IDIOSYNCRATIC SHOCKS, AGGREGATE FLUCTUATIONS AND THE REPRESENTATIVE CONSUMER*

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WP-AD 2001-07

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Editor: Instituto Valenciano de Investigaciones Econ¶omicas, S.A.
First Edition March 2001.
Dep¶osito Legal: V-1161-2001

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* I am grateful to Jordi Caballe, Finn Kydland, Lilia Maliar, Albert Marcet, Franck Portier, Morten Ravn, Michael Reiter, Xavier Sala-i-Martin, Andrew Scott and Chris Telmer for helpful comment.

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Serguei Maliar

A B S T R A C T

This paper analyzes a complete market neoclassical economy with heterogeneous agents. Agents have addilog preferences and receive idiosyncratic labor productivity shocks. We show that at the aggregate level, such an economy behaves as if there was the representative consumer who faces shocks to preferences and technology. This fact enables us to infer time-series properties of the model without specifying a process for idiosyncratic shocks. Instead, we calibrate the process for shocks to preferences and technology in the model derived from aggregation. In contrast to the standard one-shock setup, the model with two types of shocks can generate the appropriate predictions with respect to labor markets.

KEYWORDS: Heterogeneous Agents; Idiosyncratic and Preference Shocks; Aggregation.

1 Introduction

The assumption of idiosyncratic shocks to earnings has been employed recently by many researchers for addressing various questions in computable general equilibrium models, e.g., Krusell and Smith (1995), Kydland (1995), Rios-Rull (1996), Castañeda, Diaz-Gimenez and Rios-Rull (1994), etc. All these papers have two features in common: first, they calibrate the studied models by assuming a particular process for shocks so that the models match microeconomic evidence; and second, they analyze the models' implications at the aggregate level by solving explicitly for the optimal allocations of all heterogeneous consumers.

In this paper, we describe an example of a model with idiosyncratic shocks where aggregate dynamics can be inferred without making explicit assumptions about the process for idiosyncratic shocks and without solving for the equilibrium allocations at the individual level. Specifically, we consider a complete market neoclassical economy where agents differ in initial endowments of wealth and receive idiosyncratic labor productivity shocks. We show that if the preferences of agents are of the addilog type, then at the aggregate level, such an economy behaves as if there was a representative consumer who faces two types of shocks, to preferences and technology. In this case, particular assumptions about idiosyncratic uncertainty have no influence on the structure of the resulting macro model; they only affect the stochastic properties of shocks to preferences and technology at the aggregate level.

In fact, the shocks to technology and preferences of the representative consumer can be viewed as aggregate supply and demand shocks, respectively. Demand shocks have been thought for a long time to play an important role in economics, e.g., in Keynesian economics. However, some researchers have argued that this type of shocks is empirically implausible. Our aggregation result suggests that the assumption of demand shocks is not as artificial as it may seem to be. The only source of uncertainty in our heterogeneous economy are idiosyncratic shocks to individual productivity; however, at the aggregate level, it appears as if shocks affect the preferences of the representative consumer, or, in other words, aggregate demand.

The empirical part of the paper is motivated by the inability of the standard real business cycle (RBC) models to account for the behavior of labor markets. The model with homogeneous agents and technology shocks as the only source of impulses to business cycles implies that the return to working (measured either in average labor productivity or in real wage) must display a strong positive correlation with working hours. This is not the case in the real economies, where this statistic is close to zero or slightly negative (the Dunlop-Tarshis observation). Another closely related statistic which is significantly overstated in the standard RBC setup is the correlation between the return to working and output. A large body of economic research focuses on these "labor market puzzles".[1]

Maliar and Maliar (1999) analyze a heterogeneous-agent model, which is identical to that studied in this paper, except for that the levels of agents' productivity do not change over time. The paper finds that under the assumption of addilog preferences, the model can generate the correlation between labor productivity and working hours which is close to the one in the data. This is possible, however, only if the intertemporal elasticities of consumption and leisure are substantially higher than one. The latter assumption has two undesirable side effects, specifically, the model's predictions are not robust to small changes in the intertemporal elasticities and the volatility of labor productivity becomes too low. The model presented in this paper overcomes both of these problems.

To calibrate the process for shocks to preferences and technology in the constructed representative-agent model, we use the time-series data on the U.S. economy. We assume that "aggregate" shocks follow a first-order Markov process with some joint transitional probabilities. We estimate the model's parameters, including the elements of the matrix of transitional probabilities and the variances of the error terms for aggregate shocks. Subsequently, we calibrate and simulate the model.

The key findings of the paper can be summarized as follows.

- The model with shocks to preferences and technology can reproduce the feature of the data that productivity and working hours as well as productivity and output are weakly correlated.

- The model's predictions are robust to changes in the parameters including the discount factor, the individual intertemporal elasticities of consumption and leisure, and the transitional probabilities of shocks.

[1] For surveys of the literature see Christiano and Eichenbaum (1992) and Gomme and Greenwood (1995).

- The remaining statistics, including the volatility of productivity, are in line with those in the data and in the standard representative-consumer model where only technology shocks occur.

The paper is organized as follows. Section 2 describes the economy with heterogeneous agents and derives optimality conditions. Section 3 constructs the corresponding representative-consumer model. Section 4 outlines estimation and solution procedures. Section 5 discusses numerical results. Section 6 concludes.

2 The Economy

The economy consists of a set of heterogeneous agents S and a representative firm. The timing is discrete, $t \in T$, where $T = 0, 1, ..., \infty.$

The measure of agent s in the set S is denoted by $d\omega^s$, where $\int_S d\omega^s = 1.$ The agents differ in initial endowments and productivity levels. The productivity of agent $s \in S$ in a period $t \in T$ is denoted by $\beta_t^s.$ We denote the distribution of the productivities of agents in period t by $B_t \equiv \{\beta_t^s\}^{s \in S}$ and assume that B_t follows a first order Markov process with a transitional probability given by $\Pi\{B_{t+1} = B' \mid B_t = B\}_{B', B \in \Re}$, where $\Re \subseteq R_+^S$ is a bounded set. Note that this specification allows for correlation between idiosyncratic shocks to productivities of different individuals. The initial distribution of idiosyncratic shocks to productivities B_0 is given.

An infinitely-lived agent $s \in S$ seeks to maximize the expected sum of momentary utilities $u(c_t^s, l_t^s)$, discounted at the rate $\delta \in (0, 1)$, by choosing a path for consumption, c_t^s, and leisure, l_t^s. The utility function $u(\cdot)$ is continuously differentiable, strictly increasing in both arguments, and strictly concave. In period t the agent owns capital stock k_t^s and rents it to the firm at the rental price $r_t.$ Also, he supplies to the firm n_t^s units of labor in exchange for income $n_t^s \beta_t^s w_t$, where w_t is the wage paid for one unit of efficiency labor. The total time endowment of the agent is normalized to one, $n_t^s + l_t^s = 1.$ Capital depreciates at the rate $d \in (0, 1].$ When making the investment decision, the agent faces uncertainty about the future returns on capital. We assume that markets are complete: the agent can insure himself against uncertainty by trading state contingent claims, $\{m_t^s(B)\}_{B \in \Re}.$ The claim of type $B \in \Re$ costs $p_t(B)$ in period t and pays one unit of consumption good in period $t+1$ if the state B occurs and zero otherwise. Therefore, the problem solved by agent $s \in S$ is

$$\max_{\left\{c_t^s; n_t^s; k_{t+1}^s; m_{t+1}^s(B)\right\}_{B \in \Re; t \in T}} E_0 \sum_{t=0}^{\infty} \delta^t u\left(c_t^s; l_t^s; g^t\right) \tag{1}$$

$$c_t^s + k_{t+1}^s + \int_{\Re} p_t(B)\, m_{t+1}^s(B)\, dB = (1 - d + r_t)\, k_t^s + w_t g^t n_t^s \beta_t^s + m_t^s(B_t); \tag{2}$$

where g denotes the rate of labor-augmenting technological progress. Initial holdings of capital and contingent claims, k_0^s and m_0^s; are given.

The production side of the economy consists of a representative firm. The firm owns a technology which allows to transform the inputs, capital k and labor h; into output. The production function $f(k;h)$ is strictly concave, continuously differentiable, strictly increasing with respect to both arguments, has constant return to scale, satisfies the appropriate Inada conditions, and is such that $f(k;zh) = \theta(z) f(k;h)$ for $\forall k;h;z \in R_+$: Given the prices, r_t and w_t; the firm rents capital k_t and hires labor h_t to maximize period-by-period profits

$$\max_{k_t;\, h_t} \pi_t = f(k_t;h_t) - r_t k_t - w_t h_t: \tag{3}$$

The choices of the consumers and the firm must satisfy the market clearing conditions for insurance payments

$$\int_S m_{t+1}^s(B)\, d\omega^s = 0 \qquad \text{for } \forall B \in \Re; \tag{4}$$

for capital and labor

$$k_t = \int_S k_t^s d\omega^s; \qquad h_t = g^t \int_S n_t^s \beta_t^s d\omega^s; \tag{5}$$

and the economy's resource constraint

$$c_t + k_{t+1} = (1 - d)\, k_t + f(k_t;h_t); \tag{6}$$

where $c_t = \int_S c_t^s d\omega^s$ is aggregate consumption.

The equilibrium is defined as a sequence of contingency plans for allocations of the consumers, for allocations of the firm and for the prices such that given the prices, the sequence of plans for the allocations solves the utility maximization problem of each consumer and the profit maximization problem of the firm and satisfies market clearing conditions. Moreover, the plans are such that $c_t^s \geq 0$; and $1 \geq n_t^s \geq 0$ for $\forall s \in S, t \in T$ and $w_t, r_t, k_t \geq 0$ for $\forall t \in T$. It is assumed that the equilibrium exists and is interior.

Let b_t^s be the normalized productivity of agent $s \in S$, $b_t^s = \beta_t^s / \int_S \beta_t^s d\omega^s$. We introduce a new variable n_t such that

$$n_t = \int_S n_t^s b_t^s d\omega^s.$$

Labor input, h_t, and the variable n_t are related as $h_t = g^t n_t \int_S \beta_t^s d\omega^s$. In what follows, n_t is referred to as the aggregate (efficiency) number of hours worked. In terms of n_t, the profit-maximization conditions of the firm are

$$r_t = \theta_t f_1\left(k_t, g^t n_t\right), \qquad w_t = \frac{\theta_t f_2\left(k_t, g^t n_t\right)}{\int_S \beta_t^s d\omega^s},$$

where $\theta_t \equiv \theta\left(\int_S \beta_t^s d\omega^s\right)$ and $f_i(\cdot)$ denotes the first order partial derivative of the function $f(\cdot)$ with respect to the $i-$th argument. The parameter θ_t appears because the aggregate level of skills in the economy fluctuates. This parameter allows for the usual interpretation of technological innovations.

In terms of the variable n_t, the economy's resource constraint can be expressed as

$$c_t + k_{t+1} = (1 - d)\, k_t + \theta_t f\left(k_t, g^t n_t\right). \tag{7}$$

With an interior solution, the First Order Conditions (FOCs) of consumer's utility maximization problem (1), (2) with respect to insurance holdings, capital, consumption and hours worked, and the transversality condition are

$$\lambda_t p_t(B) = \delta \lambda_{t+1}(B) \cdot \Pi\left\{B_{t+1} = B' \mid B_t = B\right\}_{B', B \in \Re}, \tag{8}$$

$$\lambda_t = \delta E_t\left[\lambda_{t+1}\left(1 - d + r_{t+1}\right)\right], \tag{9}$$

$$\lambda^s u_1\left(c_t^s, l_t^s; g^t\right) = \lambda_t, \tag{10}$$

$$\lambda^s u_2\left(c_t^s, l_t^s; g^t\right) = \lambda_t w_t g^t b_t^s, \tag{11}$$

$$\lim_{t\to\infty} E_0 \left[\delta^t \lambda_t \left(k^s_{t+1} + \int_{\Re} p_t(B)\, m^s_{t+1}(B)\, dB\right)\right] = 0. \tag{12}$$

Here, we can represent the Lagrange multiplier associated with the agent's budget constraint as $\lambda^s_t = \lambda_t/\lambda^s$ because due to market completeness, the ratio of marginal utilities of any two agents remains constant in all periods and states of nature. If one formulates the associated planner's problem, then the parameters $\{\lambda^s\}^{s\in S}$ and the variable λ_t will be the welfare weights and the Lagrange multiplier associated with the economy's resource constraint. Without loss of generality, we normalize the weights to one, $\int_S \lambda^s d\omega^s = 1.$

3 The Representative Consumer

In this section, we derive aggregation results for the heterogeneous-agent economy of section 2. For the remainder of the paper, we assume that the momentary utility function of each agent $s \in S$ is of the addilog type, i.e.[2]

$$u\left(c^s_t, l^s_t; g^t\right) = \frac{(c^s_t)^{1-\gamma} - 1}{1-\gamma} + A g^{t(1-\gamma)} \frac{(l^s_t)^{1-\sigma} - 1}{1-\sigma}, \qquad \gamma, \sigma, A > 0. \tag{13}$$

Under such utility, FOCs $(10), (11)$ take the form

$$\lambda^s (c^s_t)^{-\gamma} = \lambda_t, \tag{14}$$

$$\lambda^s A g^{t(1-\gamma)} (1 - n^s_t)^{-\sigma} = \lambda_t w_t g^t b^s_t. \tag{15}$$

Solving $(14), (15)$ with respect to c^s_t and $(1 - n^s_t) b^s_t$ and integrating across agents, we obtain

$$c_t = \lambda_t^{-1/\gamma} \cdot \int_S (\lambda^s)^{1/\gamma} d\omega^s, \tag{16}$$

$$l_t = \left(\lambda_t w_t g^t\right)^{-1/\sigma} \left(A g^{t(1-\gamma)}\right)^{1/\sigma} \cdot \int_S (\lambda^s)^{1/\sigma} (b^s_t)^{1-1/\sigma} d\omega^s, \tag{17}$$

where $l_t = 1 - n_t$. From equations $(9), (16), (17)$, we get

[2] Similar aggregation results follow if the agents' preferences are quasi-homothetic.

$$c_t^{-\gamma} = \delta E_t\left[c_{t+1}^{-\gamma}\left(1 - d + r_{t+1}\right)\right]; \tag{18}$$

$$AX_t g^{t(1-\gamma)} l_t^{-\sigma} = c_t^{-\gamma} w_t g^t; \tag{19}$$

where the parameter X_t is given by

$$X_t = \frac{\left(\int_S \left(\lambda^s\right)^{1/\sigma} \left(b_t^s\right)^{1-1/\sigma} d\omega^s\right)^{-\sigma}}{\left(\int_S \left(\lambda^s\right)^{1/\gamma} d\omega^s\right)^{-\gamma}}.$$

Finally, integrating individual transversality condition (12) across agents and imposing market clearing condition for claims (4), we have

$$\lim_{t\to\infty} E_0\left[\delta^t \lambda_t k_{t+1}\right] = 0. \tag{20}$$

Using the above results, we formulate the representative-consumer model, which describes aggregate dynamics of the heterogeneous-agent economy

$$\max_{\{c_t, k_{t+1}, n_t\}_{t\in T}} E_0 \sum_{t=0}^{\infty} \delta^t \left\{ \frac{c_t^{1-\gamma} - 1}{1-\gamma} + AX_t g^{t(1-\gamma)} \frac{(1-n_t)^{1-\sigma} - 1}{1-\sigma} \right\} \quad \text{s.t.} \quad RC; \tag{21}$$

where RC denotes the economy's resource constraint (3).

Proposition 1 Under the addilog utility, the equilibrium sequence of contingency plans for aggregate quantities $\{c_t, n_t, k_{t+1}\}_{t\in T}$ in economy (1) – (4) is a solution to the representative-agent model (21).

Proof. If a solution $\{c_t, n_t, k_{t+1}\}_{t\in T}$ to problem (21) exists and is interior, then it satisfies the FOCs, the transversality condition and the budget constraint. The FOCs of this problem are (18), (19). The transversality condition is equivalent to (20). Finally, by definition, resource constraint (3) is a necessary condition for the equilibrium. ||

Unless $\gamma = \sigma$, the addilog preferences are not quasi-homothetic and, therefore, they do not lead to a representative consumer in the sense of Gorman (1953). The possibility of aggregation under the addilog utility is mentioned first by Shafer (1977). Note that, even if agents have identical time-invariant productivity and differ only in endowments, the parameter $\chi_t \equiv \chi_0$ does not vanish from problem (21). The value of this parameter depends on given distribution of endowments and affects the equilibrium marginal rate of substitution between aggregate quantities.

If productivities of agents are subject to idiosyncratic shocks, the parameters θ_t and χ_t vary with time. These parameters will be referred to as technology and preferences shocks, respectively. The parameter θ_t is exogenous both to the heterogeneous-agent model and the constructed representative-consumer setup. The parameter χ_t is exogenous to the problem of the representative consumer, but endogenous to the economy with heterogeneous agents since it depends on the welfare weights, which in turn are determined by the decisions of all heterogeneous agents.

The existence of the representative consumer makes it possible to investigate the properties of the heterogeneous-agent economy without modelling explicitly the process for idiosyncratic shocks. Instead, one can assume a law of motion for the parameters θ_t and χ_t and solve the model derive from aggregation (21). This model is sufficient to determine the sequence of contingency plans for aggregate allocations and prices.

Given that the idiosyncratic shocks to agents' productivities are assumed to follow a first-order Markov process, we presume that the aggregate shocks will also do so. Thus, the law of motion for shocks θ_t and χ_t will be

$$\begin{bmatrix} \log \theta_t \\ \log \chi_t \end{bmatrix} = \begin{bmatrix} \rho_{\theta\theta} & \rho_{\theta\chi} \\ \rho_{\chi\theta} & \rho_{\chi\chi} \end{bmatrix} \begin{bmatrix} \log \theta_{t-1} \\ \log \chi_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{\theta} \\ \varepsilon_t^{\chi} \end{bmatrix}, \tag{22}$$

where $\varepsilon_t^{\theta} \sim N(0, \sigma_{\theta}^2)$ and $\varepsilon_t^{\chi} \sim N(0, \sigma_{\chi}^2)$. For the rest of the paper, we assume that the production function is of the Cobb-Douglas type, $f(k, n) = k^{\alpha} n^{1-\alpha}$. In the remainder of the paper, we analyze quantitative implications of the constructed representative-consumer model.

4 Estimation and Solution Procedures

We now estimate the model's parameters and simulate the solutions. The estimation procedure plays an important role in our analysis as it allows us to evaluate the stochastic properties of shocks in (22).

To estimate the model's parameters, we use a version of Hansen's (1982) Generalized Method of Moments (GMM) procedure. The utility parameters, γ and σ; and the subjective discount factor, δ; are not estimated. In the baseline model, we presume $\gamma = \sigma = 1$ and $\delta = (1.03)^{-0.25}$. Later, to analyze the robustness of our results, we will also consider several alternative specifications for these parameters. The parameters under estimation are subdivided into two groups Θ_1 and Θ_2

$$\Theta_1 = \{\alpha; A; g; d\}; \qquad \Theta_2 = \left\{\rho_{\theta\theta}; \rho_{\theta X}; \rho_{X\theta}; \rho_{XX}; o_t^{\theta}; o_t^{X}\right\}:$$

The estimation procedure includes two steps. First, we estimate the parameters from the group Θ_1 from the first-moment conditions of model (21) and compute the residuals θ_t and X_t: Second, we estimate the parameters from the group Θ_2 by using the computed residuals. Given that the stochastic properties of the processes for θ_t and X_t are not known, we compute the instrumental variable estimator at both steps of the estimation procedure. As instruments, we use 8 lags of consumption, capital, output and hours worked. The first-moment conditions, employed for estimating the parameters from Θ_1 are given in the appendix. The parameters from Θ_2 are estimated according to (22):

To estimate the parameters, we use quarterly data on the U.S. economy ranging from 1959 : 3 to 1998 : 3 . The variable consumption c_t in the model is defined as real personal expenditures on nondurables and services in the data. Investment i_t in the model is real personal consumption of durables and real fixed private investment in the data. Consequently, the series for output are constructed by adding up consumption and investment, $y_t = c_t + i_t$: The variable working hours n_t in the model is defined as level of the civilian employment premultiplied by average weakly hours worked in private nonagricultural establishments in the data. The average weakly hours were previously divided by 168; which is the total number of hours per week. Before computing the estimates, the constructed series are converted in per-capita terms by using the efficiency measure of the U.S. population. The data are taken from the Federal Reserve Bank of Saint-Louis data base (mnemonics FPIC92; PCEDG92; PCENDC92; PCECS92; CE16OV; AWHNONAG). The sources for these series are U.S. Department of Labor and U.S. Department of Commerce.

The estimates of the parameters from Θ_1 in the baseline model are

$$\underset{(0.0016)}{\alpha = 0.3341}; \qquad \underset{(0.008)}{A = 3.317}; \qquad \underset{(0.0001)}{g = 1.0047}; \qquad \underset{(0.0001)}{d = 0.0209};$$

where the numbers in parenthesis are the standard deviations. These estimates are practically identical to those reported by Christiano and Eichenbaum (1992). We find that the resulting estimates are robust to modifications in the set of instruments and in the number of lags assumed. We will not report the estimates of the parameters from Ω_1 under all considered values of $(\gamma; \sigma; \delta)$. However, we will report the first moments of the model for each set of the parameters $(\gamma; \sigma; \delta)$ under which the model is simulated. The estimates of the parameters from Ω_2 will be reported in all the cases considered and discussed separately in the subsequent section.

We parametrize the model by using the values of the parameters, which are previously estimated by GMM and solve for the equilibrium. To compute numerical solutions, we employ the parametrized expectation algorithm, see, e.g., Marcet and Lorenzoni (1999). To approximate the conditional expectations, we use second order degree exponentiated polynomial. The length of simulations was 10000 and the iterations were performed until 5-digit precision in the polynomial coefficients was enforced.

In the last column of Table 1; we provide selected first and second moments of time series in the U.S. economy. The reported statistics are the sample averages of the variables provided in the first column of the table. The statistics σ_x and $corr(x; z)$ are the volatility of a variable x and the correlation between variables x and z; respectively. In the remaining columns of the table, we report the first and second moments of time series generated by the model. The model's moments are sample averages of the statistics computed for each of 400 simulations. Each simulation has the length 157 periods, as do time series for the U.S. economy. Numbers in parentheses are sample standard deviations of these statistics. Before calculating the second moments, the corresponding variables for the U.S. and artificial economies were logged and detrended by using the Hodrick-Prescott filter.

As a measure of labor productivity (wage), we use the variable y_t/n_t: To check that the constructed measure of labor productivity behaves similarly to the one in the U.S. economy, we compared this measure to the CITIBASE variable LBOUTU, which is output per-hour of all persons in the nonagricultural business sector. We find that the properties of both measures are very similar. In such a way, if instead of y_t/n_t, we use the variable LBOUTU, then we have $\sigma_{y/n} = 1.023$, $corr(y/n, n) = 0.220$ and $corr(y/n, y) = 0.543$, which are close to the corresponding statistics reported in the table.

5 Findings

We begin from a baseline standard representative the model. This corresponds to the case when the process (22) is estimated under the restriction that only technology shocks can occur in the economy.

- Model 1. $\rho_{\theta\theta}$ is estimated under the restriction $\rho_{X\theta}, \rho_{\theta X}, \rho_{XX} \equiv 0.$

This version of the model is extensively studied in the literature, e.g., Hansen (1985), and Christiano and Eichenbaum (1992). These papers use different values for the coefficient of autocorrelation $\rho_{\theta\theta}$: the first assumes AR (1) with $\rho_{\theta\theta} = 0.95$, while the second uses the random walk specification $\rho_{\theta\theta} = 1.$ As it follows from the table, our own estimate is close to the latter.[3]

Comparing the results of Hansen (1985) and Christiano and Eichenbaum (1992) shows that the key properties of the model are not substantially affected by a variation in the coefficient of autocorrelation. Specifically, in either case, the model can generate most of the statistics in line with the data, except for those with respect to labor markets. The most serious failure of the model is its inability to account for the Dunlop-Tarshis observation, which consists in that productivity (wage) and hours worked in the real economies are not significantly correlated. In fact, the quantitative expression of the Dunlop-Tarshis observation varies substantially depending on time series used. For example, Christiano and Eichenbaum (1992) calculate $corr(y/n, n)$ for the U.S. economy by using the household and the establishment time series and obtain -0.2 and 0.16, respectively. According to Gomme and Greenwood (1995), if the real wages are used as a proxy for productivity, this statistic will be around $-0.44.$

[3] We find that the estimate of the autocorrelation coefficient $\rho_{\theta\theta}$ depends significantly on which particular time series are used as a proxy for working hours. If one uses aggregate working hours, as Hansen (1985) does, then the estimates for $\rho_{\theta\theta}$ will be about $0.95.$ However, if one uses the definition suggested by Christiano and Eichebaum (1992) and adopted in this paper, the estimate for $\rho_{\theta\theta}$ will be close to one.

It turns out that the model cannot get close to any of the above numbers consistently predicting that $corr(y/n, n) \approx 1$. In addition, it overstates considerably the correlation between productivity and hours worked, and understates the volatilities of productivity and working hours compared to the data.

Next, we turn to the case when all uncertainty in the economy comes from shocks to preferences.

- Model 2. ρ_{XX} is estimated under the restriction $\rho_{X\theta}, \rho_{\theta X}, \rho_{\theta\theta} \equiv 0$.

This version of the model proves to be highly unsuccessful. It generates several serious failures such as very low volatility of consumption, output and investments and almost perfect negative correlation between productivity (output) and working hours. It is interesting to notice that in this case, the problem is exactly the opposite to the one that we had before: the productivity (output) and working hours in the model are too countercyclical compared to the U.S. data.

Next, we consider the model with two types of shocks.

- Model 3. $\rho_{\theta\theta}, \rho_{X\theta}, \rho_{\theta X}, \rho_{\theta\theta}$ are estimated without restrictions.

Once two sources of shocks are assumed, the model's performance improves considerably compared to Models 1 and 2. Model 3 generates weakly negative correlation between productivity and hours worked and, therefore, accounts for the Dunlop-Tarshis observation. Further, the correlation between productivity and output in the model is close to that in the data. Finally, incorporating two shocks adds volatility to all model's variables except for investment. In particular, the volatility of working hours in Model 3 is more than twice as large as in Model 1 and becomes close to the empirical counterpart.

In sum, the model with two types of shocks is remarkably successful in explaining the U.S. data. It is important to investigate, however, how robust our results are to modifications in the model's parameters. We begin with the sensitivity analysis with respect ot the autocorrelation coefficients by considering the following experiment.

- Model 4. $\rho_{\theta\theta}, \rho_{XX} \equiv 0.95$ and $\rho_{X\theta}, \rho_{\theta X} \equiv 0$.

As it follows from the table, this modification not only does not worsen the positive features of the previous setup but improves model's performance with respect to the volatilities of investment, output and working hours. We have done other experiments (not reported) and found that the model's implications are very robust to changes in the autocorrelation coefficients.

Consequently, we are left to explore how the model's properties are affected by changes in the values of the preference parameters $(\gamma; \sigma; \delta)$. Maliar and Maliar (1999) show an example of a heterogeneous-agent model where quantitative implications depend crucially on the intertemporal elasticities of consumption and leisure, $1/\gamma$ and $1/\sigma$. The results of this paper suggest that a sensitivity analysis with respect to the preference parameters is of potential interest. Below we report the results of experiments in which we vary the value of one of the parameters $(\gamma; \sigma; \delta)$, holding the remaining two parameters equal to the baseline values. In the remaining experiments, no prior restrictions are imposed on the values of autocorrelation coefficients; these are estimated from the data. Models 5-10 are the following.

- Models 5, 6: $\sigma = 1.0$; $\delta = 1.03^{-0.25}$ and $\gamma \in \{0.75; 1.5\}$.
- Models 7, 8: $\gamma = 1.0$; $\delta = 1.03^{-0.25}$ and $\sigma \in \{0.5; 2.0\}$.
- Models 9, 10. $\gamma = 1.0$; $\sigma = 1.0$ and $\delta \in \{1.05^{-0.25}; 1.015^{-0.25}\}$.

The results of this simulation exercise are reported in Table 2. First of all, let us notice that the fact that the estimated coefficients of the autocorrelation $\rho_{\theta\theta}$ in Models 6 and 8 are greater than one does not imply non-stationarity. In order for the process for shocks θ_t and X_t to be stationary, it is sufficient that both eigenvalues of the matrix constructed from the autocorrelation coefficients lie inside of the unit root circle, see, e.g., Hamilton (1994). This restriction is satisfied in each of the models considered.

As we can see from the table, variations in the preference parameters $(\gamma; \sigma; \delta)$ inside a reasonable range do not significantly affect the properties of the model compared to the baseline case. An exception is Model 6 in which the correlation between productivity and working hours becomes too negative. However, even this model's prediction is not entirely inconsistent with the data as it is close to the correlation between real wages and working hours in the U.S. economy. In sum, the findings obtained for the baseline model are not considerably affected by changes in the model's parameters.

6 Conclusion

This paper describes an example of a general equilibrium model with idiosyncratic uncertainty where aggregate equilibrium allocation can be characterized in a simple and economic fashion. Our economy is populated by a number of individuals who di®er in capital endowments and whose labor productivities °uctuate over time. At the aggregate level, however, it appears as if there exists a representative consumer who is hit by two types of shocks, to preferences and technology. The possibility of aggregation enables us to investigate the model's implications at the aggregate level without making explicit assumptions about unobservable idiosyncratic uncertainty.

The empirical ¯nding of the paper is that taking into account the preference shocks can enhance considerably the performance of the RBC models. In contrast to the standard setup where °uctuations in technology is the only source of impulses to business cycles, the two-shock version of the model can successfully account for such labor market stylized facts as the Dunlop-Tarshis observation and the low correlation between productivity and output.

7 APPENDIX

The conditions used for GMM estimation:

The economy's resource constraint implies that the gross investment i_t is related to capital stock k_t as

$$E\{1 - d + (i_{t+1}/k_t) - (k_{t+1}/k_t)\} = 0.$$

The hypothesis of the balanced growth implies

$$E\{\log(y_t) - \log(y_{t-1}) - \ln(g)\} = 0;$$

$$E\{\log(c_t) - \log(c_{t-1}) - \ln(g)\} = 0;$$

$$E\{\log(k_t) - \log(k_{t-1}) - \ln(g)\} = 0.$$

The intertemporal condition of problem (21) is

$$E\{1 - \delta(c_t/c_{t+1})[1 - d + \alpha(y_t/k_t)]\} = 0.$$

Taking the logarithm of FOC (19), we get

$$\ln(X_t) = -\gamma[\ln(c_t) - gt] + \sigma\ln(1 - n_t) + \ln[(1 - \alpha)y_t/n_t] - \ln(g)t - \ln(A).$$

From (3), the process for the parameter θ_t is

$$\ln(\theta_t) = \ln(y_t) - \alpha\ln(k_t) - (1 - \alpha)\ln(n_t) - (1 - \alpha)\ln(g)t - \ln(\theta),$$

where θ is the absolute level of technology.

Table 1. Baseline model: $\gamma = 1$, $\sigma = 1$, $\delta = 1.03^{-0.25}$

	Heterogeneous-agent model				U.S. economy
	Model 1	Model 2	Model 3	Model 4	
Parameters for the shocks					
$\rho_{\theta\theta}$	0.99456 (0.00939)	-	0.99602 (0.01588)	0.95000	-
$\rho_{\theta X}$	-	-	0.00594 (0.01551)	-	-
$\rho_{X\theta}$	-	-	0.10496 (0.01654)	-	-
ρ_{XX}	-	0.99886 (0.00997)	0.92671 (0.01538)	0.95000	-
v^2_{θ}	0.00652 (0.00055)	-	0.00598 (0.00058)	0.00669 (0.00055)	-
v^2_{X}	-	0.00622 (0.00051)	0.00585 (0.00046)	0.00697 (0.00045)	-
First moments					
c_t/y_t	0.750 (0.018)	0.748 (0.012)	0.750 (0.019)	0.751 (0.023)	0.745
k_t/y_t	10.365 (0.539)	10.311 (0.408)	10.359 (0.589)	10.351 (0.564)	10.237
n_t	0.211 (0.004)	0.222 (0.006)	0.212 (0.008)	0.211 (0.007)	0.213
Second moments					
σ_c	0.558 (0.067)	0.290 (0.033)	0.691 (0.081)	0.480 (0.063)	0.836
$\sigma_{y/n}$	0.679 (0.078)	0.274 (0.030)	0.764 (0.083)	0.721 (0.081)	1.011
σ_n	0.492 (0.057)	0.806 (0.090)	1.069 (0.122)	1.408 (0.153)	1.279
σ_i	3.025 (0.355)	1.304 (0.156)	2.789 (0.324)	5.220 (0.567)	4.793
σ_y	1.153 (0.127)	0.539 (0.061)	1.120 (0.130)	1.597 (0.174)	1.755
corr(c,y)	0.974 (0.006)	0.982 (0.004)	0.896 (0.031)	0.889 (0.018)	0.923
corr(y/n,y)	0.989 (0.003)	-0.963 (0.010)	0.402 (0.129)	0.471 (0.109)	0.715
corr(n,y)	0.979 (0.005)	0.996 (0.001)	0.753 (0.068)	0.890 (0.032)	0.830
corr(i,y)	0.989 (0.003)	0.990 (0.002)	0.938 (0.019)	0.986 (0.004)	0.979
corr(y/n,n)	0.939 (0.046)	-0.983 (0.004)	-0.287 (0.138)	0.026 (0.145)	0.220

Table 2. Sensitivity analysis with respect to the parameters *(γ,σ,δ)*

	Heterogeneous-agent model					
	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10
	$\gamma = 0.75$	$\gamma = 1.5$	$\gamma = 1$	$\gamma = 1$	$\gamma = 1$	$\gamma = 1$
	$\sigma = 1$	$\sigma = 1$	$\sigma = 0.5$	$\sigma = 2$	$\sigma = 1$	$\sigma = 1$
	$\delta = 1.03^{-0.25}$	$\delta = 1.03^{-0.25}$	$\delta = 1.03^{-0.25}$	$\delta = 1.03^{-0.25}$	$\delta = 1.05^{-0.25}$	$\delta = 1.015^{-0.25}$
Parameters for the shocks						
$\rho_{\theta\theta}$	0.99447 (0.01242)	1.00052 (0.02037)	0.99213 (0.01561)	1.00247 (0.01645)	0.99923 (0.01587)	0.99210 (0.01614)
$\rho_{\theta X}$	0.00979 (0.01903)	0.00453 (0.01126)	0.00196 (0.01635)	0.01127 (0.01380)	0.00879 (0.01572)	0.00310 (0.01512)
$\rho_{X\theta}$	0.07405 (0.01273)	0.15957 (0.02232)	0.08744 (0.01476)	0.13987 (0.02074)	0.10291 (0.01651)	0.10639 (0.01685)
ρ_{XX}	0.92131 (0.01904)	0.93270 (0.01148)	0.93813 (0.01483)	0.90954 (0.01627)	0.92658 (0.01555)	0.92978 (0.01498)
v^2_{θ}	0.00602 (0.00057)	0.00597 (0.00059)	0.00595 (0.00058)	0.00604 (0.00057)	0.00599 (0.00059)	0.00599 (0.00057)
v^2_X	0.00599 (0.00045)	0.00600 (0.00048)	0.00535 (0.00042)	0.00700 (0.00055)	0.00583 (0.00046)	0.00582 (0.00046)
First moments						
c_t /y_t	0.760 (0.023)	0.743 (0.033)	0.751 (0.024)	0.750 (0.020)	0.749 (0.019)	0.752 (0.024)
k_t /y_t	9.958 (0.526)	11.030 (0.872)	10.358 (0.595)	10.382 (0.570)	10.346 (0.521)	10.382 (0.643)
n_t	0.212 (0.008)	0.208 (0.011)	0.213 (0.009)	0.211 (0.007)	0.213 (0.008)	0.212 (0.008)
Second moments						
σ_c	0.631 (0.078)	0.548 0.079	0.592 (0.084)	0.675 (0.078)	0.648 (0.075)	0.638 (0.080)
$\sigma_{y/n}$	0.663 (0.069)	0.862 (0.092)	0.698 (0.072)	0.770 (0.073)	0.759 (0.080)	0.751 (0.080)
σ_n	1.272 (0.142)	1.301 (0.430)	1.311 (0.192)	1.066 (0.132)	1.068 (0.121)	1.200 (0.128)
σ_i	4.136 (0.487)	4.494 (5.271)	4.165 (1.525)	3.103 (0.435)	2.881 (0.352)	3.534 (0.401)
σ_y	1.406 ((0.161)	1.122 (0.400)	1.359 (0.172)	1.144 (0153)	1.128 (0.134)	1.256 (0.148)
corr(c,y)	0.921 (0.019)	0.717 (0.081)	0.890 (0.042)	0.841 (0.047)	0.901 (0.030)	0.850 (0.043)
corr(y/n,y)	0.421 (0.136)	0.170 (0.141)	0.320 (0.156)	0.431 (0.122)	0.408 (0.129)	0.369 (0.124)
corr(n,y)	0.880 (0.035)	0.739 (0.082)	0.858 (0.048)	0.754 (0.066)	0.757 (0.069)	0.810 (0.053)
corr(i,y)	0.975 (0.006)	0.885 (0.189)	0.964 (0.039)	0.929 (0.022)	0.951 (0.015)	0.938 (0.019)
corr(y/n,n)	-0.051 (0.158)	-0.524 (0.117)	-0.197 (0.148)	-0.255 (0.143)	-0.274 (0.141)	-0.236 (0.142)

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