , with $\in (0,1)$.

We consider as initial state a population with only one group of Altruists surrounded by egoistic groups. Numerical analysis was conducted for a sizable subset of $(c,n) \in \{\frac{1}{50}, \frac{2}{50}, ..., \frac{50}{50}\} \times \{\frac{1}{50}, \frac{2}{50}, ..., \frac{50}{50}\}$. After specifying values for (c,n) we then ran the system until convergence was achieved. In the long run, there are three possible outcomes: the population may become either totally altruistic or entirely egoistic, or both types of behavior may persist.

As we can see from Figure 23, we obtain similar results to those obtained for the linear case. An intermediate level of cohesion is the best scenario for the spreading of altruism.
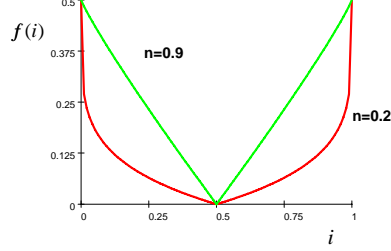
Figure 22: We represent two non-linear cohesion functions $(f(h))$ both of them with $x = \frac{1}{2}$ and $d = \frac{1}{2}$. However, one of the function with $n = 0.2$ and the other one with $n = 0.9$. The latter one is almost a linear cohesion function. The vertical axis represents the value of the cohesion function and the horizontal axis represents (without loss of generality) the individuals of group G_0, i.e., the interval $[0, 1)$.
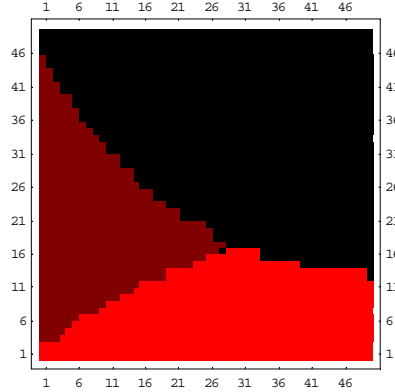

Figure 23: With $K = 4$ and $N = 40$. The vertical axis represent the Altruist's cost $c \in \{\frac{1}{50}, \frac{2}{50}, ..., \frac{50}{50}\}$ and the horizontal axis represents the level of cohesion $n \in \{\frac{1}{50}, \frac{2}{50}, ..., \frac{50}{50}\}$, when $n = \frac{50}{50} = 1$ minimal cohesion and when $n = \frac{1}{50}$ maximal cohesion. In the axes only appear the numerator of the value, e.g. if $n = \frac{1}{50}$ then in the axis appears the number 1. Black color represents that the whole population under this conditions (c and n values) becomes Egoistic in the long run, light grey Altruistic, the other color (dark grey) both behaviors persist.

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5 CONCLUSIONS

In this paper we have introduced and parameterized the concept of "group cohesion" in a model of local interaction with a population divided into groups. This allows us to control the level of "isolation" of these groups. We thus analyze if the degree of group cohesion is relevant to achieve an efficient behavior and which level would be the best one for this purpose. We are interested in situations where there is a trade off between efficiency and individual incentives. This trade off is stronger when the efficient strategy or norm is strictly dominated, as in the Prisoner's Dilemma or in some cases of Altruism. In our model we have considered that agents could choose to be Altruist or Egoist, in fact, they behave as in Eshel, Samuelson and Shaked (1998) model.

We find that:

- The concept of group cohesion plays and important role even in local-interaction models.

- We verify that an *intermediate level of group-cohesion* is the best scenario for the spread of altruism, better than the classical local-interaction scenario in which groups are inessential, and better than a scenario with almost-isolated groups (i.e., high group cohesion). Therefore, a certain level of fragmentation of the population into groups favours the persistence and spread of altruism.

The agents behave in our model as in the model of Eshel, Samuelson and Shaked (1998). However, we could have considered other situations where this trade off between efficiency and individual incentives arises, as in a Prisoner Dilemma or in some coordination games (where there is a trade off between the efficiency strategy and the "risk dominant" strategy). We have the intuition that a similar results would appear, but we will try to prove that point in future research.

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