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WP-AD 2000-11

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Editor: Instituto Valenciano de Investigaciones Económicas, s.a.
First Edition June 2000.
Depósito Legal: V-2094-2000

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in order to encourage a discussion process before sending them to scientific journals for
their final publication.

* We would like to thank Ignacio Ortúñoz, Susana Alvarez, Antonio Jiménez, Ana I. Moro
and participants in various seminars for helpful comments and suggestions.

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ABSTRACT

We focus on the consequences of a voting process on the retirement age when agents have different ages and wages. We have two groups: retired people and workers. Once the retirement age is determined through a voting process, we verify if that age will keep the same popular support in future elections. At the end, we do an exercise of comparative static in order to analyze how the retirement age is conditioned by the redistributive level of the Social Security System. In this way, we also study how elected retirement age is affected by the wage distribution.

Keywords: Social security, retirement age, median voter.

1. INTRODUCTION

The literature on political economy of public pension provide us with many studies about Social Security System and its relation with individual's retirement age.

Most of them (see, for example, Crawford and Lilien[1981] or Burbidge and Robb[1980]) have developed life-cycle models with endogenous retirement decisions. The objective of the same is to study the possible implications of Social Security System on worker's incentives to retire. However in all these models there are no collective decisions but an unique individual being affected by the Social Security.

On the other hand, there are many other models in the political economic theory of Social Security in which majority voting equilibrium is applied. In all these cases, the retirement age is always considered as an exogenously given parameter and people simply accept it. Here the majority-voting equilibrium is used to choose the tax level in order to determine the size of the Social Security program (see, for example, Breyer[1994] or Marquardt and Peters[1997]).

Our aim is to deal with the consequences of a voting process on the implemented retirement age when the agents have different ages and wages. We analyze this voting process, how it evolves and how these different ages and wages affect the retirement age. This way, we can study the popular support lying behind each retirement proposal.

Other reasons by which we consider the retirement age to be as variable to vote is that we believe this parameter to be easier to understand for voters than tax level. In other words, they will be more precise about their optimal retirement ages than about their optimal tax levels at the time of voting. In consequence, we consider retirement age as a compulsory age that will be the variable to be elected by the individuals through a majority voting system. Therefore we can deduce not only how the composition of the population may influence on the implemented retirement age but also how this election may affect in the economy.

This article is based in one (Lacomba and Lagos [1999]) in which we develop a two-stage political economy model. In the ...rst, constitutional stage, the government chooses the redistribution level of the Social Security Program according to welfare criteriums. In the second stage, we analyze the optimal retirement ages of each individual and the median voter theorem is applied. In that model all individuals belong to the same generation and are differentiated only in wage.

In the present setting we introduce heterogeneity on age. We consider a continuous distribution of agents on ages and wages. The interesting point is to analyze how people behave with regard to the retirement age not only according to their different wage levels but also to their different ages. In this model we have two groups: retired people and workers. Both have to face an unexpected voting process on retirement age.

Retired people behave as a homogeneous group and will prefer the highest possible retirement age. With regard to the working population, we show that the optimal retirement age is increasing with the wage level and the older the workers are, the closer their optimal retirement ages to the "status quo" one will be.

Once the retirement age is determined through a voting process, the next step is to verify if that age will keep the same popular support in future elections.

At the end, we do an exercise of comparative static in order to analyze how the retirement age is conditioned by the redistributive level of the Social Security System. In this way, we study how would change the elected retirement age in two identical societies except in the wage distribution.

This paper is organized as follows: the model is described in section 2; section 3 derives the majority voting process; section 4 focus on exercises of comparative static with the level of redistribution and the wage distribution; and section 5 concludes. Some proofs are in the Appendix.

2. THE MODEL

We have a constant population in which in the moment of voting the agents will be different in wage and in age. We consider this model as a continuous, uniform distribution of agents on age, with no uncertainty on the length of their lives, going from zero to a fixed age, T . In this model we focus on a continuous distribution of agents on wage that will vary from a minimum to a maximum wage level, $[w_m; w_M]$:

People face an unexpected voting process on retirement age in a period $t > 0$, so they cannot anticipate it. The elected retirement age is believed by everybody to remain valid indefinitely. Also, they just vote once in their lives.

It is necessary to define a status quo that determines the behaviour of people in the previous years of the moment of voting. This status is one in which agents

face that all the variables of the model are exogenously given. Social Security Program is a balanced budget "pay as you go" system (PAYG), defined by a redistribution degree and a tax level h^R ; i ; and a status quo retirement age, R^{sq} .

The utility function of individuals over their life-cycle is similar to Crawford and Lilien, [1981]. These individuals have a stationary and temporally independent utility function, which is separable and strictly increasing in consumption and leisure. We assume that leisure yields utility to the individual only when this individual is retired. Therefore the only way utility coming from leisure can be varied is by changing the retirement age. The pension or retirement benefits are received only after they stop working. The instantaneous utility function is, then, as follows

$$U^i c_i^t; \mu^t = u^i c_i^t + v^i \mu^t \quad (2.1)$$

where c_i^t is the consumption at period t of agent i . The utility of consumption is twice differentiable with $u^0 > 0$, $u^{00} < 0$. Let μ_w^t be the leisure, equal for all agents at period t , being the utility of leisure $v^i \mu_w^t = 0$; in their working years and $v^i \mu_R^t = v$; in their retirement years. Besides, we assume that the elasticity of consumption marginal utility $\eta_r = i c u^{00}(c) = u^0(c)$ is non-increasing and smaller than one.

Let $\pm; r$ be the subjective rate of time preference and the market rate of interest. Let p_i be the annual pension that people would get when they were retired. Then the lifetime utility of an individual i can be written as

$$\int_0^{T^i} U^i c_i^t; \mu^t dt = \int_0^{R^{sq}} u^i c_i^t e^{i \pm t} dt + \int_{R^{sq}}^{T^i} \epsilon u^i c_i^t + v^i \mu^t dt \quad (2.2)$$

$$\text{s.t: } \int_0^{T^i} c_i e^{i r t} dt = \int_0^{R^{sq}} w_i (1 - \epsilon) e^{i r t} dt + \int_{R^{sq}}^{T^i} p_i e^{i r t} dt \quad (2.3)$$

We assume that saving earns no interest and that individuals do not discount the future, so both discount rates are zero ($\pm = r = 0$): There is also a perfect capital market, so individuals may borrow at a zero interest rate. Then the utility function that an individual i has to maximize over his life-cycle (as Breyer, 1994) can be reduced to

$$U(c_i; \mu) \leftarrow Tu(c_i) + (T_i - R^{sq}) v \quad (2.4)$$

The solution to (2.4) is to consume the same amount in each period, where the consumption is given by

$$c_i = \frac{1}{T} (R^{sq} w_i (1 - \rho) + (T - R^{sq}) p_i) : \quad (2.5)$$

The annual pension p_i is defined as follows

$$p_i = \frac{R^{sq}}{T - R^{sq}} \rho [(1 - \rho) \$ + \rho w_i] \quad (2.6)$$

being $R^{sq} = (T - R^{sq})$ the ratio between working and retirement years¹ and $[(1 - \rho) \$ + \rho w_i]$ a linear combination of the mean wage, $\$$, and the individual i 's wage, w_i ($\rho \in [0; 1]$ with $\rho = 0$ meaning full redistribution, everybody receives the same pension; and $\rho = 1$, actuarially fairness, that is, the individual benefits are equal to individual contributions).²

It is easy to check that in this system the budget is annually balanced, namely, total tax contributions of workers are equal to total benefits of retired people.

At the moment of voting on retirement age, R ; all agents will have a different wealth, $\frac{1}{T} (a; w_i)$, that will depend on his age, a ; his wage, w_i and the current Social Security Program. The wealth function is given by the total earned income less the total consumption up to period t .

In summary, when the voting process on the retirement age is made, there are agents not only with different age and wage, but also, consequently to these differences, with different wealth. In order to study how the voting process will be, we can divide population in two differentiated groups. By one side, retired people, those individuals with age bigger than current retirement age, $a > R^{sq}$. By other side, working people, those with age below or equal to retirement age, $a \leq R^{sq}$:

Let $R^*(a; w)$ be the optimal retirement age of an agent of age a and wage w : In order to study the optimal retirement age of all agents in each group, we will analyze separately the problems of the retired people and the working people.

¹Since the population is constant and uniform on age, this ratio is equal to the dependency one, that is, the ratio between working and retired people.

²The case in which $\rho = 1$ is equivalent to private system, where the consumption is $c_i = (R = T) w_i$:

2.1. The retired people

Retired people are all individuals with age higher than status quo retirement age: Utility function of a retired i of age a is defined as

$$U(c_i; a) = (T_i - a)(u(c_i) + v) \quad (2.7)$$

where

$$c_i = \frac{1}{T_i - a} ((T_i - a)p_i + a^{1/4}) \quad (2.8)$$

where p_i is the annual pension that this individual is receiving, and $a^{1/4}$ is the accumulated wealth of each individual since he was born.³

The pension depends positively on R . This will imply that retired people will prefer a retirement age as high as possible, since we assume that whatever the result of voting could be they do not come back to the labour market.

We assume that the retired people will not come back to the labour market even though the elected retirement age is higher than his own age. Then due to their utility functions and their budget constraints, the higher the retirement age be, the higher the pension, the consumption and the utility will be. For this reason, the retired people will always vote for the highest eligible retirement age.⁴

2.2. The working people

We focus on the optimal retirement age of a worker; $R^*(a; w)$. This individual has his age below or equal to status quo retirement age: The utility function of a worker of age a and wage w is given by

$$U(c; a; w) = \begin{cases} \frac{8}{\gg} (T_i - a)(u(c) + v) & R \leq a \\ \frac{8}{\gg} \cdot (T_i - a)u(c) + (T_i - R)v & R > a \end{cases} \quad (2.9)$$

$$\begin{aligned} & \frac{8}{\ll} \frac{1}{T_i - a} ((T_i - a)p + a^{1/4}) & R \leq a \\ \text{where } c = & \frac{8}{\ll} \frac{1}{T_i - a} ((R_i - a)w(1 - \zeta) + (T_i - R)p + a^{1/4}) & R > a \end{aligned} \quad (2.10)$$

³The formulas of pension and wealth are very extensive and not necessary for the analysis. The only important thing is that pension depends positively on retirement age.

⁴The retired agent will be indifferent to the retirement ages in this interval $(R_{sq} + (T_i - a); T)$:

where R is the new retirement age, ζ the tax level on wage, a the agent's wealth and p the annual pension, which depends on ζ and \mathbb{P} :

The wealth comes from the difference between total earned income less total consumption until period t and it is given by

$$\frac{1}{4} = w(1 + \zeta) - \frac{1}{T} R^{\text{sq}} [w(1 + \zeta) + \zeta ((1 + \mathbb{P}) \$ + \mathbb{P} w)] \quad (2.11)$$

The annual pension is defined as follows⁵

$$p = \frac{R}{T - R} \zeta w \quad (2.12)$$

where $W = (1 + \mathbb{P}) \$ + \mathbb{P} w$. Therefore we can derive the utility function having the retirement age R as an unique variable

$$U(R; a; w) = \begin{cases} \frac{1}{4} u^{\frac{1}{T - R}} \zeta w + \frac{1}{T - a} a^{\frac{1}{4}} + v & R \leq a \\ \frac{1}{4} u^{\frac{1}{T - a}} ((R - a) w(1 + \zeta) + R \zeta w + a^{\frac{1}{4}}) + (T - R) v & R > a \end{cases} \quad (2.13)$$

Thus we have to distinguish between two different parts in the utility function according to R be lower or higher than the age of individual, a .

When retirement age is below the age of the individual, $R \leq a$, his consumption just proceeds from the pension and his wealth, given that he would have to retire at that retirement age. However, when retirement age is higher than his age, $R > a$; then besides pension and wealth, he has to take into account the wage he will earn in his remaining working years, $R - a$.

Besides, when $R \leq a$; the leisure will remain the same, that is, the remaining years of his life, $T - a$, regardless the value of R . However, when $R > a$, if R changes, then the leisure years, $(T - R)$; change and so does the utility. Those are the reasons why U consists of two parts, one for $R \leq a$ and another for $R > a$:

Proposition 2.1. Let $a \leq R^{\text{sq}}$: The utility function $U(R; a; w)$ is single peaked in R . Moreover, $R^{\text{sq}}(a; w) > a$:

⁵There is no problem although $R > R^{\text{sq}}$ since when the worker of age a is retired, all people with age $a < R$ will be working.

Proof. From the ...rst and second derivative of the utility function belonging to an individual of age a and income w , we derive:

$$\begin{aligned} \frac{\partial U(\cdot)}{\partial R} &= \begin{cases} \frac{\partial}{\partial R} u^0(c) \cdot ((1-i^0) \$ + i^0 w) & R \leq a \\ \frac{\partial}{\partial R} u^0(c) (w(1-i) + i((1-i^0) \$ + i^0 w)) & R > a \end{cases} \\ \frac{\partial^2 U(\cdot)}{\partial R^2} &= \begin{cases} \frac{1}{T_i a} u^{00}(c) i^2 ((1-i^0) \$ + i^0 w)^2 & R \leq a \\ \frac{1}{T_i a} u^{00}(c) (w(1-i) + i((1-i^0) \$ + i^0 w))^2 & R > a \end{cases} \end{aligned}$$

It can be observed that from $R = 0$ to $R = a$, the function is increasing (since $\partial U(\cdot)/\partial R > 0 \forall R \leq a$) with respect to retirement age, and from $R = a$ it is strictly concave (since $\partial^2 U(\cdot)/\partial R^2 < 0$ for all R). In other words, there will be a unique peak that will be either the own age of the individual, $R^*(a; w) = a$; or it will be to the right of his age, $R^*(a; w) > a$: This manner, we can conclude that preferences are single-peaked on retirement age. See ...gures 1 and 2. Q.E.D.

We now can obtain the optimal retirement age of an individual of age a and income w . Due to the single-peakedness of the preferences, it can be applied the median voter theorem. The next step is to know who the median voter would be. For this proposal we analyze the sign of $\partial R^*/\partial a$ and $\partial R^*/\partial w$:

2.2.1. Workers with di¤erent ages

We will focus our study on individuals having the same wage level but with di¤erent optima on retirement age depending on how old they are.

Proposition 2.2. Let $a; a^0 \in R^{sq}$: If $a^0 > a$ then $jR^*(a^0; w) < jR^*(a; w) < R^{sq}$: I.e, the older an individual is, the closer his optimal retirement age to the status quo retirement will be.

Proof. From Proposition 5.1., the optimal retirement age is higher or equal than the own age, $R^*(a; w) > a$: Then, there are two di¤erent possibilities:

- When $R^*(a; w) > a$; we calculate $\partial R^*(a; w)/\partial a$ in order to observe how the optimal retirement age changes when individuals have same wage but di¤erent

ages. We maximize the utility function (2.13) and from the F.O.C., the implicit function theorem and after some simplifications we obtain

$$\frac{\partial R^*(a; w)}{\partial a} = \frac{R^{sq} - R^*(a; w)}{T_i - a} \quad (2.14)$$

If optimal retirement age is bigger (less) than the status quo, $R^*(a; w) > R^{sq}$ ($R^*(a; w) < R^{sq}$), then $\partial R^*(a; w) / \partial a = 0 < 0 (> 0)$: Therefore increases in a leads $R^*(a; w)$ to be closer to R^{sq} .

- When $R^*(a; w) = a$; it is easy to check that increases in a leads to increases in the optimum retirement age, $R^*(a; w)$ (recall Proposition 5.1.). Q.E.D.

In summary, individuals with the same wage will have their optimal retirement ages monotonically ordered with respect to age toward status quo retirement age, R^{sq} .

It can be deduced that individuals with the same wage cannot have their optimal retirement ages to both sides of R^{sq} : In other words, if an individual has $R^*(a; w) < R^{sq}$ then any other individual of age a^0 , with the same wage cannot have his $R^*(a^0; w) > R^{sq}$:

The intuition lying behind is that wealth was made in relation to the status quo situation, so wealth acts like a magnet towards R^{sq} . Consequently, the older an individual is, the more weight his wealth has, and so his optimal $R^*(a; w)$ will be closer to R^{sq} :

2.2.2. Workers with different wages

We now focus on the behaviour of agents with the same age but different wage levels. That is, we will try to analyze the optimal retirement age of rich and poor individuals with identical ages.

Proposition 2.3. Let $a \neq R^{sq}$: If $R^*(a; w) > a$ then $\partial R^*(a; w) / \partial w > 0$.

Proof. From F.O.C. of maximization problem of utility function (2.13), also according to the implicit function theorem and after some simplifications we obtain

$$\frac{\partial R^*(a; w)}{\partial w} = i \frac{[1 - i(1 - \frac{1}{T_i})][u^0(c)(1 - \frac{1}{T_i})]}{(w(1 - i) + iW)^2 u^{00}(c) \frac{1}{T_i - a}} \quad (2.15)$$

This equation is positive since the elasticity of marginal utility $\frac{1}{\gamma_r}$ is less than one. Q.E.D.

Thus, when $R^*(a; w) > a$, the most preferred retirement age for agents of the same age will be increasing with the wage.

On the other hand, if $R^*(a; w) = a$ it may happen that $\partial R^*(a; w) / \partial w = 0$. There will be individuals who will maintain unchanged their optimal retirement age in spite of their increasing wages.⁶ See figures 3, 4 and 5

The situation of the optimal retirement age coinciding with the own age of the individual, is easier to happen when this one gets older. In these cases it may be possible that increases in wage does not lead to increases in the optimal retirement age for people of the same age. The economic intuition is that the older the individual is, the more relative weight the leisure has and there will be occasions in which, whatever the wage would be, all people of the same age will prefer to ...nish their working life in due course.

3. MAJORITY VOTING PROCESS

We next explain the voting process, the way agents vote and which is the elected retirement age.

3.1. The retired people's election

In any voting process, retired people will vote the highest retirement age, independently of their income and age.⁷

⁶We have seen that the utility function is composed of two different parts, one of them when $R \leq a$ and the other when $R > a$. When the maximum of this second part is to the left of a then the real optimum of the utility function is $R^*(a; w) = a$; since the utility function is continuous in a and to the left of a the valid part is the ...rst one, which is increasing. But we have seen that when the wage is increased the maximum of this second part goes to the right, and therefore there will be times in which this maximum surpasses the age a : The same will go from the left to the right of a ; and then will become the new optimal retirement age. Hence, there will be increases in the wage that will lead to this situation and, therefore the optimal retirement age will be higher and others where the movement to the right of this maximum of the second part will not be large enough and the optimal retirement age will keep being the same $R^*(a; w) = a$:

⁷It may exist situations in which some retired people will be indifferent to both proposed retirement ages, since we assume they behave as a homogeneous group. This manner, if some retired people prefer the highest one, all of them will vote it.

We suppose that they always represent less than ...fty per cent of the population. Thus, the median voter will belong to the working group that will be analyzed below.

3.2. The worker's election

We start analyzing people just incorporated to the labour market, that is, with age $a = 0$: These individuals will have optimal retirement ages that are increasing with the wage level (see (2.15)) and independent of the status quo retirement age, R^{sq} : That is, whatever the status quo is, the optimal retirement age of this people will not modify. Given a continuous distribution of wages, we will have a continuous distribution of optimal retirement ages from people with age $a = 0$ that will never change, regardless the status quo.

From the optimal retirement ages of the individuals with age $a = 0$ we derive the optimal ones of the rest of population depending on the age, the wage and the previous retirement age, R^{sq} : See ...gure 6

From Proposition 5.2., it follows that the older an agent is, the closer to R^{sq} his optimal retirement age is. The economic intuition lying behind is that the older an agent is, the bigger the costs of an alteration on retirement age will be. Since this agent has planned his life regarding the tax and wage level unchanged, the wealth and consumption per year and, of course, the established retirement age, R^{sq} . Consequently, it seems logical that the oldest workers have their preferences close to the status quo retirement age R^{sq} .

3.3. The voting process

Once we know how the whole population would choose the retirement age, the following step would be to analyze what would happen if government o...ers the opportunity to choose a new compulsory retirement age in a democratic process (through pairwise majority voting system).⁸

This democratic process will lead us to a retirement age that will be a Condorcet winner, which exists by the median voter Theorem. Let R^e be the retirement age elected through this majority voting process which it is considered by everybody as inde...nitely valid.

⁸In this process, R^{sq} does not need to be one of the two alternatives.

From now on we will call peaks to the optimal retirement age of each individual. For instance, R^{sq} leaving more than ...fty per cent of peaks below means that more than ...fty per cent of population have their optimal retirement ages lower than R^{sq} .

The status quo age, R^{sq} ; has a great importance in the democratic process, since it will determine the elected retirement age, R^e : If we consider that R^{sq} leaves more (less) than ...fty per cent of peaks below it, that would mean that the majority would prefer a retirement age less (bigger) than the previously established one. Then, in a democratic process, the socially elected retirement age, R^e , would move down (up) from R^{sq} and, obviously, would leave the ...fty per cent of peaks to both sides. See ...gure 7

3.4. The stability of the implemented retirement age

Once a new compulsory retirement age, R^e , is elected we may ask if R^e will keep the same popular support T years later.

We suppose that T years later there is another voting process. We will try to ...nd out if R^e remains the ...fty per cent of peaks below it (including peaks equal to R^e) in the new voting process. According to this, we analyse how the behaviour of this population changes in relation to the previous one by changing the status quo from R^{sq} to R^e :

From now on we distinguish between different periods. Let t be the period when the voting takes place and $t + 1$ the period when the following voting process takes place T years later.

The elected retirement age in period t will become status quo retirement age in period $t + 1$; $R_t^e = R_{t+1}^{sq}$:

De...nition 3.1. R_t^{sq} is stable if $R_t^e = R_t^{sq}$:

When the elected retirement age on the voting process coincides with the status quo one, R_t^{sq} ; then this age R_t^e will keep stable: It means that R_t^{sq} leaves already the ...fty per cent of the peaks in each side. In the following voting process, the status quo will be the same, $R_{t+1}^{sq} = R_t^e = R_t^{sq}$, and therefore nothing changes. There will be the same percentage of retired people and the composition of the worker's preferences will be the same, so the results will be repeated and so on. That is, it will be a stable retirement age.

Proposition 3.2. Let $R_t^e < R_t^{sq}$ then $R_{t+1}^e \leq R_{t+1}^{sq} = R_t^e$: In other words, in period t ; R_t^e leaves ...fty per cent of peaks to both sides, but in period $t + 1$, $R_t^e = R_{t+1}^{sq}$ leaves more or equal than ...fty per cent of peaks below it.

Proof. We want to show that when the status quo retirement age decreases from a voting process to the following one, $R_{t+1}^{sq} < R_t^{sq}$, the number of peaks below or equal to $R_t^e = R_{t+1}^{sq}$ never decreases.

We know that $R_t^a(0; w) = R_{t+1}^a(0; w)$ for any $w \in [w_m; w_M]$, so any agent with $a = 0$ and $R_t^a(0; w) \leq R_t^e$ will keep $R_{t+1}^a(0; w) \leq R_t^e = R_{t+1}^{sq}$: From (2.14) if an agent of age $a = 0$ has $R_{t+1}^a(0; w) \leq R_{t+1}^{sq}$ then all workers with the same wage will have $R_{t+1}^a(a; w) \leq R_{t+1}^{sq}$: Therefore individuals with $a > 0$ and $R_t^a(a; w) \leq R_t^e$ as well will keep $R_{t+1}^a(a; w) \leq R_t^e = R_{t+1}^{sq}$: Q.E.D.

So, from period t to period $t + 1$ the number of the peaks below or equal to R_t^e may increase. Therefore R_{t+1}^e may not be equal to R_{t+1}^{sq} but never higher.

In the Appendix we show some examples where the percentage of peaks below or equal to R_t^e increases in the next voting process, in order to illustrate how the popular support on elected retirement age changes from one voting to the next one, when the elected retirement age becomes the status quo one.

Proposition 3.3. Let $R_t^e > R_t^{sq}$: In this case R_{t+1}^{sq} may admit any possibility, i.e., R_{t+1}^{sq} may leave more, less or equal than ...fty per cent of peaks below it.

Proof. When the status quo retirement age increases from a voting process to the following one, $R_{t+1}^{sq} = R_t^e > R_t^{sq}$; it generates two different effects on the distribution of peaks. See ...gures 10 and 11

On one hand, the percentage of retired people will be lower in period $t + 1$ than in period t , since status quo retirement age will increase, $R_{t+1}^{sq} > R_t^{sq}$. Individuals with age $a \in (R_t^{sq}; R_{t+1}^{sq}]$ who, in period t , are retired and in the following period, $t + 1$, the people with the same age will be workers.

In period t retired people with $a \in (R_t^{sq}; R_{t+1}^{sq}]$ have $R_t^a(a; w) > R_t^e$: This group always prefers a retirement age as high as possible. Let w^e a wage level such that $R_t^a(0; w^e) = R_t^e$: In period $t + 1$ individuals with $a \in (R_t^{sq}; R_{t+1}^{sq}]$ are working and, also from (2.14), agents with this age and $w \in [w_m; w^e]$ have $R_{t+1}^a(a; w) \leq R_{t+1}^{sq} = R_t^e$: Thus the percentage of peaks below $R_t^e = R_{t+1}^{sq}$ is increased (positive effects):⁹

⁹If everybody has $R_t^a(a; w) > R_t^{sq}$ then it will not exist positive effects below $R_t^e = R_{t+1}^{sq}$.

On the other hand, there is another change in the distribution of peaks. In period t , individuals with $a = 0$ and w^e have $R_t^e(0; w^e) = R_t^e$, then from (2.14) agents with $a > 0$ and w^e will have $R_t^e(a; w^e) < R_t^e$: Besides, there are individuals with $a^0 \in (0; R_t^{sq})$ and $w \in (w^e; w_M]$ who have $R_t^e(a^0; w) = R_t^e$; then from (2.14) individuals with $a > a^0$ and $w \in (w^e; w_M]$ have $R_t^e(a; w) < R_t^e$; but in period $t + 1$ identical individuals will have $R_{t+1}^e(a; w) > R_{t+1}^{sq} = R_t^e$: Hence percentage of peaks below $R_t^e = R_{t+1}^{sq}$ is decreased (negative effects):

Since there exist positive and negative effects (see figure 12) and it can not be showed that one is always higher than the other, then R_{t+1}^e may be higher, lower or equal than R_{t+1}^{sq} : Q.E.D.

Although it is not possible to know if R_{t+1}^e will be higher or lower than R_{t+1}^{sq} , one can easily show that, when $R_t^e > R_t^{sq}$: the elected retirement age in period $t + 1$; R_{t+1}^e will be higher than R_t^{sq} :

3.5. The convergence process

In the previous section, we have showed that the elected retirement age R_t^e could not keep the same popular support, T years later, when this age becomes status quo retirement age, R_{t+1}^{sq} : In other words, R_{t+1}^e could not be equal to R_{t+1}^{sq} :

The next step in our analysis is to show if it can be obtained, through consecutive voting processes each T years, a stable retirement age. Let R^s be a retirement age remaining indefinitely once it becomes in the status quo.

Proposition 3.4. There always exists a sequence of R_t^{sq} such that $\lim_{t \rightarrow \infty} R_t^{sq} = R^s$.¹⁰

Proof. Let 1_t^{sq} be the percentage of peaks below R_t^{sq} :

1. Let $R_t^{sq} = R_t^e$, i.e., $1_t^{sq} = 50\%$: From Definition 5.1. if $R_t^{sq} = R_t^e$ then $R_t^e = R^s$:
2. Let $R_t^{sq} > R_t^e$: This implies that $1_t^{sq} > 50\%$: Besides $R_t^{sq} > R_{t+1}^{sq}$ implies that $1_t^{sq} > 1_{t+1}^{sq}$ since 1_t^{sq} is composed by people with ages $a \in (0; R_t^{sq})$ and wages $w \in (w_m; w_t^{sq}]$ being w_t^{sq} the wage such that $R_t^e(a; w_t^{sq}) = R_t^{sq}$ and 1_{t+1}^{sq} is composed by people with ages $a \in (0; R_{t+1}^{sq})$ and wages $w \in (w_m; w_{t+1}^{sq}]$ being

¹⁰ $R_t^{sq} = R^s$ may happens in a finite number of periods.

w_{t+1}^{sq} the wage such that $R_{t+1}^a \geq w_{t+1}^{sq} \geq R_{t+1}^{sq}$: From Proposition 5.5 we have $R_{t+1}^{sq} > R_{t+1}^e$; what implies that $\gamma_{t+1}^{sq} > 50\%$: Therefore always that $\gamma_t^{sq} > 50\%$ we have $\gamma_t^{sq} > \gamma_{t+1}^{sq} > \dots > \gamma_{t+n}^{sq} = 50\%$; i.e., a bounded and decreasing monotonic sequence of γ_{t+n}^{sq} which converges, what implies that R_t^{sq} converges to R^s . Moreover, since R_t^{sq} converges to R^s ; it can be found a period t^0 such that $|R_{t^0}^{sq} - R_{t^0}^e| < \epsilon$ for any ϵ , which by continuity implies that $|\gamma_{t^0}^{sq} - 50\%| < \epsilon$ for any ϵ : Therefore R^s leaves 50% of the peaks below it.

3. Let $R_t^e > R_{t+1}^{sq}$: In this case $\gamma_t^{sq} < 50\%$: Besides, as in the previous case, $R_t^{sq} < R_{t+1}^{sq}$ implies that $\gamma_t^{sq} < \gamma_{t+1}^{sq}$: From Proposition 5.6 R_{t+1}^{sq} may be higher, lower or equal to R_{t+1}^e , which implies that γ_{t+1}^{sq} may be higher, lower or equal to 50% respectively. If $\gamma_{t+1}^{sq} > 50\%$ we would be in the cases 1 and 2: If $\gamma_{t+1}^{sq} < 50\%$ you can apply the same reasoning, then you have proved that this sequence converges to the 50%: Q.E.D.

Consequently, regardless to the initial status quo retirement age, R^s will be reached.

4. RELATION BETWEEN RETIREMENT AGE, REDISTRIBUTION DEGREE AND WAGE DISTRIBUTION

An interesting exercise of comparative static is to analyze, in the short-run and in the long-run, how the retirement age is affected by different degrees of redistribution or how is related to the wage distribution.

According to this, we will focus on the elected retirement age in the first voting process, R_t^e , and on the stable retirement age, R^s :

4.1. Different degree of redistribution.

One of the most interesting aspects of the Social Security Programs is how redistributive they are. In our particular case, we are interested in studying how this redistribution degree affects the voting decision on retirement age.

We analyze two Social Security Programs with different degrees of redistribution, γ ; but with the same tax level, ζ ; and status quo retirement age, R^{sq} . We study in the two following propositions how the degree of redistribution will affect R^s and R_t^e through a democratic process.

Let R^S be the stable retirement age with a determined redistribution level, \mathbb{R} .¹¹ Let $R^*(0; \$)$ be optimal retirement age of individual of age $a = 0$ and mean wage $\$$:

Proposition 4.1. If $\mathbb{R} < \mathbb{R}$ then $jR^S < jR^*(0; \$)$. When a Social Security Program is more redistributive (lower \mathbb{R}), stable retirement age will be closer to optimal retirement age of individual of age $a = 0$ and mean wage $\$$.

Proof. We need to ...nd out the sign of $\partial R^*(a; w) / \partial \mathbb{R}$: From F.O.C. of maximiza...tion problem (2.13), the implicit function theorem and after some simpli...cations we obtain

$$\frac{\partial R^*(a; w)}{\partial \mathbb{R}} = \frac{[u^0(c)(1 - \beta_r(c))] \zeta(w + \$)}{(w(1 - \zeta) + \zeta W)^2 u^{00}(c) \frac{1}{\tau_i a}} \quad (4.1)$$

If $w < \$$ ($w > \$$); that is, if the individual has a lower (higher) wage level than the mean one, then a more redistributive system, (lower \mathbb{R}), leads to an strictly increase (decrease) in his optimal retirement age. See ...gure 13

Let $R^*(a; w)$ be the optimal retirement age related to redistribution degree \mathbb{R} : Let w^S be the wage such that for all age $R^*(a; w^S) = R^S$. Let $\mathbb{R} < \mathbb{R}$: The percentage of peaks below R^S is composed by people with age $a \in [0; R^S]$ and wage $w \in [w_m; w^S]$: See ...gure 14.1

If $w^S < \$$, from (2.15) and (4.1) $R^*(0; w^S) = R^*(0; \$)$ only if $w > w^S$. Since $R^*(0; w^S) = R^S$ leaves ...fty per cent of peaks below it (see ...gure 14.1), then if $R^*(0; w)$ were equal to R^S ; it would leave more than ...fty per cent since the percentage of peaks below it, it would be composed by people with age $a \in [0; R^S]$ and wage $w \in [w_m; w]$. Therefore $R^S = R^*(0; w^S) < R^*(0; w) = R^*(0; w^S) = R^S$:

Analogously if $w^S > \$$ then $R^S > R^S$ (see ...gure 14.2): Q.E.D.

From this proposition we derive how the stable retirement age is a...ected by the redistribution level of the Social Security Program, and how this e...ect is di...erent depending of the relation between the mean wage and the median voter's wage. Let w^S be the wage such that for all age $R^*(a; w^S) = R^S$; i.e., the wage that leaves ...fty per cent of population below it. If this wage is lower than mean wage, $w^S < \$$; then a Social Security Program with a higher redistribution degree will yield a higher retirement age, and therefore a bigger total production. Equally, when $w^S > \$$; a higher redistribution degree will yield a lower retirement age.

¹¹ It's easy to check that R^S is unique.

This section allows us to see the importance of retired people and their weight in the voting process. If these people did not vote, then the median voter's wage would be the median wage. Therefore, always that median wage were lower than mean wage, which is the realistic case, a S.S. Program with higher redistribution degree would lead to a higher stable retirement age. But when we include retired people in the voting process, this last result does not always hold. Now the important thing is not the relation between median and mean wage, but the relation between mean wage and median voter's wage, which including retired people is not the median wage any longer. So we can have a case in which median wage would be lower than mean wage, but median voter's wage would be higher than mean one, and then a higher redistribution would be related with a lower retirement age. Therefore, due to retired people, not always median wage be lower than mean wage, a more redistributive Social Security Program will be related to a higher retirement age.

With respect to the analysis of the effects induced by the redistribution degree of the S. S. Program on the elected retirement age in the ...rst voting process, we need to impose stronger conditions to get similar results.

Proposition 4.2. Let $\circledast < \circledcirc$: If $R_{t;\circledast}^e < (>) R_{t;\circledcirc}^a(a; \$)$ for all a ; then $R_{t;\circledast}^e > (<) R_{t;\circledcirc}^e$.

Proof. We are going to prove that if $R_{t;\circledast}^e$ leaves the ...fty per cent of the peaks below, then the same retirement age under \circledast will leave more than 50% of peaks below.

Let w^e be the wage such that $R_{t;\circledast}^a(a^0; w^e) = R_{t;\circledast}^e$ for any $a^0 \in \mathbb{R}_{\geq 0}^e$: We know that $R_{t;\circledast}^e$ leaves the ...fty per cent of the peaks below it, under a Social Security Program with an \circledast redistribution degree.

But, from (2.15) and (4.1), for any $a^0 \in \mathbb{R}_{\geq 0}^e$; $R_{t;\circledast}^e(a^0; w^e) < R_{t;\circledcirc}^a(a^0; w^e)$: Then, under an \circledcirc redistribution degree, $R_{t;\circledcirc}^e$ will leave more than 50% of the peaks below it, so $R_{t;\circledcirc}^e$ must be lower than $R_{t;\circledast}^e$: See ...gure 15

Analogously, if $R_{t;\circledast}^e > R^a(a; \$)$ then $R_{t;\circledast}^e < R_{t;\circledcirc}^e$. Q.E.D.

Therefore, to obtain that a higher redistribution degree will lead to a higher retirement age, this one not only has to be lower than $R_t^a(0; \$)$, but also lower than $R_t^a(a; \$)$ for any age. So, the ...nal outcome will depend on how the population is distributed.

4.2. Di¤erent wage distributions.

With respect to the e¤ects of the wage distribution on the elected retirement age in the ...rst voting process, the results are ambiguous. Therefore, here we focus on the stable retirement age. We compare two identical societies but with di¤erent wage distribution.¹² We analyze a realistic situation by considering that in both cases the median wage is lower than the mean wage.

In our model we assume that the retired people behave with regard to the optimal retirement age as if they were those who earn the highest wage level.

We consider two di¤erent settings, one in which we include the retired people and another in which we do not include them.

In the ...rst case, without retired people, the median voter is the voter with the median wage level. Then the society with the highest median wage level (always lower than the mean wage) will have a median voter with a higher wage level. Consequently, from (2.15) this society will have a higher retirement age.

However, in the second case, when we include the retired people, the median voter is no longer the agent with the median wage level. Here the median voter will have a wage level either lower or larger than the mean wage (depending on the percentage of the retired people), but always higher than the median wage.

If we suppose that in both societies the median voter's wage is larger than the mean wage, then the most egalitarian one will have the median voter's wage lower than the other one. Therefore, from (2.15) the most egalitarian one will have a lower retirement age. See ...gure 16

By this reason, it is clear the great importance of retired people since the median voter would become a voter with a higher wage level (including退休). Therefore, more egalitarian societies can have lower retirement ages.

5. CONCLUSIONS

In this article our objective is to observe the behaviour of agents with di¤erent ages and wages with regard to their optimal retirement ages. It is showed that the optimal retirement age is increasing with the wage level. Besides, the results show that the elder the workers are, the closer their optimal retirement ages to

¹²Same mean wage but di¤erent median wage.

status quo one will be. The intuition lying behind is that the increasing weight of wealth and leisure when they are getting old.

We prove that the elected retirement age could not keep the same popular support in future elections. However, it is possible to find a retirement age which leaves indefinitely the fifty per cent of the population to both sides.

On the other hand, in an exercise of comparative static, we compare two identical Social Security Programs with different degree of redistribution and two identical societies with different wage distribution. In both cases, we obtain similar conclusions. The most redistributive or the most egalitarian one may have lower retirement ages. This result is apparently contradicting the philosophy of the increasing relation between optimal retirement ages and wage levels. This might be explained by the weight of the retired people.

A natural extension of this article is to introduce the aging problem. Roughly speaking, there seems that with aging, the percentage of retired people will increase and, therefore, the retirement age will go up. Since the larger the percentage of retired people is, the higher the retirement age will be.

An intuition that can be analyzed from this setting is that this problem may be thought as a decision about the total worked hours. As a matter of fact, if the retirement age is modified, we are changing the total amount of worked hours. This issue is one of the most important problems in industrialized countries as consequence of unemployed people.

6. APPENDIX

Here we show some examples where the percentage of peaks below or equal to R_t^e (when $R_t^e < R_t^{sq}$) increases in period $t + 1$, i.e., examples where R_{t+1}^e is lower than R_{t+1}^{sq} ; in order to illustrate how changes in the status quo can affect the popular support on the elected one.

There will be cases in which there exists (see figures 8.1 and 9.1) a wage level w^e such that $R_t^a(0; w^e) = R_t^e < R_t^{sq}$. From (2.14) we know that since $R_t^a(0; w^e) < R_t^{sq}$ then $R_t^a(a > 0; w^e) > R_t^e$. However since $R_{t+1}^a(0; w^e) = R_{t+1}^{sq}$; from (2.14) $R_{t+1}^a(a > 0; w^e) = R_{t+1}^{sq} = R_t^e$. Consequently, in period t with wage $w = w^e$ just people with age $a = 0$ has $R_t^a \leq R_t^e$; but in period $t + 1$ all people with $w = w^e$ have $R_{t+1}^a = R_{t+1}^e = R_{t+1}^{sq}$:

$R_t^a(0; w^e) = R_t^e$ implies (from (2.14)) that people with age $a^0 \geq 0; R_t^e$ have $R_t^a(a^0; w^e) > R_t^e$. From (2.15) we know that there will be individuals with wage $w < w^e$ such that $R_t^a(a^0; w) < R_t^a(a^0; w^e)$ but higher than R_t^e . For example, for agents with age $a = R_t^e$; it can be found a wage $\hat{w} \geq [w_m; w^e]$ such that for any individual with $w \geq (\hat{w}; w^e]$ and $a = R_t^e$; $R_t^a > R_t^e$ but $R_{t+1}^a \leq R_{t+1}^{sq} = R_t^e$:

Another example. When w^e satisfying $R_t^a(0; w^e) = R_t^e$ does not exist. In this case, there exists an age \hat{a} (see figures 8.2 and 9.2) from which on individuals with $a \geq (\hat{a}; R_t^e]$ and the highest wage level, w_M will have $R_t^a(a; w_M) > R_t^e$ but $R_{t+1}^a(a; w_M) \leq R_{t+1}^{sq} = R_t^e$:

Moreover, individuals with $a = R_t^e$ and w_M have $R_t^a(R_t^e; w_M) > R_t^e$ and therefore (from (2.15)) there will be individuals with $w < w_M$ such that $R_t^a(R_t^e; w) < R_t^a(R_t^e; w_M)$. Thus it can be found a wage $\hat{w} \geq [w_m; w_M]$ such that for any individual with $w \geq (\hat{w}; w_M]$ and $a = R_t^e$; $R_t^a > R_t^e$ but $R_{t+1}^a \leq R_{t+1}^{sq} = R_t^e$:

Consequently, there exists a group of individuals with $a \geq (\hat{a}; R_t^e]$ and different wages such that $R_t^a(a; w) > R_t^e$ but $R_{t+1}^a(a; w) \leq R_{t+1}^{sq} = R_t^e$. Therefore the percentage of peaks below R_t^e will be higher in the next voting.

Figure 1

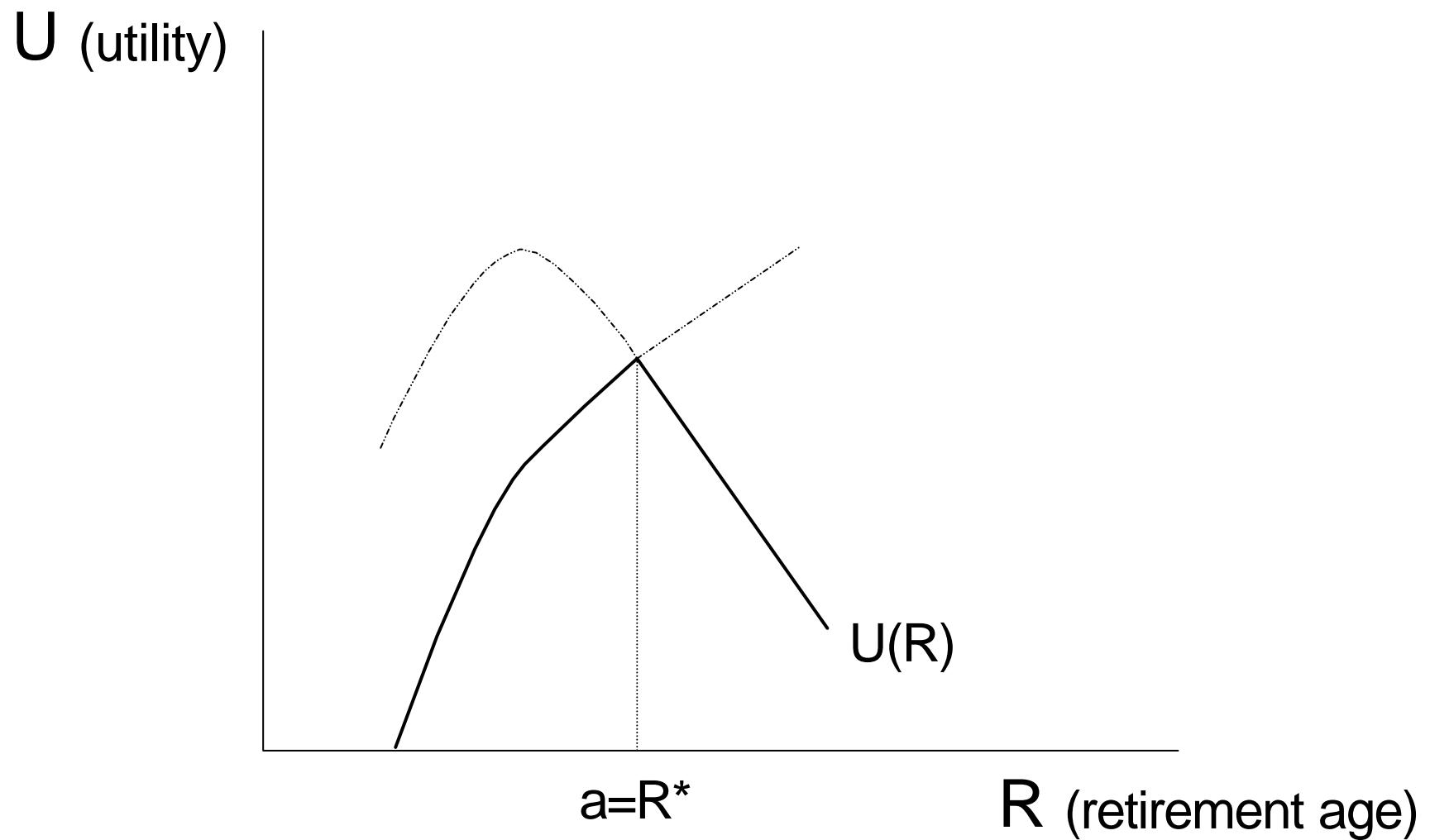


Figure 2

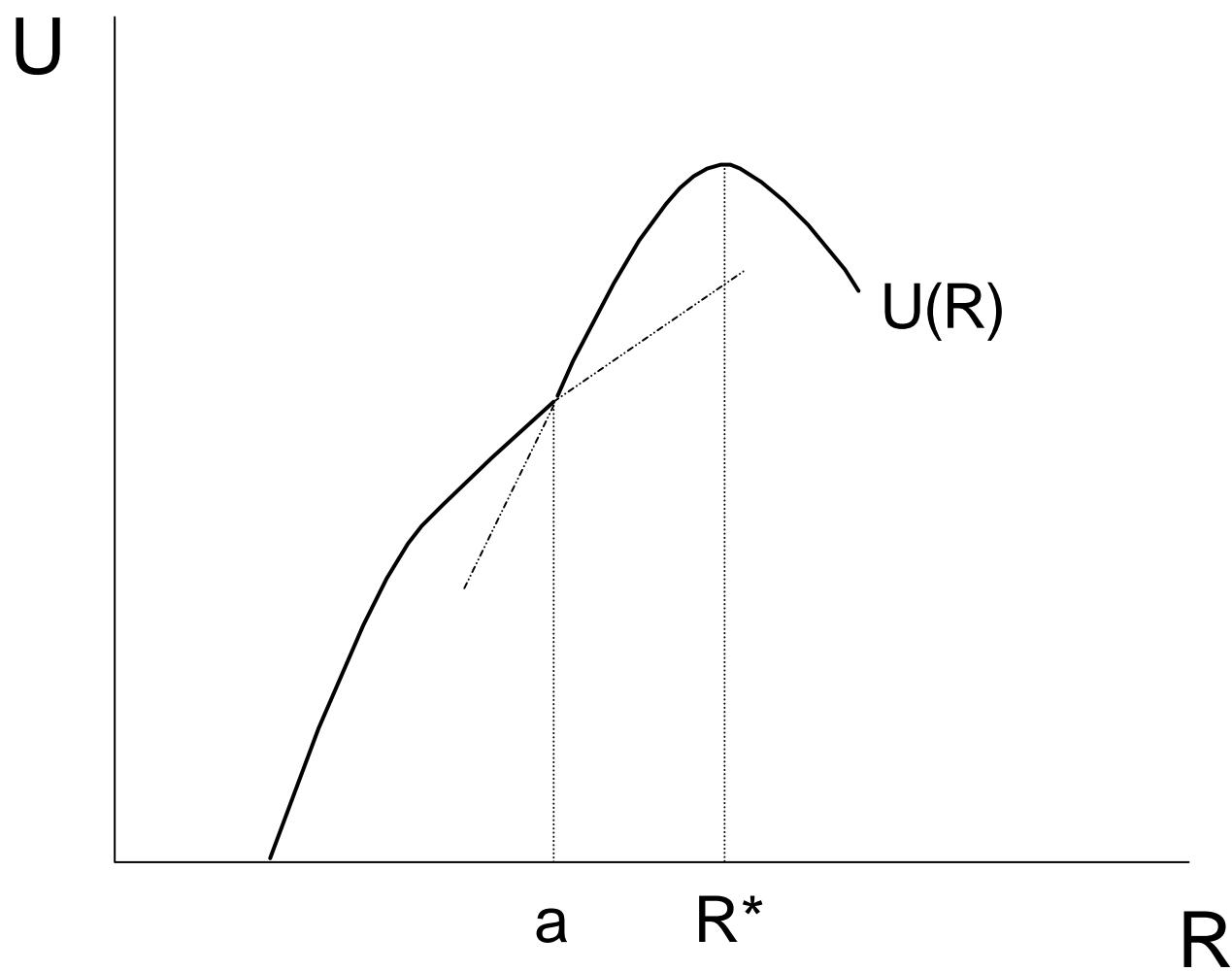


Figure 3
Utility function for a wage level, w_1

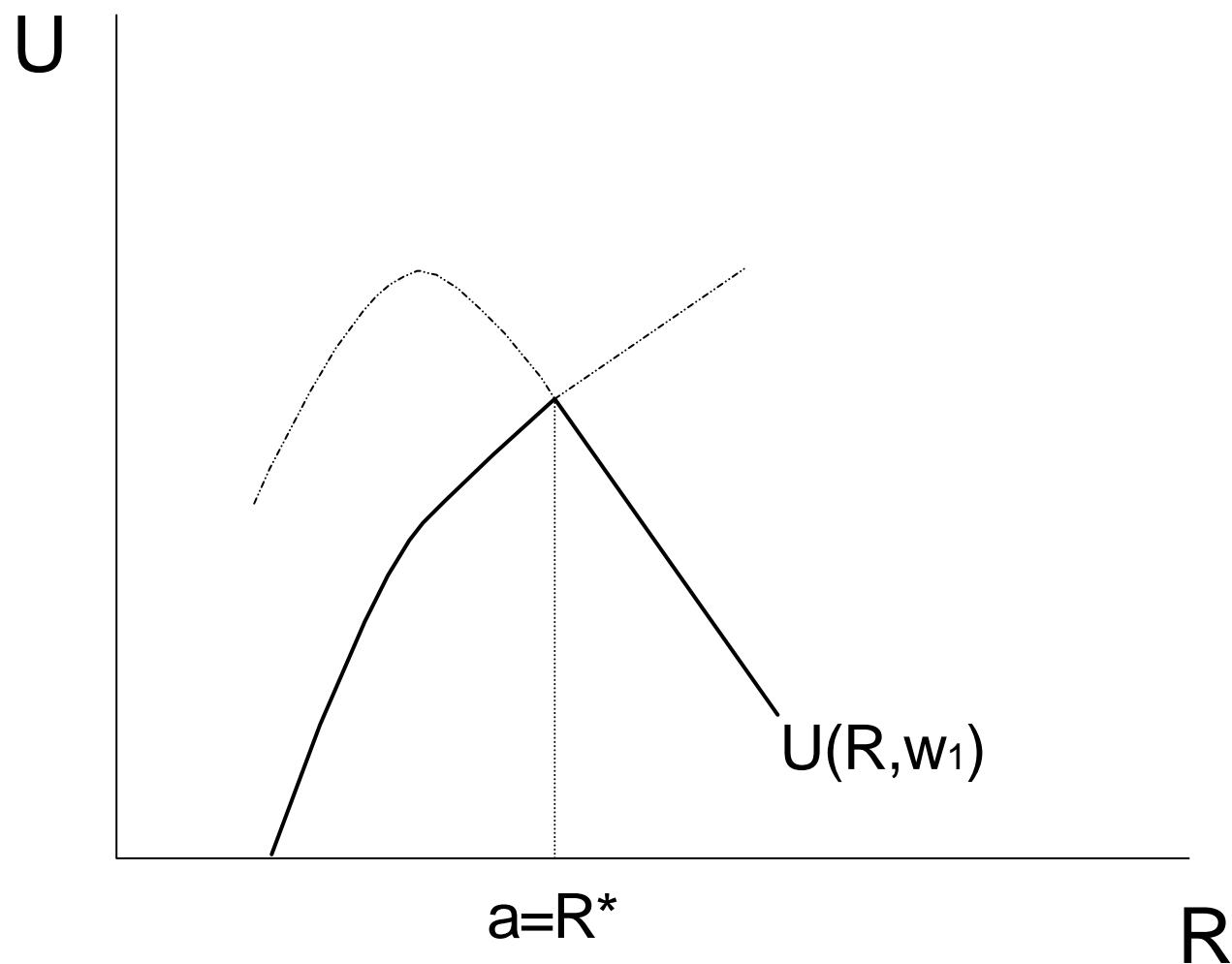


Figure 4
Utility function for a wage level, $w_2 > w_1$

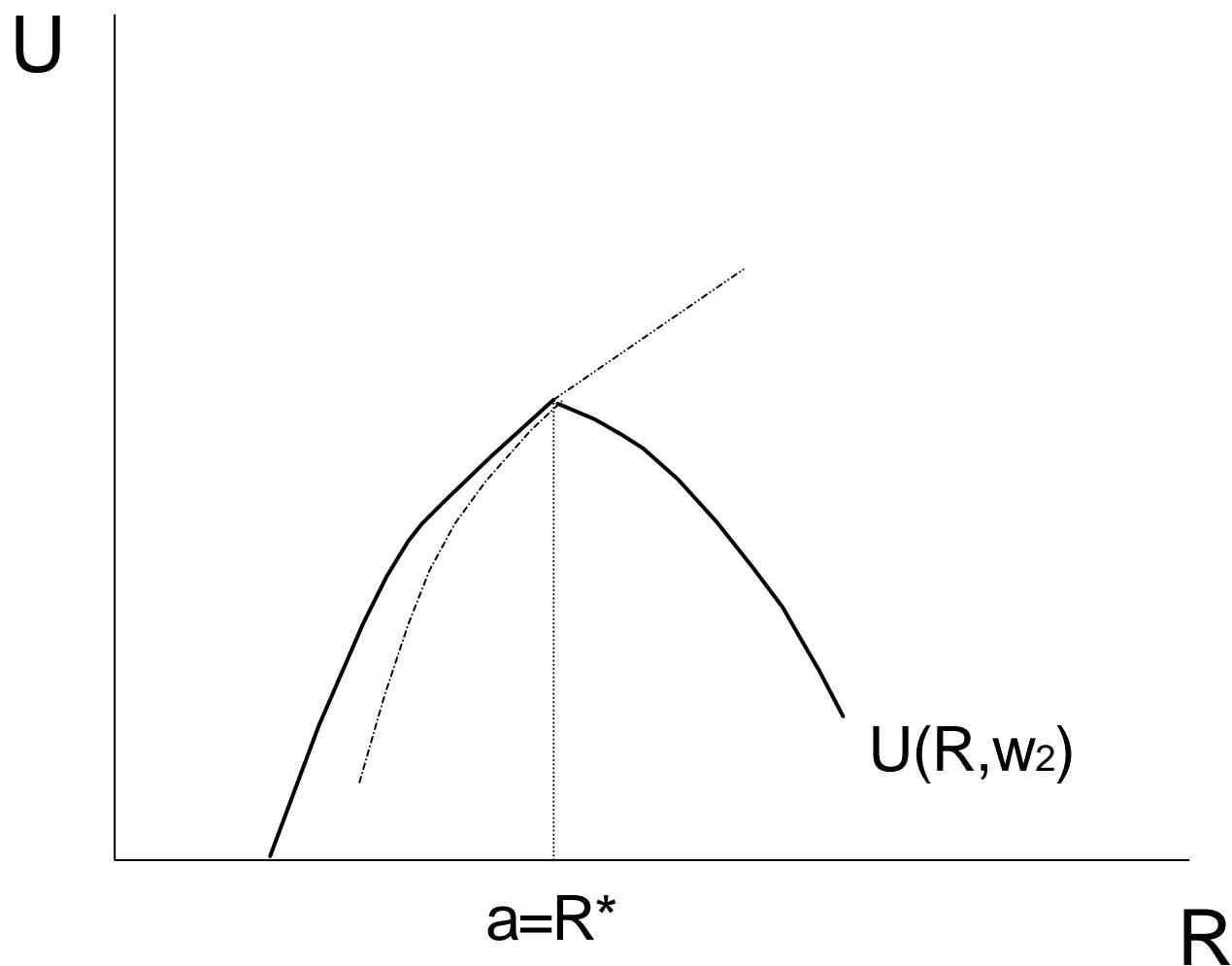


Figure 5
Utility function for a higher wage level, $w_3 > w_2 > w_1$

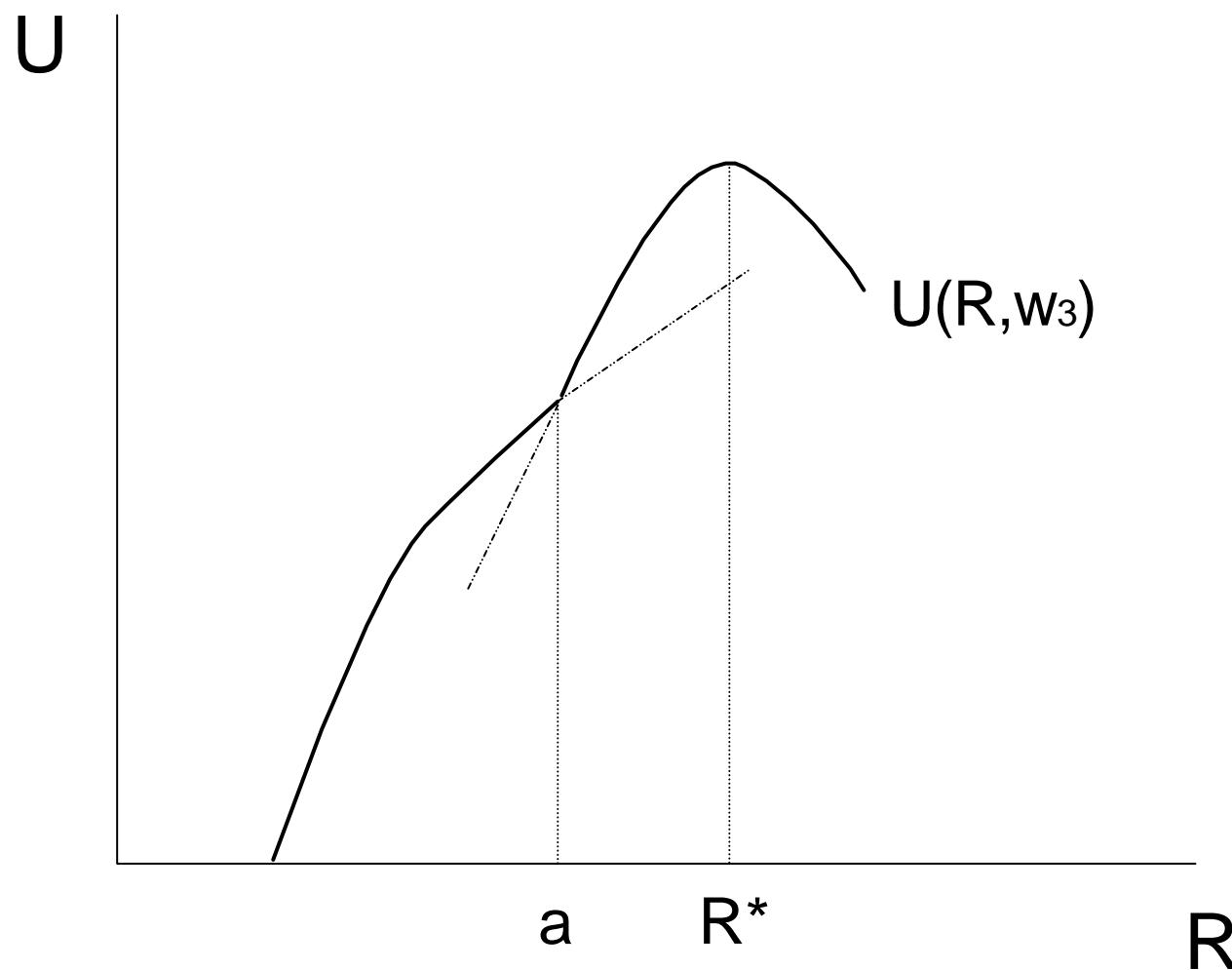


Figure 6

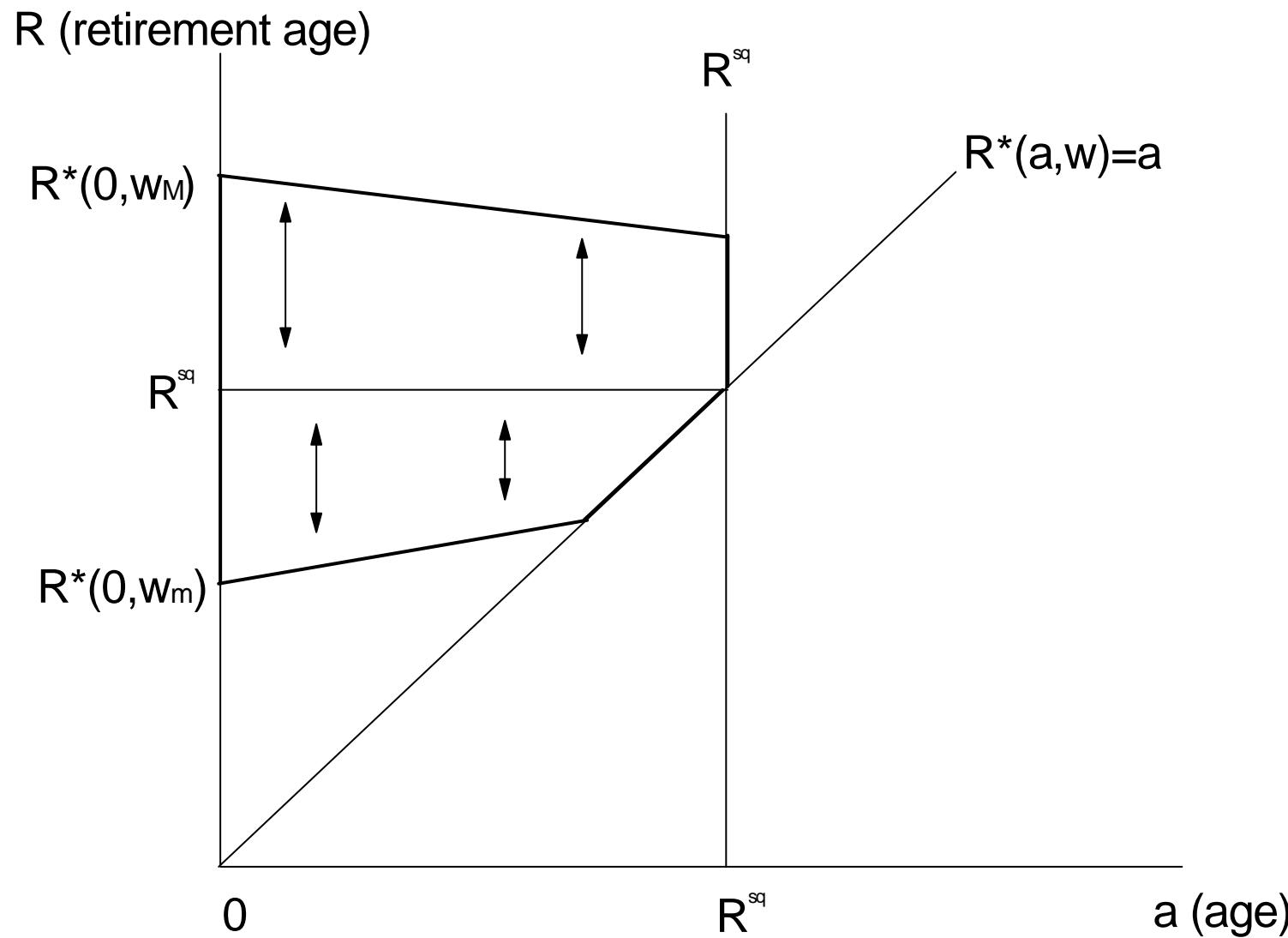


Figure 7
 R^s leaves more than 50% peaks below it

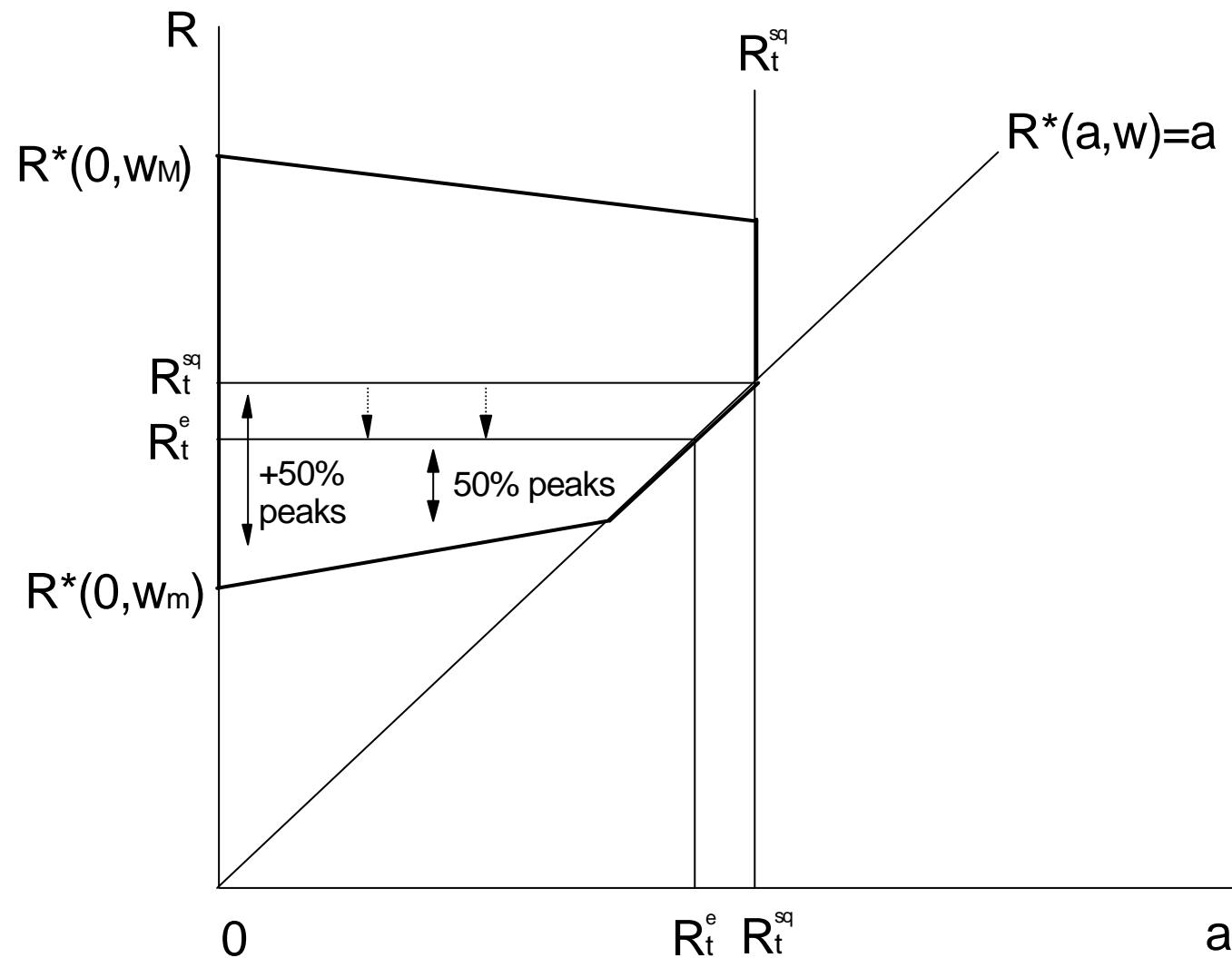


Figure 8.1
Composition of peaks in period t

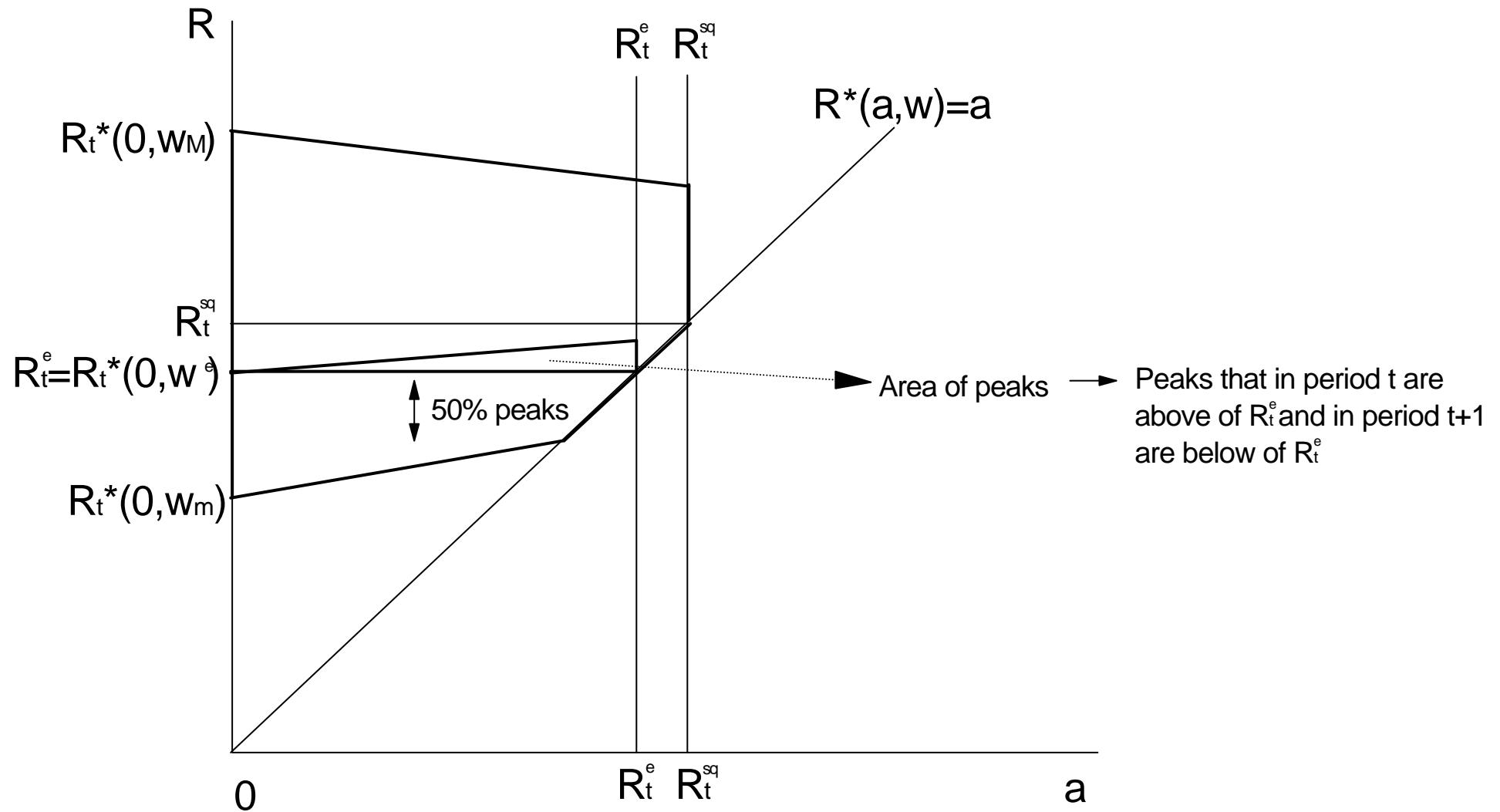


Figure 8.2
Composition of peaks in period t

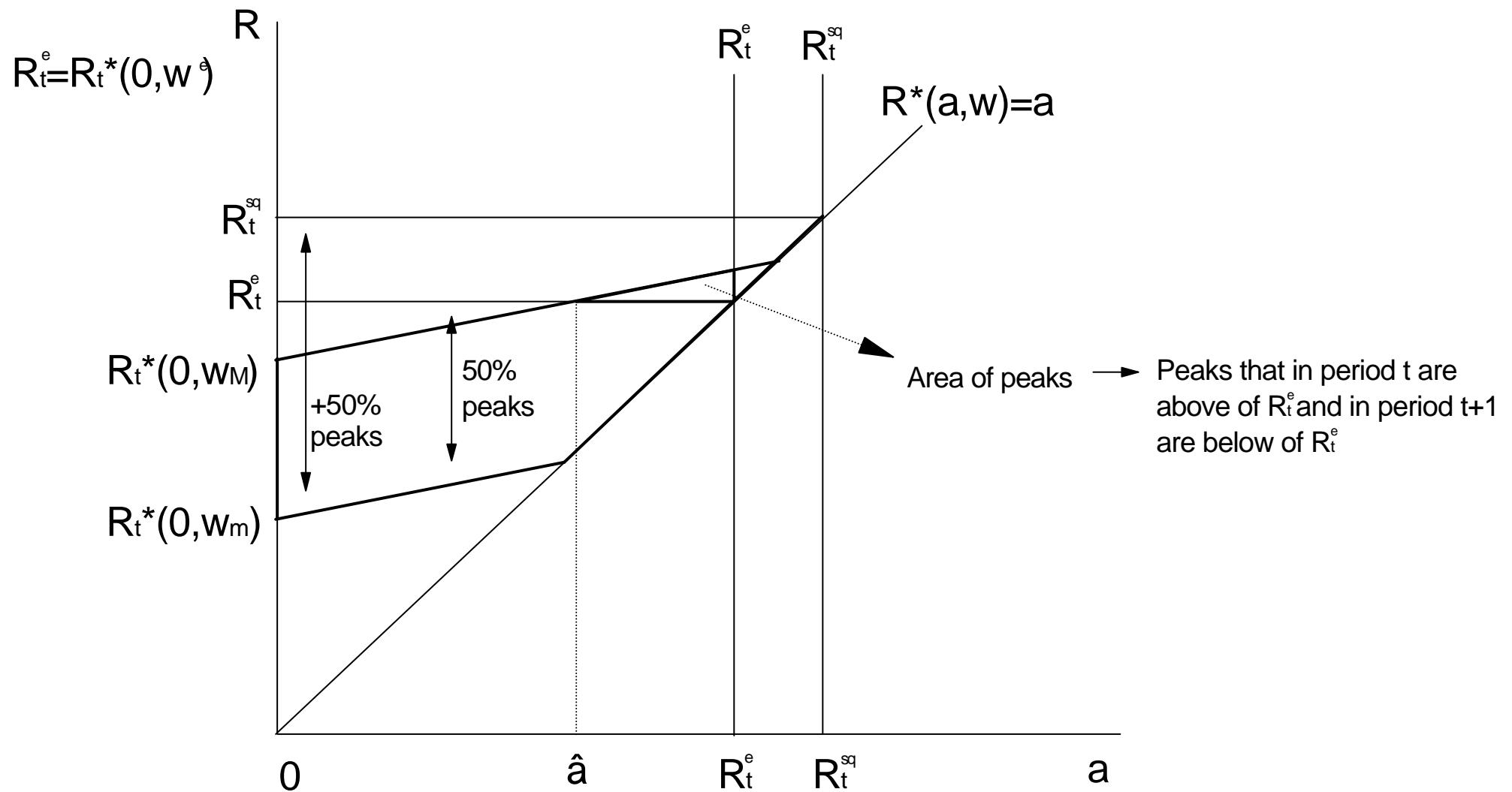


Figure 9.1
Second voting process, period $t+1$

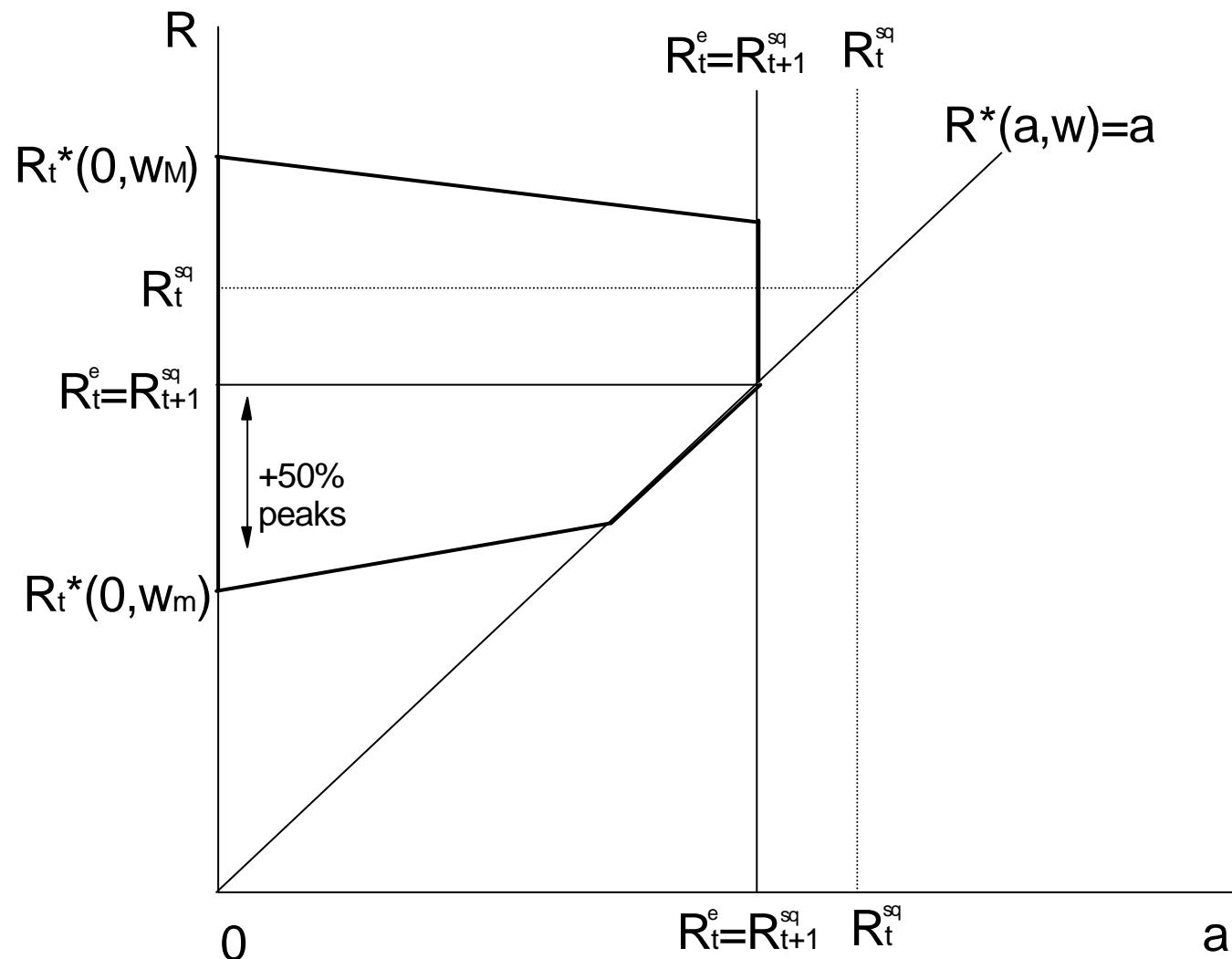


Figure 9.2
Second voting process, period $t+1$

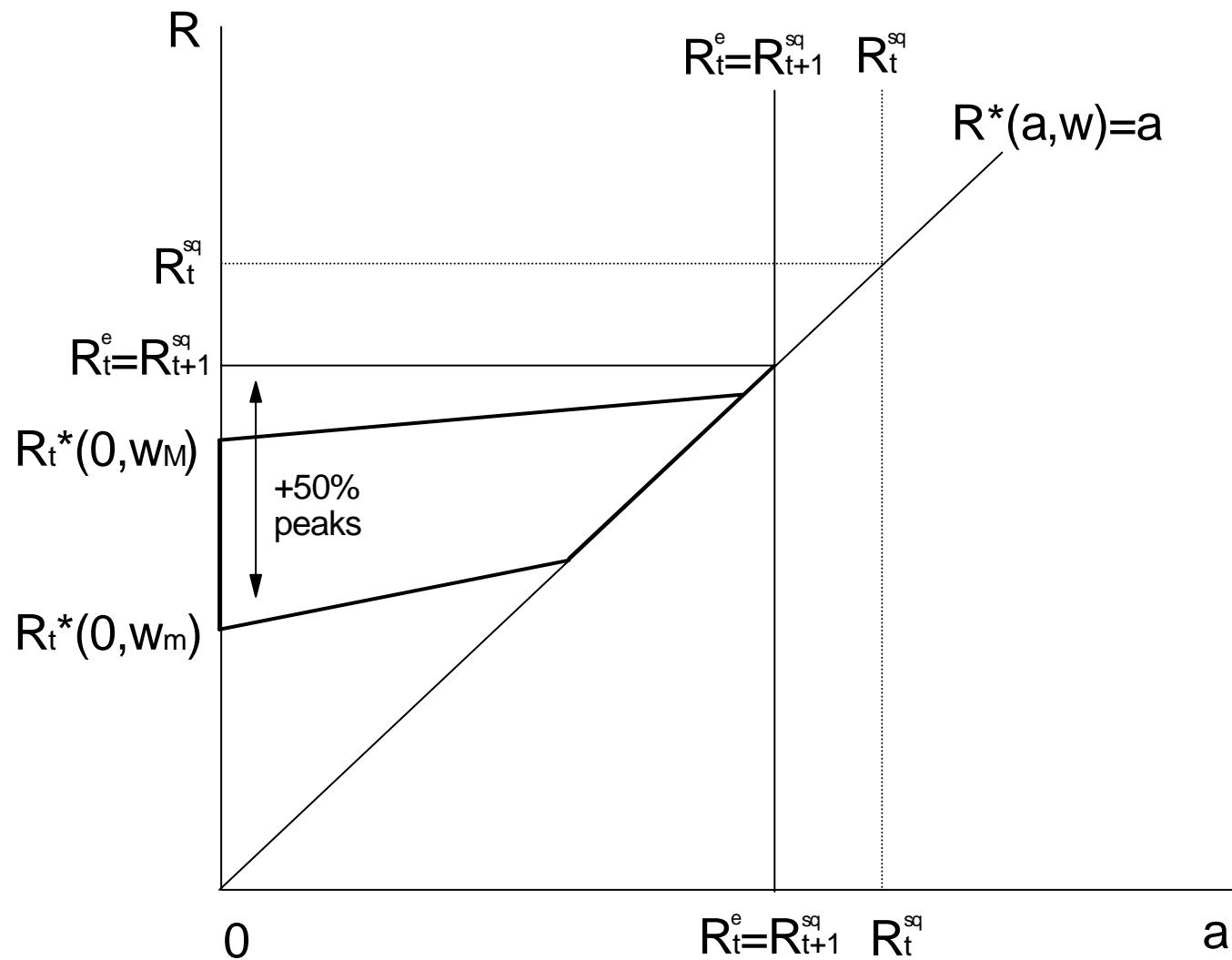


Figure 10
When R^{sq} leaves less than 50% peaks below it

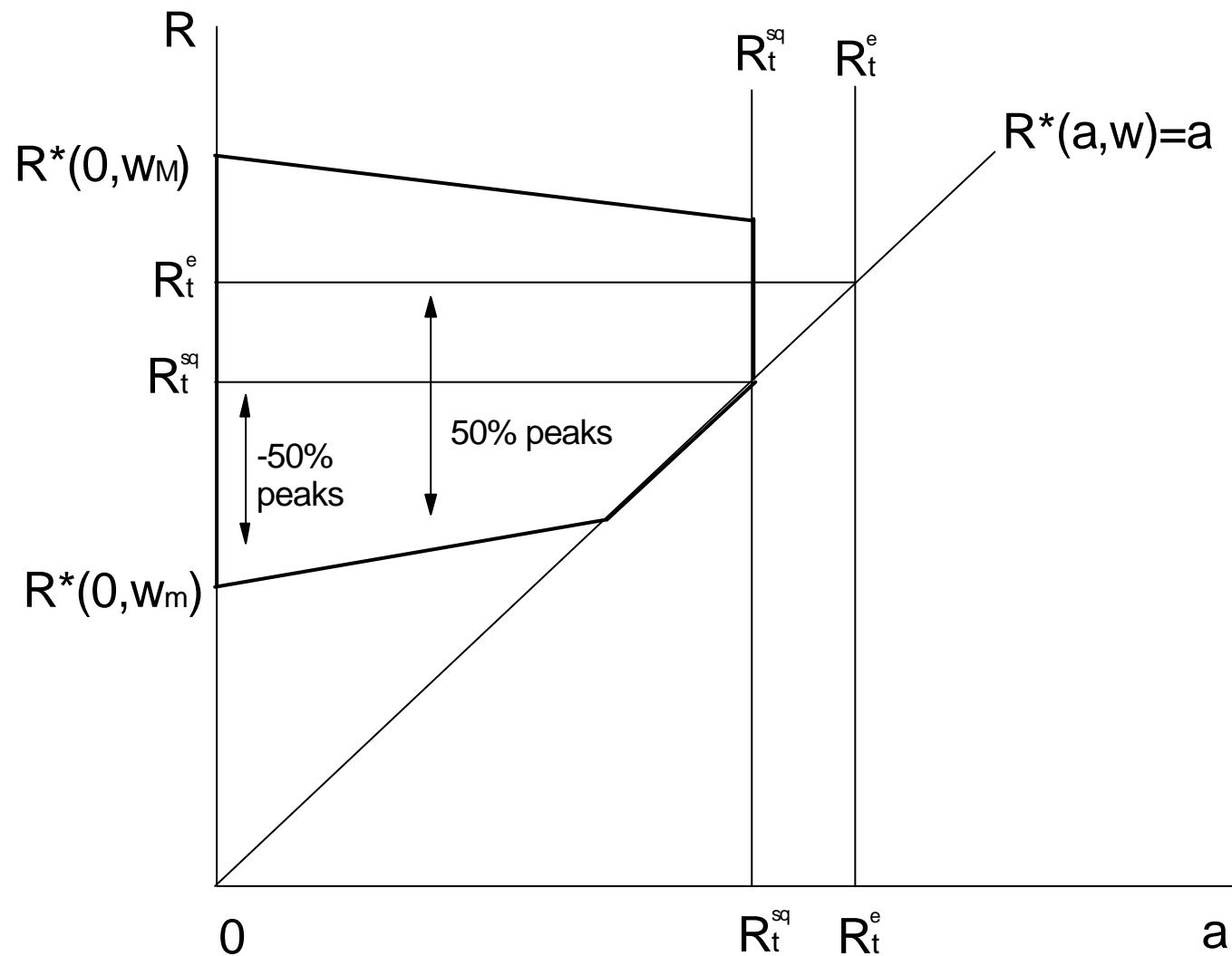


Figure 11
Composition of peaks in period t

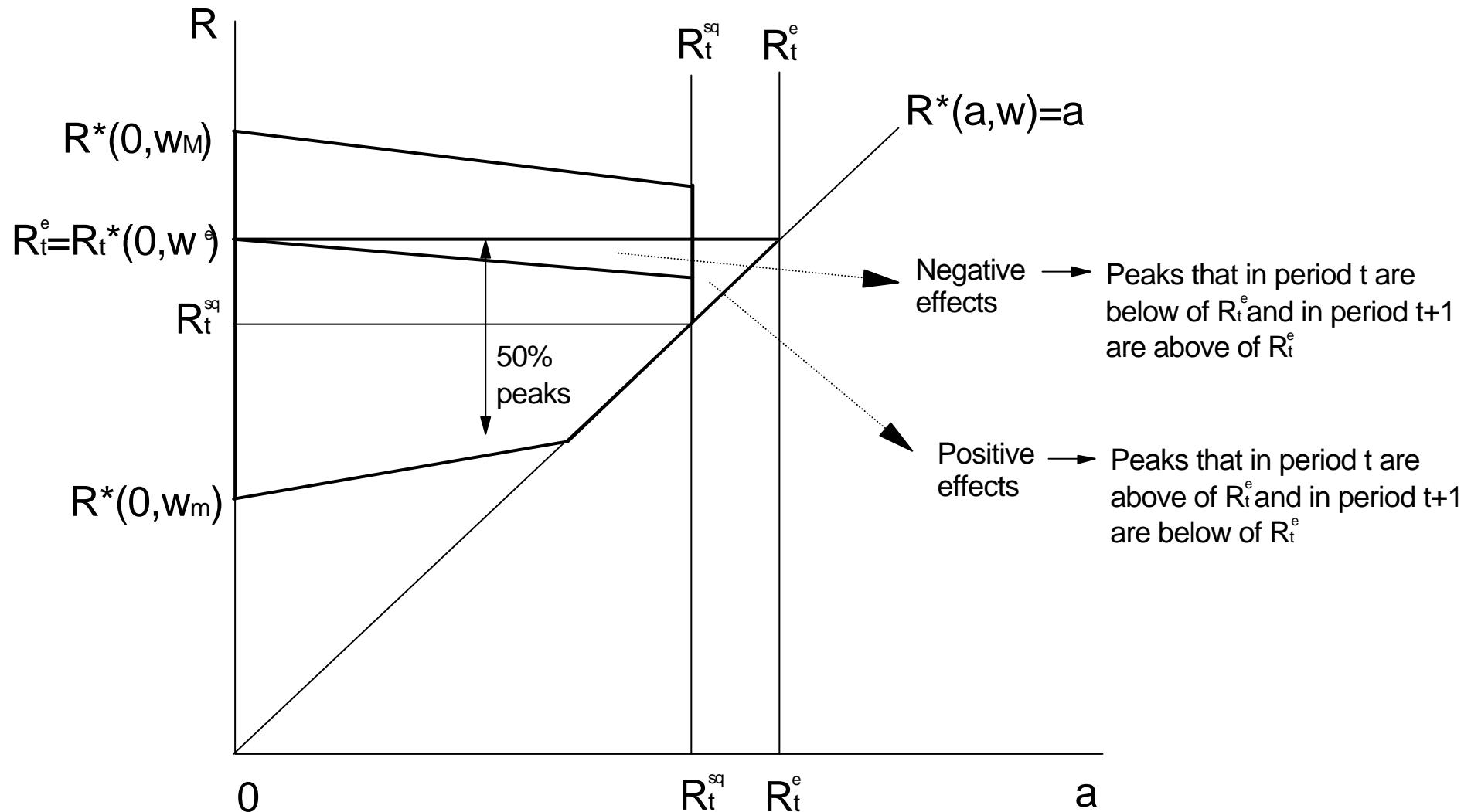


Figure 12
Second process voting, period $t+1$

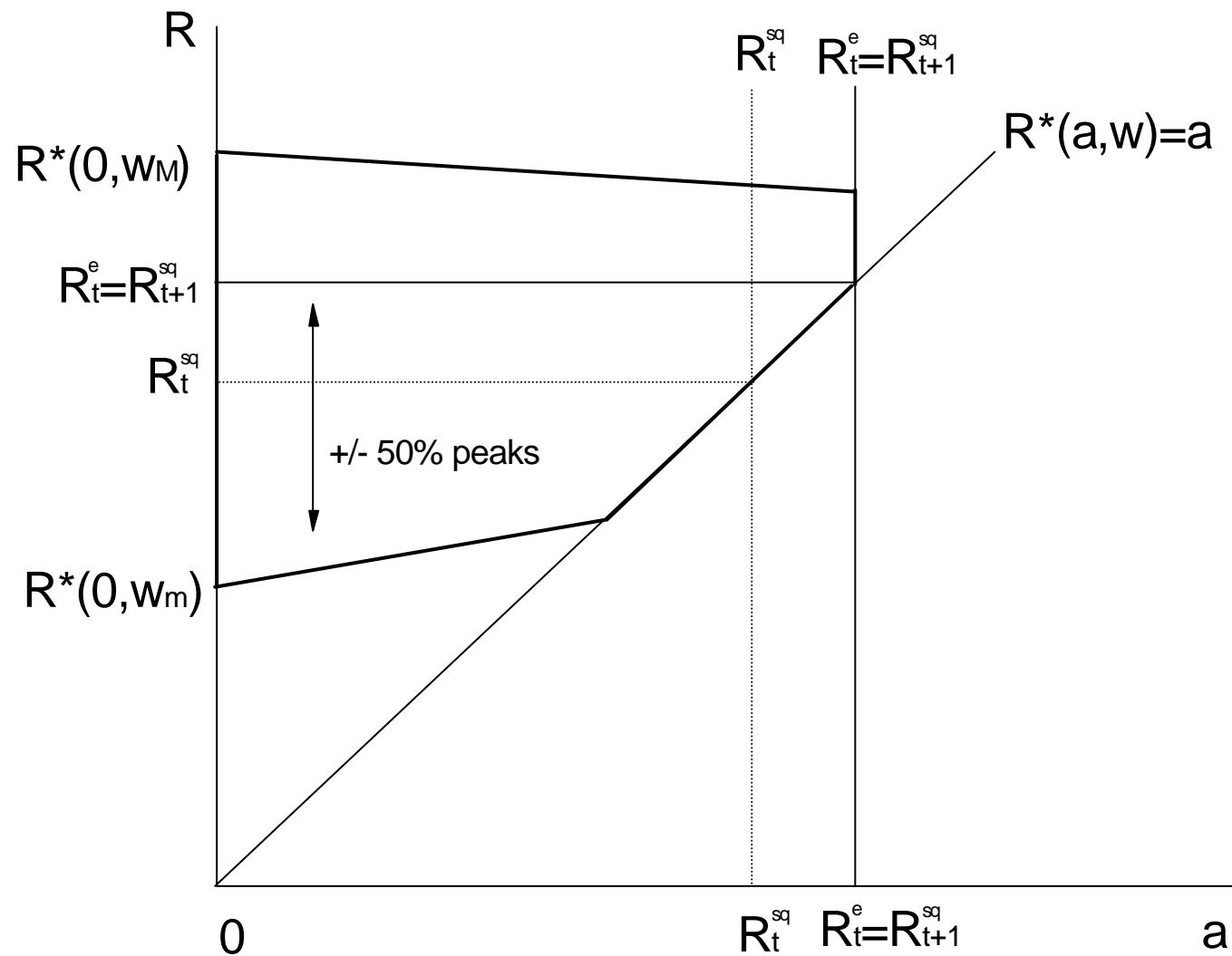


Figure 13
Two different Social Security Programs applied to the same society

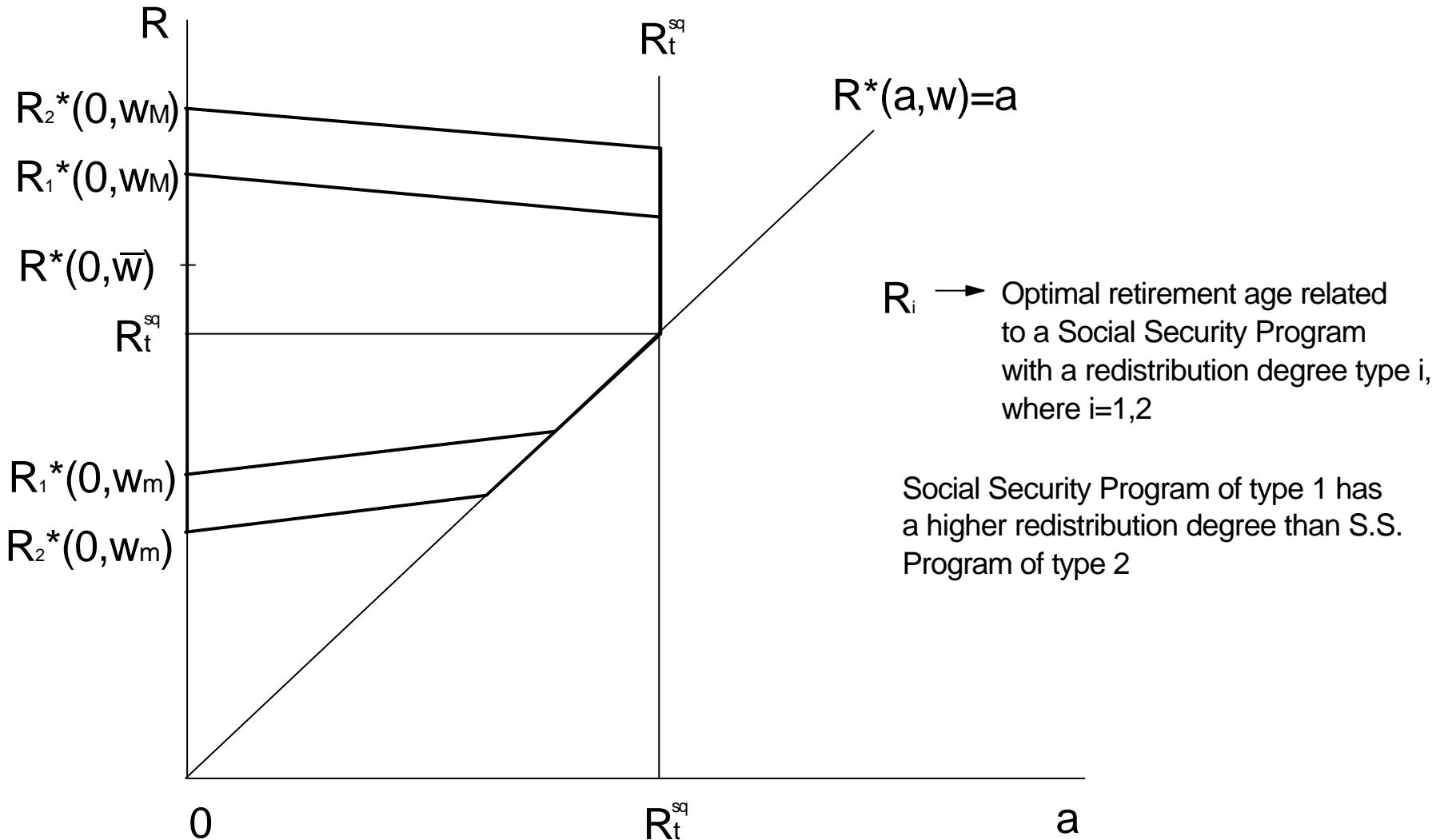


Figure 14.1
Relating different redistribution degrees and stable retirement age

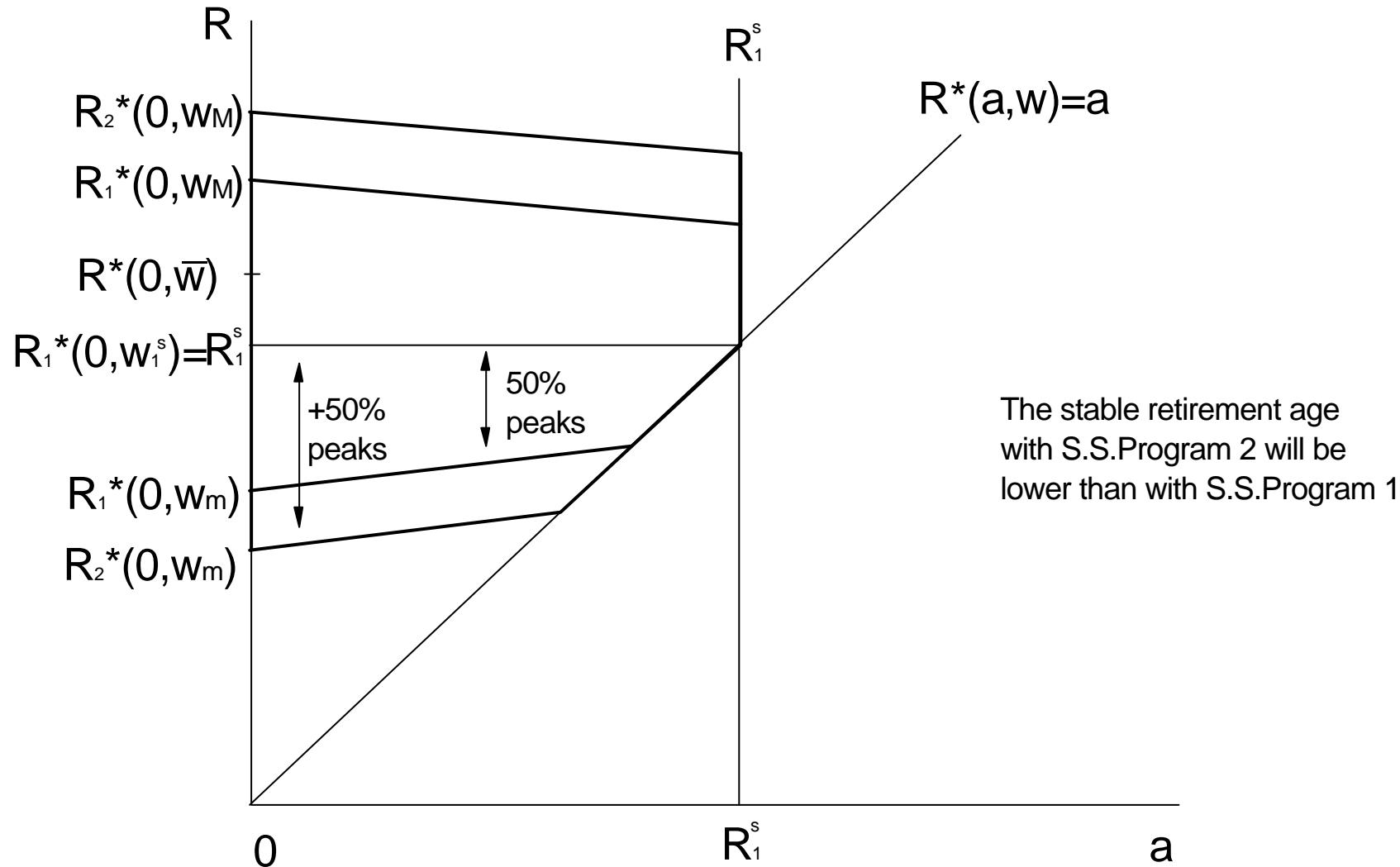


Figure 14.2
Relating different redistribution degrees and stable retirement age

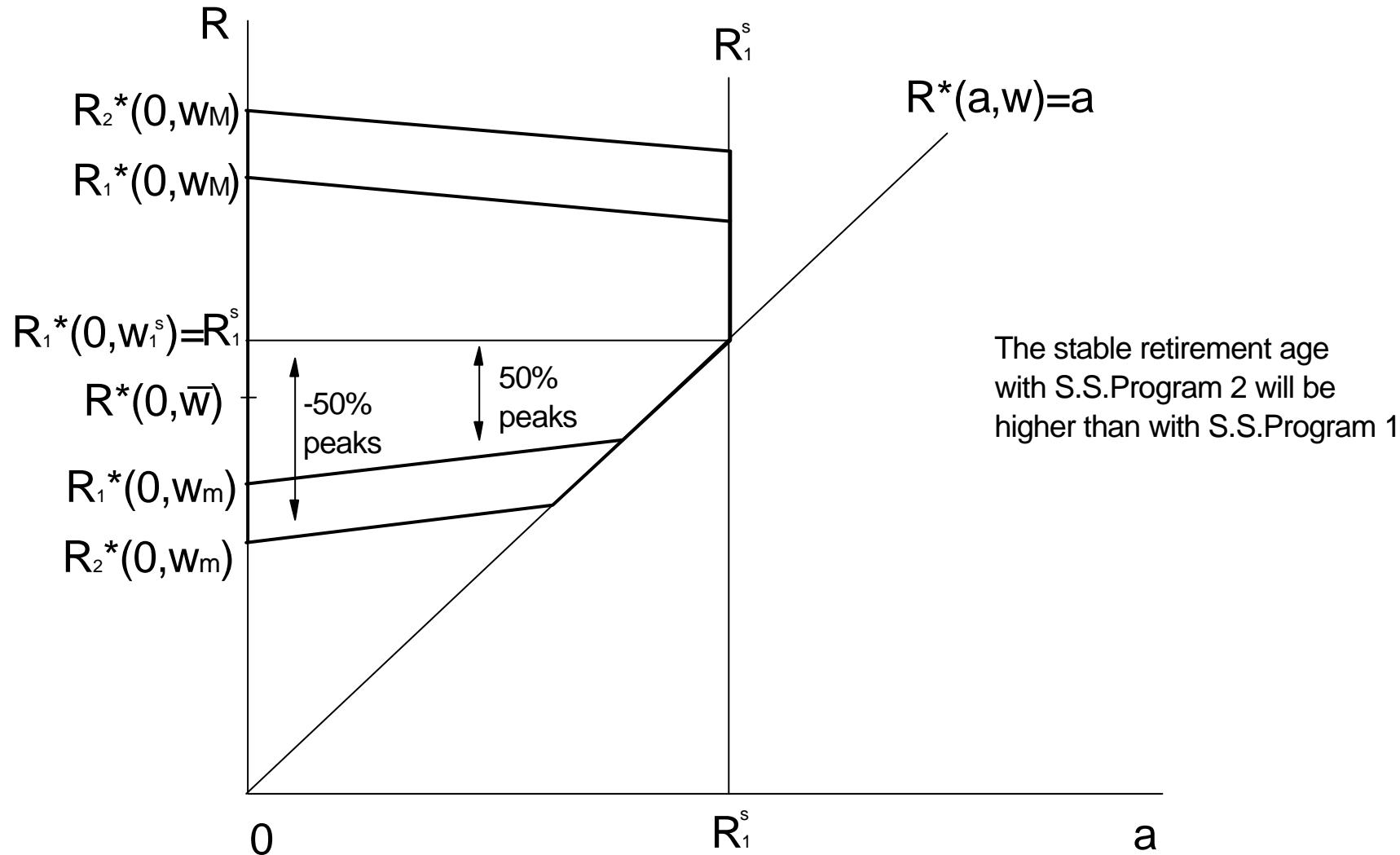


Figure 15
Relating different redistribution degrees and elected retirement age

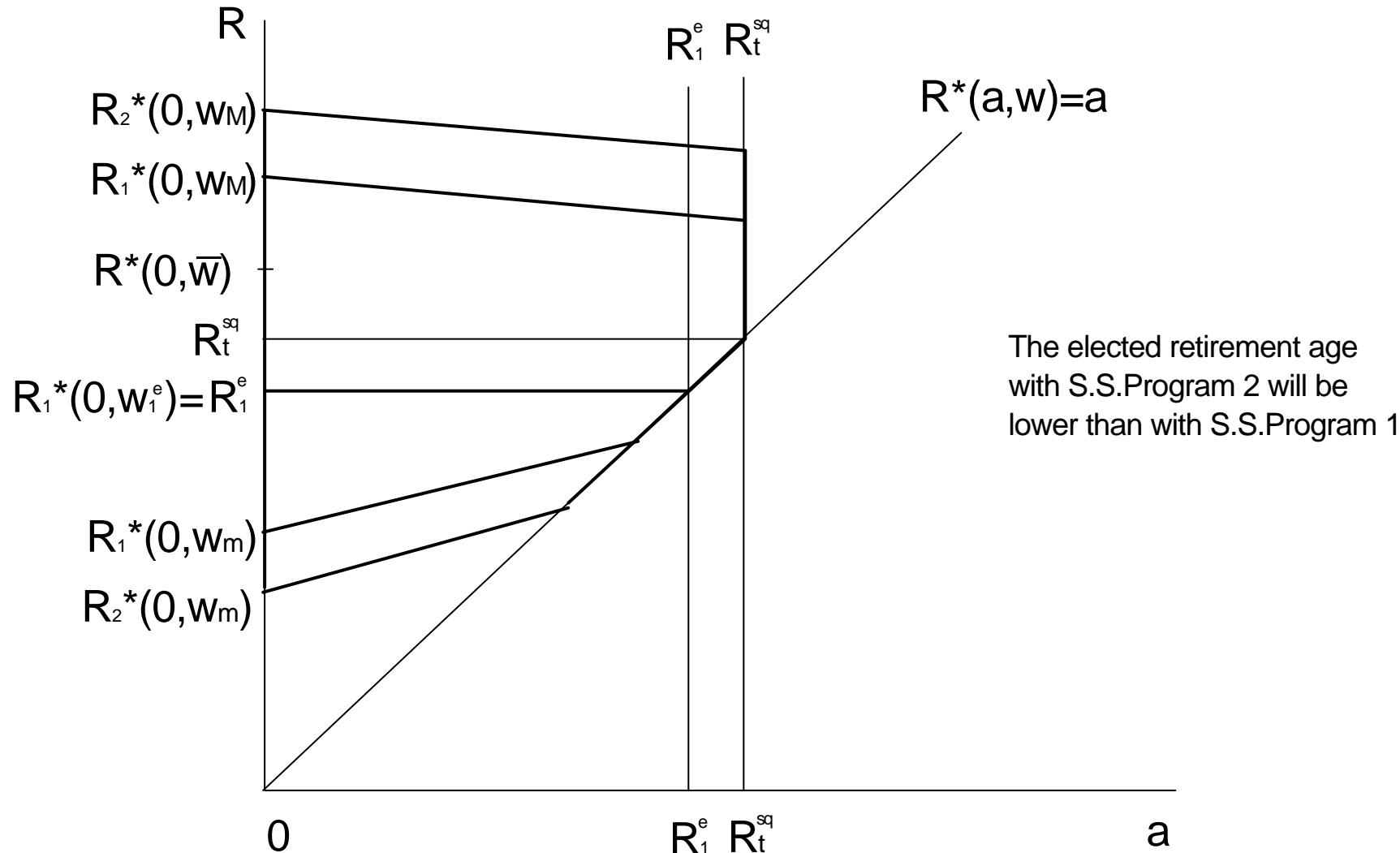
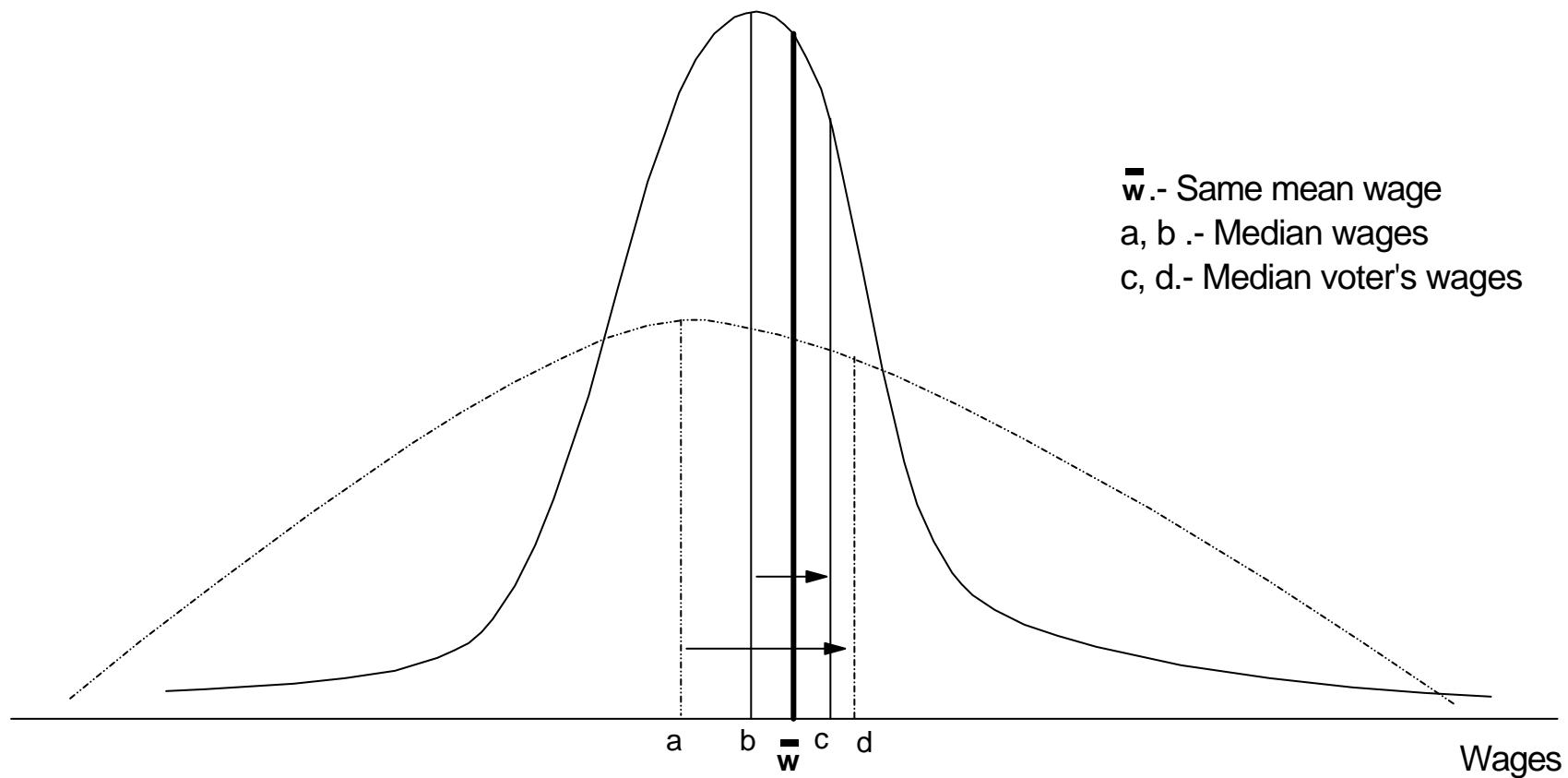


Figure 16
Different wage distributions



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