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ABSTRACT

This paper proposes a dominance approach to study well-being inequality across countries at the world level. We consider a class of well-being indices based on the three attributes considered in the HDI (Human Development Index). Indices are required to satisfy preference for egalitarian marginal distributions of income, health and education, inclination for less correlation between attributes and priority to poor countries for allocating funds to improve health and education. We exhibit sufficient conditions which are easy to implement to check dominance over the defined class of well-being indices.

Keywords: Multidimensioned Welfare; Multivariate Inequality, Well-Being Dimensions, Human Development Index

JEL Codes: O15 (Income Distribution), D31 (Personal Income Distribution)

1 Introduction

When comparing countries according to some measure of well-being, the attention has shifted from an assessment based solely on per capita GDP to one involving several attributes like life expectancy, literacy rates, mortality and morbidity statistics and other various socioeconomic indicators in addition to income per capita. Appraising inequality of the distribution of well-being across countries raises difficulties in a multidimensional context. In a casual approach, the information provided by the attributes describing a country situation is aggregated in some indicator, which is tantamount to the classical problem of assessing inequality in a one-dimensional setting. For instance, the best known measure of well-being the HDI (Human development index) published by UNDP in their Human Development Report from 1990 to date is based on three attributes: life expectancy at birth, educational attainment and real GDP per capita. The HDI[1] is obtained by placing each country on a scale of 0 to 100 with respect to any attribute and by computing a simple arithmetic mean of the attributes. But there is no uncontroversial way of aggregating carrots and potatoes. It would be more sensible to face the intrinsic multi-dimensional feature of the problem and to agree on some qualitative properties that a "well-behaved index" should satisfy. In doing so, on the one hand we leave some room for disagreement and on the other hand we must cope with a class of well-being indices[2] rather than with a unique index. This kind of problem is common in inequality analysis. The solution brought by the social dominance approach pioneered by Kolm (1969), Atkinson (1970) and Sen (1973) is to propose some comparison criteria for distributions taking into account the indeterminacy of some aggregating indicator – the utility function in the case of individuals and the well-being index in the case of countries.

Dominance analysis consists in seeking unanimity among large classes of social welfare functions over the ranking of pairs of allocations. The social welfare function, here expressed at the world level is additively separable with respect to well-being indicators defined at the country level. In short, we propose a multi-dimensional dominance analysis applied to well-being indicators of the type of those composing the HDI. Of course, in doing so we accept to be sometimes unable to make a comparison since the obtained criteria are incomplete. Nevertheless, when we can reach a conclusion, the result is robust and cannot be easily dismissed.

The literature on multi-dimensioned welfare analysis can be traced back to Kolm's paper (1977), where every attribute is considered symmetrically. There has been a bunch of papers devoted to this topic (see e.g. Marshall and Olkin (1979, chapter 15), Atkinson and Bourguignon (1982), Le Breton (1986), Koshevoy (1995, 1998), Koshevoy and Mosler (1996), Moyes (1999) and Bazen and

[1] Each of these dimensions is normalized by a simple procedure. For instance, a number at x for GDP, means that the log GDP of the country is at $x\%$ between two reference values the min and the max of log GDPs in the pool of countries registered on a given period. This normalization raises some difficulties which are touched on at the end of the paper.

[2] See, for instance the proposition made by Foster *and al.*(2003).

Moyes (2002). In particular, Atkinson and Bourguignon (1982) propose dominance relationships for various classes of utilities defined by their derivatives up to the fourth order. Nevertheless, it seems fair to say that no simple criterion has reached popular support among applied economists and even among theoretical ones to check multi-dimensional dominance.

Here we consider a three-variables distribution problem, which is rather uncommon since the main bulk of the literature has limited its attention to the case of a bivariate distribution. Section 2 presents the considered class of well-being indicators. Section 3 provides the result. Finally, Section 4 concludes.

2 A Class of Well-Being Indicators

It is assumed that well-being at a country level is based on the three attributes considered in the HDI: per capita GDP, life expectancy at birth and educational attainment. These variables[3] are supposed to be continuous and we denote the joint cumulative distribution function of these three indicators across countries: $F(x_1, x_2, x_3)$, where x_1 is per capita GDP, x_2 is life expectancy at birth, and x_3 is educational attainment. In the unweighed case, F is simply the cdf of the distribution at the country level; To make the proofs simpler, we assume that F admits a density[4] f defined on a finite support $X_1 \times X_2 \times X_3$ with $X_1 = [0, a_1]$, $X_2 = [0, a_2]$ and $X_3 = [0, a_3]$. Using the same notations as Atkinson and Bourguignon (1982), we define the social welfare function computed at the world level as

$$W_I := \int_0^{a_1} \int_0^{a_2} \int_0^{a_3} I(x_1, x_2, x_3)\, f(x_1, x_2, x_3) dx_1 dx_2 dx_3,$$

where I is the well-being function at the country level assumed to be continuously differentiable to the required degree. The partial derivatives with respect to each variable are denoted by subscripts. The change in welfare between two probability distributions f and f^* is

$$\triangle W_I := \int_0^{a_1} \int_0^{a_2} \int_0^{a_3} I(x_1, x_2, x_3) \triangle f(x_1, x_2, x_3) dx_1 dx_2 dx_3 \tag{1}$$

where $\triangle f$ denotes $f - f^*$.

Dominance is usually defined as unanimity for a family of social welfare functions based on a specific set of well-being functions.

Definition 1 *f dominates f^* for a family $\mathcal{I}$ of well-being functions if and only if $\triangle W_I \geq 0$ for all well-being functions I in $\mathcal{I}$. This is denoted $f\ D_{\mathcal{I}}\ f^*$.*

[3] These three variables are supposed to have a cardinal meaning, namely, they are defined up to any linear transformation.

[4] This assumption is made for mathematical convenience. In another paper, Muller-Trannoy (2003), we prove that similar results can be obtained if we drop this assumption.

The considered class of well-being functions is the following:

$$\mathcal{I} = \left\{ I : X_1 \times X_2 \times X_3 \to 211d \text{ such that } \begin{array}{c} I_1, I_2, I_3 \geq 0; \\ I_{11} < 0, I_{22} < 0, I_{33} < 0; \\ I_{12} \leq 0, I_{13} \leq 0, I_{23} = 0; \\ I_{121} \geq 0, I_{131} \geq 0 \end{array} \right\}$$

With this class, the marginal well-being induced by an increase in any attribute is supposed to be identical across countries. This feature captures a requirement of anonymity at the country level in the unweighed interpretation and at the individual level in the weighed interpretation. Thus, a 1$ increase of the per capita GDP in Zimbabwe is treated on an equal footing as a 1$ increase of the per capita GDP in China, irrespective of the demographic or economic size of these countries.

The marginal well-being is positive and decreasing in each dimension. Under this condition, each attribute is good for welfare and there is a social preference at the world level for more egalitarian marginal distributions. Dominance for concave and increasing utility functions in a one dimension setting is known to be equivalent to the Generalized Lorenz test introduced by Shorrocks (1983). Therefore, a more egalitarian distribution according to the Generalized Lorenz test in each dimension appears to be a prerequisite for accepting that welfare has improved at the world level. If the distributions of each attribute are independent, this requirement will be sufficient to yield a reasonable criterion. But we know that our three attributes of interest are positively correlated. Hence, some assumptions are needed to convey the intuition that a reduction in the statistical links of these variables improves welfare.

We capture the latter idea for per capita GDP and life expectancy at birth by imposing a negative sign on the second partial cross-derivative with respect to these two attributes. The same is required for the second partial cross-derivative of the well-being function with respect to per capita GDP and education attainment. Strictly speaking, these assumptions indicate that the marginal increase in well-being associated with a rise in income per capita is decreasing with the level of the other two variables. They imply that when comparing two equally poor countries with the same per capita GDP competing for international aid, priority should be given to the country with the lowest health or educative performance. A consequence of this is that transferring money from rich countries to poorer countries with worse health or worse education standards is good for welfare. After such transfers, the distribution of countries according to income per capita will be less correlated to the distribution of countries according to one or the two other attributes. Another vindication of this assumption is the equivalence between assuming that income per capita and health (or education) are substitutes in well-being. Income can compensate for deficiencies in health and education.

Finally, the nullity of I_{23} means that the well-being function is additively separable with respect to life expectancy at birth and educational attainment. Indeed for the purpose of welfare analysis, it seems sensible, at least in first approximation, to consider that the marginal gain in welfare of one year of life expectancy at birth is not affected by the level of education. In other words, if it can be defended that income can compensate for a bad health or a bad education, it is less palatable to impose that a good health is a substitute for a bad education or the opposite. Of course, it can be argued that the better educated you are, the more able you are to appreciate life but introducing such an assumption would imply some undesirable conclusion: an international program transferring resources to improve health conditions would have to be targeted to countries with the best educational level!

Another argument for ignoring the interactions of life expectancy and education in the well-being aggregate indicator is that compensatory transfers are usually thought to be implemented with money, and not with units of education or units of health. In that sense, one may want to ignore the interactions of life expectancy and education in the well-being index because one cannot directly compensate for their differences independently of monetary transfers. To close the discussion of this assumption, let us observe that the HDI which is additively separable respects this assumption. Under these conditions, the index can be written in the following form, with φ and ψ two three time differentiable functions

$$I(x_1, x_2, x_3) = \varphi(x_1, x_2) + \psi(x_1, x_3) \tag{2}$$

Even under this separability structure, it is unlikely that one can obtain dominance criteria with substantial discriminatory power without adding further assumptions. This justifies the introduction of the last two assumptions of positivity of a couple of third partial cross-derivatives. The assumptions are best understood if we note that minus the value of I_{12} (respectively I_{13}) can be interpreted as an index of priority to international aid for compensating low life expectancy (respectively education attainment). Then requiring that minus I_{121} (respectively minus I_{131}) is negative implies that the priority index decreases with the country income per capita. In other words, the countries with the highest claim to international aid for compensating health conditions are the poorest ones, a somewhat reasonable request. In contrast, it is well-known that despite a very high share of GDP devoted to health expenses, the US is far from being the leader in life expectancy among the OECD countries. Improving health conditions in this country through a programme of international aid does not seem to command a widespread support, consistently with our hypotheses.

3 The Result

In performing the integration, it is convenient to define the marginal distribution $F_1(x_1) = \int_0^{a_3}\int_0^{a_2}\int_0^{x_1} f(s, r, t)\, ds dr dt$ and the corresponding expressions for $F_2(x_2)$ and $F_3(x_3)$. Furthermore, we define some second order stochastic dominance terms:

$H_i(x_i) := \int_0^{x_i} F_i(s)ds, \qquad i = 1,2,3.$ The $H_i(.)'s$ describe the usual univariate second order stochastic dominance terms, as applied to the marginal distributions.

$H_i(x_i; x_j, x_k) := \int_0^{x_i} F(r, x_j, x_k)dr$ and $H_i(x_i; x_j, a_k) := H_i(x_i; x_j)$ for any i, j, k. These $H_i(x_i; x_j, x_k)'s$ describe univariate second order stochastic dominance terms for the distribution of the x_i restricted to the virtual sub-populations such that the second argument of the utility has value x_j and the third argument has value x_k.

The following proposition gives sufficient conditions to check dominance according to class $\mathcal{I}$.

Proposition 1 *Let f and f^* be densities.*

$$\triangle H_i(x_i) \leq 0\,, \quad \forall x_i \in X_i \quad i = 2,3 \tag{A}$$
$$\Delta H_1(x_1; x_2) \leq 0, \forall x_i \in X_i, \ i = 1,2 \tag{B}$$
$$\Delta H_1(x_1; x_3) \leq 0, \forall x_i \in X_i, \ i = 1,3. \tag{C}$$
$$\Downarrow$$
$$f D_{\mathcal{I}} f^*$$

Proof. The argument proceeds by integration by parts. The changes in ranks of integrations with respect to the different variables are justified by Fubini theorem, which is not systematically signaled in the steps of the proof.

Integrating (1) by parts the inner integral with respect to x_1 gives

$$\Delta W_I = \int_0^{a_3}\int_0^{a_2} I(a_1, x_2, x_3)\left[\int_0^{a_1} \Delta f(x_1, x_2, x_3)dx_1\right] dx_2 dx_3 \tag{3}$$
$$-\int_0^{a_3}\int_0^{a_2}\int_0^{a_1} I_1(x_1, x_2, x_3)\left[\int_0^{x_1} \Delta f(s, x_2, x_3)ds\right] dx_1 dx_2\, dx_3. \tag{4}$$

Let us call T_1 the first term in the RHS term and T_2 the second term. It is convenient to treat separately these two terms. We start by integrating T_1 by parts with respect to x_3. We get

$$T_1 = \int_0^{a_2} I(a_1, x_2, a_3)\left[\int_0^{a_3}\int_0^{a_1} \Delta f(x_1, x_2, x_3)dx_1 dx_3\right] dx_2$$
$$-\int_0^{a_3}\int_0^{a_2} I_3(a_1, x_2, x_3)\left[\int_0^{x_3}\int_0^{a_1} \Delta f(x_1, x_2, x_3)dx_1 dt\right] dx_2 dx_3.$$

We now integrate by parts the RHS of the above expression with respect to x_2. We obtain

$$T_1 = I(a_1,a_2,a_3)\Delta F(a_1,a_2,a_3) - \int_0^{a_2} I_2(a_1,x_2,a_3)\Delta F(a_1,x_2,a_3)\,dx_2$$

$$- \int_0^{a_3} I_3(a_1,a_2,x_3)\Delta F(a_1,a_2,x_3)dx_3 + \int_0^{a_3}\int_0^{a_2} I_{32}(a_1,x_2,x_3)\Delta F(a_1,x_2,x_3)\,dx_2dx_3\,.$$

The first term in the RHS term is equal to zero because $F(a_1,a_2,a_3) = F^*(a_1,a_2,a_3) = 1$. Since $I_{32} = 0$, the last term vanishes. Finally, we integrate the second term of the RHS term of the above expression with respect to x_2 and the third term with respect to x_3. One yields

$$T_1 = -I_2(a_1,a_2,a_3)\Delta H_2(a_2;a_1,a_3) + \int_0^{a_2} I_{22}(a_1,x_2,a_3)\Delta H_2(x_2;a_1,a_3)\,dx_2$$

$$-I_3(a_1,a_2,a_3)\Delta H_3(a_3;a_1,a_2) + \int_0^{a_3} I_{33}(a_1,a_2,x_3)\Delta H_3(x_3;a_1,a_2)dx_3.$$

Let us now evaluate T_2. We start by integrating T_2 by parts with respect to x_3. We get

$$T_2 = -\int_0^{a_2}\int_0^{a_1} I_1(x_1,x_2,a_3)\left[\int_0^{a_3}\int_0^{x_1}\Delta f(s,x_2,x_3)dsdx_3\right]dx_1dx_2$$

$$+\int_0^{a_2}\int_0^{a_3}\int_0^{a_1} I_{13}(x_1,x_2,x_3)\left[\int_0^{x_3}\int_0^{x_1}\Delta f(s,x_2,t)dsdt\right]dx_1dx_3dx_2.$$

Integrating T_2 once more with respect to x_1 gives

$$T_2 = -\int_0^{a_2} I_1(a_1,x_2,a_3)\left[\int_0^{a_3}\int_0^{a_1}\int_0^{x_1}\Delta f(s,x_2,x_3)dsdx_1dx_3\right]dx_2$$

$$+\int_0^{a_2}\int_0^{a_1} I_{11}(x_1,x_2,a_3)\left[\int_0^{a_3}\int_0^{x_1}\int_0^{s_1}\Delta f(s,x_2,x_3)dsds_1dx_3\right]dx_1dx_2$$

$$+\int_0^{a_2}\int_0^{a_3} I_{13}(a_1,x_2,x_3)\left[\int_0^{x_3}\int_0^{a_1}\int_0^{x_1}\Delta f(s,x_2,t)dsdx_1dt\right]dx_3dx_2$$

$$-\int_0^{a_2}\int_0^{a_3}\int_0^{a_1} I_{113}(x_1,x_2,x_3)\left[\int_0^{x_3}\int_0^{x_1}\int_0^{s_1}\Delta f(s,x_2,t)dsds_1dt\right]dx_1dx_3dx_2.$$

Finally integrating T_2 with respect to x_2 one yields

$$\begin{aligned}
T_2 = &-I_1(a_1,a_2,a_3)\Delta H_1(a_1;a_2,a_3) \\
&+\int_0^{a_2} I_{12}(a_1,x_2,a_3)\Delta H_1(a_1;x_2,a_3)dx_2 \\
&+\int_0^{a_1} I_{11}(x_1,a_2,a_3)\Delta H_1(x_1;a_2,a_3)dx_1 \\
&-\int_0^{a_2}\int_0^{a_1} I_{112}(x_1,x_2,a_3)\Delta H_1(x_1;x_2,a_3)dx_1dx_2 \\
&+\int_0^{a_3} I_{13}(a_1,a_2,x_3)\Delta H_1(a_1;a_2,x_3)\,dx_3 \\
&-\int_0^{a_2}\int_0^{a_3} I_{123}(a_1,x_2,x_3)\Delta H_1(a_1;x_2,x_3)dx_2dx_3 \\
&-\int_0^{a_3}\int_0^{a_1} I_{113}(x_1,a_2,x_3)\Delta H_1(x_1;a_2,x_3)dx_1dx_3 \\
&+\int_0^{a_3}\int_0^{a_2}\int_0^{a_1} I_{1123}(x_1,x_2,x_3)\Delta H_1(x_1;x_2,x_3)dx_1dx_2dx_3.
\end{aligned}$$

Therefore, the expression for the change in welfare becomes

$$\begin{aligned}
\Delta W_I = &-I_1(a_1,a_2,a_3)\Delta H_1(a_1) - I_2(a_1,a_2,a_3)\Delta H_2(a_2) - I_3(a_1,a_2,a_3)\Delta H_3(a_3) \\
&+\int_0^{a_1} I_{11}(x_1,a_2,a_3)\Delta H_1(x_1)\,dx_1 + \int_0^{a_2} I_{22}(a_1,x_2,a_3)\Delta H_2(x_2)\,dx_2 + \int_0^{a_3} I_{33}(a_1,a_2,x_3)\Delta H_3(x_3)dx_3 \\
&+\int_0^{a_2} I_{12}(a_1,x_2,a_3)\Delta H_1(a_1;x_2)dx_2 + \int_0^{a_3} I_{13}(a_1,a_2,x_3)\Delta H_1(a_1;x_3)dx_3 \\
&-\int_0^{a_3}\int_0^{a_1} I_{113}(x_1,a_2,x_3)\Delta H_1(x_1;x_3)dx_1dx_3 - \int_0^{a_2}\int_0^{a_1} I_{112}(x_1,x_2,a_3)\Delta H_1(x_1;x_2)dx_1\,dx_2 \\
&-\int_0^{a_2}\int_0^{a_3} I_{123}(a_1,x_2,x_3)\Delta H_1(a_1;x_2,x_3)dx_2dx_3 \\
&+\int_0^{a_3}\int_0^{a_2}\int_0^{a_1} I_{1123}(x_1,x_2,x_3)\Delta H_1(x_1;x_2,x_3)dx_1dx_2dx_3.
\end{aligned}$$

Since the well-being function satisfies $I_{23} = 0$, and consequently $I_{123} = 0$ and $I_{1123} = 0$, the conclusion follows.■

The three families of conditions which guarantee the existence of a dominance relation are easy to be implemented. Conditions (A) are equivalent to check dominance according to the Generalized Lorenz test on the marginal distributions of life expectancy at birth and educational attainment. Moreover, one has to verify conditions (B) and (C) which are symmetric. Starting from the joint cdf, one only need to consider its value at the upper bound for the health variable (respectively educational variable) in the former (respectively latter) condition. Then, one need to perform a single integration with respect to the income per capita variable and the value of the integral must be smaller for the dominant distribution than for the dominated distribution for any couple of income per capita and life expectancy (respectively educational attainment) so as to satisfy the former (latter) condition. Note that the last two conditions imply that the Generalized Lorenz test is also satisfied for the per capita GDP marginal distribution.

It remains to touch on a difficulty raised by the normalization procedure implemented in order to compute the HDI. The normalization consists on comparing the value of each variable with respect to two indicators of reference: the minimum value registered by a country in the last 38 years, x_m, and the maximum value expected for the most advanced country during the following 25 years, x_M. Hence, for the health and education dimension, the normalized variable $\widehat{x}_i$ are given by

$$\widehat{x}_i = \frac{x_i - x_m}{x_M - x_m}, \qquad i = 2, 3.$$

For the income dimension, the formula provided by Anand and Sen (2000) is the following

$$\widehat{x}_1 = \frac{\log x_1 - \log x_m}{\log x_M - \log x_m}.$$

One may ask whether to check dominance conditions A, B and C on the non-transformed variables x_i is sufficient to have the same three conditions satisfied on the normalized variables $\widehat{x}_i$. It is easy established that the answer is positive when the normalized variables are a linear transformation of the initial ones, a condition which is violated in the income dimension. Therefore the normalized procedure matters for obtaining dominance results. Nevertheless, if such a normalization may be easily vindicated when one averages variables expressed in different units, its rationale is far less obvious in the framework of a dominance approach. Thus, a dominance approach should depart from the HDI type of indices.

4 Conclusion

In this paper, we propose a dominance approach to well-being inequality across countries at the world level. We consider a class of well-being indices based on the three attributes considered in the HDI: per capita GDP, life expectancy and educational attainment. We design conditions that suit these variables: preference for more equal marginal distributions of income, health and education; inclination for less correlation between attributes and priority to poor countries in allocating international aid to improve health and education.

We derive three families of sufficient conditions which are easy to implement in order to check dominance over the defined class of well-being indices. Checking and proving that these conditions are necessary are a matter for further research. However, we do not claim that the exhibited conditions apply in any three-dimensional context. For instance, the additive separability assumption between health and education, which perhaps seems reasonable here, may be too demanding in another setting. In that case, other appropriate conditions need to be designed to fit the context of interest.

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