A discusión

A NEOCLASSICAL THEORY OF WAGE ARREARS IN TRANSITION ECONOMIES*

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ABSTRACT

The paper proposes a theory of the wage arrears phenomenon in transition economies. We build on the standard one-sector neoclassical growth model. The neoclassical firms in transition make losses and use wage arrears as the survival strategy. At the agents' level, the randomness in the timing and extent of wage payments act as idiosyncratic shocks to earnings. We calibrate the model to reproduce evidence from the Ukrainian data and assess its quantitative implications. We find that wage arrears imply substantial social costs such as the consumption loss of 8% - 16% and the welfare loss from idiosyncratic uncertainty, equivalent to an additional consumption loss of 1% - 6%.

1   Introduction

Wage arrears are wage payments that were not settled at their due date. Wage arrears are not typically observed in developed market economies, where wages are punctually paid to workers. The problem of wage arrears is, however, severe in the former Soviet Union transition economies.[1] For example, in Ukraine, the average level of wage arrears was around 22% of quarterly output during 1996-2001; in 1996, wage arrears constituted around 40% of the yearly salary (i.e., the average wage debt was equal to five monthly salaries); and they affected more than 60% of the labor force.[2] This situation continues up to now, and only recent economic growth in Ukraine seems to reduce wage arrears.

A large body of empirical literature has investigated the determinants of the wage arrears phenomenon. The existence of wage arrears was attributed to liquidity problems, such as the lack of external finance available to enterprises, and the non-payment by customers for the goods delivered (Alfandari and Shaffer, 1996, Clarke, 1998); to the opportunistic behavior of managers who practice wage payment delays to pursue their personal interests, e.g., to make workers sell their shares in enterprises (Earle and Sabirianova, 2002); to the willingness of workers to accept wage-cuts in order to preserve their jobs (Layard and Richter, 1995); to the attempt by managers to extract tax concessions from the government (Alfandari and Shaffer, 1996); to the "survival" strategy of loss-making enterprises (Desai and Idson, 2000), etc.

The empirical literature found that wage arrears play an important role in individual consumption-savings decisions. Desai and Idson (1998, 2000) emphasized that wage arrears not only reduce the household's disposable income in a given period, but also undercut the household's wealth, as wage arrears are not indexed by inflation. Lehmann and Wadsworth (2001) found that wage arrears can increase the conventional measures of earnings inequality by $20 - 30\%$. Desai and Idson (2000) provided empirical evidence that Russian families protect themselves from wage payment delays by borrowing from relatives, selling family assets, reducing savings rates, holding multiple jobs, etc. A theoretical framework for studying the impact of wage arrears

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[1]There exist other kinds of arrears in transition economies, e.g., arrears between enterprises, arrears of enterprises to banks, tax arrears of enterprises, pension arrears.

[2]Similar tendencies are observed regarding wage arrears in Russia (see Alfandari and Shaffer, 1996, Clarke, 1998, Desai and Idson, 2000, Earle and Sabirianova, 2002).

on the consumers' behavior has not yet been proposed in the literature.[3]

In this paper, we develop a theory of wage arrears, which allows us both to explain the determinants of the wage arrears phenomenon and to assess the effect of wage arrears on the individual consumption-savings behavior. In our model, wage arrears arise because firms, hit by negative shocks associated with transition, incur losses and cover the losses by underpaying wages to workers.[4] We assume that wage arrears depreciate over time, which allows us to capture the fact that wage arrears in transition economies were often not indexed by inflation. At the individual level, the randomness in the timing and extent of wage payments act as idiosyncratic shocks to earnings. Markets are incomplete: agents cannot borrow beyond a certain limit and cannot insure themselves against idiosyncratic uncertainty. Thus, the consumer side of our economy is the same as in the standard one-sector neoclassical growth model except that in our case, fluctuations in individual wages come from wage arrears shocks, while in the standard case, they come from productivity shocks, e.g., Aiyagari (1994), Huggett (1993, 1997).

The effect of wage arrears on the individual's consumption-savings behavior can be summarized by two types of costs. First, delays in wage payments lead to a reduction in the expected capital and labor income of agents, thus lowering their consumption. Capital income decreases because of the precautionary savings effect, and labor income decreases because of the depreciation of wage arrears. Secondly, the non-regularity of wage payments, together with borrowing restrictions, leads to variations in the amount of resources available to agents in each period, which induces consumption fluctuations.

We calibrate the model to reproduce empirical evidence from the Ukrainian aggregate and household data and assess the quantitative expression of the effects associated with wage arrears. Our findings are as follows: The model with the wage arrears shocks can generate approximately the same degrees of wealth, income and consumption inequality as those produced by the standard neoclassical growth model with productivity shocks (see, e.g., Aiyagari, 1994). As far as the social costs of wage arrears are concerned, in our experi-

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[3]Earle and Sabirianova (2000) presented a formal model, in which high and persistent wage arrears can arise in equilibrium as an outcome of managerial decisions. However, their paper does not explicitly consider the consumer side of the economy.

[4]This mechanism agrees with our empirical evidence from the Ukrainian data. It is also in line with the view of Desai and Idson (2000) who argue that "wage arrears in Russia could have declined and disappeared over time if nonviable units were identified, declared bankrupt, and folded or reorganized..." (Desai and Idson, 2000, p.25).

ments, the reduction in the agent's expected consumption due to wage arrears ranges from 8% to 16%. The welfare loss resulting from consumption fluctuations is equivalent to an additional consumption loss, which ranges from 1% to 6% depending on the degree of the agents' assumed risk-aversion.

The paper is organized as follows. Section 2 documents empirical evidence on wage arrears in the Ukrainian economy. Section 3 formulates the model. Section 4 discusses the methodology of the quantitative study and presents the results from simulations, and finally, Section 5 concludes.

2   Arrears in Ukraine: empirical evidence

In this section, we provide empirical evidence about arrears in Ukraine from both aggregate and household data and discuss the relation between wage arrears and inter-enterprise arrears. The aggregate time series come from the data-base of the Ukrainian-European Policy and Legal Advice Center (UEPLAC), and the household data are taken from the "Ukraine-96" survey. The description of the data used is provided in Appendix A.

The aggregate dynamics of the Ukrainian arrears over 1994-2001 are illustrated in *Figure* 1. "Gross payables" and "gross receivables" are defined as the sum of the real debt of and to economic agents, respectively; "net payables" is the difference between gross payables and gross receivables; "wage arrears" is the real wage debt of enterprises to workers. For the sake of comparison, we also plot the real output.

The first thing to notice in the figure is that the total amount of debt in Ukraine increased dramatically over the sample period: gross payables rose from a half of quarterly output in 1994 to two quarterly outputs in 2000. However, the increment in gross payables is not necessarily an indication of poor economic performance. According to estimates by the National Bank of Ukraine, three quarters of gross payables in 1996 were trade credits (i.e., inter-enterprise arrears). Trade credits are cheap and convenient substitutes for bank credits and are commonly observed in developed market economies. Trade credits are particularly useful in transition economies where financial institutions are underdeveloped. The current level of trade credits in percentage of GNP in Ukraine is comparable to that of France or Japan.[5]

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[5] See Alfandari and Schaffer (1996, Table 5) for a comparison of the size of trade credits across countries.

What appears to be a relevant problem for the Ukrainian economy is the presence of a large net debt. As is evident from $Figure$ 1, gross payables systematically exceeded gross receivables; average net payables amounted to 35.46% of quarterly output.[6] This indebtedness is a direct consequence of the adverse effects of transition on the Ukrainian producers. On one hand, the typical enterprise faced a reduction in demand for (prices of) its products because of an increase in foreign competition and because of the break-up of the Soviet Union; on the other hand, it faced an increase in the prices of inputs (especially, energy and primary resources) because of the economy's opening up. According to the Ukrainian National Bank, 84% of net debt in 1996 was concentrated in manufacturing, agriculture and the coal sector, whereas 78% of net credits belonged to input suppliers such as energy, oil and gas sectors, which indicates that many enterprises were not able to pay back for the inputs used. In the report of UEPLAC (2001), the following conclusion is reached: "...the onset of transition and the shock in relative prices resulted in a significant part of their industry producing negative value added", so that the growth of net arrears "... reflects the continuous presence of loss-making enterprises which continue their operations by financing them with a growing stock of debt" (see UEPLAC, 2001, page 60).

As can be seen from $Figure$ 1, wage arrears constituted a large fraction of Ukrainian net arrears. To be more precise, wage arrears, on average, amounted to 22.44% of quarterly output over the period 1996-2001, which corresponds to 2/3 of the Ukrainian net arrears.[7] We therefore conclude that the debt burden of the Ukrainian enterprises was essentially laid upon workers by means of wage arrears.

Similarly to the aggregate time series data, household data reveal that Ukrainian wage arrears is both a large-scale and a long-run phenomenon. We have the following evidence from the "Ukraine-96" household survey: in 1996, the wage arrears of the average Ukrainian worker amounted to 39.36% of the yearly wage, which implies almost a 5-month wages debt per year and per worker; wage arrears affected 66.3% of the agents interviewed; and 7.14% of the agents reported wage arrears that exceeded the average yearly wage. Another piece of evidence on Ukrainian wage arrears is provided by Lukya-

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[6]Alfandari and Schaffer (1996, Table 5) reported payables and receivables for different countries. For the typical developed market economy, receivables are larger than payables. The largest net receivables are observed in Japan and amount to 15% of yearly output.

[7]Except for wage arrears, net arrears include tax arrears, arrears of enterprises to banks and pension arrears.

nenko et al. (2002) from the 1999 "Ukraine Small and Medium Enterprises Survey". In this household data set, 66.4% of employees experienced wage arrears; 38% of employees faced wage delays ranging from 1 to 3 months; and 18% of employees reported wage delays exceeding one year.

To conclude this section, we summarize the key regularities observed in the Ukrainian data as follows: During the process of transition, many of the Ukrainian enterprises incurred losses. The debt burden of enterprises was mostly shifted to workers.[8] For a long time, the Ukrainian economy has had a large, roughly stationary stock of wage arrears.[9] In the next section, we present a formal model, which reproduces the above regularities.

3 The model

Time is discrete and the horizon is infinite, $t \in T$, where $T = \{0, 1, 2, ...\}$. The economy consists of a continuum of firms and a continuum of infinitely-lived consumers. Both, firms and consumers, have their names uniformly distributed on a closed interval $[0, 1]$.[10]

3.1 The firms

The firms own identical production technologies that convert capital and labor into output. Each firm chooses demand for capital input, K_t, and labor input, N_t, to maximize expected period-by-period profits, taking the interest rate, R, and wage, W, as given.[11] The level of output depends on an additive idiosyncratic shock, θ_t, which is independently and identically distributed across firms and over time. Thus, the firm solves the following

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[8]In fact, delays in wage payments also help enterprises to reduce effective wages because wage arrears were not fully indexed by inflation. Layard and Richter (1995) argued that workers were willing to accept wage-cuts in order to preserve their jobs.

[9]A similar empirical observation about wage arrears in Russia is made in Lehmann and Wadsworth (2001): "...since 1996 the stock of wage arrears has been approximately in a steady state, equivalent to two month wage bill. This means that the amount of contractual wages not paid to (some) workers in month t roughly equals to the amount of wage debts paid back to (some) workers in month t."

[10]This assumption implies that the average and aggregate quantities coincide.

[11]Here, and further in the text, variables whose equilibrium values are common for all firms are denoted by capital letters, and those whose values are firm-specific are denoted by small letters. The same type of notation will be used for consumers.

problem:

$$\max_{K_t, N_t} \quad E\left[F\left(K_t, N_t\right) - dK_t - RK_t - WN_t - \theta_t\right], \tag{1}$$

where E is the unconditional expectation, and $d \in (0, 1]$ is the depreciation rate of capital. The production function F has constant returns to scale, is strictly increasing, strictly concave, continuously differentiable and satisfies the appropriate Inada conditions.

The profit maximization conditions, which follow from (1), imply that the factor prices are equal to the corresponding marginal products:

$$R = F_1\left(K_t, N_t\right) - d, \qquad W = F_2\left(K_t, N_t\right), \tag{2}$$

where F_1 and F_2 are the first-order partial derivatives of the production function F with respect to capital and labor inputs, respectively.

With the assumption of constant returns to scale and in the absence of idiosyncratic shocks, $\theta_t = 0$, the firm makes zero profit. When $\theta_t > 0$ ($\theta_t < 0$), the firm makes negative (positive) profit. We assume that the firm first pays for capital and then for labor, and also, that the output of the firm is always sufficient to cover depreciation of capital and its rental price, i.e., $F\left(K_t, N_t\right) - \theta_t \geq (d + R) K_t$. Thus, the effective wage per unit of labor is

$$\omega_t = \left[F\left(K_t, N_t\right) - \theta_t - (d + R) K_t\right] / N_t = W - \theta_t / N_t. \tag{3}$$

Wages that are (over-paid) under-paid to workers are (subtracted from) added to the stock of wage arrears,

$$q_t = q_{t-1}\left(1 - d_q\right) + \theta_t, \tag{4}$$

where q_t denotes the stock of wage arrears at the end of period t. We assume that q_t is bounded, i.e., $q_t \in \mathbf{Q} \equiv [0, \overline{q}] \subset \mathbf{R}$ for all t. The depreciation rate of wage arrears is $d_q \in (0, 1]$.

The assumption that the stock of wage arrears depreciates, allows us to account for the previously mentioned fact that, in transition economies, wage arrears were not appropriately indexed by inflation, so that a delay in wage payment resulted in the reduction of the real value of wage debt. Furthermore, some wage arrears were not paid at all, either because firms go bankrupt or because workers quit their jobs and loose the right to claim the debt.

In order to capture the fact that the typical firm in a transition economy is a loss-maker, we shall assume that the idiosyncratic shock, θ_t, is, on average, positive, $E[\theta_t] > 0$. By assumption, the process for θ_t is exogenous and cannot be affected by actions of the firm. However, note that even if we allow the firm to reduce θ_t at zero cost, it would have no incentives to do so. This is because the firm shifts all losses to workers and makes the same effective (zero) profit, independently of the amount of wage arrears.[12]

3.2 The consumers

Each agent is matched with one firm and supplies inelastically one unit of her (non-valued) time to production, $N_t = 1$. The contractual wage of agent is W, however, her effective wage is ω_t. The evolution of the debt of the firm to the worker (wage arrears) is described by (4). Our assumptions imply that the distribution of effective wages is the same for all firms, and thus, agents have no incentives to change their jobs even if they are paid less than the amount specified in the contract.

The agent saves in the form of real assets. Income from assets is Ra_t, where a_t is the current individual asset holdings. Assets are restricted to being in the set $\mathbf{A} = [-\underline{a}, \overline{a}] \subset \mathbf{R}$, i.e., the agent is allowed to borrow only up to a certain amount $\underline{a} \geq 0$. The agent seeks to maximize the expected discounted sum of one-period utilities by choosing an optimal path for consumption, $\{c_t\}_{t=0}^{\infty}$. The period utility function $u(c)$ is continuously differentiable, strictly increasing, strictly concave and satisfies the Inada type of condition $\lim_{c \to 0} u'(c) = \infty$. Consumption is restricted to being non-negative. Consequently, the problem of the agent is as follows:

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \delta^t u(c_t) \tag{5}$$

subject to

$$c_t + a_{t+1} = \omega_t + (1 + R) a_t, \tag{6}$$

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[12]In fact, there is ample evidence in the empirical literature that managers tend to exploit the phenomenon of wage arrears in their own interests, e.g., to extract tax concessions from the government (Alfandari and Shaffer, 1996), to make the workers sell their shares in the firm (Earle and Sabirianova, 2002).

$$a_{t+1} \geq \underline{a}, \tag{7}$$

where initial condition (a_0, q_{-1}, ω_0) is given.[13] Here, E_t denotes the expectation, conditional on all information about the agent's wage payments available at t, and $\delta \in (0, 1)$ is the discount factor. To ensure the existence of a solution to the consumer's problem, we shall require that $\delta (1 + R) < 1$.

The consumer's problem, in our model with wage arrears, is identical to the one in the standard one-sector neoclassical growth model by Aiyagari (1994), except that, in our case, fluctuations in the individual wage is the result of delays in wage payments and not of idiosyncratic shocks to productivities, as assumed in Aiyagari (1994). We shall also note that, in our economy, the individual state is characterized by three state variables, (a_t, q_{t-1}, ω_t), while in Aiyagari's (1994) economy, it is described by two state variables, (a_t, s_t), where s_t is an idiosyncratic shock to productivity.

We restrict our attention to a recursive solution to the agent's problem, such that the agent makes consumption-savings decisions according to the same decision rule in all periods. It turns out that we can reduce the number of state variables from three to two with the following change of variables:

$$\widehat{a}_t = a_t + \underline{a}, \tag{8}$$

$$z_t = \omega_t + (1 + R) \widehat{a}_t - R\underline{a}. \tag{9}$$

The variable z_t can be interpreted as the total amount of resources that is available to the agent in period t. In terms of (8) and (9), we can re-write constraints (6) and (7), respectively, as follows:

$$c_t + \widehat{a}_{t+1} = z_t, \tag{10}$$

$$\widehat{a}_t \geq 0. \tag{11}$$

Let us denote by $V(z, q)$ the optimal value function for the agent with the total resources z and wage arrears q. The recursive formulation of the problem $(3) - (7)$ is then as follows:

$$V(z, q) = \max_{\widehat{a}' \in [\underline{a}, \overline{a}]} \left\{ u(z - \widehat{a}') + \delta E \left[V(z', q') \mid q \right] \right\} \tag{12}$$

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[13]The non-negativity of consumption is guaranteed by the assumption $\lim_{c \to 0} u'(c) = \infty$.

subject to

$$z' = \omega' + (1 + R)\,\widehat{a}' - R\underline{a}. \tag{13}$$

The problem (12), (13) defines the optimal asset demand function, $\widehat{a}' \equiv A(z, q)$. We assume that such a function exists and is unique, and also that it is continuous and differentiable.

By calculating the Kuhn-Tucker conditions of the problem (12), (13), we obtain the Euler equation

$$u'(c) \geq \delta E\left\{u'(c')(1 + R)\right\}, \tag{14}$$

where $c = z - \widehat{a}'$, and u' denotes the marginal utility of consumption. The Euler equation (14) holds with equality if the borrowing restriction is non-binding, $a' > \underline{a}$, and it holds with inequality if the limit on borrowing is reached, $a' = \underline{a}$.

3.3 Equilibrium

Let λ be a probability measure defined on $\mathbf{B}$, where $\mathbf{B}$ denotes the Borel subset of the set of all possible individual states $\mathbf{Z} \times \mathbf{Q}$. For all $B \in \mathbf{B}$, $\lambda_t(B)$ is the mass of agents whose individual states lie in B at time t. Given that λ_t is a probability measure, the total mass of agents is equal to 1.

Denote by $P(z, q, B)$ the conditional probability that an agent with state (z, q) will have an individual state lying in set B in the next period. The function P is defined as

$$P(z, q, B) = Prob\left(\{q' \in \mathbf{Q} : [z', q'] \in B\} \mid q\right).$$

Then, the law of motion of λ_t is: $\lambda_{t+1}(B) = \int_{\mathbf{Z} \times \mathbf{Q}} P(z, q, B)\, d\lambda_t$ for all $t \in T$ and all $B \in \mathbf{B}.$

The fact that there is a continuum of agents guarantees that the mass of agents with the shock q' at $t + 1$ and the shock q at t is equal to the conditional probability, $Prob(q' \mid q)$. Since q_{t+1} follows a first-order Markov process, such probability depends only on the recent past and is the same in all periods. Hence, the aggregate amount of wage arrears is constant.

We only study the equilibria in which the period-$t+1$ probability measure λ_{t+1} is the same as the period-t probability measure λ_t, for all $t \in T$. In this case, we say that the probability measure is stationary and denote it by λ^*.

The stationarity of λ^* implies that the aggregate capital stock is constant, $K = \int_{\mathbf{Z} \times \mathbf{Q}} a_t d\lambda^*$ for all t, (even though the asset holdings of each agent vary stochastically over time).

Definition. A stationary equilibrium is defined as a stationary probability measure λ^*, an optimal asset function $A(z, q)$, and positive real numbers (K, Q, R, W) such that:

(1) λ^* satisfies $\lambda^* = \int_{\mathbf{Z} \times \mathbf{Q}} P(z, q, B) d\lambda^*$ for all $B \in \mathbf{B}$;
(2) $A(z, q)$ solves (13), (14) for a given pair of prices (R, W);
(3) (R, W) satisfy the profit maximization conditions in (2) with $N_t = 1$;
(4) Q is the average of the agents' wage arrears: $Q = \int_{\mathbf{Q}} q_t d\lambda^*$;
(5) K is the average of the agents' asset holdings: $K = \int_{\mathbf{Z} \times \mathbf{Q}} a_t d\lambda^*$.

4   Quantitative analysis

In this section, we describe the methodology of our numerical study and present the results. The description of the Ukrainian aggregate and household data used is provided in Appendix A.

4.1   Methodology

We choose the model's period as one year and calibrate the model to reproduce several basic observations on the Ukrainian economy. With the assumption of stationarity of the wage and wage arrears distributions, we can use equations $(2) - (4)$ to show that

$$d_q = 1 - \sqrt{1 - \sigma_\omega^2 / \sigma_q^2}, \tag{15}$$

$$W = \Omega + Q d_q, \tag{16}$$

where Ω and σ_ω (Q and σ_q) are the mean and the standard deviation of the wage distribution (the wage arrears distribution), respectively.[14] Thus, the

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[14]To derive (15), we shall assume that the current individual wages are uncorrelated with past wage arrears. To check the empirical validity of this assumption, we compute this correlation with Ukrainian household data and find that it is, indeed, low (about 0.05).

ratio of the average effective wage to the contractual wage is given by

$$\psi_q \equiv \frac{\Omega}{W} = \frac{\Omega}{\Omega + Qd_q}. \tag{17}$$

We calculate Ω, Q, σ_ω and σ_q by using the data from the "Ukraine-96" household survey and compute d_q, W and ψ_q according to (15), (16) and (17), respectively. The results are reported in the upper row of $Table$ 1.

We assume that the production function is of the Cobb-Douglas type, $F(K, N) = K^\alpha N^{1-\alpha} = F(K, 1) = K^\alpha$. We calibrate the remaining parameters $\{\alpha, d, \delta\}$ so that, in the non-stochastic steady state, the model reproduces the following four statistics of the Ukrainian economy: the share of labor income in production, Y^L/Y, the consumption to output ratio, C/Y, the capital to output ratio, K/Y, and the wage arrears to output ratio, Q/Y. In the absence of wage arrears, we would have that $\alpha = 1 - (Y^L/Y)$. With wage arrears, agents do not get all the labor income earned but only a fraction of it, ψ_q, so that

$$\alpha = 1 - \frac{1}{\psi_q} (Y^L/Y). \tag{18}$$

Furthermore, equations $(2) - (4)$, (6) yield

$$d = \frac{1 - (C/Y) - d_q (Q/Y)}{(K/Y)}. \tag{19}$$

Finally, the Euler equation (14) implies

$$\delta = \frac{1}{1 - d + \alpha (Y/K)}. \tag{20}$$

We estimate the ratios Y^L/Y, C/Y, K/Y and Q/Y from the Ukrainian time series data and compute α, d and δ from equations (18), (19) and (20), respectively. We report the results in the lower row of $Table$ 1.

The debt limit is set at zero, $\underline{a} = 0$. We assume that the agent's momentary utility function is of the Constant Relative Risk-Aversion (CRRA) type, $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ with $\gamma > 0$. The coefficient of relative risk-aversion, γ, is not identified by our calibration procedure. We consider four alternative values of this parameter, such as $\gamma \in \{0.5, 1, 3, 10\}$. Our benchmark value is $\gamma = 1$, which corresponds to the limiting logarithmic case, $u(c) = \ln(c)$.

To compute numerical solutions, we assume that individual effective wages are drawn from a Normal distribution, $\omega_t \sim N\left(\Omega, \sigma_\omega^2\right)$ and approximate the process for wage arrears (4) by a 5-state Markov chain. We illustrate the resulting approximation in $Figures$ 3 and 4, and we provide the matrix of the transitional probabilities and the corresponding unconditional probabilities of states in $Table$ 2. A description of the solution procedure is elaborated in Appendix B. We illustrate the properties of the solutions obtained in $Figures$ $5 - 8$ and report the statistics generated by the model in $Table$ 3.

4.2   Results

We begin by describing the quantitative implications of our benchmark model (see column "BM" in $Table$ 3). To illustrate the distributional predictions of the model, we provide several inequality measures of the distributions of wages, assets and consumption. As we can see, idiosyncratic uncertainty associated with wage arrears leads to a significant dispersion in effective wages across workers. For instance, the bottom 40% wage group gets 20% of the total wages in the economy, while the top 1% wage group earns 2% of the total wages; the Gini coefficient of the wage distribution is 0.27. The inequality in assets and consumption occurs as a result of the wage inequality. The dispersion in assets across agents is much higher than that in consumption. For example, the bottom 40% asset group holds 21.8% of the total assets, and the top 1% asset group holds 2.6%, while the respective consumption groups consume 33% and 1.6% of the total amount of consumption. The same regularity can be appreciated by looking at the normalized standard deviations and the Gini coefficients, which are 0.496 and 0.275, respectively, for the asset distribution, and which are 0.165 and 0.105, respectively, for the consumption distribution.[15] Hence, our model predicts that risk-averse agents smooth consumption fluctuations by accumulating assets in high-wage states and dissaving in low-wage states.[16]

The fact that assets are more dispersed across agents than consumption

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[15]In fact, the degrees of wealth, income and consumption inequality in our model with wage arrears are very similar to those generated by the standard neoclassical growth model with productivity shocks (see, e.g., Aiyagari, 1994).

[16]This implication of the model agrees with the empirical findings of Desai and Idson (2000) that in Russia, agents who were owed wages were more likely to engage "in various forms of dissaving including borrowing, selling family consumer durable items and other assets, and drawing down accumulated savings" (Desai and Idson, 2000, p.218).

Table 1. Selected statistics for Ukrainian economy and the implied model's parameters.

Ukrainian household data: 1996[a]

Q, krb	σ_q, krb	Prob(q_t>0)	Ω, krb	σ_ω, krb	d_q	ψ_q	σ_q
18674.6	31574.5	0.6630	47446.5	26859.2	0.4743	0.8427	0.4770

Ukrainian aggregate time series: 1994-2001[b]

C/Y	K/Y	Y^L/Y	Q/Y	D/Y	d	α	δ
0.7261	3.1663	0.4477	0.2244	0.3546	0.0865	0.4687	0.9420

Notes: [a]Source: "Ukraine-96" Project, Kiev International Institute of Sociology.
[b]Sources: UEPLAC and Derzhcomstat.

Table 2. Markov chain and unconditional actual and fitted distribution of arrears[a].

q_{t+1}\q_t	q_1=0	q_2=0.3633	q_3=0.7266	q_4=1.0899	q_5=1.4533	Prob(q_i) fitted[b]	Prob(q_i) actual[b]
q_1=0	0.5204	0.3634	0.2267	0.1250	0.0605	0.3457	0.5164
q_2=0.3633	0.2714	0.2965	0.2780	0.2238	0.1546	0.2691	0.3106
q_3=0.7266	0.1505	0.2198	0.2756	0.2966	0.2740	0.2200	0.1016
q_4=1.0899	0.0577	0.1202	0.1572	0.2263	0.2795	0.1264	0.0283
q_5=1.4533	0	0	0.0624	0.1283	0.2314	0.0389	0.0431

Notes: [a]Wage arrears are expressed in terms of the contractual market-clearing wage, *W*.
[b]"Prob(q_i) fitted" is an unconditional probability distribution of wage arrears generated by the constructed Markov chain and "Prob(q_i) actual" is an unconditional probability distribution of wage arrears generated by Ukrainian household data. Source: "Ukraine-96" Project, Kiev International Institute of Sociology.

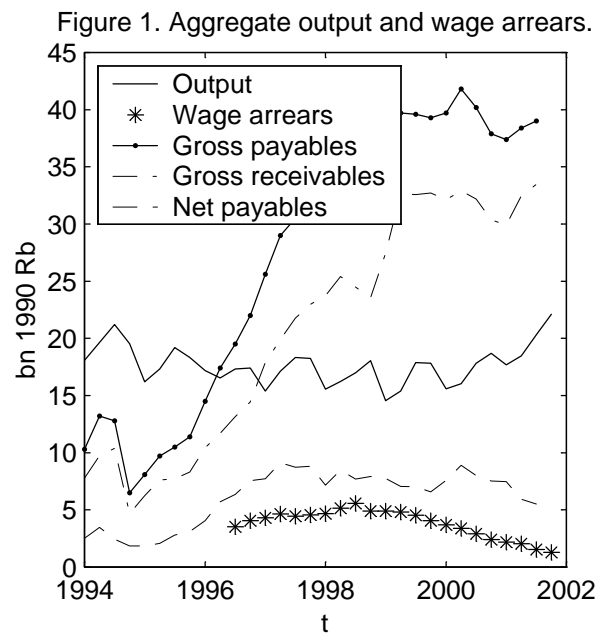Figure 1. Aggregate output and wage arrears.

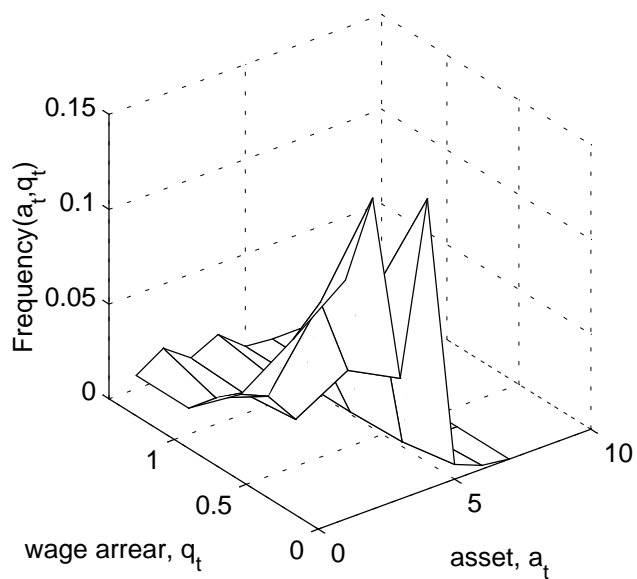Figure 2. Empirical distribution of assets and wage arrears.

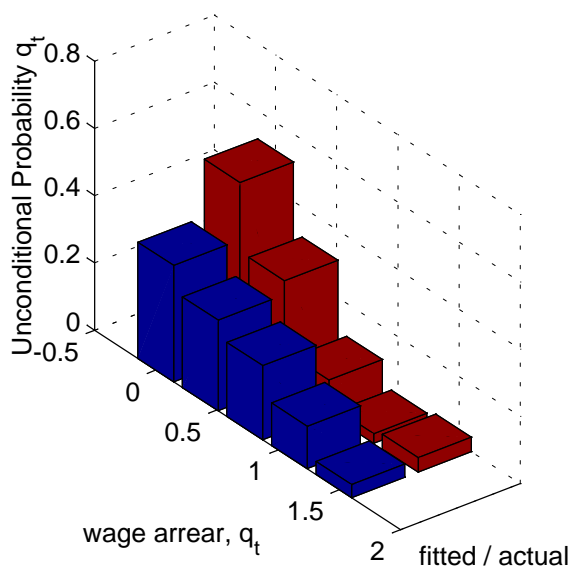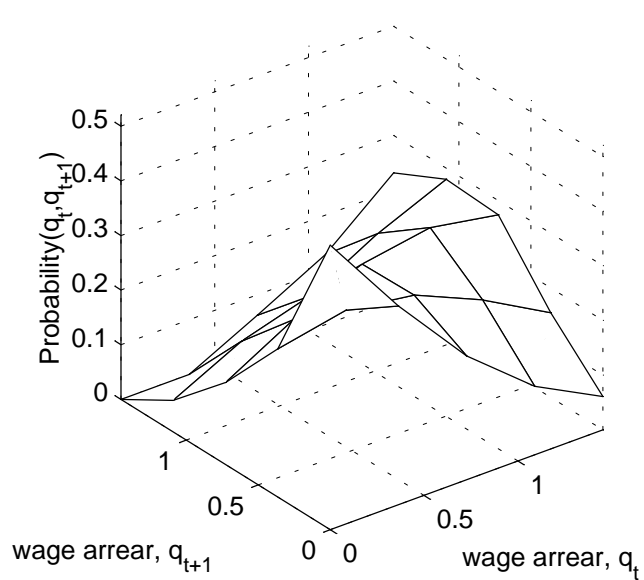Figure 3. Unconditional Probability: actual v.s. fitted.

Figure 4. Markov chain for wage arrears.

Figures 3, 4: Benchmark parameterization, d_q=0.4743, ψ_q=0.8427, σ_q=0.4770.

can be also seen from $Figures$ 7 and 8, in which we plot the simulated probability distributions of assets and wage arrears, and consumption and wage arrears, respectively. In particular, it can be seen that consumption is always strictly positive, while assets occasionally reach a zero borrowing limit. The household survey we have does not contain information about the individual consumption and asset holdings. There is, however, information about the respondents' own evaluations of their material status, relative to that of other people (see Appendix A for the description of this data). In $Figure$ 2, we plot the joint distribution of this variable and wage arrears (and we interpret it to be the empirical distribution of assets and wage arrears). The comparison of $Figure$ 2 and $Figure$ 7 shows that there is a certain degree of similarity between the empirical and simulated distributions of assets and wage arrears.

We next explore how wage arrears affect the aggregate model's variables. Let us consider the average income, which is given by the sum of capital and labor income,

$$I = \int_{\mathbf{Z} \times \mathbf{Q}} (Ra_t + \omega_t) \, d\lambda^* = RK + \Omega. \tag{21}$$

As far as capital income, RK, is concerned, uncertainty about the time and amount of wage payments, together with the restriction on borrowing, makes the agents increase their savings. As we see from $Table$ 3, the rise in the aggregate capital stock due to precautionary savings, ΔK, amounts to 2.936%.[17] The interest rate is inversely related to the aggregate capital stock, $R = \alpha K^{\alpha-1} - d$, so that its value is lower in our economy with wage arrears than in the one without wage arrears. The total effect of wage arrears on capital income is negative, $\Delta(RK) = -0.985\%$.

Labor income is given by (16) with $W = (1 - \alpha) K^{1-\alpha}$. There are two effects here. First, the marginal product of labor goes up because capital stock rises due to precautionary savings, $\Delta W = 1.387\%$. Secondly, the wage reduces by the amount of $d_q Q$ because of the depreciation of wage arrears. The second effect dominates the first one, $\Delta \Omega = -20.360\%$.

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[17] Here, and further in the text, we denote by ΔX the percentage difference between the mean of a variable X in the model with wage arrears and its non-stochastic steady state value, X^{ss}, i.e., $\Delta X \equiv \frac{X - X^{ss}}{X^{ss}} \times 100$. Note that the non-stochastic steady state can be computed analytically. In particular, the steady state capital stock, K^{ss}, can be computed from the steady state expression of the Euler equation, $1 = \delta \left(1 - d + \alpha \left(K^{ss} \right)^{\alpha-1} \right)$; the other aggregate variables can also be readily computed.

Table 3. Selected statistics generated by the model.

Statistic[a]	BM	$\gamma=.5$	$\gamma=3$	$\gamma=10$	$\alpha=.36$	$\delta=.96$	$d=.1$	$d_q=.522$	$\psi_q=.766$	$\sigma_q=.525$
Aggregate statistics										
R,%	5.928	6.042	5.447	3.780	5.813	4.027	5.926	5.920	5.900	5.915
ΔK,%	2.936	1.433	9.645	38.961	3.708	2.084	2.588	3.038	3.309	3.100
Δ(RK),%	-0.855	-0.417	-2.961	-14.664	-2.058	-1.334	-1.106	-0.885	-0.965	-0.903
ΔW,%	1.387	0.690	4.432	16.697	1.333	0.969	1.277	1.434	1.559	1.463
$\Delta\Omega$,%	-20.360	-20.907	-17.968	-8.332	-20.396	-20.670	-20.486	-20.964	-25.165	-21.095
ΔI,%	-15.120	-15.403	-13.935	-10.025	-16.914	-16.358	-15.574	-15.570	-18.666	-15.670
ΔC,%	-12.845	-13.096	-11.771	-8.229	-14.201	-14.329	-13.406	-13.257	-16.264	-13.322
Δ^uC,%	-13.803	-13.563	-14.323	-14.123	-14.723	-14.963	-14.243	-14.203	-17.243	-14.283
Δ^uC-ΔC,%	-0.958	-0.467	-2.552	-5.894	-0.522	-0.634	-0.836	-0.946	-0.979	-0.961
The distribution of wages										
std(ω)/ω	0.492	0.492	0.492	0.492	0.492	0.492	0.492	0.490	0.546	0.525
corr(ω,q)	0.113	0.113	0.113	0.113	0.113	0.113	0.113	0.106	0.091	0.126
0-40%	20.330	20.330	20.330	20.330	20.330	20.330	20.330	20.502	17.760	18.920
80-100%	32.143	32.143	32.143	32.143	32.143	32.143	32.143	31.765	34.378	32.942
90-95%	8.141	8.141	8.141	8.141	8.141	8.141	8.141	8.053	8.714	8.370
95-99%	7.178	7.178	7.178	7.178	7.178	7.178	7.178	7.050	7.744	7.390
99-100%	1.988	1.988	1.988	1.988	1.988	1.988	1.988	1.935	2.152	2.025
Gini(ω)	0.274	0.274	0.274	0.274	0.274	0.274	0.274	0.270	0.313	0.292
The distribution of assets										
std(a)/a	0.496	0.504	0.464	0.370	0.469	0.494	0.475	0.492	0.498	0.492
corr(a,q)	-0.175	-0.173	-0.182	-0.205	-0.283	-0.156	-0.224	-0.167	-0.187	-0.182
0-40%	21.777	21.512	22.874	26.155	22.820	21.921	22.558	21.945	21.686	21.929
80-100%	35.141	35.406	34.111	31.108	34.117	35.129	34.418	35.021	35.156	35.011
90-95%	8.897	8.964	8.632	7.854	8.633	8.888	8.708	8.867	8.902	8.861
95-99%	8.388	8.472	8.076	7.179	8.094	8.397	8.179	8.355	8.384	8.348
99-100%	2.647	2.678	2.524	2.181	2.542	2.659	2.570	2.633	2.641	2.629
Gini(a)	0.275	0.279	0.257	0.207	0.259	0.273	0.263	0.272	0.275	0.272
The distribution of consumption										
std(c)/c	0.165	0.168	0.153	0.116	0.126	0.139	0.141	0.166	0.174	0.166
corr(c,q)	-0.163	-0.161	-0.171	-0.193	-0.274	-0.147	-0.212	-0.159	-0.178	-0.168
0-40%	32.969	32.841	33.533	35.190	34.442	34.079	34.001	32.953	32.543	32.915
80-100%	25.423	25.539	24.929	23.535	24.029	24.553	24.546	25.410	25.711	25.423
90-95%	6.394	6.425	6.269	5.910	6.034	6.171	6.168	6.393	6.468	6.396
95-99%	5.539	5.574	5.395	4.985	5.121	5.296	5.283	5.538	5.616	5.539
99-100%	1.567	1.580	1.512	1.355	1.407	1.479	1.471	1.566	1.593	1.566
Gini(c)	0.105	0.107	0.096	0.071	0.082	0.089	0.089	0.105	0.111	0.106

Notes: [a]Statistics in column "BM" are those generated by the model under the benchmark parameterization: $\gamma=1$, $\alpha=.4687$, $\delta=.9420$, $d=.0865$, $d_q=.4743$, $\psi_q=.8427$, $\sigma_q=.4770$. Statistics in each subsequent column are obtained when the parameter on the top of the column is set to the given value, while the other parameters are set to their benchmark values. Statistic "ΔX" is the percentage difference between the mean of variable X_t and its non-stochastic steady state value, X^{ss}, i.e., $\Delta X \equiv (X-X^{ss})/X^{ss} \times 100\%$; "$\Delta^u$C" is the percentage utility cost of consumption fluctuations computed as described in the text; "x", "std(x)" and "corr(x,y)" are the mean of variable x_t, the standard deviation of variable x_t and the correlation coefficient between variables x_t and y_t, respectively; "0-40%", "80-100%", "90-95%", "95-99%", "99-100%" are the percentages of variable x_t owned by 0-40%, 80-100%, 90-95%, 95-99%, 99-100% subgroups ranked by variable x_t, respectively; and, finally, "Gini(x)" is the Gini coefficient of the density function of variable x_t.
[b]To carry out experiments reported in the last three columns, we re-compute Markov chain correspondingly.

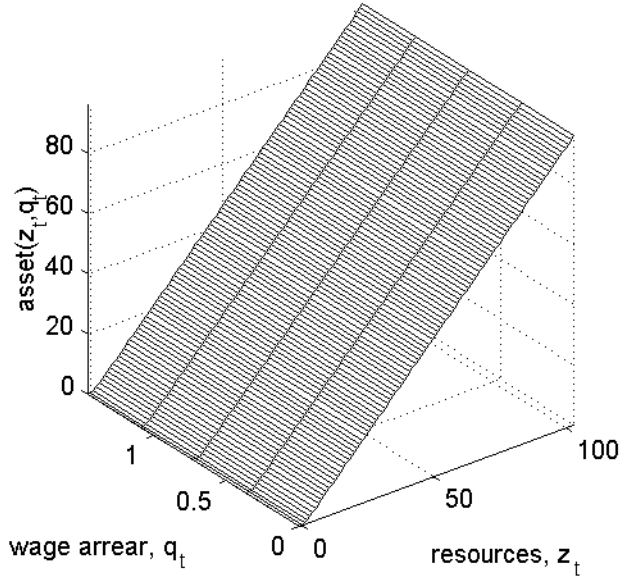Figure 5. The asset demand function.

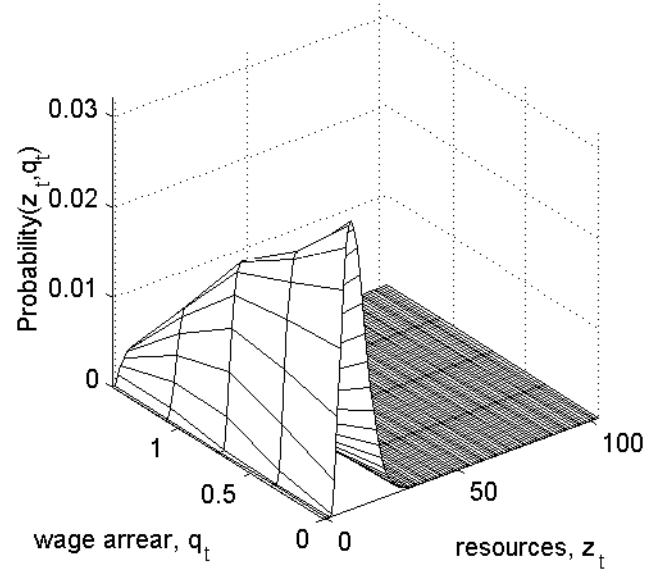Figure 6. Simulated distribution of resources and wage arrears.

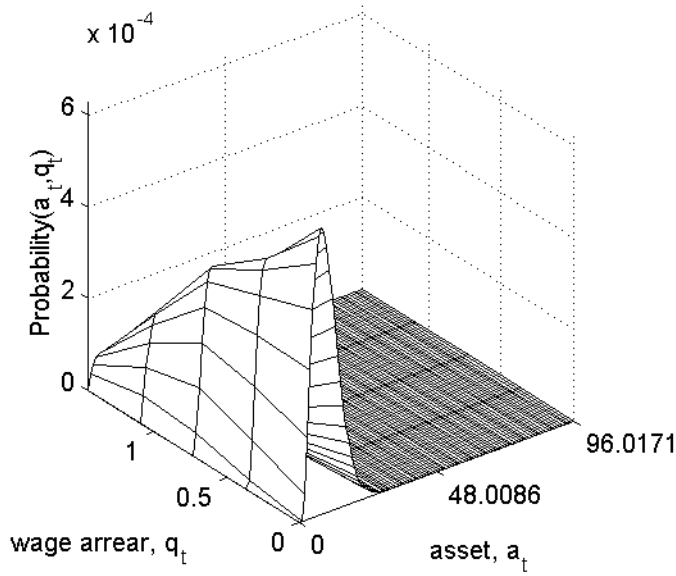Figure 7. Simulated distribution of assets and wage arrears.

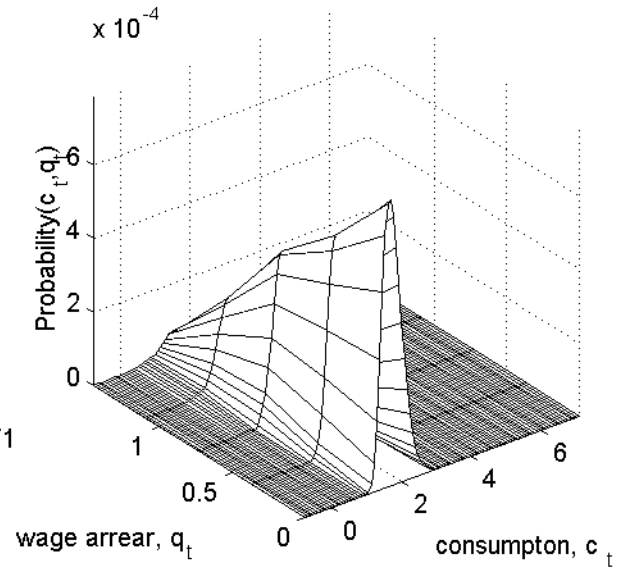Figure 8. Simulated distribution of consumption and wage arrears.

Figures 5-8: Benchmark parameterization, d=0.0865, α=0.4687, δ=0.9420, d_q=0.4743, ψ_q=0.8427, σ_q=0.4770.

It turns out that the above decrease in both capital and labor income leads to a significant reduction in total income, $\Delta I = -15.120\%$. As follows from our previous discussion, the main determinant of the income loss in our model is a very large depreciation rate of wage arrears, $d_q = 0.474$. Indeed, our estimates imply that about a half of the wage bill, which was not paid to the agent in time, is lost after a one-year delay. The high depreciation rate we obtain for wage arrears is presumably explained by a high inflation rate in Ukraine, which was, on average, equal to 63.40% per year over the period 1994-2001.[18]

We now evaluate the social welfare loss resulting from wage arrears. Social welfare falls for two reasons: First, as a result of a reduction in aggregate income, the aggregate consumption goes down. Secondly, because of idiosyncratic uncertainty about wage payments, individual consumption is volatile, which is disliked by risk-averse agents. In the benchmark case, the aggregate consumption loss is $\Delta C = -12.845\%$. To measure the cost of consumption fluctuations, we proceed as follows: We first compute a consumption premium, Γ, which is required for compensating the individual for the total welfare loss associated with wage arrears

$$E\left[u\left(c_t + \Gamma\right)\right] = u\left(C^{ss}\right), \tag{22}$$

where C^{ss} denotes the non-stochastic steady state consumption in the economy without wage arrears (in *Table* 3, we report $\Delta^u C \equiv -\Gamma/C^{ss} \times 100\%$). We then measure the cost of the consumption fluctuations as the difference between total welfare loss and aggregate consumption loss, $\Delta^u C - \Delta C, \%$. In the benchmark case, we obtain the cost of consumption fluctuations, which is equal to 0.958%.[19]

Our next step is to investigate how our results depend on the value of the risk-aversion coefficient, γ. The introspection of the results in *Table* 3

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[18]We compute the inflation rate, π, according to $CPI_{1994}\left(1 + \pi\right)^7 = CPI_{2001}$, where CPI_{1994} and CPI_{2001} are the consumer price indices in the years 1994 and 2001, respectively, as reported by UEPLAC (2001).

[19]Lucas (1987) proposed measuring the cost of business cycle fluctuations by a consumption premium, which must be given to the agent in order to provide her with the same utility level as the one that would be derived from the expected consumption. In the same way, we can measure the cost of consumption fluctuations by a consumption premium Γ' satisfying $E\left[u\left(c_t + \Gamma'\right)\right] = u\left(E\left[c_t\right]\right)$. We find that, under this measure, the cost of consumption fluctuations is slightly higher. For example, in the benchmark case, it is equal to 1.120%.

reveals the following tendencies: As agents become more risk averse, they increase their precautionary savings, which reduces the aggregate capital income, RK, and raises the aggregate labor income, Ω. The total effect of this on aggregate income and aggregate consumption is positive, i.e., the income and consumption losses reduce. In contrast, the cost of the consumption fluctuations goes up. The quantitative expression of the effects associated with variations in γ can be very significant. For instance, as γ increases from 1 to 10, precautionary savings rise from 2.936% to 38.961%; aggregate income and consumption losses are reduced from 15.120% and 12.845% to 10.025% and 8.229%, respectively; and, the welfare loss from the consumption fluctuations rises from 0.958% to 5.894%.

We study the robustness of the model's predictions with respect to changes in α, δ, d by setting these parameters to values that are standard in macroeconomic literature: $\alpha = 0.36$, $\delta = 0.96$ and $d = 0.1$. We also analyze the effects of variations in the other parameters, specifically, a 10% increase in d_q, a 10% decrease in ψ_q and a 10% increase in σ_q relative to their benchmark values. Overall, the quantitative implications of our model proved to be robust to these modifications (see the last six columns in $Table\ 3$).

One probable shortcoming in our analysis is that our calibration procedure neglects the effects associated with permanent heterogeneity in skills (productivities) by assuming that, in the absence of wage arrears, all agents would earn the same wage, W. We resort to this assumption because of the data problem. Specifically, we have data on effective but not on contractual wages. In order to evaluate the extent to which our results are contaminated by the skill heterogeneity, we compute the model's predictions under two alternative calibration procedures. One was to split the sample into groups by education and to weight the wages and wage arrears of agents by the coefficients that reflect wage differentials across the educational groups distinguished. The other was to adjust the wages and wage arrears of the agents according to the wage differentials across qualification-profession groups. The tendencies described in this section proved to be robust to these modifications as well; and the quantitative expression of the effects associated with wage arrears was comparable to that we had under the benchmark calibration procedure.

5   Conclusion

The phenomenon of wage arrears is an exceptional feature of transition economies. We show however that this phenomenon can be addressed in the context of the neoclassical growth model, which is standardly used for studying economic issues relevant to developed market economies. We argue that the effect of wage arrears on the individual consumption-savings behavior is similar to that of idiosyncratic shocks to productivity. We distinguish two types of costs associated with wage arrears. First, there is a reduction in effective wages because of the depreciation of wage arrears, and secondly, there is a welfare loss due to consumption fluctuations. In a calibrated version of the model, we find that these costs are substantial: consumption falls by $8\% - 16\%$, and welfare loss resulting from idiosyncratic uncertainty is equivalent to an additional consumption loss of $1\% - 6\%$.

In our economy, wage arrears is a survival strategy of the firms that incur losses during the transition process. In the considered setup, the firms cannot influence the amount of the losses, so that wage arrears are perpetuated forever. Our model has another interesting implication in this respect. Even if the firms were able to abandon the loss making activities at zero cost (and hence, to resolve the problem of wage arrears), they would not necessarily do so. Indeed, the firms have no incentives to reduce losses as long as they can shift such losses to their workers. The solution to the problem of wage arrears would therefore be the development of economic institutions (e.g., legislation, trade unions) which would make it difficult for the firms to practice wage arrears. On a final note, we should express our hope that wage arrears in transition economies will soon be a part of history.

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6   Appendices

In Appendices A and B, we provide the description of the data used and outline the key steps of the numerical algorithm, respectively.

6.1   Appendix A

The household survey "Ukraine 96" is carried out by Kiev International Institute of Sociology, and it contains information on 5403 Ukrainian households. From the whole sample, we select a subsample where household heads were employed. We restrict attention to wages and wage arrears of the household heads on the main job (the one that brings the largest income). We define the agent's monthly wage as the sum of both monetary and non-monetary compensation, as estimated by respondents, specifically,

$$\omega^{month} = ZH28 + ZH31 + ZH34,$$

where $ZH28$ is the net working compensation, $ZH31$ is the estimated cost of goods received, and $ZH34$ is the estimated cost of privileges.

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We construct the distribution of yearly wages by bootstrapping, specifically, we compute the yearly wage as a sum of 12 random draws from the constructed distribution of monthly wages, i.e., $\omega^{year} \equiv \sum_{i=1}^{12} \omega_i$, where $\omega_i \sim \{\omega^{month}\}$. The mean and standard deviation of wages, which we report in $Table$ 1, are computed over the sample with 1000000 observations.

The mean and standard deviation of wage arrears in $Table$ 1 are those of the variable $Z25$, which is the amount of money that the owner or the administration owes to the household head on the main job.

To draw the empirical distribution of wealth and wage arrears in $Figure$ 2, we use the variable $I20$, which is the individuals' subjective evaluations of their material status relative to the one of other people in their city (village, town). This variable takes values from 1 to 7, which correspond to evaluations "much lower than average", "lower than average", "a bit lower than average", "average", "a bit higher than average", "higher lower than average" and "much higher than average", respectively.

To construct the educational and profession-qualification groups, employed for the sensitivity experiments in Section 3.2, we use the variables $E3$ and $ZH18$, which are the education and profession (qualification) of the household head, respectively.

The Ukrainian aggregate data such as the GNP, personal consumption, wages, government expenditures, wage arrears, gross payables, gross receivables, population, CPI, and PPI are quarterly time series coming from UE-PLAC (2001). All the series except of the one for wage arrears range from 1994:1 to 2001:4; the series for wage arrears ranges from 1996:3 to 2001:4. We convert the nominal series into the real ones by using the CPI.

As a measure of output in the model, Y, we use the variable GNP in the data. We define the aggregate consumption in the model, C, as the sum of the personal and government consumption. The time series data on the government consumption are not available. However, UEPLAC (2001) provides a detailed budget of the Ukrainian government for the year 1998, which allows us to roughly estimate how the total government expenditures are subdivided between consumption and investment. We define the government investment as the sum of the government expenditures on the R&D, education, construction, health care, telecommunication, transportation and the reserve funds, and we define the government consumption as the rest of the government spendings. With this definition, we have that about half of

the government expenditures goes to investment and the other half goes to consumption (the exact estimates of the investment and consumption shares in the total government expenditures are equal to 0.48 and 0.52, respectively). We therefore construct the series for consumption by summing up personal consumption and 0.52 of the total government expenditures. The aggregate labor income in the model, Y^L, is defined as the aggregate wage bill excluding the wages paid by collective agricultural enterprises. The estimates of the Ukrainian capital stock, K, are available for the years 1996, 1998, 1999 and 2000 from Derzhcomstat (2000). We convert this variable into the real terms by using the PPI.

6.2 Appendix B

To solve for the individual asset demand function, we use an algorithm that computes a solution to the Euler equation (14) on a grid of prespecified points. We restrict wage arrears to belong to the interval $q \in \left[\underline{q}, \overline{q}\right]$ and split this interval into H equally-spaced points $\{q_1, ..., q_H\}$. We then compute the transitional probabilities associated with the autoregressive process for wage arrears (4) (i.e., the Markov chain). For each state $q \in \{q_1, ..., q_H\}$, we parametrize the asset demand by a function of the currently available resources z. The grid for the resources consists of M equally spaced points in the range $[\underline{z}, \overline{z}]$. The minimum of resources, $\underline{z}$, is achieved when the agent has no asset holdings and receives the minimal possible labor income $\underline{\omega}$ (i.e., faces the maximum possible increase in wage arrears, from q_1 to q_H). The maximum of resources is $\overline{z} = \overline{\omega} + (1 + R)\overline{a}$, where $\overline{a}$ is the maximum sustainable capital stock (i.e., the solution to $F(\overline{a}) = d\overline{a}$), and $\overline{\omega}$ is the maximal possible labor income (i.e., the maximum possible decrease in wage arrears, from q_H to q_1). In particular, our construction implies that the individual asset holdings are restricted to be in the range $[0, \overline{a}]$. Our baseline parameterization is $H = 5$ and $M = 100$. To evaluate the asset function outside the grid, we use a cubic polynomial interpolation.

We employ the algorithm, which iterates on the Euler equation. By substituting consumption from the Euler equation (14) in budget constraint (10), we obtain

$$\widehat{a}' \leq z - \left[\delta \sum_{q \in \{q_1, ..., q_H\}} \frac{(1 + R) Prob(q' \mid q)}{(A(z, q)(1 + R) + \omega' - A(A(z, q), q'))^\gamma}\right]^{-1/\gamma}, \quad (23)$$

where $\omega' = q\left(1 - d_q\right) + W - q'$.

Consequently, we implement the following iterative procedure:

- *Step* 1. Fix some asset function on the grid, $A\left(z, q\right)$.

- *Step* 2. Use the assumed decision rule for assets to calculate the right side of the Euler equation (23) in each point on the grid. The left side of the Euler equation will be the new asset function, $\widetilde{A}\left(z, q\right)$.

- *Step* 3. Compute the asset function for the next iteration $\widetilde{\widetilde{A}}\left(z, q\right)$ by using the updating:

$$\widetilde{\widetilde{A}}\left(z, q\right) = \eta \widetilde{A}\left(z, q\right) + \left(1 - \eta\right) A\left(z, q\right), \qquad \eta \in \left(0, 1\right].$$

For each point, such that $\widetilde{\widetilde{A}}\left(z, q\right)$ does not belong to $[0, \overline{z}]$, set $\widetilde{\widetilde{A}}\left(z, q\right)$ equal to the corresponding boundary value.

Iterate on *Steps* $1 - 3$ until the fixed point is achieved with a given degree of precision, $\left\| \widetilde{\widetilde{A}}\left(z, q\right) - A\left(z, q\right) \right\| < 10^{-10}$, where $\|\cdot\|$ is the L^2 distance.

We solve for the interest rate and wage corresponding to a given asset function $A\left(z, q\right)$ by computing an invariant probability distribution of the total resources and wage arrears, $Prob\left(z, q\right)$, as described in Rios-Rull (1999):

$$Prob\left(z', q'\right) = \sum_{q \in \{q_1, \ldots, q_H\}} Prob\left(A^{-1}\left(\widehat{a}', q\right), q\right) \cdot Prob\left(q' \mid q\right),$$

$$\widehat{a}' = \frac{z' - \left(q\left(1 - d_q\right) + W - q'\right)}{1 + R},$$

where $A^{-1}\left(\widehat{a}', q\right) = \{z, \ \widehat{a}' = A\left(z, q\right)\}$ is the inverse of the asset demand function $A\left(z, q\right)$.

Finally, to solve for the equilibrium fixed-point interest rate (the stochastic steady state), we use a bisection method proposed in Aiyagari (1994).