# GROWTH MIRACLES REEXAMINED*

Chris Papageorgiou and Fidel P¶erez-Sebasti¶an**

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Correspondence to: Fidel P¶erez-Sebasti¶an, Dpto. Fundamentos de An¶alisis Econ¶omico, Universidad de Alicante, Campus de San Vicente del Raspeig, 03071 Alicante, Spain, e-mail: ¯del@merlin.fae.ua.es

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** C. Papageorgiou: LouIsiana State University; F. Perez-Sebastian: University of Alicante.

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A B S T R A C T

In this paper we propose a growth model in which the combined effect of human capital and technology adoption is the key factor in replicating and explaining growth miracles. Using standard technologies and parameterization, we show that the calibrated model generates output growth paths consistent with the ones displayed by miracle economies, such as Japan and South Korea. The driving force of our result is twofold: (a) the complementarity between human capital and technological adoption; (b) the reallocation of labor across sectors along the adjustment path.

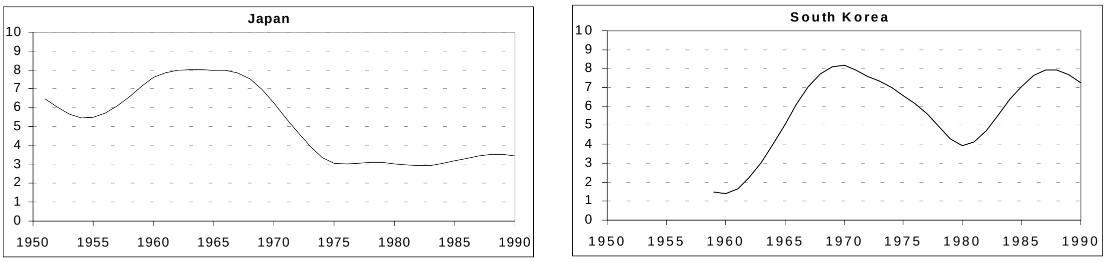
KEYWORDS: Transitional Dynamics; R&D; Input Complementarity.

# 1 INTRODUCTION

One of the most intriguing phenomena in modern economic growth is development miracles. The stylized facts concerning fast growing economies are staggering. For example, over the period 1960 ¡ 1990, Japan and South Korea averaged output growth rates over 5 percent per year. During this period, Japan's relative output per worker nearly tripled increasing from 20:6 to 60:3 percent of the U.S. level, whereas S. Korean relative output per worker almost quadrupled increasing from 11:1 to 42:2 percent. Figure 1 illustrates the growth experiences of these two miracle countries.[1]

Closer observation of ¯gure 1 reveals an interesting feature of miraculous experiences: the sharp increase of output per worker was characterized by growth rates that did not pick at the beginning of the convergence process but later on, thus giving way to a hump-shaped growth path.

Figure 1: Output per worker growth rates in Japan and S. Korea, annual data, percentage




What is even more interesting is that the underlying characteristics of the two East Asian miracle economies are distinctly di®erent. Whereas Japan started its post-War convergence path with high human capital levels, S. Korea started its convergence path with very low human capital levels. In addition, although both nations began with relatively low levels of physical capital, Japan accumulated equipment, machinery, and infrastructure at a higher rate than S. Korea.[2] Even regarding output growth rates, miraculous experiences show important di®erences. In ¯gure 1, we see that Japanese growth rates were relatively high from the beginning of the convergence process, whereas S. Korean growth rates started low and increased rapidly.

[1]All along the paper, we follow Parente and Prescott (1994) and smooth the data series using the Hodrick-Prescott ¯lter with the smoothing parameter equal to 25.

[2]The data that support these arguments are presented in section 4.4 of the paper.

The in°uential paper by Robert Lucas \Making a Miracle" concluded that improving our understanding of the mechanics of rapid growth episodes is essential in constructing a successful theory of economic development. Since Lucas (1993), there has been surging interest in theoretical research attempting to explain economic miracles, with a number of papers being able to reproduce the average convergence speed exhibited by rapidly growing nations. However, as section 2 discusses in more detail, growth models have not in general been able to predict the variable convergence speed needed to generate the observed hump-shaped adjustment path of output growth rate. Nor has the literature paid close attention to the distinct characteristics of miraculous episodes.

In this paper, we propose a model in which the complementarity between human capital and technology adoption is able to replicate and explain growth miracles. As Parente and Prescott (1994), we focus on di®erences in barriers to technology adoption. Parente and Prescott (1994) replicate the average speed of convergence implied by the Japanese and S. Korean experiences; variable convergence speeds are also possible in their model but only through exogenous variations in the degree of barriers. Whereas in Parente and Prescott's work technology barriers are exogenous and an increasing function of sociopolitical factors such as corruption, legal constraints and violence, in our paper they are endogenously determined and depend on the level of human capital.[3]

More precisely, we introduce endogenous human capital accumulation into an, otherwise, stylized R&D-based growth model. We use a version of Jones (1995) hybrid non-scale growth framework in which sustained long-run growth depends on both exogenous labor growth and endogenous technical change.[4] In our model, technical progress is enhanced through innovation and imitation, and human capital through formal schooling. Even though formal schooling is not the only source of human capital, we choose a schooling-based human capital technology because the model will ultimately be taken to the data, and by many accounts the best available data used to measure human capital across countries are the educational attainment data sets of Barro and Lee (1993) and Nehru, Swanson, and Dubey (1995). Our choice of schooling technology follows the Mincerian approach (Mincer 1974) that has recently been revived by Bils and Klenow (forthcoming).[5]

[3] Our model is also consistent with Griliches (1988a) and Nelson and Pack (1999), who emphasize the need of accumulating enough skills before being able to use new techniques and manufacture new products.

[4] Other growth models that eliminate scale e®ects include Sergerstrom (1998), Eicher and Turnovsky (1999), Young (1998), Dinopoulos and Thompson (1998), and Howitt (1999). The last three of these papers also permit sustained output growth in the absence of population growth. We choose Jones' technology speci¯cation because it lends itself more easily to our numerical analysis.

[5] For recent discussions on the advantages of the Mincerian approach in growth modeling and estimation, see Bils and Klenow (forthcoming), and Krueger and Lindahl (1998). Other papers that employ the Mincerian approach to

We then use numerical-approximation techniques based on Judd (1992) to simulate the transitional dynamics of the model, and compare their predictions to the data. Using standard technologies and parameterization, we show that the calibrated model can successfully replicate the growth experiences of Japan and S. Korea. The key factor contributing to this result is twofold: (a) the complementarity between human capital and technological adoption, (b) the reallocation of labor across sectors along the adjustment path. The model can also account for the di®erent underlying characteristics of the Japanese and S. Korean experiences. First, we show that lower schooling levels are associated with slower physical capital accumulation. A smaller human capital stock implies a larger productivity of schooling time; thus reducing the amount of labor allocated to ¯nal output production which in turn reduces physical capital formation. Second, we show that output growth rates at the beginning of the convergence process increase with the average educational attainment. This is because the rates of technology adoption and physical capital formation increase with human capital, causing output to accumulate at a faster rate.

There is a small but rapidly growing literature that investigates the relationship between human capital accumulation and technological progress, and their combined e®ect on economic growth. Eicher (1996) develops a model in which both human capital and technological innovation are endogenous. Eicher, however, is only concerned with steady-state predictions. Restuccia (1997) presents a dynamic general equilibrium model with schooling and technology adoption. But he focuses on how schooling and technology adoption may be amplifying the e®ects of productivity/policy di®erences on income disparity. Lau and Wan (1994) also reproduce the growth patterns shown by East Asian miracle countries in a model of human capital and technological catch-up, but their model's setup is rather ad hoc. Like us, Funke and Strulik (2000) study transitional dynamics in a model of physical capital, human capital, and blueprints. They, however, study the existence of threshold levels in the parameters that switch on and o® the di®erent sectors.

The remainder of the paper is organized as follows. Section 2 presents a review of existing models and their convergence implications. It also o®ers arguments in favor of a proposed model that can reproduce miracles experiences. Section 3 presents the basic model. In this section, we establish the economic environment and examine the steady-state properties of the model. The transitional dynamics analysis is presented in Section 4. Section 5 concludes discussing the main ¯ndings of our work.

model schooling include Jones (1997, 1999), Jovanovic and Rob (1998), and Hall and Jones (1999).

## 2 GROWTH MODELS AND OUTPUT CONVERGENCE RATES

This section is concerned with competing growth models in the literature and their implications for output convergence rates.[6] We first introduce a general framework which will serve as the springboard for developing several models examined thereafter. Assume that at period t, output ($Y_t$) is produced using labor ($L_{Yt}$) and physical capital ($K_t$) according to the following aggregate Cobb-Douglass technology:

$$Y_t = A_t^{\xi}(h_{Yt}L_{Yt})^{1-\alpha}K_t^{\alpha}, \quad 0 < \alpha < 1, \quad \xi > 0, \tag{1}$$

where $h_{Yt}$ represents the effectiveness of average human capital level on output labor; $\alpha$ is the share of capital; and $\xi$ is a technological externality. The economy's technological level $A_t$ evolves as follows:

$$\dot{A}_t = D\left(R_t, \frac{A_t^*}{A_t}\right), \quad \frac{\partial D}{\partial R} > 0, \quad \frac{\partial D}{\partial (A^*/A)} > 0. \tag{2}$$

where R and $A^*$ denote R&D inputs and the technology frontier, respectively. Equation (2) states that technical change is a positive function of R&D inputs, $R_t$, and a catch-up/adoption term, $\frac{A_t^*}{A_t}$. The adoption term captures the idea that the greater the technology gap between a leader and a follower, the higher the potential of the follower to catch up through imitation of existing technologies.[7][8] Finally, the capital stock evolves according to the standard motion equation

$$\dot{K}_t = Y_t - C_t - \delta K_t, \tag{3}$$

where $C_t$ is aggregate consumption; and $\delta$ is the depreciation rate.

Let $g_{xt}$ denote the growth rate of variable x in period t, and $u_{zt} \equiv \frac{L_{zt}}{L_t}$ denote the fraction of population in the production of z at t ($\forall z = Y, A, h$). Also let lower case letters denote per capita variables, and w denote per worker variables. From equations (1) and (2), we can derive the growth

[6] We are indebted to Kazuo Mino, whose comments helped build this section.

[7] The notion and formulation of the catch-up effect is due to Veblen (1915), and Gerschenkron (1962). Nelson and Phelps (1966) were the first to construct a formal model based on the catch-up term.

[8] The notion of technology adoption is empirically investigated by Coe and Helpman (1995), and Coe, Helpman, and Hoffmaister (1997). For discussion on the effects of technology adoption on East Asia see Amsden (1991) and Baark (1991). For an excellent presentation of technology adoption in Japan see Minami (1994). The author explores three categories of borrowed technology which are illustrated by examples from Japanese history. He discusses in detail the introduction of the English railway technology, the machine filature technology and the silk weaving technology. According to Minami, Japan's industrialization was revolutionary in the sense that it was accomplished by the adoption of existing foreign technology.

rate of output per worker as

$$g_{y^w t} = \xi D \left( R_t ; \frac{A_t^*}{A_t} \right) + (1 - \alpha)\, g_{h_Y t} + \alpha g_{kt} + \left[ (1 - \alpha)\, g_{u_Y ;t} - g_{u_{W;t}} \right], \tag{4}$$

where $u_W = u_Y + u_A$ is the fraction of agents in the labor force.[9] The first summand of equation (4) represents the contribution of TFP to output growth, and the last summand, in brackets, represents the net effect of population reallocation among sectors.

## 2.1 One- and two-sector neoclassical growth models

The transitional dynamics of the traditional one-sector neoclassical growth model, and two-sector endogenous growth models such as Rebelo (1991) and Mino (1996), could arguably predict fast adjustment paths which are consistent with miracles. However, under standard technologies and parameterization, one- and two-sector growth models fail to reproduce the hump-shaped convergence pattern observed in the data.[10] This is primarily because the adjustment path generated by the linearized equation system of these models is characterized by a one-dimensional stable manifold, with a constant speed of convergence.

To illustrate this point we use our general framework characterized by equations (1)-(4). In the traditional one-sector neoclassical growth model, technological progress is labor augmenting and exogenously given, implying that in our equation (1), $\xi = (1 - \alpha)$. The savings rate is also assumed exogenous. Expression (3) then gives the growth rate of physical capital per capita as

$$g_{kt} = \bar{s} \frac{Y_t}{K_t} - \delta - n; \tag{5}$$

where $n$ and $\bar{s}$ are the exogenously given population growth rate and savings rate, respectively. In addition, there is no human capital (i.e. $g_{h_Y} = 0$), although the calibrated value of $\alpha$ can be increased to make $K$ an extended measure of capital (human plus physical), and the size of

[9] Hereafter, we use the term population to denote all agents in the economy (those occupied in the final-good sector, the R&D sector and the schooling sector). We use the term labor to denote workers in the final-good and R&D sectors. Our assumption that agents engaged in human capital acquisition are not part of the labor force is based on the observation that students are in general excluded from the labor force.

[10] Exceptions are Christiano (1989), King and Rebelo (1993), and Mulligan and Sala-i-Martin (1993). However, the first two papers needed to introduce the Stone-Geary utility function into the neoclassical growth model to successfully generate a hump-shaped output growth path. The Stone-Geary preferences include a consumption subsistence level that produces a very large marginal utility of consumption for low levels of income. As consumption moves away from the subsistence level, the marginal utility rapidly decreases and capital investment rises, increasing output growth rates. Mulligan and Sala-i-Martin (1993, p. 767) also generate a hump-shaped output growth rate adjustment path, but by using an extended measure of GDP.

population equals the labor force (i.e. $g_{u_W,t} = 0$). Hence, equation (4) becomes

$$g_{y^w_t} = g_{y_t} = (1-\alpha)\bar{g}_A + \alpha\left(s\frac{Y_t}{K_t} - \delta - n\right); \tag{6}$$

where $\bar{g}_A$ denotes an exogenous rate of technological progress.

Given that less developed countries generally possess small output to capital ratios, the neoclassical framework predicts that the convergence process of these economies will be characterized by an initial growth rate of output larger than its steady-state value. Moreover, $\frac{Y}{K}$ will converge monotonically due to diminishing marginal returns to capital, thus making output display monotonically decreasing growth rates.[11]

In two-sector endogenous growth models, technology is assumed to be a constant parameter and therefore $\bar{g}_A$ equals zero. For economically meaningful parameter values, Mulligan and Sala-i-Martin (1993) show that the growth rates of physical capital and human capital are inverse functions of their relative stocks. Given that these models carry out their analyses in per capita terms, and that $g_{y_t} = g_{y^w_t} - g_{u_{W t}}$, we can now write equation (4) as

$$g_{y_t} = (1-\alpha)\left[g_h\left(\frac{h_{Y t}}{k_t}\right) + g_{u_Y}\right] + \alpha g_k\left(\frac{k_t}{h_{Y t}}\right): \tag{7}$$

It turns out that the decline of growth rate of the relatively scarce capital is faster than the increase in growth rate of the abundant capital. As a result, a decreasing pattern characterizes most of the adjustment path of output per capita growth.

## 2.2 R&D-based growth model

A variable speed of convergence is crucial if we want growth rates not to pick at the beginning of the adjustment path. Eicher and Turnovsky (1999) find that variable convergence speed is possible in non-scale versions of growth models of physical capital accumulation and technical change. The difference from the endogenous two-sector growth models is the flexibility in returns to scale permitted by the non-scale production function. In addition, Perez-Sebastian (2000) shows that the introduction of technology imitation in non-scale models of growth is critical in replicating growth miracles. Technology adoption fuelled by cheap imitation is shown to also be the key in Parente and Prescott (1994). Growth rates in these models, however, still pick at the beginning of the adjustment path because of the large productivity of imitation.

[11] This conclusion does not change when the saving rate is determined by the optimal choice of consumption over time (see, for instance, King and Rebelo (1993)).

Equation (2) implies that, when technical progress becomes endogenous, the rate at which technology is acquired depends both on R and $\left(\frac{A^{*}}{A}\right)$. The latter contributes to generate a larger convergence rate at early stages of development because the technological gap is then relatively larger. The former, however, can do the opposite because R&D inputs may rise with the level of the state variables. Which one dominates will depend on the technology used to produce R. Parente and Prescott (1994) and Perez-Sebastian (2000) present similar technology adoption structures as the one of equation (2). In the former, R takes the form of physical capital, whereas in the latter R is produced with the same technology as output. Even though both models have contributed in our understanding of development miracles, neither model induces a fast enough growth on R to revert the declining convergence rate of output. Put differently, in both models the technology catch-up engine is the dominating force of economic growth.

### 2.3 Adding human capital in R&D-based growth model

When R takes the form of either capital or output, one important limitation is that final output leads and R follows; that is, Y must grow fast before R. This is actually the opposite scenario to the one which we want to reproduce. Introducing human capital in R&D inputs can potentially solve this problem – notice that the amount of human capital can grow fast independently of output. Additionally, the endogenous determination of human capital creates a new sector that can contribute to generate larger reallocations of population, thus inducing more variability in the output growth rate (recall equation (4)).

In summary, the main argument of the analysis above is that neither the neoclassical growth model nor the existing R&D-based growth models have been, in general, able to reproduce the convergence path of development miracles. It is our conjecture that a model in which human capital and technological progress are complementary engines of growth will be more successful in reproducing miracles.

## 3 THE BASIC MODEL

In what follows, we propose an alternative model of economic growth with endogenous technological change and human capital. We assign explicit forms to the implicit functions sketched in the previous section, and embed them into a fully specified optimizing framework.

## 3.1 Economic environment

The population in this economy consists of identical infinitely-lived agents, and grows exogenously at rate $n$. Agents are involved in three types of activities: consumption-goods production, R&D effort, and human capital attainment.[12] Each period, consumers are endowed with one unit of time that is allocated between working and studying. We abstract from labor/leisure decisions and assume that agents have preferences only over consumption.

Consumption goods are produced using the aggregate Cobb-Douglass production function given by equation (1). R&D activity is the source of technological progress in the model. The R&D technology is an explicit form of equation (2) as follows:

$$A_{t+1} - A_t = \mu A_t^{\phi} (h_{At} L_{At})^{\lambda} \left( \frac{A_t^*}{A_t} \right)^{\psi}; \quad \phi < 1; \;\; 0 < \lambda \leq 1; \;\; \psi \geq 0; \;\; A_t^* \geq A_t; \tag{8}$$

where $L_{At}$ is labor employed in the R&D sector at time $t$; $h_{At}$ is a measure of effectiveness of average skill level on labor employed in the R&D sector; $\phi$ represents a positive externality due to the stock of existing technology; $\lambda$ introduces diminishing returns to effective R&D labor; and $\psi$ is a technology gap parameter. We assume that the worldwide stock of existing technology $A_t^*$ grows exogenously at rate $g_{A^*}$.

Agents increase their human capital level through formal education, provided by a schooling sector. The human capital technology is of particular interest in our model and deserves careful consideration. Since our aim is to take the model to the data then our specification ought to be one that maps the available data on average years of education to the stock of human capital. Using the Mincerian interpretation seems to deliver such a specification. This representation follows Bils and Klenow (forthcoming), who suggest that the Mincerian specification of human capital is the appropriate way to incorporate years of schooling in the aggregate production function. Following their approach, human capital per capita is given by

$$h_{jt} = e^{f_j(S_t)}; \quad j \in \{Y, A\}, \tag{9}$$

where $S_t$ is the population average years of schooling at date $t$.[13] The derivative $f_j'(S_t)$ represents

[12] Schooling is assumed to be the only source of human capital attainment in this model. Allowing for other types of human capital attainment such as learning-by-doing, studied by Stokey (1988) and Lucas (1993), would be an interesting extension of the model and worthy of future research.

[13] Our baseline assumption is that human capital augments labor productivity in R&D differently than labor productivity in output (i.e. $f_A(S) \neq f_Y(S)$). In our calibration exercises, we also examine the case where human capital effectiveness is the same on all productive labor (i.e. $f_A(S) = f_Y(S) = f(S)$).

the return to schooling estimated in a Mincerian wage regression in sector $j$: an additional year of schooling raises a worker's efficiency by $f_j^{\prime}(S_t)$.[14]

Next, we are concerned with the behavior of $S_t$. In particular, we derive a law of motion of $S_t$ that is consistent with the following two desirable properties: (a) the evolution of $S_t$ depends on the share of people in education, (b) in steady state, $S_t$ is constant. As it is counter-factual to assume that $S_t$ grows indefinitely, the second property indicates that at steady state the average years of education reaches a fixed number.[15]

We assume that, each period, agents allocate time to human capital formation only after output production has taken place. Let $L_{Ht}$ be the total amount of people that attend school in the economy at date $t$. Assume that at some point in time, say period 1, the average educational attainment equals zero. Next period, given that consumers live for ever, the average years of schooling will be $S_2 = \frac{L_{H1}}{L_2}$, where $L_t$ is the population size at date $t$. In period 3, $S_3 = \frac{L_{H1}+L_{H2}}{L_3}$, and so on. Hence, the average educational attainment can be written as

$$S_t = \frac{\sum_{j=1}^{t-1} L_{Hj}}{L_t}. \tag{10}$$

From equation (10), we can derive the law of motion of the average educational attainment as follows:

$$\begin{aligned} S_{t+1} - S_t &= \frac{\sum_{j=1}^{t} L_{Hj}}{L_{t+1}} - \frac{\sum_{j=1}^{t-1} L_{Hj}}{L_t}; \\ &= \left(\frac{1}{1+n}\right)\left(\frac{L_{Ht}}{L_t} - nS_t\right). \end{aligned} \tag{11}$$

Notice that the above motion equation has the two desirable properties mentioned above: the evolution of $S_t$ depends on the share of people in education, $\frac{L_H}{L}$, and average years of schooling at steady state, $S_{SS}$; reaches an upper bound remaining constant thereafter. The second property holds because, as will be clear later, the ratio $\frac{L_H}{L}$ is invariant at steady state; dividing expression (11) by $S_t$, we can then easily see that variable $S$ can grow at a constant rate only if $S$ is a constant.

[14] Mincer (1974) estimates the following wage regression equation:

$$\omega_i = \beta_0 + \beta_1(SCH)_i + \beta_2(EXP)_i + \beta_3(EXP)_i^2 + \varepsilon_i;$$

where $\omega_i$ is the log wage for individual $i$, $SCH$ is the number of years in school, $EXP$ is the number of years of work experience, and $\varepsilon$ is a random disturbance term.

[15] For further discussion on this issue see Jones (1996, 1997).

## 3.2 Social planner's problem

For simplicity of exposition, we focus on a centrally planned economy.[16] A central planner internalizes the externalities and chooses the sequences $\{C_t, S_t; A_t, K_t, L_{Yt}, L_{At}, L_{Ht}\}_{t=0}^{\infty}$ so as to maximize the lifetime utility of the representative consumer subject to the feasibility constraints of the economy, and the initial values $L_0; S_0; K_0$; and $A_0$. The problem is stated as follows:

$$\max_{\{C_t;S_t;A_t;K_t;L_{Yt};L_{At};L_{Ht}\}} \sum_{t=0}^{\infty} \rho^t \left[ \frac{\left(\frac{C_t}{L_t}\right)^{1-\theta} - 1}{1-\theta} \right]; \tag{12}$$

subject to,

$$Y_t = A_t^{\xi} \left[ e^{f_Y(S_t)} L_{Yt} \right]^{1-\alpha} K_t^{\alpha}; \tag{13}$$

$$I_t = K_{t+1} - (1-\delta) K_t = Y_t - C_t; \tag{14}$$

$$A_{t+1} - A_t = \mu A_t^{\phi} \left[ e^{f_A(S_t)} L_{At} \right]^{\lambda} \left( \frac{A_t^*}{A_t} \right)^{\psi}; \tag{15}$$

$$S_{t+1} - S_t = \left( \frac{1}{1+n} \right) \left( \frac{L_{Ht}}{L_t} - n S_t \right); \tag{16}$$

$$L_t = L_{Yt} + L_{At} + L_{Ht}; \tag{17}$$

$$\frac{L_{t+1}}{L_t} = 1 + n; \quad \text{for all } t; \tag{18}$$

$$\frac{A_{t+1}^*}{A_t^*} = 1 + g_{A^*}; \tag{19}$$

$$L_0; \ S_0; \ K_0; \ A_0 \ \text{given,}$$

where $\theta$ is the inverse of the intertemporal elasticity of substitution; and $\rho$ is the discount factor. Equation (14) is the economy's feasibility constraint as well as the law of motion of the stock of physical capital; it says that, at the aggregate level, domestic output must equal consumption plus physical capital investment, $I_t$. Equation (17) is the population constraint; the labor force – the number of people employed in the output and the R&D sectors – plus the number of individuals going to school must be equal to the population.

[16] It is well known that in models with externalities like ours, appropriate policies by the social planner can achieve the first best. We assume that these policies are imposed in our economy and focus on the social planner's problem.

The optimal control problem can be stated as follows:

$$
\begin{aligned}
V(A_t;K_t;S_t) \;=\; & \max_{\{L_{Ht};L_{At};I_t\}} \Bigg\{ \frac{\left[\frac{A_t^{\xi}[e^{f_Y(S_t)}(L_t - L_{Ht} - L_{At})]^{1-\alpha}K_t^{\alpha} - I_t}{L_t}\right]^{1-\theta} - 1}{1-\theta} + \\
& + \rho V\left(A_t + \mu A_t^{\phi}\left(e^{f_A(S_t)}L_{At}\right)^{\lambda}\left(\frac{A_t^{*}}{A_t}\right)^{\psi}; K_t(1-\delta) + I_t;\; S_t + \frac{1}{1+n}\left(\frac{L_{Ht}}{L_t} - nS_t\right)\right)\Bigg\}; \qquad (20)
\end{aligned}
$$

where $V(\cdot)$ is a value function; $L_{Ht}; L_{At}; I_t$ are the control variables; and $A_t; K_t; S_t$ are the state variables. Solving the optimal control problem gives the Euler equations that characterize the optimal allocation of population in human capital investment, in R&D investment, and in consumption/physical capital investment respectively as follows:

$$
\left(\frac{C_t}{L_t}\right)^{-\theta}\frac{(1-\alpha)Y_t}{L_{Yt}} = \frac{\rho}{1+n}\left(\frac{C_{t+1}}{L_{t+1}}\right)^{-\theta}\frac{(1-\alpha)Y_{t+1}}{L_{Y;t+1}}\left[1 + f_Y'(S_{t+1})\frac{L_{Y;t+1}}{L_{t+1}} + f_A'(S_{t+1})\frac{L_{A;t+1}}{L_{t+1}}\right]; \qquad (21)
$$

$$
\begin{aligned}
\left(\frac{C_t}{L_t}\right)^{-\theta}\frac{(1-\alpha)Y_t}{L_{Yt}} = \; & \frac{\rho}{1+n}\left(\frac{C_{t+1}}{L_{t+1}}\right)^{-\theta}\frac{\lambda(A_{t+1}-A_t)}{L_{At}} \,* \\
& * \left\{\frac{\xi Y_{t+1}}{A_{t+1}} + \left[1 + (\phi-\psi)\left(\frac{A_{t+2}-A_{t+1}}{A_{t+1}}\right)\right]\left[\frac{\frac{(1-\alpha)Y_{t+1}}{L_{Y;t+1}}}{\frac{\lambda(A_{t+2}-A_{t+1})}{L_{A;t+1}}}\right]\right\}; \qquad (22)
\end{aligned}
$$

$$
\left(\frac{C_t}{L_t}\right)^{-\theta} = \frac{\rho}{1+n}\left(\frac{C_{t+1}}{L_{t+1}}\right)^{-\theta}\left[\frac{\alpha Y_{t+1}}{K_{t+1}} + (1-\delta)\right]: \qquad (23)
$$

At the optimum, the planner must be indifferent between investing one additional person in schooling, R&D, and final output production. The LHS of equations (21) and (22) represent the return from allocating one additional unit of labor to output production. The RHS of equation (21) is the discounted marginal return to schooling, taking into account population growth. The RHS term in brackets arises because human capital determines the effectiveness of labor employed in output production as well as in R&D. The RHS of equation (22) is the return to R&D investment. An additional unit of R&D labor generates $\frac{\lambda(A_{t+1}-A_t)}{L_{At}}$ new ideas for new types of producer durables. Every new design increases next period's output by $\frac{\xi Y_{t+1}}{A_{t+1}}$ and R&D production by $\frac{dA_{t+2}}{dA_{t+1}}$ times $\frac{(1-\alpha)Y_{t+1}}{L_{Y;t+1}}\left[\frac{\lambda(A_{t+2}-A_{t+1})}{L_{A;t+1}}\right]^{-1}$; where $\frac{(1-\alpha)Y_{t+1}}{L_{Y;t+1}}\left[\frac{\lambda(A_{t+2}-A_{t+1})}{L_{A;t+1}}\right]^{-1}$ gives the value of one additional design that equalizes labor wages across sectors. Euler equation (23) is standard. It says that the planner is indifferent between consuming one additional unit of output today and converting it into capital, thus consuming the proceeds tomorrow.

## 3.3 Steady-state growth

We now derive the model's balanced-growth path. Solving for the interior solution, equation (17) implies that in order for the labor allocations to grow at constant rates, $L_{Ht}$, $L_{Yt}$ and $L_{At}$ must all increase at the same rate as $L_t$. This means that the ratio $\frac{L_{Ht}}{L_t}$ is invariant along the balanced-growth path. Hence, equation (16) implies that, at steady-state (ss), $S_{ss}$ is constant and equals

$$S_{ss} = \frac{u_{h;ss}}{n}; \tag{24}$$

where $u_{h;ss} = \left(\frac{L_H}{L}\right)_{ss}$: Equation (24) shows that along the balanced growth path, the economy invests in human capital just to provide new generations with the steady-state level of schooling. This is consistent with work by Jones (1996, 1997), where growth regressions are developed from steady-state predictions, and data on $S_{ss}$ acts as a proxy for $u_{h;ss}$; the estimated coefficient on $S_{ss}$ in part reflects the parameter $\frac{1}{n}$ in our framework.

Let $G_{xt} = 1 + g_{xt}$: The aggregate production function, given by equation (13), combined with the steady-state condition $g_{Y;ss} = g_{K;ss}$ delivers the gross growth rate of output as a function of the gross growth rate of technology as

$$G_{Y;ss} = (G_{A;ss})^{\frac{\xi}{1-\alpha}}(1+n): \tag{25}$$

Since $G_{A;ss}$ is constant, it follows from equation (8) that

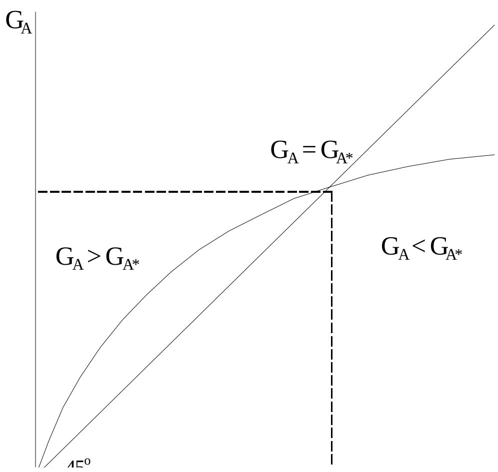
$$G_{A;ss} = \left[(1+n)^{\lambda} G_{A^{*};ss}^{\psi}\right]^{\frac{1}{1+\psi-\phi}}: \tag{26}$$

Equation (26) shows the relationship between the technology frontier growth rate and the technology growth rate of the model economy. Figure 1 illustrates this relationship.Notice that since the ratio $\frac{\psi}{1+\psi-\phi} < 1$, the function is concave with a unique point at which $G_{A;ss}$ equals $G_{A^{*};ss}$; in particular,

$$G_{A;ss} = G_{A^{*};ss} = (1+n)^{\frac{\lambda}{1-\phi}}: \tag{27}$$

The gross rate $G_{A;ss}$ cannot be larger than $G_{A^{*};ss}$ otherwise $A_t$ will eventually become bigger than $A_t^{*}$, and this has been ruled out by assumption. But $G_{A;ss}$ can be smaller than $G_{A^{*};ss}$. For simplicity, we focus on the special case in which all countries grow at the same rate at steady state;

Figure 2: Relationship between $G_{A;ss}$ and $G_{A^*;ss}$




that is, we assume that $G_{A^*;ss}$ is given by expression (27), and therefore so is $G_{A;ss}$.[17] This in turn implies that

$$G_{Y;ss} = G_{C;ss} = G_{K;ss} = (1+n)^{\frac{\lambda\xi}{(1-\alpha)(1-\phi)}}: \tag{28}$$

Consistent with Jones (1995) our balanced-growth path is free of "scale effects", and policy has no effect on long-run growth. The reason why our model's long-run growth is equivalent to that of Jones even in the presence of a schooling sector, is that at steady state the mean years of education, $S_t$; reaches a constant level $S_{ss}$.

## 3.4 Population shares in output, R&D, and schooling

Next, we derive the steady-state shares of labor in the three sectors of the economy. Euler equation (21) combined with the balanced-growth equation (28) gives

$$\frac{G_{y;ss}^{\theta-1}(1+n)}{\rho} - 1 = f_Y^0(S_{ss})\, u_{Y;ss} + f_A^0(S_{ss})\, u_{A;ss}; \tag{29}$$

[17] Alternatively, we could assume that a leading economy that is at steady-state is the one that moves the world technological frontier according to equation (8) which now reduces to

$$A_{t+1}^* - A_t^* = \mu A_t^{*\phi}(h_{At}^* L_{At}^*)^\lambda;$$

where $*$ denotes the value on which variables take in the leading country. Notice that now $\frac{A_t^*}{A_t} = 1$ because imitation is not possible at the frontier. In such case $G_A^* = 1 + g_A^* = (1+n^*)^{\frac{\lambda}{1-\phi}}$ as in Jones (1995). Assuming that $n = n^*$, and substituting $G_A^*$ into equation (26) delivers equation (27). As discuss in footnote 18, had $g_A^*$ taken on any other value, the transitional dynamics numerical analysis would become much more tedious.

where $u_{Y,ss} = \left.\frac{L_Y}{L}\right|_{ss}$ and $u_{A,ss} = \left.\frac{L_A}{L}\right|_{ss}$. The steady-state shares of R&D and output-production labor are inversely related to the return to education, other things constant.

Euler equation (22) combined with balanced-growth condition (28) deliver the steady-state labor share in R&D as

$$u_{A,ss} = \frac{u_{Y,ss}}{\left(\frac{1-\alpha}{\lambda\xi g_{A,ss}}\right)\left[\left(G_{Y,ss}^{\theta-1}\frac{G_{A,ss}}{\rho}\right) - (\phi-\psi)g_{A,ss} - 1\right]}. \tag{30}$$

Equations (29) and (30), and the population constraint

$$u_{Y,ss} = 1 - u_{h,ss} - u_{A,ss}, \tag{31}$$

give the three steady-state equilibrium shares of labor.

# 4 TRANSITIONAL DYNAMICS

This section is concerned with the transitional dynamics of the model economy. Since there is no analytical solution to our system we use numerical approximation techniques to simulate the transitional dynamics of the model, and then take their predictions to the data.

## 4.1 The normalized system

We start by redefining variables so that they take constant values along the balanced-growth path. The aggregate production function, equation (13), suggests that we normalize variables by the term $A_t^{\frac{\xi}{1-\alpha}}L_t$. We can then rewrite consumption, physical capital and output as $\hat{c}_t = \frac{C_t}{A_t^{\frac{\xi}{1-\alpha}}L_t}$, $\hat{k}_t = \frac{K_t}{A_t^{\frac{\xi}{1-\alpha}}L_t}$ and $\hat{y}_t = \frac{Y_t}{A_t^{\frac{\xi}{1-\alpha}}L_t}$, respectively. Using equation (21) gives

$$\left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)^{\theta}\left(\frac{u_{Y,t+1}}{u_{Yt}}\right)(G_{At})^{\frac{(\theta-1)\xi}{1-\alpha}}\left(\frac{\hat{y}_t}{\hat{y}_{t+1}}\right) = \left(\frac{\rho}{1+n}\right)\left[f_Y'(S_{t+1})\,u_{Y,t+1} + f_A'(S_{t+1})\,u_{A,t+1} + 1\right]. \tag{32}$$

From the R&D equation (8), we derive $G_{At}$ as

$$G_{At} = \frac{A_{t+1}}{A_t} = 1 + \upsilon\left[e^{f_A(S_t)}u_{At}\right]^{\lambda} T^{(1+\psi-\phi)}, \tag{33}$$

where $T = \frac{A_t^*}{A_t}$, and $\upsilon = \mu\,(A_t^*)^{\phi-1}L_t^{\lambda}$, which is a constant.[18] From equation (22) we get

[18] To show that $\upsilon$ is constant requires some algebra. Rewriting the equality in its gross growth form, $\frac{\upsilon_{t+1}}{\upsilon_t} = G_{A^*t}^{\phi-1}(1+n)^{\lambda}$; and given that $G_{A^*t} = G_{A,ss} = (1+n)^{\frac{\lambda}{1-\phi}}$, it follows that $\frac{\upsilon_{t+1}}{\upsilon_t} = 1$. Notice that if $A_t^*$ did not grow according to equation (27), $\upsilon$ could not be constant, making the simulation exercise more tedious.

$$\left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)^{\theta}\left(\frac{\hat{y}_t}{\hat{y}_{t+1}}\right)\left(\frac{u_{Y,t+1}}{u_{Yt}}\right) = \frac{\rho g_{At}}{G_{At}^{\frac{\xi}{1-\alpha}(\theta-1)+1}}\left(\frac{u_{A,t+1}}{u_{At}}\right) * \left[\left(\frac{\lambda\xi}{1-\alpha}\right)\left(\frac{u_{Y,t+1}}{u_{A,t+1}}\right) + \left(\frac{1}{g_{A,t+1}}\right) + (\varphi - \psi)\right]. \quad (34)$$

Finally, from equation (23) we obtain

$$\frac{1+n}{\rho}\left[\left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)(G_{At})^{\frac{\xi}{1-\alpha}}\right]^{\theta} = \alpha\frac{\hat{y}_{t+1}}{\hat{k}_{t+1}} + (1-\delta). \quad (35)$$

The system that determines the dynamic equilibrium normalized allocations are formed by the conditions associated with three control and three state variables as follows:

Control Variables:

1. Euler equation for population share in schooling, $u_{ht}$: Eq. (32).
2. Euler equation for population share in R&D, $u_{At}$: Eq. (34).
3. Euler equation for normalized consumption, $\hat{c}_t$: Eq. (35).

   Subject to the population constraint $u_{Yt} = 1 - u_{At} - u_{ht}$.

State Variables:

1. Law of motion of human capital, $S_t$: Eq. (11).
2. Law of motion of technology, $A_t$: Eq. (33).
3. Law of motion of normalized physical capital, $\hat{k}_t$:

$$(1+n)\hat{k}_{t+1}(G_{At})^{\frac{\xi}{1-\alpha}} = (1-\delta)\hat{k}_t + \hat{y}_t - \hat{c}_t, \quad (36)$$

where

$$T_{t+1} = T_t\left(\frac{G_{A^*t}}{G_{At}}\right), \quad (37)$$

and

$$\hat{y}_t = \hat{k}_t^{\alpha}\left[e^{f_Y(S_t)}u_{Yt}\right]^{1-\alpha}. \quad (38)$$

To solve the above system of dynamic equations, we follow Judd (1992), approximating the policy functions employing high-degree polynomials in the state variables.[19]

[19] In particular, the parameters of the approximated decision rules are chosen to (approximately) satisfy the Euler equations over a number of points in the state space, using a nonlinear equation solver. A Chebyshev polynomial

Table 1: Parameter values used in the simulations

| | | | | | |
|---|---|---|---|---|---|
| $\alpha$ | 0.36 | $\xi$ | 0.1 | $S_{ss}$ | 12.5 |
| $\rho$ | 0.96 | $G_y$ | 1.016 | $\eta$ | 0.69 |
| $\delta$ | 0.06 | $\lambda$ | 0.5 | $\beta$ | 0.43 |
| $n$ | 0.0116 | $\phi$ | 0.94 | $\theta$ | 1.28 |

## 4.2 Calibration

Table 1 shows the parameter values used to carry out the simulations. We choose a value of $0.06$ for the depreciation rate ($\delta$), and a value of $1.016$ for the steady-state gross growth rate of income ($G_{y,ss}$), the average number in the Bils and Klenow's (forthcoming) 91-country sample. We assign values of $0.36$ to the capital-share of output ($\alpha$), and $0.96$ to the discount factor ($\rho$). We set the population growth rate ($n$) to $0.0116$ per year, which is the average growth rate of the labor force in the G-5 countries (France, West Germany, Japan, the United Kingdom, and the United States) during the period 1965-1990. Regarding the value of the elasticity of output with respect to the technology, Griliches (1988b) reports estimates of $\xi$ between $0.06$ and $0.1$. Following Eicher and Turnovsky (1999), we choose $\xi = 0.1$.

It is not clear what the steady-state value of the average educational attainment ought to be, given that mean years of schooling have been increasing over the last decades in most developed countries. We choose to set $S_{ss}$ to $12.5$, to match the 1993 U.S. figure. To estimate the human capital equations, we first assume that

$$f_Y(S) = f_A(S) = f(S) = \eta S^{\beta}, \quad \eta > 0; \ \beta > 0. \tag{39}$$

Following Bils and Klenow (forthcoming), we use Psacharopoulos' (1994) cross-country sample on average educational attainment and Mincerian coefficients to estimate $\eta$ and $\beta$. Given equation (39), we can construct the regression equation

$$\ln(Mincer_i) = a + b \ln S_i + \varepsilon_i, \tag{40}$$

basis is used to construct the policy functions, and the zeros of the basis form the points at which the system is solved; that is, we use the method of orthogonal collocation to choose these points. Finally, tensor products of the state variables are employed in the polynomial representations. This method has proven to be highly efficient in similar contexts. For example, for the one-sector growth model, Judd (1992) finds that the approximated values of the control variables disagree with the values delivered by the true policy functions by no more than one part in 10,000. All programs are written in GAUSS and are available by the authors upon request.

Table 2: The Japanese and Korean experiences

| Country | | In 1960 | In 1963 | In 1990 |
|---|---|---|---|---|
| Japan | $Y$ per worker (%) | 20:6 | | 60:3 |
| | $K$ per worker (%) | 16:9 | | 104:6 |
| | $S$ (years) | 10:2 | | 11:0¤ |
| | $TFP(u_Y = 1)$ per worker (%) | 40:0 | | 59:5¤ |
| S. Korea | $Y$ per worker (%) | | 11:0 | 42:2 |
| | $K$ per worker (%) | | 11:6 | 50:2 |
| | $S$ (years) | | 3:2 | 7:7¤ |
| | $TFP(u_Y = 1)$ per worker (%) | | 38:8 | 58:3¤ |

¤ 1987 ¯gures.

where $Mincer_i = f^0(S_i)$ is the estimated Mincerian coe±cient for country i; a and b equal ln(´¯) and (¯ ¡ 1); respectively; and "$_i$ is a random disturbance term. We obtain estimates of ´ = 0:69 and ¯ = 0:43 that are very similar to those obtained by Bils and Klenow (forthcoming).[20] Equations (24) and (29) imply that the inverse of the intertemporal elasticity of substitution (µ) must then equal 1:28, which is well within the empirical estimates.

Estimates of ¸ found in the literature vary from 0:2 to 0:75, so we carry out a sensitivity analysis with ¸ taking the values 0:25, 0:5, and 0:75. Since the results we obtain are almost identical, we choose to use the intermediate value, ¸ = 0:5. From equation (27), we can then recover the value of Á = 0:94. We choose Ã so as to reproduce the output per worker evolution between 1960 and 1990 in Japan, and between 1963 and 1990 in S. Korea.[21] The former development experience gives a value for Ã of 0:21, whereas the latter implies that Ã equals 0:26. The initial values of the stock variables and output data used to calibrate Ã are presented in table 2; accuracy measures are presented in table 3.

## 4.3 Asymptotic stability and asymptotic speed of convergence

As a crucial ¯rst step, we establish the asymptotic stability of our model's long-run equilibrium. Linearizing the normalized system of equations around the steady state, we ¯nd that for any reasonable parameter values, the transition is characterized by a three-dimensional stable saddle-

[20] Both estimates are signi¯cantly di®erent from zero at the 1 percent level.

[21] Japan's rapid convergence toward U.S. income levels actually started right after WWII. Unfortunately, the Japanese Education Department does not possess estimates of the average educational attainment before 1960. We are grateful to Tomoya Sakagami who has attempted to obtain this data for us.

Table 3: Accuracy measures

| Country | $\psi$ | Average Error (%)* | | | Max. Error (%)* | | |
|---|---|---|---|---|---|---|---|
| | | $C$ | $u_Y$ | $u_A$ | $C$ | $u_Y$ | $u_A$ |
| Japan | 0.21 | 0.01 | 0.02 | 0.01 | 0.04 | 0.09 | 0.07 |
| S. Korea | 0.26 | 0.08 | 0.23 | 0.09 | 0.35 | 1.16 | 0.45 |
| S. Korea | 0.28 | 0.08 | 0.37 | 0.18 | 0.32 | 1.32 | 0.86 |

* We assess the Euler equation error over 10,000 state-space points using the approximated rules. For each variable, the measure gives the current value decision error that agents using the approximated rules make, assuming that the (true) optimal decisions were made in the previous period.

path. The adjustment path is then asymptotically stable and unique; furthermore, growth rates and convergence speeds can, as a consequence, vary across time and variables.

Next, we investigate the asymptotic speed of convergence – the rate by which a country's output converges to its balanced growth path once the country is sufficiently close to its long-run equilibrium. In our model, this speed is given by the largest eigenvalue among those contained in the unit circle. The one sector neoclassical growth model implies convergence speed of about 7%, which is inconsistent with most of the empirical evidence – Barro and Sala-i-Martin (1995) and Temple (1998), among others, report convergence-speed estimates for OECD nations of approximately 2%. Table 4 presents implied speeds for different values of $\lambda$ and $\psi$:

Table 4: Asymptotic speed of convergence for different values of $\lambda$ and $\psi$

| $\lambda \backslash \psi$ | 0.20 | 0.24 | 0.28 | 0.30 |
|---|---|---|---|---|
| 0.25 | 1.43% | 1.66% | 1.93% | 2.08% |
| 0.50 | 1.16% | 1.32% | 1.46% | 1.52% |
| 0.75 | 1.06% | 1.19% | 1.31% | 1.37% |

Parameter values in the neighborhood of those employed in our calibration deliver speeds of convergence that vary between 1.06% − 2.08%; consistent with most empirical evidence. In addition, our results are consistent with the finding of Eicher and Turnovsky (1999), that moving from one-sector to multi-sector non-scale growth models with endogenous technological change leads to severe reduction in the asymptotic speed of convergence.

## 4.4 Adjustment paths of Japan and S. Korea

We choose to calibrate Ã to both the S. Korean and the Japanese output paths because they represent two very di®erent \miraculous" experiences. Table 2 presents data for S. Korea and Japan on relative levels of GDP per worker (RGDPW), relative physical capital per worker, average educational attainment, and relative TFP, which is broadly de¯ned and includes everything not already captured by the other two stock variables (S and K).[22] Between 1960 and 1990, Japan's relative output per worker increased from 20:6 to 60:3 percent. GDP per worker in S. Korea started its fast growing path around 1963; during the period 1963-1990, its relative level increased from 11:0 to 42:2 percent. During these periods, Japan and S. Korea exhibited, on average, a 5:2 and a 6:5 percent annual growth rates, respectively.

Japan had lost a substantial portion of its physical capital during WWII, but its educational attainment in 1960 of 10:2 years compared well with those of the most developed nations { e.g., the U.S. educational attainment at that time was a little over 10:7.[23] What is even more interesting is that during the period 1960-1987, average years of schooling of workers increased very little, only by 0:8 years. The main engine of growth in Japan seems to have been physical capital accumulation induced in part by a very important technological catch-up process. In 1960, the Japanese physical capital stock per worker was only 16:9 percent; in 1990 reached a stunning 104:6 percent, which implies an average annual convergence rate of 6:3 percent.

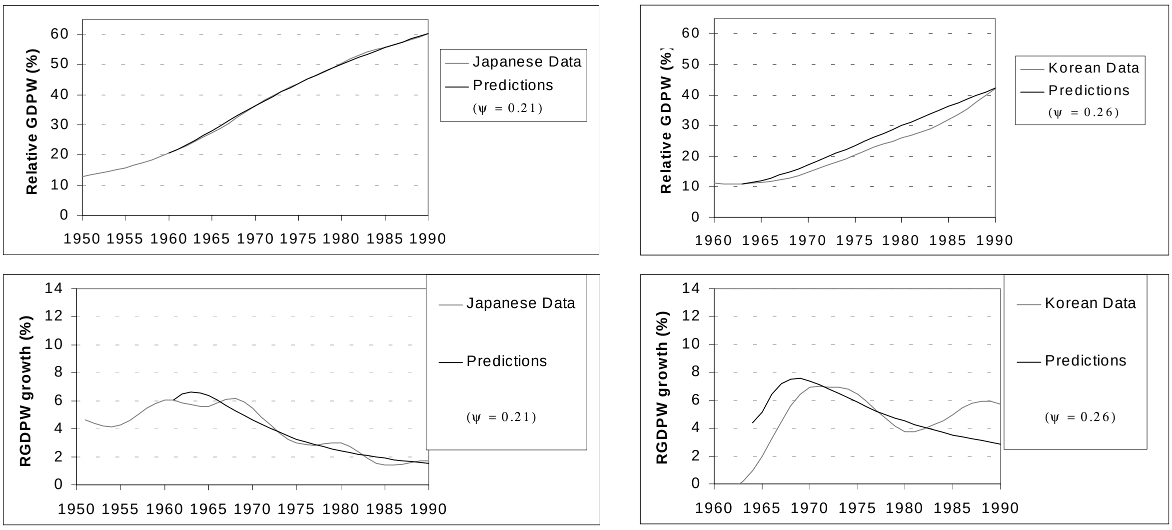
The S. Korean development experience, on the other hand, is distinctly di®erent from the Japanese experience. As shown in table 2, even though output convergence was faster in S. Korea, capital accumulated in this country at a lower rate than in Japan, growing from 11:6 to 50:2 percent during the relevant period; that is an average annual convergence rate of 5:6 percent. It is human capital accumulation that seems to have played the key role in S. Korean development process. In particular, the average educational attainment more than doubled in the period 1963-1987, increasing from 3:2 to 7:7 years.

Despite di®erent development experiences, both Japan and S. Korea obtain very close calibrated values of Ã. The adjustment paths predicted by the model for the level and growth rates of relative GDP per worker are depicted in ¯gure 3 and replicate fairly well the Japanese and the S. Korean

[22] All relative measures in the paper are with respect to U.S. levels.

[23] Human capital levels in Japan were high before WWII. After the Meiji Restoration of 1868, one of the policy priorities of the Meiji government was to introduce a nationwide education system under which all children from 6 through 13 years of age were required to attend school (see Ozawa (1985)).

Figure 3: Adjustment paths for Japan and S. Korea



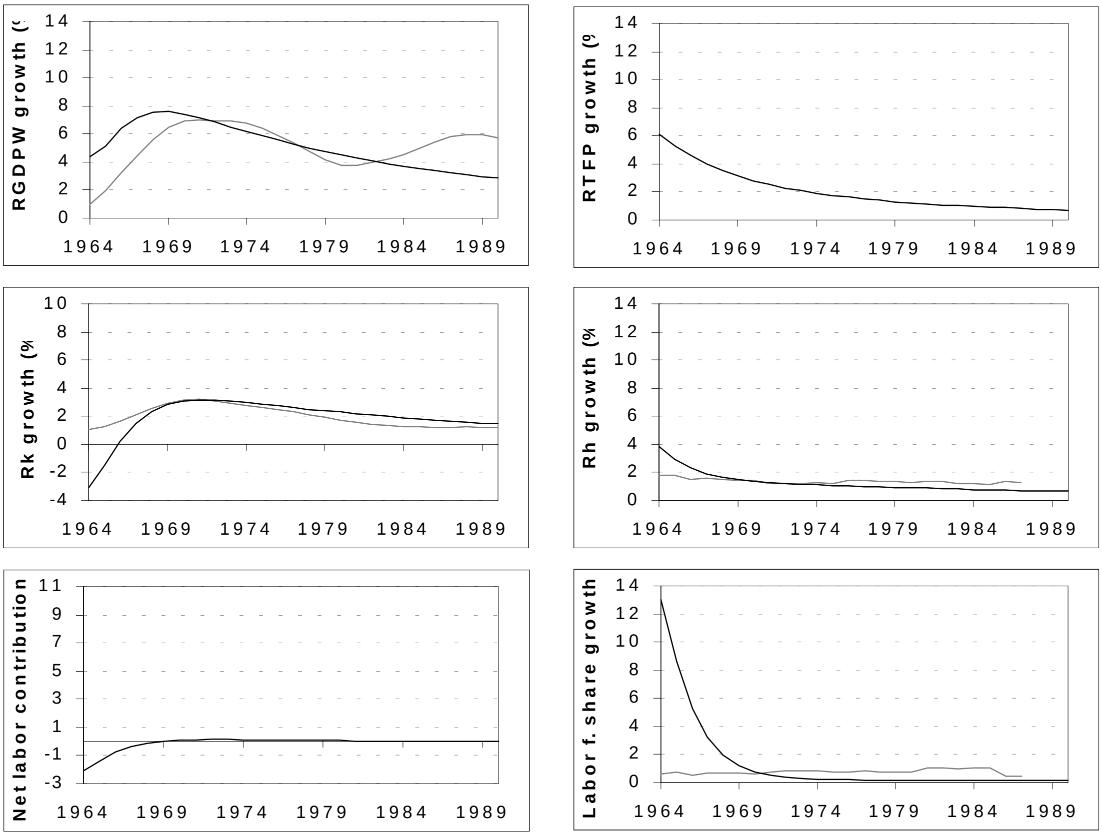

data. In particular, the model predicts that output per capita growth rates do not pick at the beginning of the adjustment path but later on.

## 4.5 Discussion of the transition results

What are the determining factors behind our results? Figures 4 and 5 present the contributions of di®erent components to S. Korean and Japanese output per worker growth, in line with equation (4). The key force that generates the hump-shape is the relatively large allocation of agents in education plus R&D activity at the beginning of the convergence process. To see this more clearly, recall that the term in brackets in equation (4) re°ects the contribution of the movement of population across sectors. More speci¯cally, this term takes into account that output growth rises with the amount of labor devoted to ¯nal-good production, but also that additional labor force de°ates output per worker. As a consequence, net labor contribution decreases with the number of students that leave school { because the amount of workers then rise { and increases as R&D e®ort declines { because part of the R&D labor is reallocated to the ¯nal output sector.

As shown in the bottom-right chart of ¯gures 4 and 5, the e®ect of students entering the labor force is larger at the beginning, and rapidly decreasing as the economy approaches the steady state, thus generating the fast declining pattern of labor force growth. This e®ect combined with a

Figure 4: Contribution of di®erent components to relative output growth, S. Korea

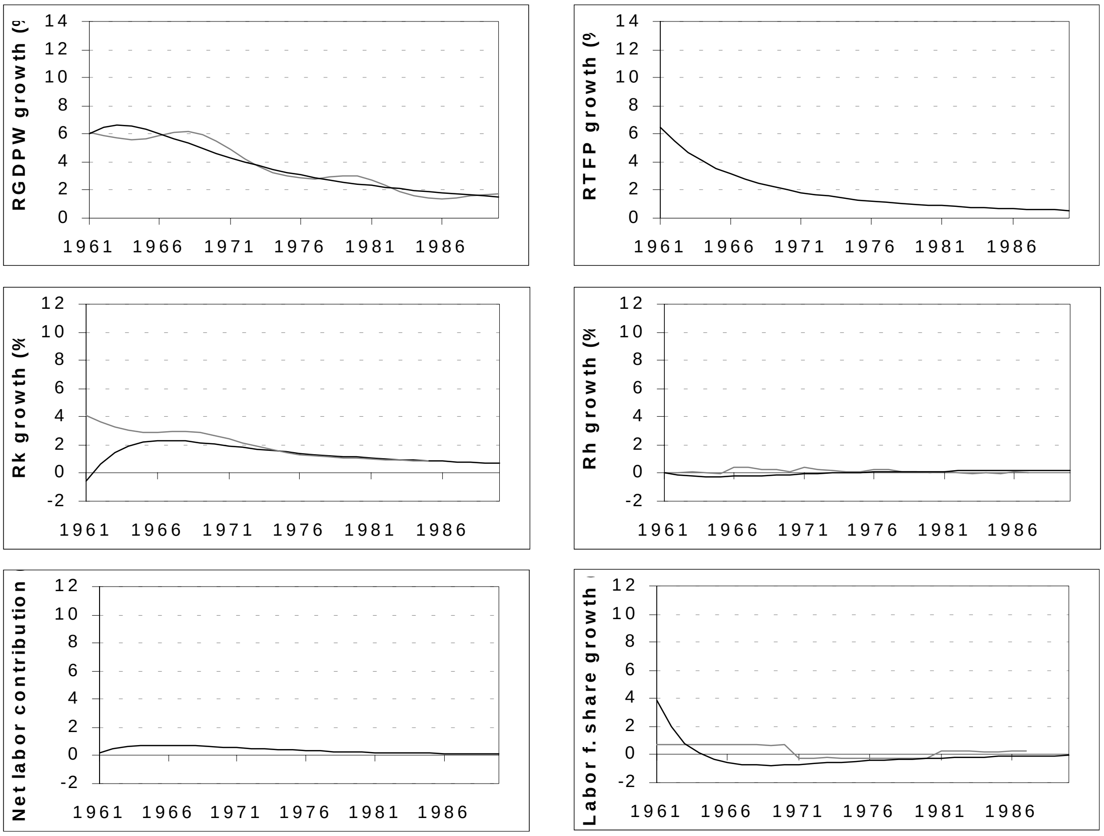



Black line: Model Predictions.

Grey line: Data.

Variables: RGDRW is relative GDP per worker; RTPF is relative TFP; Rk is relative physical capital per capita; Rh is relative human capital per capita; Net labor contribution represents the e®ect of the terms in brackets in equation (4); ¯nally, Labor f. share is labor force share in the population.

Figure 5: Contribution of di®erent components to relative output growth, Japan




Black line: Model Predictions.

Grey line: Data.

Variables: RGDRW is relative GDP per worker; RTPF is relative TFP; Rk is relative physical capital per capita; Rh is relative human capital per capita; Net labor contribution represents the e®ect of the terms in brackets in equation (4); ¯nally, Labor f. share is labor force share in the population.

decreasing R&D labor share induces the initially rising net contribution of population reallocation.

Although both Japan and S. Korea display the above pattern, the forces that cause it are di®erent in each country. In S. Korea, schooling enrollment rates sharply increase at impact due to the low human capital level. As convergence goes on, and average educational attainment rises, a rapid decrease in enrollment rates occurs, which generates the labor force growth rate decline. Japan, on the other hand, starts with relatively high human capital levels. Economy reconstruction and technology adoption then become the priority, causing students to leave the schooling sector to joint the ¯nal-good and R&D sectors. This reallocation of population is bigger at the beginning, inducing the initial decrease in labor force growth rates. As K and A achieve a su±ciently high level, some workers return to school, generating negative labor force growth.

Population reallocation has an important impact on physical capital too. Far from the steady state, R&D productivity is relatively high in the Japanese case, thus making R&D a more attractive activity. For S. Korea, high productivity in R&D is coupled with high productivity in schooling, and therefore schooling investment becomes also appealing. The consequence in both cases is a reduction in ¯nal output, with consumption smoothing pulling down the investment share, and physical capital accumulation slowing down. However, as output rises, the large marginal productivity of low physical capital eventually dominates, and the standard neoclassical diminishing returns result in declining growth rates.

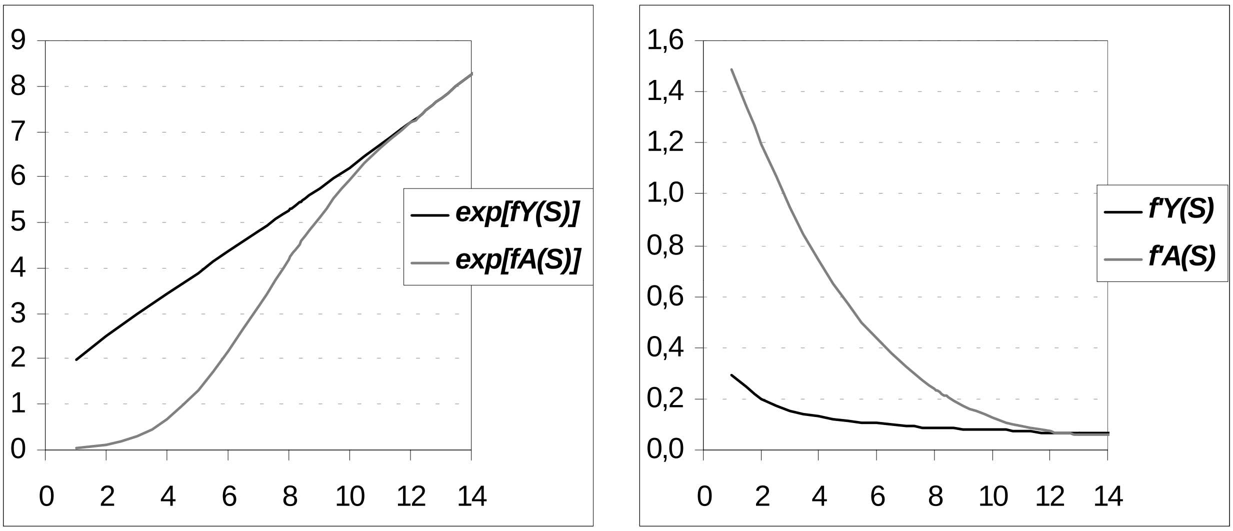
The predictions of the model are also consistent with the di®erent underlying characteristics of the Japanese and S. Korean experiences. First, lower educational attainment levels in S. Korea are associated with slower physical capital accumulation. A smaller human capital stock implies a larger productivity of schooling time, thus reducing the amount of labor allocated to ¯nal output production. In turn, as the previous paragraph explains, the lower output level slows down physical capital formation. Second, output growth rates at the beginning of the convergence process increase with the average educational attainment, both because the speed at which technology is adopted rises with human capital and because physical capital accumulates faster. In per worker terms, output growth is also faster the higher the schooling level. The reason is that the number of people that leave the schooling sector is then relatively lower, reducing the discounting e®ect caused by the rising labor force on output per worker ¯gures.

The main predictions of the model are, in general, consistent with the data as shown in ¯gures 4 and 5. In the S. Korean case, physical capital growth rates start low, pick-up after seven periods, and then decline. In the Japanese case, the labor-force share growth rate is the highest during the early sixties, then becomes negative, and declines in absolute magnitude as GDP converges. In addition, both country experiences have similar education attainment growth rates as the ones predicted by the model, su®ering only small changes along most of the adjustment path. The educational accumulation rate predicted by the model is, however, larger than the one shown by the data. In particular, simulated enrollment rates grow at impact above the observed rates, thus inducing too low physical capital convergence rates, and too high labor force growth rates, specially for S. Korea.

### 4.6 Allowing for di®erent human capital technologies

The above analyses implies that the reallocation of population across sectors and time is necessary to explain growth paths of development miracles. This reallocation is enough to replicate the Japanese data pretty well. For S. Korea, the model predictions follow the main patterns shown by the data, but the initial predicted output growth rates seem to be too high. Next, we argue

Figure 6: Distinct human capital technologies for output and R&D labor




that a stronger complementarity between human capital and technology acquisition e®ort, which requires having di®erent human capital technologies for ¯nal output and R&D ($f_Y(S) \neq f_A(S)$), may be as important as the reallocation of individuals in replicating miracles and, in particular, the S. Korean data. We consider the case in which, for low levels of human capital, the increase in years of education enhances the productivity of R&D labor at a slower pace than it enhances the productivity of output labor.[24] More specifically, we assume that $f_Y(S)$ is the same as presented before (i.e. $f_A(S) = \eta S^{\beta}$), and $f_A(S)$ behaves according to the plot in figure 6.[25]

Having two distinct human capital technologies for final output and R&D is also consistent with the data. For example, in this economy, the aggregate time-t Mincerian coefficient implied by the model is

$$Mincer_t = f_Y'(S_t)\,u_{Yt} + f_A'(S_t)\,u_{At}. \tag{41}$$

Suppose that we run an OLS regression of the type that was used in estimating $f'(S_t)$ – recall equation (40) – employing the simulated series of $S_t$ and $Mincer_t$, with the latter given by equation (41). The goal is to compare the new estimated coefficients to the ones we obtained employing the Psacharopoulos (1994) sample. Table 5 reports the estimated values for the two cases.

Table 5: Estimated coefficients from Mincer regressions

| | $f_Y(S) = f_A(S)$ | | | $f_Y(S) \neq f_A(S)$ | |
|---|---|---|---|---|---|
| | Estimated coefficients | Standard errors | | Estimated coefficients | Standard errors |
| | (Using empirical data) | | | (Using simulated numbers) | |
| $\hat{\alpha}$ | $-1.23$ | $0.32$ | $\hat{\alpha}'$ | $-1.04$ | $0.03$ |
| $\hat{\beta}$ | $-0.57$ | $0.15$ | $\hat{\beta}'$ | $-0.72$ | $0.01$ |

Notice that the estimates $\hat{\alpha}'$ and $\hat{\beta}'$ obtained from using simulated numbers are well within the 5 percent confidence intervals of $\hat{\alpha}$ and $\hat{\beta}$; obtained using empirical data. More specifically,

[24] Our assumption comply with evidence presented in Papageorgiou (2000). Regression estimates obtained from an extended Romer-type aggregate specification suggest that the relative contribution of human capital to technology adoption and final output production vary by country wealth. Furthermore, Papageorgiou finds that primary education contributes mainly to production of final output, whereas post-primary education contributes mainly to adoption and innovation of technology.

[25] The functional form of $f_A(S)$ in figure 6 is the higher order polynomial

$$\begin{aligned} f_A(S) &= -5.28314430 + 1.82378445\,S - 0.18199317\,S^2 + 0.00884479\,S^3 - 0.00019651\,S^4 \\ &\quad + 0.00000149\,S^5; \quad \text{for } S < 12.258898; \\ &= f_Y(S); \quad \text{otherwise.} \end{aligned}$$

Our choice of functional form for $f_A(S)$ only serves as an example to illustrate our point in this experiment. Differentiating between $f_A(S)$ and $f_Y(S)$ through estimation is an interesting exercise but beyond the scope of this paper.

$-1.04 \in (-1.23 \pm 1.96 \times 0.32)$ and $-0.72 \in (-0.57 \pm 1.96 \times 0.15)$. In sum, we can not reject the hypothesis that the data generating process behind the Psacharopoulos (1994) sample is the same as the one of the simulated data when we have two different human capital technologies for final output and R&D.

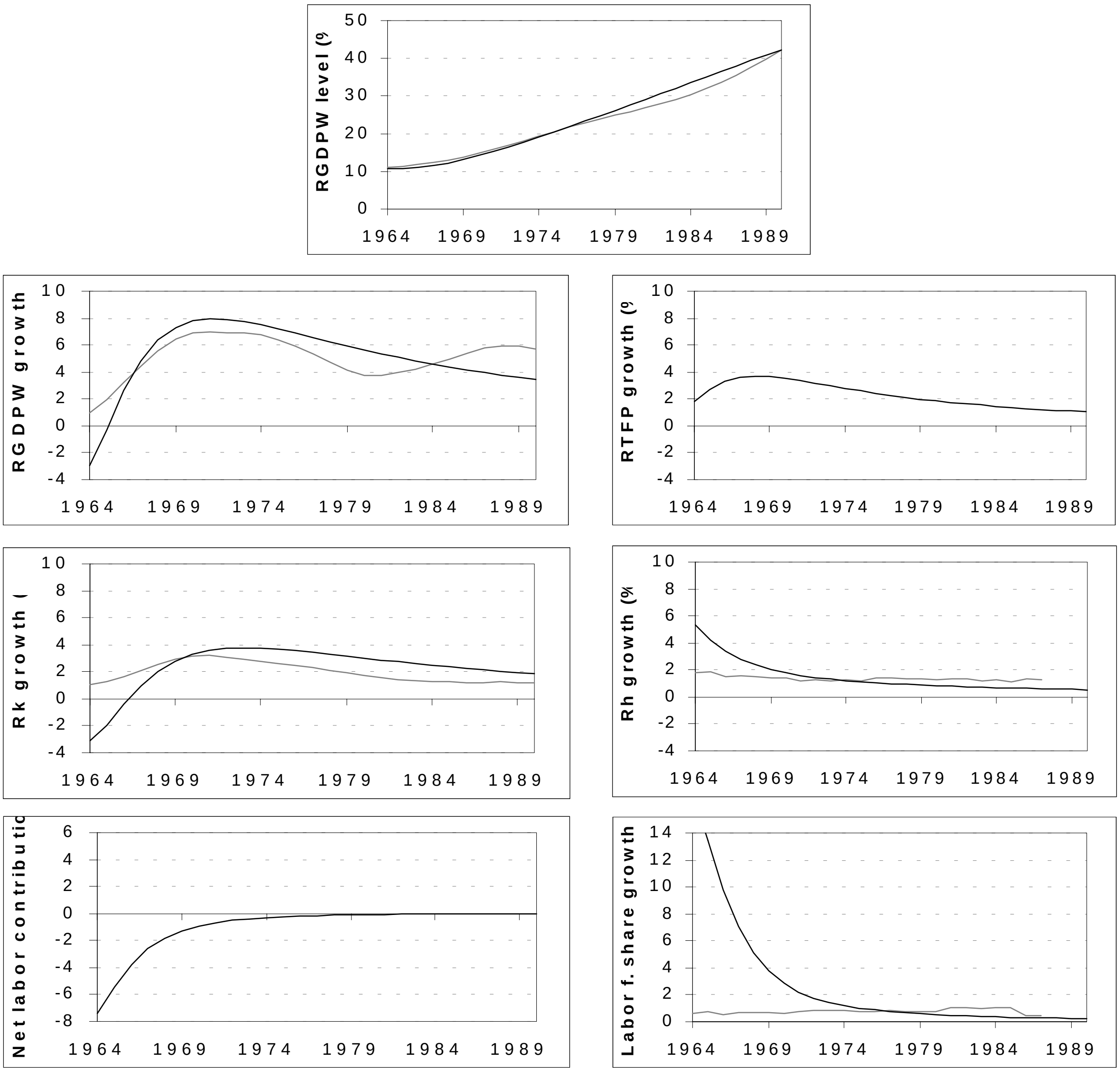
For the S. Korean case, figure 7 shows data on relative GDP per worker, its growth rate, and the contribution of the different components.[26] The smaller effectiveness of the R&D input at low levels of educational attainment shifts labor to the schooling sector. This provokes larger labor-force growth rates as a consequence of the faster increase in the R&D labor share. The result is a larger contribution of the reallocation of individuals across sectors to the inverted U-shape.

What is more important is that the effect of the complementarity between human capital and R&D labor now becomes more evident. As shown in figure 7, the relative-TFP growth initially increases with average years of schooling, clearly contributing to generating the output growth hump shape.

Overall, the predicted output level path now follows much more closely the S. Korean data. Predicted growth rates fit well the empirical observations, except for the initial years in which the simulated rates are lower than the observed ones. This last detail is important, because it could allow for a lower contribution of population reallocation as the data suggest.

[26] For Japan, given that its initial average educational attainment is above 10 years, the predicted paths are almost identical to the ones presented in figure 5.

Figure 7: S. Korean RGDPW growth with di®erent human capital technologies




Black line: Model Predictions.

Grey line: Data.

Variables: RGDRW represents relative GDP per worker; RTPF is relative TFP; Rk denotes relative physical capital per capita; Rh stands for relative human capital per capita; net labor contribution represents the e®ect of the terms in brackets in equation (4); ¯nally, labor f. share is the labor force share in the population.

# 5 CONCLUSION

In this paper we have attempted to shed new light in the making of growth miracles. We have done that by studying the transitional dynamics of a semi-endogenous growth model with physical capital, human capital and technical change. In order to compare the model predictions to the data, we have introduced human capital following the Mincerian approach suggested in recent papers. Furthermore, we have developed a law of motion for the average educational attainment that allows for endogenous human capital formation.

We have shown that the existing R&D-based growth models (and in lesser extend the one- and two-sector endogenous growth models) are capable of delivering the average convergence speed of economic miracles, but are not able to reproduce the velocity variations observed along the output adjustment path. The argument of this paper is that our proposed model is successful in generating these velocity variations observed in the output data of growth miracles.

Our work suggests that two are the main forces necessary to replicate miracle economies: a) the complementarity between human capital and technology adoption; b) the reallocation of individuals across sectors along the adjustment path. Focusing on the well-documented Japanese and S. Korean development experiences, we have shown that labor reallocation alone is su±cient to replicate the Japanese experience, but is not su±cient to fully generate the hump-shaped output per worker growth apparent in the S. Korean case. We have argued that having di®erent human capital technologies for ¯nal output and R&D, which increases the degree of complementarity between human capital and technology adoption e®ort, can be the missing part of the story. This is consistent with observation as the fraction of labor engaged in the production of new ideas is relatively small, and therefore so is its contribution to generate the observed data.

Our paper is not without limitations. In both versions, the model predicts enrollment rates that are larger than their empirical counterparts. This suggests that the model predictions could be improved if the accumulation of human capital would not necessarily imply the transfer of resources from the ¯nal-output sector. Future research could introduce leisure in the utility function, or allow for house-production. Another possibility would be to permit human capital formation though learning-by-doing or on-the-job training. In addition, we have argued that having di®erent human capital technologies for ¯nal output and R&D labor maybe important in replicating growth experiences, yet we were constraint to use a functional form that was not based on estimation.

Further research is clearly necessary in determining the appropriate weights to be assigned to the effectiveness of labor in different sectors.

The paper also has implications for other growth models. In particular, two-sector endogenous growth frameworks can also exhibit labor shifts across activities. And, as we mentioned, Mulligan and Sala-i-Martin (1993, p. 767) do generate growth rates that do not pick at the beginning of the adjustment path. To achieve it, they employ an extended measure of GDP per capita that adds a fraction of the market value of human capital labor to final output. This fraction contribution clearly depends on the amount of workers devoted to human capital formation. We then help to rationalize their result, because extending the output measure in that way raises the contribution of labor movements. In addition, two-sector growth models predict the same counterfactual (too large) share of time allocated to human capital formation along the adjustment path. The explanatory power of these models could therefore benefit from attempting some of the extensions proposed for our model in the previous paragraph.

In a general sense, we interpret our results as suggesting that a successful model of economic growth and development should include both technological progress and human capital accumulation as necessary engines, and the endogenous outcome of the economic system. In a more specific sense, our results suggest that labor reallocation and technology-human capital complementarity are crucial components in the making of miracles.

# A DATA APPENDIX

The data and programs used in this paper are available by the authors upon request.

- Income (GDP) [Source: PWT 5.6]

Cross-country real GDP per worker and real GDP per capita (chain index) are taken from the Penn World Tables (PWT), Version 5.6 as described by Summer and Heston (1991). This data set is available on-line at: http://datacentre.chass.utoronto.ca/pwt/index.html.

- Labor force [Source: PWT 5.6]

The cross-country data set on the labor force is calculated from the GDP per capita and GDP per worker numbers.

- Physical capital stocks [Source: STARS, and PWT 5.6]

Physical capital comes from PWT 5.6. The PWT, however, only provides with physical capital data from 1965. To obtain stocks back to 1963 for S. Korea, and 1960 for Japan, we used the growth rates implied by the STARS physical capital data to de°ate the 1965 PWT numbers.

- Education [Source: STARS (World Bank)]

Annual data on educational attainment are the sum of the average number of years of primary, secondary and tertiary education in labor force. These series were constructed from enrollment data using the perpetual inventory method, and they were adjusted for mortality, drop-out rates and grade repetition. For a detailed discussion on the sources and methodology used to build this data set see Nehru, Swanson, and Dubey (1995).

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