NONLINEAR PPP UNDER THE GOLD STANDARD *

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ABSTRACT

Hegwood and Papell (2002) conclude on the basis of analysis in a linear framework that long-run purchasing power parity (PPP)\ does not hold for sixteen real exchange rate series, analyzed in Diebold, Husted, and Rush (1991) for the period 1792-1913, under the Gold Standard. Rather, purchasing power parity deviations are mean-reverting to a changing equilibrium -a quasi PPP (QPPP) theory. We analyze the real exchange rate adjustment mechanism for their data set assuming a nonlinear adjustment process allowing for both a constant and a mean shifting equilibrium. Our results confirm that real exchange rates at that time were stationary, symmetric, nonlinear processes that revert to a non-constant equilibrium rate. Speeds of adjustment were much quicker when breaks were allowed.

Key Words: Purchasing Power Parity, ESTAR, Bootstrapping.

JEL classification: F31, C15, C22, C51

1. Introduction

Recent theoretical analysis of purchasing power deviations (see, e.g., Dumas 1992; Sercu, Uppal, and VanHull 1995; and O'Connell and Wei 1997) demonstrates how transactions costs or the sunk costs of international arbitrage induce nonlinear adjustment of the real exchange rate to purchasing power parity (PPP). While globally mean-reverting this nonlinear process has the important property of exhibiting near unit root behavior for small deviations from PPP since small deviations from PPP are left uncorrected if they are not large enough to cover the transactions costs or the sunk costs of international arbitrage.

A parametric nonlinear model, suggested by the theoretical literature, that capture the nonlinear adjustment process in aggregate data is the exponential smooth transition autoregression model (ESTAR) of Ozaki (1985). A smooth rather than discrete adjustment mechanism is motivated by the theoretical analysis of Dumas (1992). Also, as postulated by Terasvirta (1994) and demonstrated theoretically by Berka (2002), in aggregate data, regime changes may be smooth rather than discrete given that heterogeneous agents do not act simultaneously even if they make dichotomous decisions.[1] Recent empirical work (e.g., Michael, Nobay, and Peel 1997; Taylor, Peel, and Sarno 2001; Peel and Venetis 2002) has reported empirical results that suggest that the ESTAR model provides a parsimonious fit into a variety of data sets, particular for monthly data for the interwar and postwar floating period as well as for annual data spanning two hundred years, as reported in Lothian and Taylor (1996). In addition, nonlinear impulse response functions derived from the ESTAR models show that while the speed of adjustment for small shocks around equilibrium will be highly persistent, larger shocks mean-revert much faster than the "glacial rates" previously reported for linear models (Rogoff 1996). In this respect, the ESTAR models provide some solution to the PPP puzzle outlined in Rogoff (1996).[2]

The ESTAR model can also provide an explanation of why PPP deviations analyzed from a linear perspective appear to be described by either a non-stationary integrated I(1) process, or alternatively, described by fractional processes (see, e.g., Diebold, Husted, and Rush 1991). Taylor, Peel, and Sarno (2001), and Pippenger and Goering (1993) show that the Dickey-Fuller tests have low power against data simulated from an ESTAR model. Michael, Nobay, and Peel (1997) and Byers and Peel (2003) show that data that is generated from an ESTAR process can appear to exhibit the fractional property. That this would be the case was an early conjecture by Acosta and Granger (1995). Given that the ESTAR model has a theoretical rationale while the fractional process is a relatively nonintuitive one, the fractional property might reasonably be interpreted as a misleading linear property of PPP deviations (Granger and Terasvirta 1999).

While the empirical work employing ESTAR models provides some explanation of the glacially slow adjustment speeds obtained in linear models, there is one aspect of the empirical work that is worthy of further attention. A second way of explaining the Rogoff puzzle, raised by Rogoff himself,[3] is to relax the assumption that the equilibrium real exchange rate is a constant (see, e.g., Canzoneri, Cumby, and Diba 1996; and Chinn and Johnston 1996). Theoretical models, such as that of Balassa (1964) and Samuelson (1964), imply a non-constant equilibrium in the real exchange rate if real productivity growth rates differ between countries.[4] Nonlinear models that incorporate proxies for these effects are found to parsimoniously fit post-Bretton Woods data for the main real exchange rates (see Venetis, Paya, and Peel 2002; and Paya, Venetis, and Peel 2003). Naturally, models that ignore this effect may generate misleading speeds of PPP adjustment to shocks. In this regard, the empirical results of Hegwood and Papell (2002) for the Gold Standard period are particularly interesting. Balassa-Samuelson effects are one of the major arguments for the numerous equilibrium mean shifts found in Hegwood and Papell (2002) for the real exchange rates in the sixteen real exchange rate series analyzed in Diebold, Husted, and

Rush (1991) for the period 1792-1913 under the Gold Standard. Hegwood and Papell (2002) assume linear adjustment around an occasionally changing equilibrium determined on the basis of the Bai-Perron (1998) test for multiple structural breaks. They report that quick mean reversion around an occasionally changing mean provides a more reasonable representation of the data than does fractional integration - originally reported by Diebold, Husted, and Rush (1991) for their data set. They conclude that long-run PPP (LRPPP) does not hold but instead it is quasi-PPP (QPPP) theory-the one supported by their analysis of the data. They also state that the slow convergence of LRPPP is due to the unaccounted mean shifts in the equilibrium rate and that a reduction of more than 50% is achieved in the half-lives of shocks when those shifts are included in the model.

These results are potentially important and provide motivation for our study. Hegwood and Papell (2002) only consider the impacts of structural breaks in the context of linear adjustment. In this article, we further examine the real exchange rate adjustment mechanism in the nineteenth and early twentieth centuries under the Gold Standard by employing an ESTAR framework that allows for both a constant and structural breaks in the equilibrium real rate. Because the gold standard era was "a high point of international cooperation" (Diebold, Husted, and Rush 1991, p. 1254) and it was a symmetric arrangement (both parts were committed to maintain parities), the symmetric nonlinear ESTAR model is an appropriate model of real exchange rate behavior at that time. We find that ESTAR models incorporating the structural breaks employed by Hegwood and Papell provide a parsimonious explanation of the data. We determine the significance of the structural breaks via bootstrap and Monte Carlo analysis. We then investigate the speeds of adjustment obtained from nonlinear impulse response functions in these models and compare them to the estimated models that exclude structural breaks. Our results provide further support, on a new data set, for the hypothesis that real exchange rates are stationary, symmetric, nonlinear processes that reverted in this time period to a changing equilibrium real rate. The half-life of shocks implied by the nonlinear impulse response functions were found to be dramatically faster than those obtained in models that do not include the breaks. Clearly, our results support those of Hegwood and Papell (2002).

The rest of the article is organized as follows. In section 2, we discuss the ESTAR model considered in our empirical applications and report empirical estimates of ESTAR models where the real exchange rate long run path is modelled both as a variable or a constant. Section 3 presents the Monte Carlo simulation exercise for the confidence interval of the statistics. Section 4 presents the results of the estimated impulse response functions for the nonlinear models. Finally, section 5 summarizes our main conclusions.

2. Nonlinear PPP

We analyze properties of a set of currencies (Dollar, Pound, Deutsche Mark, French franc, Belgian franc, and Swedish krona) spanning the period 1792-1913. We use the same data set as in Diebold, Husted, and Rush (1991) and Hegwood and Papell (2002).[5] This data set includes ten real exchange rates using wholesale price index (WPI) as the deflator of the nominal exchange rate, and six real exchange rates that use the consumer price index (CPI) as deflator. We normalize all the real rates so that the first observation is set equal to zero. Hegwood and Papell (2002) could not reject the null of a unit root on the basis of the Augmented Dickey Fuller (ADF) tests for 11 out of the 16 series. Six additional series reject the null of unit root when they apply the Perron-Vogelsang (1992) test for unit root while allowing a single mean shift in the data. The five remaining real exchange rates reject the unit root test in favor of a fractionally integrated alternative as reported in Diebold, Husted, and Rush (1991). These results are all consistent with the possibility that the real exchange rate followed an ESTAR process as noted in the previous section.[6]

We assume the true data generating process for the purchasing power parity deviations (y_t) modified for structural breaks has the simple form of ESTAR model reported in Taylor, Peel, and Sarno (2001) and Paya, Venetis, and Peel (2003):

$$y_t = \alpha + d_1 + d_2 + ... + d_n + e^{\gamma(y_{t-1}-\alpha-d_1-d_2-...-d_n)^2}(\sum_{i=1}^{n}(\beta_i y_{t-i} - \alpha - d_1 - d_2 - ... - d_n) + u_t, \tag{1}$$

where y_t is the real exchange rate ($y_t = s_t - p_t + p_t^*$), s_t is the logarithm of the spot exchange rate, p_t is the logarithm of the domestic price level and p_t^* the logarithm of the foreign price level. α is the constant equilibrium level of the real exchange rate, γ is a positive constant -the speed of adjustment, β_i are constants, $d_1...d_n$ are dummies for shifts in the equilibrium rate, and u_t is a random disturbance term.[7]

The first model we estimate is the simple ESTAR model for the real exchange rate with a constant equilibrium ($d_1...d_n$ are set to zero). Tables 1 and 2 present the results of the estimations for WPI and CPI real rates, respectively. The speed of adjustment parameter is significant in all cases except for the Belgium/U.S. and Belgium/Germany WPI rates and the Belgium/Germany CPI rate. The autoregressive structure of the ESTAR model (the β_i) varies from an AR(1) to an AR(3).[8] Given the significance of γ and that in all cases we cannot reject that the sum of the autoregressive terms adds up to one, we impose this constraint in all estimations.[9]

Hegwood and Papell (2002), on the basis of the Bai and Perron (1998) test for multiple mean shifts, provide evidence that the real exchange rates do not exhibit a constant conditional mean for the whole sample, but, instead, they follow a mean reversion process to a changing mean. We include the same dummies that they found significant in the equilibrium process of the real exchange rates.[10] Tables 3 and 4 present the results for

Table 1. ESTAR estimations WPI real exchange rates 1792-1973											
	$\hat{\delta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\gamma}$	s	R^2	$q=1$	$q=4$	$q=12$
Wholesale Price Index										
US/UK	-0.076	1.28	$1-\hat{\beta}_1$		-3.12	0.075	0.71	0.70	0.89	0.69
	(0.019)	(0.13)			(0.90)					
US/Germany	-0.033	1.09	$1-\hat{\beta}_1$		-2.52	0.095	0.65	0.72	0.32	0.37
	(0.031)	(0.08)			(0.71)					
Germany/UK	-0.21	1.25	-0.56	$1-\hat{\beta}_1-\hat{\beta}_2$	-1.52	0.076	0.80	0.42	0.80	0.75
	(0.039)	(0.09)	(0.16)		(0.68)					
France/UK	0.008	1.14	$1-\hat{\beta}_1$		-11.86	0.057	0.57	0.95	0.40	0.75
	(0.014)	(0.10)			(1.03)					
France/US	-0.141	1.03	$1-\hat{\beta}_1$		-7.42	0.060	0.70	0.93	0.80	0.93
	(0.017)	(0.10)			(1.86)					
France/Germany	0.186	1.23	0.48	$1-\hat{\beta}_1-\hat{\beta}_2$	-2.55	0.051	0.89	0.16	0.26	0.55
	(0.027)	(0.06)	(0.096)		(0.98)					
Belgium/UK	0.262	1			-2.49	0.042	0.88	0.16	0.30	0.49
	(0.026)				(0.95)					
Belgium/France	0.197	1			-2.98	0.047	0.87	0.31	0.31	0.22
	(0.019)				(0.94)					

Notes: Numbers in parentheses are the Newey-West (1987) (NW) standard error estimates. s denotes residuals standard error. The test for autocorrelation for lags 1, 4, and 12 denotes the p-values of Eitrheim and Terasvirta (1996) LM test for autocorrelation in nonlinear series.

Table 2. ESTAR estimations CPI real exchange rates 1792-1913.										
	$\hat{\delta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\gamma}$	s	R^2	$q = 1$	$q = 4$	$q = 12$
Consumer Price Index										
Sweden/Germany	-0.123	1.26	$1 - \hat{\beta}_1$		-2.41	0.074	0.80	0.32	0.16	0.22
	(0.039)	(0.085)			(1.23)					
France/Sweden	0.001	1.51	$1 - \hat{\beta}_1$		-13.77	0.033	0.82	0.12	0.39	0.89
	(0.010)	(0.11)			(2.45)					
France/Germany	-0.004	1.23	$1 - \hat{\beta}_1$		-7.17	0.081	0.70	0.34	0.22	0.52
	(0.037)	(0.12)			(4.90)					
France/Belgium	0.189	1			-4.57	0.045	0.83	0.36	0.84	0.99
	(0.023)				(1.54)					
Belgium/Sweden	-0.154	0.94	$1 - \hat{\beta}_1$		-4.75	0.045	0.83	0.21	0.08	0.29
	(0.018)	(0.10)			(1.40)					

Notes: Numbers in parentheses are standard error estimates. s denotes the NW residuals standard error. The test for autocorrelation for lags 1, 4, and 12 denotes the p-values of Eitrheim and Terasvirta (1996) LM test for autocorrelation in nonlinear series.

Table 3. ESTAR estimations WPI real exchange rates 1792-1913 with dummies.

	$\hat{\delta}_0$	$\hat{\delta}_1$	$\hat{\delta}_2$	$\hat{\delta}_3$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\gamma}$	s	R^2	$q=1$	$q=4$	$q=12$	F
Wholesale Price Index													
US/UK	-0.124	0.228			1.28	$1-\hat{\beta}_1$	-4.28	0.071	0.75	0.27	0.55	0.52	
	(0.024)	(0.028)			(0.15)		(1.21)						
US/Germany	0.027	-0.137	0.113	0.157	1.07	$1-\hat{\beta}_1$	-4.43	0.088	0.71	0.43	0.39	0.61	7.33
	(0.031)	(0.037)	(0.036)	(0.043)	(0.088)		(0.78)						
Germany/UK	-0.068	-0.134	-0.237	-0.091	1.27	$1-\hat{\beta}_1$	-8.50	0.075	0.81	0.08	0.04	0.20	5.76
	(0.022)	(0.042)	(0.027)	(0.029)	(0.15)		(2.18)						
France/UK	-0.028	0.098	-0.042	0.032	1.32	$1-\hat{\beta}_1$	-30.9	0.039	0.65	0.40	0.95	0.74	8.84
	(0.001)	(0.021)	(0.017)	(0.013)	(0.10)		(9.21)						
France/US	-0.220	0.136	0.063	0.230	1.02	$1-\hat{\beta}_1$	-34.1	0.050	0.81	0.11	0.11	0.29	15.6
	(0.014)	(0.037)	(0.022)	(0.026)	(0.08)		(15.9)						
France/Germany	0.054	0.313	0.203	0.097	1.22	$1-\hat{\beta}_1$	-27.5	0.046	0.91	0.17	0.77	0.53	10.35
	(0.020)	(0.025)	(0.030)	(0.028)	(0.07)		(1.75)						
Belgium/UK	0.398	-0.320	-0.226	-0.08	1.09	$1-\hat{\beta}_1$	-186.7	0.036	0.91	0.88	0.79	0.74	10.38
	(0.017)	(0.020)	(0.018)	(0.018)	(0.12)		(64.7)						
Belgium/US	-0.002	0.295	0.251		1		-0.88	0.061	0.69	0.15	0.64	0.46	6.99
	(0.045)	(0.026)	(0.008)				(0.32)						
Belgium/Germany	0.073	0.096			1.27	$1-\hat{\beta}_1$	-8.96	0.055	0.65	0.79	0.62	0.74	
	(0.023)	(0.030)			(0.12)		(3.32)						
Belgium/France	0.292	-0.246			1		-53.3	0.043	0.90	0.70	0.76	0.81	
	(0.012)	(0.017)					(17.73)						

Notes: Numbers in parentheses are standard error estimates. s denotes the residuals standard error.

Table 4. ESTAR estimations CPI real exchange rates 1792-1913 with dummies.

	$\hat{\delta}_0$	$\hat{\delta}_1$	$\hat{\delta}_2$	$\hat{\delta}_3$	$\hat{\delta}_4$	$\hat{\delta}_5$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\gamma}$	s	R^2	q=1	q=4	q=12	F
Consumer Price Index															
Sweden/	-0.278	0.312	0.173				1.43	$1-\hat{\beta}_1$	-17.41	0.060	0.88	0.28	0.15	0.00	22.8
Germany	(0.017)	(0.028)	(0.028)				(0.13)		(4.69)						
France/	0.000	-0.023	0.015				1.50	$1-\hat{\beta}_1$	-21.18	0.033	0.82	0.11	0.35	0.91	0.93
Sweden	(0.007)	(0.012)	(0.008)				(0.11)		(4.35)						
France/	-0.030	0.241	0.048				1.31	-0.62	-7.63	0.074	0.75	0.10	0.23	0.62	3.88
Germany	(0.033)	(0.064)	(0.025)				(0.09)	(0.13)	(4.25)						
France/	0.197	-0.015					1		-5.20	0.046	0.85	0.27	0.71	0.99	
Belgium	(0.024)	(0.007)							(2.11)						
Belgium/	-0.273	0.264	0.180	0.078			1.16	$1-\hat{\beta}_1$	-255	0.037	0.89	0.30	0.67	0.59	13.16
Sweden	(0.009)	(0.011)	(0.011)	(0.012)			(0.12)		(71.8)						
Belgium/	-0.732	0.604	0.453	0.192	0.073	0.026	1.39	$1-\hat{\beta}_1$	-19.88	0.062	0.93	0.51	0.63	0.99	7.96
Germany	(0.015)	(0.017)	(0.033)	(0.040)	(0.029)	(0.022)	(0.12)		(5.67)						

Notes: Numbers in parentheses are standard error estimates. s denotes the residuals standard error. The test for autocorrelation denotes the p-values of Eitrheim and Terasvirta (1996) LM test. The $\hat{\beta}_3$ coefficient in the France/Germany rates equals $1-\hat{\beta}_1-\hat{\beta}_2$.

the estimation of the ESTAR model with changing equilibrium rates. Some of the initial dummies appeared to be insignificant when the real rates are allowed to follow a nonlinear mean-reverting process. We then removed the dummies that were insignificant in the new estimations.[11] The last column of Tables 3 and 4 display the F-test for the joint significance of all remaining dummies. In all cases, we can reject the null that all dummies were insignificant when taken all together, except in the case of the France/Sweden CPI real exchange rate. However, the residuals do exhibit significant non-normality, and, in this case, the distribution of the statistics could follow non-classical forms within a nonlinear framework. Consequently, we employ a bootstrap method in order to obtain appropriate test statistics.

3. Robustness Analysis

Our null hypothesis is that the dummy variables for breaks have zero coefficients. Accordingly we simulate an "artificial" series for y_t $(\widehat{y}_t)$, given the estimates of α and γ obtained in Tables 3 and 4, with the coefficients on the dummy variables for structural breaks set to zero. The residuals u^b are obtained from bootstrapping, with replacement, the estimated residuals obtained from the ESTAR models reported in those Tables which include the dummies.[12] The resulting "artificial" series are given by

$$\widehat{y}_t = \widehat{a} + e^{\widehat{\gamma}(\widehat{y}_{t-1}-\widehat{a})^2}(\widehat{y}_{t-1} - \widehat{a}) + u^b, \tag{2}$$

We then estimate the nonlinear ESTAR model including the pertinent dummies in each case, and we repeat this experiment 10,000 times. The distribution of the t-statistics are computed as well as the distribution of the F-test for each real exchange rate. Tables 5 and 6 present the 90% and 95% confidence interval for the t-statistics of each dummy and the

Table 5. Bootstrap confidence interval for t-stat and F-stat for dummies

Wholesale Price Index		D1	D2	D3	F
US/UK	90%	(-3.2,3.1)			
	95%	(-4.4,4.0)*			
US/Germany	90%	(-2.8,2.8)	(-2.5,2.5)	(-2.36,2.1)	3.34
	95%	(-3.5,3.4)*	(-3.2,3.2)*	(-3.0,2.6)*	4.41*
Germany/UK	90%	(-3.9,4.0)	(-3.45,3.40)	(-3.10,3.15)*	2.84
	95%	(-4.9,5.0)	(-4.35,4.25)*	(-3.90,4.0)	3.60*
US/France	90%	(-3.10,3.35)*	(-3.30,3.05)	(-2.85,2.45)	3.60
	95%	(-4.05,4.80)	(-4.40,4.15)	(-4.20,3.15)*	6.00*
France/UK	90%	(-2.24,2.36)	(-1.88,2.10)*	(-2.01,1.98)*	3.96
	95%	(-3.01,3.08)*	(-2.34,2.56)	(-2.75,2.69)	5.02*
France/Germany	90%	(-3.25,2.65)	(-2.85,2.65)	(-2.60,2.25)	3.45
	95%	(-4.25,3.25)*	(-3.80,3.45)*	(-3.35,3.00)*	4.70*
Belgium/UK	90%	(-2.46,2.36)	(-2.3,2.1)	(-2.2,2.1)	4.55
	95%	(-3.02,2.92)*	(-2.9,2.7)*	(-2.8,2.6)*	5.55*
Belgium/US	90%	(-2.65,2.55)	(-2.55,2.40)		3.71
	95%	(-3.32,3.22)*	(-2.95,3.00)*		4.75*
Belgium/Germany	90%	(-2.48,2.18)			
	95%	(-3.22,3.02)*			
Belgium/France	90%	(-1.60,1.60)			
	95%	(-1.94,1.92)*			

Table 6. Bootstrap confidence interval for t-stat and F-stat for dummies

Consumer Price Index		D1	D2	D3	D4	D5	F
Sweden/Germany	90%	(-2.28,2.91)	(-2.40,2.40)				3.25
	95%	(-2.86,3.72)*	(-3.00,3.05)*				4.25*
France/Sweden	90%	(-2.60,2.55)	(-2.20,2.12)				3.24
	95%	(-3.30,3.20)	(-2.66,2.70)				4.32
France/Germany	90%	(-2.40,2.38)	(-2.35,2.35)				5.25
	95%	(-3.10,3.02)*	(-3.25,3.05)				7.40
France/Belgium	90%	(-3.15,3.15)					
	95%	(-4.15,4.01)					
Belgium/Sweden	90%	(-2.10,2.10)	(-2.05,2.10)	(-2.04,2.05)			3.60
	95%	(-2.60,2.65)*	(-2.60,2.70)*	(-2.60,2.70)*			4.55*
Belgium/Germany	90%	(-2.80,2.95)	(-3.00,2.80)	(-2.95,2.70)	(-2.80,2.55)*	(-2.55,2.45)	2.50
	95%	(-3.46,3.78)*	(-3.84,3.60)*	(-3.60,3.30)*	(-3.45,3.20)	(-3.20,3.00)	3.12*

F-test. On the basis of the F-statistics obtained in the nonlinear estimation and the critical values from the bootstrap, we can reject the null of joint non-significance of the dummies in all cases except for the France/Sweden and France/Germany CPI real rates. With regard to the particular dummy variables, some of them cannot be considered significant within this framework.[13] Hegwood and Papell (2002) provide some historical support for some of the dummies they found significant in their study. Our results support the significance of most of those dummies: the 1864 dummy in the U.S. real exchange rate coinciding with the American Civil War, the 1866 dummy in the German real exchange rates coinciding with the dissolution of the German Confederation, and the dummies of the forties when there was "widespread protest, rebellion, and revolution in Europe" (Cook and Stevenson 1998, p.460).

4. Nonlinear Half-Lives of Shocks

In this section we compare the speed of mean reversion of the nonlinear model of real exchange rates with constant equilibrium as well as shifting equilibrium comparing with the speed of mean reversion of the linear model as in Hegwood and Papell (2002). To calculate the half-lives of PPP deviations within the nonlinear framework we must take into account that a number of properties of the impulse response functions of linear models do not carry over to the nonlinear models.[14] Koop, Pesaran, and Potter (1996) introduced the Generalized Impulse Response Function (GIRF) for nonlinear models. The GIRF is defined as the average difference between two realizations of the stochastic process $\{y_{t+h}\}$ which start with identical histories up to time $t-1$ (initial conditions). The first realization is hit by a shock at time t while the other one is not:

$$GIRF_h(h, \delta, \omega_{t-1}) = E(y_{t+h}|u_t = \delta, \omega_{t-1}) - E(y_{t+h}|u_t = 0, \omega_{t-1}), \quad (3)$$

where $h = 1, 2, ..,$ denotes horizon, $u_t = \delta$ is an arbitrary shock occurring at time t, and ω_{t-1} defines the history set of y_t.

Given that δ and ω_{t-1} are single realizations of random variables, Equation 3 is considered to be a random variable. In order to obtain sample estimates of Equation 3, we average out the effect of all histories ω_{t-1} that consist of every set $(y_{t-1}, ..., y_{t-p})$ for $t \geq p + 1$ where p is the autoregressive lag length, and we also average out the effect of future shocks u_{t+h}.[15] We set $\delta = 5\%, 10\%, 20\%, 30\%$, and 40%. The different values of δs would allow us to compare the persistence of very large and small shocks. As in Taylor, Peel, and Sarno (2001) and in Paya, Venetis, and Peel (2003), Tables 7 and 8 report the half-lives of shocks, that is, the time needed for $GIRF_h < \frac{1}{2}\delta$.[16] The last columns of both Table 7 and Table 8 correspond to the speeds of adjustment found in the linear models of Hegwood and Papell. For the nonlinear models with constant equilibrium, Table 7, the half-life of the shocks decreases significantly for shocks of around 30%, or even 40% in some cases. However, compared with the linear case, even with the smallest shock of 5% the speed of mean reversion is faster in the ESTAR model. When the equilibrium is allowed to change over time, Table 8, the "arbitrage band" in the nonlinear model, seems to lie around 20% or even 10%. In this case, ten out of the sixteen real exchange rates need a shock of 20% to achieve faster adjustment than the linear case.

5. Conclusions

Hegwood and Papell (2002) analyzed PPP adjustment under the Gold Standard. They novelly allowed for structural breaks in the equilibrium real exchange rate and suggested

Table 7. Estimated half-lives shocks in months for annual model

	$\hat{\gamma}$	*Shock*	5%	10%	20%	30%	40%	Linear
Real rate WPI								
US/UK	-3.12		5	4	3	2	2	4.04
US/Germany	-2.52		4	4	4	3	2	3.24
Germany/UK	-1.52		6	6	5	2	2	5.72
France/UK	-11.8		5	4	3	1	0	3.22
France/US	-7.42		3	3	2	1	0	7.85
France/Germany	-2.89		5	4	4	2	1	5.77
Belgium/UK	-2.49		7	7	7	6	4	9.24
Belgium/France	-2.98		5	5	5	4	2	9.66
Real rate CPI								
Sweden/Germany	-2.41		5	5	5	4	3	5.99
France/Sweden	-13.7		4	4	2	1	0	5.24
France/Germany	-7.17		3	3	2	2	2	4.08
France/Belgium	-4.53		5	5	4	3	3	7.31
Belgium/Sweden	-4.75		5	4	3	1	0	7.40

Table 8. Estimated half-lives shocks in months for annual model with dummies								
	$\hat{\gamma}$	*Shock*	5%	10%	20%	30%	40%	Linear
Real rate WPI								
US/UK	-4.28		5	4	3	2	1	2.51
US/Germany	-4.43		3	3	3	2	1	1.24
Germany/UK	-8.50		3	3	2	1	0	1.25
France/UK	-30.7		3	2	0	0	0	1.42
France/US	-34.1		2	2	0	0	0	2.54
France/Germany	-27.5		2	2	1	0	0	1.42
Belgium/UK	-186		1	0	0	0	0	0.83
Belgium/US	-0.88		6	5	4	3	2	1.68
Belgium/Germany	-8.96		4	4	3	2	1	1.26
Belgium/France	-56.9		1	0	0	0	0	1.43
Real rate CPI								
Sweden/Germany	-17.4		3	2	1	0	0	1.13
France/Germany	-7.63		2	1	1	1	0	1.13
Belgium/Sweden	-255		1	0	0	0	0	0.61
Belgium/Germany	-20		2	2	1	0	0	1.07

that relatively quick linear adjustment to an occasionally changing mean provides a more reasonable representation of the data than does the fractional process, with its long memory property, for their data set - the latter originally reported by Diebold, Husted, and Rush (1991) .

In this article we have shown that the theoretically well-motivated nonlinear ESTAR process, embodying structural breaks in the equilibrium real rate, provides a parsimonious explanation of the data set.

As conjectured by Rogoff (1996), and supported by our analysis and that of Hegwood and Papell (2002), allowance for a changing real equilibrium can have dramatic implications for the estimates of PPP adjustment speeds. On the basis of these results empirical work exploring this possibility in postwar data may help solve the Rogoff puzzle.

References

Acosta, Aparicio F.M. , and Clive W.J. Granger. 1995. A linearity test for near-unit root time series. Discussion paper no. 95-12, University of California San Diego, San Diego, CA.

Bai, Jushan, and Pierre Perron. 1998. Estimation and testing linear models with mulitple structural changes. *Econometrica* 66: 47-78.

Balassa, Bela. 1964. The purchasing-power parity doctrine: A reappraisal. *Journal of Political Economy* 72: 584-96.

Berka, Martin. 2002. General equilibrium model of arbitrage trade and real exchange rate persistence. Unpublished paper, University of British Columbia.

Byers, David, and David A. Peel. 2003. A nonlinear time series with misleading linear properties. *Applied Economics Letters* 10: 47-51.

Canzoneri, Matthew B., Robert E. Cumby, and Behzad T. Diba. 1996. Relative labor productivity and the real exchange rate in the long run: Evidence for a panel of OECD countries. *Journal of International Economics* 47: 245-66.

Chinn, Menzie, and Louis Johnston. 1996. Real exchange levels, productivity and demand shocks: Evidence from a panel of 14 countries. NBER Working Paper No. 5709.

Cook, Chris, and John Stevenson. 1998. *Longman handbook of modern European history 1763-1997,* 3rd edition. New York: Addison Wesley Longman.

Diebold, Francis X., Steven Husted, and Mark Rush. 1991. Real exchange rates under the gold standard. *Journal of Political Economy* 99: 1252-71.

Dumas, Bernard. 1992. Dynamic equilibrium and the real exchange rate in a spatially separated world. *Review of Financial Studies* 5: 153-80.

Eitrheim, Oyvind, and Timo Terasvirta. 1996. Testing the adequacy of smooth transition autoregressive models. *Journal of Econometrics* 74: 59-75.

Froot, Kenneth A., and Kenneth Rogoff. 1995. Perspectives on PPP and long-run real exchange rates. In *Handbook of International Economics,* Volume 3, edited by G.M. Grossman and K. Rogoff. Amsterdam: North-Holland.

Granger, Clive W.J., and Timo Terasvirta. 1999. Ocasional structural breaks and long memory. Discussion Paper 98-03, University of California San Diego, San Diego, CA.

Hegwood, Natalie D., and David H. Papell. 2002. Purchasing power parity under the Gold Standard. *Southern Economic Journal* 69: 72-91.

Koop, Gary, Hashem Pesaran, and Simon M. Potter. 1996. Impulse response analysis in nonlinear multivariate models. *Journal of Econometrics* 74: 119-47.

Lothian, James R., and Mark P. Taylor. 1996. Real exchange rate behavior: The recent float from the perspective of the past two centuries. *Journal of Political Economy* 104: 488-507.

Michael, Panos, Robert Nobay, and David A. Peel. 1997. Transactions costs and nonlinear adjustment in real exchange rates: an empirical investigation. *Journal of Political Economy* 105: 862-79.

Newey, Whitney, and Kenneth West. 1987. A simple positive semi-definite, heteroscedasticity and autocorrelation consistent covariance matrix. *Econometrica* 55: 703-8.

O'Connell, Paul, and Shang-Jin Wei. 1997. The bigger they are, the harder they fall: How price differences between U.S. cities are arbitraged. *Discussion Paper,* Department of Economics, Harvard University.

Ozaki, Tohru. 1985. Nonlinear time series models and dynamical systems. In *Handbook of Statistics V* pp. 25-83. E.J. Hannan, P.R. Krishnaiah, and M.M. Rao, Amsterdam: Elsevier.

Paya, Ivan, and David A. Peel. 2003. Temporal aggregation of an ESTAR process: Some implications for PPP adjustments. Unpublished paper, Cardiff Business School.

Paya, Ivan, Ioannis A. Venetis, and David A. Peel. 2003. Further evidence on PPP adjusment speeds: The case of effective real exchange rates and the EMS. *Oxford Bulletin of Economics and Statistics* 65: 421-38.

Peel, David A., and Ioannis A. Venetis. 2002. Purchasing power parity over two centuries: trends and non-linearity. *Applied Economics* 35: 609-17.

Perron, Pierre, and Timothy J. Vogelsang. 1992. Nonstationarity and level shifts with an application to purchasing power parity. *Journal of Business and Economic Statistics* 10: 301-20.

Pippenger, Michael K., and Gregory Goering. 1993. A note on the empirical power of unit root tests under threshold processes. *Oxford Bulletin of Economics and Statistics* 55: 473-81.

Rogoff, Kenneth. 1996. The purchasing power parity puzzle. *Journal of Economic Literature* 34: 647-68

Samuelson, Paul**.** 1964. Theoretical notes on trade problems. *Review of Economics and Statistics* 46: 145-54.

Sercu, Piet, Raman Uppal, and C. Van Hull. 1995. The exchange rate in the presence of transactions costs: Implications for tests of purchasing power parity. *Journal of Finance* 50: 1309-19.

Taylor, Alan M. 2001. Potential pitfalls for the purchasing power parity puzzle? Sampling and specification biases in mean-reversion tests of the taw of one price. *Econometrica* 69: 473-98.

Taylor, Mark P., David A. Peel, and Lucio Sarno. 2001. Nonlinear mean-reversion in real exchange rates: Toward a solution to the purchasing power parity puzzles. *International Economic Review* 42: 1015-42.

Terasvirta, Timo. 1994. Specification, estimation and evaluation of smooth transition autoregressive models. *Journal of the American Statistical Association* 89: 208–18.

Tse, Yi. 2001. Index arbitrage with heterogeneous investors: A smooth transition error correction analysis. *Journal of Banking and Finance* 25: 1829-55.

Venetis, Ioannis A., Ivan Paya, and David A. Peel. 2002. Do real exchange rate 'mean revert' to productivity? A nonlinear approach. Unpublished paper, Cardiff Business School.

Notes

[1]Even in high frequency asset markets the idea of heteregeneous traders facing different capital constraints or percieved risk of arbitrage has been employed to rationalise employment of the ESTAR model. See, for example, Tse (2001) for arbitrage between stock and index futures.

[2]Namely, how to reconcile the enormous short run volatility of real exchange rates with the extremely slow rate at which shocks appear to damp out-in linear models, around 3-5 years, seemingly far too long to be explained by nominal rigidities.

[3]He suggests for instance that the sustained Post Bretton Woods war appreciation of Japan's real exchange rate against the Dollar is consistent with the Balassa-Samuelson (BS) effects, in fact, he calls it the "canonical" example of BS effects.

[4]We know from the analysis of Taylor (2001) that if the true data generation process is nonlinear then the use of the linear models can severely underestimate the speed of adjustment particularly if the low frequency data is temporally aggregated.

[5]We thank Hegwood and Papell for kindly proving us with the data.

[6]A key property of some ESTAR models (also shared by some Threshold models) is that data simulated from them, although globally mean reverting, can appear to exhibit a unit root. As a consequence, the test proposed in Froot and Rogoff (1995), namely, that we impose unit coefficients and test directly, employing unit root tests, whether PPP deviations are mean reverting, can have low power if the true data generating process is nonlinear.

[7]We note in this article, as pointed out by a referee, we test for nonlinear mean reversion while assuming that PPP holds. In this article, it is assumed that there is reversion to a changing mean, and what is being tested is the form of the reversion. As a consequence, the standard errors reported for the ESTAR have classical values. There is a common misconception that research on nonlinear adjustment to PPP tests a unit root null against a nonlinear mean-reverting alternative.

[8]This variation is not surprising if we assume that the true data generating process (DGP) of the real exchange rate is generated at a higher frequency, that is, monthly. In that case, Paya and Peel (2003) show that temporal aggregation of the true monthly DGP into, for instance, annual data induces additional autoregressive terms in the ESTAR model.

[9]We observe from Equation 1 that when $\sum_{i=1}^{n}\beta_i = 1$ if $\widehat{\gamma} = 0$, PPP deviations follow a unit root. As a consequence of this point there is perhaps a conception that research on nonlinear adjustment to PPP tests, in general, a unit root null against a nonlinear mean reverting alternative. This need not be the case. Given the results reported previously, we assume, as noted above, that PPP holds and test for the form of reversion.

[10]See Hegwood and Papell (2002) for an explanation of potential causes of the different breaks. We recognise that the breaks were obtained from estimates of a linear structure. Our Monte Carlo evidence suggests the breaks are in the majority significant.

[11]In particular, for the WPI rates we removed the third dummy of the Beligium/Germany, third dummy in the Belgium/US, fourth dummy in the France/UK, and the fourth dummy in the US/Germany rate. For the CPI rates, we removed the first dummy of the France/Sweden rate and the third dummy of the Sweden/Germany rate.

[12]The bootstrapped residuals were centered on zero and scaled. The scaling factor is (n/n-k)^0.5.

[13]In particular, the first dummy of the Germany/UK WPI rate, second dummy France/US WPI rate, second dummy France/UK WPI rate, second dummy France/Germany CPI rate, fifth dummy Belgium/Germany rate, and both dummies of the France/Sweden CPI rate.

[14]In particular, impulse responses produced by nonlinear models are (i) history dependent, so they depend on initial conditions, (ii) they are dependent on the size and sign of the current shock, and (iii) they depend on the future shocks as well. That is, nonlinear impulse responses critically depend on the “past, present, and the future”.

[15]For each available history, we use 300 repetitions (500 repetitions found similar result) to average out future shocks where future shocks are drawn with replacement from the models' residuals and then we average the result across all histories. We set to $\max\{h\} = 48$.

[16]The France/Sweden and France/Belgium CPI exchange rates have not been included in Table 8 because the dummy variables were not jointly significant according to the bootstrap confidence interval presented in Table 6. In the case of the France/Germany CPI rate we include the dummy that appears significant under the bootstrap confidence interval.