

A discusión

MERGERS IN A PARTIALLY CARTELIZED MARKET*

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ABSTRACT

The paper studies a Partial Cartel model where only a subset of firms colludes. In this model, firms' ability to collude depends on the discount factor. In addition, as hardly any attention has been given by the literature to the case where mergers take place in a collusive framework, the purpose of this paper is to analyze the competitive effects of horizontal mergers on profits and welfare in a Partially Cartelized market. We show that both mergers among fringe and cartel firms increase market price. Regarding merger profitability, the discount factor decreases cartel members' merger profitability. However, the higher cartel members' discount factor, the more fringe firms will be willing to merge. An example of this could be the intense wave of mergers among oil firms that coincided with a large period of high oil prices caused by the OPEC production cuts.

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1 Introduction

Faced with the multiplicity of equilibria arising in repeated games¹, most applied papers have focused on studying the equilibrium where industry profits are maximized assuming that all firms participate in the collusive agreement. However, at least to the best of our knowledge, little attention has been paid to the situation where not all the firms of the industry join the cartel (an exception is Martin (1993) to be commented later on)².

In practice, there are many cases where collusive agreements do not involve all firms in the industry. However given that it is difficult to determine which firms really belong to the cartel, we will refer to tested cases previously investigated by antitrust authorities.

Looking at the most recent measures of the European Commission against collusion, we see that several cartels have been fined hundreds millions euros by the European Commission for price fixing and setting sales quotas. At the same time it is also verified that in many of these industries, not all firms in the industry were fined. For example, in the carbonless paper industry, the joint market share of the fined firms was between 85 and 90%. In the North Atlantic shipping industry, the market share of the cartel was calculated around 70-80%, or in the cartonboard industry where the market share of the cartel is assumed to be around 80%. Another significant example is the citric acid industry where three North-American and five European firms were convicted in United States, Canada and the European Union and fined more than one hundred million euros for fixing prices and allocating sales in the worldwide market, issuing coordinated price announcements and monitoring one another's prices and sales volumes during the period 1991-1995³. The joint market share of these eight firms was only between 50% and 60%

¹The "Folk Theorems" show that any individually rational payoff vector of a one-shot game of complete information can arise in a perfect equilibrium of the infinitely-repeated game if players are sufficiently patient (see for example Friedman (1971), Aumann, R, Shapley, L. (1976), Rubinstein, A. (1979) or Fudenberg et al. (1986)).

²There is a well-known literature where the industry structure is characterized by a small group of firms plus a competitive fringe. However this literature investigates cartel stability in static models where it is assumed that binding contracts could be signed. The seminal papers in this literature are Selten (1973) and d b4Aspremont et al. (1983), see also Shaffer (1995), Donsimoni (1986) or Thoron (1998) for example.

³U.S. Department of Justice, *Justice Department's Ongoing Probe Into Food and Feed Additives Yields Second Largest Fine Ever*, Press Release, January 29, 1997.

(see Levenstein, Suslow et al.(2002)).

Despite this empirical evidence, the Industrial Organization analysis of tacit collusion in quantity setting supergames, has generally focused on the *symmetric* subgame perfect equilibrium that maximizes industry profits. However, the continuum of equilibria in supergames allows us to select equilibria such that only a subset of firms colludes. Indeed, there is a seminal paper on partial cartels (Martin 1993 p.111) from where we get the inspiration. He considers that only a subset of firms belongs to the cartel and the rest are called fringe firms. Then, he studies conditions on the discount factor such that the outcome where cartel firms behave as a Stackelberg leader and fringe firms as followers can be sustained as an equilibrium of the repeated game using “trigger” and “stick and carrot” strategies. We extend Martin (1993) model by identifying the best equilibrium for cartel firms using “trigger” strategies for each discount factor. (This means that for any value of the discount factor of cartel firms, some degree of collusion is always achieved). It coincides with the Stackelberg-follower model for high discount factors and converges to the Cournot outcome when the discount factor tends to zero.

Although the framework with partial cartels we set can be used to study different subjects, we specialize the model to deal with the issue of mergers. Curiously, while there exists a substantial literature on the effect of mergers when pre- and post-merger behavior is noncooperative, little attention has been given by theorists to the consequences for firms of mergers when they take place in a collusive environment. In fact, despite its influence in matters of policy, there is almost no formal evidence from theory regarding the relationship between mergers and collusion⁴. Then, the purpose of this paper is to analyze the competitive effects of horizontal mergers on profits and welfare in a partial cartel model. In the sparse literature on the subject the effect of mergers is analyzed seeing which is the effect on the potential sustainability of the collusive agreements. This is reflected on the effect of mergers on the minimum discount factor required for collusion to be sustainable and it is commonly believed that mergers by reducing the number of competitors facilitate collusion. Due to the particular characteristics of our partial cartel

⁴One contribution of interest is that of Davidson and Deneckere (1984) where they do not allow for a redistribution of output among firms after the merger, so it is assumed that each merged firm conserves their pre-merger cartel quota after the merger. This makes that horizontal mergers may make more difficult for a cartel to maintain its integrity

model, the result of studying the effect of mergers on the threshold of the discount factor above which (full) collusion is sustained is ambiguous. In our case, when firms fail to sustain full collusion they can nevertheless achieve some degree of collusion. Thus, it would be more informative to study the effect of mergers on a variable that better reflects the degree of collusion achieved in the economy. It seems that price is the most indicated candidate.

Then, the first result obtained is that mergers of either cartel or fringe firms help collusion as long as they raise price. The second result obtained is that the capacity to achieve agreements, represented by the discount factor, crucially determines merger profitability. When mergers among cartel firms are considered, we obtain that the higher the discount factor, the lower merger profitability. The basic intuition is that, in the absence of cost savings derived from mergers, the main goal of merging is to reduce competition. However, if firms can achieve this goal by colluding since the discount factor is high enough, mergers are not profitable.

In contrast, as the discount factor increases, mergers among fringe firms become more profitable. In a linear oligopoly Cournot market, two firms never have an incentive to merge (see Salant et al., 1983), as non-merging firms react to the merger expanding its production. Our last result is explained by the fact that when the discount factor increases, cartel firms react expanding less their production in response to the merger of fringe firms.

We have an interesting example. Looking at what happened to the oil market in 1998-2000, we identify two different phenomena. On one hand after being elected in 1998, the Venezuelan leader Chávez worked with the OPEC president Mr. Alí Rodríguez to reinvigorate the cartel⁵. In June 1998, in its 105th meeting, the cartel agreed a cut of more than 2.5 million barrels a day. In March 1999, Mr. Rodriguez cut a deal with Saudi Arabia, the world's biggest producer, to reduce production by almost 2 million barrels a day and reverse the slide in prices. That move encouraged the rest of OPEC's 11 member nations to follow suit.

On the other hand, during this period, many oil firms merged. We have different

⁵"And now, OPEC has arisen again," President Chavez declared, adding that "its resurrection will rightly be celebrated in this land of the birth of OPEC" CNN, September 26, 2000.

examples: In 11/8/98, B.P. and Amoco. In 28/10/98, Japan Oil and Mitsubishi Oil. In 2/12/98, Exxon and Mobil. In 31/3/99, B.P. and Arco and in 14/9/99, TotalFina and Elf.

Therefore, at the end of the 90's, more or less simultaneously we could observe mergers among what we can consider fringe firms, and the success of the cartel. Our model might illustrate the basic economic intuition about the relationship between these two facts.

The structure of the paper is as follows. In the following section, the central model of the paper is set. In Section 3, the effect of mergers on the price is analyzed. In Section 4, merger profitability is considered. We conclude in Section 5. All proofs are relegated to the appendix.

2 Partial Cartels

Consider n firms, which produce the same homogenous good in the same market for infinite periods. Firms discount future at common and known factor of $\delta \in [0, 1)$. Suppose they make output decisions simultaneously at the beginning of each period.

The Stage Game description is the following: We assume that the industry inverse demand is piecewise linear:

$$p(Q) = \max(0, a - Q) \tag{1}$$

where Q is the industry output, and p is the price for the output and $a > 0$ is the demand parameter. Every firm has a constant marginal cost of production d , where $a > d$.

Let (K, F) be a partition of the player set. We assume that the subset K comprises k ($\leq n$) firms of the industry, while the remaining $(n - k)$ firms belong to the subset F . We will call hereafter firms that belong to the partition K , like cartel firms, and firms that belong to the partition F , like fringe firms.

Most of the literature has focused on studying the best (symmetric) equilibrium for all firms in the industry. We are going to study instead the best (symmetric) equilibrium for cartel firms, given that fringe firms maximize per period profits, that can be sustained as an equilibrium of the repeated game.

We are going to consider “trigger strategies”. The essence of these strategies is the following; firms join the cartel agreement given that all cartel members do so. In the event of deviation, the “loyal” members of the coalition revert forever to the static (Cournot) noncooperative equilibrium. This forces the deviant to do the same. The threat of such retaliation is a deterrent to deviation if the gain from cheating is no greater than the (discounted) per-period losses, which arise from the punishment.

Let $\sigma_{i,t}$ denote the action of player $i \in \{1, \dots, n\}$, at moment t , $t = 1, 2, \dots, \infty$. The “trigger strategies” of both partition sets of firms can be described in the following way. When the agreement is to produce q , the “trigger strategies” for cartel firms, $i \in K$, is given by:

$$\sigma_{i,1} = q \tag{2}$$

$$\sigma_{i,t} = \begin{cases} q & \text{if } \sigma_{j,h} = q \text{ for any } h < t \text{ for } j \in K, t \geq 2. \\ q_n & \text{Otherwise.} \end{cases} \tag{3}$$

For fringe firms, $i \in F$, we have:

$$\sigma_{i,1} = q^f \tag{4}$$

$$\sigma_{i,t} = \begin{cases} q^f & \text{if } \sigma_{j,h} = q \text{ for any } h < t \text{ for } j \in K, t \geq 2. \\ q_n & \text{Otherwise.} \end{cases} \tag{5}$$

We now look for the conditions on q and q^f that make these strategies conform a Subgame Perfect Nash Equilibrium of the repeated game. As far as fringe firms are concerned, we have that as the future play of their opponents is independent of how they play today, their optimal response is to play to maximize their current period’s payoff. Fringe firms do maximize per period profits if they choose:

$$q^f = \max\{0, (\frac{a - d - kq}{n - k + 1})\} \tag{6}$$

It is the Cournot equilibrium among fringe firms when the output of each cartel firm is given by q .

We proceed to look for the value of q such that, given (6), the strategies (2) and (4) are a Subgame Perfect Nash Equilibrium.

We should define first the profit functions for cartel firms:

Cartel firms profits when they play q and fringe firms play q^f are:

$$\Pi^c(q, q^f) = (a - d - kq - (n - k)q^f)q$$

Deviation profits that a cartel firm obtains when he unilaterally deviates are:

$$\Pi^d(q, q^f, q_i) = (a - d - (k - 1)q - q_i - (n - k)q^f)q_i$$

The quantity produced by the cheating cartel firm (q_i^d) is defined like follows:

$$q_i^d = \arg \max_{q_i} \Pi^d(q, q^f, q_i)$$

A trigger strategy supports noncooperative collusion if the present-discounted value of the income stream from adhering to the cartel is at least as great as the present-discounted value of the income stream from defection, which is deviating profits of one period, plus the profits of the punishment path, that is (discounted) static Nash equilibrium profits. For the case at hand, the condition for the cartel firms to be playing a Subgame Perfect Nash Equilibrium is the following:

$$\frac{\Pi^c(q, q^f)}{1 - \delta} \geq \Pi^d(q, q^f, q_i^d) + \frac{\delta \Pi^c(q^n, q^n)}{1 - \delta} \quad (7)$$

However, we have multiplicity of equilibria as for each δ , (7) holds for several cartel firms' production (q). To solve the multiplicity of equilibria we consider the allocation, which is the best for the cartel i.e. maximizes cartel firms' profits⁶. Therefore, it is the solution of the following program:

$$\max_q \Pi^c(q, q^f) \quad (8)$$

⁶This is also the widely extended way to solve the multiplicity of equilibria in infinitely repeated games with collusion at the industry level.

st.

$$\frac{\Pi^c(q, q^f)}{1 - \delta} \geq \Pi^d(q, q^f, q_i^d) + \frac{\delta \Pi^c(q^n, q^n)}{1 - \delta} \quad (9)$$

The unique maximizer of $\Pi^c(q, q^f)$ is denoted by \bar{q} . If $\delta \geq \bar{\delta}$, restriction (9) evaluated at \bar{q} is satisfied and, in those cases, \bar{q} is the solution to the whole program. For $\delta < \bar{\delta}$ the solution to the program is given by the output that satisfies (9) with equality and it is denoted by $q(\delta)$.

Thus, for cartel firms, the solution to the program is the following:

$$q^c = \begin{cases} q(\delta) & \text{if } \delta < \bar{\delta} \\ \bar{q} & \text{if } \delta \geq \bar{\delta} \end{cases}$$

The equilibrium strategies we are going to consider are (2) and (4) when:

$$q = q^c$$

$$q^f = \left(\frac{a - d - kq^c}{n - k + 1} \right)$$

Observe that as in (8) we chose the best equilibrium for the cartel, $k\bar{q}$ is the output of a unique Stackelberg leader. This is the production that cartel members want to achieve, because then their profits are maximized. It turns out that $k\bar{q} = \frac{a - d}{2}$ i.e. it does not depend on the number of firms in the fringe and it amounts to the monopoly output. Then the best for cartel firms is to sustain the monopoly output in the repeated game. It is only possible when $\delta \geq \bar{\delta}$ and then we say that full collusion is sustained. Otherwise, full collusion can not be obtained because firms discount the future too much. The question is how the cartel quota should be adjusted in those cases in order to achieve the maximal level of collusion. It turns out that it crucially depends on the number of firms in the cartel.

When most firms belong to the cartel ($k > \frac{n+1}{2}$), quotas should be adjusted upwards in order to reduce the incentives to deviate from the cartel agreement. The reason is that output from all firms except the deviator move in the same direction as the quotas. Then the residual demand left to the deviator is lower, and therefore it gains less by deviating. This adjustment should be greater the lower the discount factor, reaching the standard Cournot output when $\delta = 0$.

However, when most firms belong to the fringe ($k < \frac{n+1}{2}$), quotas must be adjusted downwards in order to reduce the incentives to deviate. The reason is that the output from all firms except the deviator move in the opposite direction than the quotas because fringe firms increase their output when the quota is reduced. And this effect now dominates because fringe firms are the majority. This adjustment should be greater the lower the discount factor, reaching again the standard Cournot output when $\delta = 0$.

Figures 1 and 2 represent the value of the quotas and the output of the fringe firms as a function of the discount factor for the two cases discussed above.

The evolution of individual outputs has a direct consequence on the evolution of price. It decreases with the discount factor when $k < \frac{n+1}{2}$ and it increases when $k > \frac{n+1}{2}$. As δ increases, cartel firms are closer to achieve their objective. The question is that it differs depending on whether they are a majority or a minority. In the former case, the objective is to reduce output in order to increase price given that they control most of the market. In the latter case, their objective is to increase market share although this leads to a price cut. In any case, profits of the cartel firms are increasing with the discount factor because as δ increases, cartel firms are closer to the joint profit maximizing outcome. However, fringe firms' profits are only increasing in δ whenever $k > \frac{n+1}{2}$. In this case, as δ increases, fringe firms expand their output in order to take advantage of the high price motivated by the output reduction agreed by cartel members. This leads fringe firms' profits to increase with δ .

An interesting comparison that could clarify how this partially cartelized market works would be to compare cartel and fringe firms profits. Thus we define the following profit functions, which are cartel and fringe firms' profits respectively when the structure of the industry is formed by n firms with k of them in the cartel:

Figure 1

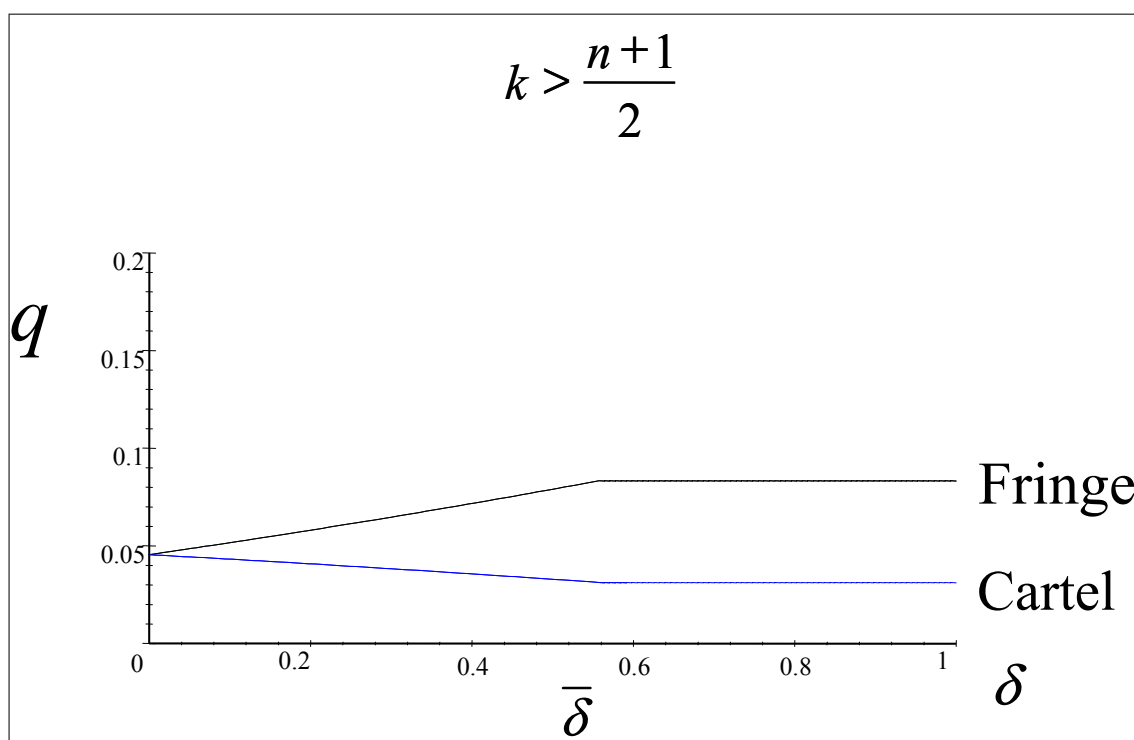
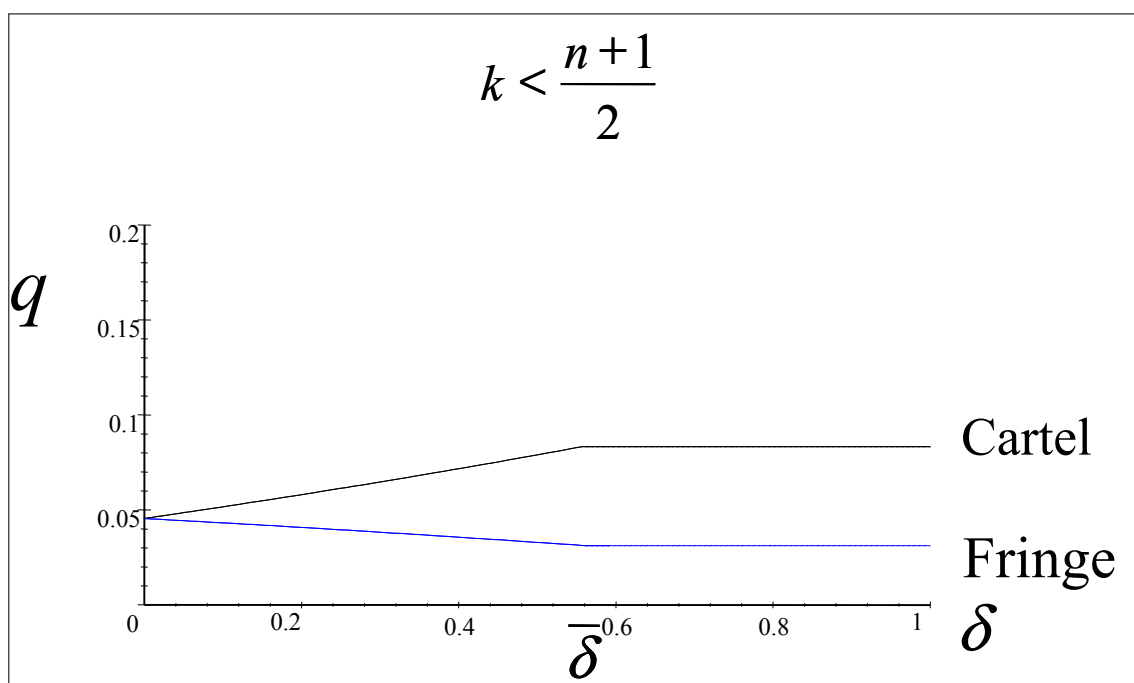


Figure 2



$$\Pi_{n,k}^c = (a - kq^c - (n - k)q^f)q^c - dq^c \quad (10)$$

$$\Pi_{n,k}^f = (a - kq^c - (n - k)q^f)q^f - dq^f \quad (11)$$

Comparing (10) and (11), we have that fringe firms obtain larger profits than cartel firms ($\Pi_{n,k}^f > \Pi_{n,k}^c$) whenever they are in a minority position ($k > \frac{n+1}{2}$). The reason can be obtained in Figure 1: fringe firms produce then more than cartel firms ($q^f > q^c$). If instead, fringe firms are in a majority position ($k < \frac{n+1}{2}$), fringe firms' profits are not only lower than cartel firms' profits but also lower than the standard Nash-Cournot equilibrium profits ($\Pi_{n,k}^f < \frac{(a-d)^2}{(1+n)^2}$).

We can see the output of fringe and cartel firms as a function of δ for a given industry size n and partition K .

3 The impact of mergers on collusion

Once we understand how this partially cartelized market works, we proceed to study how mergers among firms change the picture. There are two different types of firms in the model, the ones that belong to the cartel and the ones that belong to the fringe. We only consider mergers among firms of the same type and the merged firm has the same type as the merging partners. Furthermore, given the assumptions on costs, the merger does not bring any cost efficiency and then the merged entity is like any other independent firm of the same type. Therefore, starting from a situation with n firms and k cartel members, the merger of $m + 1$ cartel firms leads to a market with $n - m$ independent firms and $k - m$ cartel members. Similarly, the merger of $m + 1$ fringe firms turns the market into a situation with $n - k$ firms and k cartel members.

This Section is motivated by the fact that little attention has been given by theorists to the consequences of mergers when postmerger behavior is collusive. In the sparse literature on the subject the effect of mergers is analyzed seeing which is the effect on the potential sustainability of the collusive agreements. This is reflected on the effect of mergers on the minimum discount factor required for collusion to be sustainable⁷.

To focus the discussion we recall the result belonging to this literature most closely related to our setting. In the repeated symmetric Cournot game with linear demand and costs (Vives (1999)), the monopoly outcome can be sustained if $\delta \geq \frac{(n+1)^2}{(n+1)^2+4n} = \delta(n)$, where n is the number of firms. Then as $\frac{\partial \delta(n)}{\partial n} > 0$, we have that it is harder to collude with more firms. Then, mergers by reducing the number of competitors facilitate collusion⁸.

My model differs from those models in two accounts. On the one hand, I do not study

⁷Compte, Jenny and Rey (2002) study collusion in a setting with firms with asymmetric capacities. They show that the main problem for collusion is to prevent the largest firm from deviating. Then the effect of mergers is ambiguous whenever it involves the largest firm. On the one hand, it reduces the number of competitors what tends to hurt collusion. But on the other hand it increases the size of the largest firm, what increases its incentives to deviate. In their case, restricting attention to the sustainability of (full) collusion is validated by the fact that (full) collusion is sustainable whenever some collusion is sustainable.

⁸In the same setting, Davidson and Deneckere (1984) obtain the opposite result by assuming that firms are not allowed to redistribute output after a merger, which means that whenever the cartel is active, merged firms produce their pre-merger cartel quota. If $m + 1$ firms merge, then full collusion is obtained if $\delta \geq \delta(n, m) = \frac{(n-1)^2(n-m+2)^2}{(n-1)^2(n-m+2)^2+4n((n-m+2)^2-4n)}$, where $\frac{\partial \delta(n, m)}{\partial m} > 0$.

collusion at the industry level but the ability of a subset of firms to reach a collusive agreement (although the first case is obtained in the limit case when $k = n$). On the other hand, for any δ some degree of collusion is always achieved (outright competition is only obtained when $\delta = 0$).

When $\delta \geq \bar{\delta}(n, k)$ cartel firms can sustain full collusion i.e. their best preferred market outcome. But for $\delta < \bar{\delta}(n, k)$ profits of cartel firms are always greater than the ones obtained with competition as cartel firms achieve the maximum degree of collusion they are able given their discount factor.

Following the logic of the aforementioned models we could study the effect of mergers on $\bar{\delta}(n, k)$. Surprisingly, the result is ambiguous. Mergers can make either easier or more difficult for cartel firms to sustain full collusion.

On the one hand, as

$$\bar{\delta}(n, k) = \frac{(n+1)^2}{(n+1)^2 + 4k(n+1-k)}$$

the merger of $m+1$ cartel firms helps (full) collusion if

$$m < (n+1) \frac{2k - n - 1}{k} \quad (12)$$

because then $\bar{\delta}(n-m, k-m) < \bar{\delta}(n, k)$. It hinders (full) collusion if the inequality in (12) is reversed. On the other hand, the merger of $m+1$ fringe firms helps (full) collusion if:

$$m < (n+1) \frac{n+1-2k}{n-k+1} \quad (13)$$

because then $\bar{\delta}(n-m, k) < \bar{\delta}(n, k)$. It hinders (full) collusion if the inequality in (13) is reversed.

These disappointing results may not be so important in my case where firms when they fail to sustain full collusion can nevertheless achieve some degree of collusion. In my case, it would be more informative to study the effect of mergers on a variable that better reflects the degree of collusion achieved in the economy. It seems that price is the most indicated candidate for that purpose. And then, a robust conclusion is obtained: mergers always increase price. This result is developed in the following Propositions that deal respectively with the case of mergers of cartel firms and mergers of fringe firms. This result seems to invalidate the approach taken so far to focus on the ability of firms to achieve (full) collusion.

Proposition 1 *Any merger of cartel members increase price.*

The increase is always strict except for high values of the discount factor such that (full) collusion is sustained both pre-merger and post-merger. If $m + 1$ cartel firms merge, price does not change with the merger if

$$\delta \geq \max\{\bar{\delta}(n - m, k - m), \bar{\delta}(n, k)\}$$

Then both pre-merger and post-merger full collusion is sustained what means that the joint output of the firms in the cartel amounts to $\frac{a - d}{2}$. Then fringe firms do not change either their output and price is unaffected by the merger.

Observe that when $k = n$ the above Proposition encompasses the case previously analyzed in the literature where all firms participate in the collusive agreement. My model is richer though because, for any δ , collusion is exploited at its maximum level. Nevertheless my model confirms the results obtained in previous models that mergers help to sustain higher prices.

The most intriguing part of the previous Proposition is to understand why even when full collusion is more difficult to sustain after merger, price increases. We are considering a situation where

$$\bar{\delta}(n, k) < \bar{\delta}(n - m, k - m) \quad (14)$$

and then for $\delta \in (\bar{\delta}(n, k), \bar{\delta}(n - m, k - m))$ (full) collusion was sustainable before merger but it is not possible after the merger. (14) holds if and only if

$$m > (n + 1) \frac{2k - n - 1}{k} \quad (15)$$

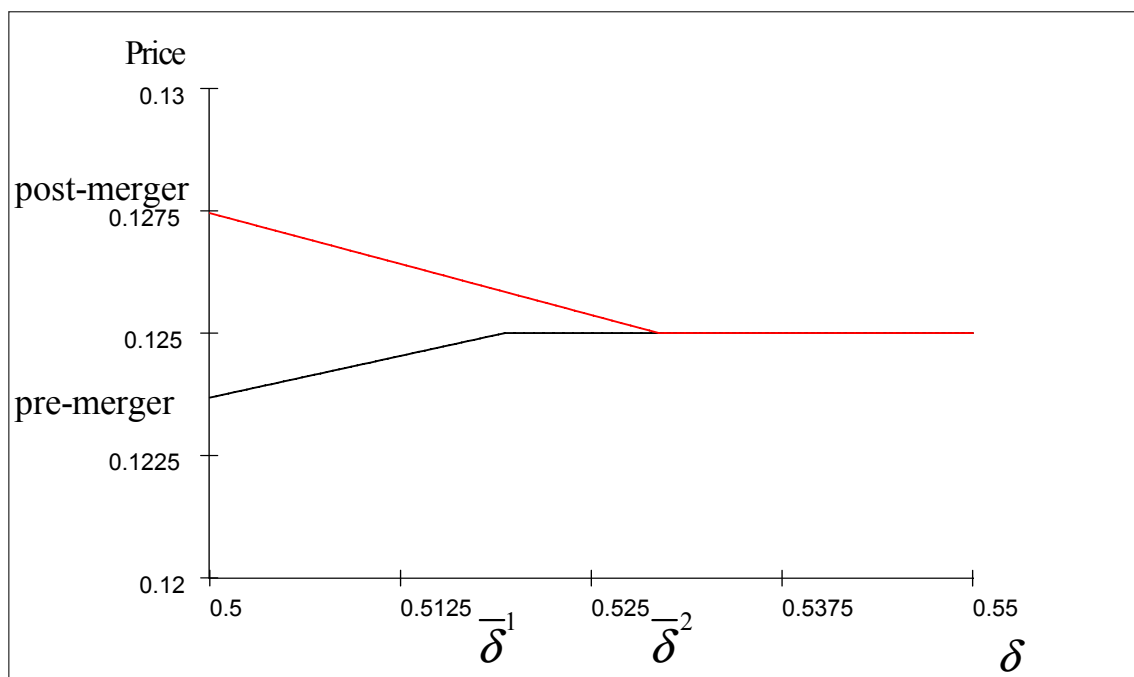
The key point to solve the puzzle is to realize that (15) implies $k - m < \frac{n - m + 1}{2}$ i.e. cartel firms are in a minority position after the merger. Then Figure 2 shows that when full collusion is not sustainable cartel firms adjust their quotas downwards what drives the price upwards. So the puzzle created because we may have simultaneously that full collusion becomes more difficult with the merger and price increases is explained by the anomalous feature of my model that price can decrease with the discount factor when cartels find themselves in a minority position.

The following picture illustrates the issue. It depicts the evolution of price pre-merger and post-merger for the specific case that $n = 10$, $k = 7$ and $m = 5$, with $a = 1$ and $d = 0$. Observe that in this case (15) holds.

Regarding mergers among fringe firms, the following proposition establishes its impact on price

Proposition 2 *Any merger of fringe firms strictly increases price.*

As opposed to what happened in the previous Proposition, in this case, price strictly increases even when the discount factor is such that (full) collusion is sustained both pre-merger and post-merger. We have just seen that, in this case, the joint output of the cartel firms does not change with the merger. However, as the merger reduces the number of fringe firms their production is lower after the merger what explains that price strictly increases.



4 The Effect of Collusion on Mergers

In the last Section, we studied the effect of mergers on price. In the present Section, we focus on the private incentives to merge. It is well-known that the price increase by itself does not guarantee that a merger is profitable. For example, in a Cournot setting (see the seminal paper Salant et al. (1983)), although mergers increase price, they are (generally) not profitable, because non merging firms react to the merger expanding their production⁹. To understand this, recall that the output sold in equilibrium in the standard linear Cournot model is given by: $q_i = \frac{a-c}{n+1}$. When n decreases (because of a merger), q_i increases, harming the newly merged firm, reducing incentives to merge.

If as in Salant et al. (1983). we refer to the subset of firms that do not participate in the proposed mergers as “outsiders”, we have that the present model (that encompasses the Cournot case when $\delta = 0$) shares the same characteristic that outsiders increase their market share and therefore merger profitability can not be taken for granted. In our model the outputs of cartel and fringe firms when $\delta < \bar{\delta}(n, k)$ are given respectively by:

$$q^c(n, k, \delta) = \frac{(a-d)((1+n)^2 + (1-2k+n)(3-2k+3n)\delta)}{(1+n)^3 - (1+n)(1-2k+n)^2\delta}$$

$$q^f(n, k, \delta) = \frac{(a-d)(1-n(2+n)(-1+\delta) - \delta + 4k^2\delta)}{(1+n)^3 - (1+n)(1-2k+n)^2\delta}.$$

Both the merger of cartel firms and the merger of fringe firms increase the individual output of nonmerging firms. For the merger of cartel firms the result follows from:

$$\frac{\partial q^c(n-m, k-m, \delta)}{\partial m} > 0, \text{ and } \frac{\partial q^f(n-m, k-m, \delta)}{\partial m} > 0$$

⁹Salant et al. (1983) shows that at least 80% of the firms must merge in order to make the merger profitable. See also Faulí-Oller (1997) or Hennessy (2000) for an extension to a wider range of demand functions.

and for the mergers of fringe firms from:

$$\frac{\partial q^c(n-m, k, \delta)}{\partial m} > 0, \text{ and } \frac{\partial q^f(n-m, k, \delta)}{\partial m} > 0.$$

The greater the reaction of outsiders to merger the lower its profitability. We check below that it crucially depends on the discount factor and therefore the discount factor will be a key determinant of merger profitability.

With these preliminaries at hand we are going to present the results on how merger profitability depends on the discount factor. First of all, I am going to define what I understand by merger profitability. Using the profit functions defined in (10 and 11), the profitability of a merger of $m + 1$ cartel firms is given by:

$$\Pi_{n-m, k-m}^c - (m + 1) \Pi_{n, k}^c \tag{16}$$

and the profitability of a merger of $m + 1$ fringe firms by:

$$\Pi_{n-m, k}^f - (m + 1) \Pi_{n, k}^f. \tag{17}$$

They are simply the difference between the profits obtained by merging firms after and before the merger. The following Propositions represent our main results. Proofs can be found in the appendix. We restrict attention to the case where full collusion is not sustained either before or after the merger.

Proposition 3 *Merger profitability among cartel firms decreases with the discount factor.*

Proposition 3 shows that the following condition holds:

$$\frac{\partial(\Pi_{n-m,k-m}^c - (m+1)\Pi_{n,k}^c)}{\partial\delta} < 0. \quad (18)$$

The intuition for the result is as follows. In this model, firm's ability to collude depends on the discount factor: the greater the discount factor the greater the scope for collusion. When mergers do not involve any cost saving, firms merge basically to restrict competition. However, when competition is already low because firms are sustaining collusive agreements, mergers lose attractiveness as an anticompetitive device. Thus, the more firms can collude, the less they are interested in merging.

We turn now our attention to the private incentives of fringe firms to merge. The following Proposition makes clear that they are closely related with the value of the discount factor that parametrizes the ability of cartel firms to reach collusive agreements. The key point of Proposition 5 is that it establishes a connection between the level of collusion achieved by cartel firms and the profitability of mergers among firms not belonging to the cartel. This connection will be used in the next Section to present an example where we simultaneously have that cartel firms reduce their production and that fringe firms find profitable to merge. This example will be related with the evolution of the oil market at the end of 90's that is characterized by the same two features, namely, the success of the cartel and a wave of mergers of firms not belonging to the cartel.

Proposition 4 *The profitability of mergers among fringe firms increases with the discount factor.*

Proposition 4 shows that the following condition holds:

$$\frac{\partial(\Pi_{n-m,k}^f - (m+1)\Pi_{n,k}^f)}{\partial\delta} > 0. \quad (19)$$

The key point to understand the result is that as δ increases, cartel firms increase less their output after a merger of fringe firms. In algebraic terms, this means that:

$$\frac{\partial^2 q^c(n, k, \delta)}{\partial n \partial \delta} > 0. \quad (20)$$

In this case, the negative effect on the profitability of mergers because outsiders increase their production is attenuated as an increase in the discount factor reduces the reaction of firms belonging to the cartel.

4.1 Example

We present below a numerical example that illustrates the implications of the last Proposition. In the first place, we show that given a market structure the profitability of the merger of fringe firms is positive for high values of the discount factors whereas it is negative for low values of the discount factor. In the second place, we show that the reason of this result is that when δ is high cartel firms increase their output after merger much less than when δ is low.

The exact specification of the example is the following. Assume market demand is given by $P = 1 - Q$ and the marginal cost of firms is $d = \frac{1}{2}$. We have $n = 6$ firms and $k = 4$ firms belong to the cartel. We are going to consider the effect of the merger of fringe firms for the case $\delta = 0.1$ and $\delta = 0.5$.

If $\delta = 0.1$ we have that without the merger each cartel firm produces $q^c = 0.07$ and with the merger $q^c = 0.08$. The profitability of the merger is given by $\Pi_{6-1,4}^f - 2\Pi_{6,4}^f = -2.6338 \times 10^{-3} < 0$.

If $\delta = 0.5$ we have that without the merger each cartel firm produces $q^c = 0.0625$ and with the merger $q^c = 0.063$. The profitability of the merger is given by $\Pi_{6-1,4}^f - 2\Pi_{6,4}^f = 1.1703 \times 10^{-3} > 0$ (Results are summarized on the table).

$$\delta = 0.1 \implies \begin{cases} q^c = 0.07 & \text{Without merger.} \\ q^c = 0.08 & \text{With merger.} \\ \Pi_{6-1,4}^f - 2\Pi_{6,4}^f = -2.6338 \times 10^{-3} < 0 \end{cases} \quad (21)$$

$$\delta = 0.5 \implies \begin{cases} q^c = 0.0625 & \text{Without merger.} \\ q^c = 0.063 & \text{With merger.} \\ \Pi_{6-1,4}^f - 2\Pi_{6,4}^f = 1.1703 \times 10^{-3} > 0 \end{cases}$$

Table 1. Numerical Example.

The merger is only profitable when δ is high. Cartel firms increase their output with the merger by 0.01 if $\delta = 0.1$ and by 0.0005 if $\delta = 0.5$. This explains the different results on profitability.

The example shows that if for some reason unexpectedly the discount factor jumped from $\delta = 0.1$ to $\delta = 0.5$ we would observe the following adjustment in the strategies played by firms: on the one hand, fringe firms would decide to merge and, on the other hand, cartel firms would reduce their output from 0.07 to 0.063. This situation resembles what happened in the oil market at the end of the 90's: OPEC members agreed to cut their quotas and private firms not belonging to the cartel decided to merge.

5 Conclusion

The main aim of this paper has been to develop a theoretical foundation of the impact of collusion on merger profitability and the effect of horizontal mergers on a previously collusive market.

We show that although the critical discount factor above which joint profit maximization could be sustained may increase (due to a merger), the effect of a merger on price is unambiguous, and price increases.

There exists a traditional view in Industrial Organization, following Salant al. (1983), according to which there is little scope for merger profitability when mergers do not involve any cost saving and firms are in a Cournot environment. Our partial cartel model predicts a positive (negative) correlation between the degree of collusion among cartel firms and merger profitability of fringe (cartel) firms. This can reinforce the tendency of some groups of firms to merge in a non-competitive environment.

In a simple example, we have shown that some mergers are only profitable whenever the degree of collusion in the market is large enough.

Hence, the model predicts that in a Partial Cartel model, fringe firms would have an incentive to merge whenever the cartel is successful enough. Therefore, it should be taken into account that collusion may not only directly increase price but also indirectly by rendering profitable mergers among fringe firms. The closest real example to our model comes from the oil market. In the oil market at the end of the 90b4s, more or less simultaneously, mergers among, what we considered fringe firms, and production cuts by the cartel, the OPEC, took place. That could perhaps explain the wave of mergers among oil firms at the end of the 90b4s.

Several issues have been left for future research. It could be fruitful to consider different types of mergers, like in Huck et al.(2001), considering mergers among firms of different types.(i.e. mergers within fringe and cartel firms). Second, as we let cartel firms pick up the best equilibrium for them, it could be also considered other punishment phases that could lead the cartel to better outcomes than using trigger strategies like for example using an Optimal punishment, following Abreu (1986) and Abreu (1988).

6 Appendix

Proof of Proposition 1: The market price is given by the following expression:

$$p(n, k, \delta) = \begin{cases} f(n, k, \delta) = \frac{-(1+n)^2(a+dn)+(1-2k+n)(a(1+2k+n)+d(n+n^2-2k(2+n)))\delta}{-(1+n)^3+(1+n)(1-2k+n)^2\delta} & \text{if } \delta < \bar{\delta} \\ \frac{a+d(1+2(n-k))}{2(n-k+1)} & \text{if } \delta \geq \bar{\delta} \end{cases} \quad (22)$$

Observe that

$$p(n, k, \bar{\delta}) = f(n, k, \bar{\delta})$$

Define $\bar{\delta}^1$ and $\bar{\delta}^2$ as the pre-merger and post merger cutoffs respectively.

First we will prove that the following is true:

$$f(n-m, k-m, \delta) > f(n, k, \delta) \text{ if } \delta < \min\{\bar{\delta}^1, \bar{\delta}^2\} \quad (23)$$

First, we see that $f(n-m, k-m, 0) > f(n, k, 0)$. On the other hand, we can see that there exists only one $\delta \in (0, 1)$ that solves $f(n-m, k-m, \delta) = f(n, k, \delta)$. We will call it δ^0 .

We can check that if $\bar{\delta}^1 < \bar{\delta}^2$, then $\delta^0 > \bar{\delta}^1$, but if $\bar{\delta}^1 > \bar{\delta}^2$, then $\delta^0 > \bar{\delta}^2$. If $\bar{\delta}^1 = \bar{\delta}^2$, then $\delta^0 = \bar{\delta}^2 = \bar{\delta}^1$. Therefore, (23) is true.

If $\delta < \min\{\bar{\delta}^1, \bar{\delta}^2\}$, (23) ensures that a merger (strictly) increases market price.

If $\delta \geq \max\{\bar{\delta}^1, \bar{\delta}^2\}$ the market price pre and post merger is the same, as $\frac{a+d(1+2(n-k))}{2(n-k+1)} = \frac{a+d(1+2(n-m-(k-m)))}{2(n-m-(k-m)+1)}$.

For the remaining values of δ , we have two different relevant cases to consider: the first one is when $m > (n+1)\frac{2k-n-1}{k}$ and $\bar{\delta}^1 < \bar{\delta}^2$. The second case is when $m < (n+1)\frac{2k-n-1}{k}$ and $\bar{\delta}^1 > \bar{\delta}^2$.

We know that $p(n, k, \bar{\delta}^1) = p(n-m, k-m, \bar{\delta}^2) = \frac{a+d(1+2(n-k))}{2(n-k+1)}$.

In the first case, it is enough to check that $f(n-m, k-m, \delta)$ is strictly decreasing with δ . This is true if $m > 2k-n-1$, and this holds as $m > (n+1)\frac{2k-n-1}{k}$.

In the second case it is enough to check that $f(n, k, \delta)$ is strictly increasing with δ , and this is true if $2k-n-1 > 0$, and this holds as otherwise $m < (n+1)\frac{2k-n-1}{k}$ could never hold. ■

Proof of Proposition 2: Market price is given by (22). We can easily see that the following holds:

$$\frac{\partial f(n, k, \delta)}{\partial n} < 0 \quad (24)$$

Define $\bar{\delta}^1$ and $\bar{\delta}^2$ like pre-merger and post-merger cutoff respectively

If $\delta < \min\{\bar{\delta}^1, \bar{\delta}^2\}$, (24) is enough to prove that price strictly increases.

If $\delta \geq \max\{\bar{\delta}^1, \bar{\delta}^2\}$, we can see that as $p(n, k, \bar{\delta}^1) = \frac{a+d(1+2(n-k))}{2(n-k+1)} < p(n-m, k, \bar{\delta}^2) = \frac{a+d(1+2(n-m-k))}{2(n-m-k+1)}$, price strictly increases.

For the remaining cases, when, as we have seen, we have two relevant cases: the first is if $m > (n+1)\frac{n+1-2k}{n-k+1}$, then we have then that $\bar{\delta}^1 < \bar{\delta}^2$. The second case is if $m < (n+1)\frac{n+1-2k}{n-k+1}$, then we have then that $\bar{\delta}^1 > \bar{\delta}^2$.

In the first case, as $p(n-m, k, \bar{\delta}^1) > p(n, k, \bar{\delta}^1)$, it is enough to check that $f(n-m, k, \delta)$ is increasing with δ , which is true if $m > n-2k+1$, and this holds as $m > (n+1)\frac{n+1-2k}{n-k+1}$.

In the second case, as $p(n-m, k, \bar{\delta}^2) > p(n, k, \bar{\delta}^2)$ (remember 24), it is enough to check that $f(n, k, \delta)$ is decreasing with δ , which is true if $n+1 > 2k$, and this holds as otherwise $m < (n+1)\frac{n+1-2k}{n-k+1}$ could never be true. ■

Proof of Proposition 3: Basically what we have to do is proving that (18) holds. If we consider the following expression:

$\Pi_{n,k}^c = \frac{(a-d)^2(1-n(2+n)(-1+\delta)-\delta+4k^2\delta)((1+n)^2+(1-2k+n)(3-2k+3n)\delta)}{(1+n)^6-2(1+n)^4(1-2k+n)^2\delta+(1+n)^2(1-2k+n)^4\delta^2}$. Therefore, it is tedious but straightforward to show that (18) holds. ■

Proof of Proposition 4: As in the last Proof, we just have to take the following expression:

$\Pi_{n,k}^f = \frac{(a-d)^2(-1+n(2+n)(-1+\delta)+\delta-4k^2\delta)^2}{(1+n)^6-2(1+n)^4(1-2k+n)^2\delta+(1+n)^2(1-2k+n)^4\delta^2}$. We can also see that it is easy but tedious to see that (19) holds. ■

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