ON THE EFFICIENCY OF MARKET EQUILIBRIUM IN PRODUCTION ECONOMIES

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A B S T R A C T

This paper introduces the notion of Market Equilibrium With Active Consumers (MEWAC), in order to characterize the e¢ciency of market outcomes in production economies. We show that, no matter the behaviour followed by the …rms, a market equilibrium is e¢cient if it is a MEWAC. And also that every e¢cient allocation can be decentralized as a MEWAC in which …rms follow the marginal pricing rule.

Keywords: E¢ciency, Market Equilibrium, Production Economies, Imperfect Competition

1 Introduction

It is well established that perfectly competitive markets are institutions that permit the spontaneous coordination of economic agents, and that they do it efficiently. The standard general equilibrium model shows that decentralization is neither chaotic nor wasteful. This summarizes the Invisible Hand Theorem, according to Adam Smith's poetic expression. This Theorem relies on three key axioms about the nature of the economy: complete markets, price-taking behaviour and the convexity assumption.

The axiom of complete markets postulates that each commodity has associated with it a price, where commodities are distinguished by their characteristics (physical properties, uses, etc.) and their availability (where, when and in which state of the world are they available). Therefore, each agent faces as many relative prices as necessary to solve her individual optimization problem once and for all. In a broader sense, as it will be used here, complete markets also indicate the absence of spill-overs (including public goods as a particular case). That is, all non-price variables affecting individual agents' decision problems belong to their individual choice sets so that efficiency only requires the equalization of private marginal rates to relative prices.

Price taking behaviour means that those variables conditioning agents' choices are independent on individual actions. Therefore, individual maximization entails the equalization of marginal rates with the corresponding prices. When markets are complete, this axiom implies that in equilibrium all commodities will have the same marginal value in all possible uses and for all agents.

The convexity assumption says that agents choose actions within convex sets, guided by a convex criterion (a quasi-concave objective function). From this it follows that marginal conditions are sufficient to characterize the behaviour of individual agents, because all local maxima turn out to be global, and the necessary conditions for a maximum are also sufficient. Moreover, agents' behaviour can be described in terms of upper hemicontinuous correspondences, with non-empty, closed and convex values. This, together with to the other axioms, permits one to apply a fixed-point argument in order to prove the existence of equilibrium.

Complete markets, parametric prices and convexity, together, imply that the set of competitive equilibria of a given economy is non-empty (existence of equilibrium) and coincides with the set of efficient allocations (the two welfare theorems). The equalization of private marginal rates is a sufficient condition to ensure the efficiency of equilibrium allocations, because the local properties that characterize the maximization of individual objective functions imply global maximization of the aggregate payoffs. These three axioms also provide us with precise guidelines about the environments in which we can expect market failures: Incomplete markets (including here the cases of externalities and public goods), monopolistic competition, and increasing returns to scale or other forms of non-convexities.

The purpose of this paper is to provide some insights on the presence of market failures in economies in which production sets are not assumed to be convex and/or firms do not behave competitively, while keeping the assumption of complete markets. The analysis relies on two key methodological features: (i) Modelling the behaviour of firms by means of pricing rules. (ii) Analyzing the role played by consumers in production decisions. Let us comment on these two aspects.

A pricing rule is a mapping from the firm's set of efficient production plans to the price space. The graph of such a mapping describes the prices-production pairs which a firm finds acceptable. An equilibrium for the economy is defined in this setting as a price vector, a list of consumption allocations, and a list of production plans such that: (a) consumers maximize their preferences subject to their budget constraints; (b) each individual firm is in "equilibrium" at those prices and production plans (namely, the prices-production combination is in the graph of the firm's pricing rule); and (c) the markets for all goods clear. It is the nature of the equilibrium condition (b) which establishes the difference with respect to the Walrasian model.

There are very general existence results for equilibrium models in which firms' behaviour is described in terms of abstract pricing rules [see for instance Bonnisseau & Cornet (1988), Villar (1999, ch. 5)]. Moreover, Guesnerie (1975) has shown that every Pareto optimal allocation can be obtained as a marginal pricing equilibrium, regardless of the convexity of production sets. Under very mild regularity conditions marginal pricing is actually a necessary condition for optimality. Yet, marginal pricing does not ensure efficient outcomes. It may well be that there is an inadequate number of active firms in equilibrium, so that the resulting production lies in the interior of the aggregate production set [Beato & Mas-Colell (1985)]. Even if we take the simplified case of a single firm, there are economies in which no marginal pricing equilibrium is Pareto optimal [Guesnerie (1975), Brown & Heal (1979)], and economies in which marginal pricing is Pareto dominated by average cost pricing [Vohra (1988)]. This is so because, contrary to the standard convex world, the mapping associating efficient allocations to income distributions is not onto [see the discussion in Guesnerie (1990), Vohra (1990), (1991) and Quinzii (1992, ch. 4)]. These conclusions indicate the presence of an impossibility result: marginal pricing is a necessary condition for optimality, but it does not yield efficient outcomes.

It is interesting to observe the role played by the consumers in this impossibility result. A characteristic feature of those models with non-convex production sets is that consumers' income is defined as a mapping whose domain is the Cartesian product of the price space and the space of production allocations. That is to say, prices and production plans are treated as two independent sets of variables, regarding the consumer's choice problem. The strategy of including more variables to define the restrictions faced by consumers helps demonstrating the existence of equilibrium. Yet, this procedure generates many equilibria in which efficiency cannot be achieved because reallocating the resources devoted to production activities might yield higher incomes at given prices (which clearly implies the inefficiency of the original allocation).

We propose the notion of Market Equilibrium With Active Consumers (MEWAC) as a way of retrieving the link between consumers' wealth and production decisions. A MEWAC is a situation in which all consumers agree on the production plans that firms are to develop, and all firms agree on the prices that are to be associated with these production plans. The "price agreement" is the standard requirement for an equilibrium, in those models where firms' behaviour is modelled in terms of pricing rules. The "production agreement" among consumers is new. It indicates that we abandon here the assumption that consumers adjust passively to the decisions made by the firms. They participate in production decisions trying to maximize their net income (the owners of the firms have a say on firms' decisions).

The purpose of this paper is the analysis of the relationship between equilibrium and efficiency in an imperfectly competitive scenario.. We show that, no matter the behaviour followed by the firms, a market equilibrium is efficient if it is a MEWAC. And also that every efficient allocation can be decentralized as a MEWAC in which firms follow the marginal pricing rule. This equilibrium concept is an application to this scenario of the concept of valuation equilibria, as used in Hammond & Villar (1998), (1999), (2000) for the analysis of economies with spillovers (incomplete markets).

The paper is organized as follows: Section 2 contains the model, section 3 the results and a few final comments are gathered in section 4.

2 The model

Consider an economy with ℓ commodities, m consumers and n firms. The vector $\omega \in \mathbb{R}^{\ell}$ represents the aggregate initial endowments. There are n firms in the economy. We denote by $Y_j \subset \mathbb{R}^{\ell}$ the jth firm's production set, and by F_j the jth firm's set of weakly efficient production plans, that is,

$$F_j \equiv \{y_j \in Y_j \mid y_j' >> y_j \Rightarrow y_j' \notin Y_j\}$$

Let $F \equiv \prod_{j=1}^{n} F_j$ stand for the Cartesian product of the n sets of weakly efficient production plans. Points in F are denoted by

$$\tilde{y} = (y_1, y_2, ..., y_n)$$

A pricing rule is an abstract construct that provides a general way of describing the behaviour of firms. A pricing rule for the jth firm can be defined as a mapping ϕ_j applying the set of efficient production plans F_j into $\mathbb{R}^{\ell}_{+}$. For a point y_j in F_j, $\phi_j(y_j)$ has to be interpreted as the set of price vectors found "acceptable" by the jth firm when producing y_j. In other words, the jth firm is in equilibrium at (p, y_j), if $p \in \phi_j(y_j)$. Formally:

Definition 1 A Pricing Rule for the jth firm is a correspondence, $\phi_j : F_j \to \mathbb{R}^{\ell}_{+}$.

Some familiar examples of pricing rules that exhibit good operational properties under standard assumptions are the following:

(i) Profit maximization, ϕ_j^{PM}: Assuming that production sets are convex, this pricing rule associates with each efficient production plan the corresponding set of supporting prices. Namely, $\phi_j^{PM}(y_j) = \{p \in \mathbb{R}^{\ell}_{+} \; / \; py_j \geq py_j', \; \forall y_j' \in Y_j\}$.

(ii) Average cost pricing, ϕ_j^{AC}. This is a pricing rule that associates with each efficient production plan those prices that make the firm to break even. Formally (taking $y_j \neq 0$), $\phi_j^{AC}(y_j) = \{p \in \mathbb{R}^{\ell}_{+} \; / \; py_j = 0\}$.

(iii) Marginal pricing, ϕ_j^{MP}. This pricing rule describes a situation in which firms sell their output at prices that satisfy the necessary conditions for optimality. That is, $\phi_j^{MP}(y_j)$ corresponds to the Clarke Normal Cone to Y_j at the boundary point y_j.

(iv) Two-part marginal pricing. This is a non-linear price structure which combines marginal and average cost pricing, by charging an entrance fee plus a proportional one, to those consumers who buy positive amounts of the goods produced by non-convex firms.

(v) Constrained profit maximization. This is actually a family of pricing rules that describe a situation in which firms maximize profits at given prices, subject to some quantity constraints.

Remark 1 One can define more generally a pricing rule as a correspondence, $\Phi_j : F \times \mathbb{R}^{\ell}_{+} \to \mathbb{R}^{\ell}_{+}$. In that case, the jth firm's pricing rule depends on the "market conditions", as summarized by all firms' production plans and a reference price vector.

There are m consumers in the economy, each characterized by a triple (X_i, u_i, r_i) where $X_i \subset \mathbb{R}^\ell$, $u_i : X_i \to \mathbb{R}$, denote the ith consumer's consumption set and utility function, and $r_i : \mathbb{R}^\ell_+ \times F \to \mathbb{R}$ is the ith consumer's net income mapping, as a function of market prices and the firms' production plans. The net income mapping may include private income as well as taxes and transfers.

For the sake of illustration, we can think of the income mapping as given by:

$$r_i(p, \bar{y}) = p\omega_i + \sum_{j=1}^{n} \theta_{ij} p y_j + \tau_i(p, \bar{y}) \qquad [1]$$

where $\omega_i \in \mathbb{R}^\ell$ is the ith consumer's vector of initial endowments, θ_{ij} her share in the jth firm's profits (or losses, if negative), and $\tau_i : \mathbb{R}^\ell_+ \times F \to \mathbb{R}$ describes the ith consumer's tax-subsidy mapping. It is assumed, by the very definition of a market economy, that $\sum_{i=1}^m \omega_i = \omega$ (that is, total resources are fully distributed among the consumers) and $\sum_{i=1}^m \theta_{ij} = 1$, for all j (that is, firms are owned by the consumers). This income mapping corresponds to a private ownership market economy with taxes and transfers. We shall refer here and there to this particular case.

Consider now the following definition:

Definition 2 An income schedule is a collection of mappings $(r_i)_{i=1}^m$, with $r_i : \mathbb{R}^\ell_+ \times F \to \mathbb{R}$ for all i, such that, for all $(p, \bar{y}) \in \mathbb{R}^\ell_+ \times F$:

(i) $r_i(\lambda p, \bar{y}) = \lambda r_i(p, \bar{y})$, for all $\lambda > 0$.

(ii) $\sum_{i=1}^m r_i(p, \bar{y}) \leq p\omega_i + \sum_{j=1}^n p y_j$.

Definition 3 An income schedule is balanced when it satisfies $\sum_{i=1}^m r_i(p, \bar{y}) = p\omega_i + \sum_{j=1}^n p y_j$ for all $(p, \bar{y}) \in \mathbb{R}^\ell_+ \times F$.

An income schedule is a collection of mappings, one for each consumer, that satisfy two simple and intuitive properties. First, that each mapping is homogeneous of degree one in prices. Second, that total income cannot exceed the worth of the aggregate resources plus total profits. When part (ii) holds with equality the income schedule is said to be balanced. In the particular case illustrated by equation [1] above, an income schedule requires all tax-subsidy mappings being homogeneous of degree one in prices and self-financing, that is, $\sum_{i=1}^m \tau_i(p, \bar{y}) \leq 0$ (resp. $\sum_{i=1}^m \tau_i(p, \bar{y}) = 0$ if balanced)

Remark 2 Note that taking r_i as a mapping defined on the Cartesian product $\mathbb{R}^\ell_+ \times F$ amounts to considering prices and production plans as two separate sets of variables, from the ith consumer's viewpoint (contrary to the case of standard competitive economies).

An economy is a collection of: (a) m consumers, each characterized by her consumption set, her utility function and her income mapping; (b) n firms, each characterized by its production set and its pricing rule; and (c) A vector ω of initial endowments. This can be written shortly as:

$$\mathcal{E} = [(X_i, u_i, r_i)_{i=1}^m, (Y_j, \phi_j)_{j=1}^n, \omega].$$

Note that the very definition of an economy permits different firms to follow different patterns of behaviour (embodied in their corresponding pricing rules).

The following definitions make precise the equilibrium notions:

Definition 4 A *market equilibrium* for an economy $\mathcal{E}$, is a price vector $p^* \in \mathbb{R}^\ell_+$, and an allocation $[(x_i^*), \mathbf{y}^*)]$ such that:

(i) For every $i = 1, 2, ..., m$, x_i^* maximizes u_i over the set

$$\{x_i \in X_i / p^* x_i \leq r_i(p^*, \mathbf{y}^*)\}$$

(ii) $p^* \in \bigcap_{j=1}^n \phi_j(y_j^*)$.

(iii) $\sum_{i=1}^m x_i^* - \omega = \sum_{j=1}^m y_j^*$.

A market equilibrium is a price vector and a feasible allocation such that: (a) All consumers maximize utility at given prices within the budget sets that result from a passive adjustment to firms decisions; and (b) All firms are in equilibrium according to their pricing rules (which can differ from one another).

Market equilibria can be shown to exist when the following conditions hold [e.g. Bonnisseau & Cornet (1988, th. 2')]: (1) X_i is a non-empty, closed, and convex subset of $\mathbb{R}^\ell$, bounded from below; (2) u_i is continuous, quasi-concave and locally non-satiable; (3) r_i is continuous and satisfies the cheaper point requirement on the set of production equilibria;[1] (4) Y_j is closed and comprehensive; (5) ϕ_j is upper hemicontinuous, with non-empty, closed and convex values, and bounded losses;[2] and (6) The set of attainable allocations is compact. The proof of this general existence result relies very much on the treatment of income mappings as functions that are defined on the Cartesian product $\mathbb{R}^\ell_+ \times \mathcal{F}$, that is, functions that treat prices and production plans as two separate sets of variables (see Remark 2 above).

In order to present our equilibrium notion, let us define a mapping R_i : $\mathbb{R}^\ell_+ \to \mathbb{R}$ as follows:

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[1] That is to say, $r_i(p, \mathbf{y}) > \min pX_i$ for all $(p, \mathbf{y}) \in \mathbb{R}^\ell_+ \times \mathcal{F}$ such that $p \in \bigcap_{j=1}^n \phi_j(p, \mathbf{y})$.

[2] That means that $qy_j \geq \alpha$, for all $q \in \phi_j(p, \mathbf{y})$, all $y_j \in \mathcal{F}_j$, some given scalar $\alpha \leq 0$.

$$R_i(p) = \sup_{\mathbf{y} \in F} r_i(p; \mathbf{y}) \qquad [2]$$

This is the income that the ith consumer "demands" at prices p, by choosing those production plans that maximize the net revenue of her assets at these prices. This notional income mapping re-establishes the relationship between prices and production in the consumers' choice problem. Indeed, this is the analog of competitive budget sets, that is, $R_i(p)$ is precisely the income that consumers obtain when convex firms maximize profits at given prices.

Let $Y^i(p)$ stand for the set of solutions $\mathbf{y}^i$ to program [2], that may differ from one consumer to another. Clearly, $R_i(p) = r_i(p; \mathbf{y})$ whenever $\mathbf{y} \in Y^i(p)$. Also observe that, for an arbitrary $\mathbf{y} \in F$,

$$\sum_{i=1}^{m} R_i(p) = \sum_{i=1}^{m} r_i(p; \mathbf{y}^i) \geq \sum_{i=1}^{m} r_i(p; \mathbf{y})$$

with the equality holding if and only if $\mathbf{y} \in \bigcap_{i=1}^{m} Y^i(p)$. Note, however, that $\sum_{i=1}^{m} R_i(p) \geq \sum_{i=1}^{m} p y_j + p\omega$, for any given $\mathbf{y} \in F$, can only be ensured when the income schedule is balanced.

Remark 3 The notional income mapping R_i can actually be defined as the supremum of $r_i(p; \mathbf{y})$ on an arbitrary compact set $\overline{F} = \prod_{j=1}^{n} \overline{F}_j$, where $\overline{F}_j$ is a compact subset of F_j that contains in its (relative) interior all the efficient production plans that are attainable for the jth firm.[3]

Definition 5 A market equilibrium with active consumers (MEWAC, for short) for an economy E, is a price vector $p^* \in \mathbb{R}^{\ell}_{+}$ and an allocation $[(x_i^*), \mathbf{y}^*)]$ such that:

(i) For every $i = 1, 2, ..., m$, x_i^* maximizes u_i over the set

$$\{x_i \in X_i \,/\, p^* x_i \leq R_i(p^*)\}$$

(ii) $p^* \in \bigcap_{j=1}^{n} \phi_j(y_j^*)$.

(iii) $\sum_{i=1}^{m} x_i^* - \omega = \sum_{j=1}^{m} y_j^*$.

(iv) $\sum_{i=1}^{m} R_i(p^*) = p^*\omega + \sum_{j=1}^{m} p^* y_j^*$.

[3] Let us recall here that the set of attainable production plans for an economy is given by the set of points $[(x_i)_{i=1}^m, (y_j)_{j=1}^n] \in \prod_{i=1}^m X_i \times \prod_{j=1}^n Y_j$ such that $\sum_{i=1}^m x_i - \sum_{j=1}^n y_j \leq \omega$. The projection of this set on F_j gives us the jth firm's set of efficient and attainable production plans.

A market equilibrium with active consumers is a price vector and a feasible allocation such that: (a) No consumer finds it individually beneficial to choose an alternative consumption plan that is affordable, with respect to the maximum income achievable at the equilibrium prices; (b) Firms follow their corresponding pricing rules; and (c) Total income equals the worth of total resources plus total profits, and there is no alternative collection of production plans that yields higher income for some consumer. That is to say, $\bar{y}^* \in \bigcap_{i=1}^{m} Y^i(p^*)$ with $\sum_{i=1}^{m} R_i(p^*) = \sum_{i=1}^{m} r_i(p^*; \bar{y}^*)$: Note that, when consumers are not satiable, (iv) follows from (i) and (iii). Moreover, a consumption plan x_i that maximizes utility subject to the restriction $px_i \leq R_i(p)$ can be identified with the ith consumer's notional demand, whereas those x_i that maximize utility subject to $px_i \leq r_i(p; \bar{y})$ can be identified with her effective demand. From this point of view a MEWAC is a market equilibrium in which effective and notional demands coincide.

We can think of a MEWAC as the equilibrium of a market mechanism that works as follows. There is an auctioneer who calls out a price vector $p \in \mathbb{R}^{\ell}_{+}$. For all $i = 1, 2, ..., m$; the ith consumer calculates the income she can achieve at these prices, by choosing the most convenient production plans for the firms she owns (taking into account the tax-subsidy rule included in her income mapping).[4] This determines the set $Y^i(p)$: Then she solves her demand problem by maximizing utility at these prices within the budget set associated with her wealth estimated in that way. For all $j = 1, 2, ..., n$; the jth firm aggregates the orders given by all the incumbent consumers in a target production plan (e.g. the point in F_j which is closest to the weighted average of the consumers' proposals, where the weights correspond to their respective property shares). Then, it chooses those prices that make that production plan agree with its objectives (as defined through its pricing rule). The auctioneer compares all these actions and modifies the reference price vector when these decisions are inconsistent, until an equilibrium is reached. An equilibrium is a fixed point of this process.

Remark 4 It follows from definitions 3 and 4 that a MEWAC is a market equilibrium $[p^*; (x_i^*); \bar{y}^*]$ in which the additional restriction $\bar{y}^* \in \bigcap_{i=1}^{m} Y^i(p^*)$ holds. Therefore, the set of economies for which a MEWAC exists is smaller than those for which one can ensure the existence of market equilibria.

[4] We assume implicitly here that the ith consumer's tax subsidy rule is independent on the production plans of those firms in which she has no participation.

3 The results

We now proceed to analyze how this equilibrium concept fares with respect to the two welfare theorems.

Our first result establishes that a MEWAC yields an efficient allocation. Formally:

Theorem 1 Let E be an economy with locally non-satiable consumers. A MEWAC for E yields an efficient allocation, provided the income schedule is balanced.

Proof.

Let $[p^*; (x_i^*); \mathbf{y}^*]$ be a MEWAC and suppose that $[(x_i); \mathbf{y}]$ is a feasible allocation such that $u_i(x_i) \geq u_i(x_i^*)$ for all i; with a strict inequality for some agent. As this allocation is feasible, it must be the case that $\sum_{i=1}^m x_i \leq \omega + \sum_{j=1}^n y_j$; and consequently,

$$\sum_{i=1}^{m} p^* x_i \leq p^* \omega + \sum_{j=1}^{n} p^* y_j \qquad [3]$$

It follows from the non-satiation hypothesis, the definition of MEWAC and the balancedness condition, that:

$$\sum_{i=1}^{m} p^* x_i > \sum_{i=1}^{m} R_i(p^*) \geq p^* \omega + \sum_{j=1}^{n} p^* y_j$$

But this contradicts expression [3] above. Therefore, such an allocation cannot exist. ■

Theorem 1 establishes that a market equilibrium $[p^*; (x_i^*); \mathbf{y}^*]$ in which $r_i(p^*; \mathbf{y}^*) = R_i(p^*)$; for all i; yields an efficient allocation, no matter the pricing policies followed by the firms. Therefore, the agreement of consumers on the production plans induces efficiency whenever an equilibrium is reached. From this it follows that the usual inefficient equilibria one obtains in general equilibrium models correspond to a situations in which $\mathbf{y}^* \notin \bigcap_{i=1}^m Y^i(p^*)$: That is, there are consumers that would like to change the production decisions in order to achieve a higher net income.

Now consider the following axioms, that are needed in order to prove the second welfare theorem:

Axiom 1 For all $i = 1, 2, \ldots, m$;

(i) $X_i = \mathbb{R}^\ell_+$.

(ii) $u_i : X_i \to \mathbb{R}$ is continuous, quasi-concave, and satisfies local non-satiation.

Axiom 2 For all $j = 1, 2, \ldots, n$; Y_j is a nonempty and closed subset of $\mathbb{R}^\ell$ such that $Y_j - \mathbb{R}^\ell_+ \subset Y_j$.

Axiom 1 establishes that every consumer is characterized by a closed convex choice set bounded from below, that we take to be $\mathbb{R}^\ell_+$ for the sake of simplicity in exposition, and a preference relation that is complete, transitive, continuous, convex and locally non-satiable. Axiom 2 refers to the firms. It postulates that production sets are closed and comprehensive (but we assume neither the convexity of choice sets nor the feasibility of inaction).

The following result is obtained:

Theorem 2 Under axioms 1 and 2, let $[(x^*_i), y^*]$ be a Pareto optimal allocation such that $x^*_i \in \mathrm{int} X_i$ for all i. Then, there exist $p^* \in \mathbb{R}^\ell_+ - \{0\}$, and an income schedule $(r_i)_{i=1}^m$ such that $[p^*, (x^*_i), y^*]$ is a MEWAC in which firms follow the marginal pricing rule.

Proof.

First, apply the standard theorem that ensures that $[(x^*_i), y^*]$ can be decentralized as a marginal pricing equilibrium [e.g. Quinzii (1992, ch. 2)]. This theorem proves the existence of a price vector $p^* \in \mathbb{R}^\ell_+ - \{0\}$ such that $[p^*, (x^*_i), y^*]$ is a marginal pricing equilibrium relative to some income distribution.

To show that $[p^*, (x^*_i), y^*]$ is a MEWAC one has to find a suitable income schedule $(r_i)_{i=1}^m$ and to check that parts (i) and (iv) of definition 4 are satisfied (parts (ii) and (iii) being satisfied by construction). Let $\beta_i(p)$ denote the ratio between the ith consumer's cost of acquiring x^*_i at prices p, and the total worth of $\sum_{i=1}^m x^*_i$ also evaluated at prices p. That is,

$$\beta_i(p) = \frac{p x^*_i}{p \sum_{i=1}^m x^*_i}$$

Now consider the following wealth function for the ith consumer, for $i = 1, 2, \ldots, m$:

$$r_i(p, y) = \min \left\{ \beta_i(p) \left(\sum_{j=1} p y_j + p\omega \right), \; p x^*_i \right\}$$

Therefore, $r_i(p;\bar{y})$ is the minimum between a share $\beta_i(p)$ of the aggregate wealth at $(p;\bar{y})$, and the cost of x_i^* at prices p. Each r_i is clearly homogeneous of degree one in prices.

Summing over i, we get:

$$\sum_{i=1}^{m} r_i(p;\bar{y}) = \sum_{i=1}^{m} \min\left\{ px_i^* \frac{\sum_{j=1}^{n} py_j + p\omega}{p\sum_{i=1}^{m} x_i^*},\ px_i^* \right\}$$

Now observe that if $p(\sum_{j=1}^{n} y_j + \omega) \geq p\sum_{i=1}^{m} x_i^*$, all the terms of the sum take on the value px_i^* so that $\sum_{i=1}^{m} r_i(p;\bar{y}) = p\sum_{i=1}^{m} x_i^* \leq p(\sum_{j=1}^{n} y_j + \omega)$. If, alternatively, $p(\sum_{j=1}^{n} y_j + \omega) < p\sum_{i=1}^{m} x_i^*$, then $\sum_{i=1}^{m} r_i(p;\bar{y}) = p(\sum_{j=1}^{n} y_j + \omega)$. Therefore, $\sum_{i=1}^{m} r_i(p;\bar{y}) \leq \sum_{j=1}^{m} py_j + p\omega$, that is, $(r_i)_{i=1}^{m}$ is an income schedule, according to definition 2. Moreover, when evaluated at $(p^*;\bar{y}^*)$, we get:

$$r_i(p^*;\bar{y}^*) = \min\left\{ p^*x_i^* \frac{\sum_{j=1}^{n} p^*y_j^* + p^*\omega}{p^*\sum_{i=1}^{m} x_i^*},\ px_i^* \right\} = p^*x_i^*$$

Let $\pi : \mathbb{R}_+^{\ell} \to \mathbb{R}$ be a mapping given by $\pi(p) = \sup_{\bar{y}\in F} \sum_{j=1}^{m} py_j$, with $\pi(p) = +\infty$ if no maximum exists at prices p. The function R_i associated with the income mapping r_i is given by:

$$R_i(p) = \min\{\beta_i(p)\,[\pi(p) + p\omega],\ px_i^*\}$$

Clearly, if $\beta_i(p)[\pi(p) + p\omega] > px_i^*$ it follows that $r_i(p;\bar{y}) = R_i(p) = px_i^*$. Suppose now that $\beta_i(p)[\pi(p) + p\omega] \leq px_i^*$. We can rewrite this expression as:

$$\pi(p) + p\omega \leq \frac{1}{\beta_i(p)} px_i^* = p(\sum_{j=1} y_j^* + \omega)$$

which is possible only if $\pi(p) + p\omega = p(\sum_{j=1}^{n} y_j^* + \omega)$, by the very definition of $\pi(p)$. Therefore, when evaluated at $(p^*;\bar{y}^*)$ we find that $r_i(p^*;\bar{y}^*) = R_i(p^*) = p^*x_i^*$, so part (iv) of definition 5 is satisfied.

Finally, take a consumer i and a consumption plan $x_i \in X_i$ such that $u_i(x_i) > u_i(x_i^*)$. It is immediate to see that this consumption plan is not affordable because, by definition, $p^*x_i > R_i(p^*) = p^*x_i^*$, which is the minimum expenditure that is required to attain a utility greater than or equal to $u_i(x_i^*)$ at prices p^*. From this and the interiority assumption it is routine to show that part (i) of the definition is also satisfied, so that the proof is completed. ■

Theorem 2 establishes that, under fairly general assumptions, any Pareto efficient allocation can be decentralized as a MEWAC in which firms follow the marginal pricing rule. Note that when u_i is differentiable for some i, on a neighborhood of $x_i^* \in \text{int}X_i$, the (normalized) vector of marginal rates of substitution is unique, so that the (normalized) price vector supporting that allocation is unique as well.[5] This amounts to saying that a market equilibrium is efficient only if it corresponds to a MEWAC in which firms follow the marginal pricing rule. Note, however, that:

(i) Taking $x_i^* \in \text{int}X_i$ for all i is too strong an assumption (that is used here for the sake of simplicity in exposition). The only thing which is required in order to derive utility maximization from expenditure minimization is that $p^* x_i^* > 0$ for all i. Therefore, when there are commodities that do not enter the preferences of consumers, the efficient equilibrium price vector must be a marginal pricing vector in the subspace of commodities that are effectively consumed, and we find some degrees of freedom in the complementary subspace.

(ii) When firms experience quantity constraints, the restriction imposed on firms by marginal prices is less tight because the cone of normals at y_j in the truncated production set is larger than the usual normal cone to Y_j at the boundary point y_j.

4 Final comments

We have shown in the former sections that giving a more active role to the consumers in production decisions permits one to ensure the efficiency of equilibrium outcomes (Theorem 1). And also that each efficient allocation corresponds to a MEWAC in which firms follow the marginal pricing rule (Theorem 2).

It follows from those results that the efficiency of market equilibria calls for two restrictions to be satisfied:

(i) The equilibrium allocation must be supportable as a marginal pricing equilibrium. This restriction introduces local properties on the relationship between agents' choices and equilibrium price systems.

(ii) The income schedule must be rich enough to induce global optimization. This is a global condition on the income generated by the economy.

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[5] This implies that $\frac{\partial u_i/\partial x_{ik}}{\partial u_i/\partial x_{ih}} = \frac{p_k^*}{p_h^*}$, for all $k, h = 1, 2, \ldots, \ell$, is a necessary condition for the efficiency of market equilibria. This condition is naturally satisfied in our model because $\frac{\partial r_i(\cdot)}{\partial x_{ik}} = 0$, according to definition 2 (i.e. in an interior allocation the income function is independent of the consumption level of x_{ik}).

A MEWAC is a market equilibrium $[p^*; (x_i^*); y^*]$ in which $y^* \in \bigcap_{i=1}^{m} Y^i(p^*)$. Therefore, the set of economies for which a MEWAC exists is smaller than those for which one can ensure the existence of market equilibria. In a standard private ownership market economy those two notions coincide, because marginal pricing implies profit maximization when production sets are convex and because the competitive income mappings are precisely the functions R_i (as firms maximize profits at given prices and there are neither taxes nor subsidies). But we cannot count on this in general. Only particular income schedules can ensure that $r_i(p^*; y^*) = R_i(p^*)$ for all i; when firms are not price-taking profit-maximizing entities. Therefore, the usual conditions under which the existence of market equilibrium is postulated, typically valid for any arbitrary given income schedule, may well be incompatible with the efficiency requirements.

These results suggest that efficiency requires private wealth to be supplemented by a suitable system of taxes and transfers. Clearly the presence of a tax-subsidy rule does not ensure efficiency (the inefficiency of marginal pricing equilibria is well known). But without such a system there is little hope of achieving efficient outcomes through a market mechanism. To put it in a more provocative way: Our analysis points out that pure market mechanisms are generally inefficient because, except in the extremely rare case of perfect competition, some public intervention is necessary (though not sufficient) for the achievement of optimal allocations. Note that the key purpose of this tax-subsidy scheme would be to induce the right allocation of resources, rather than performing a redistribution policy.

There are nevertheless some particular models of imperfectly competitive economies in which the existence and efficiency of equilibrium hold. This is the case in the following examples: (1) Models with a single firm that follows marginal pricing and "suitable" consumers [Brown & Heal (1979), Vohra (1988), Quinzii (1991)]. (2) Models with particular types of marginal pricing, such as two-part tariffs [Brown, Heller & Starr (1992), Moriguchi (1996)], other forms of non-linear marginal pricing [Vega-Redondo (1987)], or "personalized commodities" [Edlin, Epelbaum & Heller (1998)]. (3) Models in which firms maximize profits subject to an input restriction [Scarf (1986), Villar (1999, chs. 8, 9), (2000)].

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