

A discusión

RISK-SHARING AS A DETERMINANT OF CAPITAL STRUCTURE: INTERNAL FINANCING, DEBT, AND (OUTSIDE) EQUITY*

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ABSTRACT

This paper proposes a historically-grounded mechanism-design model of corporate finance, with two-side risk aversion under limited contract enforceability, where (inside) equity held by entrepreneurs, debt and (outside) equity coexist. This capital structure shares optimally the non-diversifiable risk associated with costly and risky ventures. Furthermore, it uniquely sustains the optimal risk allocation if agents' personal wealth is contractible at a higher enforcement cost than the projects' returns. Otherwise, the irrelevance theorem of Modigliani and Miller applies. Consistent with the theoretical predictions, we observe that (i) risk-averse merchants-entrepreneurs financed part of their ventures (hold inside equity) and raised additional funds from risk-averse investors through debt-like sea loan and equity-like commenda contracts when long-distance medieval trade was indeed highly costly and risky and that (ii) maritime insurance, with higher protection against the non-diversifiable "risk of loss at sea or from the action of men" but higher enforcement costs, did not develop until the mid-fourteenth century, when the ventures' costs and risk had decreased significantly. Whereas the model emphasizes the entrepreneurs' equity holdings and the limited-liability aspects of debt and equity, the choice between debt or equity derives from simple, although historically backed, information assumptions. The analysis is therefore complementary to other capital-structure theories based on agency costs, information asymmetries, signalling, transaction costs and incomplete contracting.

Keywords: debt contracts, capital structure, creditworthiness, enforceability, inside and outside equity, insurance, limited liability, private information, risk-sharing

JEL classification numbers: D81, D82, G22, G32, N23.

1 Introduction

“[T]he emphasis [of corporate finance] on large companies with dispersed investors has underemphasized the role that different financing instruments can play to provide investors better risk diversification. If all companies’ stock is held by well-diversified investors, there is little need for additional diversification. Unfortunately, this convenient assumption does not seem to hold in practice.”

[Zingales, *Journal of Finance*, 55.4 (2000), p.1628]

“To judge the extent to which today’s methods of dealing with risk are either a benefit or a threat, we must know the whole story, from its very beginnings.”

[Bernstein, “Against the Gods. The Remarkable Story of Risk,” 1996, p.7]

For the last fifty years or so Corporate Finance has focused on various departures from the Modigliani and Miller’s theorem that make capital structure relevant to a firm’s value under the assumption that economic agents are risk neutral (Harris and Raviv, 1991, Hart, 1995, and Freixas and Rochet, 1997, among others, survey this literature). Some models take the form of the securities (contracts) issued by the firm, mainly debt and equity, as exogenously given; others endogenously derive the characteristics of debt and equity in terms of cash flows and/or control rights; but all predict a maximum inside participation rate to the firms’ investments. Since contracts are chosen or designed exclusively to minimize the costs associated with agency relations, information asymmetries, signaling, incomplete contracting or transacting, the best an entrepreneur can do is to finance the project he manages on its entirety, unless restricted to rely on external funds because of budget constraints. However, the evidence indicates that entrepreneurs/firms both are risk averse and resort to external funds before investing 100 percent of their own wealth (for evidence on these respects, see Zingales, 2000 and Zingales, 1995, table IV, p. 1439).

A notable exception is Leland and Pyle (1977). They exploit managerial risk aversion to obtain a signaling equilibrium in which entrepreneurs retain a higher fraction of (inside) equity than they would hold in the absence of adverse selection, but still rely on external funds because of diversification. Leland and Pyle thus rationalize the observed fact that

entrepreneurs self-finance only a part of their ventures, but do not address the fundamental question of why debt and equity are designed the way they are.

This paper offers a mechanism-design model where (inside) equity held by entrepreneurs, debt and (outside) equity emerge optimally under two-side risk aversion and limited contract enforceability. To evaluate the empirical relevance of the model, the paper confronts its assumptions and predictions with evidence from a historical episode in which risk-sharing was unquestionably important. In particular, it investigates the financial relations between merchants and their potential financiers in Venice during the Commercial Revolution from the eleventh to the fourteenth centuries.

Like in Leland and Pyle (1977) and more generally the optimal portfolio selection theory under risk aversion, entrepreneurs rely on external funds because of risk-sharing. Unlike traditional capital structure theories, the specific form of contracts (securities) under which funds are supplied is endogenously derived, but only in terms of cash-flows. In fact, the model is closest to the costly state verification/falsification literature (Townsend, 1979; and Gale and Hellwig, 1985/Lacker and Weinberg, 1989) which, in a one-period comprehensive contracting framework, derives the optimal income rights of debt or equity. However, it differs in its main assumptions and results.

In the model, all agents are risk-averse; each might be endowed with the resources required to finance the venture by his own— so that entrepreneurs must not rely on external funds because of budget constraints; there are both a downside risk and a commercial risk, the former accounting for the possibility of losing part or all of the (fixed) capital invested; and there is no verification/falsification cost. On the contrary, the model considers two exogenous information structures and introduces enforcement costs. Contracts are thus designed to attain the optimal allocation of risk, given the information structure, while minimizing enforcement costs.

Whereas the model emphasizes the entrepreneurs' equity holdings and the limited-liability aspects of debt and equity, the choice between debt or equity is exogenously driven by simple information assumptions: under hidden information debt emerges as a second-best because contracts cannot be contingent on the non-verifiable commercial return; under full information first-best equity contracts are possible. In contrast to the costly state-verification literature, in which the threat of verification is not credible because verification is ex-post inefficient, debt is optimal under hidden information even if stochastic verification-bankruptcy schemes, like auditing, are allowed because the verification technology is exogenously given. Moreover, debt and equity can co-exist under different information structures and are robust to the allocation of bargaining power. However, the model does not account for the equity's linearity features, which would require introducing some sort of enforcement (falsification) cost. Last but not least, the model rationalizes the observed fact that entrepreneurs hold inside equity but rely on debt and outside equity before investing all their wealth in the projects they manage.

The paper first considers the case in which the entrepreneur is constrained to rely on external funds because of a shortage of his own and delivers debt and equity as corner solutions if and only if ventures are highly costly and risky. Because of the binding budget constraint, the entrepreneur cannot repay any amount above the venture's return, which in case of loss is dreadfully insufficient to reward the financier's capital investment. To compensate for the possibility of losing his capital, the (risk-averse) financier calls for soaring payments otherwise. As a consequence, his consumption varies widely, whereas that of the entrepreneur is invariably small. The lesser the amount paid out in the case of failure, the higher the repayment required otherwise and the more volatile the financier's event-consumption relative to the merchant's. Because both agents are assumed to be equally risk averse, the financier values consumption in the event of loss relatively more than the

entrepreneur, and it is optimal that the investor recoup as much of his capital as possible from the venture's return. In other words, debt and equity, with repayment equal to all what is saved from a firm's assets in case of failure, emerge optimally, one or the other depending on the information structure. Yet, these contractual forms provide too much insurance to the entrepreneur against the downside risk. Both the entrepreneur and his potential financier would prefer a contract with higher repayment in the case of loss, but this is not feasible when the merchant lacks the resources required to fund the venture (debt and equity are corner solutions).

The model is then extended to consider the case in which the entrepreneur is endowed with sufficient capital to fund the venture on his own. From the observation that debt and equity are corner solutions, it follows that a well-endowed entrepreneur will not optimally raise funds for the whole trading venture through debt and equity, but will rather finance part of the venture, thereby bearing the liability of loss for the corresponding amount and effectively receiving less insurance against the downside risk. In other words, a capital structure with (inside) equity held by the entrepreneur, debt and (outside) equity shares risk efficiently.

Yet, debt and equity are not the only contracts through which the optimal risk allocation can be attained. The Modigliani and Miller irrelevance theorem, though, does not apply because of enforcement cost. In particular, a capital structure with (inside) equity holdings and limited liability debt and/or (outside) equity is preferable to a combination of risk-free debt plus premium insurance because the cost of enforcing contracts contingent on the ventures' returns is assumed to be lower than from the agents' personal wealth.

The organization of the paper is as follows. Section 2 sets the model and characterizes various historically observed contracts. Sections 3 to 5 solve the contracting problem for various parameter values and evaluate empirically various theoretical predictions generated

by the model. Section 6 comments on the contract's nominal indeterminacy and proposes enforcement cost to determine a unique capital structure. Finally, section 7 concludes.

2 The Model

Consider a one-period two-dates contracting economy with a merchant-entrepreneur and a financier who face the possibility to undertake a single welfare-enhancing investment project, say a trading venture. Both agents are risk averse with preferences represented by continuous and twice-differentiable utility functions $U_i(c_i)$ exhibiting a decreasing absolute risk aversion (DARA) coefficient with $U'_i(\cdot) > 0$, $U''_i(\cdot) < 0$, where c denotes consumption and the subscript $i = 1, 2$ stands for the merchant and the financier, respectively.¹ Each agent i is initially endowed with k_i units of good and maximizes his expected utility from second-date consumption. Goods can be costlessly self-stored or invested in the trading venture. Any amount of the good can be stored but the investment is lumpy. The venture requires k units of investment, plus the entrepreneurship of the merchant. It yields a random return $s \in S = \{y, \underline{x}, \bar{x}\}$ with probability p_s , where y denotes the loss due to the downside risk and $\{\underline{x}, \bar{x}\}$ stand for the risky commercial return, with $0 \leq y \ll k \ll \underline{x} < \bar{x}$ and $k < E[s]$. In the historical context under study, the downside risk refers to the so-called “risk of sea and people” and includes the risk of natural shipwreck, piracy and confiscation of property by foreign rulers. The commercial risk accounts for wide variations in profits depending on the tariffs and bribes paid in customs, transportation and storage fees, the conditions of the goods upon arrival after hazardous trips, fluctuations in prices, and so on and so forth.

To account for the optimality of undertaking the investment project we assume that

$$E[U_i(s)] > U_i(k) \quad \forall i, \tag{1}$$

¹Note that Innada conditions are not assumed to hold, so that we do not force an interior solution.

which in combination with DARA ensures that the higher productivity of the trading venture ($E[s] > k$) compensates for its risk. Therefore, agents will only self-store and consume their autarkic endowments, k_i , if they are unable to mobilize funds in long-distance trade. For ease of notation, let $w(s) = k_1 + k_2 - k + s$ be the second-date total risky endowment of the economy when the venture is undertaken. We first study the case in which the merchant is constrained to rely on external funds because of budget constraints and, for simplicity, he is assumed to have zero initial wealth, $k_1 = 0$. Then, the model is extended to capture the situation in which the merchant can finance the venture entirely on his own, $k_1 > k$. The financier is always endowed with enough resources to fund the venture, $k_2 > k$.

At the first date, when investment decisions are taken, there is no information whatsoever on the true realization of the venture return. At the second date the return is realized and revealed to the merchant. Based on historical evidence, the model assumes that the State both provides contract enforcement and can verify losses “at sea or from the action of enemies,” and considers two extreme scenarios regarding the ability of the State to verify the commercial return (González de Lara, 2004a). Under the hidden information structure the commercial returns is non-verifiable, at any cost, but the State can force the merchant pay out the minimum possible commercial return, \underline{x} . Under the full information structure the commercial return is verifiable without any private cost for the financier.

Before proceeding, several comments are in order. First, to focus on the role that financial contracts can play in providing better risk-sharing, the model characterizes the non-diversifiable risk associated with pre-modern long-distance trade. As a result, the model does not reveal diversification although, in reality, each trading venture entailed an idiosyncratic risk that could be and actually was diversified to some extent. This leading assumption is justified by the historical evidence, which clearly indicates that the presence

of significant indivisibilities and aggregate risk limited the degree of risk spreading that the economy could achieve and prevented agents from effectively becoming risk neutral.

Since evidence on the agents' limited ability to diversify has been presented in detail somewhere else (González de Lara, 2004a), a few examples will suffice here. The high start-up cost associated with manning and outfitting a typical medieval ship— with a crew of a hundred and about 200 tons capacity during the thirteenth century— prevented the undertaking of a sufficiently large number of ventures during a period in which capital was the more scarce. Furthermore, since the few ventures that were actually financed concentrated on the most profitable Levantine trade, they were exposed to the aggregate risk of having their proceeds impounded by a politically hostile ruler, as it happened in 1171 when the Byzantine Emperor ordered the arrest of all the Venetians in his territory (estimated to sum up to 20,000 over a total population beneath 100,000) and the confiscation of all their goods.

Second, to keep the structure as simple as possible, the model assumes that the State functioned as an enforcement and information-transmission mechanism. As a result the model ignores the role of adverse selection, moral hazard with hidden effort/action, incomplete contracting and private-order institutions in shaping contracts, although in reality these forces might play an important role. The assumption that the legal system can enforce contracts contingent on verifiable information is standard in the Finance and Contract Theory literatures. In late-medieval Venice the State, in addition to developing legal institutions, generated the stream of rents required to induce the Venetians to keep their affiliation with the city, thereby enabling merchants to commit not to flee with their investors' capital despite the limited ability of a medieval court to exercise its coercive power over a merchant who emigrated.² Transaction Costs Economics, as well as the Costly State-

²For a historical institutional analysis of the emergence of the Venetian State as an institution for

Verification literature, further assumes that the legal system can enforce contracts but at a cost. The model described above does not consider any enforcement cost, although section 6 relaxes this unrealistic assumption.

The information structures reflect the changing State's ability to verify information. In far-off markets merchants were the only ones to know the bribes they had paid to pass customs, the damages (if any) suffered by their wares while on transit or the true price received and paid for their goods. However, in well-established markets public officials carefully monitored commercial ventures in each and all of their phases. Thus, institutional arrangements that enabled the Venetians to consolidate their colonial and commercial empire in the East enhanced the State's ability to verify information, leading from hidden information to full information in the model (see González de Lara, 2004a).

2.1 Contracts

We consider allocations that can be obtained by means of a *contract* enabling the funding of the venture and specifying transfers from the merchant to the financier both at the first and at the second date. Whereas first-date transfers $\tau \in \mathfrak{R}$ are prior to the realization of the state and, accordingly, are independent of it, second-date transfers $\tau(s)$ are contingent on the state. A contract will result in *consumption schedules* for the merchant and the financier, c_1 and c_2 respectively, where $c_1 := c_1(s) = k_1 - k + s - [\tau + \tau(s)]$ and $c_2 := c_2(s) = k_2 + [\tau + \tau(s)]$.³

Like today's entrepreneurs, medieval merchants typically self-financed only part of their

contract enforcement, see González de Lara, 2004b.

³These consumption schedules capture the fact that the venture's returns first accrued to the merchant, who need to cover the fix cost k to generate the return s . Moreover, the evidence suggests that in pre-modern Venice the merchant acquired the ownership of the venture returns and that he was the residual claimant. Yet, ownership in the model is irrelevant because (i) this study is robust to the allocation of property rights (it considers *all* levels of bargaining power subject to individual rationality) and (ii) there is no incomplete contracting and, consequently, the allocation of control rights is unimportant. Formally, nothing would change if the financier, agent 2, had the right to the venture's returns and consumption were expressed as $c_1(s) = k_1 + \check{\tau} + \check{\tau}(s)$ and $c_2(s) = k_2 - k + s - \check{\tau} - \check{\tau}(s)$, with these new transfers to be understood as a contingent salary payment to the merchant for his entrepreneurship. The optimal transfers will still give rise to the same optimal allocation of consumption, with $\check{\tau} = -k - \tau$ and $\check{\tau}(s) = s - \tau(s)$.

ventures— ϕk , with $\phi \in (0, 1)$ — and raised the remaining capital— $\tau = -(1 - \phi)k$ — through debt-like sea loan and/or equity-like commenda contracts. Both the sea loan and the commenda established repayment equal to all what was saved from misfortune on the event of loss “at sea or from the action of hostile people.” Thus, merchants-entrepreneurs were partially insured against the downside risk, for they were exempted from repayment beyond the amount retrieved from a loss — $\tau(y) = y < k$ — but they did not enjoy full insurance, which would have required a coverage payment— $\tau(y) < 0$ instead of $\tau(y) = y \geq 0$ — to offset the merchant’s lack of gain in the event of loss. The sea loan and the commenda differed only in the transfers they established in the case the ship arrived safe and sound in port. Whereas the sea loan paid out a fixed constant, the commenda divided the commercial profit according to a ratio agreed upon, and thus shared not only the downside risk but also the commercial risk between the merchant and his financier. In short, the sea loan and the commenda established the same income-rights as today’s debt and equity contracts. They were, however, silent about the allocation of control rights.

Definition 1 *A sea loan contract (debt) establishes*

$$-k \leq \tau < 0 \text{ and } \tau(y) = y < \tau(\underline{x}) = \tau(\bar{x})$$

Definition 2 *A commenda contract (equity) establishes*

$$-k \leq \tau < 0 \text{ and } \tau(y) = y < \tau(\underline{x}) < \tau(\bar{x})$$

But, if the debt-like sea loan sustained a worse risk allocation than the equity-like commenda (risk averse merchants bore all the commercial risk), why was it used? Standard contract theory informs us that in a one-period model contracts can only be made contingent on states whose occurrence can be verified to the satisfaction of all contracting parties (incentive compatibility). In other words, financial contracts in the model under hidden information cannot depend on merchant’s private information, thereby ruling out commenda contracts. González de Lara (2004a) identifies empirically various situations in

which the State could not verify the true venture return and others in which the State could, and find that the sea loan and the commenda were used as predicted: the sea loan when and where hidden information prevented the merchant from credibly committing not to misappropriate some of the return; the better-risk sharing commenda when and where various institutional developments alleviated this hidden information problem.

Yet, many questions should be addressed. Why were the financiers entailed to all the capital saved from misfortune in case of loss— $\tau(y) = y$ — or, put it differently, why were both the entrepreneur and the financier “protected” with limited liability? Why did the merchants finance part of their ventures and raise additional funds through debt-like sea loan and equity-like commenda contracts before investing 100 percent of their own resources— $\phi \in (0, 1)$? Did the observed capital structure provide the optimal allocation of risk? And why did the Venetians provided insurance through the credit market? Indeed, later in the fifteenth and sixteenth centuries, credit and insurance were furnished through two distinct contracts. Overseas trade was then typically funded through risk-free bills of exchange and insured through premium contracts, with the particularity that the premium was commonly paid after the insured cargo was safely arrived to destination.⁴ The formal model provides the foundations for addressing these questions.

Definition 3 *A bill of exchange (risk-free debt) establishes*

$$-k \leq \tau < 0 \text{ and } \tau(y) = \tau(\underline{x}) = \tau(\bar{x})$$

Definition 4 *A pure insurance contract establishes either $\tau > 0$ with at least one $\tau(s) < 0$ (as today’s premium insurance contracts) or $\tau = 0$ with at least one $\tau(s) < 0$ and other $\tau(s) > 0$ (as commonly happened in pre-modern Europe). In brief, a pure insurance contract is such that, at date 1, the entrepreneur receives in some states a positive payment from his financier-insurer, i.e. $\exists s : \tau(s) < 0$.*

⁴The bill of exchange was an order to pay in one place in one kind of money because of a payment received in a different place in a different kind of money. There was always a time lag between receipt and payment, so that one of the parties was extending credit to the other in the meantime.

2.2 Optimal Contracts

A contract is optimal if and only if it sustains an optimal allocation of consumption, which is defined as the solution to the following (concave) problem for some given value of \bar{U}_2 .⁵ Further, we restrict our attention to contractual forms that sustain the optimal allocations for *all* individually rational levels of \bar{U}_2 , i.e. regardless of the competitive structure of the capital market. Thus, the model can explain the variety of financial relations observed in Venice, where it was as common that non-noble merchants of low means raised capital from the leading aristocracy as that rich and politically-powerful merchants got funds from medium-class artisans. In other words, the model— in contrast with Lacker and Weinberg (1989), Bolton and Scharfstein (1990) and Hart and Moore (1998), where optimal contracts are not robust to the allocation of bargaining power— can be applied to both venture and well-functioning capital markets.

Program 1:

$$\begin{aligned} \max_{\{c_i(s)\}_{i=1,2}} \quad & E\{U_1[c_1(s)]\} \\ \text{s.t.} \quad & E\{U_2[c_2(s)]\} \geq \bar{U}_2 \end{aligned} \tag{2}$$

$$c_i(s) \geq 0 \quad \forall i, \forall s \tag{3}$$

$$\sum_i c_i(s) \leq w(s) \quad \forall s. \tag{4}$$

Restrictions (3)-(4) are feasibility constraints. Consumption cannot be negative and cannot exceed total resources. Constraint (4) holds with equality, since resources are not spared optimally. Therefore, $c_1(s) = w(s) - c_2(s)$ and the problem can be solve on $c_2(s)$ for $s \in S$. It also follows that (2) holds with equality, so that the concave utility possibilities frontier can be traced out as the parameter \bar{U}_2 is varied. Individual rationality imposes

⁵The Revelation Principle ensures that any contingent allocation that satisfies the self-selection property, i.e. which is incentive compatible, can be achieved under a mechanism (a contract). This allows us to convert a problem of characterizing efficient contracts into a simpler one of characterizing efficient allocations, as in a pure exchange economy. Then, we only have to find a contract that sustains the optimal allocation and respects all the restrictions.

both a lower and upper bound on \bar{U}_2 . Hereafter, this parameter is restricted to lie in this interval.

The lower bound is given by the financier's ex-ante participation constraint

$$\bar{U}_2 \geq U_2[k_2], \quad (5)$$

which ensures that he is as well off under the contract as he is under autarky. The upper bound of the parameter \bar{U}_2 varies depending on the merchant's initial endowments. If he is endowed with zero wealth, $k_1 = 0$, $\bar{U}_2 \leq E\{U_2[k_2 - k + s]\}$ because the merchant will always agree to undertake the venture. His ex-ante participation constraint, $E\{U_1[c_1(s)]\} \geq E\{U_1[k_1]\}$, can thus be ignored, for his best alternative is consuming nothing in all the states, which gives him no more utility than he can get through contracting, as constraint (3) reads. If, on the contrary, the merchant is initially endowed with the necessary resources to finance the venture himself, $k_1 > k$, he can undertake the venture alone and consume $c_1(s) = k_1 - k + s$. In this case, his individually rational constraint

$$E\{U_1[c_1(s)]\} \geq E\{U_1[k_1 - k + s]\} \quad (6)$$

is more restrictive than his ex-ante participation constraint, because of (1) for $i = 1$ and utility functions exhibit decreasing absolute risk aversion (DARA). This implies a smaller upper bound on the parameter \bar{U}_2 for $k_1 > k$ than for $k_1 = 0$.

Also, the restricted set over which contracts are optimally chosen changes depending on the assumed information structure. First, incentive-compatibility restriction (8) must be fulfilled when the merchant possesses hidden information, but it is relaxed when the commercial return is verifiable. Second, hidden information restricts the upper bound of the parameter \bar{U}_2 to the expected value of the verifiable second-date endowment.

$$\bar{U}_2 \leq p_y U_2[k_2 - k + y] + [p_{\underline{x}} + p_{\bar{x}}] U_2[k_2 - k + \underline{x}] < E\{U_2[k_2 - k + s]\}. \quad (7)$$

3 Optimal risk-sharing with $k_1 = 0$ under hidden information: a debt-like sea loan contract

Let us first examine the case in which the merchant-entrepreneur is constrained to rely on external funds because of a shortage of his own ($k_1 = 0$) and has hidden information about the realization of the venture's return. Incentive compatibility imposes fixed consumption for the financier on the event that the ship with its cargo arrived at port safe and sound (see Townsend, 1982):

$$c_2(x) = c_2(\underline{x}) = c_2(\bar{x}). \quad (8)$$

Thus, hidden information naturally explains fix repayment in debt-like sea loan contracts: $\tau(x) = \tau(\underline{x}) = \tau(\bar{x})$. Yet, an explanation for the optimality of the sea loan needs also to account for the fact that it established repayment equal to all the capital retrieved from a navigation accident, just like today's standard debt contract entails the financier to recoup as much of his credit as possible from the firm's assets if the firm fails, $\tau(y) = y$.

Imposing restriction (8) under hidden information, the programming problem becomes

Program 2:

$$\begin{aligned} \max_{\{c_2(y), c_2(x)\}} \quad & E\{U_1[w(s) - c_2(s)]\} \\ \text{s.t.} \quad & E\{U_2[c_2(s)]\} = \bar{U}_2 \end{aligned} \quad (9)$$

$$c_2(y) \leq w(y), \quad c_2(x) \leq w(\underline{x}), \quad c_2(s) \geq 0, \quad (10)$$

where \bar{U}_2 , satisfying (5) and (7), defines each individually-rational optimal allocation and restriction (10) combines the feasibility constraints (3) and (4) with the information constraint (8).

3.1 Graphical analysis

Let us depict the optimal event-contingent allocations of consumption in an Edgeworth Box, whose dimensions are given by the total resources of the economy in each state,

$w(s) = k_2 - k + s$. Under hidden information incentive compatibility imposes $c_2(\underline{x}) = c_2(\bar{x}) = c_2(x)$. Therefore, there are two instead of three independent variables and the analysis can be drawn in a two dimensional picture, with a trick: the consumption of the financier is two event-contingent, but the consumption of the merchant is three state-contingent. Thus, figures 1 to 4 present an Edgeworth Box in which the consumption of the merchant has two different origins. The presence of hidden information, and restriction $c_2(x) \leq w(x)$ in particular, also accounts for the shaded noncontractible area. This means that $c_1(\bar{x}) \geq \bar{x} - \underline{x} > 0$ because the merchant can expropriate ex-post the non-verifiable component of the profit, so that contracts with $\tau(x) > \underline{x}$ are non enforceable and the maximum utility that the financier can achieve is given by the value of \bar{U}_2 in (7).

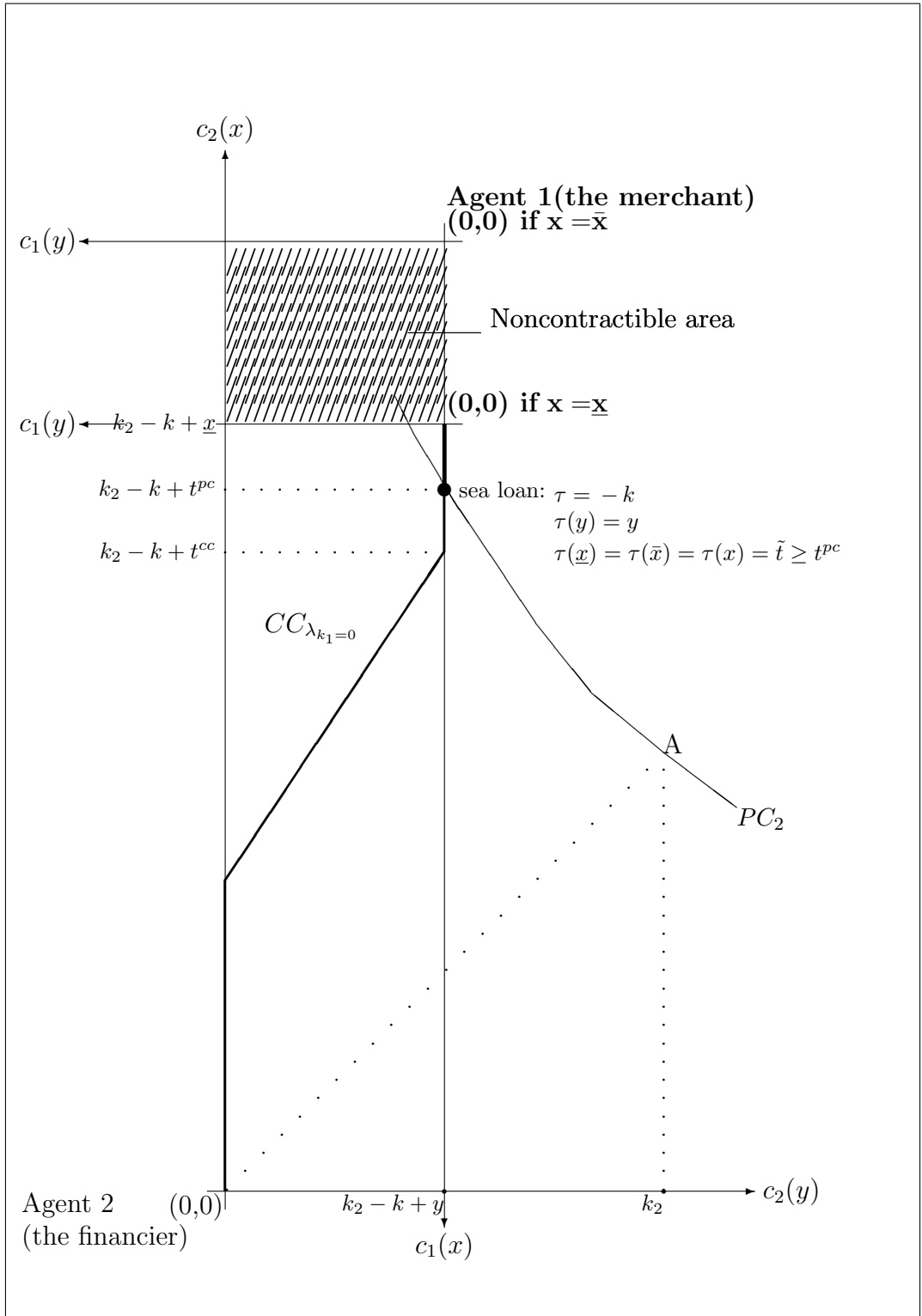
From figure 1 it is clear without further analysis that risk-free debt contracts providing constant consumption to the financier does not satisfy the financier's ex-ante participation constraint— PC_2 in figure 1— as long as the venture involves risk: $y < k$. In other words, risk-free loans sustaining individually-rational allocations, $c_2(x) = c_2(y) \geq k_2$, are not feasible (they lie outside the Edgeworth Box; see, for example, point A) because the merchant is short of resources to pay out a non-contingent transfer on the event of a loss (i.e. $\tau(y) = \tau(x) \geq k$ is not feasible). Therefore, the mobilization of capital in long-distance trade required the sharing of the downside risk between the merchant and his financier. How much insurance would the financier optimally provide to the merchant?

3.2 Risk preferences

Let imagine for a moment a large number of independent and identically distributed ventures and merchants. If long-distance trade had been so characterized, the financier could have diversified the idiosyncratic risks faced by each poor merchant ($k_1 = 0$). Therefore,

FIGURE 1

(Hidden Information, $k_1 = 0$, Two-side Risk Aversion)



it would have been optimal for the financier to provide full downside insurance— with $\tau(y) < 0 \leq y$ — to each merchant, whose consumption would have thus been smoothed.⁶ In other words, on the margin the financier would have behaved as if he were risk neutral (each merchant being risk averse) and the sea loan— with $\tau(y) = y$ — would have provided too little insurance to the merchant. Figure 2 shows that, under these risk preferences, the sea loan would have never been optimal, since any efficient and individually rational allocation would have been then characterized by $c_2(y) < w(y)$, which implies $\tau(y) < y$.

Alternatively, if the merchant had been risk-neutral and the financier risk-averse, the sea loan— which provides as little insurance to the merchant as feasibility constraints impose— would have been optimal (see figure 3) and it would have also been so if both agents had been risk-neutral, since then any division of risk would have been optimal.

Let us now consider the most realistic case in which both the merchant and his financier are risk averse. Figure 4 illustrates different possible contract curves: CC' and CC . For parameter values leading to CC' , debt-like sea loans are not optimal. To sustain any allocation in the core,⁷ the financier would need to provide more downside insurance than the merchant enjoys through a sea loan contract: $c_2(y) < w(y) = k_2 - k + y$. On the contrary, the sea loan is optimal for parameter values leading to a contract curve like CC in figure 1 and 4. The contract curve CC is such that at the corner point $(c_2(y), c_2(x))$, with $c_2(y) = k_2 - k + y$ and $c_2(x) \geq k_2 - k + t^{pc}$, the slope of the merchant's indifference curve is smaller

⁶Yet, merchants would have borne the commercial risk, because of hidden information. Therefore, the merchant's consumption would have not been constant across states: $c_1(\underline{x}) = w(\underline{x}) - c_2(x) < w(x) - c_2(x) = c_1(x) = c_1(y) = w(y) - c_2(y) < w(\bar{x}) - c_2(x) = c_1(\bar{x})$. The first and the last equalities derive from the incentive-compatibility constraint (8) and the inequalities from $w(y) < w(\underline{x}) < w(x) < w(\bar{x})$, with $w(x)$ and $c_1(x)$ defined such that $\frac{p_{\bar{x}}}{p_{\underline{x}} + p_{\bar{x}}} U'_1[w(\bar{x}) - c_2(x)] + \frac{p_{\underline{x}}}{p_{\underline{x}} + p_{\bar{x}}} U'_1[w(\underline{x}) - c_2(x)] = U'_1[w(x) - c_2(x)]$ and $c_1(x) = w(x) - c_2(x)$. Then, the first order conditions for a risk neutral financier can be expressed as $U'_1[w(x) - c_2(x)] = U'_1[w(y) - c_2(y)]$. This implies an optimal allocation such that $c_1(x) = c_1(y)$.

⁷The core in this simple two-agent economy is standardly defined as the Pareto efficient individually-rational event-contingent consumption allocations. It is represented as the Pareto set that lies between the indifference curves that pass through the initial endowments, meaning the shaded interval of the Contract Curve CC or CC' in figures 1 to 4.

FIGURE 2

(Hidden Information, $k_1 = 0$, Merchant Risk Averse, Financier Risk Neutral)

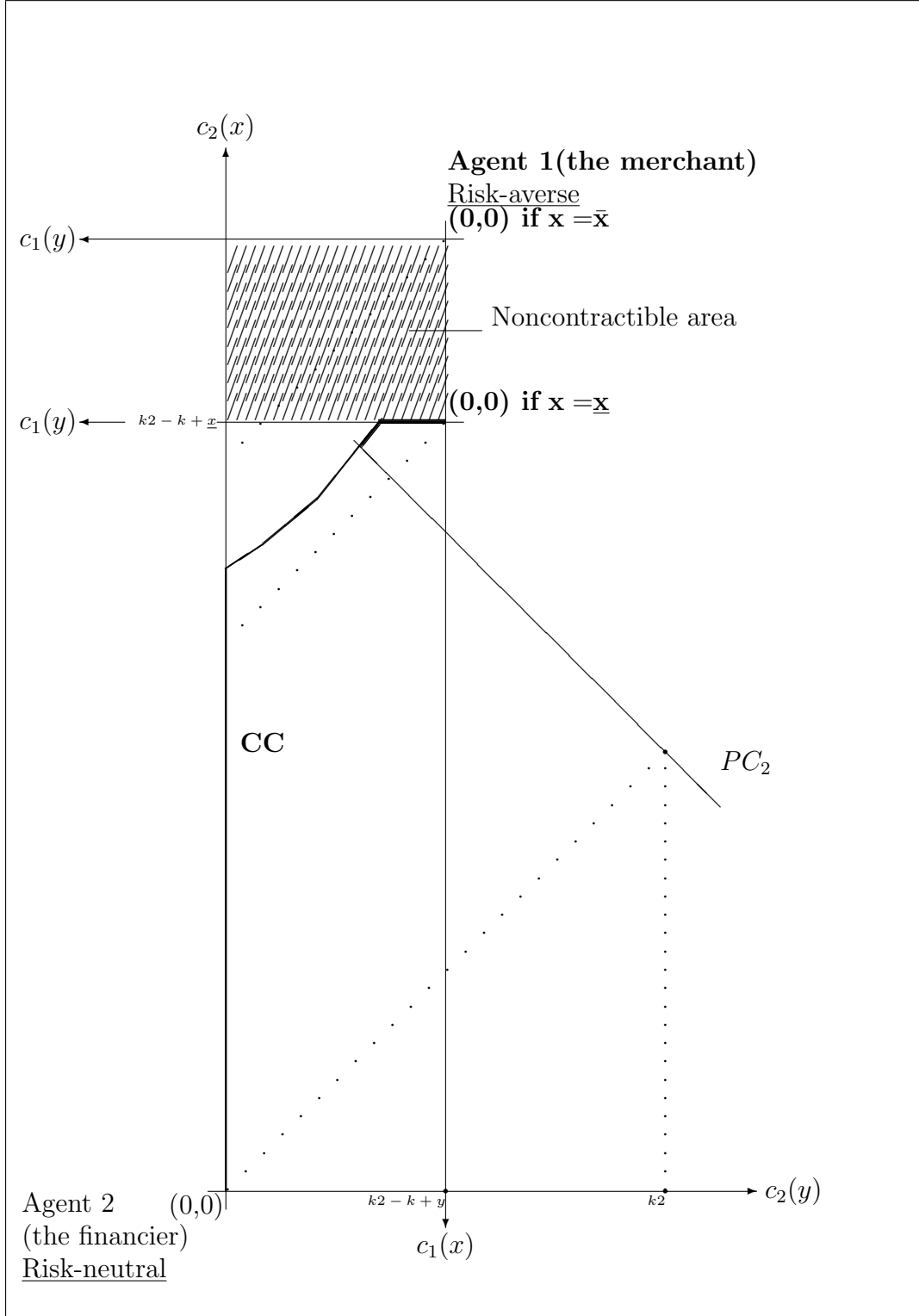


FIGURE 3

(Hidden Information, $k_1 = 0$, Merchant Risk Neutral, Financier Risk Averse)

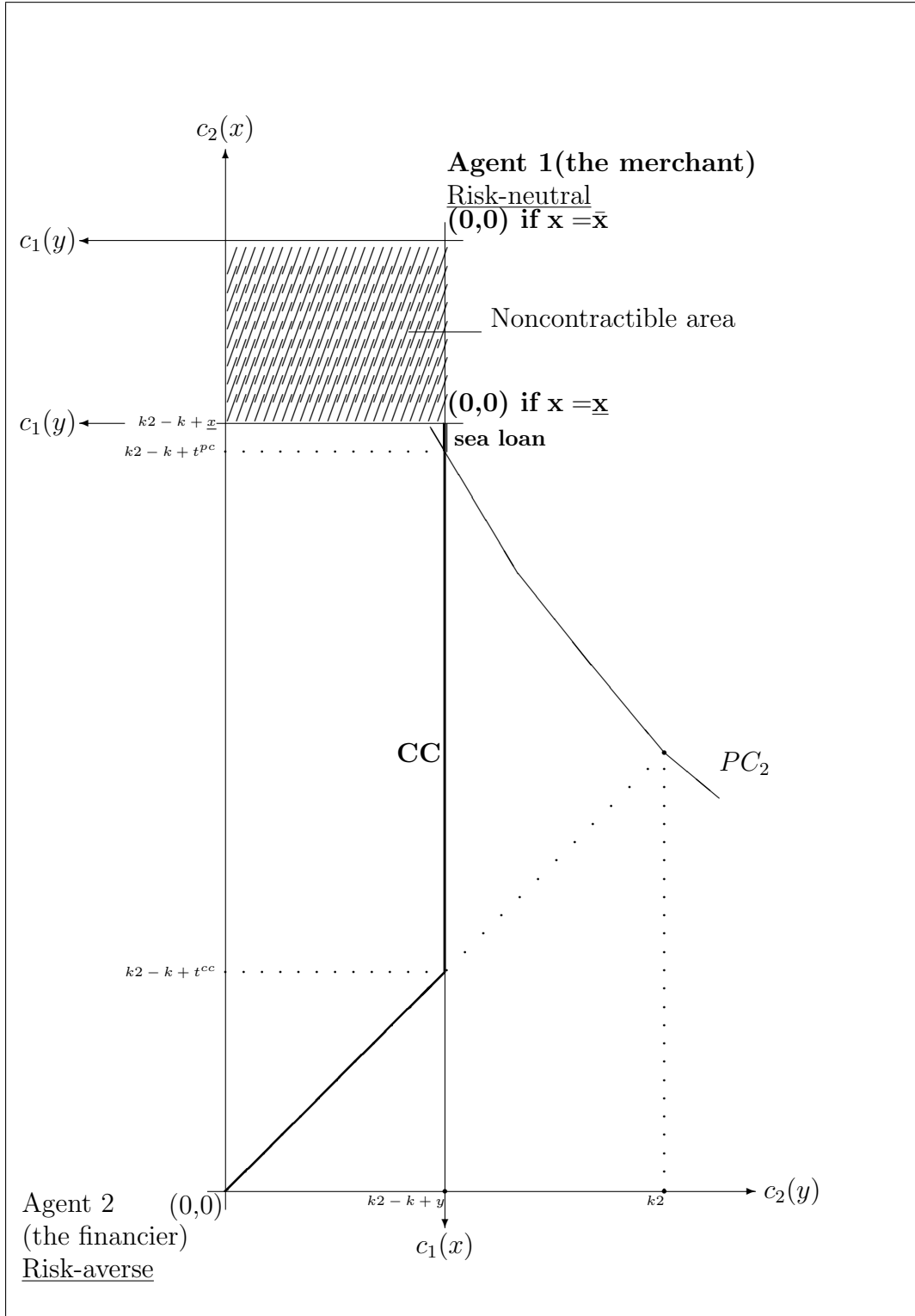
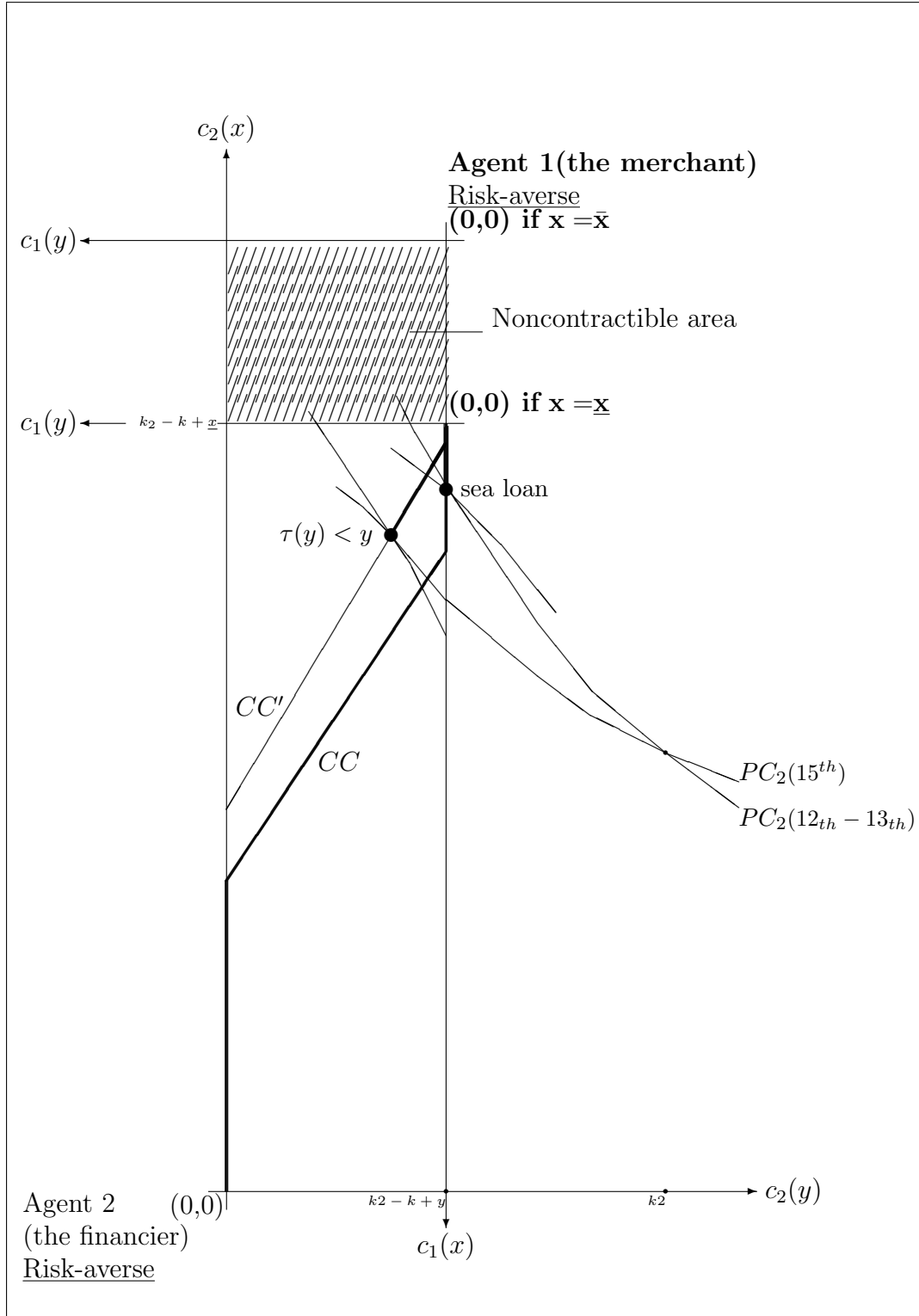


FIGURE 4

(Hidden Information, $k_1 = 0$, Two-side Risk Aversion)



than the slope of the financier’s indifference curve, meaning that, to renounce to a marginal unit of state- y -consumption, the financier wants more units of event- x -consumption than the merchant values that extra marginal unit of state- y -consumption. A corner sea loan contract, establishing $\tau = -k$, $\tau(y) = y$, and $\tau(\underline{x}) = \tau(\bar{x}) = \tilde{t}$, will then sustain the unique optimal allocation of risk, with $\tilde{t} \in [t^{pc}, \underline{x}]$ taking different values for each individually-rational level of bargaining power \bar{U}_2 . In sum, because of the merchant’s lack of resources, $k_1 = 0$, the financier needs to fund the venture on its entirety ($\phi = 0$ so that $\tau = -k$) and bears at least the downside risk of losing the capital he contributes minus the amount saved from failure “at sea or from the action of the enemy” (feasibility constraints impose $\tau(y) \leq y$). Under parameter values leading to CC , this transfer is optimally set at its bound, $\tau(y) = y$, whereas for parameter values leading to CC' , it optimally takes smaller values, $\tau(y) < y$. The point is that contracting through debt-like sea loan contracts— with $\tau(y) = y$ — is not unambiguously efficient under two-side risk aversion. Under what conditions/parameter values, if any, is the sea loan optimal?

3.3 Comparative Statics

A simple exercise in comparative statics leads to proposition 1, whose rationale is explained below and proved in appendix A.

Proposition 1 *The more risky and costly the venture is (the higher p_y and the lower y , the higher k relative to k_2 , and the lower $E[x] = p_{\underline{x}} \underline{x} + p_{\bar{x}} \bar{x}$ given $\underline{x} \geq t^{pc}$), the more likely it is that the risk averse merchant raises funds from the risk averse financier through a debt-like sea loan contract, optimally.*

On the likely event of a navigation incident (p_y high), the financier could not recoup any amount beyond what was saved from misfortune, which was dreadfully insufficient (y very low) to reward the capital investment: $\tau(y) \leq y \ll k$. This implied a very significant loss to the financier, for he invested a major part of his scarce initial endowment in the venture

(k high relative to k_2). To compensate for the possibility of loss, the financier required soaring payments when the merchandise arrived at port safe and sound. This payment \tilde{t} was very high relative to the expected value of the commercial return ($E[x] = p_{\underline{x}} \underline{x} + p_{\bar{x}} \bar{x}$ low, implying a low $p = \frac{p_{\bar{x}}}{p_{\underline{x}} + p_{\bar{x}}}$, \underline{x} , and/or \bar{x}). Indeed, sea loan contracts usually charged yearly interest rates above 33 percent for relatively safe ventures. For particularly risky voyages, interest rates rose up to well above 40 percent, not yearly but for the few months a voyage lasted (González de Lara, 2004a).⁸

As a result, voluntary financial contracting (for any level of bargaining power subject to the financier's ex-ante participation constraint) calls for commercial transfers $\tau(x) = t \geq \tilde{t} \geq t^{pc}$ such that the merchant consumes very little in all the states: $0 \leq y - \tau(y) = c_1(y) < E[c_1(x)] = E[x] - t \leq E[x] - \tilde{t} \leq E[x] - t^{pc}$, with t decreasing as $\tau(y) \leq y$ approximates y (the financier's indifference curves, and PC_2 in particular, have negative slope). On the contrary, the financier's event-consumption varies widely: $c_2(y) = k_2 - k + \tau(y) \leq k_2 - k + y$ is very low while $c_2(x) = k_2 - k + t \geq k_2 - k + \tilde{t} \geq k_2 - k + t^{pc}$ is very high. Thus, along the relevant indifference curve in the Edgeworth Box (to the North-East of PC_2), the merchant's consumption is smoother than the financier's consumption

$$\max\{E[c_1(x)] - c_1(y)\} = E[x] - \tilde{t} \ll \tilde{t} - y = \min\{c_2(x) - c_2(y)\}, \quad (11)$$

where the volatility of agents' consumption is the least by setting $\tau(y) = y$ and $\tau(x) = \tilde{t}$, with $\tilde{t} \in [t^{pc}, \underline{x}]$ depending on each party's bargaining power. Consequently, the risk averse financier values consumption in the case of failure relatively more than the merchant and it is optimal that the merchant provides as much downside-risk insurance to the financier as possible, $\tilde{\tau}(y) = y$. So, the sea loan is optimal in proposition 1.

⁸Likewise, commenda contracts—which provided exactly the same downside insurance than the sea loan, $\tau(y) = y$ —customarily remunerate capital with three fourths of the commercial profit.

Inequality (11) is thus a necessary and sufficient condition under two-side risk aversion for the optimality of the sea loan (debt contract). The sea loan will more likely be optimal the lower $E[x]$, the higher $\tilde{t} \geq t^{pc}$, and the lower y , i.e. the more risky and costly the venture is. First, the higher p_y and the lower y , the higher the transfer $\tilde{t} \in [t^{pc}, \underline{x}]$ required to compensate for the possibility of losing the capital advanced, by construction. Second, the more costly the venture (the lower $k_2 - k$), the more absolute risk averse the financier is and the higher the transfer $\tilde{t} \geq t^{pc}$ he would require to compensate for the risk of loss— because of DARA, the financier’s indifference curves become more sloppy. Third, a decrease in y and $E[x]$ reduces the dimensions of the Edgeworth Box and shifts the merchant’s indifference curves towards its flatter region (the origin of the merchant’s consumption shifts to the south or to the west for smaller values of the parameters y and $E[x]$, respectively). It is worth noting, however, that $E[x]$ cannot take extremely low values because the minimum verifiable return \underline{x} needs to be high enough to compensate the financier for the possibility of losing his capital. In other words, the financier would refuse funding a venture on which he took the risk of loss, $\tau(y) = y < k$, unless $\underline{x} \geq \tau(x) = \tilde{t} \geq t^{pc}$.

3.4 Empirical evaluation of the theoretical predictions

This theoretical explanation for the use of the debt-like sea loan (and the equity-like commenda, see next section) lends itself to empirical evaluation. First, as predicted, sea loan (and commenda) contracts, with repayment equal to all the capital saved from loss at sea or from the action of hostile people and very high repayment otherwise, prevailed during the twelfth and thirteenth centuries, when trading ventures were indeed highly risky and costly.

Second, the sea loan (and the commenda) lost their popularity when “commerce lost much of its adventurous and almost heroic features” (p_y lower and y higher) and “tended to

become a routine” ($E[x] = p_{\underline{x}} \underline{x} + p_{\bar{x}} \bar{x}$ higher),⁹ and wealth accumulation from commerce reduced the former scarcity of capital (k_2 higher, at least with respect to k).¹⁰ Consistent with the model’s predictions, the Venetians responded to this environmental change—the new parameter values violated (11), leading to a contract curve such as CC' in figure 4—by developing new contractual forms such that the merchants enjoyed more insurance against the downside risk than they used to receive through sea loan and commenda contracts: $\hat{c}_2(y) < w_2(y)$. From late in the fourteenth century, sea ventures were typically funded through risk-free bills of exchange, with $\hat{\tau}(y) = \hat{\tau}(\underline{x}) = \hat{\tau}(\bar{x})$, and insured through premium policies, with $\hat{\tau}(y) < -k + y < 0$.

Finally, the severe drop on insurance rates lends empirical support to the theoretical explanation for the selection of alternative contracts. The model predicts the use of the sea loan (and the commenda) because the high risk and cost initially involved in sea ventures made insurance too expensive, thereby leading the merchant to buy as little insurance as he could, $\tau(y) = y \geq 0$. Likewise, the model predicts the use of premium insurance, with $\tau(y) < 0$, in response to a fall in the insurance cost. In real truth, underwriters during the fifteenth century charged much lower premiums than their counterparts from the twelfth and thirteenth centuries used to obtained through sea loan and commenda contracts. The insurance rates varied widely according to the type of vessel, the port of call, the state of war or peace, the season of the year, and other circumstances, but insurance policies remained consistently cheaper than the insurance rates paid above the safe interest on sea loans. As already mentioned, rates of return on sea loans, repayable upon safe arrival of

⁹The quotations are from the great historian of the Commercial Revolution Lopez (1976, p. 97), who also noted that “high risks and high profits were predominant in the early stages of the Commercial Revolution; they were instrumental in forming the first accumulations of capital.”

¹⁰That capital became relatively less scarce is indicated by the sharp drop in interest rates. During the twelfth century Venice’s “common use of our land” established a 20 percent yearly interest on risk-free loans, subjected to double penalty if payment was delayed and secured by a general lien on the debtor’s property (González de Lara, 2004a). By the mid fourteenth century average interest rates on commercial loans within Venice had declined to 5 percent (Luzzatto, 1943, pp. 76-79) and the 1482’s new issue of government bonds promised to pay 5 percent yearly interest on perpetuity (Mueller, 1997, p. 420).

the ship, varied from 25 to 50 percent for the duration of the voyage, while yearly interest rates on risk-free loans— although high— only amounted to 20 percent, thereby implying an insurance cost from around 15 to 40 percent on sea loans. The premium paid during the fifteenth century for the safest voyages amounted to a mere 1-1,5 percent and raised up to 20 percent for exceptionally risky ventures (see, for example, De Roover, 1945, and Tenenti, 1991).

4 Optimal risk-sharing with $k_1 = 0$ under full information: an equity-like commenda contract

The previous section shows that, when ventures are highly risky and costly, the sea loan emerges optimally under hidden information. However, the (risk-averse) merchant undesirably undertakes all the commercial risk, $\tau(\underline{x}) = \tau(\bar{x})$ because of his inability to commit to reveal a commercial return other than the minimum verifiable (\underline{x} by assumption). The provision of verifiable information relaxes incentive-compatibility constraints and enables better risk-sharing commenda contracts, with $\tau(y) < \tau(\underline{x}) < \tau(\bar{x})$. Yet, to explain the optimality of the commenda, one need to account for the fact that it shares the downside risk in the very same manner as the sea loan, with $\tau(y) = y$.

With a sea loan, the financier bears as little downside risk as possible, $\tau(y) = y$, while taking none of the commercial risk, $\tau(\underline{x}) = \tau(\bar{x})$. If this is the efficient agreement under hidden information, it naturally follows that the financier will not optimally bear more downside risk once he is optimally assuming part of the commercial risk, $\tau(\underline{x}) < \tau(\bar{x})$ under full information. Thus the optimality of the commenda derives from that of the sea loan and can hence be linked to the venture's risk and cost characteristics. Proposition 2, which is proven in Appendix B, follows.

Proposition 2 *If the sea loan sustains all the optimal individually-rational allocations under hidden information, the commenda supports the corresponding allocations under full information.*

4.1 Empirical evaluation of the theoretical predictions

In the model, debt-like sea loan and equity-like commenda contracts can coexist under various information structures, which is in sharp contrast with the costly state verification/falsification literature. In reality, the sea loan and the commenda were simultaneously used to finance different kinds of sea ventures. For example, from the thirteen documented ventures taken in 1190, when Venice had not yet consolidated her commercial empire in the East, eight were funded through sea loans but the five round voyages from Venice to her well-established colonies in Constantinople and the Adriatic Sea were funded through commenda contracts. Moreover, various financiers who invested through the sea loan later used the commenda and some of those who relied on the commenda had previously used the sea loan (see González de Lara, 2004b , for more details).

In fact, the commenda substituted the sea loan as the predominant way of funding sea ventures by the turn of the twelfth century. This observation can be accounted for as reflecting changes in the State's ability to verify information. González de Lara (2004a) documents various institutional developments— such as the edification of a commercial empire in the Levant, the public organization of round convoys from Venice to her colonies, and the implementation of very detailed regulations and State's direct controls over trade— that enhanced the State's ability to adjudicate disputes and punish cheaters. The resulting enhanced State's ability to verify information, however, did not crystallized all at once, but it rather developed incrementally as overseas trade became well-established throughout the various Venetian colonies in the Levant, and as trading voyages were organized in State's round convoys from Venice to the Venetian enclaves in the East. As a result, the commenda

contract progressively acquired prominence as the twelfth century turned to its close and prevailed by the third decade of the thirteenth century, when Venice had consolidated her commercial and colonial Empire in the East.

The observed transition from the sea loan to the commenda in response to changes in the State's ability to verify information lends empirical support to the theoretical assumption that the State provided contract enforcement under two distinct information structures.

5 Risk-sharing through Credit Contracts, $k_1 > k$

The previous two sections show that when the merchant must rely on external funds for all the venture's capital requirements because of a shortage of his own ($k_1 = 0$), the sea loan or the commenda— one or the other depending on the information structure— are corner solutions. Not only is the financier consuming all the second-date endowment in the case of loss, $\tilde{c}_2(y) = w(y)$, but both parties would have been better off if more resources had been allocated to his consumption in this state. This implies that a contract with higher repayment in case of navigation loss— which provides less downside insurance to the merchant than the sea loan and the commenda— would have been Pareto improving. However, such contract is not feasible because the merchant is already paying with everything he has, meaning the very low or zero proceeds of the venture, $\tau(y) = y$. Limited liability is thus understood in terms of optimal risk-sharing when feasibility constraints are binding.

When the merchant is initially endowed with more resources, $k_1 > k > 0$, the total downside endowment $w(y) = k_1 + k_2 - k + y$ gets larger and the constraint $c_2(y) \leq w(y)$ cease to be binding in the optimum. An interior optimal allocation $(c_2^*(y), c_2^*(x))$ can then be achieved, with $c_2^*(y) > \tilde{c}_2(x) = w(y) = k_2 - k + y$ and $c_2^*(x) < \tilde{c}_2(x)$. Therefore, individually-rational optimal allocations with $k_1 > k > 0$ will no longer entail raising funds

for the whole trading venture ($\tau^* \neq -k$) through sea loan or commenda contracts. Yet, these allocations $(c_2^*(y), c_2^*(x))$ can still be achieved through a sea loan or a commenda providing only part of the capital requirements. The merchant finances part of the venture himself and bears its corresponding downside risk, thus effectively receiving less downside insurance. In particular, proposition 3 holds.

Proposition 3 *The optimal event-contingent allocation of consumption when $k_1 > k$ can be attained by self-financing part of the venture $\phi^* \in (0, 1)$ and raising external funds through a debt-like sea loan or an equity-like commenda, one or the other depending on the information structure, for the rest of the capital requirements, $\tau^* = -(1 - \phi^*)k > -k$.*

5.1 Graphical analysis

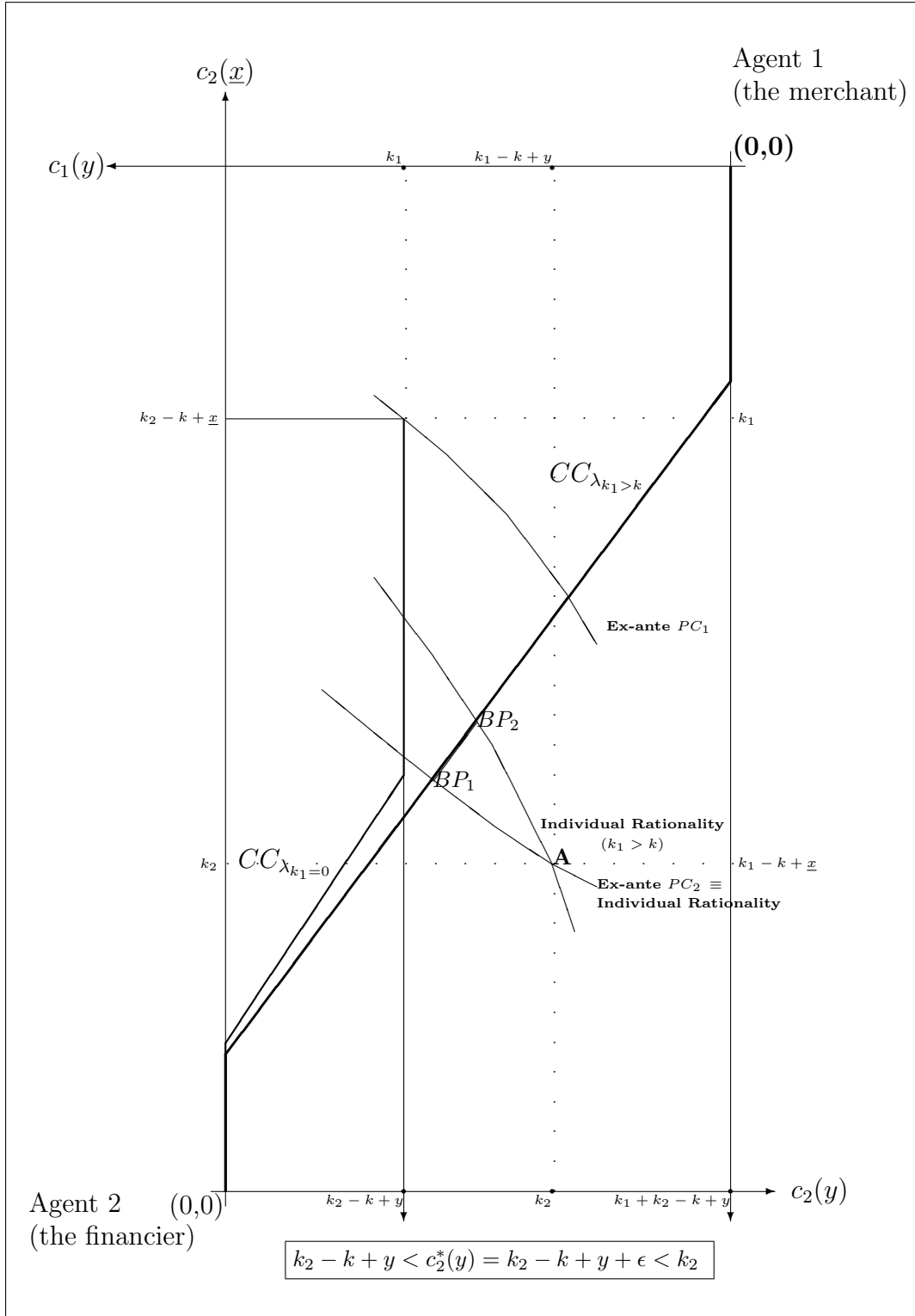
When the merchant is initially endowed with $k_1 > k > 0$, there are more resources in the economy than when he does not have any initial wealth, $k_1 = 0$, so that the corresponding Edgeworth Box is larger. Figure 5 represents both the Edgeworth Boxes for parameters $\lambda_{k_1=0}$ (with $k_1 = 0$) and $\lambda_{k_1>k}$ (with $k_1 > k > 0$), although for ease of exposition the noncontractible area is not depicted under the hidden information scenario.¹¹

Because of DARA, the indifference curves of the merchant become flatter as he receives more initial wealth, while those of the financier are unaltered by any increase of the merchant's wealth. Therefore, the contract curve for interior points for parameters $\lambda_{k_1>k}$ lies below the contract curve for $\lambda_{k_1=0}$. The previous subsections shows that the contract curve for $k_1 = 0$ can be represented by $CC_{\lambda_{k_1=0}}$ in figure 5 when a venture is risky and costly and both agents are risk-averse. Therefore, the contract curve for $k_1 > k$ will be characterized by $CC_{\lambda_{k_1>k}}$ in figure 5.

¹¹Alternatively, figure 5 can be thought of as representing a face of the Edgeworth Boxes for a given value of $c_2(\bar{x})$ under full information. In this case there are three events for both agents and the contract curve must be represented in a 3-dimensional Edgeworth Box. However, as we are looking at a binding restriction for only one event, we can abstract from the optimal relationship between $c_2(\bar{x})$ and $c_2(\underline{x})$ given by the Kuhn-Tucker conditions of program 3 (in the appendix) and draw the optimal allocation in an intuitive 2-dimensional Edgeworth Box with coordinates $c_i(y), c_i(\underline{x})$.

FIGURE 5

($k_1 > k$, Two-side Risk Aversion)



The individually-rational allocations, in addition to belonging to the contract curve, must satisfy the individually-rational participation constraints, which, as noted before, are more restrictive than the ex-ante participation constraint for the merchant. The merchants' individually rational constraint (6) takes into account the technological and financial ability of the merchant to undertake the project alone. Therefore, any individually rational allocation must provide him with at least the same utility he would get by financing the venture himself and taking all its profits and risks. Yet, the merchant will voluntarily exchange with the financier because financial contracting leads to mutually beneficial risk-sharing.

The core of this simple economy will be thus characterized by the interval $[BP_1, BP_2]$ on the contract curve, where the point BP_i is the individually-rational optimal allocation when agent i has all the bargaining power (see figure 5). In other words, the lower bound of the interval to which \bar{U}_2 belongs remains $U_2(k_2)$ but its upper bound is now given by the merchants' individually-rational constraint (6). When the financier has all the bargaining power, the optimal event-consumption allocation BP_2 lies on the indifference curve of the merchant providing him with his minimum individually rational expected utility $E\{U_1[k_1 - k + s]\}$ (see point A and BP_2 in figure 5). By construction of the contract curve, this merchant's indifference curve intersects at BP_2 with the financier's indifference curve assigning him the individually-rational maximum value of \bar{U}_2 .

From inspection of figure 5, it follows that the optimal allocation BP_1 reached when the merchant exercises all the bargaining power is such that $c_2^*(y) > k_2 - k + y$ (see appendix C). Also, the optimal allocation BP_2 reached when the financier exercises all the bargaining power is such that $c_2^*(y) < k_2$. Thus, any allocation in the core satisfies lemma 1, which is proved in appendix C.

Lemma 1 $k_2 - k + y < c_2^*(y) = k_2 - (1 - \phi^*)k + \tau^*(y) < k_2$

In sum, the optimal allocation of risk can be attained by letting the merchant provide a part $\phi^* \in (0, 1)$ of the capital requirements and raise additional funds ($\tau = -(1 - \phi^*)k$) through a sea loan (under hidden information) or a commenda (under full information) with $\tau^*(y) = y$.

This result is novel in the optimal security design literature based on agency costs (both in the costly state-verification and the incomplete contracting ones) which predicts an extreme inside participation rate: either the (risk-neutral) merchant-manager does not resort to outside funding until he has invested 100 percent of his personal wealth ($\phi = 1$) or he (being risk-averse) does not finance the project at all ($\phi = 0$). However, we observe in reality that the merchants invested part of their wealth in their businesses and raised additional funds in the form of sea loan and commenda contracts. In this simple model, debt-like sea loan and equity-like commenda contracts optimally exhibit limited liability because of risk-sharing even when financial constraints are not binding ($k_1 > k$).

6 Enforcement Costs

The optimal allocation of risk $c_2^*(y) \in (k_2 - k + y, k_2)$, though, can be attained through a multiplicity of contracts, for example through a combination of riskless debt— with $\tau = -k$ and $\tau(y) = \tau(\underline{x}) = \tau(\bar{x}) \geq k$ — and premium insurance— with $\tau > 0$, $-\tau(y) < k - y$ and $\tau(\underline{x}) = \tau(\bar{x}) = 0$.¹² When the merchant is initially endowed with zero capital ($k_1 = 0$), a risk-free debt contract is not enforceable (feasible) unless associated with insurance, which might explain the observed provision of credit and insurance within the same contract during the twelfth and thirteenth centuries. However, even when merchants were endowed with the resources required to finance the venture on their own, they relied on external funds and

¹²The possibility to make transfers both at date 0 and 1 creates a nominal indeterminacy, as in Bolton-Bolton and Schartfstein [1990] and Hart and Moore [1998]. What matters is not so much the particular security structure but the linear space it involves. Any linear combination of various points in the Edgeworth box that span the optimal event consumption allocation can be thought of as optimal contracts.

used sea loan and commenda contracts, with $\tau(y) = y$, as both credit and insurance devices. It was only from late in the fourteenth century that insurance began slowly to develop as an independent form of business, when— as section 3 suggests— overseas trade became relatively less risky and costly and, accordingly, the optimal allocation of the downside risk could no longer be achieved through the sea loan or the commenda. Thus, one needs to understand why the sea loan and the commenda prevailed during the twelfth and the thirteenth centuries, respectively, despite the possibility of using other contracts to achieve the very same optimal allocation of risk.

One possible explanation focuses on the transaction costs of thinking, negotiating and writing new contracts, and thus on the additional administrative and/or regulating costs of enforcing these new contract. Moreover, since debt-like sea loan and equity-like commenda contracts are necessary to attain the optimal allocation of risk when the merchant lacks sufficient resources to finance the venture himself, the theory of standardization (see Allen and Gale, 1994) predicts the use of these very same contracts even when the merchant is endowed with the required resources.

In the light of the incomplete contracting literature (see Hart, 1995) one would expect, as observed, that optimal contracts ensure that, whatever happens, each side has some protection against opportunistic behavior by the other party. Because both the merchant and his potential financier-insurer can default on their promised future payments, the optimal contract will minimize those payments, while guaranteeing the optimal allocation of risk. This is achieved by making both parties commit part of their wealth to the funding of the venture, $\phi^* \in (0, 1)$, and financing the rest through sea loan and commenda contracts. In this respect, it is meaningful that insurance premiums were agreed to be paid after the conclusion of the voyage. Not only could the merchant pay the premiums from the venture's proceeds, but he also received some "power" against the possibility that the underwriters

defaulted on the coverage payments. However, as the insured not seldom delayed paying the premiums due on their policies long after their ships were safely returned, so the insurers were dilatory about payment in case of loss (Tenenti, 1991, p.675).

Furthermore, one can reasonable argue that enforcing payments from the parties' initial wealth, k_1 and k_2 , if any, was much more arduous than from the venture's returns, for example, because the underwriter might have actually failed in the intervening time from the signing of the insurance contract to the payment of the coverage or because the absence of an effective registration system enabled defaulters to hide their real property, particularly if abroad. In Venice as elsewhere, marine insurance remained in the hands of private underwriters, operating alone or with partners but with very limited capital, until the end of the eighteenth century. They were commonly involved in all kind of trades and it was all but rare that they were unable or unwilling to meet the coverage payments, as indicated by the rulings of the Venetian magistracy on insurance, who found that "there was no money that more gladly is touched and more difficultly is paid than that of the maritime insurance" (Tenenti, 1991, p. 677; see also De Roover, 1945, p. 197).

Since merchants could not use collateral to fulfill their obligations without incurring in a high enforcement cost, risk-free debt and more generally any contract with $\tau(y) > y \geq 0$ was hardly used. Similarly, because it was costly to identify and confiscate agents' future wealth at date 1, pure insurance contracts with $\tau(y) < 0$ did not develop until the changing character of long-distance trade made them the only means through which the optimal allocation of risk could be achieved. In sum, the model predicts uniqueness in the optimal use of the sea loan and the commenda (one or the other depending on the information structure) under the realistic assumption that the (minimum) venture's returns are contractible at a lower cost than the agents's initial wealth.

7 Conclusions

This paper combines a mechanism-design model and historical evidence to examine the financial structure in pre-modern long-distance trade. Merchants-entrepreneurs raised funds both internally, through (inside) equity holdings, and externally, through debt-like sea loan and equity-like commenda contracts. This capital structure can be rationalized as being determined to achieve the optimal allocation of risk while minimizing enforcement costs. The extent to which these factors are relevant today is indicated by the generality of the model's assumptions and results.

First, two-side risk aversion. Historically, significant indivisibilities and aggregate risk limited the agents' ability to diversify, preventing them from effectively becoming risk neutral. Today, most companies have a large share-holder who is not well-diversified (Zingales, 2000, p. 1628). Risk-aversion, because of limited risk-pooling, is typically assumed when dealing with the myriad of young and small firms who do not have access to public markets, such as individual proprietorships, partnerships and closely-held corporations. Even when the financial capital is held by well-diversified investors, large widely-held corporations might still act in a risk-averse manner because the human capital invested in the firm is not well-diversified (Zingales, 2000); the firm's managers faces some kind of bankruptcy costs (Greenwald and Stiglitz, 1990); managerial compensation is aligned with shareholders wealth maximization (Smith and Stulz, 1985); or there exist proprietary information (De Marzo and Duffie, 1991). Furthermore, although various works on Transaction Costs Economics have recently criticized risk-sharing arguments on account of their lack of empirical significance (mainly for labor contracts), Akerberg and Botticini (2002) shows that, controlling for endogenous matching, risk aversion appears to influence contract choice.

Assuming more tractably risk neutrality would be innocuous had the optimal capital

structure been robust to risk preferences. However, the entrepreneurs' partial funding of the project (a non-extreme inside participation rate) can hardly be explained unless considering managerial risk aversion. Further, the paper's simple model also shows that neither debt nor equity— with repayment equal to all the capital saved from the project in the case of failure— would be optimal if (on the margin risk-neutral) financiers could diversify the idiosyncratic risks faced by each (risk-averse) merchant; rather, optimal risk-sharing would require that the financier provide full insurance to the merchant against the downside risk, say through a fix salary. Current capital structure's theories can therefore be enriched by taking into account optimal risk-sharing, as already suggested by both Townsend (1978) and Gale and Hellwig (1981), who, in their theory of debt based on costly state verification, recognized that standard debt contracts are not efficient if the parties are risk averse.

Second, limited contract enforceability. Historically, enforcing payments from the agents' private wealth was more costly than from the venture's returns since, on the one hand, neither effective registration systems nor joint-stock insurance companies had yet developed and, on the other hand, the State fairly controlled sea ventures. Today, enforcement costs arguably cause banks' reluctance to go through bankruptcy procedures for recovering loans from the debtors' personal wealth (collateral). Similarly, an insurance company runs administrative cost greater than those of a capital market where bonds and equity are traded.

It can therefore be reasonably argued that credit and insurance are optimally provided through the capital structure, rather than separately through risk-free debt from a bank and premium policies from an insurance company, because of enforcement costs. Thus, the model complements the optimal portfolio selection theory, which explains the holding of equity by both entrepreneurs and outsiders in terms of managerial risk aversion but ignores other contracts through which an outsider can potentially provide the same risk allocation. Furthermore, limited contract enforceability can inform more complex capital structure's

theories, like the incomplete contracting models of Bolton and Scharfstein (1990) and Hart and Moore (1998), where debt emerges as one of many optimal contracts. In our simple model, uniqueness is achieved by imposing reasonable assumptions on enforcement costs. In other words, the irrelevance theorem of Modigliani and Miller does not apply because of enforcement constraints.

Third, although the model is tailored for the historical example, it can also be interpreted as representing today's financial relations between an entrepreneur and a potential investor-insurer. The entrepreneur can undertake a risky investment project with a fix cost, k , and a random return, $s = \{y, x, x\}$. The project is risky in the sense that, with some positive probability, it loses money, $y < k$, although it has a positive expected net present value, $k < E[s]$. The event of loss, $s = y$, can be reasonably assumed to be observable and verifiable, while the profitability of a well-functioning firm, $s = \{x, x\}$, might or might not be verifiable—thereby enabling or ruling out equity contracts—depending on the availability of effective disclosure mechanisms, auditing, informative economic press, and so on and so forth. Also, failure can be identified with a natural catastrophe or a political breakdown and the sea loan, with a catastrophe bond.

Finally, the model delivers debt and (outside) equity as corner solutions to the optimal contracting problem when, in addition to assuming two-side risk aversion and considering the case in which the merchant must rely on external funds because of budget constraints, investment projects are costly and risky. Debt and equity are corner contracts in the sense that, if feasible, both parties would prefer that the risk-averse investor recouped part of his capital investment if the project failed, for limited liability associated with debt and equity results in very high financing costs. To avoid these cost, rich merchants optimally finance part of their ventures while relying on debt and equity, one or the other depending on the information structure. However, if the venture is relatively less risky and costly, insurance

against the downside risk becomes relatively cheaper and entrepreneurs optimally respond by buying more insurance through premium policies, even if at a higher enforcement costs.

Consistent with this explanation for the selection of alternative contracts, we observe that debt-like sea loan and equity-like commenda contracts actually gave way to premium insurance when commerce lost many of its adventurous features and wealth accumulation from commerce reduced the former scarcity of capital. During the fourteenth and fifteenth centuries, sea ventures were commonly funded through risk-free bills of exchange, insured through premium contracts, and organized by means of commission agents that resided permanently abroad. Empirical confirmation of these theoretical predictions lends support to the view that risk-sharing and enforcement costs are indeed relevant in determining firms' financial structure.

Yet, the model ignores other important forces shaping contracts. In particular, the choice between debt and (outside) equity, which have motivated most research on corporate finance, is determined by the simplest information structures: under hidden information regarding the project's profitability, equity is not incentive-compatible and fix-repayment debt contracts arise as a second-best; under full information, however, better risk-sharing commenda contracts become possible. The model can thus be extended to incorporate various types of agency costs, multiple margins for moral hazard, adverse selection, signaling, transactions costs, and/or incomplete contracting.

A Proof of proposition 1

Let parameter values $\hat{\lambda} = (\hat{\theta}, \hat{k}_1, \hat{k}, \hat{y}, \hat{p}_y, \hat{x}, \hat{x}, \frac{\hat{p}_x}{\hat{p}_x + \hat{p}_x})$ with $\hat{\theta} = 0$ and $\hat{k}_1 = 0$ be such that the sea loan contract does not sustain program 2's optimal allocation of consumption, $(\hat{c}_2(y), \hat{c}_2(x))$. Then, $\hat{c}_2(y) < \hat{w}(y)$, as draw in figure 2 for CC' , and necessary and sufficient

Kuhn-Tucker conditions lead to

$$\begin{aligned}
\frac{\partial \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial \eta} &= -\bar{U}_2 + E[U_2[c_2(s)]] = 0 \\
\frac{\partial \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial c_2(y)} &= -p_y U_1'[w(y) - c_2(y)] + \eta p_y U_2'[c_2(y)] - \mu_y = 0 \\
\frac{\partial \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial c_2(x)} &= -[p_x U_1'[w(\underline{x}) - c_2(x)] + p_{\bar{x}} U_1'[w(\bar{x}) - c_2(x)]] \\
&\quad + \eta(1 - p_y) U_2'[c_2(x)] - \mu_x = 0
\end{aligned} \tag{12}$$

evaluated at $\hat{\eta}$, $\hat{\mu}(y) = \hat{\mu}(x) = 0$, $\hat{c}_2(y)$, $\hat{c}_2(x)$, and $\hat{\lambda}$, where η , μ_y , and μ_x are the Lagrangian multipliers associated with restrictions (9) and (10), respectively, and $\mathcal{L}(\cdot)$ is the Lagrange function associated with Program 2.¹³

Let λ varies in an open neighborhood of $\hat{\lambda}$ such that the pattern of binding and slack constraints of program 2 does not change. Totally differentiating (12) and applying the Implicit Function Theorem, one can calculate the comparative statics effects of each parameter value λ_i on $c_2(y)$ at a solution point $(\hat{c}_2(y), \hat{c}_2(x))$ and parameter values $\hat{\lambda}$ with $\hat{\eta} > 0$, $\hat{\mu}(y) = \hat{\mu}(x) = 0$, and $d\mu_y = d\mu_x = 0$:

$$\frac{dc_2(y)}{d\lambda_i} = - \frac{\left| \hat{H}_{\lambda_i} \right|}{\left| \hat{H} \right|},$$

where

$$\left| \hat{H}_{\lambda_i} \right| = \begin{vmatrix} \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{(\partial \eta)_2} & \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial \eta \partial \lambda_i} & \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial \eta \partial c_2(x)} \\ \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial c_2(y) \partial \eta} & \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial c_2(y) \partial \lambda_i} & \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial c_2(y) \partial c_2(x)} \\ \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial c_2(x) \partial \eta} & \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial c_2(x) \partial \lambda_i} & \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{(\partial c_2(x))_2} \end{vmatrix}$$

and the (binding-constraints) border hessian \hat{H} evaluated at $\hat{\lambda}$ and $(\hat{c}_2(y), \hat{c}_2(x))$ is negative definite: $\left| \hat{H} \right| > 0$.

It is proved below that at a solution point $(\hat{c}_2(y), \hat{c}_2(x))$,

$$\begin{aligned}
\frac{dc_2(y)}{d(k_2 - k)} &> 0 \quad \text{but} \quad dc_2(y) < dw(y) = d(k_2 - k) \\
\frac{dc_2(y)}{dy} &> 0 \quad \text{but} \quad dc_2(y) < dw(y) = dy \\
\frac{dc_2(y)}{dp_y} &> 0 \quad \text{and} \quad dw(y) = 0 \\
\frac{dc_2(y)}{d\underline{x}} < 0, \quad \frac{dc_2(y)}{d\bar{x}} < 0, \quad \frac{dc_2(y)}{d\frac{p_x}{p_x + p_{\bar{x}}}} < 0 \quad \text{and} \quad dw(y) = 0.
\end{aligned} \tag{13}$$

Therefore, beginning with parameter values $\hat{\lambda}$ for which the sea loan contract is not optimal ($\hat{c}_2(y) < \hat{w}(y)$), a change in parameters such that the venture becomes more costly—

¹³This assumes that for parameter values $\hat{\lambda}$ the solution of program 2 is such that $\hat{c}_2(x) < \hat{w}(\underline{x})$. The case for which $\hat{c}_2(x) = \hat{w}(\underline{x})$ can be easily derived by adding restriction $\frac{\partial \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial \mu_x} = 0$ to (12).

$d(k_2 - k) < 0$ — and/or risky— $dy < 0$, $dp_y > 0$ and/or $dE[x] < 0$ — leads the new solution to get closer and closer to the border $c_2(y) = w(y)$. At a point, the solution will reach the border and the sea loan will be optimal.

However, this tricky point with $c_2(y) = w(y)$ and $\mu_y = 0$ is on the boundary between two regions with different patterns of binding and slack constraints. As a result, deviations to the left side will have to be treated using the set of equations (12) with $\mu_y = \mu_x = 0$ and $d\mu_y = d\mu_x = 0$ while totally differentiating, whereas those to the right side using a different set: equation $\frac{\partial \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial \mu_y} = w(y) - c_2(y) = 0$ must be added to (12), which now hold for $\mu_y \geq 0 = \mu_x$ and $d\mu_y \neq 0 = d\mu_x$. If parameter values keep on changing so that the venture becomes even more risky and costly, the sea loan will keep on being optimal, because at a solution point $(\tilde{c}_2(y), \tilde{c}_2(x))$ with $\tilde{c}_2(y) = \tilde{w}(y)$, and parameter values $\tilde{\lambda}$ with $\tilde{\eta} > 0$, $\tilde{\mu}(y) \geq 0$, $\tilde{\mu}(x) = 0$, and $d\mu_x = 0$,

$$\frac{dc_2(y)}{d\lambda_i} = - \frac{\left| \tilde{H}_{\lambda_i} \right|}{\left| \tilde{H} \right|},$$

where

$$\left| \tilde{H}_{\lambda_i} \right| = \begin{vmatrix} \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{(\partial \eta)^2} & \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial \eta \partial \mu_y} & \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial \eta \partial \lambda_i} & \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial \eta \partial c_2(x)} \\ \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial \mu_y \partial \eta} & \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{(\partial \mu_y)^2} & \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial \mu_y \partial \lambda_i} & \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial \mu_y \partial c_2(x)} \\ \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial c_2(y) \partial \eta} & \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial c_2(y) \partial \mu_y} & \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial c_2(y) \partial \lambda_i} & \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial c_2(y) \partial c_2(x)} \\ \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial c_2(x) \partial \eta} & \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial c_2(x) \partial \mu_y} & \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{\partial c_2(x) \partial \lambda_i} & \frac{\partial^2 \mathcal{L}(\eta, \mu_y, \mu_x, c_2(y), c_2(x); \lambda)}{(\partial c_2(x))^2} \end{vmatrix},$$

the relevant border hessian \tilde{H} evaluated at $\tilde{\lambda}$ and $(\tilde{c}_2(y), \tilde{c}_2(x))$ is negative definite— $\left| \tilde{H} \right| > 0$ — and

$$\begin{aligned} \frac{dc_2(y)}{d(k_2 - k)} &> 0 \quad \text{but} \quad dc_2(y) = dw(y) = d(k_2 - k) \\ \frac{dc_2(y)}{dy} &> 0 \quad \text{but} \quad dc_2(y) = dw(y) = dy \end{aligned} \quad (14)$$

$$\frac{dc_2(y)}{dp_y} = \frac{dc_2(y)}{d\bar{x}} = \frac{dc_2(y)}{d\bar{x}} = \frac{dc_2(y)}{d\frac{p\bar{x}}{p_x + p\bar{x}}} = 0 \quad \text{and} \quad dw(y) = 0.$$

It is worth noting that for parameters values such that $\mu_y > 0$, the comparative statics effects given by (14) holds for both parameters's increases and decreases. Therefore, the sea loan contract is optimal for a robust set of parameters: let parameter values $\tilde{\lambda}$ be such that the sea loan is optimal, with $\tilde{c}_2(y) = \tilde{w}(y)$, then for parameter values in an open neighborhood of $\tilde{\lambda}$, $\tilde{c}_2(y) + dc_2(y) = \tilde{w}(y) + dw(y)$.

Show that $\left| \widehat{H} \right| > 0$ evaluated at $\hat{\lambda}$ and $(\hat{c}_2(y), \hat{c}_2(x))$.

$$\begin{aligned} \left| \widehat{H} \right| &= \begin{vmatrix} 0 & p_y U_2'[c_2(y)] & (1-p_y) U_2'[c_2(x)] \\ p_y U_2'[c_2(y)] & h_{22} & 0 \\ (1-p_y) U_2'[c_2(x)] & 0 & h_{33} \end{vmatrix} = \\ &= - [p_y U_2'[c_2(y)]]^2 h_{33} - [(1-p_y) U_2'[c_2(x)]]^2 h_{22} > 0 \end{aligned}$$

because both h_{22} and h_{33} are negative.

$$\begin{aligned} h_{22} &= p_y U_1''[w(y) - c_2(y)] + \eta p_y U_2''[c_2(y)] = \\ &= -p_y R_1[w(y) - c_2(y)] U_1'[w(y) - c_2(y)] - \eta p_y R_2[c_2(y)] U_2'[c_2(y)] = \\ &= -p_y R_1[w(y) - c_2(y)] U_1'[w(y) - c_2(y)] - p_y R_2[c_2(y)] U_1'[w(y) - c_2(y)] = \\ &= -p_y [R_1[w(y) - c_2(y)] + R_2[c_2(y)]] U_1'[w(y) - c_2(y)] < 0 \end{aligned} \quad (15)$$

and

$$\begin{aligned} h_{33} &= p_x U_1''[w(x) - c_2(x)] + p_x U_1''[w(\bar{x}) - c_2(x)] + \eta (1-p_y) U_2''[c_2(x)] = \\ &= -R_1[w(x) - c_2(x)] U_1'[w(x) - c_2(x)] - \eta (1-p_y) R_2[c_2(x)] U_2'[c_2(x)] = \\ &= -R_1[w(x) - c_2(x)] U_1'[w(x) - c_2(x)] - R_2[c_2(x)] U_1'[w(x) - c_2(x)] = \\ &= -[R_1[w(x) - c_2(x)] + R_2[c_2(x)]] U_1'[w(x) - c_2(x)] < 0, \end{aligned} \quad (16)$$

where the first equalities are simply the definitions of h_{22} and h_{33} ; the second equalities derive from applying the definition of absolute risk-aversion coefficient, $R(z) = -\frac{U''(z)}{U'(z)}$, and defining

$$R_1[w(x) - c_2(x)] = -\frac{U_1''[w(x) - c_2(x)]}{U_1'[w(x) - c_2(x)]} = -\frac{p_x U_1''[w(x) - c_2(x)] + p_x U_1''[w(\bar{x}) - c_2(x)]}{p_x U_1'[w(x) - c_2(x)] + p_x U_1'[w(\bar{x}) - c_2(x)]} \quad (17)$$

to make the notation more compact; the third equalities follow from, respectively, the second and third row in (12) with $\mu_y = \mu_x = 0$; the fourths, from simply operating; and the inequality, from $U'(\cdot) > 0$ and $U''(\cdot) < 0$, so that $R_i(\cdot) > 0$, for $i = 1, 2$. **QED**

Note that $\left| \widehat{H} \right|$ can be expressed as

$$\begin{aligned} \left| \widehat{H} \right| &= [p_y U_2'[c_2(y)]]^2 U_1'[w(x) - c_2(x)] R_1[w(x) - c_2(x)] + \\ & [p_y U_2'[c_2(y)]]^2 U_1'[w(x) - c_2(x)] R_2[c_2(x)] + \\ & [(1-p_y) U_2'[c_2(x)]]^2 p_y U_1'[w(y) - c_2(y)] R_1[w(y) - c_2(y)] + \\ & [(1-p_y) U_2'[c_2(x)]]^2 p_y U_1'[w(y) - c_2(y)] R_2[c_2(y)]. \end{aligned} \quad (18)$$

Show that $\frac{dc_2(y)}{d(k_2-k)} = -\frac{\left|\hat{H}_{(k_2-k)}\right|}{\left|\hat{H}\right|} \in (0, 1)$ **evaluated at** $\hat{\lambda}$ **and** $(\hat{c}_2(y), \hat{c}_2(x))$.

$$\begin{aligned} \left|\hat{H}_{(k_2-k)}\right| &= [(1-p_y)U_2'[c_2(x)]]^2 p_y U_1'[w(y)-c_2(y)] R_1[w(x)-c_2(x)] - \\ & [(1-p_y)U_2'[c_2(x)]]^2 p_y U_1'[w(y)-c_2(y)] R_1[w(y)-c_2(y)] < 0 \end{aligned} \quad (19)$$

because $U'(\cdot) > 0$, $w(y) - c_2(y) < w(\underline{x}) - c_2(x) < w(x) - c_2(x)$,¹⁴ and utility functions exhibit DARA, so that $R_1[w(y) - c_2(y)] > R_1[w(x) - c_2(x)]$.

Comparing (19) with (18), it follows that $0 < \left|\frac{\hat{H}}{\hat{H}}\right| + \left|\frac{\hat{H}_{(k_2-k)}}{\hat{H}}\right|$. **QED**

Show that $\frac{dc_2(y)}{dy} = -\frac{\left|\hat{H}(y)\right|}{\left|\hat{H}\right|} \in (0, 1)$ **evaluated at** $\hat{\lambda}$ **and** $(\hat{c}_2(y), \hat{c}_2(x))$.

$$\begin{aligned} \left|\hat{H}(y)\right| &= [(1-p_y)U_2'[c_2(x)]]^2 p_y U_1''[w(y)-c_2(y)] = \\ &= -[(1-p_y)U_2'[c_2(x)]]^2 p_y U_1'[w(y)-c_2(y)] R_1[w(y)-c_2(y)] < 0 \end{aligned} \quad (20)$$

Comparing (20) with (18), it follows that $0 < \left|\frac{\hat{H}}{\hat{H}}\right| + \left|\frac{\hat{H}_{(k_2-k)}}{\hat{H}}\right|$. **QED**

Show that $\frac{dc_2(y)}{dp_y} = -\frac{\left|\hat{H}_{p_y}\right|}{\left|\hat{H}\right|} > 0$ **evaluated at** $\hat{\lambda}$ **and** $(\hat{c}_2(y), \hat{c}_2(x))$.

$$\begin{aligned} \left|\hat{H}_{p_y}\right| &= [U_2[c_2(y)] - U_2[c_2(x)]] p_y U_2'[c_2(y)] U_1'[w(x)-c_2(x)] [R_1[w(x)-c_2(x)] + R_2[c_2(x)]] \\ & - (1-p_y) U_2'[c_2(x)] p_y U_2'[c_2(y)] \frac{U_1'[w(x)-c_2(x)]}{1-p_y} < 0 \end{aligned}$$

because $R_i(\cdot) > 0$, $U_i'(\cdot) > 0$, and $c_2(y) \leq c_2(x)$, so that $[U_2[c_2(y)] - U_2[c_2(x)]] < 0$. **QED**

Show that $\frac{dc_2(y)}{d\underline{x}} = -\frac{\left|\hat{H}(\underline{x})\right|}{\left|\hat{H}\right|} < 0$ **and** $\frac{dc_2(y)}{d\bar{x}} = -\frac{\left|\hat{H}(\bar{x})\right|}{\left|\hat{H}\right|} < 0$ **evaluated at** $\hat{\lambda}$ **and** $(\hat{c}_2(y), \hat{c}_2(x))$.

$$\left|\hat{H}(\underline{x})\right| = -(1-p_y)U_2'[c_2(x)] p_y U_2'[c_2(y)] p_{\underline{x}} U_1''[w(\underline{x})-c_2(x)] > 0.$$

$$\left|\hat{H}(\bar{x})\right| = -(1-p_y)U_2'[c_2(x)] p_y U_2'[c_2(y)] p_{\bar{x}} U_1''[w(\bar{x})-c_2(x)] > 0.$$

These follow from $U'(\cdot) > 0$ and $U''(\cdot) < 0$. **QED**

¹⁴The optimal sharing rule gives some insurance to both parties, for both are risk-averse. Appendix B proves this result under full information. The proof under hidden information runs similarly.

Show that $\frac{dc_2(y)}{d\frac{p_{\bar{x}}}{p_{\underline{x}}+p_{\bar{x}}}} = -\frac{\left|\frac{\hat{H}}{p_{\underline{x}}+p_{\bar{x}}}\right|}{\left|\hat{H}\right|} < 0$ **evaluated at** $\hat{\lambda}$ **and** $(\hat{c}_2(y), \hat{c}_2(x))$.

$$\left|\frac{\hat{H}}{p_{\underline{x}}+p_{\bar{x}}}\right| = (1-p_y)U_2'[c_2(x)]p_yU_2'[c_2(y)][U_1'[w(\underline{x})-c_2(x)]-U_1'[w(\bar{x})-c_2(x)]] > 0$$

because $U'(\cdot) > 0$, $U''(\cdot) < 0$ and $w(\underline{x}) < w(\bar{x})$, so that $U_1'[w(\underline{x})-c_2(x)]-U_1'[w(\bar{x})-c_2(x)] > 0$. **QED**

Show that $\left|\frac{\tilde{H}}{\tilde{H}}\right| > 0$ **evaluated at** $\tilde{\lambda}$ **and** $(\tilde{c}_2(y), \tilde{c}_2(x))$.

$$\left|\frac{\tilde{H}}{\tilde{H}}\right| = \begin{vmatrix} 0 & 0 & p_y U_2'[c_2(y)] & (1-p_y) U_2'[c_2(x)] \\ 0 & 0 & -1 & 0 \\ p_y U_2'[c_2(y)] & -1 & h_{22} & 0 \\ (1-p_y) U_2'[c_2(x)] & 0 & 0 & h_{33} \end{vmatrix} = [(1-p_y) U_2'[c_2(x)]]^2 > 0,$$

where h_{22} and h_{33} are defined by (15) and (16), respectively. **QED**

Show that $\frac{dc_2(y)}{d(k_2-k)} = -\frac{\left|\frac{\tilde{H}}{\tilde{H}}\right|_{(k_2-k)}}{\left|\tilde{H}\right|} = 1$ **evaluated at** $\tilde{\lambda}$ **and** $(\tilde{c}_2(y), \tilde{c}_2(x))$.

$$\left|\frac{\tilde{H}}{\tilde{H}}\right|_{(k_2-k)} = -[(1-p_y)U_2'[c_2(x)]]^2 < 0. \quad \text{QED}$$

Show that $\frac{dc_2(y)}{dy} = -\frac{\left|\frac{\tilde{H}}{\tilde{H}}\right|(y)}{\left|\tilde{H}\right|} = 1$ **evaluated at** $\tilde{\lambda}$ **and** $(\tilde{c}_2(y), \tilde{c}_2(x))$.

$$\left|\frac{\tilde{H}}{\tilde{H}}\right|(y) = -[(1-p_y)U_2'[c_2(x)]]^2 < 0. \quad \text{QED}$$

Show that $\frac{dc_2(y)}{dp_y} = \frac{dc_2(y)}{d\underline{x}} = \frac{dc_2(y)}{d\bar{x}} = \frac{dc_2(y)}{d\frac{p_{\bar{x}}}{p_{\underline{x}}+p_{\bar{x}}}} = 0$ **evaluated at** $\tilde{\lambda}$ **and** $(\tilde{c}_2(y), \tilde{c}_2(x))$.

$$\left|\frac{\tilde{H}}{\tilde{H}}\right|_{p_y} = \left|\frac{\tilde{H}}{\tilde{H}}\right|_{\underline{x}} = \left|\frac{\tilde{H}}{\tilde{H}}\right|_{\bar{x}} = \left|\frac{\tilde{H}}{\tilde{H}}\right|_{\frac{p_{\bar{x}}}{p_{\underline{x}}+p_{\bar{x}}}} = 0. \quad \text{QED}$$

B Proof of proposition 2

The commenda contract solves the relevant optimization problem under full information with $k_1 = 0$

Program 3:

$$\begin{aligned} \max_{\{c_2(s)\}_{s \in S}} \quad & E[U_1[w(s) - c_2(s)]] \\ \text{s.t.} \quad & E[U_2[c_2(s)]] = \bar{U}_2 \end{aligned} \quad (21)$$

$$w(s) - c_2(s) \geq 0 \quad \forall s \quad (22)$$

$$c_2(s) \geq 0 \quad \forall s. \quad (23)$$

Step 1. Show that all individually-rational efficient allocations are characterized by

$$c_2(y) < c_2(\underline{x}) < c_2(\bar{x})$$

Let η and μ_s be the Kuhn-Tucker multipliers of restrictions (21) and (22) for each s , respectively. Define the Lagrangian function for program 3 as

$$\mathcal{L}(c_2(s), \eta, \mu_s) = E[U_1[w(s) - c_2(s)]] + \eta(-\bar{U}_2 + E[U_2[c_2(s)]]) + \sum_s \mu_s[w(s) - c_2(s)].$$

Program 3 is concave so that Kuhn-Tucker conditions are necessary and sufficient for optimality:

$$\begin{aligned} \frac{\partial \mathcal{L}(\cdot)}{\partial c_2(s)} &= -p_s U_1'[w(s) - c_2(s)] + \eta p_s U_2'[c_2(s)] - \mu_s \leq 0 & c_2(s) \geq 0 & c_2(s) \frac{\partial \mathcal{L}(\cdot)}{\partial c_2(s)} = 0 & \forall s \\ \frac{\partial \mathcal{L}(\cdot)}{\partial \eta} &= -\bar{U}_2 + E[U_2[c_2(s)]] \geq 0 & \eta \geq 0 & \eta \frac{\partial \mathcal{L}(\cdot)}{\partial \eta} = 0 \\ \frac{\partial \mathcal{L}(\cdot)}{\partial \mu_s} &= w(s) - c_2(s) \geq 0 & \mu_s \geq 0 & \mu_s \frac{\partial \mathcal{L}(\cdot)}{\partial \mu_s} = 0 & \forall s. \end{aligned} \tag{24}$$

Let $c_2(s) > 0 \forall s$, then complementary slackness in (24) ensures that $\frac{\partial \mathcal{L}(\cdot)}{\partial c_2(s)} = 0$. Then, $c_2(y) < c_2(\underline{x})$ because

$$\begin{aligned} \text{if } \mu_y = \mu_1(\underline{x}) = 0 & \quad \Psi(c_2(y), c_2(\underline{x}); \lambda_1) = \frac{U_2'[c_2(y)]}{U_2'[c_2(\underline{x})]} - \frac{U_1'[w(y) - c_2(y)]}{U_1'[w(\underline{x}) - c_2(\underline{x})]} = 0 \\ \text{if } \mu_y > 0 \text{ and } \mu_1(\underline{x}) = 0 & \quad \Psi(c_2(y), c_2(\underline{x}); \lambda_1) = \frac{U_2'[c_2(y)]}{U_2'[c_2(\underline{x})]} - \frac{U_1'[w(y) - c_2(y)]}{U_1'[w(\underline{x}) - c_2(\underline{x})]} > 0 \\ \text{if } \mu_y = 0 \text{ and } \mu_1(\underline{x}) > 0 & \quad c_2(\underline{x}) = w(\underline{x}) > w(y) \geq c_2(y). \end{aligned}$$

$\Psi(c_2(y), c_2(\underline{x}); \lambda_1) = \frac{U_2'[c_2(y)]}{U_2'[c_2(\underline{x})]} - \frac{U_1'[w(y) - c_2(y)]}{U_1'[w(\underline{x}) - c_2(\underline{x})]} \geq 0$ derives from operating the Kuhn-Tucker conditions for $\mu_y \geq 0$ and $\mu_1(\underline{x}) = 0$. From applying $U_i'(\cdot) > 0$ and $U_i''(\cdot) < 0$ to this expression, it follows that optimal event-contingent consumption must satisfy $c_2(y) < c_2(\underline{x})$. The equality $c_2(\underline{x}) = w(\underline{x})$ in the third expression derives from complementary slackness in (24) with $\mu_1(\underline{x}) > 0$; $w(\underline{x}) > w(y)$ by assumption; and $w(y) \geq c_2(y)$ is restriction (22) for $s = y$. Likewise, it can be proved that $c_2(\underline{x}) < c_2(\bar{x})$.

The case $c_2(y) = c_2(\underline{x}) = c_2(\bar{x}) = 0$ lies in the contract curve, but does not satisfy the participation constraint of agent 2. The case $c_2(s) = 0$ and $c_2(s') > 0$ for $s, s' \in S$ and $s > s'$ leads to contradiction.

Step 2. Show that all individually-rational efficient allocations are characterized by

$$c_2(y) = w(y) = k_2 - k + y.$$

Assume they are not and, then, let $(k_2 - k + \hat{t}^y, k_2 - k + \hat{t}^x, k_2 - k + \hat{t}^{\bar{x}})$ be the solution of program 2, with $\hat{t}^y \neq y$. From imposing non-negative consumption, it follows that $\hat{t}^y < y$

and it is possible to write

$$\hat{t}^x = \underline{a} + d$$

$$\hat{t}^{\bar{x}} = \bar{a} + d$$

with $\bar{a} \neq 0 \neq \underline{a}$, $E(a) = 0$ and $d = \frac{p_{\bar{x}}}{p_x + p_{\bar{x}}} \hat{t}^{\bar{x}} + \frac{p_x}{p_x + p_{\bar{x}}} \hat{t}^x$. For notational purposes, let us define $E[u_1[\tau(y), \tau(\underline{x}), \tau(\bar{x})]] = E[U_1[s - \tau(s)]]$ and $E[u_2[\tau(y), \tau(\underline{x}), \tau(\bar{x})]] = E[U_2[k_2 - k + \tau(s)]]$.

Let the event-contingent allocation $(k_2 - k + y, k_2 - k + \tilde{t}, k_2 - k + \tilde{t})$ be the solution to program 2. Then, for $d \neq \tilde{t}$, it holds that if

$$E[u_2[\hat{t}^y, d, d]] \geq E[u_2[y, \tilde{t}, \tilde{t}]] = \bar{U}_2,$$

$$E[u_1[\hat{t}^y, d, d]] < E[u_1[y, \tilde{t}, \tilde{t}]].$$

Let also define $(k_2 - k + y, k_2 - k + \tilde{t}^x, k_2 - k + \tilde{t}^{\bar{x}})$ as the solution to a restricted version of program 3 in the sense that $c_2(y) = k_2 - k + y$ is imposed. If $(k_2 - k + \hat{t}^y, k_2 - k + \hat{t}^x, k_2 - k + \hat{t}^{\bar{x}})$ is efficient, then, by definition,

$$E[u_2[\hat{t}^y, \underline{a} + d, \bar{a} + d]] = \bar{U}_2 = E[u_2[y, \tilde{t}, \tilde{t}]] \quad (25)$$

$$E[u_1[\hat{t}^y, \underline{a} + d, \bar{a} + d]] > E[u_1[y, \tilde{t}^x, \tilde{t}^{\bar{x}}]]. \quad (26)$$

It can be proved that (26) leads to a contradiction and, therefore, the optimal allocation of consumption establishes $c_2(y) = k_2 - k + y$. Indeed,

$$E[u_1[\hat{t}^y, \underline{a} + d, \bar{a} + d]] < E[u_1[\hat{t}^y, d, d]] < E[u_1[y, \tilde{t}, \tilde{t}]] < E[u_1[y, \tilde{t}^x, \tilde{t}^{\bar{x}}]], \quad (27)$$

where the first inequality derives from the utility function being concave and $E(a) = 0$. The second one follows from $(k_2 - k + y, k_2 - k + \tilde{t}, k_2 - k + \tilde{t})$ solving program 1 and from

$$E[u_2[\hat{t}^y, d, d]] > E[u_2[\hat{t}^y, \underline{a} + d, \bar{a} + d]] = \bar{U}_2 = E[u_2[y, \tilde{t}, \tilde{t}]],$$

where the inequality derives from the utility function being concave and $E(a) = 0$ and the equalities are imposed by (25). The last inequality of (27) holds because the allocation $(k_2 - k + y, k_2 - k + \tilde{t}, k_2 - k + \tilde{t})$ could have been chosen—program 2 is a restricted version of program 3— but it was not. Therefore, it must give less expected utility to agent 1 than the optimally chosen allocation $(k_2 - k + y, k_2 - k + \tilde{t}^x, k_2 - k + \tilde{t}^{\bar{x}})$.

Step 3. The individually-rational efficient allocations of consumption

$$(k_2 - k + y, k_2 - k + \tilde{t}^x, k_2 - k + \tilde{t}^{\bar{x}})$$

can be sustained by a commenda contract establishing $\tau = -k$, $\tau(y) = y$, $\tau(\underline{x}) = \tilde{t}^x$, and $\tau(\bar{x}) = \tilde{t}^{\bar{x}}$. Just note that contracts are defined such that $c_2(s) = k_2 + \tau + \tau(s)$. **QED**

C Proof of proposition 3 and lemma 1

The relevant problem when the merchant is initially endowed to fund the venture on his own ($k_1 > k$) becomes a restricted version of program 2 (under hidden information) or program 3 (under full information) in which restriction (6) is added. The proof is developed for the hidden information structure, but can be extended to full information easily.

Step 1: Show that $c_2^*(y) > k_2 - k + y$.

When the entrepreneur lacks the wealth to finance his project in its entirety ($k_1 = 0$), subsection 3, and proposition 1 in particular, states the conditions under which the optimal allocation is a binding solution with a positive Kuhn-Tucker multiplier μ_y associated with the restriction $w(y) - c_2(y) \geq 0$. This means that for the (binding) solution $(\tilde{c}_2(y), \tilde{c}_2(x))$,

$$\Upsilon(\tilde{c}_2(y), \varphi(\tilde{c}_2(y), \lambda_1); \lambda_1) > 0, \quad (28)$$

where $\tilde{c}_2(y) = k_2 - k + y$ and $\tilde{c}_2(x) = k_2 - k + \tilde{t}$, and λ_1 represents parameter values representing a risky and costly venture, and $\theta = 0$, $k_1 = 0$. From (28) and utility functions exhibiting DARA, it follows that the allocation $(\tilde{c}_2(y), \tilde{c}_2(x))$ is not optimal for parameters λ_2 such that $k_1 > k$ either.

Lemma 2 $\Upsilon(\tilde{c}_2(y), \tilde{c}_2(x); \lambda_2) > 0$.

Proof of Lemma 2: $\Upsilon(\tilde{c}_2(y), \tilde{c}_2(x); \lambda_1) > 0$, and

$$\begin{aligned} \frac{\partial \Upsilon(c_2(y), c_2(x); \lambda)}{\partial k_1} &= - \frac{U_1''[w(y) - c_2(y)] [p U_1'[w(\bar{x}) - c_2(x)] + (1-p) U_1'[w(\underline{x}) - c_2(x)]]}{[p U_1'[w(\bar{x}) - c_2(x)] + (1-p) U_1'[w(\underline{x}) - c_2(x)]]^2} + \\ &+ \frac{U_1'[w(y) - c_2(y)] [p U_1''[w(\bar{x}) - c_2(x)] + (1-p) U_1''[w(\underline{x}) - c_2(x)]]}{[p U_1'[w(\bar{x}) - c_2(x)] + (1-p) U_1'[w(\underline{x}) - c_2(x)]]^2} = \\ &= \frac{R_1[w(y) - c_2(y)] U_1'[w(y) - c_2(y)]}{[p U_1'[w(\bar{x}) - c_2(x)] + (1-p) U_1'[w(\underline{x}) - c_2(x)]]} - \frac{R_1[w(x) - c_2(x)] U_1'[w(y) - c_2(y)]}{[p U_1'[w(\bar{x}) - c_2(x)] + (1-p) U_1'[w(\underline{x}) - c_2(x)]]} = \\ &= \frac{U_1'[w(y) - c_2(y)]}{p U_1'[w(\bar{x}) - c_2(x)] + (1-p) U_1'[w(\underline{x}) - c_2(x)]} [R_1[w(y) - c_2(y)] - R_1[w(x) - c_2(x)]] > 0, \end{aligned}$$

where the first equality derives from simple derivation; the second one from applying the definition of absolute risk-aversion coefficient, $R(z) = -\frac{U''(z)}{U'(z)}$, and (17); the third equality derives from operating and the inequality from $U'(\cdot) > 0$, $U''(\cdot) < 0$ and the utility function exhibiting DARA, with $w(y) - c_2(y) < w(\underline{x}) - c_2(x) < w(x) - c_2(x)$, so that

$$R_1[w(y) - c_2(y)] > R_1[w(x) - c_2(x)].$$

QED

Therefore, the optimal allocation is not $(\tilde{c}_2(y), \tilde{c}_2(x))$ but $(c_2^*(y), c_2^*(x)) = (\tilde{c}_2(y) + \epsilon, \tilde{c}_2(x) - \delta(\epsilon))$, where different values of ϵ define distinct points in the core.

Lemma 3 *There exists an $\epsilon > 0$ and a $\delta(\epsilon) > 0$ such that all the restrictions are satisfied and $\Upsilon(\tilde{c}_2(y) + \epsilon, \tilde{c}_2(x) - \delta(\epsilon); \lambda_2) = 0$.*

Proof of Lemma 3: The result follows from applying the sign of the following derivatives to lemma 2. $\frac{\partial \Upsilon(c_2(y), c_2(x); \lambda)}{\partial c_2(y)} < 0$ and $\frac{\partial \Upsilon(c_2(y), c_2(x); \lambda)}{\partial c_2(x)} > 0$. The proof is straightforward. Just find the derivatives and note that $U'(\cdot) > 0$ and $U''(\cdot) < 0$. **QED**

Lemma 4 *The contract curve for interior points, defined by $\varphi(c_2(y), \lambda)$ for parameters λ_2 , such that $k_1 > k > 0$, lies below the contract curve for λ_1 , with $k_1 = 0$: $\varphi(c_2(y), \lambda_2) < \varphi(c_2(y), \lambda_1) \forall c_2(y)$.*

Proof of Lemma 4: The *Implicit Function Theorem* also gives the first-order comparative statics of any parameter on $\varphi(c_2(y))$, for any $c_2(y)$ at a solution. In particular,

$$\frac{d\varphi(c_2(y), \lambda)}{dk_1} = -\frac{\frac{\partial \Upsilon(c_2(y), c_2(x); \lambda)}{\partial k_1}}{\frac{\partial \Upsilon(c_2(y), c_2(x); \lambda)}{\partial c_2(x)}} < 0 \quad \forall c_2(y) \text{ in the restricted set,} \quad (29)$$

since $U'(\cdot) > 0$, $U''(\cdot) < 0$ and the utility functions exhibit decreasing absolute risk-aversion (DARA). **QED**

Therefore, the optimal allocation establishes $c_2^*(y) = k_2 - k + y + \epsilon$ with $\epsilon > 0$.

Step 2: Show that $c_2^*(y) < k_2$.

This follows from imposing the optimal sharing rule $c_2^*(y) < c_2^*(x)$ to the ex-ante participation constraint of agent 1 holding with equality and noting that all individually rational allocations of agent 2's consumption are upper-bounded by the ex-ante participation constraint of agent 1.

Let $c_1(y) = w(y) - c_2(y) \leq k_1 - k + y$, and, consequently, $c_2(y) \geq (k_1 + k_2 - k + y) - k_1 + k - y = k_2$. Then, for $E[U_1[w(s) - c_2(s)]] = E[U_1[k_1 - k + s]]$ to be satisfied, $c_1(\bar{x}) = w(\bar{x}) - c_2(x) > k_1 - k + \bar{x}$. These, in turn, imply that $c_2(x) \leq k_2$, which is in contradiction with the optimal sharing rule, $c_2^*(y) < c_2^*(x)$.

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