SOLOMON'S DILEMMA: AN EXPERIMENTAL STUDY ON DYNAMIC IMPLEMENTATION*

Giovanni Ponti, Anita Gantner, Dunia López-Pintado and Robert Montgomery**

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Correspondence to: Giovanni Ponti, Departamento de Fundamentos del Análisis Económico – Universidad de Alicante - 03071 Alicante - SPAIN – Tel.: +34 96 590 3619 - Fax: + 34 96 5903898, e-mail: giuba@merlin.fae.ua.es.

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ABSTRACT

This paper reports an experimental investigation on two mechanisms for the so-called *King Solomon Dilemma*, where one of them fails to implement the social choice rule *dynamically*. We compare the two mechanisms in terms of their welfare, incentive and learning properties.

JEL Classification Numbers: C70, C78

Keywords: experiments, implementation, backward induction, bounded rationality.

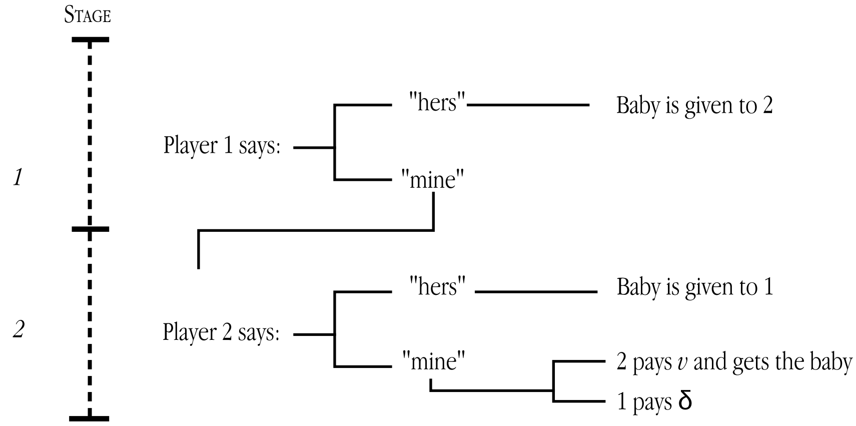

Figure 1: Glazer and Ma's mechanism (GMM).

1 Introduction

This paper reports an experimental study on *King Solomon's Dilemma*, a simple allocation problem inspired by the biblical episode.

1.1 Solomon's Dilemma

King Solomon is called to resolve a dispute between two women, "Anna" and "Betta", who both claim to be the mother of a baby. In economic terms, an indivisible prize is to be allocated between two individuals who hold different evaluations. The identity of the "true mother" (i.e. the agent with the highest evaluation) is common knowledge between Anna and Betta, but unknown to Solomon, whose objective is to rightfully resolve the dispute at no cost for the true mother.

To solve this dilemma, Glazer and Ma [12] propose the simple mechanism sketched in Figure 1, to be used when there are only two admissible evaluations, $\underline{v}$ and $\bar{v}$, with $\underline{v} < \bar{v}$. Denote by $v_i \in \{\underline{v}, \bar{v}\}; i = A, B$, i's evaluation and call player k, $k = 1, 2$, the player moving at stage k. According to the game-form of Figure 1 (labeled as "GMM" in what follows), Anna and Betta sequentially announce the identity of the true mother. The mechanism is designed in such a way that a statement in which Anna (Betta) attributes the baby to Betta (Anna) is never challenged. If both claim to be the true mother, then player 1 pays a penalty $\delta > 0$, while the prize goes to player 2, subject to a lump-sum transfer to Solomon whose utility value, $v \in (\underline{v}, \bar{v})$ lies somewhere in between the two evaluations.

It is not difficult to show that GMM implements the first-best as the unique subgame-perfect equilibrium of the induced game. To see this, assume (without loss of generality) that Anna is the true mother (i.e. $v_B = \underline{v} < \bar{v} = v_A$).[1] If Anna is player 2, she has an incentive to claim, since $v_A > v$, whereas Betta as player 1 would have to pay the penalty δ if she claims as well. If Betta is player 2, she has no incentive to claim, since $v_B < v$, which, in turn, makes it worth for Anna to claim as player 1. Thus, the unique subgame-perfect equilibrium of the game induced by the mechanism requires Betta to attribute the baby to Anna, independent of the order in which the two mothers are called to speak.

However, the game induced by GMM has many other "social inefficient" Nash equilibria. Precisely, there is a component (i.e. a closed and connected set) of Nash equilibria in which Anna, conditional on being selected as player 1, gives up the baby under the ("incredible") threat that Betta will claim in return (leaving Anna without the baby and with a penalty to pay).[2] Thus, to ensure that the first-best is achieved by way of GMM, we need to assume that both mothers are rational (in the sense that they would never use a dominated action), and they know that their opponent is also rational.

1.2 (Monotonic) Dynamic Implementation

Whether these are to be considered as demanding assumptions is, essentially, an empirical matter. In this respect, there is already substantial experimental evidence that casts doubts on the use of standard game-theoretic equilibrium notions to describe how people play games in the lab. What we learn from experiments is that subjects often *fail* to play the equilibrium, especially if the equilibrium notion is fairly refined (as is the case of subgame perfection).[3] Better results are observed when subjects can acquire some experience through repeated play.[4]

Prompted by these experimental findings, Ponti [21] approaches Solomon's dilemma taking *bounded rationality* into account. The underlying theory is based upon an alternative definition of implementation. Among the variables that specify the "environment" in which the mechanism is supposed to operate, this definition includes the learning protocols agents may use, as well as initial conditions of the learning process. According to this alternative approach, a social choice rule will be said to be *dynamically implemented* by a mechanism if, for all possible environments (i.e. preferences, adjustment processes, initial

[1] Throughout the paper, we shall always associate Anna with the *role* of the agent with the high valuation (i.e. the "true mother").

[2] This threat is to be consider "incredible" in the spirit of subgame-perfection insofar, by claiming the baby, Betta would choose a dominated action.

[3] See McKelvey and Palfrey [15] and Binmore *et al.* [3].

[4] See Güth *et al.* [13] and Cooper *et al.* [7].

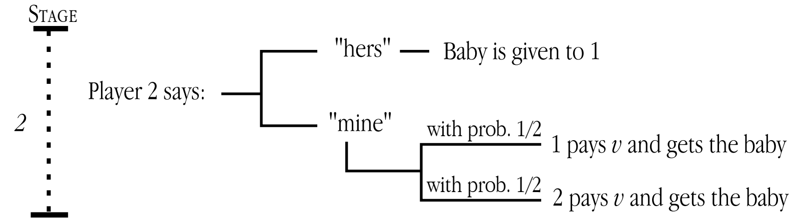

Figure 2: Ponti's mechanism (PM)

conditions), the limiting set of outcomes *i*) coincides with the set of outcomes of the social choice rule and *ii*) is also *asymptotically stable*, that is, robust to arbitrarily small perturbations.[5]

As for the dynamic implementation of GMM, Ponti [21] shows that if the learning dynamics satisfy Nachbar's [18] *monotonicity* condition,[6] then many equilibria in the inefficient Nash equilibrium component can be limit points of the adjustment process. In other words, GMM fails to implement the first-best *dynamically*, if repeated play evolves according to monotonic dynamics.

1.3 An Alternative Solution

To solve this problem, Ponti [21] proposes an alternative mechanism ("PM" hereafter). Its game-form is shown in Figure 2. According to the biblical story, Solomon was able to solve the dilemma by threatening to "split the baby in two". By using PM, Solomon can still achieve his goal by introducing a lottery in which splitting occurs "in expected terms", with no risk of blood being spilled.[7] Since no penalty is levied to either player (i.e. no threat is possible), Anna has now a weakly dominant strategy at her disposal (i.e. claiming the baby under all possible contingencies). This, in turn, implies that PM is not only *subgame-perfect* implementable, but also *Nash*-implementable, hence every Nash equilibrium of the induced game is outcome-equivalent to the first-best. As far as its dynamic implementation, PM is also dynamically implementable with monotonic dynamics.

[5] Cabrales and Ponti [6] discuss in detail the rationale behind this alternative approach (see also Sandholhm [22]).

[6] This condition essentially implies higher growth rates for those strategies which perform better and is generally satisfied by all the adjustment processes applied in the evolutionary literature to model bounded rationality. One particularly well known member of the class of monotonic dynamics is the *Replicator Dynamics* of evolutionary game theory (Taylor and Jonker [24]). These dynamics have been given a learning theoretic foundation by Börgers and Sarin [4] and they can also be interpreted as a model of imitation (Schlag [23]).

[7] Although non standard, the use of lotteries is not new in the implementation literature (see, e.g., Gibbard [11], Abreu and Matsushima [1] and Perry and Reny [20]).

From the above analysis, a testable theory evolves: does PM perform better than GMM? The aim of this paper is to compare experimentally the performance of these two mechanisms and to explore (whether and) how subjects modify their response when the game-form is played repeatedly.

1.4 Related Studies and Hypothesis

Among the experimental literature on implementation,[8]. the paper more related to ours is certainly that of Elbittar and Kagel [8]. They compare the performance of the mechanisms proposed by Moore [17] and Perry and Reny [20] to solve Solomon's Dilemma.[9] An important finding of their investigation is the *lack of efficiency* in both mechanisms mainly due to Betta's behavior. These features of the experimental data do not differ significantly between the two mechanisms.

Elbittar and Kagel [8] distinguish between two different sources of 'irrational behavior" for low-value players:

- *Rivalrous behavior.* Betta is willing to pay in order to make costly for Anna to get the prize.
- *Level of aspiration.* Betta is willing to loose money to get the prize.

They conclude that neither effect seems to be relevant in explaining Betta's (out-of-equilibrium) behavior.

As we just explained, there are both static and dynamic grounds to prefer PM to GMM. This is what we investigate first. However, our hypotheses for the experimental investigation concern not only *outcomes*, i.e. whether we can observe a significant difference in the frequency of first-best outcomes, but also *behavior*, that is, whether *i)* there is evidence that Anna takes advantage of the weakly dominant strategy provided by PM and *ii)* if the evolution of subjects' -individual and aggregate- behavior is monotonic. A third question that we pursue refers to a more general issue of applicability of mechanisms: does a quantitative change in the *payoff structure* matter? In particular, will it affect players' behavior if we *(i)* vary the levels of players' evaluations and *(ii)* vary the difference between their valuation and the price they might have to pay? Since qualitative incentives remain unchanged, theory would predict no difference in either player's behavior.

[8]See, for example, Cabrales *et al.* [5] and Katok *et al.* [14]

[9]Moore's [17] mechanism is a modification of GMM when evaluations are drawn from a finite set. In this case, if both players claim, then the prize is allocated by way of an (out-of-equilibrium) auction. Perry and Reny's [20] mechanism employs the iterated elimination of weakly dominated strategies and is supposed to be used in presence of incomplete information of the agents' evaluations.

1.5 Results

Our experiment shows a practically identical relative frequency of first-best outcomes, together with a significant evidence of inefficient outcomes in both mechanisms (about $\frac{1}{3}$ of the total observations). This is mainly due to Betta's behavior: she claims significantly more often than what theory would predict. Consistently with Elbittar and Kagel [8], we cannot explain this evidence by irrationality alone, rivalrous behavior or level of aspiration.

However, we also see that subjects react "strategically" to the alternative incentive structures provided by the two game-forms. Consistently with our theoretical conjecture, Anna as player 1 claims the prize significantly more often in PM than in GMM. In consequence, *second-best* efficiency (that is, the ability to allocate the prize to the highest evaluation player) is significantly higher in PM than in GMM.

The experimental data also show a significant difference in behaviors across *games* (that is, varying monetary payoffs). In particular, Betta's ("irrational") choice of claiming the prize is more likely to occur when its opportunity cost (that is, the amount of money Betta is going to loose) is lower. This effect is more evident in PM rather than in GMM.

As for our dynamic (monotonicity) assumption, this is consistent with the evolution of play, both at the aggregate and at the individual level. In other words, monotonicity seems the appropriate dynamic property to describe the evolution of subjects' behavior. On the other hand, the analysis of individual behavior also shows that, contrary to what monotonicity assumes, *memory matters*, that is, players' behavior is sensitive not only to current payoffs but also to cumulative payoffs. From the analysis of the individual data we also observe that individual fixed effects -that is, related to each individual subject- are stronger in Anna's population, while common fixed effect -that is, related to all subjects in the same role- are stronger in Betta's population. In other

1.6 Things to come

The remainder of the paper is arranged as follows. Section 2 provides a brief synopsis of the theory underlying the experiment, as developed in Ponti [21]. Section 3 describes the experimental design, while section 4 summarizes the results. Finally, section 5 concludes, followed by an Appendix containing some explanation of the test on aggregate learning and the experimental instructions.

2 The Theoretical Background

Let G (Γ) be the game induced by GMM (PM) when Anna is the true mother, with G^k (Γ^k) denoting the subgame of G (Γ) in which Anna is player k.

Let also $x \equiv (x_i^k) \in \Theta \equiv [0,1]^4, i \in \{A, B\}, k \in \{1, 2\}$ denote the vector collecting all behavioral strategies for game G (Γ). In other words, $x_i^k \in [0,1]$

is the probability with which a player in i's role claims the prize.

The evolution of $x(t)$ is given by the following system of continuous-time differential equations:

$$\dot{x}_i^k = f_i^k(x), \tag{1}$$

with $f_i^k : \Theta \to \Re$ satisfying standard regularity conditions.[10] We specify our *boundedly rational environment* as the set of dynamics (1) which satisfy the following

Assumption 1 (Behavioral monotonicity) *A regular dynamic (1) is called behavioral monotonic if, for all x, i and k,*

$$f_i^k(x) \geq 0 \iff u_i^k(C, x) \geq u_i^k(\bar{C}, x), \tag{2}$$

where $u_i^k(C, x)$ ($u_i^k(\bar{C}, x)$) denotes the (VNM) utility associated to (not) claiming in role i and at stage k if the state is x.

First introduced by Nachbar [18], condition (2) is commonly used in the evolutionary literature to capture the essence of a *selective* process. Given the current state of the system, $x(t)$, behavioral strategies with higher expected payoff should grow faster.

As we explained in the introduction, we shall employ *asymptotic stability* and *global convergence* as sufficient conditions for the first-best outcome to be *dynamically implementable.*[11]

Proposition 1 *If repeated play evolves according to monotonic dynamics, then the first-best can(not) be dynamically implemented by PM (GMM).*

Proof. See Ponti [21].[12]■

To understand this result, notice that, unlike PM, GMM is *not* Nash implementable. This is because GMM induces a game which has a *component* of Nash equilibria in which Anna, conditional on being player 1, gives up the baby under the ("incredible") threat that Betta will claim in return. This inefficient Nash equilibrium component is "reachable" by a a non-zero measure of interior initial conditions (the exact measure depending on the dynamics). This, in turn, implies that GMM fails to implement the first best dynamically, insofar the requirement of *global convergence* is violated. However, both mechanisms pass the *stability test,* that is, for initial conditions starting sufficiently close to the first-best, both mechanisms display the desired stability properties.

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[10] See Weibull [25].

[11] By asymptotic stability, every trajectory starting arbitrarily close stays sufficiently close and eventually converges to the solution. By global convergence, every trajectory converges to the first-best. For formal definitions, see Weibull [25].

[12] In Ponti [21], the evolutionary properties of the two games are proved for a two-population model (one population of Bettas and one population of Annas) in which agents play *pure* strategies. It can be shown that the same result also holds in the current framework, that is, with (1-population) behavioral strategies.

3 The experimental design

In what follows, we describe the features of the experiment in detail. To compare the experimental properties of the two mechanisms, we run a total of eight experimental sessions. Four of those (the *American sessions* hereafter) were conducted in March 1999 at the University of California, Santa Barbara. Four additional sessions (the *Spanish sessions* hereafter) were run in May 2001 at the Universidad de Alicante, with the specific aim to study learning effects. In what follows, we shall discuss the experimental design of the American sessions.[13]

Subjects A total of 160 students (20 per session) were recruited among the undergraduate student populations of the Santa Barbara and Alicante.[14] Each session lasted for approximately one hour.

Treatment. All experimental sessions were run in a computer lab.[15] As for the American sessions subjects participated to a single session only, playing 20 rounds of GMM or PM. We run two PM session and two GMM sessions. Subjects were informed that their opponent would change in each round, that they would be in different roles and positions over the 20 rounds and that their evaluations would change from one round to the next.

Instructions were provided by a self-paced, interactive computer program that introduced and described the experiment. Four dry rounds of the respective mechanism were played during the instructional phase, so as subjects could familiarize with the basic rules of the game and the computer interface. In each round, before they were asked to make a decision, subjects were informed about their own evaluation, their opponent's evaluation, the price v and the penalty δ (in the case of GMM). Since the game was played sequentially, subjects selected as player 2 were also informed about player's 1 action. At the end of each round, each player knew about the the game outcome and the monetary payoff associated with it.

Payoffs. All subjects received eight dollars ($ 8) to show up. Among the 20 rounds of each session, subjects experienced 4 times each of the payoff treatments as shown in Figure 3.[16] In this way we could evaluate how subjects played under different incentive schemes. In GMM, the penalty δ was equal to $3 in all payoff treatments.

[13]As for the specific design features for which the Spanish sessions were different, see subsection *Learning* below.

[14]Mainly, undergraduate students from the Economics Departments with no (or very little) prior exposure to game theory.

[15]The experiment was programmed and conducted with the software z-Tree (Fischbacher [9]). The complete set of instructions can be found in the Appendix.

[16]All payoffs are expressed in USD.

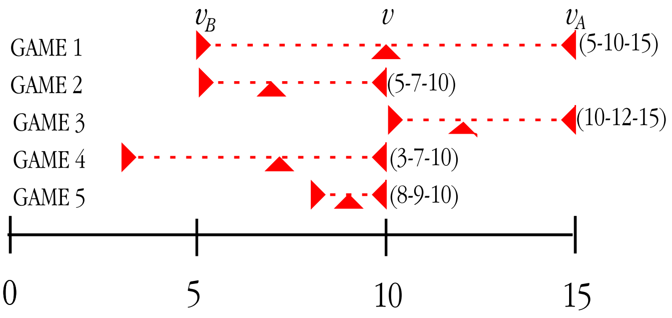

Figure 3: The payoff treatments

At the end of each session, a subject was asked to publicly draw one round, and the monetary payoff of that round was then added (or subtracted, in the case of a negative payoff) to the show-up fee.[17] Average earnings were \$ 12, including the participation fee.

Matching. All four American sessions shared the same matching assignment, according to the following rules.

Among the 20 rounds, subjects experienced both roles (10 times as Betta and 10 times as Anna) as well as both positions (10 times as player 1 and 10 times as player 2). The reason for this design is that we wanted to evaluate how subjects formed a *full* contingency plan, experiencing every possible role and position.[18] Moreover, we wanted to avoid the possibility that being consistently assigned Betta's role in the game would cause a bias, e.g., frustration.

Moreover, all subjects played subgames G^1 or Γ^1 (G^2 or Γ^2) within rounds 1-5 and 11-15 (6-10 and 16-20). We also kept the same order of the 5 different payoff treatments. GAME 1 was played in all matches of rounds 1,6,11,16; GAME 2 of rounds 2,7,12,17 and so on.

These features were not publicly announced, and we shall analyze the experimental data under the assumption that subjects did not recognize those patterns. Since roles, positions, opponents and payoffs were changing at each round, we believe it was very difficult for subjects to keep track of any systematic assignment.

4 Results

In analyzing the experimental data, we first look at the distribution of outcomes, behavior and incentive effects induced by the two mechanisms. We shall use the

[17]This feature of the experiment was also announced in advance.

[18]This feature of the experimental design differs from that of Elbittar and Kagel [8] in which subjects always play either as Betta or Anna in all repetitions.

American sessions for this purpose. Later, we also check for learning effects in the Spanish sessions.

4.1 Outcomes.

The aggregate statistics of the experimental outcomes are summarized in Figure 4. GMM (PM) results are reported in the bottom-left (top-right) corner of each cell.

We begin by noting that, relative frequency of first-best outcome (row 1a)) was virtually identical, 68 % (69 %) of the cases in GMM (PM). With this difference not being statistically significant,[19] we can consider both mechanisms as successful in implementing the first-best, if compared with the outcome of a random mechanism in which the price is assigned to either player with equal probability.[20]

Things are different if we look at second-best efficiency, measured as the relative frequency with which the prize is assigned to Anna, even for those cases in which she has to pay $v < v_A$ to get it (row 1)). Again, this frequency is higher in PM (84%) than in GMM (79%), but now the null hypothesis can be rejected at the 10 % confidence level.[21] We also see that the two mechanisms differ in the relative frequency with which first and second-best outcomes are achieved conditional on players' positions. While in PM the differences in frequencies of

[19]variable which is 1 if the $i-$th match of experiment j is first-best efficient and 0 otherwise, with $j = 0$ for GMM and $j = 1$ for PM. Thus, the distribution of X_i^j is Binomial $B(1, p^j)$. If we assume that all 400 repetitions of each experiment correspond to independent and identically distributed (i.i.d.) observations, then $\bar{p}^j \equiv \frac{\sum_{i=1}^{400} X_i^j}{400}$ has normal asymptotic distribution $N(p^j, \frac{p^j(1-p^j)}{400})$. If the null hypothesis is true (i.e. first-best efficiency in GMM at least as high as in PM), then

$$Z \equiv \frac{\bar{p}^1 - \bar{p}^0}{\sqrt{\frac{\bar{p}^1(1-\bar{p}^1)}{400} + \frac{\bar{p}^0(1-\bar{p}^0)}{400}}}$$

is asymptotically distributed as a standard normal random variable. Hence, Z can be used as test statistic. Since we hold a directional hypothesis, we use a one-tailed test, rejecting the null hypothesis at three different significance levels. If the null hypothesis is rejected at the 10 % (5 %, 1% significance level, resp.) the symbol * (**, *** resp.) appears in the top-right corner of the corresponding cell. Unless otherwise stated, all other tests are conducted with entirely similar arguments.

[20]In both cases, the corresponding Z tests reject the null hypothesis at the 1 % confidence level.

[21]Our data on GMM can be compared with Elbittar and Kagel's [8] evidence on Moore's [17] mechanism, to be considered as the natural extension of GMM in the case of multiple evaluations. While [8] reports a significantly lower frequency of first-best outcomes (37 %) than in GMM (possibly due to the higher complexity of that game), the relative frequency of second-best-outcomes (82 %) is not statistically different from that of GMM.

	Total	As Player 1	As Player 2
Observations	400 400	200 200	200 200
1) Anna gets the prize	.84* .79	.85 *** .65	.825 .95
1a) At no cost	.69 .68	.7 .65	.68 .72
1b) Paying v	.15* .11	.15 0 (N/A)	.14 ** .22
2) Betta gets the prize	.16 .21	.11 0	.24 *** .41
2a) At no cost	.05* .08	.01 0	.09 ** .15
2b) Paying v	.11 .13	.1 0 (N/A)	.15 *** .26

Figure 4: Outcomes for the two mechanisms

first and second-best outcomes conditional on players' position are not statistically significant, in GMM the probability of achieving second-best efficiency is higher (statistically significant at the 1 % significance level) when Anna is player 2. This clearly depends on how GMM is designed (if both players claim, the prize goes to player 2).[22]

As for the inefficient outcomes, our particular interest is to check the case in which Betta as player 2 gets the prize at no cost (i.e. Anna as player 1 gives up the prize), which is a Nash equilibrium in GMM but not in PM. In this respect, we find (see Figure 4, row 2a)) that this outcome occurs in 15 % of the cases in GMM, compared to only 9 % for PM. This difference is significant at the 5 %-level. Thus, the important difference in the set of equilibria provided by the two mechanisms, which allows for an inefficient Nash equilibrium only in GMM, leads to a corresponding difference in the experimental data.

4.2 Aggregate behavior

We now investigate on whether and how the two mechanisms induce subjects to behave differently at the aggregate level. Figure 5 displays the relative frequencies of subjects claiming at all information sets.

We begin by looking at player 2's behavior. Remember that both mechanisms provide Anna (Betta) with a dominant action when she is player 2: she should always (not) claim, since her (in the case of PM: expected) payoff is greater (less) than zero, which is what she would get if she doesn't claim. Therefore, *first-order rationality* requires (Betta) Anna (not) to claim in the single-person decision problem she faces at stage 2. As Figure 5 shows, this prediction is violated much more frequently by players in Betta's role, suggesting that *bounded rationality alone* cannot explain the variability of the experimental data. For example, in the case of PM, the relative frequency with which Betta as player 2 does not behave rationally, i.e. claims, is 28 % whereas, Anna, as player 2, does not behave rationally, i.e does not claim, only with a relative frequency of $100 - 98 = 2\%$.[23] Since, in GMM, Betta claims with almost equal probability both as player 1 and player 2, we cannot explain Betta's irrational behavior neither by rivalrous behavior nor level of aspiration.

We now move on to player 1's behavior. In this case, claiming is a weakly dominant action for Anna only in PM. As Figure 5, shows, this feature seems

[22] The fact that in GMM player 1 can never pay the price v is highlighted in Figure 4 by the symbol "N/A" in the corresponding cells.

[23] For both mechanisms, the null hypothesis that player 2 violates first-order rationality in both roles with equal frequency can be rejected at the 1 % confidence level. Here the null hypothesis is tested using use a χ^2 distribution, insofar our theoretical model does not yield a directional hypothesis.

to have a significant impact on Anna's behavior: she claims significantly less in GMM (87 %) than in PM (94 %) as player 1. This difference is significant at the 1 %-level.

4.3 Do Payoffs Matter?

We varied the levels of v_A, v_B and v according to five treatments, as shown in Figure 3. In comparing these five payoff parameterizations ("GAMES"), notice that

- GAME 1 and GAME 3 (GAME 3) have the highest evaluations v_A (v_B). In other words, if Anna (Betta) gets the prize at no cost, her gain would be highest in GAME 1 and GAME 3 (GAME 3). GAME 4 and GAME 5 (GAME 4) have the lowest value for v_A (v_B).
- GAME 1 has the highest payoff differences $|v_B - v|$ and $|v_A - v|$. In other words, if Anna (Betta) gets the prize and has to pay the price v, her gain (loss) would be highest. GAME 5 has the lowest payoff differences $|v_B - v|$ and $|v_A - v|$ of all games.
- GAME 2 and GAME 3 share the same payoff differences. However, GAME 3 differs from GAME 2 insofar all stakes are \$ 5 higher.
- GAME 2 and GAME 4 look identical from Anna's viewpoint (v_A and v are the same), but not for Betta's (v_B is lower in GAME 4).

We now check whether these different incentive schemes might have affected the performance of the two mechanisms, as well as subjects' behavior. The idea behind the analysis is the following: given that we observe the significant presence of inefficient outcomes (i.e. "irrational play"), are there "games" in which these phenomena are more likely to occur? If so, are subjects more sensitive to the levels of their evaluations, v_A and v_B, or rather to the differences $|v_A - v|$ and $|v_B - v|$?

The summary statistics of outcomes disaggregated for games are shown in Figure 6.[24]

Looking at first-best efficiency, Figure 6 (row 1a) shows that relative frequencies differ strongly (significant at the 1 %-level) within the respective mechanism. GAME 4 shows the highest relative frequency of first-best outcomes for both mechanisms: .85 for GMM and .81 for PM. First-best frequencies in the remaining four games all lie within the interval [.6, .68] for GMM, and [.59, .72]

[24] Also for Figure 6 p-values are evaluated from a χ^2 distribution, testing the null hypothesis that, within each mechanism, outcome distributions are independent from payoff treatments.

	Observations	x_i^k
x_A^1	200 200	.94 ** .87
x_A^2	55 46	.9 .98
x_B^1	200 200	.27 .23
x_B^2	195 182	.29 .28

Figure 5: Aggregate behavior

	Game 1	Game 2	Game 3	Game 4	Game 5	$H_0 : \chi^2 = 0$
Obs.	80 80	80 80	80 80	80 80	80 80	N/A
1)	.86 .79	.85 .75	.82 .78	.88 .9	.77 .79	$p < .1$ $p < .01$
1a)	.72 .67	.675 .6	.66 .62	.81 .85	.59 .69	$p < .01$ $p < .01$
1b)	.14 .11	.17 .15	.16 .15	.06 .05	.19 .1	$p < .05$ $p < .05$
2)	.14 .21	.15 .25	.17 .22	.12 .1	.27 .25	$p < .01$ $p < .01$
2a)	.02 .09	.05 .09	.04 .09	.01 .04	.12 .09	$p < .01$ $p > .1$
2b)	.11 .12	.1 .16	.14 .14	.11 .06	.15 .16	$p > .1$ $p < .05$

Figure 6: Outcomes for the two mechanisms disaggregated for games

for PM. Thus, in both mechanisms, GAME 4 really stands out for its relative efficiency. For PM, besides the best-performing GAME 4, we find that GAME 5 has the lowest rate of first-best outcomes, with a relative frequency of .59. Also notice that ranking games according to their efficiency does not change moving from first-best to second-best.

What does it make GAME 4 and GAME 5 so different for subjects who played PM? Remember that GAME 4 (GAME 5) has the highest (lowest) difference $|v_B - v|$ of all games. Thus, given that we observe a *violation of first order rationality* - Betta as player 2 claims the prize with a negative (expected) payoff even though she would loose nothing by not claiming- we might expect subjects to prefer a smaller loss to a bigger one. In other words, the rival's rule behavior occurs more frequently when is cheaper. This intuition is supported by our experimental data: we observe a much higher first- and second-best efficiency in GAME 4 rather than in GAME 5 . Also notice that the outcome distributions of GAMES 2 and 3 are not significantly different within the respective mechanism. That is, subjects' behavior does not seem to be sensitive to *income effects* payoff difference matter.

Again, we look at players' behavior disaggregated for games in Figure 7 to see how these differences come about. Here we find much more responsiveness to payoffs in GMM rather than in PM. In fact, the null hypothesis of no difference in behavior across games is always rejected by the experimental data in GMM and only for Betta's behavior as player 1 in PM.

Figure 7 also explains why GAME 4 performs best in both mechanisms: Anna claims more than in any other game and Betta claims less. On the other hand, the lack of efficiency in GAME 5 for PM can be explained by two facts. First, Anna as player 2 claims in only 87 % of the cases, whereas she always claims in GAME 4. Second, Betta claims 40 % of the times as player 1 in GAME 5, and never more than 30 % of the times in all other games. Similarly, Betta as player 2 claims considerably more often in GAME 5 (36 %) compared to all other games, leading to such a low rate of first- and second-best outcomes of this game in PM.

The fact that GAME 5 stands out in PM for its bad performance especially through Betta's behavior may suggest that subjects used this game mostly to try to "win the object over Anna", even in the disadvantageous position of the low-value player.[25] If so, GAME 5 would have been the one in which they could do so without losing too much, given that GAME 5 has the lowest $|v_B - v|$ of all games. In this case, the presence of a lottery in PM (and the consequent reduction of Betta's expected loss) may have played against efficiency especially

[25] This idea of *rivalrous behavior* is well-known in the experimental literature (see, for example, [10]).

in PM. To explore this conjecture, Figure 8 tests the correlation between the ranking of claiming frequencies at all information sets and the ranking of v_A, v_B, $|v_A - v|$ and $|v_B - v|$ using Spearman's correlation test.

Figure 8 confirms that in PM, Betta's costs (benefits) are negatively (positively) correlated with her relative likelihood to claim. Both these correlations are statistically significant. This feature of the experimental data may suggest an easy modification in the structure of PM to increase its efficiency. Remember that, for the mechanism to work, Solomon is assumed to know Anna's and Betta's true valuations. Clearly, those values are not subject to Solomon's control. However, he can set the discriminatory price v so as to maximize Betta's (expected) loss, measured by the the difference $|v_B - v|$, since Anna's observed behavior appears to be less sensitive to $|v_A - v|$.

4.4 Learning

In this section we shall address the following question: what (if at all) did subjects learn through the course of the experiment? In particular: was their behavior consistent with our monotonicity assumption? This assumption has already been described in Section 2 and heuristically reflects the idea that a strategy that has been profitable in the past will be followed by a higher proportion of subjects in the future (see equation (2)).

Consistently with our theoretical framework, this is done by looking at the evolution of subjects' *aggregate* behavior at each information set. In this case, testing for monotonicity simply consists in checking whether the evolution of behavioral strategies "moves in the direction" of current best-replies. We shall also consider learning at the individual level. That is, we adjust our theoretical assumptions to test whether subjects individual behavior was also monotonic.

The relevant statistics on outcome distributions disaggregated for periods for the American sessions are shown in Figure 9.

We shall refer to Period 1 (2,3, and 4 resp.) as the aggregate distribution of rounds 1-5 (6-10, 11-15 and 16-20 resp.). As noticed earlier,[26] subgames G^1 and Γ^1 -where Anna is player 1- were always played in Period 1 and Period 3 and subgames G^2 and Γ^2 -where Anna is player 2- were always played in Period 2 and Period 4. As for first and second-best efficiency, we find that the null hypothesis (that is, no significant difference in the relative frequency of efficient outcomes across periods) is rejected for both mechanisms.[27]

[26] See section 3.

[27] Again, a χ^2 statistics is used here to test this hypothesis.

	Game 1	Game 2	Game 3	Game 4	Game 5	$H_0 : \chi^2 = 0$
x_A^1	.95 .82	.92 .82	.94 .85	.98 .93	.9 .92	$p > .1$ $p < .05$
x_A^2	1 1	.92 1	.85 1	1 1	.88 .9	$p > .1$ $p < .05$
x_B^1	.25 .22	.3 .3	.28 .25	.11 .11	.4 .25	$p < .01$ $p < .05$
x_B^2	.26 .3	.3 .39	.38 .37	.23 .12	.36 .32	$p > .1$ $p < .01$

Figure 7: Aggregate behavior disaggregated for games

	v_A	v_B	$\lvert v_A - v \rvert$	$\lvert v_B - v \rvert$
x_A^1	.25 -.375	-.6 -.15	.6 -.5	$-.85^{**}$ -.22
x_A^2	-0.12 0.25	-0.87^{**} -0.25	0.5 0.5	0.75^{*} 0.5
x_B^1	-0.25 -0.12	.6 .5	-.6 -.3	$-.85^{**}$ -.62
x_B^2	0 -0.25	$.85^{**}$ .2	-.6 -.2	$-.85^{**}$ -.45

Figure 8: Testing for correlation between claiming rates and payoff incentives

	Period 1	Period 2	Period 3	Period 4	$H_0 : \chi^2 = 0$
Obs.	100 / 100	100 / 100	100 / 100	100 / 100	N/A / N/A
1)	.77 / .58	.88 / .97	.83 / .66	.87 / .99	$p < .05$ / $p < .01$
1a)	.64 / .58	.68 / .7	.69 / .66	.76 / .81	$p < .1$ / $p < .01$
1b)	.13 / N/A	.2 / .27	.14 / N/A	.11 / .18	$p < .1$ / $p > .1$
2)	.23 / .42	.16 / .06	.17 / .34	.13 / .01	$p < .1$ / $p < .01$
2a)	.08 / .15	.07 / .05	.04 / .1	.01 / .01	$p < .05$ / $p < .01$
2b)	.15 / .27	.09 / N/A	.13 / .24	.12 / N/A	$p > .1$ / $p > .1$

Figure 9: Outcomes for the two mechanisms disaggregated for periods

This is mainly due to two interacting effects.

- In PERIODS 2 and 4 outcome distributions are more efficient than in PERIODS 1 and 3. In other words, for both mechanisms, first and second-best efficiency is more likely to be achieved when Anna is player 2.
- Conditional on the subgame being played, for both mechanisms, efficiency increases over time (i.e. the relative frequency of first and second-best outcomes is higher in PERIODS 3 and 4 than in PERIODS 1 and 2).[28]

To dissentangle between these two effects, we run four additional sessions that differ in the design for some crucial aspects.[29] These *Spanish Sessions* were held in May 2001 at the Universidad de Alicante. In two of the sessions subjects played 40 rounds GMM and, in the other two sessions subjects played 40 rounds of PM. Notice that we have increased the number of rounds from 20 to 40 in order to have more evidence on learning. The basic motivation for the changes in the design is to reduce the variability of the treatment parameters.

We have only considered one payoff treatment, GAME 1. We have also fixed player positions: subjects selected as player 1 (or player 2) would keep their position for the entire experiment. The only thing that could change from one period to the next was whether subjects' role was that of Anna or Betta, that is, which of the two subgames G^1 (Γ^1) or G^2 (Γ^2) was being played. This was decided by an *aggregate* random variable (that is, the same for all 10 groups) by which either subgame was selected with equal probability. This made easier the analysis of subjects' behavior in the two subgames separately.

Figure 10 traces the evolution of aggregate behavior in subgame G^1 (Γ^1) in all four experimental sessions. We have concentrated on G^1 (Γ^1) mainly for two reasons. First, because subjects' population is the same in the two subgames and we observed a very similar learning pattern. Second, and more importantly, for subgame G^2 (Γ^2) we have very few observations of players 2's behavior. This is because, when Betta is player 1, her best response is not to claim.[30]

Figure 10 shows four different diagrams, the top (bottom) diagrams refer to GMM (PM) data. The evolution of $x^1(x^2)$ is reported on the y axe of the left (right) diagrams. Each diagram traces two trajectories, one for each session. Thus, in the left (right) diagrams, the efficient subgame-perfect strategy corresponds to $y = 1$ ($y = 0$). Remember that Betta as player 2 has always a

[28] With the sole exception of subgame Γ^2.

[29] We thank an anonymous referee for suggesting changes in the original design to test learning effects more effectively.

[30] Since we are focusing on a single subgame, we shall abuse our notation by abolishing the subscript i hereafter.

dominant action: she should not claim. The same occurs for Anna in PM, since, for this mechanism, claiming corresponds to a weakly dominant action. On the contrary, Anna's best response in GMM depends on the probability with which she is to face a claiming Betta (if this probability is sufficiently high, she should not claim). Precisely, given that we only used one payoff parameter (GAME 1), this threshold value for x^2 is equal to 0.63 (the horizontal line in the top-right diagram of Figure 10).

However, Figure 10 shows that the relative frequency of subjects claiming as Betta are always lower than the threshold. This makes claiming for Anna the *expected payoff maximizing action for all sessions and periods*. In other words, the state of the system of all the adjustment processes always stays in the "basin of attraction" of the efficient (subgame-perfect) Nash equilibrium component. In consequence, monotonicity simply implies that x^1 (x^2) should rise (fall) over time. As Figure 10 shows, this theoretical prediction is somehow consistent with the experimental evidence. To proceed in the analysis, we constructed a test. This test implicitly assumes that, for any given mechanism and session, $x^k(t)$ is the sample mean of an i.i.d. distribution of subjects who claim with probability $p^k(t)$.

To see how the test is constructed, let us consider player 1's population. In this case, the test is constructed as follows:

$$H_0 : p^1(t) \leq p^1(t+1) \forall t \in \{1, ..., T\};$$

$$H_1 : p^1(t) > p^1(t+1),$$

for some $t \in \{1, ..., T\}$, where T is the number of restrictions, that is, the number of times that subgame was played in that particular session.[31] Given that, within each period, we deal with small samples we cannot use an asymptotic approximation of the statistic's distribution. In particular, we have 10 observations for player 1 and, given that player 2 moves only if player 1 claims, we have at most 10 observations for player 2. Therefore, We have employed an hypergeometric conditional distribution in order to construct the statistic.[32]

The results for all sessions $[h, j]$ are summarized in Figure 11, where $h = 0$ ($h = 1$) for GMM (PM) and $j = 1, 2$ stands for the particular session within mechanisms. The row variable k refers to player k's position. Each number in the table indicates the minimum p-value obtained computing the test across all T restrictions separately. For example, when $k = 1$ and $[h, j] = [0, 1]$ (i.e. the top-left cell in Figure 11) the minimum p-value is .23. This means that, for all restrictions, H_0 is accepted with at least a confidence level of 23%.

[31] Analogous considerations hold for player 2's population, once inequalities are reversed.

[32] See Appendix A.

	$[0,1]$	$[0,2]$	$[1,1]$	$[1,2]$
$k=1$	.23	.5	.77	.5
$k=2$	.23	.27	.23	.18

Figure 11: Individual monotonicity tests

What can we say about the test with multiple restrictions? We apply the so-called Bonferroni's method to this purpose.[33] This method says that, if we want to accept a null hypothesis that involves T restrictions with a significant level of α, then we must accept each single restriction with a significant level of $\alpha_I = \frac{\alpha}{(T-1)}$. For example, fix $\alpha = 0.1$, that is, the highest confidence level used in the paper. In session $[1,1]$, we have that T=19, which corresponds to the smallest number of restrictions across all mechanisms and sessions. To accept H_0 at a significant level of 0.1 in session [1,1] the p-values of all 19 restrictions must be higher than $\alpha_I = \frac{0.1}{18} = .005$. This corresponds to the worst possible case to accept H_0. Figure 11 shows that all values are far above this threshold. In consequence, monotonicity is overwhelmingly accepted in all sessions.

We now move to *individual learning*, that is, we consider whether individual subjects adjusted their behavior over time consistently with our monotonicity assumption. To check this, we are forced to modify monotonicity to take into account that subjects only receive a very limited information of the the current population strategy by way of their experience in the matches they were involved. Therefore, we shall assume subjects' behavior to be sensitive (in the direction monotonicity requires) *to their individual experience only.* Following Mookherjee and Sopher [16], we shall consider a model in which subjects use information regarding payoffs realized in the past, and increase over time the probability to select a strategy which yielded higher payoffs.

At any given $t \in \{2, T\}$, let $\pi_s^k(t-1)$ the payoff subject s received in the previous period she was playing as player k. Let also $\Pi_s^k(t-1)$ denote the *cumulative payoff* acquired by subject s as player k up to period $t-1$. In other words, we assume subjects modify their individual propensity to claim not only conditional to their single previous experience (as monotonicity assumes), but also to their *extended memory* of all previous observations (using $\Pi_s^k(t)$ as a proxy). Letting f denote the logistic function, this hypothesis postulates

$$p_s^k(t) = f(\beta^k + \gamma^k \pi_s^k(t-1) + \delta^k \Pi_s^k(t-1)), \tag{3}$$

where $p_s^k(t)$ is the probability with which subject s claims in period t and β^k

[33]See. for example, [19].

	Obs.	Coef.	Std. Err.	p-value	95% CI
β^1 eq. (3)	790	-1.9647	1.568	.210	[-.5.0379, 1.1085]
γ^1 eq. (3)	790	.0181	.0084	.031	[.0017, .0345]
δ^1 eq. (3)	790	.0074	.0028	.007	[.002, .0129]
γ^1 eq. (4)	230	.0149	.009	.098	[-.0028, .0325]
δ^1 eq. (4)	230	.0059	.003	.049	[0, .0118]

Figure 12: Individual learning for player 1

measures the (aggregate) fixed effect associated to player position k. An alternative approach is to include in (3) also *fixed individual effects*. This yields the following:

$$p_s^k(t) = f(\beta^k + \gamma^k \pi_s^k(t-1) + \delta^k \Pi_s^k(t-1) + \sigma_s), \tag{4}$$

where σ_s measures the (individual) fixed effect associated to subject s.

The test statistics for player 1 are summarized in Figure 12.

First, notice that estimates of γ^1 and δ^1 are all significant and of the right sign, whether we consider individual effects or not. Moreover we observe that, for both regressions (3) and (4), the explanatory power of δ^1 is higher. In other words, contrary to what monotonicity requires, *memory matters.* On the other hand, regression (4) uses a significant smaller number of observations (230 *vs.* 790) insofar, among the 40 subjects selected as player 1 (i.e. Anna), 28 of them *claimed at every period.* Remember that the estimation of (4) is only based on the observations of those subjects who used both actions throughout the experiment. For this reason we can say that, for player 1 population, *individual fixed effects are relevant*,[34] but learning effects are also relevant, even when fixed effects are taken into account.

Things are different when we look at player 2, as Figure 13 shows.

[34]Another evidence of this effect comes from the fact that, for player 1, the confidence intervals for γ^1 and δ^1 estimated from equations (3) and (4) are not included one another.

	Obs.	Coef.	Std. Err.	p-value	95% CI
β^2 eq. (3)	765	1.5808	.5093	.002	[.5821, 2.5795]
γ^2 eq. (3)	765	-.0411	.0097	0	[-.0601, .022]
δ^2 eq. (3)	765	-.004	.0008	0	[-.0055, .0025]
γ^2 eq. (4)	538	-.0089	.0107	.408	[-.0299, .0122]
δ^2 eq. (4)	538	-.003	.0009	.001	[-.0047, .0013]

Figure 13: Individual learning for player 2

Here again estimates of γ^2 and δ^2 are all of the right sign, with the impact of δ^2 "significantly higher." However, considering fixed effects makes γ^2 no longer significant.[35] Unlike the regressions for player 1, there are fewer subjects playing the same action (i.e. not claiming) throughout the session (12 out of 40).[36] When individual fixed effects are taken into account, we see that player 2's behavior is much more sensitive to cumulative payoffs.

There is another clear evidence of difference in behavior due to player's position. If we look at the estimates of β^k, i.e. the constant in regression (3), we see that only β^2 is significant, with positive sign. In other words, we observe *a tendency to claim of subjects in Betta's role which is explained neither by learning nor by individual effects.*

5 Conclusion

One of the aims of implementation theory is to provide a formal dress to the choice among competing mechanisms. In this respect, our theoretical assumption was justified by the simple claim that, *ceteris paribus*, a Nash-implementable mechanism should be preferred to a subgame-perfect implementable one. To our surprise, the experimental results reported here do *not* support this claim. Our dynamic analysis provides an explanation for this. Here we find that *stability* (rather than *convergence*) seems to be the key to understand subjects' behavior, insofar the states of the system are always sufficiently close to the first-best.

[35]Notice that this does not yield a rejection of monotonicity. By analogy with the test on aggregate learning, we would reject monotonicity if γ^2 were positive and significant.

[36]Another evidence of this comes from the fact that, for player 2, the confidence intervals for γ^2 and δ^2 estimated from equation (3) *always* contain the confidence intervals estimated from equation (4).

In this respect, the two mechanisms display very similar properties and almost identical rates of first-best outcomes.

This evidence notwithstanding, our experiment also shows that, despite a significant evidence of out-of-equilibrium ("irrational") play, *incentives matter* in the characterization of the aggregate play and that subjects react "strategically" to the competing implementation schemes. In other words, our experimental evidence can be fruitfully applied to reduce inefficiency in the presence of bounded rationality.

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Appendix A. Testing for monotonicity.

Fix a mechanism and a session $[h, j]$ and denote by by

$$y_s^k(t) \equiv \begin{cases} 1 & \text{if subject } s \text{ claims at round } t, \\ 0 & \text{if player } s \text{ does not claim at round } t. \end{cases}$$

We assume that, for all s and t, $y_s^k(t)$ follows a binomial distribution $B(1, p^k(t))$. Let $n^k(t)$ denote the number of observations of player k at time t. More precisely, $n^1(t) = 10$ for all t, since we have 10 groups per sessions and player 1 always moves. On the contrary, $n^2(t) \leq 10$ is endogenously given by the number of players 1 that have claimed at time t.

Given that we deal with small samples, we apply the following method.[37]

Consider the variables: $v = n^k(t)\chi^k(t)$, and $w = n^k(t+1)\chi^k(t+1) + n^k(t)\chi^k(t)$, where $\chi^k(t)$ is the theoretical distribution of $x_s^k(t)$. Under the null hypothesis, the variable v/w follows a hypergeometric distribution with the density function being

$$f(v/w) = \frac{\begin{bmatrix} n^k(t+1) \\ v \end{bmatrix} \begin{bmatrix} n^k(t) \\ w-v \end{bmatrix}}{\begin{bmatrix} n^k(t) + n^k(t+1) \\ w \end{bmatrix}}.$$

In this case, v/w will be the statistic we will use to test the null hypothesis H_0, but considering each of the T restrictions individually. The p-value for each of these tests can be calculated by $P(v \geq \overline{v}/w = \overline{w})$ where $\overline{v}$ and $\overline{w}$ are the statistics evaluated at the samples. The results are summarized in Figure 11, in which we present the minimum p-value across periods.

[37]See Arnold [2]

Appendix B. The experimental instructions (*American Sessions*).

Screen 1: Welcome to the Experiment

This is an experiment to study how people bargain over a prize. We are only interested in what people do on average and keep no record at all of how our individual subjects behave. Please do not feel that any particular behavior is expected from you.

On the other hand, keep also in mind that your behavior will affect the sum of money you may win during the course of this experiment.

On the following pages you will find a series of instructions explaining how the experiment works and how to use the computer during the experiment.

When you are ready to continue, please Click Here

Screen 2: How you can make money (I)

You will be playing two different experiments for 20 rounds each. In both experiments, you and another person will have to determine which of you is to receive a prize.

Please keep in mind that

- at the beginning of each round each player will be matched with a different opponent;
- at the beginning of each round a new value for the prize will be assigned to each player;
- the values that will be assigned will always differ, so that there will always be one **high-value** player and one **low-value player**;[38]
- your value determines your payoff in the game. If you get the prize, you will be awarded of the sum of money the prize is worth to you (and the same holds for your opponent).

When you are ready to continue, please Click Here

[38]The terminology of **high-value** (**low-value player**) was always associated with the color blue (red). Also payoffs in the experiments were highlighted by the corresponding color.

Screen 3: How you can make money (II)

Sometimes you will be the **high-value** player, sometimes you will be the **low-value** player. Remember that this information is PUBLIC. You and your opponent will ALWAYS know how much the prize is worth to each of you.

At the end of today's experiments, one round will be selected at random. The payoff you obtained in that particular round will be added (or subtracted, if the payoff was negative) to your showing-up fee of $ 8.

When you are ready to continue, please Click Here

Screen 4: The experimental setting

This experiment is played in Stages.

- At the beginning of each round, the computer will select at random whether you or your opponent will play at Stage 1.
- We will refer to this player as Player 1, while the player who is selected to play at Stage 2 will be referred as Player 2.
- Sometimes you will play the game as Player 1, sometimes you will play the game as Player 2.
- Remember that you and your opponent have equal chance, at the beginning of each round, to be selected to play first.

When you are ready to continue, please Click Here

Screen 5(GM): The Game

In this game Player 1 chooses either to claim the prize or not to claim the prize. If Player 1 does not claim the prize, the prize automatically goes to Player 2. If Player 1 claims the prize, then Player 2 must decide whether or not to also claim the prize.

The payoffs in the game are as follows:

- If Player 1 does not claim the prize, the prize goes to PLAYER 2. Neither player has to pay anything. If Player 1 claims the prize and PLAYER 2 does not claim the prize, the prize goes to Player 1. Neither player has to pay anything.
- If Player 1 and Player 2 both claim the prize, the prize goes to PLAYER 2. But Player 2 has to pay a price of \$ v for the prize and Player 1 has to pay \$ 3 even though s/he doesn't get the prize. The value of \$ v may differ between rounds. However, it will always be in between the **high value** and the **low value**.

We shall now practice through the various instructions, stage by stage.

When you are ready to continue, please Click Here

Screen 5(PM): The Game

In this game Player 1 chooses either to claim the prize or not to claim the prize. If PLAYER 1 does not claim the prize, the prize automatically goes to Player 2. If Player 1 claims the prize, then Player 2 must decide whether or not to also claim the prize.

The payoffs in the game are as follows:

- If Player 1 does not claim the prize, the prize goes to Player 2. Neither player has to pay anything.
- If Player 1 claims the prize and Player 2 does not claim the prize, the prize goes to Player 1. Neither player has to pay anything.
- If Player 1 and Player 2 both claim the prize, the outcome is determined by a lottery.

The payoffs in the lottery are as follows:

- With probability 1/2 Player 1 pays a price of \$ v and gets the prize (in this case, Player 2 neither gets nor pays anything).
- With probability 1/2 Player 2 pays the price of \$ v and gets the prize (in this case, Player 1 neither gets nor pays anything).
- The value of \$ v may differ between rounds. However, it will always be in between the **high value** and the **low value**.

We shall now practice through the various instructions, stage by stage.

When you are ready to continue, please Click Here

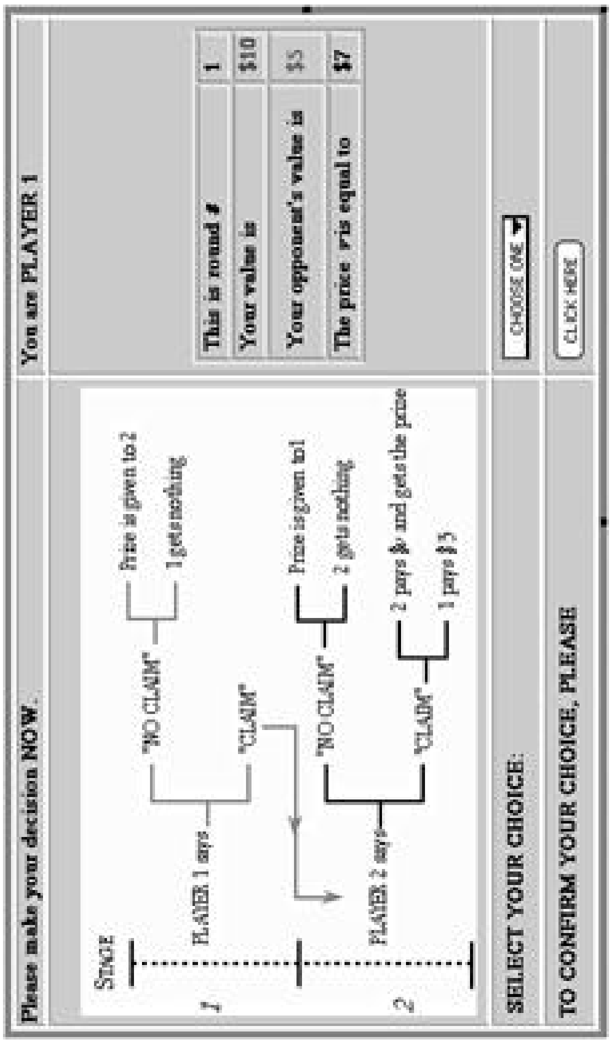

Figure 14: GMM: User Interface

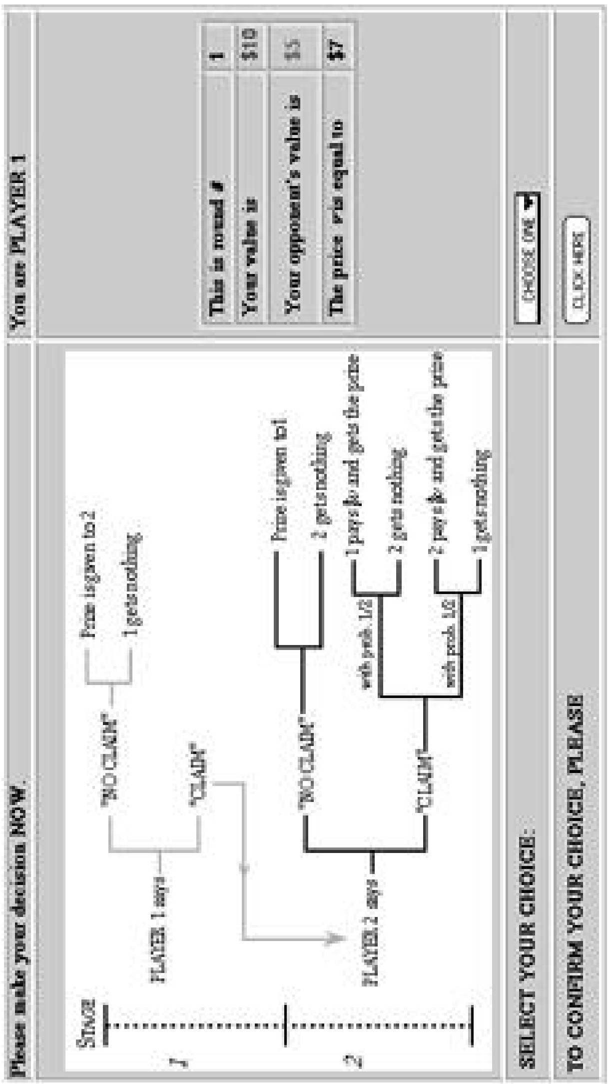

Figure 15: PM: User Interface