A discusión

ON THE COMPETITIVE EFFECTS OF VERTICAL INTEGRATION UNDER PRODUCT DIFFERENTIATION*

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ABSTRACT

The result of neutrality of vertical integration for competition postulated by the Chicago School can be supported by a benchmark model with (1) an upstream monopolist, (2) homogeneous goods downstream and (3) observable (two-part tariff) contracts. The result does not hold however, whenever any of the three assumptions is relaxed. Rey and Tirole (1999) show that, with secret contracts, vertical integration is profitable and anticompetitive. The present paper shows that, adding an alternative supplier and product differentiation to the benchmark model, the effects of vertical integration depend on the efficiency level of the alternative supplier. When the alternative supply is relatively efficient, we also obtain that vertical integration is profitable and anticompetitive. However, when the alternative supplier is relatively inefficient, vertical integration becomes unprofitable and increases social welfare.

JEL Classification: L22, K21, D4

Keywords: Vertical integration, market foreclosure, two-part tariff contracts.

1. Introduction

There has been a long debate on the competitive effects of vertical integration. One of the central issues involves vertical foreclosure: the vertically integrated firm may have an incentive not to supply the input to the non-integrated downstream rivals or, at least, to worsen the supply conditions for those firms. The traditional market foreclosure theory, which was accepted in leading court cases in the 1950s through the 1970s, viewed vertical mergers as harming competition by denying competitors access to either a supplier or a buyer (Chen, 2001). This informal version of the foreclosure theory was criticized by The Chicago School. They revealed the logical flaws of the traditional theory and argued that a vertically integrated firm has no incentive to exclude its rivals and, if it did try to exclude them, rivals could protect themselves by contracting with other unintegrated firms. They subsequently defended that vertical integration was most likely to be pro-competitive or competitive neutral.(e.g., Bork, 1978 and Posner, 1976). Their criticism had a major influence on antitrust activities and was largely responsible in the 1970s and 1980s for the dormancy of antitrust enforcement with vertical elements (Riordan, 1998).

The idea of neutrality of vertical integration can be supported by a benchmark model with an upstream monopolist, homogeneous goods downstream and two-part tariff observable supply contracts. In this case, both the integrated and unintegrated structures lead to full monopolization.

In a very influential paper, Rey and Tirole (1999) show, among other things, that the result of neutrality of vertical integration does not hold if we relax the assumption of observable contracts. Under secret contracts (and passive conjectures) the upstream firm cannot commit to restrict supplies to competitors, being unable to get the monopoly profits. This commitment capacity is shown to be restored by vertical integration.

The present paper shows that, adding an alternative supplier and product differentiation to the benchmark model, the effect of vertical integration depends on the efficiency level of the alternative supplier. When the alternative supplier is relatively efficient, we obtain the same result as Rey and Tirole (1999), namely, that vertical integration is profitable and anticompetitive. However, when the alternative supplier is relatively inefficient, vertical integration is not profitable and increases social welfare.

The intuition to understand the result on profitability is the following: we know that the higher the input prices the higher the profits of the vertical structure. However, the effect of vertical integration on input prices is ambiguous. On the one hand, it increases the wholesale price paid by non-integrated downstream firms. On the other hand, it reduces the price "paid" by the integrated downstream firm: vertical integration eliminates double marginalization and input is priced at marginal cost.[1] When the alternative supplier is very efficient wholesale prices are already low without integration. Then, the elimination of double marginalization implied by vertical integration is not very important and the effect that dominates is the increase in the input price paid by non-integrated firms and, therefore, vertical integration is profitable. When the alternative supplier is inefficient, input prices are high without vertical integration. Then, the reduction in the wholesale price paid by the downstream subsidiary of the integrated firm is large and, therefore, this effect dominates and vertical integration is not profitable.

The argument to explain the evolution of welfare follows the same lines. The same reason that explains when vertical integration is profitable (higher average input prices) also explains its negative effect on welfare. Our model shows cases where a vertical merger increases social welfare. However, it can not be used to defend mergers submitted for approval from antitrust authorities, because we show that whenever a merger is profitable it reduces social welfare. In other words, private incentives and social incentives move in opposite directions.

Our paper can be placed in the so called *new* market foreclosure theory. Following the Chicago School criticism, this new school of thought attempts to place vertical foreclosure theory on a firmer theoretical ground with game theoretic foundations (Choi and Yi, 2000). Among the most influential papers Ordover, Saloner and Salop (1990) present a successive oligopoly game that includes the possibility of a countermerger by the foreclosed firm and the holdout incentives that the target upstream firm may have in the acquisition process. Their

[1] The negative strategic effect of the elimination of double marginalization induced by vertical integration was studied in Bonanno and Vickers (1988).

model however, has been criticized because in order for foreclosure to be an equilibrium they need that the merged firm is able to commit not to compete aggressively with the remaining unintegrated supplier to supply the other downstream firms. That is, it is supposed to be a Stackelberg price leader, changing the nature of the input pricing game.

On the other hand, in Salinger (1988) market foreclosure is also obtained as an equilibrium outcome of a successive oligopoly game. The result is obtained by assuming that integrated firms commit not to supply the remaining unintegrated downstream firms.[2]

More recently, some authors obtain market foreclosure as a consequence of a technological decision made by integrated firms regarding the use of a specific, non-standard input that commits those firms not to supply the remaining unintegrated downstream firms (See Choi and Yi (2000), Avenel and Barlet (2000) and Church and Gandal (2000).

It is interesting to notice the strong analogy between our model and the patent licensing literature. In particular, the problem faced by the integrated firm is akin to that of an internal to the industry patentee who licenses a cost reducing innovation to rival firms (see Faulí-Oller and Sandonís (2002b)). On the other hand, the case of the unintegrated upstream firm resembles that of an external patentee (for an application of this model to the patent licensing literature see Sandonís and Faulí-Oller (2002)).

The rest of the paper is organized as follows. Next section presents the model with an alternative supplier and product differentiation. Section 3 discusses the competitive effects of vertical integration. We conclude in section 4.

2. The Model.

We are going to consider the following vertical structure conformed by firm U producing an input and two firms (firm 1 and firm 2) operating in the downstream part of the market. Firm U produces an input at cost c_u. The downstream firms transform this input on a one-for-one basis to produce two differentiated products. Downstream firm i produces good

[2] Other important contributions are Bolton and Whinston (1993) and Hart and Tirole (1990)

i whose demand is given by:

$$p_i = 1 - q_i - \gamma q_j, i, j = 1, 2, i \neq j,$$

where $\gamma \in (0, 1)$ represents the degree of product differentiation.[3] These demands are derived from the maximization problem of a representative consumer (see Singh and Vives (1984)), endowed with a utility function separable in money (denoted by m) given by:

$$u(q_1, q_2) = q_1 + q_2 - \frac{q_1^2}{2} - \frac{q_2^2}{2} - \gamma q_1 q_2 + m.$$

Let us define the social welfare function by:

$$W(q_{1,}q_2) = u(q_1, q_2) - c_u(q_1 + q_2).$$

There also exists an alternative second source (competitive) supply of the input at a price equal to c where $1 > c \geq c_u$. We are going to compare in terms of profits and social welfare the case in which the firms remain independent and the one in which firms U and one of the downstream firms (say firm 1) decide to merge. We start by the first scenario.

2.1. The unintegrated case

The timing of the game is the following: in the first stage firm U offers two-part tariff supply contracts to downstream firms. In the second stage, they decide whether to accept or reject the contract. Finally, both firms compete à la Cournot in the final market. We look for the subgame perfect Nash equilibrium of the proposed game, solving it by backward induction. A contract offered to firm i includes a fixed fee f_i and a per unit charge w_i. We do not consider negative fixed fees.

If both firms accept the contract, in the third stage the equilibrium outputs and market profits are given by:

$$q_i(w_i, w_j) = \max\{\min\{\frac{1 - w_i}{2}, \frac{(2 - \gamma) - 2w_i + \gamma w_j}{4 - \gamma^2}\}, 0\}, i, j = 1, 2, i \neq j,$$
$$\pi_i(w_i, w_j) = q_i^2.$$

[3] Observe that we do not consider either the case of homogenous goods ($\gamma = 1$) or the case of independent goods ($\gamma = 0$) because the general results obtained do not apply to those particular cases. They will be dealt separately at the end of the paper.

In order to obtain the equilibrium if firm i has not accepted the contract, one has to replace w_i by c in the expressions above.

In the second stage, firm i accepts the contract whenever $f_i \leq \pi_i(w_i, g_j) - \pi_i(c, g_j)$, where $g_j = w_j$ if firm j accepts and $g_j = c$ otherwise. In the first stage firm U designs the supply contracts taking into account that it is never optimal that a downstream firm is supplied by the competitive market. Therefore, it will set a pair of contracts that will be accepted by both firms, in order to maximize:

$$\begin{array}{l} Max_{w_i,f_i} \ \sum_{i=1}^{2}\{(w_i - c_u)q_i(w_i, w_j) + f_i\} \\ s.t. 0 \leq \ f_i \leq \pi_i(w_i, w_j) - \pi_i(c, w_j), \ i, j = 1, 2, \ i \neq j. \end{array} \tag{2.1}$$

Observe that the restriction that f_i cannot be negative implies that $w_i \leq c$. On the other hand, the right hand side of the constraints implies that both firms accept the contract.[4] As it is in firm U's interest to charge as high an f_i as possible, the right hand side of the constraints is binding and the previous maximization program can be rewritten as follows:

$$\begin{array}{l} Max_{w_i,f_i} \sum_{i=1}^{2}\{(w_i - c_u)q_i(w_i, w_j) + \pi_i(w_i, w_j) - \pi_i(c, w_j)\}, \\ s.t. \ w_i \leq c, \ i, j = 1, 2, \ i \neq j. \end{array}$$

Solving the maximization program we obtain the following result[5].

Proposition 2.1. *The optimal supply contracts are given by:*

$$\begin{array}{l} w_1^* = w_2^* = w^* = \min\{\dfrac{\gamma + c_u(2+\gamma)}{2(1+\gamma)}, \dfrac{c_u(2+\gamma)(2-\gamma)^2 + \gamma(4c - \gamma(2-\gamma))}{2(4 - 2\gamma^2 + \gamma^3)}\}, \\ f_1^* = f_2^* = f^* = \pi_i(w^*, w^*) - \pi_i(c, w^*). \end{array} \tag{2.2}$$

[4] Given that $\dfrac{\partial^2 \pi_i}{\partial w_i \partial w_j} < 0$, we have that $\pi_i(w_i, c) - \pi_i(c, c) > \pi_i(w_i, w_j) - \pi_i(c, w_j)$. Then, the constraint $f_i \leq \pi_i(w_i, w_j) - \pi_i(c, w_j)$ implies that the only equilibrium in the second stage is that both firms accept the contract.

[5] The only situation we have not excluded as optimal is the one in which firm U offers two contracts such that only one firm accepts and the other does not produce. However, this situation is equivalent to another where the firm that was not producing is offered a contract including a zero fixed fee and the lowest wholesale price that keeps that firm out of the market. But those contracts are already considered in the maximization program we solve.

Observe that the upstream firm faces a maximization program (2.2) which is composed of market profits minus the external options of downstream firms. Solving that program involves the balance of two opposite effects: by increasing the wholesale prices market profits increase but, at the same time, the profits to be obtained by downstream firms by using the alternative supply ($\pi_i(c, w_j)$, $i = 1, 2$) also increase. The balance of the two effects leads to an optimal wholesale price lower than the one that would maximize market profits ($w^* = \dfrac{\gamma + c_u(2+\gamma)}{2(1+\gamma)}$), except when the competitive supply is so inefficient ($c \geq \dfrac{4 + 2\gamma(1+c_u) - \gamma^2(1-c_u)}{4(1+\gamma)} = c^M$) that any downstream firm would get zero profits when using the competitive supply. In this case, the maximization problem implies maximizing market profits and the upstream firm can obtain the full monopoly profits.

Finally, let us compute the equilibrium profits obtained by the upstream firm and downstream firms, which will be useful to study the profitability of vertical integration. They are given respectively by[6]:

$$\Pi_U = (w^* - c_u)(q_1(w^*, w^*) + q_2(w^*, w^*)) + 2f^*,$$
$$\Pi_1 = \Pi_2 = \pi_1(w^*, w^*) - f^*.$$

2.2. The integrated case

Next, we analyze the case where the upstream firm is vertically integrated with firm 1. In this case, the timing of the game is as follows: first, the integrated firm $U - D_1$ offers a supply contract to downstream firm 2. In the second stage this firm decides whether to accept or reject the contract. Finally, both firms compete à la Cournot in the final market.

The third stage equilibrium outputs and profits are given by the same expressions as in the unintegrated case where we replace w_1 by c_u. In the second stage, firm 2 accepts the contract whenever $f_2 \leq \pi_2(w_2, c_u) - \pi_2(c, c_u)$.

In the first stage, the vertically integrated firm looks for the contract to be offered to firm 2 in order to maximize:

[6] Actual expressions are relegated to Appendix A

$$Max_{w_2,f_2}\{\pi_1(c_u,w_2)+(w_2-c_u)q_2(w_2,c_u)+f_2\}$$
$$s.t.\ 0\leq f_2\leq \pi_2(w_2,c_u)-\pi_2(c,c_u).$$

Observe that the restriction that f_2 cannot be negative implies that $w_2 \leq c$. Furthermore as the right hand side of the constraint is binding in equilibrium, the previous maximization program can be rewritten as follows:

$$Max_{w_2}\{\pi_1(c_u,w_2)+(w_2-c_u)q_2(w_2,c_u)+\pi_2(w_2,c_u)-\pi_2(c,c_u)\},$$
$$s.t. \qquad w_2\leq c.$$

Next proposition gives us the optimal contract.

Proposition 2.2. *The optimal contract is given by:*

$$\overline{w}_2=\min\{c,\frac{\gamma(2-\gamma)^2+c_u(8-4\gamma-2\gamma^2-\gamma^3)}{2(4-3\gamma^2)}\},\ \overline{f}_2=\pi_2(\overline{w}_2,c_u)-\pi_2(c,c_u).$$

Vertical integration worsens the supply conditions of firm 2 because $\overline{w}_2 > w^*$. This is the main anticompetitive effect of vertical integration. However, it never leads to complete market foreclosure of firm 2 because it is always active ($q_2(\overline{w}_2,c_u)>0$), even when it would be possible for the integrated firm to drive the rival out of the market.[7] The reason is that, due to product differentiation[8], the revenues obtained in market 2 compensates the integrated firm from the increase in market competition. In other words, the integrated firm prefers a duopoly rather than a monopoly in its own market.

Finally, we can compute the profits of the integrated firm. They are given by (actual expressions can be seen in Appendix A):

$$\Pi_{U1}=\pi_1(c_u,\overline{w}_2)+\overline{w}_2q_2(\overline{w}_2,c_u)+\pi_2(\overline{w}_2,c_u)-\pi_2(c,c_u).$$

[7] This would be the case if $q_2(c,c_u)=0$. In this case, the cost difference between the integrated firm and firm 2 is "drastic" according to the classical meaning of the word from the patent licensing literature. This happens for $c\geq c^N=\frac{2-\gamma(1-c_u)}{2}$.

[8] This is one of the features that dramatically separates the case with product differentiation from the one with homogenous goods. In this latter case, the integrated firm imposes complete market foreclosure when the cost difference is drastic (see the previous footnote).

3. The competitive effects of vertical integration

Next, we proceed to analyze the profitability of vertical integration as well as its effect on social welfare, with the aim to derive the optimal competition policy.

Regarding profitability we have to sign the difference between the profits of the integrated firm and the sum of the profits of the upstream firm and firm 1, namely, the sign of $\Pi_{U1} - (\Pi_1 + \Pi_U)$. We obtain the following result:

Proposition 3.1. *A threshold value for the level of efficiency of the alternative supply c_Π always exists such that vertical integration is profitable whenever $c \leq c_\Pi$ and not profitable if $c > c_\Pi$.*

Proof. See Appendix B.

In order to grasp the main intuition of the proposition it is useful to discuss what happens when the alternative supply is so inefficient that the external option of downstream firms in the unintegrated case becomes zero. In this case, the upstream firm maximizes market profits by choosing two instruments (one contract for each firm), which allows the upstream firm to implement the monopoly outcome and get the monopoly profits. As the integrated firm is not able to implement the monopoly outcome given that it can only use one instrument (a contract for firm 2), a vertical merger between the upstream firm and firm 1 cannot be profitable.

When the alternative supply is not so inefficient, the comparison is not clear because a trade-off arises. Now, in the unintegrated case the upstream firm also cares about the profits that downstream firms can obtain when rejecting the contracts, namely, their external options. The size of the external option effect is decreasing in c. Thus, when the alternative supply is efficient enough the objective of the upstream firm is so distorted from profit maximization that, in spite of its lower flexibility, vertical integration becomes profitable.

One implication of the result that for inefficient alternative supplies vertical integration becomes unprofitable is that the vertically integrated firm would find profitable to divest from its subsidiary firm. Although this may seem surprising, Rey and Tirole (1999) report the case of AT&T's 1995 voluntary divestiture of its manufacturing arm, AT&T (now Lucent) Technology, that took place when it began to compete with the regional operating companies both in the local and the long distance market. Observe that the divestiture can be explained in terms of our model as a commitment to treat all downstream firms equally in order to restrict the sales of its subsidiary.

So far we have analyzed the private incentives of firms for vertical integration. Given that any merger has to be approved by the competition authorities, it is very useful to know the effect of a vertical merger on social welfare. This is done in the next proposition.

Proposition 3.2. *A threshold value for the level of efficiency of the alternative supply c_{SW} always exists such that vertical integration reduces welfare whenever $c < c_{SW}$ and increases welfare if $c > c_{SW}$.*

Proof. See appendix B.

Vertical integration increases the wholesale price paid by firm 2 ($\overline{\overline{w}}_2 > w^{**}$). This is the negative welfare effect of a vertical merger. There is however, a positive effect in that the subsidiary firm 1 is "supplied" at marginal cost c_u whereas, in the unintegrated case, it faces a wholesale price (w^{**}) higher than c_u. Whereas the positive effect of vertical integration ($w^{**} - c_u$) is increasing in c, the negative one ($\overline{\overline{w}}_2 - w^{**}$) is decreasing in c[9]. This explains that for high values of c vertical integration turns out to be welfare improving.

In the particular case where the alternative supply is so inefficient that it is not a profitable option for downstream firms, we know from the above proposition that a vertical merger would increase welfare. There is a nice application of this result to horizontal merger policy by considering that a licensing contract and a merger are two substitutive instruments to transfer technologies. Assume we have a duopoly where one of the firms owns a patented process innovation. This firm could either license the technology to its competitor or to take the rival over. The licensing option is equivalent in that model to our integrated case whereas the merger option leads to the same market outcome as the unintegrated case

[9]Notice that this is not the case when $w^{**} = c$ (the restricted case) but, in this case, vertical integration is always welfare reducing.

whenever $c \geq c^M$. Faulí-Oller and Sandonís (2002 a) obtain the counterpart of the result of Proposition 4.2 applied to the case of very inefficient alternative supplies, by showing that licensing is welfare superior to a merger.

If we consider that the antitrust authorities can approve or reject only mergers that are proposed by the merging partners (i.e., profitable mergers), in order to derive the optimal competition policy we have to combine the above proposition on welfare with the previous result on profitability.

It is direct to see that $c_{WS} > c_{\Pi}$, which means that profitable vertical mergers are never welfare improving. Thus, the following corollary emerges:

Corollary 3.3. *In our framework, vertical mergers should be forbidden.*

Observe that it is possible that some unprofitable mergers increase welfare. However, compulsory action or subsidies to carry them through would go against the normal practices of antitrust policy.

Before concluding two short comments concerning the extreme cases of homogenous goods ($\gamma = 1$) and independent goods ($\gamma = 0$).

In the former case, we have that $c_{WS} = c_{\Pi} = c^M$. Then, the two Propositions above imply that for $c < c^M$ vertical integration is profitable and reduces welfare. The different result appears for $c > c^M$. For such an inefficient alternative supplier, the unintegrated firm can obtain the monopoly profits by simply foreclosing the rival. Then both the integrated and unintegrated cases lead to the same market outcome, namely, full monopolization. Then with homogenous goods we can not get the result that vertical integration either reduces profits or increases welfare.

In the latter case, we have two separate markets downstream and we can check that $w^* = \overline{w}_2 = c_u$. Then, as wholesale prices are unaffected by integration[10] we are back to the result of neutrality of vertical integration i.e. it has no effect either on profits or on welfare.

[10] The downstream division of the integrated firm is also supplied at marginal cost.

4. Conclusions

The crucial effects of vertical integration take place in the input market. On the one hand, a vertical merger allows the integrated firm to charge higher input prices to the remaining unintegrated downstream firms. On the other hand, the integrated firm loses its commitment capacity to restrict its own output, as it cannot credibly increase the marginal cost of its subsidiary firm. The balance of these two opposite forces determines the sign of the final effect of vertical integration. Interestingly enough, the existence of product differentiation, usually ignored in the literature, plays a key role in determining the relative weight of each effect on input prices. As product differentiation increases, competition becomes less intense and, given that consumers value variety, the integrated firm is more interested in supplying the remaining unintegrated downstream firms a positive amount of input, charging a lower wholesale price in the contract. This means that product differentiation puts more weight on the second effect. This explains that vertical integration may reduce the profits of the vertical structure and increase social welfare. To the best of our knowledge, these results are new in the literature and represent our main contribution.

In the paper we have mentioned the example of AT&T's 1995 voluntary divestiture of its manufacturing arm, AT&T (now Lucent) Technology, that took place when it began to compete with the local telephone companies. This example illustrates a case where vertical integration becomes unprofitable.

The surprising result that vertical mergers may be unprofitable hinges crucially on the fact that contracts are observable and they can be used as a commitment to restrict output. If contracts were unobservable as in Rey and Tirole (1999), in the unintegrated case we would have that inputs are priced at marginal cost and that vertical integration raises input prices. Then, we would have that vertical mergers increase profits and reduces social welfare as in the case with homogenous goods. Therefore, it is the combination of observable contracts and product differentiation that leads to our surprising results.

To conclude, we have seen that vertical integration is not neutral for competition but has important effects on the upstream market and they should be taken into account by antitrust authorities when evaluating its desirability. Of course, in order to take a final decision these effects should be compared with its potential efficiency advantages.

5. Appendix

5.1. Appendix A

In the unintegrated case, the upstream and downstream firms' equilibrium profits are given respectively by:

If $c \leq c^M$,

$$\begin{aligned}
\Pi_U &= \tfrac{1}{2(2+\gamma)^2(4-2\gamma^2+\gamma^3)} \left(\begin{array}{c} \gamma^4 - 16c^2(1+\gamma) + 8c(4+2\gamma-\gamma^2) + c_u^2(4-\gamma^2)^2 + \\ +8c_u c(2\gamma+\gamma^2) - 2c_u(16+8\gamma-4\gamma^2+\gamma^4) \end{array} \right), \\
\Pi_1 &= \Pi_2 = \frac{(2-\gamma)^2(4c(1+\gamma) - \gamma c_u(2+\gamma) - 4 - 2\gamma + \gamma^2)^2}{4(2+\gamma)^2(4-2\gamma^2+\gamma^3)^2}.
\end{aligned}$$

If $c > c^M$,

$$\begin{aligned}
\Pi_U &= \frac{(1-c_u)^2}{2(1+\gamma)}, \\
\Pi_1 &= \Pi_2 = 0,
\end{aligned}$$

where $c^M = \dfrac{4+2\gamma(1+c_u) - \gamma^2(1-c_u)}{4(1+\gamma)}$.

The integrated firm's equilibrium profits are given by:

If $c \leq c_r$, then

$$\Pi_{U1} = \frac{(2-\gamma)^2 + c^2(-8+3\gamma^2) + c(8-4\gamma^2+\gamma^3)}{(4-\gamma^2)^2}.$$

If $c_r < c \leq c^N$, then

$$\Pi_{U1} = \frac{\begin{array}{c} -16c(2+\gamma(-1+c_u))(-4+3\gamma^2) + (2-\gamma)^2(16-8\gamma^2-4\gamma^3+\gamma^4) + \\ +16c^2(-4+3\gamma^2) - 2c_u(128-96\gamma-64\gamma^2+40\gamma^3+12\gamma^4-8\gamma^5+\gamma^6) + \\ +c_u^2(128-128\gamma-64\gamma^2+64\gamma^3+12\gamma^4-8\gamma^5+\gamma^6) \end{array}}{4(4-\gamma^2)^2(4-3\gamma^2)}.$$

Finally, if $c > c^N$, then $\Pi_{U1} = \dfrac{(1-c_u)^2(8-8\gamma+\gamma^2)}{4(4-3\gamma^2)}$,
where $c_r = \dfrac{\gamma(2-\gamma)^2 + c_u(8-4\gamma-2\gamma^2-\gamma^3)}{2(4-3\gamma^2)}$ and $c^N = \dfrac{2-\gamma(1-c_u)}{2}$.

5.2. Appendix B

Proof of Proposition 4.1

For $c \geq c^M$, the external option of downstream firms in the unintegrated case becomes zero, which implies that Firm U maximizes market profits by choosing two instruments (one contract for each firm). This allows Firm U to implement the monopoly outcome and get the monopoly profits. As the integrated firms is not able to implement monopoly given that he can only use one instrument (a contract for firm 2), the vertical merger cannot be profitable.

For $c^N \leq c < c^M$, the difference[11] $(\Pi_1 + \Pi_U) - \Pi_{U1}$ is a concave function of c with two roots c^+ and c^-. We have that $c^+ > c^M$ and $c^N \leq c^- < c^M$ whenever $\gamma \geq 0.94$ and $c^- < c^N$ whenever $\gamma < 0.94$. Therefore, a vertical merger is profitable in this region only when $\gamma \geq 0.94$ and $c < c^-$, where

$$c^- = \frac{\begin{array}{c}-64-32\gamma-32c_u\gamma+80\gamma^2-16c_u\gamma^2+16\gamma^3+32c_u\gamma^3-36\gamma^4+ \\ +8c_u\gamma^4+10\gamma^5-10c_u\gamma^5+9\gamma^6+3c_u\gamma^6-3\gamma^7+3c_u\gamma^7+ \\ +(1-c_u)\gamma(8+4\gamma-4\gamma^2+\gamma^4)\sqrt{16-16\gamma-16\gamma^2+20\gamma^3-\gamma^4-6\gamma^5+3\gamma^6}\end{array}}{4(-16-16\gamma+16\gamma^2+12\gamma^3-7\gamma^4+3\gamma^6)}.$$

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For $c_r \leq c < c^N$, the difference $(\Pi_1 + \Pi_U) - \Pi_{U1}$ is a convex function of c with two roots $\widetilde{c}$ and $\widehat{c}$. We have that $\widetilde{c} < c_r$ and $c_r < \widehat{c} \leq c^N$ whenever $\gamma \leq 0.94$. For $\gamma > 0.94$, we have that $\widehat{c} > c^N$. Therefore, a vertical merger is profitable in this region whenever $\gamma \leq 0.94$ and $c \leq \widehat{c}$, or when $\gamma > 0.94$, where

[11] In this region the difference $\Pi_{u1} - (\Pi_1 + \Pi_U)$ is characterized by the fact that the outside option of firm 2 in the integrated case ($\pi_2(c, c_u)$) becomes zero, that is, the threshold value c^N bounds the region where the difference in marginal costs becomes "drastic".

$$\widehat{c} = \frac{1}{4\gamma^2(-4+3\gamma^2)} \left(\begin{array}{c} 64 - 64c_u - 64\gamma + 64c_u\gamma - 64\gamma^2 + 48c_u\gamma^2 + 88\gamma^3 - 88c_u\gamma^3 - 8\gamma^4 + 20c_u\gamma^4 - \\ -26\gamma^5 + 26c_u\gamma^5 + 15\gamma^6 - 15c_u\gamma^6 - 3\gamma^7 + 3c_u\gamma^7 + (-16 + 16c_u + 16\gamma - 16c_u\gamma + \\ +4\gamma^2 - 4c_u\gamma^2 - 12\gamma^3 + 12c_u\gamma^3 + 6\gamma^4 - 6c_u\gamma^4 - \gamma^5 + c_u\gamma^5)\sqrt{16 - 20\gamma^2 + 6\gamma^4} \end{array} \right)$$

.

Finally, for $c_u \leq c < c_r$, the difference $(\Pi_1 + \Pi_U) - \Pi_{U1}$ is a convex function of c with two roots c' and c''. We have that $c' < 0$ and $c'' > c_r$. Therefore, a vertical merger is always profitable.

Summing up, the threshold value c_Π that appears in Proposition 4.1 is given by: $c_\Pi = \widehat{c}$ whenever $\gamma < 0.94$ and $c_\Pi = c^-$ otherwise.

Proof of Proposition 4.2.

If $c_r \leq c \leq c^M$, the difference between welfare under both the unintegrated and the integrated scenarios is given by the expression:

$$W_n = \frac{\gamma}{8(2+\gamma)^2(4-3\gamma^2)(4-2\gamma^2+\gamma^3)^2} \left[\begin{array}{c} 256 + 64\gamma - 384\gamma^2 - 16\gamma^3 + 192\gamma^4 - 16\gamma^5 - 32\gamma^6 - 2\gamma^7 + 2\gamma^8 + \\ +\gamma^9 + 32c^2\gamma(-4 - 4\gamma + 3\gamma^2 + 3\gamma^3) - c_u^2(2+\gamma)^2(-64 + 16\gamma + \\ +96\gamma^2 - 56\gamma^3 - 16\gamma^4 + 30\gamma^5 - 14\gamma^6 + 3\gamma^7) + 2c_u(256 + 64\gamma - \\ -320\gamma^2 - 48\gamma^3 + 112\gamma^4 + 8\gamma^5 - 8\gamma^6 - 6\gamma^7 - 2\gamma^8 + \gamma^9) + \\ +16c(-4 + 3\gamma^2)(8 - \gamma^2 + \gamma^3 + \gamma^4 + c_u(2-\gamma)^2(2 + 3\gamma + \gamma^2)). \end{array} \right]$$

We have that W_n is a concave function of c with two roots c^+ and c^-. We have that $c^- < 0$ and $c_r < c^+ < c^M$. Therefore, a vertical merger is welfare improving whenever $W_n \leq 0$. This holds when $c \geq c^+$, where

$$c^+ = \frac{1}{8\gamma(-4-4\gamma+3\gamma^2+3\gamma^3)} \left(\begin{array}{c} -\sqrt{-(1-c_u)^2(8 + 4\gamma - 4\gamma^2 + \gamma^4)^2(-32 + 16\gamma + 36\gamma^2 - 16\gamma^3 - 9\gamma^4 + 3\gamma^5)} \\ \sqrt{2} + 64 - 64c_u - 32c_u\gamma - 96\gamma^2 + 64c_u\gamma^2 + 8\gamma^3 + 16c_u\gamma^3 + 44\gamma^4 - \\ -20c_u\gamma^4 - 6\gamma^5 + c_u\gamma^5 - 6\gamma^6 + 6c_u\gamma^6 \end{array} \right.$$

When $c > c^M$, in the unintegrated case we have the monopoly outcome, whereas in the integrated case, outputs do not depend on c because the wholesale prices do not depend on c either. Therefore, the difference in welfare becomes constant in c and amounts to W_n

evaluated in $c = c^M$. But we know from the analysis of the previous interval that a vertical merger is welfare improving at that point, which means that it is also welfare improving in the whole interval.

If $c_u \leq c < c_r$, we have that the difference between welfare under both the unintegrated and integrated scenarios is given by the expression:

$$W_r = \frac{1}{4(2-\gamma)^2(2+\gamma)^2(4-2\gamma^2+\gamma^3)^2}\left(\begin{array}{c} (2-\gamma)^3\gamma^2(16-10\gamma^2+3\gamma^3+\gamma^4)+2c_u(2-\gamma)^2(-32+32\gamma- \\ -24\gamma^3+16\gamma^4-4\gamma^5-\gamma^6+\gamma^7)+2c^2(64-144\gamma^2+32\gamma^3+88\gamma^4- \\ -48\gamma^5-8\gamma^6+12\gamma^7-3\gamma^8+c_u^2(384-512\gamma-96\gamma^2+384\gamma^3- \\ -160\gamma^4-16\gamma^5+16\gamma^6+\gamma^8-\gamma^9)-4c(-128+128c_u+ \\ +192\gamma-128c_u\gamma+96\gamma^2-128c_u\gamma^2-256\gamma^3+160c_u\gamma^3+56\gamma^4+ \\ +92\gamma^5-60c_u\gamma^5-50\gamma^6+22c_u\gamma^6-2\gamma^7+2c_u\gamma^7+6\gamma^8-2c_u\gamma^8- \\ -\gamma^9+(2-\gamma)^2(16-32\gamma^2+16\gamma^4-4\gamma^5-3\gamma^6+\gamma^7)). \end{array}\right).$$

In order to show that W_r is positive in the whole interval, it is sufficient to check first, that it is a quadratic, continuous function of c, which implies that it is either a convex or a concave function of c; second, that W_r is positive at both extremes of the interval. When W_r is a concave function of c both points imply that it is positive in the whole interval. When W_r is convex, we have additionally to check that its first derivative is positive at the origin of the interval, which completes the proof.

Summing up, the threshold value c_{SW} that appears in Proposition 4.2 is given by $c_{SW} = c^+$.

6. References

Avenel, E. and Barlet, C., 2000, "Vertical foreclosure, technological choice and entry on the intermediate market", Journal of Economics and Management Strategy, Vol. 9, No. 2, 211-230.

Bolton, P. and Whinston, M. D., 1991, "The foreclosure effects of vertical mergers", Journal of Institutional and Theoretical Economics, Vol. 147, 207-226.

Bonanno, G. and Vickers, J. (1988) "Vertical Separation" Journal of Industrial Economics. Vol 36(3), 257-65

Bork, R. H., 1978, The Antitrust Paradox: a policy at war with itself, New York: Basic Books.

Chen, Y., 2001, "On vertical mergers and their competitive effects", RAND Journal of Economics, Vol. 32, No. 4, 667-685.

Choi, J. P. and Yi, S. S., 2000, "Vertical foreclosure with the choice of input specifications", RAND Journal of Economics Vol. 31, No 4, 717-743.

Church, J. and Gandal, N., 2000, "Systems competition, vertical merger and foreclosure", Journal of Economics and Management Strategy Vo 9, No 1, 25-51.

Faulí-Oller, R. and J. Sandonís, 2003, "To merge or to license: implications for competition policy", International Journal of Industrial Organization 21, 655-672.

Faulí-Oller, R. and J. Sandonís, 2002, "Welfare reducing licensing", Games and Economic Behavior, 41, 192-205.

Hart, O. and Tirole, J., 1990, "Vertical integration and market foreclosure", in Brooking Papers on Economic Activity: Microeconomics, M. Bradly and C. Whinston, eds., Washington: Brookings Institution, 205-276.

Ordover, J., Saloner, G. and Salop, S., 1990, "Equilibrium vertical foreclosure", American Economic Review, Vol. 80 (1), 127-142.

Posner, R. A., 1976, "Antitrust Law", Chicago: University of Chicago Press.

Rey, P. and J. Tirole, 1999, "A primer on foreclosure", forthcoming in Handbook of Industrial Organization, vol. III. New York. Elsevier- North-Holland.

Riordan, M., 1998, "Anticompetitive vertical integration by a dominant firm", American Economic Review 88, 1232-48.

Salinger, M., 1988, "Vertical mergers and market foreclosure", Quarterly Journal of Economics 77, 345-356.

Sandonís, J. and R. Faulí-Oller, 2002, "Merging to license: internal vs. external patentee", working paper, University of Alicante.

Singh, V. and Vives, X., 1984, “Price and quantity competition in a differentiated duopoly”, Rand Journal of Economics 15, 546-554.