A discusión

BIDDING BEHAVIOR IN MULTI-UNIT AUCTIONS — AN EXPERIMENTAL INVESTIGATION AND SOME THEORETICAL INSIGHTS*

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ABSTRACT

We present laboratory experiments of five different multi-unit auction mechanisms. Two units of a homogeneous object were auctioned off among two bidders with flat demand for two units. We test whether expected demand reduction occurs in open and sealed–bid uniform–price auctions. Revenue equivalence is tested for these auctions as well as for the Ausubel, the Vickrey and the discriminatory sealed–bid auction. Furthermore, we compare the five mechanisms with respect to the efficient allocation of the units. We also provide some theoretical insights concerning the equilibria of uniform–price auctions with incomplete information.

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Keywords: Multi–Unit Auctions, Demand Reduction, Experimental Economics.

1 Introduction

Motivated by high profile auctions such as the FCC or the Treasury bill auctions, theoretical research has recently been extended from single to multi-unit auctions. Friedman (1960) proposed to change the rules of the Treasury bill auctions from a discriminatory to a uniform–price format which was thought by some authors to be a generalization of the incentive compatible Vickrey auction to the multi-unit case. Vickrey (1961), however, has already indicated that this is not the case. In the uniform–price auction any bidder has an incentive to reduce demand on all except for the first unit, since one of his bids may determine the price he has to pay for inframarginal units. A formal proof was provided more recently by Ausubel and Cramton (2002) who, moreover, showed that in many cases the discriminatory auction outperforms the uniform-price auction. Similarly, Katzmann (1995), Noussair (1995), Engelbrecht–Wiggans and Kahn (1998), and Grimm et al. (2003) analyze auctions where bidders have demand for multiple units and give examples of equilibria that involve demand reduction.

A sealed–bid mechanism that generalizes the Vickrey auction for single units to the multi-unit case has already been presented in Vickrey (1961). It is basically a special case of the revelation mechanisms developed independently by Clarke (1971) and Groves (1973). Ausubel (2002) proposed an open auction that implements the outcome of the (incentive compatible) multi–unit Vickrey auction in a way that is possibly most transparent to bidders.

In this paper we experimentally investigate bidding behavior in five different multi–unit auction formats: the discriminatory auction (DA), the uniform–price sealed–bid auction (UPS), the uniform–price open auction (UPO), the Vickrey Auction (VA), and the Ausubel Auction (AA). Our experiment consists of a series of two-unit, two bidder auctions. Bidders have a flat demand for two units. In this framework, in the most extreme case, demand reduction in equilibrium involves a zero bid on the second unit in the uniform–price auctions. This implies a maximum difference between the theoretical prediction for the uniform–price auction and the other auction formats in terms of revenue.

We find that demand reduction is more frequent in UPO than in UPS, but, interestingly, does also occur in AA. As a consequence efficiency is substantially lower in UPO than in the other auctions. Demand reduction decreases substantially in AA over time and as a consequence, efficiency in AA is significantly higher than in DA, UPO, and UPS in the second half of the experiment. Revenue equivalence for the two uniform–price auctions and for the non–uniform–price auctions, respectively, does not hold. In contrast to the theoretical prediction, revenues depend less on the pricing rule than on whether the auction is open or sealed–bid and they are higher in the latter case. On the one hand, this is due to the different extent of demand reduction in open and sealed-bid auctions. On the other hand, first, bidders more frequently overbid their valuation in VA and UPS where it is less clear that overbidding is dominated than in the more transparent open auctions. Second, in DA average bids frequently are above the equilibrium prediction. In clear contrast to the theoretical prediction, in DA bidders place substantially different bids on the first and the second unit, which might be caused be a myopic zero-profit aversion of the bidders.

In the uniform–price treatments bidders played both, UPO and UPS. Here we found that even pairs that in UPO coordinated on the payoff-dominant equilibrium involving complete demand reduction, only rarely managed to do so in the subsequent UPS. We observe, however, some tendency towards the payoff dominant equilibrium.

Closely related experiments were run by Alsemgeest et al. (1998), Kagel and Levin (2001), List and Lucking–Reiley (2000), and Porter and Vragov (2000). Our experiments are, however, the first to compare all these five standard auction formats in the same framework. Alsemgeest et al. (1998) compare UPO and a version of UPS (with a different pricing rule). They find that the revenue is higher in the sealed bid auction and that bidders reduce demand in UPO. Kagel and Levin compare uniform–price sealed–bid and open auctions and the Ausubel auction and find systematic demand reduction in the uniform–price auctions. Their subjects also have flat demand for two units but bid against robot bidders with unit demand. List and Lucking–Reiley conduct field experiments, comparing the uniform–price sealed–bid and the Vickrey sealed–bid auction by selling sportscards in two–unit, two–person auctions. They also find demand reduction in uniform–price auctions, compared to Vickrey auctions. They cannot, however, control for the bidders' valuations. Their experiment is replicated in the laboratory by Porter and Vragov (2000), who find substantial deviations from demand revelation in both, UPS and VA, but also more demand reduction in UPS.

The paper is structured as follows. Section 2 presents the equilibria of our auction games and the implied hypotheses. The experimental design is presented in section 3, followed by the experimental results in section 4, and the conclusions. In the appendix we present in more detail an analysis of equilibria of the uniform–price auctions and show that there is a continuum of equilibria both in UPS and UPO, but that even under incomplete information the demand reduction equilibrium is the unique equilibrium of UPO that satisfies certain refinements.

2 Theoretical Background and Hypotheses

2.1 Equilibrium Analysis

We investigate bidding behavior in independent private value auctions with two bidders and two indivisible identical objects for sale. Each bidder demands at most two units. A bidder places the same value v_i on each unit. The bidders' valuations are drawn independently from the same uniform distribution on the interval $[0, V]$.

We consider five different auction formats. In the three sealed–bid auctions the bidders simultaneously submit sealed–bids for each of the units demanded and prices and allocations are determined according to the auction rules. The two open auctions start out with a price of zero and active bids on all units demanded. The price is increased and units are traded according to the rules of the mechanism as bidders drop out. In all auctions the two highest bids each win a unit.

Uniform–Price Sealed–Bid Auction [UPS] and Uniform–Price Open Auction [UPO]

In the uniform–price auctions the price for all units equals the highest rejected bid. In our experiment, this is the third highest bid. In the uniform–price sealed–bid auction, each bidder places two bids and the units are allocated to the two highest bids (or randomly in case of a tie). The uniform-price open auction starts out with a price of zero, with the price increasing continuously thereafter. Bidders start out actively bidding on two units each and may choose the price(s) where they drop out on one unit, or on both. Dropping out is irrevocable so that a bidder can no longer bid on a unit he has dropped out on. As soon as the number of active bids equals the number of units available, both items are sold to the bidder(s) holding the active bids at the price at which the last bidder dropped out. Thus, the price is determined either by a second dropout of a bidder on one unit or by a bidder's simultaneous dropout on both units.

In both uniform–price formats it is a weakly dominant strategy to bid one's valuation v_i on the first unit[1] (i. e. the higher bid always equals the true valuation). A bid on the first unit will only determine the price if it is the highest rejected bid, i. e. if the bidder does not get a unit. Therefore, lowering the bid implies the risk of missing a profitable deal whereas overbidding might result in buying a unit at a loss. This is even more obvious in the open auction. If bidder i has already dropped out on one unit, dropping out on the other unit before his valuation v_i is reached guarantees a profit of 0, whereas continuing might yield a positive profit, if the other bidder drops before v_i is reached. Staying in at prices above v_i causes a loss as soon as the other bidder drops out.

Lowering the bid on the second unit, however, presents a trade-off. A lower bid on the second unit lowers the chance of winning two units but, at the same time, may reduce the price paid for the first unit. As it turns out, the uniform-price auctions have multiple equilibria. All equilibria that do not involve truthful bidding on the first unit are weakly dominated. Among those equilibria that involve truthful bidding on the first unit the following are the extreme cases: Truthful revelation on both units,

$$b_1(v_i) = b_2(v_i) = v_i, \tag{1}$$

(where b_1 denotes the first unit bid and b_2 the second unit bid) and full demand reduction on the second unit such that the bid on the second unit is zero,

$$\begin{aligned} b_1(v_i) &= v_i, \\ b_2(v_i) &= 0. \end{aligned} \tag{2}$$

In the following we will refer to these equilibria as the truth–telling (TT) and the demand reduction (DR) equilibrium, respectively.

The remaining equilibria in undominated strategies are of the following form: Let K be an integer ≥ 1, $x_{K+1} = V$ and $[x_k, y_k)$, $k = 1, \ldots K$ be a sequence of non–overlapping intervals with $x_1 \geq 0$, $x_k < y_k$, and $y_k \leq x_{k+1}$. Then, the equilibrium strategies are:

[1] "First unit" ("second unit") always refers to the unit on which the bidder places the higher (lower) bid.

$$
\begin{aligned}
b_1(v_i) &= v_i \\
b_2(v_i) &= \begin{cases} x_k & \text{if} \quad v_i \in [x_k, y_k), \\ v_i & \text{otherwise.} \end{cases}
\end{aligned}
\tag{3}
$$

This implies that a bidder bids truthfully if his valuation lies in a truth–telling interval $[y_k, x_{k+1})$ and partially reduces demand if his valuation lies in a demand reduction interval $[x_k, y_k)$.[2]

For the sealed–bid auction we show in appendix A.1 that it is not profitable to deviate from this strategy given the other bidder plays it. For the open auction, the strategy is sequentially rational if beliefs have the following form (i. e. together they constitute a weak perfect Bayesian equilibrium, wPBE, according to the definition of Mas-Colell et al., 1995):[3]

(a) As long as the other bidder does not drop out on any unit bidder i believes that the other bidder's valuation is uniformly distributed on the interval of valuations for which equilibrium behavior does not prescribe dropping out at a lower price. Hence, if the current price p is in the interior of a demand reduction interval (x_k, y_k), then the belief is uniformly distributed on $[y_k, V]$ (because equilibrium behavior prescribes for all valuations in $[x_k, y_k)$ to drop out at x_k) and otherwise the belief is uniformly distributed on $[p, V]$.

(b) If the other bidder drops out on one unit at $z_k \in [x_k, y_k)$ bidder i believes that the other bidder's valuation is uniformly distributed[4] in the interval $[p, y_k)$ for any current price $p \in [z_k, y_k)$,

(c) If the other bidder drops out on one unit at $z_k \in [y_k, x_{k+1})$ bidder i believes that the other bidder's valuation is uniformly distributed in the interval $[p, \min\{x_{k+1}, v_i\})$ for any current price $p \in [z_k, x_{k+1})$,

[2] Note that if $y_K = V$ then the Kth interval can also be closed. Furthermore, equilibria can be constructed in the same way with left open instead of right open intervals.

[3] In particular bidders stick to their equilibrium bids even if the other bidder deviated from the equilibrium path. See also appendix A.2 for a detailed analysis.

[4] The uniform distribution is only one example that implies that the suggested equilibrium strategy is indeed a best response. For example, in this case, any symmetric distribution would work. A non-uniform distribution would, however, require an unintuitive updating process.

(d) If the other bidder has already dropped out on one unit in an interval but has not dropped out on the other unit in the same interval, then bidder i believes that the other bidder's valuation is uniformly distributed in $[p, y_l)$ if $p \in [x_l, y_l)$ and in $[p, \min\{x_{l+1}, v_i\})$ if $p \in [y_l, x_{l+1})$, p being the current price.

It seems, however, highly unlikely that bidders can coordinate on one of these more sophisticated equilibria. There is no particular incentive to do so and they are more difficult to determine than the TT– and the DR–equilibrium. Note that the latter are extreme cases in the sense that DR requires $K = 1$, $x_1 = 0$, and $y_1 = V$ and TT results for $K = 0$.

Note that any equilibrium that involves bidding truthfully on the second unit requires type dependent beliefs. While Mas-Colell et al. (1995) do not explicitly rule out type dependent beliefs in their definition of wPBE,[5] Fudenberg and Tirole (1991) do so in their definition of a Perfect Bayesian Equilibrium (PBE).[6] Type-dependent beliefs do not appear plausible, because this assumes that the beliefs of one player concerning another player depend on a random event (namely that which determines his own valuation). They are also inconsistent with the "no signalling what you do not know" condition (Fudenberg and Tirole, 1991, p. 332), because an action of player B would signal something to player A about player A's valuation (since B's action would, according to A's belief, differ with A's valuation), which B does not know. Type dependent beliefs are definitely precluded in a Sequential Equilibrium (Kreps and Wilson, 1982), because the beliefs are derived as a limit of beliefs resulting from completely mixed strategies of the players.

If type dependent beliefs are not permitted, the only remaining equilibria are those that involve only DR–intervals (but arbitrarily many). By a sequence of infinitesimally small DR–intervals, however, a TT–interval can be approximated. In particular, in our experiment, where valuations are restricted to integers in [0,100], a sequence of m DR–intervals of length 1 induces the same behavior as a TT–interval of length m (but the beliefs supporting these equilibria are different).

Uniqueness of the DR–equilibrium results if we require that a sequential equilibrium satisfies support restriction (Madrigal et al., 1987). The latter amounts to requiring that a player does not assign positive

[5] Their criterion for a wPBE to be a PBE does not rule out such beliefs either.

[6] The lack of a generally accepted definition of a PBE precludes a definite answer as to whether type dependent beliefs are admissible.

probability to another player's type if he has assigned probability 0 to this type before (see appendix A.4).[7]

Furthermore, the DR–equilibrium is the only Perfect Bayesian equilibrium of the open auction if the beliefs strictly follow Bayes' rule also off the equilibrium path (that is, if a bidder observes a dropout he infers only that the opponent's valuation is higher than the dropout price and updates the initial distribution accordingly). In particular, such beliefs imply that whenever one bidder drops out on one unit, the other immediately follows (see appendix A.4).

All other equilibria of the open auction require that the bidders believe they are able to infer information from the other bidder's actions (off the equilibrium path) exceeding the minimal requirement that bidders only play undominated strategies. Intuitively, such equilibria may not seem completely implausible: If, for example $K = 1$, $x_1 = 0$, and $y_1 = V/2$, equilibrium beliefs would be that only bidders with low valuations drop out early, whereas bidders with high valuations always behave rather competitively. However, to make truthful bidding above $V/2$ an equilibrium strategy, a bidder's beliefs have to depend on his own type (i. e. valuation), as in (c) and (d) above.

We show in appendix A.3 that among all equilibria of the uniform–price auction the DR–equilibrium yields the highest expected payoff to the bidders.

Discriminatory Auction [DA]

In the discriminatory auction, the two highest bids win a unit each and the respective prices equal these bids.

An important observation in order to derive the optimal strategy is that with flat demand a bidder places the same bid on both units.[8] Suppose the other bidder placed two different bids. Then, in order to win one unit a bidder has to overbid only the other bidder's lower bid and in order to get two units both his bids have to exceed the other bidder's higher bid. Therefore, a bid on the first unit solves the optimal

[7] In our design, however, where valuations are restricted to integers (and hence, as argued above, TT–intervals can be constructed through a sequence of DR–intervals of length 1 without requiring type dependent beliefs), support restriction only precludes DR–intervals of length 2 or larger that do not include V. Hence sequential equilibria satisfying support restriction are of the form implying truth–telling up to some valuation v^* and one DR–interval $[v^*, V]$.

[8] See Lebrun and Tremblay (2000) for a formal proof of this fact for much more general demand functions.

trade-off between the probability of winning (against the other bidder's lower bid) and profit in this case. Now observe that the probability of winning the second unit is even lower (one has to overbid the other bidder's higher bid) and therefore, the optimal trade-off for the second unit cannot be solved at a lower bid. Thus, both bids will be equal since by definition the bid for the second unit cannot be higher than the bid for the first unit. If the other bidder chooses identical bids, the argument is even more obvious, since the trade-off is the same for both units.

Thus, the equilibrium bid function on each unit solves

$$\max_b F(\sigma(b))[v_i - b], \tag{4}$$

where $\sigma(b)$ is the inverse of the equilibrium strategy $b^\star(v)$. In the case of uniformly distributed valuations on $[0, V]$ and two bidders the equilibrium bid functions are

$$b_1(v_i) = b_2(v_i) = \frac{1}{2}v_i. \tag{5}$$

Vickrey Auction [VA]

In the multi unit generalization of the Vickrey auction the total price a bidder pays for the units he obtains equals the sum of the bids (other than his own) that are displaced by his successful bids. In our framework this means that, if one bidder places the two highest bids, he pays the two bids of the other bidder. If each bidder places one of the two highest bids, each pays the lower bid of the other bidder because his higher bid displaces the lower bid of the other bidder.

Thus, a bidder cannot influence the price he pays for any unit he obtains by changing his bids. Changing an unsuccessful bid has no effect unless it displaces another bid. In that case one obtains another unit and pays the displaced bid. This, of course, increases profits if and only if the displaced bid is lower than the bidder's own valuation v_i. Thus it is clearly weakly dominated to bid below v_i. But bidding above v_i on any unit is also dominated, since one might displace a bid that is also higher than v_i and hence incur a loss. Therefore, each bidder has the weakly dominant strategy of bidding truthfully on both units. (For the general case see also Vickrey, 1961.)

Ausubel Auction [AA]

The Ausubel (or dynamic Vickrey) auction (Ausubel, 2002) is an open mechanism that implements the same outcome as the multi-unit Vickrey auction in a way that has a great potential for transparency to bidders. The auction starts out at a price of zero which is then increased continuously. In the general case, at any price it is checked for each bidder whether the aggregate demand of the other bidders is smaller than the available number of units. If this is the case, he receives the available units at the current price.

In our case, the price is raised until one bidder (say, bidder i) drops out on one unit. At this point bidder j gets one unit for sure (in other words: he has "clinched" one unit). This unit is traded immediately and bidder j pays the price at which he has clinched it. Then the auction continues at this price for the remaining item that is still unsold. From that point on the two bidders are involved in a single-object English clock auction.

Under these rules the bidders have an incentive for full demand revelation on both units since the price paid for the first unit does not affect the price paid for the other unit. Thus,

$$b_1(v_i) = b_2(v_i) = v_i. \tag{6}$$

This equilibrium is obtained by iterated elimination of weakly dominated strategies. If one bidder has already dropped out it is weakly dominated to drop out at a price other than v_i, since the dropout price only determines the price for the remaining unit. One can only lose by staying in above v_i and can miss a possible gain by dropping out before v_i is reached. Eliminating these strategies then implies that the price of the first dropout does not influence the result of the subsequent bidding process. Hence it is also weakly dominated to drop out first at a price other than v_i since this dropout price only determines the price for this unit. To make not dropping out at a price lower than v_i optimal, however, requires knowing that the other bidder will not play a dominated strategy (e.g. will not drop out immediately after). Hence the equilibrium is not in weakly dominant strategies, but the game is only dominant solvable. The solution concept is thus weaker than in VA. In contrast the mechanism appears to be more transparent, which might compensate, in terms of efficiency, for the weaker equilibrium concept (see also Kagel et al., 2001).

2.2 Hypotheses Derived from the Theory

The theoretical analysis gives us several hypotheses to test.

(1) First unit bids in UPO, UPS, AA, and VA should equal the valuation (see the results in section 4.1).

(2) We expect to observe demand reduction on the second unit in UPO and UPS (see section 4.2).

(3) There should be no demand reduction in AA, VA, and DA. In particular, the bids should be equal on both units in all three auction formats (see section 4.3).

(4) In equilibrium, all units should be allocated efficiently in VA, AA, and DA, whereas only half of the units should be allocated efficiently if the DR–equilibrium is played in UPO and UPS (see section 4.4).

(5) Revenues are expected to be significantly lower in the uniform–price auctions than in the other three auctions. Revenues in AA, VA, and DA are theoretically equivalent in our setting (see section 4.5).[9]

(6) The bidders' expected payoffs are equal in AA, VA, and DA on the one hand, and in the DR–equilibrium of UPO and UPS on the other hand, and they are higher in the latter two (see section 4.6).

(7) Furthermore, we have an equilibrium selection problem in the uniform-price auctions. These auctions have several equilibria, one of which (the DR–equilibrium) payoff dominates the others from the bidders' viewpoint. The DR–equilibrium involves a zero bid on the second unit. It seems more likely that the DR–equilibrium is chosen in UPO than in UPS, since, as we argued above, it is the only equilibrium of UPO that satisfies certain refinements.[10] Furthermore, in UPO one bidder can initiate it by dropping out on one unit immediately. From these considerations two interesting questions arise: Do the bidders select the equilibrium that guarantees them the highest expected payoff, and do

[9] In DA, the price for each unit is $\frac{1}{2}\max\{v_i, v_j\}$ and $E[\max\{v_i, v_j\}] = \frac{2}{3}V$. In AA and VA the price is $\min\{v_i, v_j\}$ and $E[\min\{v_i, v_j\}] = \frac{1}{3}V$, so that the expected revenue is $\frac{2}{3}V$ in both cases. In contrast, the expected revenue in the uniform-price auctions is between 0 (if the DR–equilibrium is played) and $\frac{2}{3}V$ (if TT is played).

[10] See also appendix A.4.

the bidders select this equilibrium more often in UPS if they have played UPO before, i. e. do they manage to transfer the DR they may learn in UPO to UPS? (see section 4.7)

3 Experimental Design

In each auction two units of a homogeneous object were auctioned off among two bidders with flat demand for two units. Choosing to auction off two units per auction yields a simple payoff-dominant equilibrium in the case of uniform–price auctions, as described above. This creates the most significant difference between equilibrium bidding in the uniform–price auctions and the discriminatory, Vickrey, and Ausubel auctions. In each auction the bidders' private valuations for both units were drawn independently from the same uniform distribution on $[0, 100]$.[11] The bidders were undergraduate students from Humboldt University Berlin, the University of Zürich, and the ETH Zürich. Pairs of bidders were randomly formed. In DA, VA and AA each pair played ten auctions under the same rules. In the uniform–price auctions, in treatment UPOS each pair first played ten open auctions and then ten sealed–bid, in treatment UPSO vice versa.[12] Apart from this, in each session only one type of auction was conducted. For each treatment we had ten pairs, except for treatment DA, where we had nine.

Subjects were placed at isolated computer terminals, so that they could not determine whom they formed a pair with. Then the instructions (see appendix B for a translated sample) were read aloud. Before the start of a sequence of ten auctions, subjects played three dry runs, where they knew that their partner was simulated by a pre-programmed strategy. These strategies and the valuations of the subjects in the three dry runs were chosen in such a way that it was likely that each subject was exposed to winning 0 units in one auction, 1 unit in another and 2 units in the third. The pre-programmed strategies did not reflect any characteristics of the equilibria (in particular complete demand reduction in the uniform–price auctions) and the subjects were explicitly advised that they should not see these strategies as examples of

[11] Valuations were in fact drawn from the set of integers in [0,100] and also bids were restricted to integers. As argued above, this affects the equilibria in UPO. It does not, however, influence the predictions in the other treatments.

[12] This is why we only played ten auctions per pair and auction type. We wanted the total number of periods not to exceed 20 to avoid subjects getting bored. We also wanted to keep the incentives in each auction relatively high with a limited budget.

a good or a bad strategy (because they only observed the bids, they could not really copy the programmed strategy in any case). In the uniform–price sessions subjects were informed that after the first ten auctions, ten further auctions under a different rule would be conducted, without further details being given at that point. After all pairs had finished the first ten auctions, the instructions for the second part were again read aloud.

In the open auction formats the price stayed at 0 for four seconds and then increased at a rate of 1 per second. Bidders could drop out on one or both (if no bidder had dropped out before) units at any time. After one bidder dropped out on one unit and the other bidder was informed about this, the price stayed at the dropout level for four seconds and increased at a rate of 1 per second thereafter. If a bidder dropped out during these four seconds, the dropout is regarded as at the same price but later than the first dropout. At any time during the bidding process, the bidders could observe the current price, the number of items for sale and the number of active bids. If there were more than two active bids when the price rose above the maximum price of 100, then in UPO both bidders received one unit for a price of 100. In AA both bidders received one unit for a price of 100 if both still had two active bids, while the remaining unit was randomly allocated if one bidder had dropped out on one unit before. The sealed–bid auctions were run in a straightforward way, i. e. both bidders simultaneously placed two bids. Subjects were informed that the order of the bids was irrelevant.

After each auction bidders were informed about the observed dropout prices in the open auctions, or all four bids in the sealed–bid auctions, as well as the resulting allocation, their own gains or losses and their aggregate profits.

The experimental software was developed in zTree (Fischbacher, 1999). The sessions lasted for about 60 to 80 minutes in the uniform–price auctions and for about 30 to 50 minutes in the other treatments. At the end of each session, experimental currency units were exchanged in real currency at a rate of DM 0.04 (Berlin) or CHF 0.04 (Zürich) per ECU. In addition subjects received DM 5 (Berlin) or CHF10 (Zürich) as show-up fee.[13] Average total payoffs were 342 ECU in AA, 270 ECU in DA, 290 ECU in VA, 350 ECU in

[13] In order to relate the earnings, the exchange rates are 1 CHF = 0.65 Euro and 1 DM = 0.51 Euro. Cost of living is higher in Zurich, which justified the higher returns. The higher show-up fee in Zurich is based on a longer average commute to the laboratory than in Berlin.

UPO, 351 in UPoS (amounting to an average total payoff in UPOS of 701 ECU), 312 ECU in UPS, and 351 in UPsO (amounting to an average total payoff in UPSO of 663 ECU). This resulted in average earnings (including show-up fees) of DM 25.23 (about EURO 12.90) in Berlin and CHF 27.29 (about EURO 17.75) in Zuerich.

4 Experimental Results

As stated above, in treatments UPOS and UPSO the subjects played both uniform–price auctions in sequence. For the general comparison of all five auctions we only consider the first set of auctions out of these sessions (denoted by UPO and UPS) since the behavior in the second set of auctions is not independent of the behavior in the first one. We analyze behavior in the second set of auctions (denoted by UPsO and UPoS) separately in subsection 4.7, looking in particular at whether bidders move closer to the DR–equilibrium in the sealed–bid auction if they played the open version first.

The scatter diagrams in figures 1.1 through 2.4 provide a first impression of the behavior of the bidders in the five different auctions. Figure 1.1. through 1.6 show the bids in the three different sealed–bid auctions, where "unit1 bids" refers to the (weakly) higher, and "unit2 bids" to the (weakly) lower bid of a bidder. Figures 2.1 through 2.4 show dropout prices in the open auctions, AA and UPO. "Double dropouts" are simultaneous dropouts of one bidder on both units. "First dropouts" are the first dropout of a bidder on a unit while "second dropouts" refer to the second dropout in one auction, i. e. the price where the auction ends, not necessarily to the second dropout of one bidder. While figures 2.1 and 2.3 depict the overall behavior, 2.2 and 2.4 depict the behavior of pairs that almost followed equilibrium behavior. In what follows we will refer to these figures in order to illustrate the results.

Below, we generally use non-parametric Mann–Whitney tests for comparisons between treatments. These are always based on aggregate data per pair. The aggregate is computed over all periods unless explicitly behavior in only the second five of the ten auctions is compared. For comparisons within a treatment (between the first five and the second five auctions or between the first and the second set of auctions in UPOS and UPSO), as well as for comparisons with equilibrium predictions, we generally use non-parametric Wilcoxon signed-rank tests, because the data are paired. Again the tests are based on aggregate data per pair.

4.1 Are First-Unit Bids Truthful in VA, UPO, UPS, and AA?

Result 1 (First Unit Bids) *In AA, UPO, and VA, first-unit bids generally resemble truthful bidding. In UPS, overbidding is frequent and substantial.*

In **AA** most of the observed first unit bids were truthful except for one pair where one of the bidders tried to cooperate by dropping out on both units immediately and then expecting the other bidder to do the same in the next round (see the double dropouts at 0 in Figure 2.1). Of 83 observable first-unit bids, 53% were exactly equal to the valuation and another 18% just one ECU above or below. Overbidding was very rare (9 bids exceeded the valuation by more than 1 ECU), probably because it is easy to see for bidders that this is dominated.

In **UPO** we only have few cases where we can observe both bids (56 out of 200 possible cases). If the bidders play close to the DR–equilibrium, they both drop out immediately and thus, we do not know what they would have bid on the first unit. However, 30.4% of observable first unit bids were exactly equal to the valuation and an additional 21.4% were one ECU above or below. According to Wilcoxon signed-rank tests (for the hypothesis that first-unit bids relative to valuations equal 1), relative underbidding on the first unit (where the first unit bids are observable) was not significantly different from 0 in AA ($p = 0.76$), but was significant in UPO ($p = 0.012$). In particular, a Mann-Whitney test reveals that relative underbidding was significantly larger in UPO than in AA ($p = 0.016$). Overbidding occurred even less in UPO than in AA.

In **VA** a high fraction of first unit bids was at the valuation (29.5% of bids were exactly equal to the valuation and an additional 10.5% were one ECU above or below), as can be seen in Figure 1.3. The average bid on the first unit exceeded the valuation by only 0.78 ($p = 0.3329$, Wilcoxon signed-rank test).

In **UPS** bidders frequently over- or underbid their valuations on the first unit. Overbidding is substantial (33.5 % of first unit bids where above the valuation), as can also be seen in Figure 1.1. Bidders bid truthfully on the first unit in about a third of the cases (19.5% of bids were exactly equal to the valuation and an additional 10% were one ECU above or below). Average overbidding on the first unit (5.55) just fails to be significant ($p = 0.114$, Wilcoxon signed-rank test). Overbidding even increased (insignificantly) over time. One subject in UPS bid his valuation on the first unit in all auctions.

Two additional observations in UPS might be interesting in this context: First, out of 67 instances where bidders overbid on at least the first unit, only 11 led to a loss for the bidder. This illustrates quite well that bidders in UPS hardly learn that overbidding is dominated. Moreover — quite surprisingly — only in one case a bidder revised his behavior after suffering a loss.

4.2 Demand Reduction in UPO/UPS

Result 2 (Demand Reduction) *Complete demand reduction occurs frequently in UPO, but only rarely in UPS.*

Figures 2.3 and 2.4 show dropout prices in UPO.[14] Figure 2.4 shows that in three pairs both bidders almost always dropped out on one unit at price 0, independently of their valuation.[15] These three pairs almost played the (DR–) equilibrium strategy, while the other pairs either bid roughly consistent with the TT–equilibrium or did not seem to have found a reasonable strategy. Figure 2.3 depicts the overall behavior. One can also observe whether bidders play according to the requirement of the DR–equilibrium to drop out on one unit immediately once the other bidder has dropped out, which was violated in 55 % of the observable cases.

Figures 1.2 shows the (weakly) lower bids in UPS ("unit2 bids"). As expected we observe substantially fewer cases of complete demand reduction. In particular, we observed only 9 zero–bids on the second unit (from bidders with positive valuations) in UPS, which were all placed by the same subject, while we observed 33 such bids in UPO (notably, in UPO, the number of zero bids increases from 12 in periods

[14] Recall that "second dropouts" refer to the second dropout in one auction, that is, the price where the auction ends, not necessarily to the second dropout of one bidder.

[15] Note that for the open auctions we can only include the observed bids in the Figures. For the unobserved bids, a lower threshold is given by the price at which the auction ended. Hence, the figures for the open auctions should not be directly compared to those for the sealed–bid auctions, because the latter show all bids, whereas the former show only the two lowest bids in each auction.

1 to 5 to 21 in periods 6 to 10).[16] The number of zero–bids is significantly higher in UPO than in UPS (Mann–Whitney test, $p = 0.085$). In UPS, only one subject consistently chose the TT–equilibrium strategy, which was, however, not part of an equilibrium either, since the other subject was underbidding on the second unit most of the time.

4.3 Equality of Bids and Bid Spreads

According to the equilibrium prediction, in AA, VA, and DA the bidders should place equal bids on both units. In this section, we study the deviation from this prediction and also compare it to the bid spreading in UPS, where this is consistent with equilibrium behavior.

Result 3 (Bid Spreading) *(i) Bid spreads are small in AA and VA.*

(ii) In sharp contrast, they are substantial and persistent in DA, and of similar magnitude in UPS.

AA Five pairs played almost exactly according to the equilibrium prediction, i. e. double dropouts at the valuation (see Figure 2.2). In some pairs bidders tried to cooperate by bidding 0 on both units if their valuation was relatively low (see the double dropouts at zero in Figure 2.1), or by using strategies that resembled the DR–equilibrium strategies in a uniform–price auction. Overall we observed 26 extremely low (0 or 1) bids on the second unit, which is substantially (but not significantly) lower than in UPO (48), but higher than in UPS (13) and VA (6). These attempts to cooperate were in general not successful (or, in one case successful for only three periods) and were abandoned after some auctions. As a consequence, the number of very low bids drops from 17 in periods 1 to 5 to 9 in periods 6 to 10 (while in UPO this number increases from 19 to 29).

In about half of the cases we can observe (or infer lower or upper bounds for) the difference between maximal and minimal bids in AA. In 64% of decidable cases the bids were exactly equal, in 68% the difference was smaller than 10% of the equilibrium bid (i. e. the valuation) and in only 18% of the cases the difference was greater than or equal to 40% of the valuation (see Table 1). Most unobservable cases are those where the other bidder dropped out on two units, hence there is no indication that the undecidable

[16] In addition, there were 15 bids equal to 1 (by bidders with valuations larger than 1) in UPO, but only 4 in UPS.

maxbid-minbid	UPO	UPS	AA	VA	DA
$= 0$	27%*	18%	64%*	49%	12%
$< 10\%$ Equ.	43%*	34%	68%*	62%	15%
$\geq 40\%$ Equ.	32%*	33%	18%*	14%	49%

Table 1: Share of bid pairs (*: of observable bid pairs) that are exactly equal, where the difference is smaller than 10, or larger than 40 percent.

cases correspond to large bid spreads. Rather the contrary is true, because not dropping out on one unit up to this price reduces the maximal possible bid spread of the remaining bidder.

VA Though only two out of ten pairs were very close to the equilibrium in VA, eight subjects played close to the weakly dominant strategy to place bids equal to the valuation for both units. 2 subjects bid exactly the valuation on both units in all auctions. The first and second unit bids are shown in Figures 1.3 and 1.4, respectively.

Similarly to AA, in VA most of the bid differences (62%) were smaller than 10% of the equilibrium bid (i. e. the valuation), with 49% of bid pairs being exactly equal. In only 14% of cases the difference exceeded 40% of the valuation. The bid differences were thus substantially smaller than in DA and UPS (see Table 1). The aggregate bid spread (sum of bid differences divided by sum of equilibrium bids over all pairs and periods) is 13%. Analyzing the data for the individual bidders, according to a Kolmogorov-Smirnov test, the hypothesis that both, the higher and the lower bid (relative to equilibrium bids) are drawn from the same distribution can be rejected at the 5%-level for only 4 bidders. Equal bids in all 10 auctions were placed by 4 (out of 20) bidders. A linear regression of the bidspread yields, over all subjects and periods (with robust standard errors taking the dependence within a pair into account) a significantly ($p = 0.057$) negative coefficient (-0.87) for the variable "period". Hence the bid spreading in VA clearly decreased over time. In fact, the aggregate bid spread is 17% in periods 1 to 5 and 8% in periods 6 to 10. Moreover, the aggregate bid spread decreases from the first to the second half of the experiment in nine pairs, but increases in only one (the decrease is significant,Wilcoxon signed-rank test $p = 0.028$).

Estimating bid functions that are linear in the valuation yields over all subjects very similar results in regressions of the higher (coefficient for valuation 0.859, constant 7.74) and the lower (coefficient for valuation 0.826, constant 3.09) bid.[17]

DA In DA bidders rarely choose equal bids. In only 12% of cases the bids were exactly equal and in only 15% (including the 12% equal bids) the difference was smaller than 10% of the equilibrium bid (i. e. 5% of the valuation, see Table 1). About half of these nearly equal bids were submitted by only two subjects. 49% of the bid spreads were larger than or equal to 40% of the equilibrium bid. The aggregate bid spread is 37%. Thus, bid spreads were of comparable magnitude as in UPS, where bid spreading is implied by demand reduction. The bid spreading is also vividly illustrated by comparing Figures 1.5 and 1.6. As shown by Figure 1.5, the majority of the first unit (i. e. higher) bids lies between the valuation and the equilibrium bid (valuation divided by 2), whereas a large share of the second unit (i. e. lower) bids lies substantially below the equilibrium bid. According to Wilcoxon signed-rank tests, first-unit bids were significantly higher ($p = 0.021$) than the equilibrium bid (average difference 5.48). The average second-unit bid is 3.73 smaller than the equilibrium bid ($p = 0.139$). As can also be seen in Figures 1.5 and 1.6, except for one subject in one auction, we observed overbidding of the valuation only for very small valuations and to a very small degree. It seems that it is obvious to bidders in DA that overbidding is dominated.

A Kolmogorov-Smirnov test shows that the hypothesis that both, the higher and the lower bids (relative to equilibrium bids) are drawn from the same distribution, can be rejected at the 5%-level for 12 out of 18 bidders. Hence, bid spreading (relative to equilibrium bids) was clearly more prominent in DA than in VA, which is also confirmed by a Mann-Whitney test ($p = 0.0025$). A linear regression of the bidspread yields, over all subjects and periods (with robust standard errors) a negative coefficient (-0.05) for period, which is, however, not significantly smaller than 0 ($p = 0.83$). Hence the bidspread on average decreased over time, but the effect is very small and insignificant. In fact, the aggregate bid spread is 38% in periods 1 to 5 and 36% in periods 6 to 10. Moreover, the aggregate bid spread increases from the first to the second half of the experiment in five pairs, but decreases in only four.

[17] We also estimated bid functions for the individual subjects. The coefficient for valuation in a regression of the higher bid is within 10% deviation of the equilibrium prediction (i. e. 1) for 10 subjects. For the lower bid this is the case for 11 subjects.

These results are supported by estimating bid functions that are linear in the valuation. Over all subjects, in a regression of the higher bid the coefficient for the valuation is 0.516 which is close to the equilibrium value of 0.5, while it is substantially smaller in a regression of the lower bid (0.379). In bid functions estimated for individual subjects the coefficient for valuation in a regression of the higher bid is within 10% deviation of the equilibrium prediction only for 7 out of 18 subjects. In a regression of the lower bid, this is the case for only 5 subjects.

At a first glance, a possible explanation for the bidspreading seems to be risk aversion. However, for constant or increasing absolute risk aversion a lower bid on the second unit cannot be an equilibrium strategy since the lower bid competes with the higher bid of the other bidder, which implies bidding higher rather than lower on the second unit. Strongly decreasing risk aversion could explain bid spreading in a single auction but would imply that bids decrease over time, i. e. after winning some auctions, which they do not. In addition second unit bids below the risk-neutral equilibrium strategy ($b = \frac{v}{2}$) cannot be consistent with any form of risk aversion, because risk aversion always predicts bidding higher than the risk-neutral equilibrium on both units, since a risk averse person is willing to trade off expected payoff for a higher probability of winning a unit.[18] The data are consistent with some statements in the questionnaires that suggest that subjects were placing one bid as if they were highly risk averse and the other as if they were risk seeking ("a high secure bid and a lower bid that could yield a higher profit"). This seems to describe a highly myopic zero-profit aversion (since subjects try to secure a unit in each single auction). This behavior did not only lead to first bids being substantially higher than the equilibrium bid, but also to average bids above equilibrium.

UPS In UPS, 34% of bid spreads were below 10% of the valuation (including 18% of the pairs that were exactly equal). Only 33% of bid differences exceeded 40% of the valuation (see Table 1). The aggregate bid

[18] Furthermore, according to Rabin (2000) and Rabin and Thaler (2001) risk aversion on such small stakes cannot be reconciled with the maximization of the expected utility of wealth. (According to Cox and Sadiraj, 2001, however, it is consistent with the maximization of the expected utility of income). Since no losses are involved (at least as long as the valuation is not overbid), the explanation of myopic loss aversion given by Rabin and Thaler for low stake risk aversion does not explain our results either.

spread is 41%. Bid spreading (relative to equilibrium bids) was clearly larger in UPS than in VA (Mann-Whitney test, $p = 0.0082$), but indistinguishable from that in DA ($p = 0.807$). A Kolmogorov-Smirnov test applied to the individual bidders reveals that the hypothesis that both, the higher and the lower bids (relative to equilibrium bids), are drawn from the same distribution can be rejected at the 5%-level for 13 out of 20 bidders. There is no clear time trend concerning the bid spread. A linear regression of the bidspread yields, over all subjects and periods (with robust standard errors) an insignificantly ($p = 0.571$) negative coefficient (-0.23) for period, suggesting that the bid spreading in UPS slightly decreased over time. However, the aggregate bid spread relative to the equilibrium bid is 40% both in periods 1 to 5 and in periods 6 to 10. It decreases from the first to the second half of the 10 auctions in six pairs, and increases in four.

Estimating bid functions that are linear in the valuation, yields over all subjects a coefficient for the valuation of 0.83 (constant 13.15) in a regression of the higher bid and 0.63 (constant 4.29) in a regression of the lower bid. The coefficient for valuation in regressions of the higher bid is for 6 subjects within 10% deviation of the equilibrium prediction.

4.4 Efficiency

Result 4 (Efficiency) *(i) Due to demand reduction, efficiency is lower in UPO than in the other auction formats, that do not differ substantially in terms of efficiency.*

(ii) Efficiency increases, however, significantly over time in AA and hence in periods 6 to 10, efficiency in AA is significantly higher than in DA, UPO, and UPS.

In equilibrium both units are allocated to the bidder with the higher valuation in AA, VA, and DA, but only one unit in the DR–equilibrium of the uniform-price auctions. An efficient allocation requires allocating both units to the bidder with the higher valuation, because independent of the price this maximizes social welfare, the sum of the bidders' profits and the auctioneer's revenue.[19]

[19] This notion of efficiency is common in experimental auctions. It corresponds to non-experimental auctions, where the valuation represents, for example, an intrinsic valuation of the buyer for the good (in case of works of art) or prospective profits in a market (in case of licenses). In the experiment, however, both the valuation and the prices paid correspond to

Because valuations are randomly and independently drawn in our experiment, simply comparing treatments with respect to the achieved total welfare would be biased by these random draws. We compare the auction formats with respect to three different efficiency measures that are aimed to minimize this bias: allocative efficiency, relative efficiency loss and relative efficiency. The term *allocative efficiency* refers simply to the number of efficiently (i. e. to the bidder with the higher valuation) allocated units. Measuring allocative efficiency in this way does not reflect the actual magnitude of efficiency losses due to misallocations. If the "wrong" bidder obtains a unit, his valuation may be substantially or only slightly below the other bidder's valuation, causing either dramatic or small welfare losses. Our second and third measures take this into account. With *relative efficiency loss* we refer to the loss in terms of total welfare relative to the maximum possible efficiency loss. *Relative efficiency* measures the achieved total welfare relative to maximum possible welfare. In Table 2 we report for each measure aggregate results over all pairs and periods, as well as separated into the first five periods and the second five periods.[20]

Concerning allocative efficiency over all periods, treatments do not differ much (AA 84%, VA 82,5 %, DA 83,3 %, UPO 74 % UPS 81 %, see Table 2). None of the pair-wise differences is significant ($p > 0.2$). In particular, in UPS the allocative efficiency was only slightly below that in AA, DA, and VA, although the predicted allocative efficiency in the DR–equilibrium is only half of that predicted in the three other

transfers between experimenter and subjects. Hence the total payoff (including the experimenter) is constant. This would not be reason to worry, if experimental subjects did not care for efficiency. There is, however, evidence that experimental subjects directly care for efficiency and not only for their own payoff and the fairness of the allocation (see Charness and Rabin, 2002, and Engelmann and Strobel, 2002). It therefore matters whether the subjects include the experimenter in their considerations. If not (which is usually assumed), the efficiency measure corresponds to the joint payoff of the bidders, i. e. $(v_1-p_1)+(v_2-p_2)$ (with v_i being the valuation of the bidder who obtained unit i, and p_i the price paid for this unit), whereas the standard measure of efficiency simply amounts to v_1+v_2. This could lead efficiency minded subjects to cooperate in the experiment (causing efficiency losses according to our standard measure), but not outside the laboratory. If they include the experimenter, efficiency concerns are irrelevant since the total payoff is constant, unless subjects assume that they have higher marginal utility from the payoff than the sponsor of the experiment, or if they also consider the experimenters' future salary increases due to ground-breaking publications. In the latter case, they should try to produce interesting results.

[20] Aggregates are not averages over the relative measures, but relative measures computed with respect to aggregate data. For example, relative efficiency for one treatment corresponds to the total achieved welfare by all pairs in this treatment over all (respectively the first or second five) periods, devided by the aggregated maximum possible welfare. This minimizes the impact of outliers based on small valuations.

	Periods	all	1 – 5	6 – 10
AA	allocative efficiency	84 %	77 %	91 %
	relative efficiency loss	9.2 %	18 %	0.8 %
	relative efficiency	95.6 %	91.4 %	99.6 %
VA	allocative efficiency	82.5 %	77 %	88 %
	relative efficiency loss	13.8 %	16.8 %	11.1 %
	relative efficiency	93.3 %	92.4 %	94.2 %
DA	allocative efficiency	83.3 %	81.1 %	85.5 %
	relative efficiency loss	7.1 %	9.8 %	4.5 %
	relative efficiency	96.5 %	95.3 %	97.8 %
UPS	allocative efficiency	81 %	80 %	82 %
	relative efficiency loss	12.2 %	11.5 %	12.9 %
	relative efficiency	92.9 %	93.5 %	92.2 %
UPO	allocative efficiency	74 %	72 %	76 %
	relative efficiency loss	25.9 %	25.8 %	25.9 %
	relative efficiency	87.6 %	88.5 %	86.6 %

Table 2: Efficiency, measured by allocative efficiency, relative efficiency loss, and relative efficiency.

auctions. In each of AA, VA, and UPS for exactly one pair all units were allocated efficiently. The low allocative efficiency in UPO is due to the coordination of some pairs on the DR–equilibrium. According to relative efficiency losses and relative efficiency, in UPO efficiency is (marginally) significantly lower than in both AA ($p = 0.059$ resp. $p = 0.041$) and DA ($p = 0.086$ resp. $p = 0.06$).

Differences in efficiency are far more pronounced in the second five periods (allocative efficiency: AA 91%, VA 88 %, DA 85,5 %, UPO 76 % UPS 82 %). Indeed, efficiency increases over time are substantial only in DA, VA, and AA, and significant only in AA ($p = 0.039$, $p = 0.011$, and $p = 0.025$ for allocative efficiency, relative efficiency loss and relative efficiency, respectively; in all other treatments $p > 0.2$ with respect to each measure). Efficiency in periods 6 to 10 is significantly higher in AA than in DA ($p = 0.054$, and $p = 0.045$ for relative efficiency loss and relative efficiency, respectively), than in UPO ($p = 0.073$, $p = 0.008$, and $p = 0.011$ for allocative efficiency, relative efficiency loss and relative efficiency, respectively), and than in UPS ($p = 0.085$, $p = 0.004$, and $p = 0.004$). None of the other treatments differ significantly at a 10% level with respect to any efficiency measure in periods 6 to 10.

A more detailed look at the efficiency losses in AA reveals two main underlying reasons for inefficient allocations. First, attempts to collude by demand reduction or by dropping at price 0 with both units in the case of low valuations, apparently with the hope of reciprocation in later periods. Second, situations where the valuations of both bidders were very close, so that small deviations from the equilibrium strategy could result in misallocations. Since attempts to collude are largely unsuccessful, misallocations due to the first reason have disappeared in the second half of the experiment. The remaining misallocations all lead to very small losses in terms of welfare. Consequently, the decrease of the relative efficiency loss (from 18.3 % to 0.8 %) and the increase of the relative efficiency (from 91.4 % to 99.6 %) are much stronger than the increase in allocative efficiency (from 77 % to 91 %), because exactly those misallocations that cause substantial efficiency losses disappear over time. In contrast, in all other mechanisms, even in periods 6 to 10, the relative efficiency loss is larger than 4% and the relative efficiency below 98 %. In particular, while the allocative efficiency also increases substantially in VA (from 77 % to 88 %), with respect to relative efficiency losses (16.8 % to 11 %) and relative efficiency (92.4 % to 94.2 %) the increase in efficiency is much weaker than in AA. In contrast to AA, in VA it is not the misallocations with substantial efficiency losses that disappear over time. Hence while according to our overall results, none of the mechanisms appears to be clearly preferable in terms of efficiency, AA clearly is preferable in case of experienced bidders.

The disappearance of misallocations due to collusive attempts also suggests that AA would have proved superior in terms of efficiency if we had conducted more periods or with random matching, which makes collusion less likely. In UPO, in contrast, under random matching, one bidder might teach a series of other bidders the DR–equilibrium. Hence we suspect that the advantage of AA over UPO with respect to efficiency would even be larger under random matching than under fixed matching, so that the fixed matching employed in our experiment is a tougher test for the efficiency superiority of AA.

While efficiency also increases (though not significantly) over time in DA (and is second highest with respect to relative efficiency in periods 6 to 10), the misallocations in DA were primarily caused by bid spreading, a robust effect in most pairs (indeed it increases from the first five to the second five periods in five out of nine pairs). The increase in efficiency appears to be due primarily to the reduction in erratic behavior, but the remaining misallocations due to bid spreading are likely to persist even in experiments over substantially more periods. We suspect that increasing the number of bidders would emphasize the advantage of AA over DA. On the one hand, underbidding in attempts to cooperate is likely to decrease in AA for more than two bidders. On the other hand, if bid spreading in DA prevails for more bidders, the probability that first unit bids of bidders with low valuation are higher than second unit bids of bidders with high valuation (and hence for misallocations) increases in the number of bidders.

4.5 Auctioneer's Revenues

Result 5 (Revenue) *(i) Revenue equivalence is rejected for AA and DA, and it is also clearly rejected for the uniform-price auctions.*

(ii) Revenues are generally higher in sealed-bid than in open auctions.

The theoretical results predict equal expected revenues in equilibrium for VA, DA and AA. The empirical (see Table 3.1) revenues in AA ranged from 44 % to 116 % of the equilibrium revenues in the individual pairs with 84,74 % over all pairs. In VA empirical revenues were between 41 % and 131 % with 95,58 % over all pairs. In contrast, in DA the empirical revenues reached between 83 % and 145 % of the equilibrium revenues in the individual pairs and 110,72 % over all pairs. The difference in relative revenues between AA and DA is significant (Mann-Whitney test , $p = 0.034$), but both do not differ significantly from the equilibrium (Wilcoxon signed-rank test, $p = 0.139$ for DA and $p = 0.203$ for AA) or from VA (Mann-Whitney, $p = 0.545$ for AA and $p = 0.165$ for DA).

In the uniform–price auctions the (DR–) equilibrium revenues are 0. The empirical revenues were naturally higher. To compare the revenues in the two uniform–price auctions, we measure the revenues relative to the TT–equilibrium revenues (which correspond to the expected equilibrium revenues in the other auctions). In UPO the revenues ranged from 1 % to 108 % of the (TT–) equilibrium revenues, with 68.97 % over all pairs. In UPS they ranged from 80 % to 161 % and reached 106,74 % over all pairs. The difference between UPO and UPS is clearly significant (Mann-Whitney test, $p = 0.019$). Furthermore, the revenues were significantly smaller than in TT–equilibrium in UPO (Wilcoxon sign-rank test, $p = 0.028$) but not significantly different in UPS ($p = 0.507$). Thus revenue equivalence does not hold for the two uniform–price auctions either.

In line with the equilibrium prediction the relative revenues in UPO were significantly lower than in VA and DA (Mann-Whitney, $p = 0.070$ and $p = 0.006$, respectively), but only insignificantly lower than in AA ($p = 0.290$). In contrast to the equilibrium prediction, the relative revenues in UPS were even slightly larger than those in AA ($p = 0.199$).

4.6 Bidder Payoffs

Result 6 (Bidder Payoffs) *(i) Bidder payoffs are significantly lower than the DR-equilibrium prediction in UPO and UPS.*

(ii) In DA, bidder payoffs are significantly lower than in equilibrium, in UPO, and in AA.

The pair that played closest to the (DR–) equilibrium in UPO naturally received almost the (DR–) equilibrium payoff (see Table 4.1, UPO, pair 4), while the other pairs and all pairs in UPS obtained payoffs substantially below the equilibrium payoff in most of the auctions. In some auctions, however, the latter pairs obtained above equilibrium payoffs due to underbidding of subjects with low valuations. In UPO the bidder payoffs ranged from 38% to 102% of the DR–equilibrium payoff, with an average of 68%. In

UPS they ranged from 51% to 88% with an average of 69%. While bidder profits in both UPS and UPO were significantly lower than the DR–equilibrium payoff (Wilcoxon signed-rank test, $p = 0.007$ in UPO, $p = 0.005$ in UPS), in UPS they were even lower than the TT–equilibrium payoffs (Wilcoxon signed-rank test, $p = 0.059$) and payoffs relative to the TT–equilibrium payoffs were significantly smaller in UPS than in UPO (Mann-Whitney test, $p = 0.034$).[21]

In AA the collusive attempts resulted in payoffs that exceeded equilibrium payoffs in five pairs (see Table 4.1, the difference is not significant: Wilcoxon signed-rank test, $p = 0.445$). The extreme excess profits of pair 4 (they achieved 289% of equilibrium payoffs) were partly a coincidence. In several auctions the valuations of both bidders in this pair were very close, so that the equilibrium payoffs were very small. Attempts to cooperate through demand reduction or generous dropping out at a low price with both units led to payoffs substantially above the equilibrium. In addition, the low bidder payoffs in pair 10 (54% of equilibrium payoffs) were partly driven by a chance event. One bidder always had the lower valuation. This seems to have caused some frustration which resulted in overbidding, which may have been driven by spite or just by a desire to experiment. The other pairs' payoffs ranged from 91% to138% with an overall average of 107%.

In VA the bidder profits were close to the equilibrium with an average of 91% over all pairs (difference is not significant: Wilcoxon signed-rank test, $p = 0.203$). While four pairs were within 5% deviation of the equilibrium payoff, one pair only achieved 43%, and another 164% (see Table 4.1).

In DA, since average bids were above equilibrium, the bidders' payoffs were consistently lower than under the equilibrium prediction in most pairs (see Table 4.1). Seven out of nine pairs were below 90% of the equilibrium payoffs, one pair obtained 113% and the average was 82% of the equilibrium. The bidder profits in DA were significantly lower than the equilibrium payoffs (Wilcoxon signed-rank test, $p = 0.028$). Furthermore, profits (relative to the (TT–) equilibrium) were significantly lower in DA than in UPO (Mann-Whitney test, $p = 0.028$) and AA ($p = 0.041$).

[21] The comparison between UPS and UPO yields different results depending on which equilibrium is used as a benchmark because the valuations were randomly drawn and hence different in the two treatments.

4.7 Effects UPO $\leftrightarrow$ UPS

One might suspect that the subjects would learn how to play the payoff dominant DR–equilibrium of the uniform–price auction better in the open version. They might, however, be able to transfer the DR they learn in UPO to UPS, whereas playing UPS before UPO should not help them finding the DR-equilibrium in UPO. In order to check whether the intuition is right, we let the subjects who played UPO and UPS play another ten auctions in the other uniform–price format. We will refer to the ten open auctions that were played after the sealed–bid auctions as UPsO and to the sealed–bid auctions played after the open auctions as UPoS.

Result 7 *(i) Bidders who have played UPO first, play closer to the DR-equilibrium in the sealed-bid auction than those in UPS. They exhibit, however, less DR than they did in UPO.*

(ii) There appear to be slight hysteresis effects for bidders who first play UPS and then the open auction. They exhibit less demand reduction than those who started with UPO.

Three of the pairs that played UPO first cooperated almost from the start and continued to do so until the end of the open auctions. Figure 5 depicts for UPOS the percentage of the (DR–) equilibrium profit each of those three pairs reached per auction. While the pairs realized roughly the DR–equilibrium profit most of the time in UPO, this did not carry over to the subsequent sealed–bid auctions with the same pricing rule. There, the bidders' profits differed substantially from equilibrium profits across all pairs.

If one looks at all pairs, the bidder profits between UPS and UPoS (see Table 4.2) did not differ significantly (Mann-Whitney test, $p = 0.258$ for profits relative to TT-equilibrium, $p = 0.762$ for profits relative to DR-equilibrium), suggesting that bidders did not learn to play the DR–equilibrium in UPS even if they had played UPO before. However, the auctioneer's revenue was much lower in UPoS than in UPS (71,33 % vs. 106,74 % of the TT–equilibrium revenue, see Tables 3.2 and 3.1, Mann-Whitney test, $p = 0.013$). In UPoS it was also significantly lower than in the TT–equilibrium (Wilcoxon signed-rank tests, $p = 0.047$). Furthermore, efficiency was significantly lower in UPoS than in UPS (allocative efficiency 67.5% vs. 81%, according to Mann-Whitney tests $p = 0.031$ for allocative efficiency, $p = 0.049$ for relative efficiency loss, $p = 0.096$ and for relative efficiency).[22] In addition, the number of extremely low (0 or 1) bids was significantly higher in UPoS than in UPS (36 vs. 13, Mann-Whitney test, $p = 0.047$). These results indicate that behavior got closer to playing the DR–equilibrium in the sealed-bid auction if the open auction was played first.

Moreover, analysis of the scatter diagrams provides a better understanding of UPoS. Figure 3.1 and 3.2 depict the first and second unit bids in UPoS. The average first-unit bid was 5.02 below the equilibrium but a regression of the difference between the higher bid and the valuation shows that underbidding decreased significantly ($p = 0.068$) over time. In total, even fewer first unit bids were truthful in UPoS than in UPS (20.5% of bids were exactly equal to the valuation and an additional 10.5% were one ECU above or below). Compared with Figure 1.1 and 1.2 (UPS), bids seem to be closer to equilibrium behavior. This is even more true for those pairs that seem to have found the DR–equilibrium in the open auction they played before (see Figures 3.3 and 3.4). The aggregate bid spread in UPoS (44%) was larger than in UPS (40%), but not significantly (Mann-Whitney test, $p = 0.545$).

For the pairs that played UPS first and then UPsO we got an unexpected result. As one might predict, bidders did not learn to play the DR–equilibrium in the UPS design. Surprisingly, though, this seems to have partially extended to UPsO. On the one hand, in UPsO two pairs played close to the DR–equilibrium and bidder profits were on average close to those in UPO (69,71% vs. 67,83% of the DR–equilibrium profits, see Tables 4.2 and 4.1). Bidder profits (relative to the TT–equilibrium) increased significantly from UPS to UPsO ($p = 0.028$, Wilcoxon signed-rank test), as did the number of extremely low (0 or 1) bids on the second unit (13 vs. 36, Wilcoxon signed-rank test, $p = 0.079$), while the auctioneer's revenues decreased significantly (Wilcoxon signed-rank test, $p = 0.013$).

On the other hand, the number of extremely low bids is considerably (though far from significantly) smaller than in UPO (48). Furthermore, the auctioneer's revenues were higher (but not significantly, $p = 0.706$) in UPsO than in UPO (80.22 % vs. 68.97 % of the TT–equilibrium revenue, see Tables 2.2 and 2.1) and the efficiency is higher in UPsO than in UPO with respect to all three measures but far from significantly so and it is only marginally smaller than in UPS. The allocative efficiency in UPsO is even

[22] Interestingly, the allocative efficiency was even slightly (but insignificantly) lower in UPoS than in UPO.

larger than in UPoS (Mann-Whitney, $p = 0.072$) opposed to what one would expect from a comparison of the mechanisms and suggesting that indeed behavior from the first set of auctions carries to some extent over to the second set. Finally, bidders violated the requirement of the DR–equilibrium to drop out on one unit immediately once the other bidder had dropped out, more often in UPsO (66 % of the cases where it was possible). In UPO, it was violated in only 55 % of the cases. This does not seem to be attributable to a lower rationality of the bidders in UPsO than in UPO, because truthful bidding on the first unit was more frequent in UPsO than in UPO (out of 59 observable first-unit bids, 45.8% were exactly equal to the valuation and an additional 25.4% were one ECU above or below).[23]

Figure 4.1 depicts the observed bids in the UPsO. As Figure 4.2 shows, there are still two pairs who played close to the equilibrium prediction. Figure 4.3 depicts the behavior of those pairs in the preceding UPS treatment.

4.8 Learning

It is striking that there seems to be almost no learning within one auction format. In most cases where bidders played close to equilibrium strategies, they started doing so in the first three auctions. Also time trends led partially towards the equilibrium (e.g. bid spreading in VA decreased and increased in UPoS, underbidding on the first unit decreased in UPoS), but partially away from the equilibrium (e.g. bid-spreading decreased and overbidding increased in UPS). The major exception appears to be that bidders learn that attempts to collude do not pay in AA and that these are consequently abandoned.

That we did not find much learning is certainly in part because we only played ten auctions. The virtual absence of learning trends after the third auction is still surprising, though. Furthermore, most subjects played the auctions very fast and took very little time to review the results. Hence it appears to us, that simply increasing the number of auctions would not have resulted in significantly more learning. This would rather require the slowing down of the subjects, for example by imposing a minimal time they are shown the feedback.

[23] Alsemgeest et al. (1998) find similar hysteresis effects in open auctions. Subjects who play the open auction with multi-unit demand after playing the open auction with single unit demand exhibit substantially less demand reduction than those who played the multi-unit demand auction first.

4.9 Questionnaires

In all treatments there were participants who indicated in post–experimental questionnaires that they tried to cooperate, as well as participants who explicitly behaved competitively or even spiteful. There is no indication that subjects realized that demand reduction is an equilibrium in the uniform-price auctions. In the uniform-price auctions as well as in the Vickrey and Ausubel auctions, several subjects realized that complete demand reduction is (weakly) payoff dominating all (other) equilibria and some realized that in UPO cooperation is easier than in UPS, while none made an explicit reference to equilibrium logic. Many subjects cited avoiding losses as a primary aim or as a constraint on their attempts to maximize their payoffs.

5 Conclusions

The results of our experiments are in line with some of the theoretical predictions, while they clearly contradict others. Demand reduction occurs in the uniform-price auctions, though it also does to a lesser extent in the Ausubel auction. The allocative efficiency is lowest in UPO, and highest in AA, where the latter differs only slightly from UPS, VA, and DA over all periods with respect to the number of efficiently allocated units, but the causes of misallocations appear to be least robust in AA. As a consequence, efficiency is significantly higher in AA than in DA, UPO, and UPS in the second half of the experiment. The revenue equivalence of AA and DA is clearly rejected, as it is for the two uniform-price auctions. In clear contrast to the theory, the auctioneer's revenues do not primarily depend on the pricing-rule, but whether the auction is open or sealed–bid.

Some of the results do not come as a surprise, though not predicted by the equilibrium analysis. Overbidding is more frequent in UPS and in VA than in UPO and AA, apparently since in the sealed–bid auctions it is less clear that overbidding is dominated. Coordination on the DR–equilibrium seems to be much easier in UPO than in UPS, because one bidder can signal by dropping out. Bidding above the equilibrium strategy is much more frequent in DA than in VA, and in particular in AA, since in the latter cases this involves overbidding, and it is easier to recognize that this is not optimal, than it is to calculate the optimal bids in DA. These behavioral effects cause the auctioneer's revenues to be higher in the sealed–bid auctions than in the open auctions.

Our primary results are qualitatively in line with those of Kagel and Levin (2001), though some differences apparently result from our design involving two human players. They also find more demand reduction in the uniform–price open auction than in the uniform–price sealed–bid auction. However, while they find much less demand reduction in the Ausubel auction, we find more extremely low bids in early stages of AA than in UPS. Apparently these extremely low bids were the results of attempts to elicit collusion, which is impossible in their design with simulated opponents.

In accordance with our results, Kagel and Levin (2001) also find much more overbidding in the uniform–price sealed–bid auction than in the two open auctions. Furthermore, in their experiment as well as in ours UPS yields higher revenues to the auctioneer but lower allocative efficiency than AA, although the difference in efficiency is smaller in our experiment, again probably resulting from attempts to collude in AA (in the second half of our experiment, when attempts to collude have ceased, efficiency is substantially and significantly larger in AA than in UPS). Hence we provide some further indication for this theoretically unanticipated trade-off between revenue and efficiency in AA and UPS. Thus, their main results do not seem to depend critically on the simulation of other participants by computers, although the superior performance of the Ausubel auction seems to be weakened in our interactive environment. In contrast to Kagel and Levin, in our experiment there seems to be surprisingly little learning both within and across auction rules (with the exception of AA where bidders learn that collusion does not work). Those subjects who manage to determine the equilibrium do so almost at once. This is particularly surprising given that our interactive environments seem to be more complicated and that we did not provide hints against overbidding.

In line with our observation that the pricing-rule is less important for revenues than whether the auction is sealed–bid or open, List and Lucking-Reiley (2000) find little differences in revenues between VA and UPS. They also find more overbidding on the first unit in UPS compared to VA, as we do.[24] Our results

[24] In a related experiment, Engelbrecht-Wiggans, List and Lucking-Reiley (1999), find that this effect disappears with 3 or 5 bidders. They also find, consistent with the theoretical prediction, that demand reduction is still present, but reduced if the number of bidders is increased.

also confirm the observation of List and Lucking-Reiley that the bid spreading is larger in UPS than in VA, and confirm that this leads to (slightly) more misallocations.

What is surprising, though, in our experiment, is that bid spreading is very strong in DA, where it is not consistent with equilibrium behavior. This seems to be caused by a dislike for zero profits which leads subjects to increase the probability of acquiring at least one unit at the expense of expected profits. This zero profit aversion has no distorting effect in the other auction mechanisms, since the probability of acquiring at least one unit (without making losses) is maximized by bidding the valuation on the first unit, consistent with equilibrium behavior. The bid spreading in DA cannot be completely explained by risk aversion, because the majority of second-unit bids is below the equilibrium, a finding inconsistent with risk aversion. Since the observed bidding behavior at first glance looks perfectly consistent with the usual overbidding in first-price single unit auctions but cannot be explained by risk aversion, one might even argue that this raises doubts about the explanatory adequacy of risk aversion in accounting for overbidding in single unit auctions.

This bid spreading in DA as well as the overbidding on the first unit in UPS might be rationalized by a utility function that accounts for the bidder's expected payoff on the one hand, and, on the other hand, explicitly values the probability of winning at least one unit. Maximizing such a utility function would imply a higher bid on the first unit than suggested by expected utility maximization, even if this led to making losses with positive probability.

Another interesting observation is that the total allocative efficiency is almost identical in VA and AA. Inefficient allocations in AA seem partially caused by bidders hoping that the second bidder will play the weakly dominated strategy of dropping out after a dropout of the first bidder (which the second bidder then sometimes does), whereas inefficient allocations in VA result from a higher number of bids that deviate from the valuation, though only slightly. The latter observation may possibly be due to the fact that in VA it is less transparent to the bidders that bidding their own valuation is dominant. Hence we find that the possibly more transparent mechanism in AA can compensate for the weaker equilibrium concept compared to VA, a finding in agreement with the results in Kagel et al. (2001). After some experience, though, the collusive attempts in AA are given up and efficiency is higher than in VA.

Manelli et al. (2001) compare VA and AA in a design where three bidders compete for three units and each can buy three units, but the third unit has a value of 0. They study both auctions with and without a common value component. In one of their experiments (treatment ASU), they find that due to overbidding in VA, the revenues are higher than in AA. Since some bidders in AA bid aggressively on the third unit, causing efficiency losses, the total efficiency is roughly equal. These results are in line with ours. In contrast, in their second experiment (treatment UI), where overbidding is weaker, in VA the efficiency is higher but the revenues are lower than in AA.

One interesting feature of our research is that statements in the post-experimental questionnaires are similar after the uniform-price auctions and after the AA. Several participants tried to cooperate by reducing demand and they observed that this worked well in UPO, but less so in UPS and even less in AA. It seems, however, that they all failed to realize that cooperation was stable when it was an equilibrium. Hence, the equilibrium prediction organizes the data well for some pairs although they do not think in these terms. This is, of course, interesting from a general perspective. Equilibria can yield good predictions even if they are possibly too sophisticated for subjects to determine, given that equilibrium choices can result from less sophisticated thought processes.

Finally, there are some policy conclusions to be drawn from the research. If the objective of the auctioneer is a combination of the maximization of efficiency and of his revenues, the uniform–price open auction is clearly not preferable. Demand reduction leads both to a reduction of revenues and to a misallocation of one unit.[25] If the primary aim is the efficient allocation, AA seems to be best suited, in particular if bidders have time to gain experience, while if the focus is on revenues, the sealed–bid auctions perform best due to frequent overbidding in the case of UPS and to bids generally exceeding equilibrium bids in DA. A mechanism that is easy to understand seems best suited to allocating the units efficiently, while a sealed–bid mechanism where bids have to be determined completely unaware of the other bidders' choices may raise higher revenues.

[25] Goswami, Noe, and Rebello (1996) find that in a setting with a high number of bidders (11) and units (100) and identical valuations, non–binding pre–play communication facilitates demand reduction in a uniform-price sealed–bid auction and shifts behavior towards the equilibrium in a discriminatory auction. This indicates that outside the laboratory, where communication is more likely, the differences between auction formats may even be stronger than in our (and others') results.

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A Equilibria of UPS and UPO

The setting we analyze corresponds to example 3 in Ausubel and Cramton (2002). Whereas they present the DR– and the TT–equilibrium as two examples of equilibria of the uniform–price auction, we characterize a continuum of equilibria of both, the sealed–bid and the open version of the uniform–price auction in this setting.

In the UPO we show uniqueness of the demand reduction equilibrium under certain refinements, which, we believe, contributes to the literature on multi–unit auctions. Uniqueness of low price equilibria has so far been shown only in a framework with complete information on valuations.[26]

[26] See e. g. Grimm et al. (2003).

A.1 Equilibria of UPS

We first show that the strategies

$$\begin{aligned} b_1(v_i) &= v_i \\ b_2(v_i) &= \begin{cases} x_k & \text{if} \quad v_i \in [x_k, y_k), \\ v_i & \text{otherwise.} \end{cases} \end{aligned} \tag{7}$$

with $K \geq 1, x_{K+1} = V$, and $[x_k, y_k)$, $k = 1, \ldots K$, a sequence with the following properties: $x_1 \geq 0$, $x_k < y_k$, and $y_k \leq x_{k+1}$, are an equilibrium of UPS. In order to simplify the calculations we show this for $V = 1$. Clearly, this also extends to arbitrary values of V.

First note that in both uniform–price formats it is a weakly dominant strategy for the bidders to bid their valuation v_i on the first unit (i. e. their higher bid always equals their true valuation). Their bid on the first unit will only determine the price if it is the highest rejected bid, i. e. if the bidder does not get a unit. As in the standard argument for the second price auction for the single unit case, lowering the bid implies the risk of missing a profitable deal, whereas overbidding might result in buying a unit at a loss. This is even more obvious in the open auction. If bidder i has already dropped out on one unit, dropping out on the remaining unit before his valuation v_i is reached guarantees a profit of 0, whereas continuing might yield a positive profit. Staying in above v_i causes a loss as soon as the other bidder drops out. It remains to be shown that the bid on the second unit as stated in (7) is indeed part of an equilibrium strategy.

In the following we show that if bidder j plays according to (7), it is indeed a best reply for bidder i to play (7) as well. We proceed in two steps. First, suppose bidder i's valuation is in a demand reduction interval, i. e. $v_i \in [x_k, y_k)$. Then

(a) If $v_j \geq y_k$ bidder i's profit is zero if he plays (7) and he cannot increase his profit by deviating.

(b) If $v_j \in [x_k, y_k)$ then for bidder i any bid below x_k or above v_i clearly does not pay. Consider $b_2(v_i) = z$

with $z \in [x_k, v_i)$. We get

$$\pi_i(b_2(v_i) = z) = Pr\{v_j > z | x_k \le v_j < y_k\}(v_i - z)$$

$$+ Pr\{v_j \le z | x_k \le v_j < y_k\} 2\,(v_i - E[v_j | x_k \le v_j \le z])$$

$$= \frac{y_k - z}{y_k - x_k}(v_i - z) + \frac{z - x_k}{y_k - x_k} 2 \left(v_i - \frac{z + x_k}{2}\right)$$

$$= (v_i - z) + \frac{(z - x_k)}{y_k - x_k}(v_i - x_k).$$

Taking the derivative with respect to z yields $-1 + \frac{v_i - x_k}{y_k - x_k}$ which is negative for $v_i < y_k$. Therefore, it does not pay to deviate from $b_2 = x_k$ if the competitor's valuation is in the same interval, which yields $\pi_i(b_2(v_i) = x_k) = v_i - x_k$.

(c) if $v_j \in [y_{l-1}, x_l)$, $l \le k$, the price on any unit bidder i obtains is always v_j. Since bidder i clearly prefers getting two units at price v_j to getting one unit, it does not pay to deviate from (7) either.

(d) if $v_j \in [x_l, y_l)$, $l < k$, then for bidder i bidding below x_l clearly does not pay. Any bid greater or equal to y_l yields the same outcome, namely obtaining both units for v_j. Consider $b_2(v_i) = z$ with $z \in [x_l, y_l)$. We get

$$\pi_i(b_2(v_i) = z) = Pr\{v_j > z | x_l \le v_j < y_l\}(v_i - z)$$

$$+ Pr\{v_j \le z | x_l \le v_j < y_l\} 2\,(v_i - E[v_j | x_l \le v_j \le z])$$

$$= \frac{y_l - z}{y_l - x_l}(v_i - z) + \frac{z - x_l}{y_l - x_l} 2 \left(v_i - \frac{z + x_l}{2}\right)$$

$$= (v_i - z) + \frac{(z - x_l)}{y_l - x_l}(v_i - x_l).$$

Taking the derivative with respect to z yields $-1 + \frac{v_i - x_l}{y_l - x_l}$ which is positive for $v_i > y_l$. Since any bid greater or equal to y_l yields the same outcome (and $v_i \ge y_l$), it does not pay to deviate from $b_2 = x_k$ if the competitor's valuation is in a lower demand reduction interval.

Hence, independent of the interval where the other bidder's valuation lies, deviating from (7) does not pay for a bidder with a valuation in a demand reduction interval. It remains to be shown that there is also no incentive to deviate from (7) if the bidder's valuation is in a truth–telling interval, i. e. $v_i \in [y_{k-1}, x_k)$. For $v_j \ge x_k$, $v_j \in [y_{l-1}, x_l)$, $l < k$, and $v_j \in [x_l, y_l)$, $l < k$ the same arguments as in (a), (c), and (d) apply.

If $v_j \in [y_{k-1}, x_k)$, bidder i will receive no unit if $v_i < v_j$. If $v_i \geq v_j$ the price on any unit bidder i obtains will be v_j. So he prefers getting two units to getting one unit. Therefore, deviation from (7) never pays if the other bidder plays this strategy.

A.2 Perfect Bayesian Equilibria of UPO

In order to show that the equilibria of UPS, as given in (7), are also perfect Bayesian equilibria (PBE) of UPO, we need to specify beliefs about the other bidder's valuation that make the strategy (7) sequentially rational and that are derived from Bayes' rule at least on the equilibrium path. Let $\mu_i(v|\cdot)$ denote the subjective probability assigned by bidder i to the event that the other bidder's valuation is (weakly) smaller than v. The following beliefs support the strategy (7) as a PBE. In particular with these beliefs bidders stick to their strategy even if they observe a deviation by the other bidder.

(a) As long as the other bidder does not drop out on any unit, bidder i believes that the other bidder's valuation is uniformly distributed on the interval of valuations for which equilibrium behavior does not prescribe dropping out at lower prices. Formally, if the current price $p \in (x_k, y_k)$, then

$$\mu_i(v|p) = \begin{cases} \frac{v-y_k}{1-y_k} & \text{for} \quad v \in [y_k, 1], \\ 0 & \text{for} \quad v \in [0, y_k), \end{cases}$$

and if $p \in [y_k, x_{k+1}]$, then

$$\mu_i(v|p) = \begin{cases} \frac{v-p}{1-p} & \text{for} \quad v \in [p, 1], \\ 0 & \text{for} \quad v \in [0, p). \end{cases}$$

(b) If the other bidder drops out on one unit at $z_k \in [x_k, y_k)$, $k = 1, \ldots K$, bidder i updates his subjective distribution function as follows:

$$\mu_i(v|z_k, p) = \begin{cases} 1 & \text{for} \quad v \in [y_k, 1], \\ \frac{v-p}{y_k-p} & \text{for} \quad v \in [p, y_k), \\ 0 & \text{for} \quad v \in [0, p), \end{cases}$$

with $p \in [z_k, y_k)$ being the current price in the auction.

(c) If the other bidder drops out on one unit at $z_k \in [y_k, x_{k+1})$, $k = 1, \ldots K$, bidder i updates his subjective distribution function as follows:

$$\mu_i(v|z_k,p) = \begin{cases} 1 & \text{for} \quad v \in [\min\{x_{k+1}, v_i\}, 1], \\ \frac{v-p}{\min\{x_{k+1},v_i\}-p} & \text{for} \quad v \in [p, \min\{x_{k+1}, v_i\}), \\ 0 & \text{for} \quad v \in [0,p). \end{cases}$$

with $p \in [z_k, x_{k+1})$ being the current price in the auction.

(d) If the current price p exceeds the upper limit of the interval where the other bidder dropped out on one unit, bidder i believes the other bidder's valuation to be in the current interval. In particular, bidder i updates his subjective distribution function as follows:

$$\mu_i(v|z_k,p) = \begin{cases} 1 & \text{for} \quad v \in [y_l, 1], \\ \frac{v-p}{y_l-p} & \text{for} \quad v \in [p, y_l), \\ 0 & \text{for} \quad v \in [0,p), \end{cases}$$

if the current price in the auction is in a demand reduction interval, $p \in [x_l, y_l)$, $l > k$, and

$$\mu_i(v|z_k,p) = \begin{cases} 1 & \text{for} \quad v \in [\min\{x_{l+1}, v_i\}, 1], \\ \frac{v-p}{\min\{x_{l+1},v_i\}-p} & \text{for} \quad v \in [p, \min\{x_{l+1}, v_i\}), \\ 0 & \text{for} \quad v \in [0,p), \end{cases}$$

if the current price in the auction is in a truth–telling interval, $p \in [y_l, x_{l+1})$, $l > k$.

(e) If the other bidder drops out on both units the auction is over and beliefs can be arbitrary.

Note that if there are truth–telling intervals, bidder i's beliefs depend on his own type (i. e. valuation). This violates requirements of some definitions (e. g. Fudenberg and Tirole, 1991) of a Perfect Bayesian Equilibrium. Furthermore, type dependent beliefs are definitely precluded in a Sequential Equilibrium (Kreps and Wilson, 1982), because the beliefs are derived as a limit of beliefs resulting from completely mixed strategies of the bidders. It is possible but tedious to show that equilibria involving TT–intervals always require type dependent beliefs.[27]

If type dependent beliefs are not permitted, the only remaining equilibria are those that involve solely DR-intervals (but arbitrarily many). To establish an equilibrium with several DR-intervals as a sequential

[27] The idea is that for any possible type the belief has to assign most of the mass on values lower than bidder i's own value. For a belief that does not depend on i's own type, one obtains a contradiction (namely that the total mass exceeds 1) by letting i's own value approach p.

equilibrium one needs to choose a sequence of completely mixed strategies that assign a higher (by an order of magnitude) probability to dropping out "too late" than "too early" (and a lower probability to overbidding on any unit).[28]

A.3 Payoff Dominance of the DR–Equilibrium

We show that among all equilibria of the uniform–price auction, the DR–equilibrium yields the highest expected profit to the bidders. We first compare the expected profits of the DR– and the TT–equilibrium:

$$\begin{aligned}\pi_i^{DR} &= v_i \qquad \text{and}\\ \pi_i^{TT} &= Pr\{v_j \leq v_i\} 2\left(v_i - E[v_j | v_j \leq v_i]\right)\\ &= v_i^2.\end{aligned}$$

Thus, $\pi_i^{DR} \geq \pi_i^{TT}$ whenever $v_i \leq 1$ which is always true. The expected payoff difference is $\pi_i^{DR} - \pi_i^{TT} = v_i(1 - v_i)$. Now consider one of the intermediate equilibria. They yield the same payoff as the TT–equilibrium unless both valuations are in the same demand reduction interval $[x_k, y_k)$. In the latter case, the payoff exceeds the TT-equilibrium payoff by $\Delta\pi_i$

$$\begin{aligned}\Delta\pi_i = &\, Pr\{v_j \in [x_k, v_i]\}\left(v_i - x_k - 2\left(v_i - E[v_j | v_j \in [x_k, v_i]]\right)\right)\\ &+ Pr\{v_j \in [v_i, y_k]\}(v_i - x_k)\\ &= (v_i - x_k)\left(v_i - x_k - 2\left(v_i - \frac{v_i + x_k}{2}\right)\right) + (y_k - v_i)(v_i - x_k)\\ &= (y_k - v_i)(v_i - x_k).\end{aligned}$$

Therefore, we always get $\Delta\pi_i \leq \pi_i^{DR} - \pi_i^{TT}$, which proves that the DR–equilibrium is payoff dominant.

[28] Consider for example a completely mixed strategy that is given by a density $f_v^2(p)$ for a second drop-out at p given that the bidder's own value is v with $f_v^2(p) = \varepsilon^{K+1}$ for $p \neq v$ (and all the remaining probability going to a drop-out at v) and a density $f_v^1(p)$ for a first drop-out at p of the following form: if $v \in [x_k, y_k)$ then $f_v^1(p) = \varepsilon$ for $p \in (x_k, v]$, $f_v^1(p) = \varepsilon^K$ for $p > v$, $f_v^1(p) = \varepsilon^{k-m+1}$ for $p \in [x_m, y_m)$, $m < k$ (and all the remaining probability going to a drop-out at x_k). The beliefs derived by Bayesian updating then converge for any observed drop-out price p_1 and any current price p towards the given belief and the mixed strategy converges to the equilibrium strategy for $\varepsilon \to 0$.

A.4 DR–Equilibrium in UPO

We can establish the uniqueness of the DR–equilibrium by requiring that a sequential equilibrium satisfies support restriction (Madrigal et al., 1987). The latter amounts to requiring that a player does not assign positive probability to another player's type if he has assigned probability 0 to this type before. In UPO, if bidder i observes that bidder j does not drop out on one unit at price x_k, he concludes (in equilibrium) that bidder j's valuation is not in $[x_k, y_k)$. Support restriction implies that even after observing that bidder j drops out on one unit at $z \in (x_k, y_k)$, bidder i sticks to this belief, inconsistent with the beliefs needed to support the equilibrium above. If beliefs satisfy support restriction it would be subjectively optimal to follow a dropout at z immediately even for some bidders with $v_i > y_k$. This in turn makes it optimal for some bidders with valuations above y_k to drop out slightly above x_k and hence earlier than prescribed by their DR-interval. Thus equilibria with multiple DR–intervals break down.

Madrigal et al. (1987) and Nöldeke and van Damme (1990) argue that support restriction is not an appealing refinement because in some games the only Nash–equilibrium does not satisfy support restriction. Furthermore, it is not always intuitive. In our setting, for example, it is not clear why after an off–equilibrium dropout it should be more plausible that a bidder erred by dropping out too early than by dropping out too late. While uniqueness of an equilibrium is certainly appealing, the argument in our case does not appear entirely convincing.

The DR–equilibrium is, however, also the only Perfect Bayesian equilibrium that has another appealing property, namely that beliefs strictly follow Bayes' rule also off the equilibrium path in the sense that if a bidder observes an off-equilibrium dropout on one unit he infers only that the opponent's valuation is higher than the dropout price and updates the initial distribution accordingly. Put differently, this requirement means that a bidder does not assume that he learns anything from another bidder's move that no type should take, but maintains, if possible, the assumption that the other bidder is not playing a weakly dominated strategy.

To see that only the DR–equilibrium satisfies this requirement, recall that in the uniform-price open auction it is a weakly dominant strategy to remain active on the first unit until one's true value v_i is reached, and that bidder i's payoff in the DR–equilibrium is $v_i - 0$. Furthermore, note that in equilibrium a drop-out on a single unit only occurs at the lower boundary x_k of a demand reduction interval.

Consider an equilibrium as in Appendix A.2 but where the beliefs fulfill the above requirement, i. e. if bidder j drops out on one unit at any current price $p \in [0, 1]$ that is not equal to an x_k, bidder i believes bidder j's valuation to be uniformly distributed on $[p, 1]$.[29]

Assume bidder j drops out at price $y \geq 0$ (and $y \neq x_k$ for all k). Now bidder i has two options: Dropping out on one unit as well guarantees him profit $v_i - y$. In contrast, staying active with the second unit until some price $x > y$ yields, (if the other bidder uses the weakly dominant strategy to bid his valuation on the first unit) with (subjective) probability $\frac{1-x}{1-y}$ a profit of $v_i - x$ (if bidder j's valuation exceeds x), whereas with probability $\frac{x-y}{1-y}$ bidder j will drop out at a price below x which implies an expected price of $\frac{x+y}{2}$ and thus an expected profit $2(v_i - \frac{x+y}{2})$. Hence the total expected payoff from staying active until a price $x > y$ is

$$\begin{aligned}
\pi_i \left(b_2(v_i) = x\right) &= \frac{1-x}{1-y}\left(v_i - x\right) + 2\frac{x-y}{1-y}\left(v_i - \frac{x+y}{2}\right) \\
&= v_i - x + \frac{x-y}{1-y}\left(v_i - y\right) < v_i - y \\
\text{for } x &> y \text{ and } v_i < 1.
\end{aligned}$$

Thus, whenever bidder j drops out at y, it is optimal for i to drop out at y as well (unless $v_i = 1$ which happens with probability 0). Hence in any continuation game that is reached after one bidder drops out,[30] the other drops out as well.

Hence, if bidder j drops out at 0 (or if $x_1 = 0$, at a price which is an arbitrarily small amount larger than 0), bidder i drops out as well and j receives $\pi_j = v_j - 0$.

It has been shown in Appendix A.3 that this payoff dominates the expected payoff in any other equilibrium. Hence given our assumption of updating after off-equilibrium drop-outs, in any equilibrium other than the DR–equilibrium a bidder can increase his profit by deviating from his equilibrium strategy to

[29] One could also consider the requirement that bidder i uses strict Bayesian updating of his last belief. In that case if $p \in (x_k, y_k)$, i would believe the valuation of j to be uniformly distributed in $[y_k, 1]$. But this implies support restriction, which leads to a breakdown of equilibria as described above.

[30] Note that there are no real subgames, since there is incomplete information about the valuation of the other bidder.

dropping out at 0 on one unit (or if $x_1 = 0$, at a price which is an arbitrarily small amount larger than 0). Hence all Perfect Bayesian equilibria other than the DR–equilibrium break down. For the DR–equilibrium, in contrast, the beliefs in Appendix A.2 correspond to the condition of strict Bayesian updating.

B Instructions (Ausubel Auction)

Please read these instructions carefully. If there is something you do not understand, please raise your hand. We will then answer your questions privately. The instructions are identical for all participants.

In the course of the experiment you will participate in 10 auctions. In each auction you and another bidder will bid for two units of a fictitious good. This other bidder will be the same in each auction. Each unit that you acquire will be sold to the experimenters for your private resale value v. Before each auction this value **per unit**, v, will be randomly drawn independently for each bidder from the interval $0 \leq v \leq 100$ ECU (Experimental Currency Unit). Any number between 0 and 100 is equally probable. The private resale values of different bidders are independent. **In each auction any unit that you acquire will have the same value for you. This value will be drawn anew before each auction.**

Before each auction you will be informed about your resale value **per unit**, v. Each participant will be informed only about his or her own resale value, but not about the other bidder's resale value.

After a short break the auction starts:

The price **per unit** will be increased successively in steps of 1, beginning at a price of 0. At the beginning of the auction you are active on both units. At any time you can drop out on *one* unit by clicking the button *"dropout 1"* or you can drop out on *both* units simultaneously by clicking the button *"dropout 2"*.

If one of the bidders clicks the button *"dropout 2"*, the other bidder obtains both units for the price where the first bidder dropped out and the auction is finished (since then there are only two active bids left).

If one bidder drops out on one unit, the other immediately obtains one unit (since the first bidder has only one active bid left and can thus acquire at most *one* unit) for the price at which the first bidder dropped out.

Then the auction continues at the price at which the first unit was given away. Now only **one** unit is auctioned off and both bidders have only **one** active bid. If now one bidder drops out on this unit, the other bidder obtains this unit for the price at which the bidder dropped out and the auction is finished.

If upon reaching the maximal price of 100 ECU there are four active bids left, both bidders receive one unit for a price of 100 ECU. If upon reaching the maximal price of 100 ECU there is only one unit given away, (both bidders still have one active bid), then the other unit will be randomly allocated for a price of 100 ECU among the two bidders.

Your profit per unit acquired is your resale value minus the price at which you obtained the unit.

If you do not obtain a unit you neither receive nor pay anything. Hence your profit is 0.

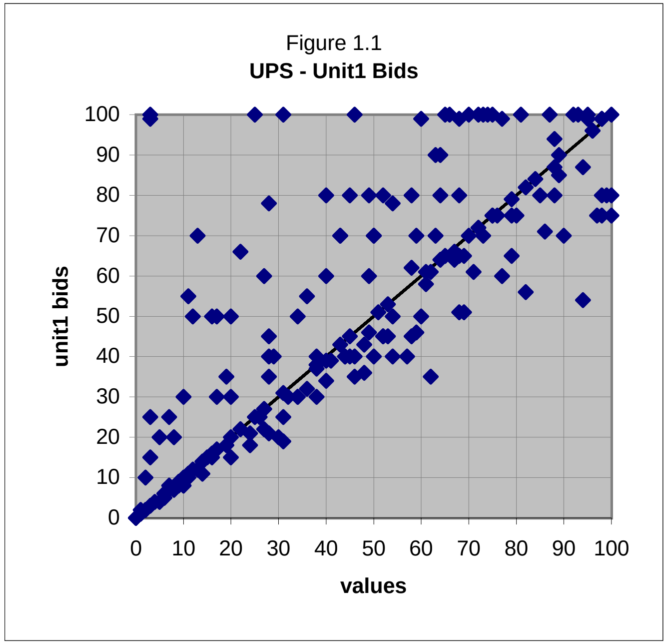
Note that you can make losses as well. It is always possible, however, to bid in such a way that you can prevent losses for sure.

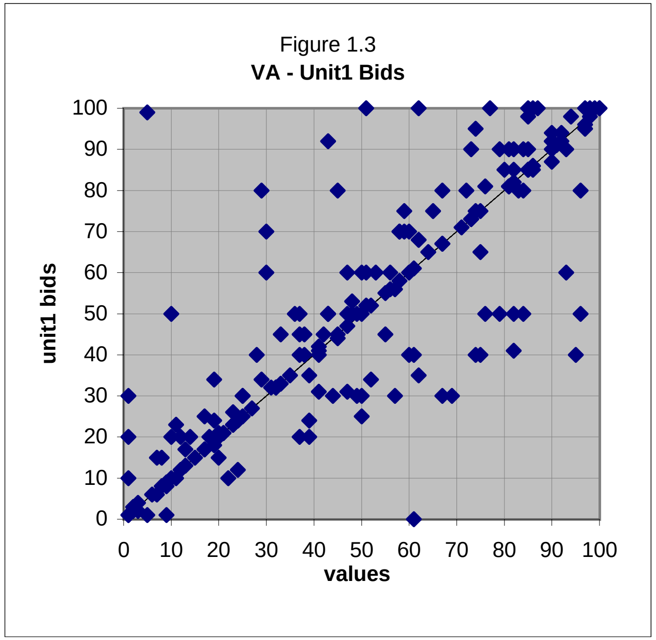
You will make your decision via the computer terminal. You will not get to know the names and code numbers of the other participants. Thus all decisions remain confidential.

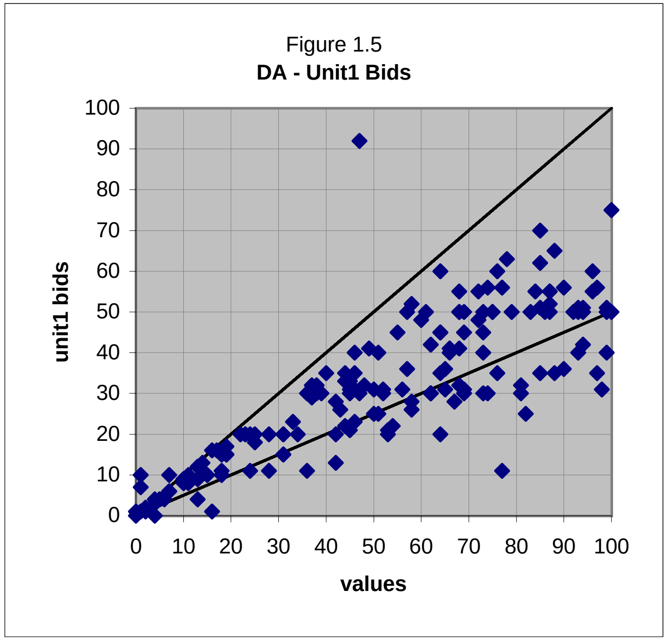
One ECU corresponds to 0,04 DM. You will obtain an initial endowment of 5 DM. If you make losses in an auction these will be deducted from your previous gains (or from your initial endowment). You will receive your final profit in cash at the end of the experiment. The other participants will not get to know your profits.

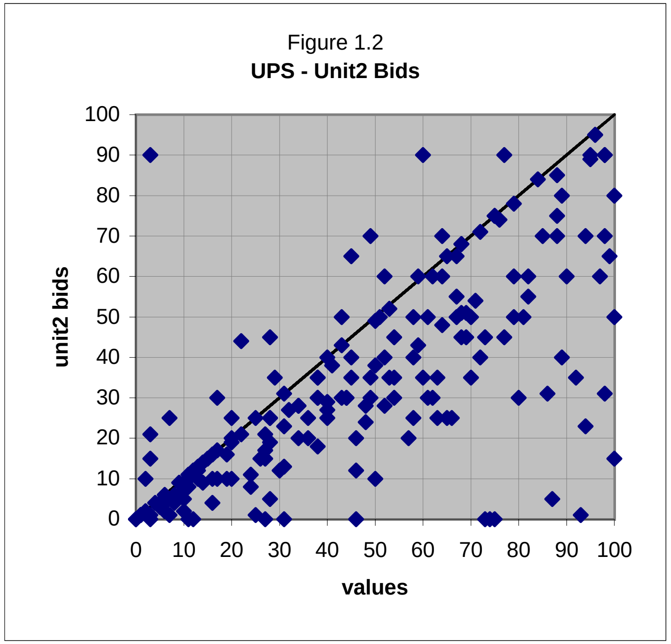
If there is something you have not understood, please raise your hand. We will then answer your questions privately.

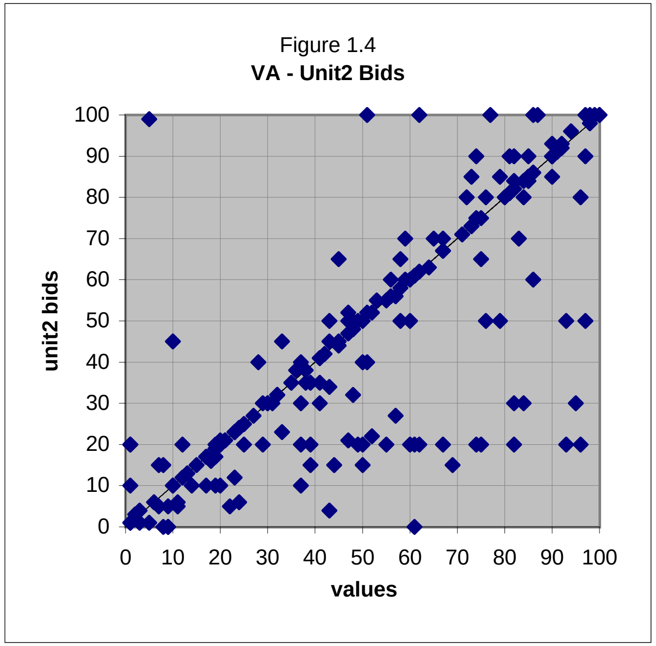
Scatter Diagrams, Sealed-Bid Auctions

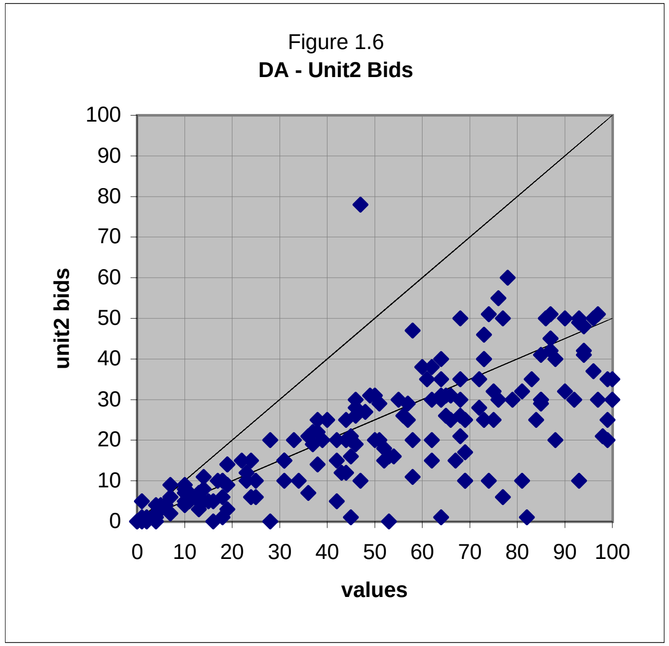




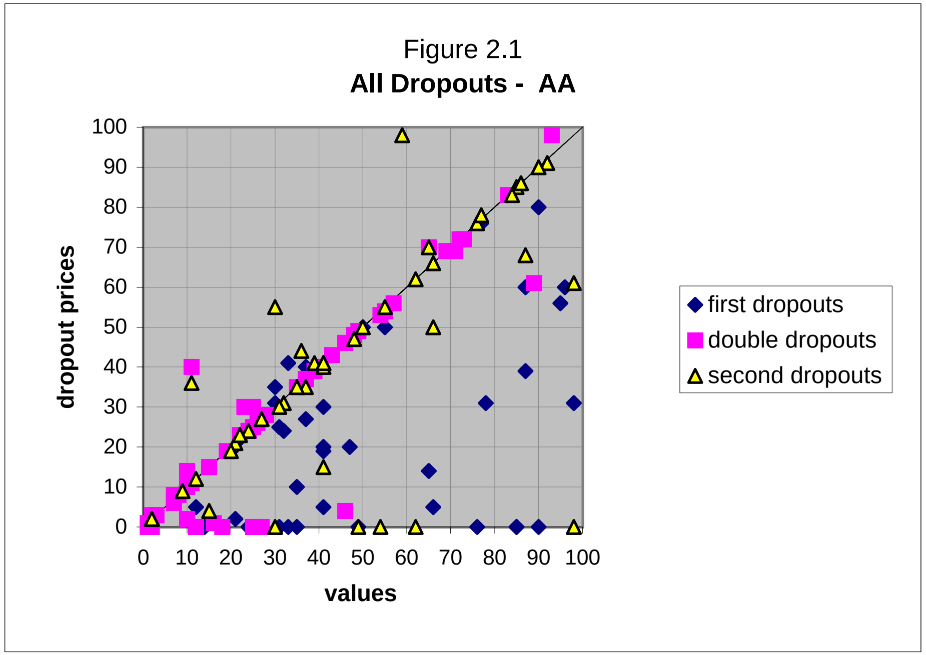


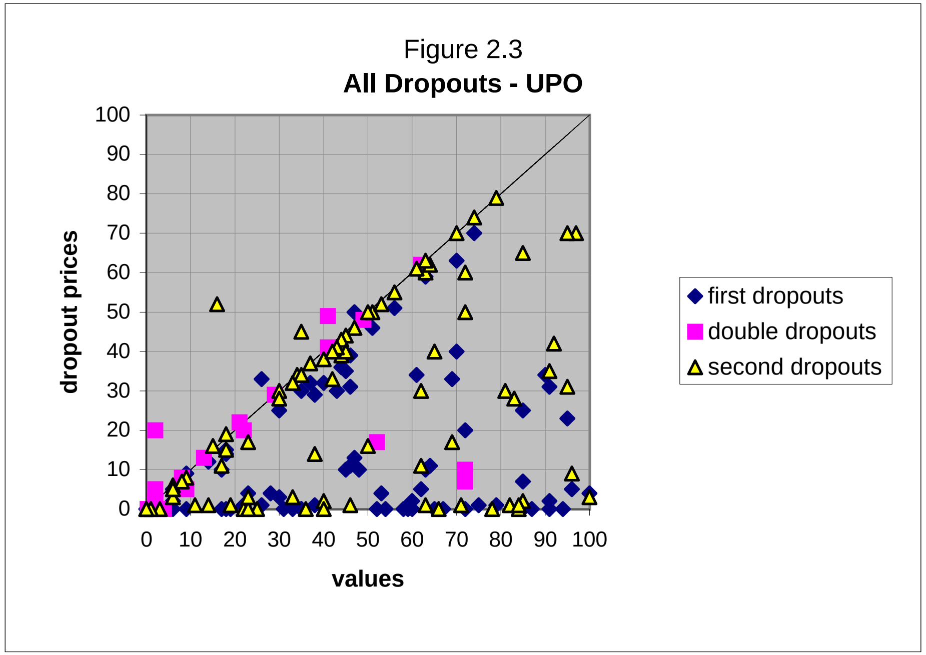


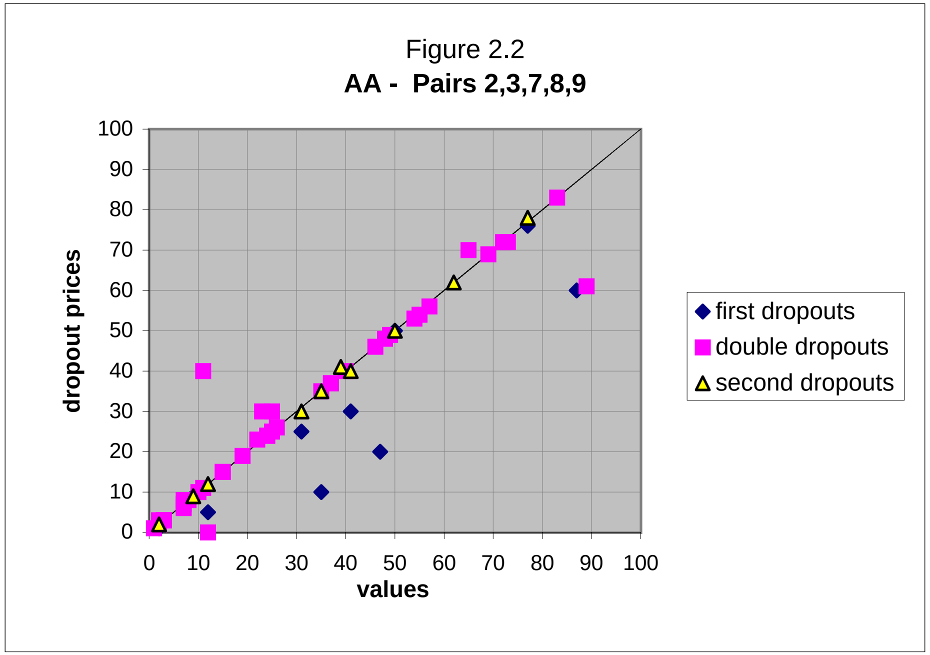


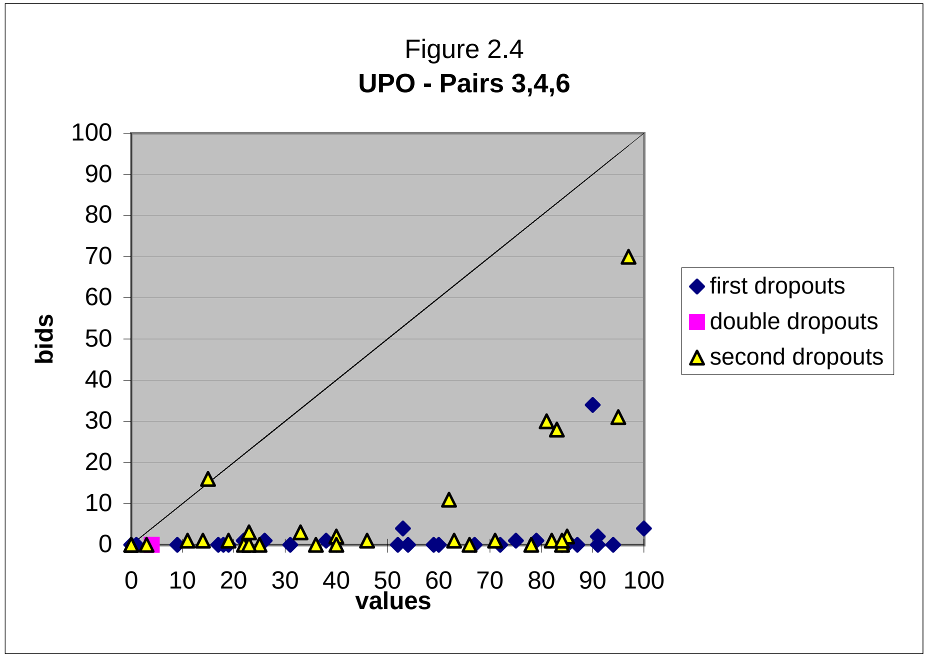


Scatter Diagrams - Open Auctions









UPoS - All Pairs

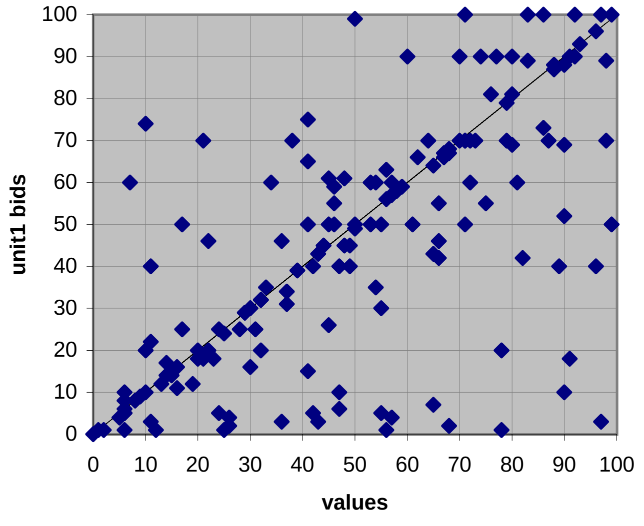


Figure 3.1
UPoS - Unit1 Bids

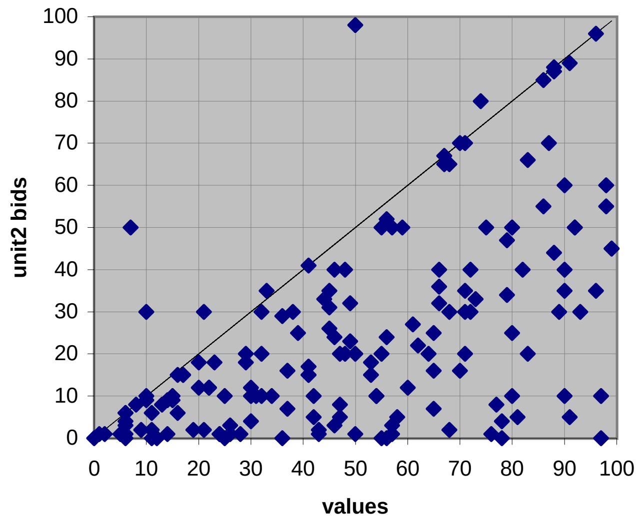


Figure 3.2
UPoS - Unit2 Bids

UPoS - pairs that played close to DR in the open auction

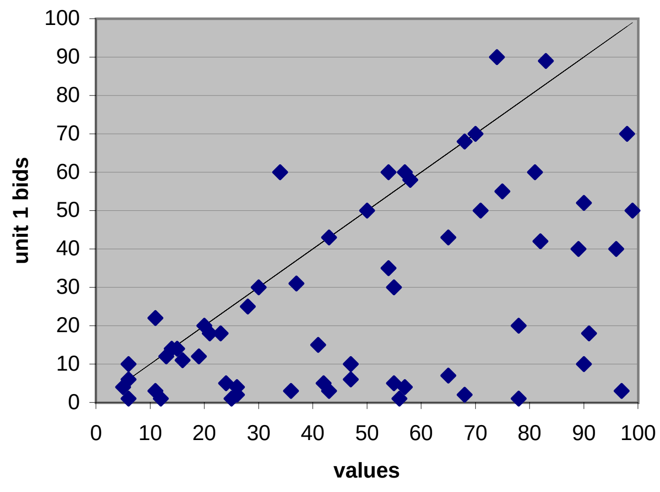


Figure 3.3
UPoS - Unit1 Bids (Pairs 3,4,6)

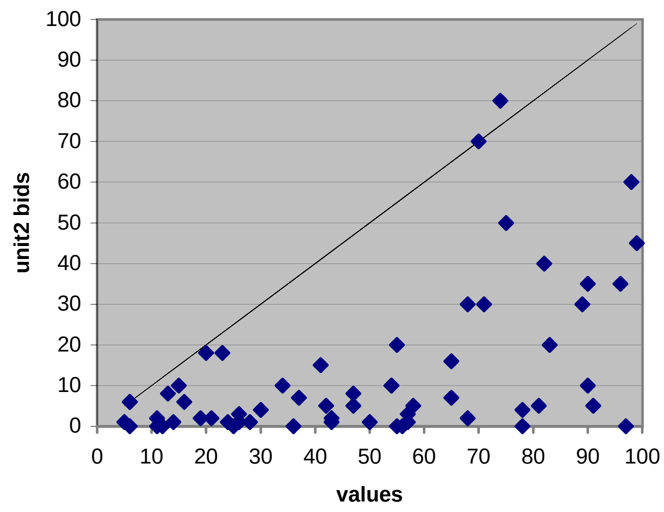
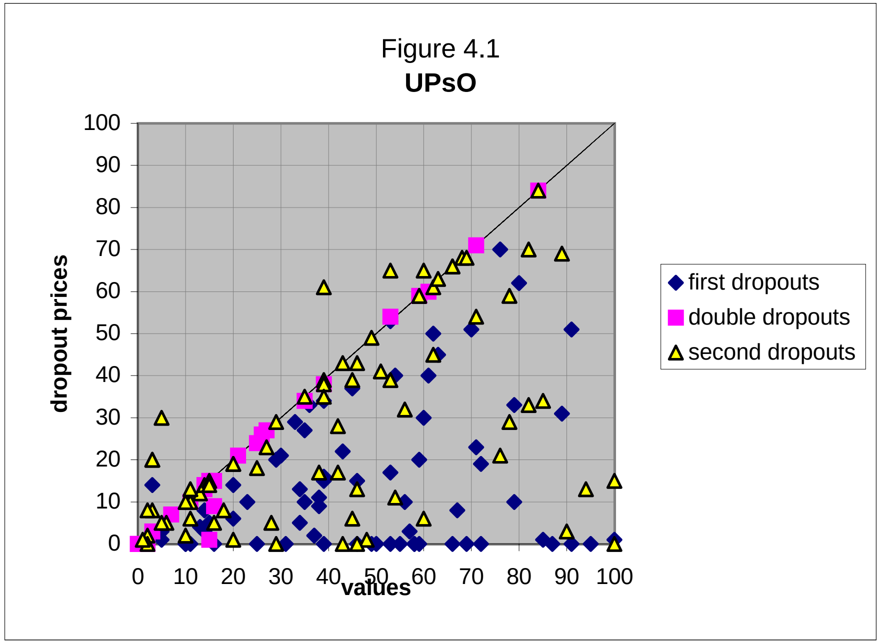
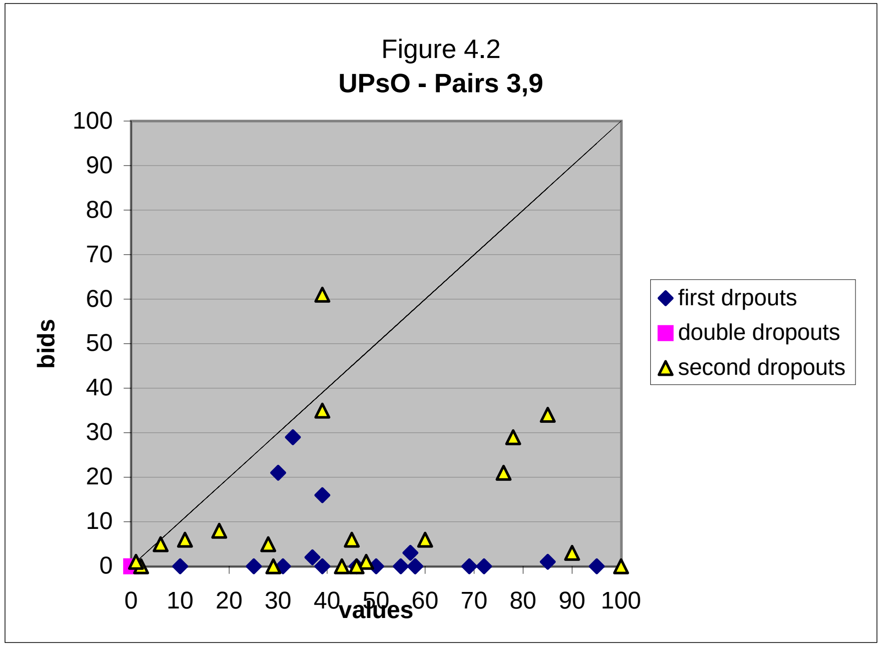


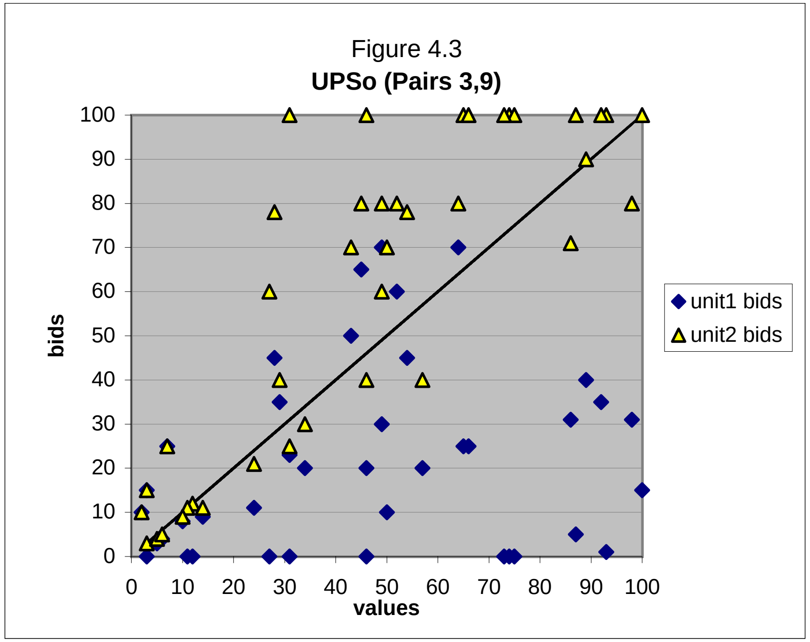
Figure 3.4
UPoS - Unit2 Bids (Pairs 3,4,6)

UPsO - all pairs and pairs that played close to DR





UPSo - bids in the preceeding sealed-bid auction by pairs that played close to DR in the open auction



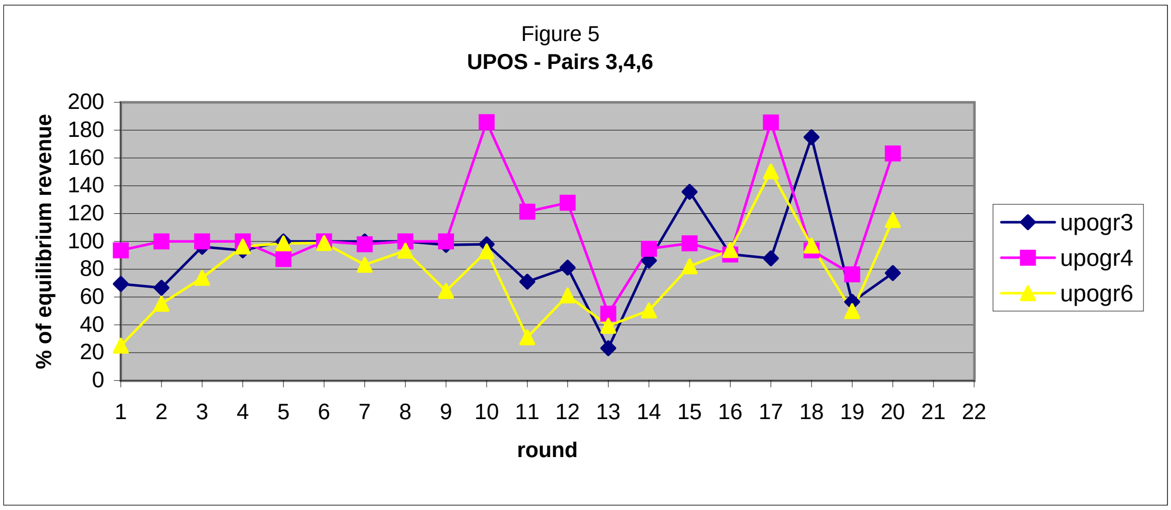


Figure 5
UPOS - Pairs 3,4,6
% of equilibrium revenue
200
180
160
140
120
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
round
upogr3
upogr4
upogr6

Table 3.1 **Auctioneer's revenues** (Equilibrium Revenue = TT Eq. Revenue in all auctions to make revenues comparable)

UPS

Pair	TT-Equilibrium Revenue (ER)	Revenue (R)	R - ER	(R * 100)/ER
1	488	682	194	139.75
2	548	486	-62	88.69
3	498	478	-20	95.98
4	356	572	216	160.67
5	496	502	6	101.21
6	618	760	142	122.98
7	760	710	-50	93.42
8	498	576	78	115.66
9	546	498	-48	91.21
10	502	404	-98	80.48
TOTAL	5310	5668	358	**106.74**

VA

Pair	Equilibrium Revenue (ER)	Revenue (R)	R - ER	(R * 100)/ER
1	614	568	-46	92.51
2	848	861	13	101.53
3	550	604	54	109.82
4	608	252	-356	41.45
5	770	643	-127	83.51
6	520	547	27	105.19
7	858	829	-29	96.62
8	576	753	177	130.73
9	476	444	-32	93.28
10	872	895	23	102.64
TOTAL	6692	6396	-296	**95.58**

DA

Pair	Equilibrium Revenue (ER)	Revenue (R)	R - ER	(R * 100)/ER
1	734	879	145	119.75
2	615	655	40	106.50
3	700	1017	317	145.29
4	628	639	11	101.75
5	714	829	115	116.11
6	687	835	148	121.54
7	652	542	-110	83.13
8	571	612	41	107.18
9	592	517	-75	87.33
TOTAL	5893	6525	632	**110.72**

UPO

Pair	TT-Equilibrium Revenue (ER)	Revenue (R)	R - ER	(R * 100)/ER
1	814	584	-230	71.74
2	562	592	30	105.34
3	654	128	-526	19.57
4	614	8	-606	1.30
5	732	740	8	101.09
6	778	272	-506	34.96
7	1014	868	-146	85.60
8	546	448	-98	82.05
9	588	634	46	107.82
10	768	602	-166	78.39
TOTAL	7070	4876	-2194	**68.97**

AA

Pair	Equilibrium Revenue (ER)	Revenue (R)	R - ER	(R * 100)/ER
1	618	271	-347	43.85
2	518	525	7	101.35
3	770	723	-47	93.90
4	638	279	-359	43.73
5	914	630	-284	68.93
6	892	674	-218	75.56
7	600	658	58	109.67
8	530	562	32	106.04
9	636	613	-23	96.38
10	804	929	125	115.55
TOTAL	6920	5864	-1056	**84.74**

Table 3.2 **Auctioneer's Revenues - UPsO and UPoS**
(Eq. Revenue = TT Eq. Revenue in all auctions to make revenues comparable)

UPoS

Pair	TT-Equilibrium Revenue (ER)	Revenue (R)	R - ER	(R * 100)/ER
1	594	762	168	128.28
2	782	672	-110	85.93
3	480	180	-300	37.50
4	646	42	-604	6.50
5	772	570	-202	73.83
6	618	478	-140	77.35
7	682	412	-270	60.41
8	682	776	94	113.78
9	540	492	-48	91.11
10	608	184	-424	30.26
TOTAL	6404	4568	-1836	**71.33**

UPsO

Pair	TT-Equilibrium Revenue (ER)	Revenue (R)	R - ER	(R * 100)/ER
1	628	500	-128	79.62
2	828	654	-174	78.99
3	474	190	-284	40.08
4	752	754	2	100.27
5	608	438	-170	72.04
6	928	950	22	102.37
7	712	658	-54	92.42
8	508	464	-44	91.34
9	738	252	-486	34.15
10	478	478	0	100.00
TOTAL	6654	5338	-1316	**80.22**

Table 4.2 **Bidders' Profits - UPsO and UPoS**
(Eq. Profit = DR Eq. Profit)

UPoS

Pair	DR Equilibrium Profit (EP)	Profit (P)	P - EP	(P * 100)/EP
1	962	380	-582	39.50
2	1007	400	-607	39.72
3	942	825	-117	87.58
4	943	1029	86	109.12
5	1090	647	-443	59.36
6	1046	795	-251	76.00
7	989	829	-160	83.82
8	1072	629	-443	58.68
9	787	540	-247	68.61
10	1052	951	-101	90.40
TOTAL	9890	7025	-2865	**71.03**

UPsO

Pair	DR Equilibrium Profit (EP)	Profit (P)	P - EP	(P * 100)/EP
1	1075	895	-180	83.26
2	963	375	-588	38.94
3	837	758	-79	90.56
4	1154	757	-397	65.60
5	966	691	-275	71.53
6	1150	402	-748	34.96
7	984	530	-454	53.86
8	909	807	-102	88.78
9	1067	874	-193	81.91
10	958	926	-32	96.66
TOTAL	10063	7015	-3048	**69.71**

Table 4.1 **Bidder Profits** (Equilibrium Profit = DR Equilibrium Profit in UPO and UPS)

UPS

Pair	DR-Equilibrium Profit (EP)	Profit (P)	P - EP	(P * 100)/EP
1	969	637	-332	65.74
2	916	738	-178	80.57
3	978	726	-252	74.23
4	772	497	-275	64.38
5	866	734	-132	84.76
6	1077	682	-395	63.32
7	1003	533	-470	53.14
8	803	409	-394	50.93
9	852	548	-304	64.32
10	826	726	-100	87.89
TOTAL	9062	6230	-2832	**68.75**

VA

Pair	Equilibrium Profit (EP)	Profit (P)	P - EP	(P * 100)/EP
1	898	941	43	104.79
2	564	391	-173	69.33
3	584	470	-114	80.48
4	514	843	329	164.01
5	392	410	18	104.59
6	832	805	-27	96.75
7	484	316	-168	65.29
8	648	281	-367	43.36
9	832	790	-42	94.95
10	636	555	-81	87.26
TOTAL	6384	5802	-582	**90.88**

DA

Pair	Equilibrium Profit (EP)	Profit (P)	P - EP	(P * 100)/EP
1	734	553	-181	75.34
2	615	545	-70	88.62
3	700	329	-371	47.00
4	628	520	-108	82.80
5	714	535	-179	74.93
6	687	489	-198	71.18
7	652	737	85	113.04
8	571	490	-81	85.81
9	592	656	64	110.81
TOTAL	5893	4854	-1039	**82.37**

UPO

Pair	DR-Equilibrium Profit (EP)	Profit (P)	P - EP	(P * 100)/EP
1	1183	617	-566	52.16
2	898	538	-360	59.91
3	1011	883	-128	87.34
4	933	954	21	102.25
5	1072	592	-480	55.22
6	1055	783	-272	74.22
7	1168	443	-725	37.93
8	884	764	-120	86.43
9	981	688	-293	70.13
10	1121	729	-392	65.03
TOTAL	10306	6991	-3315	**67.83**

AA

Pair	Equilibrium Profit (EP)	Profit (P)	P - EP	(P * 100)/EP
1	826	1017	191	123.12
2	462	453	-9	98.05
3	728	767	39	105.36
4	188	543	355	288.83
5	458	632	174	137.99
6	582	644	62	110.65
7	652	592	-60	90.80
8	1008	976	-32	96.83
9	922	920	-2	99.78
10	540	293	-247	54.26
TOTAL	6366	6837	471	**107.40**