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HETEROGENEITY IN THE DEGREE OF QUASI-GEOMETRIC DISCOUNTING: THE DISTRIBUTIONAL IMPLICATIONS*

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ABSTRACT

This paper modifies the standard one-sector growth model with uninsurable idiosyncratic risk and liquidity constraints to include multiple types of quasi-geometric consumers. For a calibrated version of the model, we show that a modest difference between the quasi-geometric discounting parameters of types can lead to large differences in their marginal propensities to consume. Unlike the standard one-sector growth model, the model with heterogeneous quasi-geometric consumers can generate realistic degrees of wealth inequality.

Keywords: Time inconsistency, Quasi-geometric discounting, Hyperbolic discounting, Idiosyncratic shocks, Wealth inequality.

1 Introduction

Many recent studies deviate from the assumption of a constant discount factor emphasizing the importance of short-run urges to consume (to save) for the individual consumption-savings decisions. Moreover, it is argued in the literature that the real world consumers are heterogeneous in the degrees of the short-run patience. Specifically, Krusell, Kuruşçu and Smith (2002b) use Gul and Pesendorfer's (2001) framework to construct an asset-pricing model with two types of agents differing in short-run patience and show that such heterogeneity can help to explain the U.S. asset-pricing data. Collado, Maliar and Maliar (2003) use Spanish household data to estimate the Euler equation derived from Harris and Laibson's (2001) model with quasi-geometric discounting and find that the degrees of short-run patience significantly differ across consumers.[1]

In this paper, we investigate how heterogeneity in the degree of short-run patience can affect the distributions of income and wealth across agents.

[1] The concept of quasi-geometric (hyperbolic) discounting was developed by Strotz (1955-1956), Pollak (1968), and Phelps and Pollak (1968). The recent literature on quasi-geometric discounting includes, e.g., Laibson (1997), Laibson, Repetto and Tobacman (1998), Hall (1998), Barro (1999), Harris and Laibson (2001), Krusell and Smith (2000, 2003), Angeletos, Laibson, Repetto, Tobacman and Weinberg (2001), Krusell, Kuruşçu and Smith (2002a), Collado, Maliar and Maliar (2003), Maliar and Maliar (2003a, 2003b).

Our analysis is carried out in the context of a general equilibrium variant of Harris and Laibson's (2001) model augmented to include multiple types of quasi-geometric consumers. The types differ in the dimension of the short-run discount factor (one used between today and tomorrow). The long-run discount factor (one used between any two adjacent periods other than today and tomorrow) is identical for all consumers.[2] We refer to a consumer whose short-run discount factor is lower (higher) than the long-run discount factor as a short-run impatient (short-run patient) one.[3]

Our study is motivated by the fact that the standard one-sector growth model with uninsurable idiosyncratic risk and liquidity constraints fails to explain the inequality of the income and wealth distributions existing in actual economies. To be precise, such a model severely overpredicts income and wealth held by the poor and underpredicts income and wealth held by the rich (see Aiyagari, 1994, and Quadrini and Ríos-Rull, 1997). As argued in Krusell and Smith (1998) and Carroll (2000), the underling problem is that the model produces too small variations in the Marginal Propensities to Consume (MPCs) across agents. These studies propose one way to magnify

[2] Thus, a consumer discounts all future utilities, except of tomorrow's utility, according to geometrically declining sequence of weights. Krusell and Smith (2000) were first to call such discounting quasi-geometric.

[3] The assumption of short-run impatient consumers is advocated by Laibson (1997). Hall (1998) argues that short-run patience is also empirically plausible.

differences in the MPCs across agents, which is to assume heterogeneity in the (long-run) discount factors.

In this paper, we show that the introduction of heterogeneity in the degree of short-run patience is another way of generating large differences in the MPCs across agents. The mechanism here is as follows: Quasi-geometric discounting leads to time-inconsistency in preferences. When making plans about consumption in a distant future, consumers discount the utilities in any two adjacent periods by using the long-run discount factor. However, as time to fulfill the consumption plan arrives, the preferences change: now, the tomorrow's utility is discounted by using the short-run discount factor. The preference reversal makes the agents to deviate from the original consumption plans: the short-run impatient agents consume more and the short-run patient agents consume less than they would have committed to in the previous period if commitment had been available. The consequence is that the MPCs of short-run patient consumers tends to be lower than the MPCs of short-run impatient ones.

We analyze the quantitative implications of the calibrated two-type version of the model. We find that a relatively small difference between the degrees of quasi-geometric discounting of the types can lead to large differences in their consumption-savings decisions. For example, under the baseline parametrization, the MPC of a short-run patient consumer is on average almost 10 times lower than the MPC of a short-run impatient one. Given this foregoing result, it is no surprise that our model predicts a sharp polarization of the population: short-run impatient agents get very poor, whereas short-run patient agents get very rich. We find that if the population is composed of few short-run patient and many short-run impatient agents, then the degrees of wealth inequality in our model are comparable to those in the data. Also, our model generates more inequality in income than the standard one-sector growth model does, however, the improvement is not sufficient to account for the data.

The rest of the paper is organized as follows. Section 2 formulates the model. Section 3 describes the calibration and solution procedures and discusses the results, and finally, Section 4 concludes.

2 The model

We extend Aiyagari's (1994) model to include ex-ante heterogeneous quasi-geometric consumers. Time is taken in discrete intervals, $t = 0, 1, 2, ...$ The economy is populated by $H \geq 1$ types of infinitely-lived agents indexed by $h = 1, 2, ..., H$. Within a type h, there is a continuum of agents with names on a closed interval $[0, \lambda_h]$, where $\sum_{h=1}^{H} \lambda_h = 1$. The parameter $\lambda_h > 0$ reflects the relative size of type h. Heterogeneity across types is in the dimension of the discounting parameter β_h. In period t, an agent puts the weight 1 on the utility of period t and the weight $\beta_h \delta^{\tau+1-t}$ on the utility of period $\tau > t$, where the discounting parameters β_h and δ are such that $\beta_h > 0$ and $0 < \delta < 1$. Agents are subject to idiosyncratic labor productivity shocks. The process for shocks is a first-order Markov one; it is identical for all agents and uncorrelated across agents.

On each date t, an agent of type h solves the following problem

$$\max_{\{c_{h,\tau}, a_{h,\tau+1}\}_{\tau=t}^{\infty}} \left\{ u\left(c_{h,t}\right) + E_t \sum_{\tau=t}^{\infty} \beta_h \delta^{\tau+1-t} u\left(c_{h,\tau+1}\right) \right\} \tag{1}$$

subject to

$$c_{h,\tau} + a_{h,\tau+1} = w s_{h,\tau} + (1+r)\, a_{h,\tau}, \tag{2}$$

$$a_{h,\tau+1} \geq -b, \tag{3}$$

$a_{h,\tau} \in \mathcal{A}$ and $s_{h,\tau} \in \mathcal{S}$ are given, where $\mathcal{A} = [-b, \infty) \subset R$ and $\mathcal{S} = [s_{\min}, s_{\max}] \subset R_+$. Here, E_t denotes the expectation, conditional on all information available at t; $c_{h,\tau}$, $a_{h,\tau}$ and $s_{h,\tau}$ are consumption, asset holdings and idiosyncratic shock to labor productivity, respectively; r is the interest rate and w is the wage per unit of efficiency labor; b is the borrowing limit. The momentary utility function $u(c)$ is continuously differentiable, strictly increasing, strictly concave and satisfies $\lim_{c \to 0} u'(c) = \infty$.

The assumption of $\beta_h \neq 1$ generates time-inconsistent choices. If $\beta_h < 1$, the short-run discount factor (the one between the periods t and $t+1$), $\beta_h \delta$, is smaller than the long-run discount factor (the one between any two adjacent periods further in the future), δ. As a result, an agent systematically undersaves relative to what she would have committed to in the past if commitment had been available. If $\beta_h > 1$, then the opposite is true: an agent saves more than she would have committed to in the past. If $\beta_h = 1$, then the preferences are time-consistent: a choice perceived to be optimal in the past remains to be optimal in all subsequent periods.

Output is produced according to a Cobb-Douglas production function, $K_t^{\alpha} N_t^{1-\alpha}$, with $\alpha \in [0, 1]$, where K_t and N_t are the aggregate capital and labor inputs, respectively. The depreciation rate of capital is $d \in (0, 1]$. Therefore, the production technology is given by $K_t^{\alpha} N_t^{1-\alpha} + (1 - d) K_t$.

2.1 Equilibrium

Harris and Laibson (2001) show that the problem $(1) - (3)$ without commitment can be represented recursively. In our economy, an agent of type h with current state (a_h, s_h) solves:

$$W_h(a_h, s_h) = \max_{c_h} \{u(c_h) + \beta_h \delta E[V_h(a'_h, s'_h) \mid s_h]\}, \tag{4}$$

where given a_h, s_h, the value function $V_h(a_h, s_h)$ solves the functional equation

$$V_h(a_h, s_h) = u[C_h(a_h, s_h)] + \delta E\{V[ws_h + (1+r)a_h - C_h(a_h, s_h); s'_h] \mid s_h\}, \tag{5}$$

subject to the budget constraint

$$a'_h = ws_h + (1+r)a_h - c_h, \tag{6}$$

and the borrowing constraint

$$a'_h \geq -b. \tag{7}$$

Here, primes on variables indicate their values one period from now.

We restrict attention to an interior solution. If such a solution exists, the optimal choice of an agent of type h must satisfy the quasi-geometric Euler equation

$$u'(c_h) \geq \delta E\left\{u'(c'_h)\left[1 + r - (1-\beta_h) \cdot \frac{\partial C_h(a'_h, s'_h)}{\partial a'_h}\right]\right\}, \tag{8}$$

where $C_h(a'_h, s'_h)$ is the optimal consumption function. If the borrowing constraint is not binding, i.e., $a'_h > -b$, then (8) holds with equality.

Let $P_h(a_h, s_h, B)$ be the conditional probability that an agent of type h with state (a_h, s_h) will have a state lying in set $B \in \mathcal{B}$ in the next period

$$P_h(a_h, s_h, B) = Prob\left(\left\{s'_h \in \mathcal{S} : \left[A_h(a_h, s_h), s_h\right] \in B\right\} \mid s_h\right),$$

where $\mathcal{B}$ denotes the Borel subset of the set of all possible individual states $\mathcal{A} \times \mathcal{S}$, and $A_h(a_h, s_h) \equiv a'_h = ws_h + (1+r)a_h - C_h(a_h, s_h)$ is the decision function for assets (the asset function).

Definition. An equilibrium is defined as a set of stationary probability measures $\{x_h\}_{h\in H}$, a set of optimal consumption functions $\{C_h(a_h, s_h)\}_{h\in H}$ and four positive real numbers (K, N, r, w) such that

(1) $x_h = \int_{\mathcal{A}\times\mathcal{S}} P_h(a_h, s_h, B)\, dx_h$ for all $B \in \mathcal{B}$ and $h \in H$;

(2) $C_h(a_h, s_h)$ solves (2), (3), (8) for all $h \in H$;

(3) $K = \sum_{h=1}^{H} \lambda_h \int_{\mathcal{A}\times\mathcal{S}} A_h(a_h, s_h)\, dx_h$ and $N = 1$;[4]

(4) r and w are equal to the corresponding marginal products

$$r = \alpha K^{\alpha-1} - d, \qquad w = (1-\alpha) K^{\alpha}.$$

Thus, in the economy studied, the aggregate quantities and prices are constant even though the individual quantities vary stochastically.

[4] Given that there is a continuum of agents of each type h, the mass of agents with shocks $s_{h,t} = s$ and $s_{h,t+1} = s'$ is equal to the conditional probability $Prob(s_{h,t+1} = s' \mid s_{h,t} = s)$. The process for labor productivity shocks is stationary and, therefore, such probability is the same in all periods. Hence, $N_t = \sum_{h=1}^{H} \lambda_h \int_{\mathcal{A}\times\mathcal{S}} s_{h,t} dx_h$ is constant; for convenience, we normalize it to unity, $N_t = 1$ for all t.

3 Quantitative analysis

In this section, we study the implications of the model by simulation. First, we describe the calibration procedure, secondly, we discuss some computational issues and finally, we present the results.

3.1 Calibration

We calibrate the model following Aiyagari (1994). The model's period is one year. We assume $\delta = 0.96$, $\alpha = 0.36$ and $d = 0.08$. We set the borrowing limit at $b = 0$. The momentary utility function is $u(c) = \log(c)$. The process for idiosyncratic shocks is $AR(1)$,

$$\log s_{h,t+1} = \rho \log s_{h,t} + \sigma \left(1 - \rho^2\right)^{1/2} \varepsilon_{h,t+1}, \qquad \varepsilon_{h,t+1} \sim N(0,1),$$

where $\rho \in [0,1]$ is the autocorrelation coefficient, and $\sigma \geq 0$ is the unconditional standard deviation of the variable $\log s_{h,t}$. Our baseline parameterization is $\rho = 0.6$ and $\sigma = 0.2$. We also study the case $\rho = 0.9$ and $\sigma = 0.4$.

Regarding the discounting parameter β_h, we consider several alternatives. The Benchmark Model (BM) is the one studied in Aiyagari (1994): all consumers have identical time-consistent preferences, $H = 1$ and $\beta_1 = 1$. We then consider two homogeneous agents economies with time inconsistent preferences: one, populated by short-run patient consumers, $H = 1$ and $\beta_1 = 1.2$, and another, populated by short-run impatient consumers, $H = 1$ and $\beta_1 = 0.8$. We finally analyze the economies populated by two types of consumers, $H = 2$, such that $\beta_1 = 0.8$ and $\beta_2 = 1.2$.[5] The shares of the first and second types are λ and $1 - \lambda$, respectively; we consider $\lambda \in \{0.25,\ 0.5,\ 0.75\}$.

3.2 Solution algorithm

Krusell and Smith (2000) study the standard infinite-horizon deterministic neoclassical growth model with quasi-geometric discounting and find that value iterative methods produce cycles. Krusell and Smith (2000, 2003) show that the computational problems are related to the indeterminacy of equilibrium: in addition to a smooth interior solution, the model has a continuum of step-function equilibria. Krusell et al. (2002a) argue that one can work around the indeterminacy by restricting attention to the interior solution, which is a unique limit of the finite-horizon equilibrium. Nonetheless, the indeterminacy in the infinite-horizon model tends to make the computation of equilibria difficult since many algorithms are "attracted" to other equi-

[5] Laibson et al. (1998) argue that empirically plausible value of β_h for the short-run impatient consumers will be around 0.6. Given this estimate, the assumed difference in short-run patience between the two types seems to be modest.

libria besides the limit of the finite-horizon equilibrium. To deal with this problem, Krusell et al. (2002a) develop a perturbation method, which can systematically pick out the limit of the finite-horizon equilibrium.

In this paper, we solve the model by a parameterized expectations algorithm implemented on a grid of prespecified points. (The description of the algorithm is provided in Maliar and Maliar, 2003b). Maliar and Maliar (2003a) study the convergence properties of this algorithm in the context of the one-agent neoclassical growth model with quasi-geometric discounting and show that it yields the same solutions as those obtained by the perturbation method proposed by Krusell et al. (2002a). The drawback of our algorithm is that it works not for all parameters' values: it converges only if β_h is not very different from one (specifically, $\beta_h \in [0.8, 1.2]$) and if the grid is not too fine.

3.3 Results

Before presenting the results, we shall discuss how the degree of the agent's short-run patience affects her consumption-savings decisions. As can be seen from the Euler equation (8), the future rate of return on assets, $R_h\left(a_h', s_h'\right)$, from the perspectives of an agent of type h is

$$R_h(a'_h, s'_h) \equiv r - (1 - \beta_h) \cdot \frac{\partial C(a'_h, s'_h)}{\partial a'_h}.$$

In all our simulations, the Marginal Propensity to Consume (MPC) out of assets was strictly positive, $\frac{\partial C(a'_h, s'_h)}{\partial a'_h} > 0$ for all a'_h, s'_h. Thus, if $\beta_h > 1$ ($\beta_h < 1$), the individual subjective rate of return on assets, $R_h(a'_h, s'_h)$, is higher (lower) than the actual one, r, which induces the agent to save more (less) relative to the case $\beta_h = 1$. Moreover, if the consumption function is strictly concave, which was the case in our simulations, then $R_h(a'_h, s'_h)$ is strictly decreasing (increasing) in the level of wealth under $\beta_h > 1$ ($\beta_h < 1$). This implies that if $\beta_h > 1$ ($\beta_h < 1$), the rich have a lower (higher) savings rate than the poor.[6]

To appreciate the quantitative expression of the effects associated with quasi-geometric discounting, in *Table* 1, we provide the Gini coefficient of the wealth distribution and the percentages of wealth held by different groups of the population in the artificial and the U. S. economies.

We first consider the economy populated by one type of agents, $H = 1$, under $\beta_1 = 0.8$ $(\lambda = 1)$, $\beta_1 = 1.0$ (BM) and $\beta_1 = 1.2$ $(\lambda = 0)$. The main thing to notice here is that with one type of agents, the wealth is much

[6] See Maliar and Maliar (2003b) for a discussion.

Table 1. **Selected statistics of the wealth distribution in the U.S. and artificial economies.**

Model		*MPC*[1]	*MPC*[2]	*r, %*	*Gini*	*0-40%*	*80-100%*	*90-95%*	*95-99%*	*99-100%*
ρ =0.6	BM	.0523 (.0192)	-	3.98	0.33	17.0	38.6	9.5	9.5	3.1
σ=0.2	λ=0	.0493 (.0080)	-	3.09	0.23	22.8	31.2	8.1	7.5	2.3
	λ=1	.0576 (.0368)	-	5.05	0.39	12.0	42.2	10.4	10.4	3.5
	λ=.25	.3695 (.1561)	.0450 (.0066)	3.20	0.42	8.9	39.6	10.1	9.4	2.9
	λ=.50	.3563 (.1567)	.0402 (.0040)	3.31	0.60	0.2	53.3	13.7	13.1	4.1
	λ=.75	.3436 (.1573)	.0362 (.0027)	3.42	0.78	0	86.5	22.7	22.4	7.1
ρ =0.9	BM	.0716 (.0344)	-	3.33	0.45	10.4	46.6	11.7	12.0	4.1
σ=0.4	λ=0	.0621 (.0223)	-	2.38	0.41	13.3	43.2	11.0	11.0	3.6
	λ=1	.0820 (.0576)	-	4.40	0.47	9.7	48.6	12.0	12.6	4.4
	λ=.25	.2689 (.1456)	.0556 (.0175)	2.59	0.51	6.4	50.1	12.7	12.8	4.3
	λ=.50	.2337 (.1376)	.0486 (.0132)	2.85	0.62	1.5	60.7	15.4	15.8	5.4
	λ=.75	.2137 (.1305)	.0444 (.0127)	3.01	0.73	0.8	77.8	21.0	23.5	8.3
U.S. [(a)]					0.76	2.2	77.1	12.6	23.1	28.2

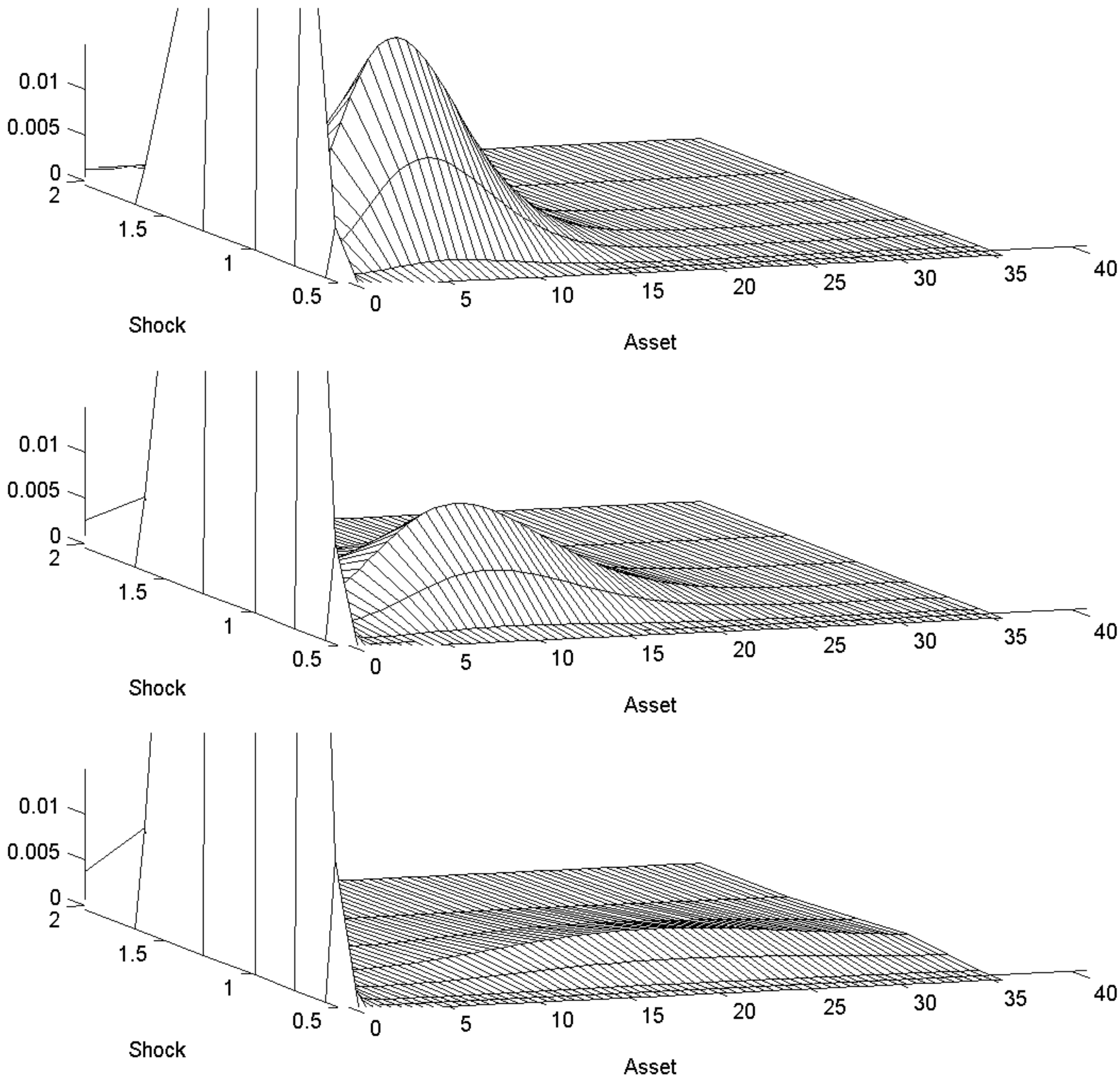
[(a)] *Source: Quadrini and Ríos-Rule (1997)* more equally distributed in the model than in the data. A lower degree of quasi-geometric discounting, β_1, leads to a larger dispersion of wealth across agents. This effect is modest, however.[7]

We now turn to the economy with two types of agents, $H = 2$. In *Figure* 1, we plot the stationary probability distributions of shocks and assets for the economies with $\lambda = 0.25$, $\lambda = 0.5$ and $\lambda = 0.75$ (we again assume $\rho = 0.6$ and $\sigma = 0.2$). As we can see, the probability distribution is two-peaked. Agents who are short-run patient ($\beta_2 = 1.2$) are distributed around the high-mean peak whereas those who are short-run impatient ($\beta_1 = 0.8$) are concentrated tightly around zero. Thus, most of the short-run impatient agents are liquidity-constrained.

The results in *Table* 1 demonstrate that the introduction of two types of agents can substantially increase the wealth inequality in the model. The noteworthy case is one where the economy is composed of many short-run impatient and few short-run patient agents ($\lambda = 0.75$). Compare, for example, this economy with the one populated by short-run impatient agents only ($\lambda = 1$) under $\rho = 0.6$ and $\sigma = 0.2$. After incorporating 25% of the short-run patient population, the percentages of wealth held by the richest $99 - 100\%$

[7] The quantitative implications of the economy with one type of quasi-geometric consumers are extensively studied in Maliar and Maliar (2003).

Figure 1. The stationary distribution with two types of agents: λ=0.25, λ=0.50 and λ=0.75, respectively.



and $95 - 99\%$ of the population increase from 3.5 to 7.1 and from 10.4 to 22.4, respectively; the percentage of wealth held by the bottom 40% reduces from 12 to 0; the Gini coefficient rises from 0.39 to 0.78. As we see, all the statistics get closer to their empirical counterparts except of the percentage of wealth held by $90 - 95\%$ of the population, which is now too high relative to the data.[8]

A significant increase in wealth inequality occurs because the assumed two types of agents have very different consumption-savings behavior. To illustrate this fact, in $Table$ 1, we provide the mean and the standard deviation (in brackets) of the MPC out of assets. In the economies with two types, the average MPC of the short-run impatient consumers (MPC_1) is almost 10 times higher than that of the short-run patient consumers (MPC_2). The consequence is that consumers of the first type are poor, whereas those of the second type are rich.[9]

Why do we observe a non-monotonic relation between λ and the degrees of wealth inequality? The explanation for this result lies in the relation be-

[8] This drawback is related to the fact that the wealth distribution has two peaks. Including more than two types in the model will help to generate a more realistic shape of the wealth distribution.

[9] Hence, the mechanism that produces a large dispersion of wealth in our model is similar to one advocated by Krusell and Smith (1998) and Carroll (2000). In particular, Carroll (2000) argues: "the crucial requirement for many purposes is likely to be simply that the model have multiple classes of households, some with little wealth and a high MPC and some with substantial wealth and a low MPC...".

tween an individual's effective discount rate and the equilibrium interest rate. In a two-type economy, $0 < \lambda < 1$, the interest rate is determined mostly by the discount rate of the short-run patient agents. The gap between the equilibrium interest rate and the effective discount rate of the short-run impatient agents is therefore large enough that such agents choose not to accumulate much wealth. Consequently, the MPCs of the short-run impatient agents are much larger than those of the short-run patient agents. If, instead, all agents become short-run impatient, $\lambda = 1$, or all agents become short-run patient, $\lambda = 0$, then the gap between the individual effective discount rates and the interest rate becomes small as the interest rate adjusts in equilibrium to clear asset markets.[10]

We next focus on income inequality. *Table* 2 summarizes the statistics on the income distribution in the artificial and the U. S. economies. The tendencies here are parallel to those established for the distribution of wealth. In a one-type economy, variations in the degree of the agents' short-run patience do not significantly affect the size of income inequality. The introduction of two types makes the income distribution more unequal as a higher wealth inequality leads to a higher dispersion of capital income. The increase in

[10] A similar phenomenon occurs in the model by Krusell and Smith (1998) where agents are heterogeneous in the (long-run) discount factors.

Table 2. **Selected statistics of the income distribution in the U.S. and artificial economies.**

Model		*Gini*	*0-40%*	*80-100%*	*90-95%*	*95-99%*	*99-100%*
$\rho=0.6$	BM	0.12	32.2	26.2	6.5	5.8	1.7
$\sigma=0.2$	λ=0	0.12	31.8	26.5	6.6	5.9	1.7
	λ=1	0.13	31.2	26.9	6.7	6.0	1.8
	λ=.25	0.13	31.5	26.7	6.6	5.8	1.7
	λ=.50	0.14	30.2	27.5	7.0	6.1	1.8
	λ=.75	0.18	25.2	30.7	7.8	7.1	2.1
$\rho=0.9$	BM	0.23	24.9	33.3	8.3	8.1	2.3
$\sigma=0.4$	λ=0	0.23	25.1	33.2	8.3	8.0	2.3
	λ=1	0.23	24.8	33.2	8.4	8.0	2.3
	λ=.25	0.23	25.1	33.1	8.4	8.1	2.3
	λ=.50	0.23	24.6	33.5	8.5	8.1	2.4
	λ=.75	0.27	22.9	35.6	9.2	8.7	2.9
U.S. [(a)]		0.51	10.3	53.6	10.7	13.5	14.1

[(a)] *Source: Quadrini and Ríos-Rule (1997).*

income inequality is modest, however.

We finally assess the robustness of our results to variations in the parameters ρ and σ. The tendencies described above proved to be robust to all modifications considered. As an example, in *Tables* 1 and 2, we report the model's predictions under $\rho = 0.9$ and $\sigma = 0.4$. As we see, the difference between the MPCs of the short-run impatient and short-run patient agents is now lower than that in the baseline case $\rho = 0.6$ and $\sigma = 0.2$. The MPCs of the short-run impatient agents reduce because higher idiosyncratic uncertainty increases their precautionary savings. A precautionary motive for savings is practically missing for the short-run patient agents who are rich enough not to face the liquidity constraint. The MPCs of such agents increase because the interest rate goes down in response to higher precautionary savings of the short-run impatient population. Although the differences between the MPCs of types get smaller relative to the baseline case, the resulting degrees of wealth inequality are of roughly the same magnitude. Regarding the income distribution, we observe that higher labor income uncertainty leads to a higher dispersion of income across agents. As in the baseline case, introducing two types of agents makes the income distribution even more unequal. Still, the model's predictions on income inequality are far from the data.

4 Conclusion

This paper investigates the quantitative implications of a general equilibrium model with two types of quasi-geometric consumers. We find that a modest difference between the types' short-run discount factors can lead to very differing consumption-savings behavior: in the baseline case, the average MPCs of the types differ by almost the factor of 10! We show that the constructed model is capable of generating the degrees of wealth inequality which are much larger than those predicted by the standard one-sector growth model and which are comparable to those observed in the data.

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