TEMPORAL AGGREGATION OF AN ESTAR PROCESS: SOME IMPLICATIONS FOR PURCHASING POWER PARITY ADJUSTMENT *

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ABSTRACT

Nonlinear models of deviations from PPP have recently provided an important, theoretically well motivated, contribution to the PPP puzzle. Most of these studies use temporally aggregated data to empirically estimate the nonlinear models. As noted by Taylor (2001), if the true DGP is nonlinear, the temporally aggregated data could exhibit misleading properties regarding the adjustment speeds. We examine the effects of different levels of temporal aggregation on\ estimates of ESTAR models of real exchange rates. Our Monte Carlo results show that temporal aggregation does not imply the disappearance of nonlinearity and that adjustment speeds are significantly slower in temporally aggregated data than in the true DGP. Furthermore, the autoregressive structure of some monthly ESTAR estimates found in the literature is suggestive that adjustment speeds are even faster than implied by the monthly estimates.

Keywords: ESTAR, Real Exchange Rate, Purchasing Power Parity, Aggregation.

JEL classification: F31, C22, C51

1 Introduction

At the time of publication of Rogoff's (1996) influential survey on Purchasing Power Parity (PPP), almost without exception, the vast amount of empirical work reported was based on a linear framework. The empirical results concerning the mean-reverting properties of PPP deviations were contradictory. Whilst on the basis of standard unit root tests, the majority of studies reported that real exchange rate deviations were mean reverting, I(0) processes, a significant number of studies reported that they were described by non mean-reverting, I(1), processes. Other studies reported that PPP deviations were parsimoniously described by mean reverting fractional processes that exhibited the long memory associated with this process (see e.g., Diebold et al., 1991; and Cheung and Lai, 1993)[2].

Though the literature on this issue disagreed on the appropriate degree of integration of PPP deviations it did have a common feature. Short run PPP deviations were undoubtedly very persistent. In his review, Rogoff (1996) concluded that, on balance, the implied speed of adjustment of PPP deviations to shocks was "glacially slow". This fact constituted a puzzle given the half life of shocks was some 3-5 years, seemingly far too long to be explained by nominal rigidities.

Since 1996 a number of papers have reported empirical results which provide some explanation of the contradictory empirical results obtained when employing the linear methodology. In these papers PPP deviations are parsimoniously modelled employing two nonlinear models, namely the threshold model of Tong (1990), and the Exponential Smooth Autoregressive model (ESTAR) model of Ozaki (1985) (see e.g., Michael et al., 1997; Obstfeld and Taylor, 1997; Taylor et al., 2001; and Kilian and Taylor, 2003). Whilst globally mean-reverting these nonlinear processes have the property of exhibiting near unit root behavior for small deviations from PPP. This type of nonlinear adjustment process captures the adjustment derived in the theoretical analyses of PPP by a number of authors (see e.g., Dumas, 1992; Sercu et al., 1995; O'Connell, 1998; and Berka, 2002). These theoretical analyses of

[2]Fractional processes allow the degree of integration of a series to be non-integer. Standard unit root tests may exhibit low power against the fractional alternative. The fractional processes are given by members of the class of ARFIMA(p, d, q) processes, $x_t(1-L)^d = u_t$, where u_t is a stationary ARMA(p, q) process, and d is non-integer. The autocorrelations of the fractional process exhibit hyperbolic rather than geometric decay. For $0.5 \leq d < 1$ the process is non-stationary but mean reverting.

PPP demonstrate how transactions costs, transport costs or the sunk costs of international arbitrage induce nonlinear adjustment of the real exchange rate. Essentially small deviations from PPP are left uncorrected if they are not large enough to cover the costs of international arbitrage.

The nonlinear models can provide both an explanation of the contradictory results reported on the integration properties of PPP deviations as well as providing a different perspective on adjustment speeds. Taylor et al. (2001) and Pippenger and Goering (1993) demonstrate that the standard Dickey Fuller unit root tests have low power against data simulated from an ESTAR model. Byers and Peel (2000, 2003) show that data that is generated from an ESTAR process can appear to exhibit the fractional property. That this would be the case was an early conjecture by Acosta and Granger (1995). Given that the ESTAR model has a theoretical rationale, whilst the fractional process is, from an economic perspective, a relatively non intuitive process, the apparent fractional property of PPP deviations might reasonably be interpreted as a misspecification of a linear model relative to the DGP.[3]

An important property of the nonlinear models is that the impulse response functions derived from them show that, whilst the speed of adjustment for small shocks around equilibrium can be extremely slow, larger shocks mean-revert much faster than the "glacial rates" obtained in the linear estimates (see e.g., Taylor et al., 2001; and Paya et al., 2003). Consequently nonlinear models provide a theoretically well motivated and therefore important contribution to explaining the PPP speed of adjustment puzzle outlined in Rogoff (1996).

One issue not discussed in previous analysis is how temporal aggregation will impact on the adjustment speed obtained from estimates of non linear models. Taylor (2001) demonstrates that if the true data generating process is a nonlinear threshold process, and if the data employed in estimation is temporally aggregated, then linear estimates of adjustment speeds can be substantially downward biased. He does not consider the implications of temporal aggregation for estimates of adjustment speeds if the true data generating process is a nonlinear process but a nonlinear model is estimated on the temporally aggregated data. This issue seems to be of interest for two reasons. First, as noted by Taylor (2001), much of the data employed in PPP empirical work is temporally aggregated. Second, in the recent PPP literature estimates of nonlinear models have been reported employing data

[3]See Granger and Terasvirta (1999).

sampled at different levels of aggregation. For example, employing the ESTAR form, Baum et al. (2001), and Taylor et al. (2001) report results employing monthly data, Kilian and Taylor (2003) quarterly data whilst Michael et al. (1997), and Paya and Peel (2003a) report results employing annual data.[4] If the true data generating process (DGP) is nonlinear, it is clearly of interest to examine the nature of the nonlinear models that occur in temporally aggregated data. Conditional on the assumed DGP the results obtained may shed light on the appropriate frequency of the adjustment process in actual data. In addition, we can compare and contrast the estimated speeds of adjustment obtained in the DGP and the temporally aggregated process.[5]

These are the purposes in this article. We assume, based on a priori considerations, that at the very highest data frequency the true DGP is given by a particular, theoretically well motivated, ESTAR model. A Monte Carlo analysis is conducted where we estimate ESTAR models derived from this DGP at various degrees of temporal aggregation. The important point to emerge is that, for the particular DGP assumed, temporal aggregation does not lead, in general, to the disappearance of nonlinearity. In addition, the temporally aggregated models exhibit a different ESTAR form to that in the DGP and which is precisely that found in empirical estimates of quarterly and annual data. Given that the DGP is as postulated we can infer from estimates based on monthly data whether temporal aggregation has occurred with consequent implications for adjustment speeds. We find that adjustment speeds implied in the temporally aggregated data are much slower than the adjustment speeds obtained in the true DGP and that true adjustment speeds may be faster than those derived from monthly data, the highest frequency available to us.

The rest of the paper is organized as follows. In section 2 we set out the DGP for highest frequency data, our Monte Carlo methodology, and the effect of temporal aggregation on nonlinear estimates of an ESTAR model. Section 3 compares the Monte Carlo results with actual estimates. In section 4 we examine, employing nonlinear impulse response functions, the speeds of adjustment to shocks obtained in the DGP and the estimated temporally

[4] In related work on the monetary model of exchange rate determination Taylor and Peel (2000) estimate an ESTAR model on quarterly data.

[5] Very little work has been done on the effects of aggregation on Non-linear time series models. Granger (1991), and Granger and Lee (1999) are notable exceptions. However their analysis does not consider nonlinear processes involving symmetric adjustment.

aggregated ESTAR models. Finally, section 5 summarizes our main conclusions.

2 The True Structural Model and the Effect of Time Aggregation on estimated nonlinear parameters

We assume that at the highest frequency the DGP is given by an Exponential Smooth Autoregressive model (ESTAR) model of Ozaki (1985). A smooth rather than discrete adjustment mechanism is chosen for two reasons. First a smooth adjustment process is suggested by the theoretical analysis of Dumas (1992). Second, as postulated by Terasvirta (1994) and demonstrated theoretically by Berka (2002), in aggregate data, regime changes may be smooth rather than discrete given that heterogeneous agents do not act simultaneously even if they make dichotomous decisions.[6] The relevant ESTAR model which is, a priori, appropriate for modelling PPP deviations at the highest data frequency has the simplest possible structure within the class of ESTAR models and is given by

$$y_t = e^{-\gamma y_{t-1}^2} y_{t-1} + u_t \tag{1}$$

where γ is a positive constant and u_t is a white noise disturbance term.

Figure 1 is a deterministic plot of the relationship between $\Delta y = y_t - y_{t-1}$ and y_{t-1} obtained from (1). We observe in Figure 1 that for small deviations from equilibrium, adjustment may be modelled as a unit root process - "the optimality of doing nothing" - but for large deviations from equilibrium there is mean reversion. If the process spends a significant proportion of time in or near the unit root region, it will exhibit strong persistence and near unit root behavior. It is also interesting to note that the form of ESTAR model we assume is precisely that which has been found to provide a parsimonious fit to many monthly data sets.

[6] Even in high frequency asset markets the idea of heteregeneous traders facing different capital constraints or percieved risk of arbitrage has been employed to rationalise employment of the ESTAR model. See, e.g. Tse (2001) for arbitrage between stock and index futures.

We simulate data from the ESTAR model (1) where the disturbance term, u_t, is assumed to be normally distributed[7].

Following Taylor (2001) we create arithmetic temporal aggregates from the simulated data as[8]

$$y_t^* = \frac{(y_t + y_{t-1} + y_{t-2} + \ldots\ldots + y_{t-i})}{i} \tag{2}$$

where $i = 2, 3, 12$.

Two different assumptions about the true DGP are made. First, we assume that the true DGP is a nonlinear 'monthly' ESTAR process and simulate from this 120,000 observations. We replicate this experiment 1,000 times.

The range of standard deviations of the disturbance term is calibrated on the monthly estimates of equation (1), the highest aggregate data frequency available to researchers (see e.g., Taylor et al., 2001; and Venetis et al., 2002). These studies report standard deviations (σ) of around 0.035. For purposes of comparison we also simulate series with a much lower standard deviation than found in the monthly data and employ values of σ of 0.01 and 0.035. The adjustment parameter is given the values of $\gamma = 0.5, 1$. The estimates obtained in actual monthly data tend to fall in this range.

Aggregating these observations three times, $i = 3$ (quarterly), or twelve times, $i = 12$ (annual), yields 1,000 samples of 40,000 and 10,000 observations respectively. These samples will be used to analyze the 'large sample' behavior of aggregated nonlinear 'monthly' ESTAR models. To analyze the small sample properties, we employ the same method but limit the sample

[7] We also consider nonnormal disturbances such as t-Student with 18 degrees of freedom that in previous research appears to match the nonnormality of residuals (see Paya and Peel 2003b). Results were qualitatively unchanged.

[8] If the data is in logarithmic form, then y_t^* is the geometric mean instead of the arithmetic mean of the real exchange rates. We compared the correlation between the arithmetic and geometric means conditional on some price processes. The correlations were close to unity and the results qualitatively similar. Given this for simplicity we follow Taylor (2001) and employ the arithmetic mean for the temporally aggregated data.

sizes to 120 for 'quarterly' ($i = 3$) aggregation and 200 for 'annual' ($i = 12$) as these span the most common used samples in the literature.[9]

The second assumption made is that the true DGP given by (1) is for data generated at either a fortnightly or ten days frequency that is aggregated to monthly data ($i = 2$, or 3) respectively. Again 120,000 observations are simulated from (1) 1,000 times. In this case the standard errors of the residuals in the true DGP are chosen as $\sigma = 0.024$, 0.028 so that the standard errors of the residuals in the temporally aggregated data, $i = 2, i = 3$ match those found in actual monthly estimates. Values of $\gamma = 0.3, 0.4$ were employed which produced values of the speed of adjustment parameter in the aggregate data similar to those observed in empirical work. In this exercise, the large sample analysis was done with 10,000 observations of the aggregated data[10] and the small sample analysis with 360 observations matching the sample size of monthly data on real exchange rates available from the post Bretton Woods period and around the length of sample that has typically been employed in previous empirical analysis.

On the aggregated data we estimate by nonlinear least squares the following model

$$y_t^* = a + B(L)y_{t-1}^* e^{-\gamma(y_{t-1}^* - a)^2} + v_t \tag{3}$$

where B(L) is a polynomial lag operator of order up to five which rendered the disturbance term v_t empirical white noise,[11] and a is a constant. Empirical marginal significance levels of the estimated parameter γ has to be obtained through Monte Carlo simulation as it is not defined under the null. In particular, the model is assumed to follow a unit root linear autoregressive process[12] and then a nonlinear ESTAR specification (equation 3) is estimated, computing the appropriate confidence interval of significance for γ.

[9] Samples of real exchange rates of 120 at quarterly data are available for the post Bretton-Woods period. At annual frequency the longest data set available is from 1792 in the case of Dollar/Pound and Dollar/French Franc (see Lothian and Taylor 1996).

[10] The results employing 60,000 or 40,000 appeared essentially the same than on a sample using 10,000. We report results on samples of 10,000 as it was computationally much less time consuming.

[11] On the basis of the LM test of Eitrheim and Terasvirta (1996).

[12] The number of autoregressive terms and the calibration of the slope parameters and variance were obtained through a first estimate of a linear autoregressive process (restricting the parameters to add up to one).

First, we examine the results obtained in the case of the large samples described above. We observe in the results reported in Tables 1, 2a, 2b, and 3 that time aggregation induces higher order autoregressive terms in the fitted models at lower frequencies than occur in the DGP. Moreover, the additional autoregressive structure induced by time aggregation seems to have a limiting number of terms. The second order autoregressive term is always significant. Terms in an autoregressive process of order three are significant at least 95% of the time except for i=2 when it falls to 59%. Higher order terms exhibit a steep fall in significance. The significance of the AR(4) parameter varies between 37% and 7% with that of the AR(5) parameter between 5-7%. The order of the autoregressive structure appears to be independent of the range of standard errors of the disturbance term and the speed of adjustment parameters imposed in the true DGP in our simulations.

The regression standard error and the point estimate of the speed of adjustment parameter, γ, increase with the degree of aggregation. The speed of adjustment parameter is always significant in the large sample estimates. Another feature of the time aggregation is the finding of significant LM test for ARCH. The greater the degree of aggregation and the higher the standard error of the disturbance term in the DGP the more accentuated the finding of a significant LM test for ARCH. Noting that the LM test for ARCH is a test for model misspecification and that the errors in the DGP do not exhibit ARCH, this suggests that specification (3) may become less parsimonious as an appropriate way of modelling the temporally aggregated process (1) as the degree of aggregation increases. We also note that the lower the frequency and the higher the standard error of the disturbance term the lower the goodness of fit parameter R^2.

When the estimations are undertaken with smaller samples of observations of 120, 200 and 360,corresponding to quarterly, annual and monthly data employed in empirical studies, the nonlinear estimates of (3) show the following features. The fitted ESTAR exhibits significant AR(2) structure between 50 and 89 percent of the time for i=2,3,12 dependent upon the noise and the speed of adjustment in the true DGP. Autoregressive terms of order greater than two are significant less than ten percent of times. Significant LM tests for ARCH are not found in 90% of the fitted models. The estimated speed of adjustment parameter are higher than in the large sample simulations with larger standard errors and approximately forty percent are significant at the 5% significance level. Consequently, small sample estimations of nonlinear ESTAR models on temporally aggregated data could erroneously reject that the true DGP follows a nonlinear process.[13]

Nonlinear ESTAR models have been reported at various levels of aggregation and the reported empirical results conform with those obtained on the simulated data. Kilian and Taylor (2003) report AR(2) structure in all ESTAR models fitted to quarterly data for seven OECD economies. Michael et al. (1997) report AR(2) structure employing annual data. No one has reported AR structure greater than two. Also significant LM tests for ARCH are rarely reported.

3 Further Comparison between Simulated Data and Empirical Estimates from Actual Data

We now proceed to compare further the empirical results obtained from simulated data with those obtained from actual data. Table 4 presents monthly estimates of ESTAR models for seven bilateral real exchange rates against the Dollar in the post Bretton Woods era taken from Venetis et al. (2002).[14] The estimated model corresponds to that of Equation (3). The estimates of γ are between 0.16 and 0.8 and the standard deviation of the regressions is around 0.033. We added the last column, where the p-value of the second AR term in the estimates is included. For the majority of the cases, PPP deviations appear parsimoniously described by the simple ESTAR structure given by equation (1). However, it appears that in the case of the Dollar/Yen at the five percent, the Dollar/Pound and Dollar/Lira at the fifteen percent, the second AR term plays a significant role. Simulations presented above show that time aggregation induces AR(2) structure in the estimated nonlinear process. The fact that some of the monthly models have significant second AR terms could imply that the true ESTAR model is appropriate at even higher frequency than monthly. Of course, if there are already AR lags of order two at the high-frequency DGP then this will not be the case. However we feel that AR lags of higher order than one at the high frequency DGP

[13]Granger and Lee (1999) examine the effects of time aggregation on nonlinearity tests drawing a similar conclusion. Nonlinearity could be rejected when the model has been temporally aggregated.

[14]This represents the highest possible frequency on actual data on real exchange rates as prices are provided monthly.

do not make apriori sense. The significance of this point will be ultimately determined by whether higher order lags can be generated theoretically in the DGP.

Empirical results at different levels of aggregation ($i = 3, i = 12$) are reported in Tables 5 and 6. The quarterly estimates are taken from Kilian and Taylor (2003) and we also present annual estimates of Equation (3) for Dollar/Pound and Dollar/Franc for two hundred years derived by Lothian and Taylor (1996) and analyzed by Michael et al. (1997). The Dollar/Deutsche Mark is for the Gold Standard -data source- reported in Paya and Peel (2003a). We observe that the estimates of γ are higher than at monthly frequency and similar to those suggested by the simulation exercise above. We also note that the autoregressive structures have a significant AR(2) component.[15] This is interesting given our Monte Carlo showed that in over fifty percent of simulations at "quarterly aggregation" and seventy five percent of simulations at "annual aggregation" gave rise to this specification.

4 Generalized impulse response functions

A number of properties of the impulse response functions of linear models do not carry over to the nonlinear models. In particular, impulse responses produced by nonlinear models are; a) history dependent, so they depend on initial conditions, b) dependent on the size and sign of the current shock, and c) they depend on future shocks as well. That is, nonlinear impulse responses critically depend on the "past", "present" and "future".

The Generalized Impulse Response Function (GIRF) introduced by Koop, Pesaran and Potter (1996) successfully confronts the challenges that arise in defining impulse responses for nonlinear models. The impulse response is defined as the average difference between two realizations of the stochastic process $\{y_{t+h}\}$ which start with identical histories up to time $t-1$ (initial conditions) but one realization is "hit" by a shock at time t while for the other (the benchmark profile) no shock occurs. In a context similar to ours, Taylor and Peel (2000) conduct GIRF analysis on the deviations of real exchange rates from monetary fundamentals, and Baum et al. (2001), and Taylor et

[15] In the case of the Dollar/Franc the AR(2) term is insignificant but the residuals exhibit better properties. It is worth noting that these estimations span a long period of time with different exchange rate regimes. However, those nonlinear estimates have recently been proved to be robust (see Lothian and Taylor, 2004; and Paya and Peel, 2004).

al. (2001) use impulse response functions to gauge how long shocks survive in real exchange rate nonlinear models. The GIRF of Koop et al. (1996) is defined as,

$$GIRF_h(h, \delta, \omega_{t-1}) = E(y_{t+h}|u_t = \delta, \omega_{t-1}) - E(y_{t+h}|u_t = 0, \omega_{t-1}) \quad (4)$$

where $h = 1, 2, ..,$ denotes horizon, $u_t = \delta$ is an arbitrary shock occurring at time t and ω_{t-1} defines the history set of y_t. Given that δ and ω_{t-1} are single realizations of random variables, expression (4) is considered to be a random variable. In order to obtain sample estimates of (4), we average out the effect of all histories ω_{t-1} that consist of every set $(y_{t-1}, ..., y_{t-p})$ for $t \geq p+1$, where p is the autoregressive lag length, and we also average out the effect of future shocks u_{t+h}. In particular, for each available history we use 300 repetitions to average out future shocks, where future shocks are drawn with replacement from the models residuals, and then we average the result across all histories.[16] We set the shocks on the log real exchange rate y_t (equal to $\ln(1 + k/100)$ with $k = 10, 20, 30$) which correspond roughly to 10%, 20% and 30% shocks, respectively. The speed of real exchange rate convergence will be measured with the half-life of shocks. In this case, the half-life of shocks will be computed as the number of time periods that PPP deviations need to settle below 50% the size of the shock.[17] Accordingly, we will report the half-lives of shocks defined as the time needed for $GIRF_h < \frac{1}{2}\delta$.[18]

We examine the implied speeds of adjustment to shocks for the Monte Carlo experiments in Section 2 using the following procedure. First, we generate ESTAR DGP's with given parameters γ, and standard deviations of error term, se. These correspond to the highest frequency of the 'true' process followed by PPP deviations. Following our discussion of the Monte Carlo simulations and empirical results in previous sections, we consider three cases: monthly, fortnightly and ten days. Parameters γ and se are fixed accordingly. We estimate the models without aggregation and calculate the average half-life of shocks for those processes.

[16] We found out that the difference with using 500 repetitions was quantitatively insignificant. Without loss of generality, the impulse response horizon is set to $\max\{h\} = 48$ in the future.

[17] For a full discussion on different measures of half-life shocks and estimating procedures see Murray and Papell (2002) and Kilian and Zha (2002).

[18] This is the same definition as in Taylor et al. (2001). However, please note that half-life of shocks could also, in theory, be oscillatory.

Second, we aggregate the 'true' DGP at different levels: $i = 3$ and $i = 12$ in the case of the 'monthly' true DGP; $i = 2$ in the case of 'fortnight' true DGP; and $i = 3$ in the case of 'ten days' true DGP. We estimate the ESTAR models as in Equation (3) and calculate the average half-life of shocks for the cases where a significant estimate of γ is found.

Table 7 reports the results of applying this procedure in the case of the same large and small samples outlined above. For large samples, the half life of shocks, for a 10% shock, is between 15 and 17 months for the true DGP (lines 1, 4, and 6). However, the half-life of shocks of the temporally aggregated data lie between 20 and 36 months (lines 2, 3, 5, and 7). In the case of small samples, for a 10% shock, the true DGPs exhibit half-lives between 6 and 13 months (lines 8, 11, and 13). The aggregated models show much lower adjustment responses, between 16 and 36 months (lines 9, 10, 12, and 14).

In order to compare our simulation results with actual estimates, Tables 8, 9 and 10 show the half-life shocks for the nonlinear models estimated on actual data reported in Tables 4, 5 and 6. Employing the simulations results as a benchmark, we can then use these empirical estimates of half-lives of shocks to try to approximate the nature of the true DGP of PPP deviations. We will concentrate on the speed of adjustment to shocks of the Dollar/Pound, Dollar/French Franc, and Dollar/Deutsche Mark. The difference between the speed of adjustment to shocks in the monthly and annual data is around twenty four months for the three different currencies. The difference between the adjustment at quarterly and annual data is either zero or twelve months. This pattern is the one followed by the Monte Carlo results when we aggregate a true DGP from monthly to quarterly and annual data.

5 Conclusions

Taylor (2001) demonstrated that if the DGP is a nonlinear threshold process, and if the data employed in estimation is temporally aggregated, then linear estimates of adjustment speeds can be substantially downward biased. We have demonstrated in this article that a similar result holds for ESTAR models estimated on temporally aggregated data when the true DGP is ESTAR. In addition, if the true DGP is as postulated, the significance of more than one autoregressive term in some monthly estimates raises the possibility that adjustment speeds are even faster than obtained in monthly estimates. Consequently, temporal aggregation provides a further potential solution to the speed of adjustment puzzle raised by Rogoff.

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Table 1. Results for simulated aggregated data of ESTAR model

True DGP:	$y_t = e^{-\gamma y_{t-1}^2} y_{t-1} + u_t$			
Estimated model:	$y_t^* = a + B(L) y_{t-1}^* e^{-\gamma (y_{t-1}^* - a)^2}$			
	Aggregation i=12 (annual aggregation)			
	$\gamma = 1$		$\gamma = 1$	
	$se = 0.035$		$se = 0.01$	
	$sample 10,000$	$sample$ 200	$sample 10,000$	$sample$ 200
$Mean\ \widehat{\gamma}$	4.50	5.00	7.62	10.50
$sd\ \widehat{\gamma}$	0.45	3.85	0.70	6.80
$t(\widehat{\gamma})$	1.000	0.240	1.000	0.370
R^2	0.60	0.60	0.86	0.85
se	0.077	0.077	0.025	0.025
$LM\ Arch$	1.000	0.183	0.300	0.070
$AR(2)$	1.000	0.750	1.000	0.860
$AR(3)$	0.995	0.095	0.990	0.120
$AR(4)$	0.220	0.075	0.270	0.065
$AR(5)$	0.070	0.070	0.060	0.060
	$\gamma = 0.50$		$\gamma = 0.50$	
	$se = 0.035$		$se = 0.01$	
	$sample 10,000$	$sample$ 200	$sample 10,000$	$sample$ 200
$Mean\ \widehat{\gamma}$	2.78	3.30	4.07	5.95
$sd\ \widehat{\gamma}$	0.24	2.00	0.38	4.58
$t(\widehat{\gamma})$	1.000	0.360	1.000	0.340
R^2	0.69	0.69	0.90	0.89
se	0.082	0.082	0.026	0.026
$LM\ Arch$	0.995	0.010	0.157	0.058
$AR(2)$	1.000	0.770	1.000	0.840
$AR(3)$	0.996	0.120	1.000	0.123
$AR(4)$	0.220	0.080	0.290	0.067
$AR(5)$	0.070	0.070	0.077	0.043

Notes: sd denotes standard deviation of coefficient γ. $t(\widehat{\gamma})$ denotes ratio of significant γ parameter where empirical significance level is obtained through Monte Carlo. se denotes standard error of equation. $LM\ Arch$ is the ratio of rejection of the Lagrange Multiplier test for ARCH in the residuals. $AR(p)$ denotes ratio of significant autoregressive term of order p

Table 2a. Results for simulated aggregated data of ESTAR model				
True DGP:		$y_t = e^{-\gamma y_{t-1}^2} y_{t-1} + u_t$		
Estimated model:		$y_t^* = a + B(L) y_{t-1}^* e^{-\gamma (y_{t-1}^* - a)^2}$		
	Aggregation i=3 (quarterly aggregation)			
	$\gamma = 1$		$\gamma = 1$	
	$se = 0.035$		$se = 0.01$	
	sample 40,000	*sample* 120	*sample* 40,000	*sample* 120
$Mean\ \widehat{\gamma}$	2.14	3.50	2.37	13.80
$sd\ \widehat{\gamma}$	0.09	3.00	0.17	19.50
$t(\widehat{\gamma})$	1.000	0.390	1.000	0.270
R^2	0.86	0.85	0.96	0.92
se	0.047	0.047	0.013	0.013
$LM\ Arch$	0.584	0.092	0.128	0.085
$AR(2)$	1.000	0.460	1.000	0.510
$AR(3)$	1.000	0.090	1.000	0.110
$AR(4)$	0.370	0.066	0.350	0.083
$AR(5)$	0.055	0.055	0.077	0.066
	$\gamma = 0.50$		$\gamma = 0.50$	
	$se = 0.035$		$se = 0.01$	
	sample 40,000	*sample* 120	*sample* 40,000	*sample* 120
$Mean\ \widehat{\gamma}$	1.10	2.20	1.20	11.70
$sd\ \widehat{\gamma}$	0.05	2.54	0.09	19.25
$t(\widehat{\gamma})$	1.000	0.375	1.000	0.240
R^2	0.90	0.88	0.97	0.92
se	0.048	0.047	0.014	0.014
$LM\ Arch$	0.310	0.070	0.127	0.087
$AR(2)$	1.000	0.510	1.000	0.490
$AR(3)$	1.000	0.100	1.000	0.090
$AR(4)$	0.350	0.050	0.380	0.062
$AR(5)$	0.070	0.070	0.068	0.078

Notes: see notes in table 1

Table 2b. Results for simulated aggregated data of ESTAR model		
True DGP:		$y_t = e^{-\gamma y_{t-1}^2} y_{t-1} + u_t$
Estimated model:		$y_t^* = a + B(L) y_{t-1}^* e^{-\gamma (y_{t-1}^* - a)^2}$
	Aggregation i=3 (monthly aggregation)	
	$\gamma = 0.3$	
	$se = 0.024$	
	$sample\ 10,000$	$sample\ 360$
$Mean\ \widehat{\gamma}$	0.71	1.09
$sd\ \widehat{\gamma}$	0.08	0.75
$t(\widehat{\gamma})$	1.000	0.390
R^2	0.95	0.93
se	0.033	0.033
$LM\ Arch$	0.114	0.100
$AR(2)$	1.000	0.890
$AR(3)$	0.950	0.110
$AR(4)$	0.130	0.080
$AR(5)$	0.052	0.055

Notes: see notes in table 1

Table 3. Results for simulated aggregated data of ESTAR model		
True DGP:		$y_t = e^{-\gamma y_{t-1}^2} y_{t-1} + u_t$
Estimated model:		$y_t^* = a + B(L) y_{t-1}^* e^{-\gamma (y_{t-1}^* - a)^2}$
	Aggregation i=2 (monthly aggregation)	
	$\gamma = 0.4$	
	$se = 0.028$	
	$sample\ 10,000$	$sample\ 360$
$Mean\ \widehat{\gamma}$	0.67	1.01
$sd\ \widehat{\gamma}$	0.08	0.76
$t(\widehat{\gamma})$	1.000	0.390
R^2	0.95	0.93
se	0.033	0.033
$LM\ Arch$	0.126	0.084
$AR(2)$	1.000	0.700
$AR(3)$	0.590	0.080
$AR(4)$	0.070	0.050
$AR(5)$	0.050	0.045

Notes: see notes in table 1

Table 4. Results from ESTAR models of real exchange rates montly observations 1973-2001.

	$\hat{\delta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\gamma}$	s	$p\ AR(2)$
FRF	-0.025	1.037	$\beta_2 = 0$	0.779	0.031	0.55
	(0.031)	(0.022)		(0.313)		
BEF	0.005	1.018	$\beta_2 = 0$	0.331	0.033	0.46
	(0.048)	(0.020)		(0.185)		
DEM	-0.027	1.033	$\beta_2 = 0$	0.625	0.033	0.27
	(0.036)	(0.021)		(0.248)		
ITL	-0.045	1.017	$\beta_2 = 1 - \beta_1$	0.336	0.030	0.15
	(0.043)	(0.022)		(0.194)		
JPY	0.479	1.105	$\beta_2 = 1 - \beta_1$	0.155	0.033	0.05
	(0.059)	(0.053)		(0.082)		
NLG	0.041	1.022	$\beta_2 = 0$	0.481	0.033	0.48
	(0.046)	(0.022)		(0.236)		
GBP	0.109	1.094	$\beta_2 = 0$	0.595	0.031	0.16
	(0.059)	(0.069)		(0.361)		

Notes: Numbers in parentheses are Newey-West standard error estimates.. s denotes the residuals standard error. $pAR(2)$ denotes p-value of second autoregressive term.in the ESTAR estimation.
Source: Table from Venetis et al. (2002)

Table 5. Results from ESTAR models of real exchange rates quarterly observations 1973.I-1998.IV

	$\hat{\delta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\gamma}$	s
FRF	0.095	1.32	$\beta_2 = 1 - \beta_1$	0.964	0.047
	(0.033)	(0.096)		(0.152)	
DEM	0.096	1.233	$\beta_2 = 1 - \beta_1$	0.794	0.053
	(0.036)	(0.099)		(0.125)	
CAN	0.00	1.181	$\beta_2 = 1 - \beta_1$	0.706	0.019
		(0.078)		(0.043)	
ITL	0.00	1.154	$\beta_2 = 1 - \beta_1$	0.909	0.054
		(0.113)		(0.247)	
JPY	0.00	1.350	$\beta_2 = 1 - \beta_1$	0.725	0.057
		(0.103)		(0.094)	
SW	0.00	1.292	$\beta_2 = 1 - \beta_1$	0.724	0.059
		(0.099)		(0.139)	
GBP	0.00	1.144	$\beta_2 = 1 - \beta_1$	1.069	0.052
		(0.103)		(0.324)	

Notes: Numbers in parentheses are Newey-West standard error estimates..
s denotes the residuals standard error
Source: Table from Kilian and Taylor (2003)

Table 6. Results from ESTAR models of real exchange rates on annual data.

	$\hat{\delta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\gamma}$	s
Dollar/FrF	-0.083	1.12	$\beta_2 = 1 - \beta_1$	4.03	0.076
1804-1992	(0.025)	(0.15)		(1.54)	
Dollar/Pound	-0.210	1.18	$\beta_2 = 1 - \beta_1$	2.43	0.069
1792-1992	(0.019)	(0.069)		(0.54)	
Dollar/DM	-0.033	1.09	$\beta_2 = 1 - \beta_1$	2.52	0.095
1795-1913	(0.032)	(0.08)		(0.60)	

Notes: Numbers in parentheses are Newey-West standard error estimates..
s denotes the residuals standard error

Table 7: Estimated half-lives of shocks in months from Temporally aggregated Data: True DGP $y_t = e^{-\gamma y_{t-1}^2} + u_t$ where $u_t = N(0, se)$

Shock:		10% Months	20% Months	30% Months
Line	Temporal Aggregation Large Sample			
1	True DGP $\gamma = 1, se = 0.035$: $y_t = e^{-y_{t-1}^2} y_{t-1}$	17	15	12
2	i=3: $\gamma = 2.14, se = 0.047$	24	18	12
3	i=12: $\gamma = 4.5, se = 0.077$	36	24	12
4	True DGP $\gamma = 0.4, se = 0.028$	17	16	14
5	i=2 $\gamma = 0.67, se = 0.035$	21	19	16
6	True DGP $\gamma = 0.3, se = 0.024$	15	13	11
7	i=3 $\gamma = 0.71, se = 0.035$	20	17	14
	Temporal Aggregation Small Sample			
8	True DGP $\gamma = 1.65, se = 0.035$	13	11	8
9	i=3: $\gamma = 3.5, se = 0.047$	21	15	12
10	i=12: $\gamma = 5, se = 0.077$	36	24	12
11	True DGP $\gamma = 0.7, se = 0.028$	10	8	7
12	i=2: $\gamma = 1, se = 0.035$	19	17	14
13	True DGP $\gamma = 0.5, se = 0.024$	6	4	3
14	i=3: $\gamma = 1, se = 0.035$	16	13	9

Table 8. Estimated half-lives shocks in months for monthly model 1973-2001

	$\hat{\gamma}$	*Shock* 10%
Real rate		Months
FRF EST-AR(1)	0.78	12
BEF EST-AR(1)	0.33	25
DEM EST-AR(1)	0.62	13
NLG EST-AR(1)	0.48	18
JPY EST-AR(2)	0.15	40
ITL EST-AR(1)	0.30	39
ITL EST-AR(2)	0.34	36
GBP EST-AR(1)	0.50	24
GBP EST-AR(2)	0.54	22

Source: Table from Venetis et al. (2002)

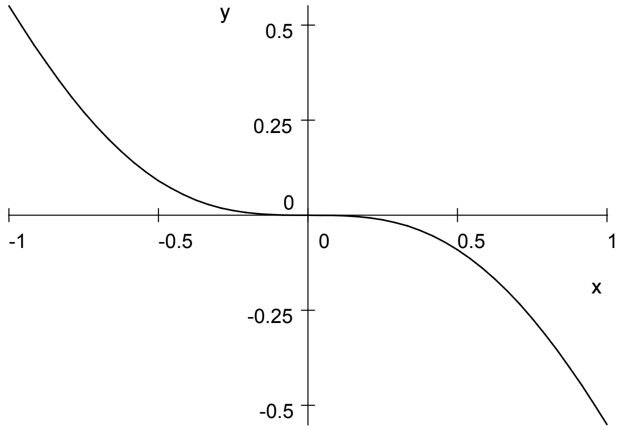
Table 9. Estimated half-lives shocks in months for quarterly model. 1973.I-1998.IV

	$\hat{\gamma}$	*Shock*	10%
Real rate			Months
FRF EST-AR(2)	0.86		36
DM EST-AR(2)	0.79		36
CA EST-AR(2)	0.71		40
IT EST-AR(2)	0.91		36
JP EST-AR(2)	0.73		40
SW EST-AR(2)	0.72		40
GBP EST-AR(2)	1.07		36

Source: Table from Kilian and Taylor (2003)

Table 10. Estimated half-lives shocks in months for annual model

	$\hat{\gamma}$	*Shock*	10%
Real rate			Months
FRF EST-AR(2) 1802-1992	4.04		36
GBP EST-AR(2) 1792-1992	2.44		48
DM EST-AR(2) 1794-1913	2.52		48



Deterministic plot of Δy (vertical axis), y_{t-1} (horizontal axis) from ESTAR with γ = 0.8.