EQUILIBRIUM IN MOBILITY AND REDISTRIBUTION MODELS*

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A B S T R A C T

We develop a simple model in which individuals migrate freely among regions. In each region there is a majority voting procedure that determines a proportional taxation on earnings which finances a uniform transfer for each of the agents. Equilibrium requires that no one wishes to migrate and that the taxation policy be the result of majority voting in each region.

In this setting Caplin and Nalebuff (1997) show that there is no equilibrium, since all the agents want to live in a region where their income is below the mean income.

Furthermore, we assume that in each region there is a competitive labor market that endogenously determines the wages. Then, if the agents when migrating take into account the taxation and the wages of each region, we find that equilibrium exists. This result is obtained for two different kinds of agents' rationality: myopic and sophisticated agents.

Keywords: Tiebout Model; Redistribution; Policy-Based Equilibrium; Membership-Based Equilibrium.

1. INTRODUCTION

The paper we present is based on the well-known model of Tiebout (1956). Tiebout points out that the profit from a public good is limited to people who live in a small geographic area. One can suppose that people move among different regions without any restriction, looking for the schemes of taxes and public goods that best suits their preferences and their level of earnings (or income). Additional literature in this area is due to Westhoff (1977), Epple, Filimon and Romer (1984), Rose-Ackerman (1979) among others. In the same spirit as Tiebout, we shall consider in this paper redistributive taxation. In the literature, we find several papers analyzing the question of whether mobility and redistribution are compatible (see Cremer et al. (1997) for a survey of this literature).

Labor mobility is generally considered to restrict the government's ability to implement redistributive policies. If we consider the case of a region with skilled agents and non-skilled agents, redistribution in which skilled agents pay taxes and less skilled agents receive a transfer is not possible as the skilled agents would have a strong incentive to migrate to regions where the tax rate is lower. Stigler (1957), Musgrave (1969) and Oates (1972), have shown that the free mobility of factors decreases the government's ability to implement a redistributive policy. The reason for this is that such a policy would attract individuals who wish to receive benefits from the policy whereas individuals with high levels of income would want to leave.

Epple and Romer (1991) present a model in which the price of housing is subject to a tax and the proceeds are distributed uniformly among the residents of the city. They show that in this case, mobility would not prevent the existence of different redistributive taxations in each of the regions. However, different prices of housing prevent from a result where all agents are willing to migrate to the same region to avoid taxation.

On the other hand, Hansen and Kessler (1996) describe a model in which there is a tax on labor's income that finances a uniform transfer to the agents in that region. They get the result that, in equilibria, all the regions have the same taxation policy and all the regions are equal in the sense that they have the same tax rate and the same transfer as well. They also deduce that under some assumptions on the agents' rationality, it is not possible, in equilibrium, to find regions with a positive tax rate. They therefore show what we have already mentioned mobility and redistribution are not compatible.

In this paper, we develop a simple model with a continuum of individuals that differ in their skill and that are free to migrate among regions. In each region there is a political process that determines a proportional taxation on earnings which finances a uniform transfer for each of the agents. Equilibrium requires that no one wishes to migrate (external equilibrium) and that the taxation rate in each region be the result of majority voting in that region (internal equilibrium).

In this setting, Caplin and Nalebuff (1997), Hansen and Kessler (1996) show that there is no equilibrium in which different regions have different redistributive policies, since all the agents would want to live in a region where their incomes are below the mean income.

We assume additionally, that in each region there is a competitive labor market that determines the wages. We consider that labor is supplied inelastically and that labor demand is determined by the technology of each region. A similar assumption is also introduced by Wildasin (1991). He points out the fact that labor-market conditions are very important determinants of location. He considers a common labor market for all the jurisdictions (regions in our model). However, in our model, we consider separated labor markets that determine each region' wage[1]. Since each region' wage adjusts in response to changes in labor supply, the wages adjustments serve to equilibrate migration flows. A basic difference between our model and that of Wildasin is that he considers that redistribution is justified by altruism from the rich to the poor. Whereas in our model, redistributive taxation is a political decision.

Following Caplin and Nalebuff (1997), we are going to study our model for two different kinds of agents' rationality: myopic agents and sophisticated agents. By sophisticated agents we mean those who anticipate the migration flows induced by the tax rates (this case is considered by Epple and Romer (1991)). Hence, the sophisticated agents consider that each region's population depends on the tax rate of her own region as well as those of the other regions. When agents take the population of each region as given, we consider that agents are myopic.

In this paper we show that: a) when each region' technology displays constant returns to scale, an equilibrium where the regions have different taxation policies is not sustainable. (the same result as Caplin and Nalebuff (1997), Hansen and Kessler (1996)), b) when the technology displays strictly decreasing returns to scale, the price of labor depends on each region' labor supply (that is, the total population who decides to live in that region) we find that, in equilibrium, it is sustainable to have regions with different taxation policies. Furthermore, we prove that this result holds to whether we consider myopic or sophisticated agents. We

[1] This assumption is justified on the basis that labor demand is not mobile.

show, that in those equilibria where the regions have different taxation policies, there is a cutting skill agent such that all the agents with lower skill live in a region that is more redistributive and the agents with higher skill live in other region that is less redistributive and where they obtain a greater net income.

We next give the main intuition of our result based on a two-region model.

Let us suppose that the migration flows stop at a point where agents with skill below the cutting point have gone to region 1, and the rest have moved to region 2. Then, if in region 1 the price of labor is greater (that is, the labor supply is lower) and the taxation policy is more redistributive, we find that equilibrium exists. The explanation is as follows: on the one hand, the net losers of the redistributive taxation in region 1 have no incentive to migrate since the price of labor in region 2 is too low. On the other hand, the agents living in region 2, although they observe a higher price of labor in region 1, do not migrate because of the higher taxation policy. Moreover, we shall show that the described taxation schemes, are obtained as a result of the political process of each region, for the two versions of agents' rationality: myopic, or sophisticated agents.

The paper is organized as follows: In section 2, we introduce the model. In section 3, we provide the properties that characterize the external equilibria. In section 4 we show that when the price of labor is fixed and equal across regions, there is no equilibrium where at least two regions are different. In section 5, we provide the conditions that guarantee that equilibrium exists for the two versions of agents' rationality. There, some examples illustrate the existence of equilibrium. Finally in section 6 we summarize the results and we compare them with the closely related literature.

2. THE MODEL

We consider an economy that consists of a continuum of agents in the set $N = [0,1]$, denoted by $i \in [0,1]$, that are free to migrate among K regions. Each region is indexed by a number k in $\{1, 2, ..., K\}$ and N_k is the set of agents living in region k.

The regions:

In each region there is a majority voting procedure that determines a proportional income tax $t_k \in [0,1]$. This taxation finances a uniform transfer among the agents in N_k.

We assume that each region has a competitive labor market that determines the equilibrium price of labor w_k, and we consider that each agent' labor supply is inelastic, and so, in each region the total labor supply L_k is inelastic.

The production technology of each region may be equal or different among regions, and is described by a function $T_k : \mathbb{R}_+ \to \mathbb{R}_+$ that uses homogenous labor L as an input. We assume that T_k is increasing, concave, and twice continuously differentiable. The labor demand in each region is determined by the marginal productivity of labor. Consequently, the equilibrium price of labor w_k is given by

$$w_k = T_k' (L_k) .$$

We consider that the benefits of the production process are shared among a small group of agents which may or may not live in that region. We assume that this group of agents is sufficiently small so that they can not influence the region's policy decision with their action. We assume that the benefits are not taxed.

The agents:

We consider that agents of this economy differ in their skills λ. Each agent i, is endowed with λ_i units of efficient homogenous labor that is entirely devoted to work. Skill λ_i may also be interpreted as the qualification achieved by each agent i. We consider that λ is distributed by means of a continuous and strictly positive density function f on λ with range $[1, 0]$.

Hence, for a given tax vector $t = (t_1, ..., t_K)$ and labor price w_k, welfare level of agent i if he chooses to live in region k, is given by the net income:

$$Y_k (\lambda_i, t) = w_k \lambda_i (1 - t_k) + t_k \bar{w}_k,$$

where $\bar{w}_k$ is the mean labor earnings in region k. Thus, $\bar{w}_k = \bar{\lambda}_k \; w_k$ where $\bar{\lambda}_k$ is the mean skill in region k and $t_k \bar{w}_k$ is the uniform transfer for each agent living in region k.

Finally, we assume that migration is costless. An agent i migrates from region k to region ℓ when he strictly increases his net income, i.e. when $Y_k (\lambda_i, t) < Y_\ell (\lambda_i, t)$. Furthermore, we assume that when an agent decides to migrate he takes the other regions' price of labor as fixed.

The description of an economy that satisfies all the assumptions listed above is denoted by E where $E = \{K, T, f\}$, T is a list of technologies $T = (T_1, ..., T_K)$.

The Equilibrium Notion

Following Westhoff (1977), Epple et al. (1984), Epple and Romer (1991) our definition of equilibrium takes the migration flows among regions and the political process within each region into account.

Definition 2.1. *An equilibrium for the economy* $E = \{K, T, f\}$ *is a pair* $\{t, P\}$ *where* $t = (t_1, ..., t_K)$ *is a vector of tax policies and* $P = \{N_1, ..., N_K\}$ *is a partition of the population satisfying:*
i) (Internal Equilibrium) Each t_k*, where* $k \in \{1, ..., K\}$ *is the result of region k's majority voting procedure.*
ii) (External Equilibrium) No agent wants to migrate to another region, i.e. for every $k, \ell \in \{1, ..., K\}$ *and every* $i \in N_k$, $Y_k(\lambda_i, t) \geq Y_\ell(\lambda_i, t)$.

Part i) and part ii) of the equilibrium fit what Epple et al. (1984) and Epple and Romer (1991) call internal and external equilibrium respectively.

Authors like Westhoff (1977), Epple et al (1984), Epple and Romer (1991), Rose-Ackerman (1979), Hansen and Kessler (1996), use majority rule within each region to determine the internal equilibrium. In order to apply majority voting rule, we require a positive mass of population in every region. Consequently, in equilibrium, all the regions must be inhabited.

Regarding internal equilibrium we shall study two different kinds of agents' rationality: sophisticated agents and myopic agents. When the agents decide on their most preferred tax rate we consider them to be sophisticated if they anticipate the migration flows induced by the tax rates. Hence, the sophisticated agents understand that each region's population depends on the tax rate of that region as well as that of the other regions. When the agents do not anticipate the migration flows we consider them to be myopic agents. Thus, when they decide on their most preferred tax rate, they consider the population of each region as fixed.[2].

In the terminology of Caplin and Nalebuff (1997), we shall refer to **membership-based equilibrium** (the population is assumed to be fixed) when we consider the agents to be myopic and to **policy-based equilibrium** when we consider the agents to be sophisticated.

3. THE EXTERNAL EQUILIBRIUM

In this section we provide a characterization of the pairs $\{t, P\}$ that qualify as external equilibria. It is important to point out that the characterization of the

[2] A discussion on agent's rationality and time structure is provided by Epple and Romer (1991, p. 837), Caplin and Nalebuff (1997, p. 316). The former authors defend the assumption of sophisticated agents on the basis that this is more widely applicable. On the other hand, the latter authors do not share the same point of view and argue that assuming that the agents are myopic is more widely applicable.

external equilibria is independent of the agents' rationality that we consider.

In the same spirit than Westhoff (1977) and Epple and Romer (1991), in our model the agent's net income can be ordered according to the skill of the agents. As we next show, it generates considerable structure on the equilibrium's partition of the population. In order to determine the equilibrium structure, we consider the following properties on $\{t, P\}$.

Stratification (S): each region's population belongs to a single interval of skill so that if $\lambda_i < \lambda_{i'}$ and both agents i, i' live in the same region, every agent i'' with skill $\lambda_{i''} \in (\lambda_i, \lambda_{i'})$ also lives in that region.
Boundary Indifference (BI): regions can be ordered from the lowest to the highest skill levels and when they are ordered in this way, between each two adjacent regions, there is a boundary agent with skill $\hat{\lambda}$ who is indifferent between them.
Increasing Net Labor Prices (INLP): let $\bar{\lambda}_k$, $\bar{\lambda}_\ell$ be the mean skill of region k and ℓ respectively and $\bar{\lambda}_k < \bar{\lambda}_\ell$ then, $w_k(1 - t_k) \leq w_\ell(1 - t_\ell)$.

In the sequel, we shall assume that the boundary skilled agent lives in the region where the mean skill is lower.

In our analysis we focus attention on equilibria where some regions differ.

Definition 3.1. *We say that* ***two regions differ*** *when in equilibrium either the net price of labor $w_k(1 - t_k)$ or the transfer $t_k \bar{w}_k$ differ across these two regions.*

Let us next show that the mentioned properties are in some cases necessary and sufficient conditions for an external equilibrium.

Proposition 3.2. *Let $E = \{K, T, f\}$, then,*
I) S, BI and INLP are necessary conditions for $\{t, P\}$ to be an external equilibrium where all the regions are different
II) S, BI and INLP are sufficient conditions for $\{t, P\}$ to be an external equilibrium.

Proof. (Is in the appendix).

The above proposition characterizes all the external equilibria where all the regions are different. When two regions are identical we find that the agents are indifferent between living in them. Then, stratification within these identical regions is not any more a necessary condition for an external equilibrium.

By Proposition 3.2, the external equilibria with different regions are characterized by certain relations among the exogenous distribution of skills and the degree of redistribution obtained in each region. The following corollary yields a relation among the equilibrium values of t_k and w_k.

Corollary 3.3. *Let $E = \{K, T, f\}$ and let $\{t, P\}$ be an external equilibrium of E where all the regions are different. Then, if in equilibrium we order the regions by skill levels, so that $\bar{\lambda}_1 < \bar{\lambda}_2 < ... < \bar{\lambda}_K$, then,*
a) $w_1 > w_2 > ... > w_K$
b) $t_1 > t_2 > ... > t_K$
c) $\bar{w}_1 t_1 > \bar{w}_2 t_2 > ... > \bar{w}_K t_K$
d) $\bar{w}_1 < \bar{w}_2 < ... < \bar{w}_K$

Proof. (See appendix)

From this Corollary we deduce some interesting characteristics of the equilibria with different regions.

From a), in equilibrium the price of labor is higher in the low skill region, which favors the agents with lower skill. Note as well, that a) implies that equilibrium can not be efficient since some of the regions have a higher productivity of labor than others[3].

By b) and c), the redistributive policy is positively related to low skilled regions and so higher skill agents tend to skip high taxation rates, which is coherent with the conventional wisdom on redistributive taxation.

From d), we observe that it is always the case that the mean labor earnings increase as we move from regions with low skill to regions with higher skill.

4. NON EXISTENCE OF EQUILIBRIUM

In this section we analyze an example inspired by Caplin and Nalebuff (1997)[4] and that illustrates the fact that when the productivity of labor is constant and equal across regions there is no equilibrium where at least two regions are different.

Let us assume that in each region there is an equal technology with constant returns to scale. It implies that the equilibrium price of labor is the same across

[3] For a study on efficiency of equilibrium see Fernández and Rogerson (1996).

[4] The example is adapted from Epple and Romer (1991). A similar example is also analyzed in Hansen and Kessler (1996).

regions. In this case, normalizing the price of labor, the net income obtained by an agent with skill λ_i is given by $Y_k(\lambda_i, t) = \lambda_i(1 - t_k) + t_k\bar{\lambda}_k$. We denote by $E' = \{K, \bar{T}, f\}$ an economy where $\bar{T} = (\bar{T}_1, ..., \bar{T}_K)$ and each $\bar{T}_k$ is a constant returns to scale technology, such that $\bar{T}_1 = \bar{T}_2 = ... = \bar{T}_K$.

We find that the economy described in the example of Caplin and Nalebuff (1997) fits properly in this setting. They consider a two town model (regions in our case) where agents differ in their income and where each region sets a proportional tax rate and also a level of transfer. Note that for economies as E' the distribution of skill levels can be also interpreted as a distribution of income levels.

In the following proposition we generalize Caplin and Nalebuff's two regions result to a K region model. We show that there is no equilibrium where at least two regions are different. Note that in this reduced model two regions are different when in equilibrium either the tax rate t_k or the transfer $t_k\bar{\lambda}_k$ differ across them.

Proposition 4.1. *Let $E' = \{K, \bar{T}, f\}$. Then, there is no equilibrium for the economy E' with at least two different regions.*

Proof. Suppose to the contrary that $\{t, P\}$ is an equilibrium with at least two different regions k and ℓ. Then, by Proposition 3.2, it satisfies S, BI and INLP. By S we can order the regions by skill levels. Suppose that k is a lower skill region than ℓ. By BI there is an indifferent agent with skill $\hat{\lambda}$ such that $\hat{\lambda}(1 - t_k) + \bar{\lambda}_k t_k = \hat{\lambda}(1 - t_\ell) + \bar{\lambda}_\ell t_\ell$. Simplifying and rearranging terms

$$t_k(\bar{\lambda}_k - \hat{\lambda}) = t_\ell(\bar{\lambda}_\ell - \hat{\lambda}). \tag{4.1}$$

Since region k and ℓ are different it can not be the case that $t_k = t_\ell = 0$. Hence, given that $(\bar{\lambda}_k - \hat{\lambda}) < 0$ and $(\bar{\lambda}_\ell - \hat{\lambda}) > 0$, (4.1) is a contradiction. **Q.E.D.**

Proposition 4.1 shows that for the economy E' there is no pair $\{t, P\}$ satisfying BI. The intuition behind is as follows: when the lowest skill region has a positive taxation rate $t_k > 0$, the agents living in region k with skill levels such that $\lambda_i > \bar{\lambda}_k$, are net loser of the redistributive taxation and they always take advantages from migrating to a higher skill region, that is region ℓ. And when $t_k = 0$, since region k and ℓ are different, $t_\ell > 0$, and then, the agents living in region ℓ with skill $\lambda_i > \bar{\lambda}_\ell$, are net contributors to the redistributive taxation and they benefit from migrating to region k. A similar intuition for our result is also provided by Caplin and Nalebuff (1997, p. 318), Hansen and Kessler (1996).

As a consequence of the above Proposition we deduce that for the economies described by E', the only possible equilibria are the trivial ones where all the regions are identical, i.e. $w_k(1-t_k)$ and $t_k w_k$ are the same for every region (this implication is shown in Hansen and Kessler (1996)).

If each region has a technology with constant returns to scale which is different for each of the region, we can provide some examples where an equilibrium fails to exist:

Example 4.2. : *Consider an economy consisting of two regions. Suppose that the skill distribution is such that $t=(1,0)$ is the unique internal equilibrium*[5]*. Since the technology displays constant returns to scale, the price of labor in each region is fixed. Normalizing this price, let $w_1=1$ and $w_2=\gamma$. Then, BI requires that $\bar{\lambda}_1=\gamma\hat{\lambda}$ and clearly for $\gamma>1$ an equilibrium does not exist.*

5. EQUILIBRIUM EXISTENCE

In this section, we are going to restrict attention to a two region model.

We firstly show that under certain assumptions on the technology, an external equilibrium exists.

In subsection 5.1, we are going to consider the case where the agents are myopic. There, we characterize the equilibria with different regions, and we provide a numerical example that shows existence.

In subsection 5.2, we assume that the agents are sophisticated. There, we show existence of equilibrium and we discuss a numerical example.

In the sequel we are going to consider that the technologies display decreasing returns to scale. In this case, we show that under the following assumption it is always possible to find some pairs $\{t,P\}$ satisfying S and BI.

Assumption 1: Each region technology T is strictly concave and $\lim_{L\to 0} T'(L)=\infty$.

Note that the following lemma is independent of whether we consider myopic or sophisticated agents.

Lemma 5.1. *Let $E=\{2,T,f\}$ where Assumption 1 holds. Then, for every $t\in[0,1]\times[0,1]$, there exists a partition of the population P such that $\{t,P\}$ satisfies S, BI and where the skill of the indifferent agent $\hat{\lambda}\in(0,1)$.*

[5] We shall show in next section that when the agents are myopic, there are several density functions such that $t=(1,0)$ is the unique tax vector that qualifies as an internal equilibrium.

Proof. Let us consider the following partition of the population $P = \{N_1, N_2\}$ with $N_1 = \left\{i : \lambda_i \leq \hat{\lambda}\right\}$, $N_2 = \left\{i : \lambda_i > \hat{\lambda}\right\}$ where $\hat{\lambda} \in (0,1)$. Then, by continuity of f, the labor supply and the mean skill in each region can be obtained as a continuous function of $\hat{\lambda}$. And since $w_k(L_k) = T_k'(L_k)$ where T is continuously differentiable, the price of labor can be also obtained as a continuous function of $\hat{\lambda}$, $w_k\left(L_k(\hat{\lambda})\right)$. Hence, the net income obtained by each agent can be expressed as a continuous function of $\hat{\lambda}$, $Y_k\left(\lambda_i, t, \hat{\lambda}\right) = w_k\left(\hat{\lambda}\right)(1-t_k)\lambda_i + \bar{w}_k\left(\hat{\lambda}\right)t_k$.

Let the function ϕ such that

$$\phi\left(\hat{\lambda}, t\right) \equiv \hat{\lambda}\left[w_1(\hat{\lambda})(1-t_1) - w_2(\hat{\lambda})(1-t_2)\right] + \bar{w}_1(\hat{\lambda})t_1 - \bar{w}_2(\hat{\lambda})t_2 \tag{5.1}$$

BI requires that the agent with skill $\hat{\lambda}$ be indifferent between migrating to the other region or not migrating so that $\phi\left(\hat{\lambda}, t\right) = 0$. Moreover by Assumption 1, for every $t \in [0,1] \times [0,1]$, we have that $\lim_{\hat{\lambda} \to 0} \phi\left(\hat{\lambda}, t\right) > 0$ and $\lim_{\hat{\lambda} \to 1} \phi\left(\hat{\lambda}, t\right) < 0$. Then, since $\phi\left(\hat{\lambda}, t\right)$ is continuous in $\hat{\lambda}$, by the intermediate value theorem there exists $\hat{\lambda} \in (0,1)$ such that (5.1) is satisfied and the stratified partition $P = \{N_1, N_2\}$ with $N_1 = \left\{i : \lambda_i \leq \hat{\lambda}\right\}$, $N_2 = \left\{i : \lambda_i > \hat{\lambda}\right\}$ always exists. **Q.E.D.**

The intuition behind Lemma 5.1 is related to a necessary assumption used by Caplin and Nalebuff (1997) to show existence of equilibrium in a similar context. They call this assumption *viability of small institutions* and it requires that when the population of a region tends to zero, there must be some agents with incentive to move into that region. Their assumption ensures a strictly positive mass of population in every region. In our model, when the population of a region tends to zero, there are always some agents with incentive to migrate to that region since by Assumption 1 the price of labor is extremely high. Consequently, there exists an indifferent agent such that both regions are inhabited. When we consider constant returns to scale technologies, we have that the price of labor is fixed and then, as we showed in Proposition 4.1, all agents prefer one of the regions to the other, and consequently, BI is not satisfied.

Remark 1. *Note that if additionally to Lemma 5.1, we want to guarantee that*

$\hat{\lambda}$ is unique, it is sufficient to show that $\frac{\partial\phi(\hat{\lambda},t)}{\partial\hat{\lambda}} \neq 0$. Solving this partial derivative

$$\frac{\partial\phi(\hat{\lambda},t)}{\partial\hat{\lambda}} = I + \hat{\lambda}\left(\frac{dw_1(\hat{\lambda})}{d\hat{\lambda}}(1-t_1) - \frac{dw_2(\hat{\lambda})}{d\hat{\lambda}}(1-t_2)\right) + \frac{d\bar{w}_1(\hat{\lambda})}{d\hat{\lambda}}t_1 - \frac{d\bar{w}_2(\hat{\lambda})}{d\hat{\lambda}}t_2$$

where $I = w_1(1-t_1) - w_2(1-t_2)$. In the next section we shall refer to this point when discussing equilibrium existence.

5.1. The Membership-Based Equilibrium

Here we assume that the agents are myopic and so, when each agent decides his most preferred tax rate, he takes the population of each region as given and the net income function that he faces is $Y_k(\lambda_i, t_k) = w_k\lambda_i + w_k t_k(\bar{\lambda}_k - \lambda_i)$ where w_k and $\bar{\lambda}_k$ are considered to be fixed. Remember that the policy of a region is determined by majority voting. Hence, depending on whether each agent skill is below or above the mean skill, his most preferred tax rate is either $t_k = 1$ or $t_k = 0$ respectively. Note that with myopic agents, the location of the median skill agent with respect to the mean skill agent in each region is crucial in order to define a majority voting equilibrium. And so, when the median skill agent do not coincide with the mean agent, the equilibrium tax policy is $t = 0$ or $t = 1$. But when the median and the mean skill agent coincide, all $t \in [0,1]$, can be obtained as an internal equilibrium. It causes some problems when trying to characterize the equilibria. Think for instance that $t_1 < t_2$, then, by Corollary 3.3, we have that there is no external equilibrium. In order to skip it, we introduce the following assumption, where we denote by λ_1^m, λ_2^m the skill of the agent in the median of region 1 and 2 respectively..

Assumption 2: When $\lambda_1^m = \bar{\lambda}_1$, we select $t_1 = 1$ to be the tax policy of region 1, and when $\lambda_2^m = \bar{\lambda}_{2.}$, we select $t_2 = 0$ to be the tax policy of region 2.

In the following proposition we show two properties that characterize the membership-based equilibria.

Proposition 5.2. *Let $E = \{2, T, f\}$, then, under Assumption 2, a membership-based equilibrium $\{t, P\}$ with different regions exists if and only if*
i) there is an indifferent agent $\hat{\lambda} \in (0,1)$
ii) in the interval $\left[0, \hat{\lambda}\right]$, we have that $\lambda_1^m \leq \bar{\lambda}_1$ and in the interval $\left(\hat{\lambda}, 1\right]$ we have that $\lambda_2^m \geq \bar{\lambda}_2$.

Proof. Let us prove necessity. Since $\{t, P\}$ is an external equilibrium, by Proposition 3.2 it satisfies BI and so, in equilibrium property i) holds.

Suppose to the contrary that there is an equilibrium $\{t, P\}$ with different regions where ii) does not hold. By part b) of Corollary 3.3 we have that in equilibrium $t_1 > t_2$ then, by Assumption 2 and given that we consider myopic agents, $t = (1, 0)$ is the unique vector of tax policies compatible with an external equilibrium with different regions. Since ii) does not hold, it follows that either in the interval $[0, \lambda_i)$ we have that $\lambda_1^m > \bar{\lambda}_1$ or in the interval $[\lambda_i, 1]$ we have that $\lambda_2^m < \bar{\lambda}_{2.}$ It implies that in the internal equilibrium $t_1 = 0$ and $t_2 = 1$ which contradicts that in equilibrium $t = (1, 0)$.

Let us prove sufficiency. By i), there is an indifferent agent with skill $\hat{\lambda} \in (0, 1)$. Consider the following partition of the population $P = \{N_1, N_2\}$ where $N_1 = \left\{i : \lambda_i \leq \hat{\lambda}\right\}$, $N_2 = \left\{i : \lambda_i > \hat{\lambda}\right\}$. Clearly, P satisfies S. By ii) and Assumption 2, in region 1 more than half of the population prefers $t = 1$ to any other tax rate and in region 2 more than half of the population prefers $t = 0$ to any other tax rate, it implies that $t = (1, 0)$ is an internal equilibrium of E.

And then, since

$$w_1 (1 - t_1) = 0 \leq w_2 (1 - t_2) = w_2, \tag{5.2}$$

INLP holds. Then, by Proposition 3.2, since $\{t, P\}$ satisfies S, BI and INLP, we have that $\{t, P\}$ is an external equilibrium. Finally since by i) $\hat{\lambda} \in (0, 1)$, we have that $w_2(\hat{\lambda}) > 0$ and consequently inequality (5.2) is strict, and so, in this equilibrium the regions are different. **Q.E.D.**

Let us next discuss how restrictive are properties i) and ii).

Property i): by Lemma 5.1 we know that under Assumption 1 there exists $\hat{\lambda} \in (0, 1)$ such that $\hat{\lambda}$ is the skill of the indifferent agent. Clearly, for every technology $T(L) = L^\alpha$ where $\alpha \in (0, 1)$, Assumption 1 is satisfied.

Property ii): we find that in general there are many common density functions that do not satisfy this property, think for instance of the normal distribution. However, we find some example that may be a good approximation to some regions' skills or qualification distribution and that satisfy this property, think for instance of a uniform distribution, some bimodals distribution, some U-shaped distributions among others.

Let us next provide a numerical example of a membership-based equilibrium.

Example 5.3. *Let the economy $E = \{2, T, f\}$ where $T_1 = T_2 = L^\alpha$ with $\alpha \in (0,1)$ and where f is a uniform density function with range $[0,1]$.*
The skill of the agent in the mean of the distribution is given by $\bar{\lambda}_1 = \frac{\hat{\lambda}}{2}$, $\bar{\lambda}_2 = \frac{(1+\hat{\lambda})}{2}$. By S we have that $L_1 = \frac{\hat{\lambda}^2}{2}$ and $L_2 = \frac{1-\hat{\lambda}^2}{2}$. Consequently, the price of labor in terms of the endogenous variable $\hat{\lambda}$ is given by

$$w_1 = \alpha \left(\frac{\hat{\lambda}^2}{2} \right)^{\alpha - 1}, \; w_2 = \alpha \left(\frac{1 - \hat{\lambda}^2}{2} \right)^{\alpha - 1}$$

We are going to consider the case of $\alpha = .6$.
Since f is a uniform density function, property ii) of Proposition 5.2 holds and by Assumption 2, we select $t = (1,0)$ to be the internal equilibrium. Then, by BI we have that in equilibrium:

$$w_1 = 2w_2. \tag{5.3}$$

From (5.3) we obtain that there is a unique $\hat{\lambda} = .38758$, and then, we can calculate the value of all the endogenous variables that we summarize in the following table:

t_1	t_2	w_1	w_2	$\bar{w}_1$	$\bar{w}_2$	$\hat{\lambda}$	L_1	L_2
1	0	1.69	.84497	. 32751	. 58623	.38758	.07511	.42489

(5.4)

Finally, note that INLP also holds, and so, the above table describes an external equilibrium with different regions.
The intuition behind this equilibrium is the following: On the one hand, the net losers of the redistributive taxation in region 1 do not have incentive to migrate since the price of labor in region 2 is too low. And, on the other hand, the agents living in region 2, although they observe a higher price of labor in region 1, do not migrate since the tax rate is higher in region 1.
Clearly the above table describes a membership-based equilibrium with different regions for the described economy.

5.2. The Policy-Based Equilibrium

In this section we are going to consider the case where the agents are sophisticated. We firstly analyze the result of majority voting in each region. We then show, that under similar conditions to the ones that characterize the membership-based equilibria, a policy-based equilibrium where the regions are different exists.

Since agents are sophisticated, when they choose their most preferred t_k they take as given the taxation policy of the other regions $\bar{t}_{-k}$, and they consider that the price of labor and the mean labor earnings vary as the tax vector changes. In particular, the agents know that when the tax rate of each region is modified, the population migrates and the agent who is indifferent varies. Consequently, the labor supply of each region varies and it modifies the price of labor.

Hence, the net income function that they face is

$$Y_k\left(\lambda_i, t_k, \bar{t}_{-k}\right) = w_k\left(\hat{\lambda}\left(t\right)\right)\lambda_i\left(1-t_k\right) + \bar{w}_k\left(\hat{\lambda}\left(t\right)\right)t_k.$$

Let us next show how to obtain the price of labor as a function of the tax rates. By Lemma 5.1, we know that Assumption 1 guarantees that there is $\hat{\lambda} \in (0,1)$ such that BI holds. Moreover, by Remark 1, if $\frac{\partial\phi\left(\hat{\lambda},t\right)}{\partial\hat{\lambda}} \neq 0$, we have that $\hat{\lambda}$ is unique. We shall show that in equilibrium and also around equilibrium, it is always the case that $\frac{\partial\phi\left(\hat{\lambda},t\right)}{\partial\hat{\lambda}} \neq 0$ and so, around equilibrium we have that $\hat{\lambda}$ can be obtained as a function of t. Let us denote this function by $\Phi\left(t\right) = \hat{\lambda}$. For simplicity, suppose at this point that for every t the function Φ can be defined (this technical problem is solved in the existence prove). And by the Implicit Function Theorem, Φ is continuous[6], differentiable and

$$\frac{\partial\Phi(t)}{\partial t_k} = -\frac{\partial\phi\left(\hat{\lambda},t\right)}{\partial t_k} \Big/ \frac{\partial\phi\left(\hat{\lambda},t\right)}{\partial\hat{\lambda}}. \quad (5.5)$$

Since agents anticipate the migration flows produced by the tax rate, for an agent living in region k, his most preferred tax rate is given by

$$t_k \in \underset{t_k\in[0,1]}{\arg\max} \quad w_k\left(\Phi\left(t\right)\right)\lambda_k\left(1-t_k\right) + \bar{w}_k\left(\Phi\left(t\right)\right)t_k \quad (5.6)$$

The above objective function is continuous[7] in the interval $t_k \in [0,1]$ so that a maximum always exists.

We next want to know whether the solutions to problem (5.6) is interior or is in the boundary of the interval $[0,1]$. First order condition for an interior solution is given by

$$\frac{\partial Y_k(\lambda_k,t)}{\partial t_k} = \frac{\partial\Phi(t)}{\partial t_k}\left(\frac{dw_k(\Phi(t))}{d\Phi}\left(1-t_k\right)\lambda_k + \frac{d\bar{w}_k(\Phi(t))}{d\Phi}t_k\right) - w_k\lambda_k + \bar{w}_k\left(\Phi\left(t\right)\right) = 0. \quad (5.7)$$

[6]In order to show existence of equilibrium it will be sufficient to show that for a fixed t_2, as t_1 varies, it is possible to select a path where $\hat{\lambda}$ varies in a continuous way.

[7]By continuity of Φ,of f and given that T_1 and T_2 are continuously differentiable.

Let us analyze the sign of the derivatives in (5.7). Since T_1 and T_2 are strictly concave, we have that $\frac{dw_1(\Phi(t))}{d\Phi} < 0$ and $\frac{dw_2(\Phi(t))}{d\Phi} > 0$. Solving the derivative $\frac{d\bar{w}_k(\Phi(t))}{d\Phi}$

$$\frac{d\bar{w}_k(\Phi(t))}{d\Phi} = \frac{dw_k(\Phi(t))}{d\Phi}\bar{\lambda}_k + \frac{d\bar{\lambda}_k(\Phi(t))}{d\Phi}w_k \tag{5.8}$$

And since in region 1, we have that $\frac{d\bar{\lambda}_1(\Phi(t))}{d\Phi} > 0$, the sign of $\frac{d\bar{w}_1(\Phi(t))}{d\Phi}$ is not determined. The intuition is the following: in region 1, as the skill of the indifferent agent decreases, the labor supply decreases and the price of labor in region 1 rises. However, as the skill of the indifferent agent decreases, the mean skill in region 1 decreases. Consequently, the mean labor earnings may increase or decrease depending on what effect is dominant.

For region 2, since $\frac{d\bar{\lambda}_2(\Phi(t))}{d\Phi} > 0$, from (5.8) we have that $\frac{d\bar{w}_2(\Phi(t))}{d\Phi} > 0$. The intuition is the following: as the skill of the indifferent agent increases, the labor supply in region 2 decreases and the skill of the agent in the mean increases, consequently, the price of labor and the mean labor earnings increase.

In the following lemma, we identify the properties that guarantee that the equilibrium tax policy is not obtained as an interior solution to the problem (5.6).

Let us denote by $\varepsilon_{w_1\hat{\lambda}} = \frac{dw_1(\hat{\lambda})}{d\hat{\lambda}}\frac{\hat{\lambda}}{w_1(\hat{\lambda})}$ the elasticity of the price of labor in region 1 with respect to the skill of the indifferent agent. And by $\varepsilon_{\bar{\lambda}_1\hat{\lambda}} = \frac{d\bar{\lambda}_1(\hat{\lambda})}{d\hat{\lambda}}\frac{\hat{\lambda}}{\bar{\lambda}_1(\hat{\lambda})}$ the elasticity of the mean skill in region 1 with respect to the skill of the indifferent agent.

Lemma 5.4. *Let $E = \{2, T, f\}$ an economy and $\{t, P\}$ a policy-based equilibrium, if i') and ii) are satisfied, where*
i') $\varepsilon_{\bar{\lambda}_1\hat{\lambda}} \leq \left|\varepsilon_{w_1\hat{\lambda}}\right|$
ii) in the interval $\left[0, \hat{\lambda}\right]$, we have that $\lambda_1^m \leq \bar{\lambda}_1$ and in $\left(\hat{\lambda}, 1\right]$ we have that $\lambda_2^m \geq \bar{\lambda}_2$.
then $t = (1, 0)$.

Proof. (It is in the appendix).

Note that ii) is the same property used in Proposition 5.2.

By i'), we have that a marginal (percentage) change in the skill of the indifferent agent has a smaller marginal (percentage) effect on the skill of the agent in the mean of region 1, than on the price of labor of region 1. In other terms, property i') requires that the slope of the production function be steeper in relative terms than the slope of the distribution function[8]. Consequently, it implies that as the skill of the indifferent agent decreases, the mean labor earnings increase.

The intuition of Lemma 5.4 is the following. By property i'), we find that those agents with skill below $\bar{\lambda}_1$, always benefit from reducing the labor supply of his region. Consequently, by means of increasing the tax rate to $t_1 = 1$, they avoid up to the maximum that the agents from region 2 migrate to region 1. In region 2, as $\hat{\lambda}$ increases, the price of labor and the mean labor earnings increase, then, in equilibrium the agent with skill above $\bar{\lambda}_2$, always benefit from reducing the labor supply of his region. Hence, the tax policy $t_2 = 0$ avoids up to the maximum that the agents from region 1 migrate to region 2.

We shall show that when $\varepsilon_{\bar{\lambda}_1 \hat{\lambda}} > \left|\varepsilon_{w_1 \hat{\lambda}}\right|$, the equilibrium tax policy differs from $t_1 = 1$. Then, a majority voting equilibrium in region 1 requires that the net income $Y_1(\lambda_i, t)$ be single peaked in t_1. Taking it into account, in the following proposition, we show that property ii) guarantees that a policy-based equilibrium exists.

Proposition 5.5. *Let $E = \{2, T, f\}$ where Assumption 1 holds then, if either i') and ii) or i") and ii) are satisfied, where*
i') $\varepsilon_{\bar{\lambda}_1 \hat{\lambda}} \leq \left|\varepsilon_{w_1 \hat{\lambda}}\right|$, ii') $\varepsilon_{\bar{\lambda}_1 \hat{\lambda}} > \left|\varepsilon_{w_1 \hat{\lambda}}\right|$ and $\frac{\partial^2 Y_1(\lambda_i, t)}{\partial^2 t_1} < 0$,
ii) in the interval $\left[0, \hat{\lambda}\right]$, we have that $\lambda_1^m \leq \bar{\lambda}_1$ and in $\left(\hat{\lambda}, 1\right]$ we have that $\lambda_2^m \geq \bar{\lambda}_2$, a policy-based equilibrium with different regions exists.

Proof. (It is in the appendix)

The intuition of this result is provided in the following example:

Example 5.6. *Let the same economy than in Example 5.3: $E = \{2, T, f\}$ where $T_1 = T_2 = L^{.6}$ and f is a uniform density function.*
Since $\bar{\lambda}_1 = \frac{\hat{\lambda}}{2}$ and $w_1 = 0,6\left(\frac{\hat{\lambda}^2}{2}\right)^{-.4}$, calculating the elasticities we have that $\varepsilon_{\bar{\lambda}_1 \hat{\lambda}} = 1$ and $\varepsilon_{w_1 \hat{\lambda}} = -0,8$. Then, it follows that $\left|\varepsilon_{w_1 \hat{\lambda}}\right| < \varepsilon_{\bar{\lambda}_1 \hat{\lambda}}$, and so, t_1 is obtained as an interior solution to problem 5.6, and we have that $t_2 = 0$.

[8] The elasticity $\varepsilon_{w_1 \hat{\lambda}}$ is given by the concavity of T_1. Hence, when the technology has constant returns to scale we have that $\varepsilon_{w_1 \hat{\lambda}} = 0$. The elasticity $\varepsilon_{\bar{\lambda}_1 \hat{\lambda}}$, refers to the density function f, in particular it measures the relative curvature of the slope of the distributibution function generated by f.

Let us next calculate the equilibrium. Since $\bar{\lambda}_1 = \frac{\hat{\lambda}}{2}$ *and* $\bar{\lambda}_2 = \frac{1+\hat{\lambda}}{2}$*, by BI, we have that*

$$w_1\left(1-\frac{t_1}{2}\right)=w_2 \tag{5.9}$$

and since the mean and the median skill agent coincide, solving the maximization problem (5.6) of the median agent, we have that

$$t_1=\frac{-\varepsilon_{w_1\hat{\lambda}}}{\varepsilon_{\bar{\lambda}_1\hat{\lambda}}} \tag{5.10}$$

Solving (5.10) we obtain that $t_1 = .8$ *is the equilibrium tax rate for region 1. That* $t_1 = .8$ *is a majority voting equilibrium follows from verifying second order conditions* $\frac{\partial^2 Y(\lambda_i,t)}{\partial^2 t_1} < 0$ *for all* $\lambda_i \in \left[0, \hat{\lambda}\right]$*, which are satisfied here. And substituting in (5.9) we obtain that* $\hat{\lambda} = .46696$ *is the unique value for the skill of the indifferent agent. We summarize the value of all the endogenous variables in the following table*

t_1	t_2	w_1	w_2	$\bar{w}_1$	$\bar{w}_2$	$\hat{\lambda}$	L_1	L_2
.8	0	1.4559	.87356	. 33993	. 64074	.46696	.10903	.39097

Clearly, since INLP also holds, the above table describes a policy-based equilibrium with different regions.
Since in the example we consider a uniform skill distribution, the obtained equilibrium also qualifies as a membership-based equilibrium, in particular it belongs to the set of membership-based equilibrium excluded by Assumption 2.

We refer to the policy-based equilibrium with the subindex P, and to the membership-based equilibrium with the subindex M. Note that for those skill distributions satisfying property ii) and such that the mean and the median skill agent do not coincide, we have that $t_1^P \leq t_1^M$. The explanation is as follows, when the agents are sophisticated, they take into account how the migration flows affect their income, and so, if as $\hat{\lambda}$ increases, the mean labor earnings increase, the agents with lower skill prefer reducing t_1 below 1 in order to attract higher skill agents.

Finally, it is important to point out that even when property ii) does not hold, we can always find a partition of the population satisfying S and BI, but then, it is not always possible to show that INLP is satisfied, since it depends on each region population and on the majority voting taxation policy. However, note that when the agents are sophisticated, property ii) is not any more a necessary condition for equilibrium.

6. FINAL COMMENTS

Our results are related with those obtained by Epple and Romer (1991) (E-R in the sequel). E-R establish the properties of equilibria with different regions, but they do not study existence of those equilibria. Indeed, authors like Caplin and Nalebuff (1997), show by means of an example, in a version simpler than E-R, that an equilibrium fails to exist[9]. In this paper, we have investigated what are the precise properties that lead us to this negative existence result.

We have firstly shown the necessary and sufficient conditions for existence of an external equilibrium with different regions. These conditions are basically the same as in E-R' paper. In a context simpler than E-R, we then show, that these conditions are not compatible (Proposition 4.1). It is important to point out, that the example provided by Caplin and Nalebuff and the model presented by Hansen and Kessler (1996), fit properly in our context.

Secondly, we have assumed that in each region there is a labor market with congestions[10] that endogenously determines the price of labor. Under this assumption, we show that the conditions that characterize the external equilibria are now compatible (Lemma 5.1). We have shown, basically, that the migration flows vary the price of labor until reaching the point where no agent wants to migrate.

Finally, we have established some conditions that guarantee existence of equilibrium (internal and external) with different regions. For the case where the agents are myopic, we have provided the necessary and sufficient conditions for an equilibrium. These conditions require that the skill distribution be uniform or similar to a U-shaped distribution, i.e. we require that a big mass of population be low-skilled and another big mass be high-skilled (Proposition 5.2). For the case where the agents are sophisticated, we have shown (Proposition 5.5) that the same kind of skill distribution guarantees equilibrium existence, but then, if the slope of the distribution function is steeper in relative terms than the slope of the production function, equilibrium existence requires that the agents' net income be a concave function of the tax rate.

[9] See De Donder and Hindriks (1998) for a wider discussion.

[10] That the technology displays decreasing returns to scale plays a similar role to considering that there are congestions in each of the regions.

7. APPENDIX

Proof of Proposition 3.2:

Let us first prove statement I):

Stratification: Suppose to the contrary that there is an external equilibrium $\{t, P\}$ where all the regions are different and where S does not hold. Hence in equilibrium there must be at least two agents i and i' with skill $\lambda_i < \lambda_{i'}$ that lives in region k and an agent i'' with skill $\lambda_{i''} \in (\lambda_i, \lambda_{i'})$ that lives in region ℓ. Then, the following inequalities hold

$$\lambda_i [w_\ell(1-t_\ell) - w_k(1-t_k)] + [t_\ell \bar{w}_\ell - t_k \bar{w}_k] \leq 0 \quad (7.1)$$
$$\lambda_{i''} [w_\ell(1-t_\ell) - w_k(1-t_k)] + [t_\ell \bar{w}_\ell - t_k \bar{w}_k] \geq 0 \quad (7.2)$$
$$\lambda_{i'} [w_\ell(1-t_\ell) - w_k(1-t_k)] + [t_\ell \bar{w}_\ell - t_k \bar{w}_k] \leq 0. \quad (7.3)$$

Since (7.1), (7.2) and (7.3) are only compatible when they hold with equality, it implies that in equilibrium $w_\ell(1-t_\ell) = w_k(1-t_k)$ and $t_\ell \bar{w}_\ell = t_k \bar{w}_k$. But then, region k and ℓ are identical which is a contradiction.

Boundary Indifference: Suppose to the contrary that there is an external equilibrium $\{t, P\}$ where BI is not satisfied. Then, by S the regions can be ordered by skill levels and so, for some λ, λ', $\lambda'' \in (0,1)$, the agents with skill in the interval $(\lambda, \lambda']$ live in region k and the agents with skill in the interval $(\lambda', \lambda'']$ live in region $k+1$. Since there is no indifferent agent, the agent with skill λ' strictly prefers region k to region $k+1$, i.e. $Y_k(\lambda', t) > Y_{k+1}(\lambda', t)$ and the agent with skill $\lambda' + \varepsilon$ (where $\varepsilon > 0$ and close enough to zero) strictly prefers region $k+1$ to region k, i.e. $Y_k(\lambda' + \varepsilon, t) < Y_{k+1}(\lambda' + \varepsilon, t)$. But then, since f is strictly positive and continuous, there is an agent i with skill $\hat{\lambda} \in [\lambda', \lambda' + \varepsilon]$ such that $Y_k\left(\hat{\lambda}, t\right) = Y_{k+1}\left(\hat{\lambda}, t\right)$. Consequently, the agents with skill $\hat{\lambda}$ are indifferent between region k and $k+1$, which is a contradiction.

Increasing Net Labor prices: Suppose to the contrary that there is an external equilibrium $\{t, P\}$ not satisfying INLP. Then, in equilibrium there are at least two regions k and ℓ, which mean skill levels satisfy that $\bar{\lambda}_k < \bar{\lambda}_\ell$, and where $w_k(1-t_k) > w_\ell(1-t_\ell)$. Consequently

$$\bar{\lambda}_k [w_k(1-t_k) - w_\ell(1-t_\ell)] < \bar{\lambda}_\ell [w_k(1-t_k) - w_\ell(1-t_\ell)]. \quad (7.4)$$

And since the agents with skill $\bar{\lambda}_\ell$ live in region ℓ, $Y_\ell\left(\bar{\lambda}_\ell, t\right) \geq Y_k\left(\bar{\lambda}_\ell, t\right)$ and so

$$\bar{\lambda}_\ell [w_k(1-t_k) - w_\ell(1-t_\ell)] + t_k \bar{w}_k - t_\ell \bar{w}_\ell \leq 0. \quad (7.5)$$

Hence, from (7.4) and (7.5) we deduce that

$$\bar{\lambda}_k \left[w_k \left(1-t_k\right)-w_\ell \left(1-t_\ell\right)\right]+t_k \bar{w}_k-t_\ell \bar{w}_\ell<0$$

i.e. $Y_k\left(\bar{\lambda}_k, t\right)<Y_\ell\left(\bar{\lambda}_k, t\right)$ which is a contradiction.
Let us second prove statement II):
Suppose that $\{t, P\}$ satisfies S, BI and INLP. By S the regions can be ordered by skill levels,and so, for $\lambda,\ \lambda',\ \lambda'' \in (0,1)$, the agents with skill in the interval $\left(\lambda, \hat{\lambda}\right]$ live in region k and the agents with skill in the interval $\left(\hat{\lambda}, \lambda'\right]$ live in region $k+1$ and so on. By BI,

$$w_k(1-t_k)\hat{\lambda}+t_k \bar{w}_k=w_{k+1}\left(1-t_{k+1}\right)\hat{\lambda}+t_{k+1} \bar{w}_{k+1}. \tag{7.6}$$

Rearranging terms

$$\hat{\lambda}\left[w_k(1-t_k)-w_{k+1}\left(1-t_{k+1}\right)\right]+t_k \bar{w}_k-t_{k+1} \bar{w}_{k+1}=0. \tag{7.7}$$

By INLP the term in brackets is negative or equal to zero. Consequently, since the skill of every agent $i \in N_k,\ \lambda_i$ is smaller or equal to $\hat{\lambda}$,

$$\lambda_i\left[w_k(1-t_k)-w_{k+1}\left(1-t_{k+1}\right)\right]+t_k \bar{w}_k-t_{k+1} \bar{w}_{k+1} \geq 0. \tag{7.8}$$

And since the skill of every agent $i \in N_{k+1},\ \lambda_i'$, is greater or equal to $\hat{\lambda}$,

$$\lambda_i'\left[w_k(1-t_k)-w_{k+1}\left(1-t_{k+1}\right)\right]+t_k \bar{w}_k-t_{k+1} \bar{w}_{k+1} \leq 0. \tag{7.9}$$

Clearly by (7.8) and (7.9) no agent in region k wants to migrate to region $k+1$ and vice versa.

Let $\hat{\lambda}'$ be the indifferent agent among region $k+1$ and $k+2$, from BI we deduce that

$$\hat{\lambda}'\left[w_{k+1}(1-t_{k+1})-w_{k+2}\left(1-t_{k+2}\right)\right]+t_{k+1} \bar{w}_{k+1}-t_{k+2} \bar{w}_{k+2}=0.$$

By INLP the term in brackets is negative or equal to zero. Consequently, every agent with skill strictly smaller than $\hat{\lambda}'$, prefers living in region $k+1$ than in region $k+2$, in particular, for the agent with skill $\hat{\lambda}$,

$$w_{k+1}(1-t_{k+1})\hat{\lambda}+t_{k+1} \bar{w}_{k+1} \geq w_{k+2}\left(1-t_{k+2}\right)\hat{\lambda}+t_{k+2} \bar{w}_{k+2}. \tag{7.10}$$

And so, from (7.6) and (7.10), we have that

$$\hat{\lambda}\left[w_k(1-t_k)-w_{k+2}\left(1-t_{k+2}\right)\right]+t_k\bar{w}_k-t_{k+2}\bar{w}_{k+2}\geq 0.$$

And since by INLP the term in brackets is negative, this inequality is also satisfied for every agent $i\in N_k$, with skill λ_i smaller or equal to $\hat{\lambda}$. And so, no agent in region k wants to migrate to region $k+2$. Following a similar argument it can be shown that the agents in region $k+2$ do not want to migrate to region k. Since it is satisfied for every two regions, we have that $\{t,P\}$ is an external equilibrium. **Q.E.D.**

Proof of Corollary 3.3:

a) Since we are in equilibrium, $Y_k(\lambda_i,t)\geq Y_{k+1}(\lambda_i,t)$ for all $i\in N_k$. In particular, this is true for the mean agent of N_k, $Y_k\left(\bar{\lambda}_k,t\right)\geq Y_{k+1}\left(\bar{\lambda}_k,t\right)$. Therefore, $w_k\bar{\lambda}_k\geq w_{k+1}\bar{\lambda}_k\left(1-t_{k+1}\right)+t_{k+1}w_{k+1}\bar{\lambda}_{k+1}$. Since $\bar{\lambda}_k<\bar{\lambda}_{k+1}$,

$$w_k\bar{\lambda}_k>w_{k+1}\bar{\lambda}_k\left(1-t_{k+1}\right)+t_{k+1}w_{k+1}\bar{\lambda}_k=w_{k+1}\bar{\lambda}_k$$

Consequently, $w_k>w_{k+1}.$

b) Since by a) $w_k>w_{k+1}$, we have that INLP implies that $(1-t_k)<(1-t_{k+1})$, and so $t_k>t_{k+1}$.

c) By BI, $\hat{\lambda}\left[w_k\left(1-t_k\right)-w_{k+1}\left(1-t_{k+1}\right)\right]=\bar{w}_{k+1}t_{k+1}-\bar{w}_kt_k$. Then, by INLP and since the regions are different, $w_k\left(1-t_k\right)-w_{k+1}\left(1-t_{k+1}\right)<0$ and so $\bar{w}_{k+1}t_{k+1}<\bar{w}_kt_k$.

d) By BI, $Y_k\left(\hat{\lambda},t\right)=Y_{k+1}\left(\hat{\lambda},t\right)$. Since f is strictly positive, we have that $\bar{\lambda}_k<\hat{\lambda}$, then, by INLP $Y_k\left(\bar{\lambda}_k,t\right)<Y_k\left(\hat{\lambda},t\right)$ where $Y_k\left(\bar{\lambda}_k,t\right)=w_k\bar{\lambda}_k$. And given that $\bar{\lambda}_{k+1}>\hat{\lambda}$, from a similar argument, $Y_{k+1}\left(\hat{\lambda},t\right)<Y_{k+1}\left(\bar{\lambda}_{k+1},t\right)$. So that

$$Y_k\left(\bar{\lambda}_k,t\right)<Y_k\left(\hat{\lambda},t\right)=Y_{k+1}\left(\hat{\lambda},t\right)<Y_{k+1}\left(\bar{\lambda}_{k+1},t\right),$$

which implies that $w_k\bar{\lambda}_k<w_{k+1}\bar{\lambda}_{k+1}$. **Q.E.D.**

Proof of Lemma 5.4

From problem (5.6), we have that

$$\frac{\partial Y_k(\lambda_k,t)}{\partial t_k}=\frac{\partial\Phi(t)}{\partial t_k}\left(\frac{dw_k(\Phi(t))}{d\Phi}\left(1-t_k\right)\lambda_k+\frac{d\bar{w}_k(\Phi(t))}{d\Phi}t_k\right)-w_k\lambda_k+\bar{w}_k\left(\Phi\left(t\right)\right),$$

let us analyze the sign of this derivative.

For region 1, the derivative (5.8) can be expressed in terms of elasticities:

$$\frac{d\bar{w}_1(\hat{\lambda})}{d\hat{\lambda}} = \frac{\bar{w}_1}{\hat{\lambda}}(\varepsilon_{w_1\hat{\lambda}} + \varepsilon_{\bar{\lambda}_1\hat{\lambda}}) \tag{7.11}$$

and then, $\varepsilon_{\bar{\lambda}_1\hat{\lambda}} \leq \left|\varepsilon_{w_1\hat{\lambda}}\right|$ implies that $\frac{d\bar{w}_1(\hat{\lambda})}{d\hat{\lambda}} < 0$. By (5.5)

$$\frac{\partial\Phi(t)}{\partial t_1} = -w_1(\bar{\lambda}_1 - \hat{\lambda})/\frac{\partial\phi(\hat{\lambda},t)}{\partial\hat{\lambda}}$$

and since $w_1\left(\bar{\lambda}_1 - \hat{\lambda}\right) < 0$ and calculating the derivative $\frac{\partial\phi(\hat{\lambda},t)}{\partial\hat{\lambda}}$

$$\frac{\partial\phi(\hat{\lambda},t)}{\partial\hat{\lambda}} = I + \hat{\lambda}\left(\underset{-}{\frac{dw_1(\hat{\lambda})}{d\hat{\lambda}}}(1-t_1) - \underset{+}{\frac{dw_2(\hat{\lambda})}{d\hat{\lambda}}}(1-t_2)\right) + \underset{-}{\frac{d\bar{w}_1(\hat{\lambda})}{d\hat{\lambda}}}t_1 - \underset{+}{\frac{d\bar{w}_2(\hat{\lambda})}{d\hat{\lambda}}}t_2 \tag{7.12}$$

where $I = w_1(1-t_1) - w_2(1-t_2)$ and by INLP we have that $I < 0$. Hence, we obtain that $\frac{\partial\phi(\hat{\lambda},t)}{\partial\hat{\lambda}} < 0$. Consequently, by (7.12), $\frac{\partial\Phi(t)}{\partial t_1} < 0$. Finally, since $\frac{dw_1(\hat{\lambda})}{d\hat{\lambda}} < 0$, $\frac{d\bar{w}_1(\hat{\lambda})}{d\hat{\lambda}} < 0$, $\frac{\partial\Phi(t)}{\partial t_1} < 0$ and by hypothesis $(-w_1\lambda_i + \bar{w}_1) \leq 0$ for all $\lambda_i \in \left[0, \bar{\lambda}_1\right]$, we have that by (5.7), the solution to the maximization problem is not interior, in particular $\frac{\partial Y_1(\lambda_i,t)}{\partial t_1} > 0$ for all $\lambda_i \in \left[0, \bar{\lambda}_1\right]$. Since $\lambda_1^m \in \left[0, \bar{\lambda}_1\right]$, we have that $t_1 = 1$ is a majority voting equilibrium.

In region 2, by (5.8) we have already shown that $\frac{d\bar{w}_2(\hat{\lambda})}{d\hat{\lambda}} > 0$. And by (5.5) we have that

$$\frac{\partial\Phi(t)}{\partial t_2} = w_2(\bar{\lambda}_2 - \hat{\lambda})/\frac{\partial\phi(\hat{\lambda},t)}{\partial\hat{\lambda}}$$

Since $(\bar{\lambda}_2 - \hat{\lambda}) > 0$ and $\frac{\partial\phi(\hat{\lambda},t)}{\partial\hat{\lambda}} < 0$, we obtain that $\frac{\partial\Phi(t)}{\partial t_2} < 0$. Furthermore, $(-w_2\lambda_i + \bar{w}_2) \leq 0$ for all $\lambda_i \in \left[\bar{\lambda}_2, 1\right]$, and so, from (5.7) we have that $\frac{\partial Y_2(\lambda_i,t)}{\partial t_2} < 0$ for all $\lambda_i \in \left[\bar{\lambda}_2, 1\right]$. Since $\lambda_2^m \in \left[\bar{\lambda}_2, 1\right]$, we have that $t_2 = 0$ is a majority voting equilibrium. **Q.E.D.**

Proof of Proposition 5.5

First, suppose that E satisfies i') and ii). Then, by Lemma 5.4, in equilibrium $t = (1, 0)$. By Lemma 5.1, we know that under Assumption 1, for all t, there exists a partition of the population P satisfying S and BI. In particular, it exists for

$t = (1,0)$. Furthermore, by Remark 1, if $\frac{\partial \phi(\hat{\lambda},t)}{\partial \hat{\lambda}} \neq 0$, there is a unique indifferent agent. Evaluating this partial derivative in $t = (1,0)$ we have that

$$\frac{\partial \phi(\hat{\lambda},t)}{\partial \hat{\lambda}} = -w_2 - \hat{\lambda} \underset{+}{\frac{\partial w_2(\hat{\lambda})}{\partial \hat{\lambda}}} + \frac{\partial \bar{w}_1(\hat{\lambda})}{\partial \hat{\lambda}}$$

and since $\varepsilon_{\bar{\lambda}_1 \hat{\lambda}} \leq \left| \varepsilon_{w_1 \hat{\lambda}} \right|$, it follows that $\frac{\partial \bar{w}_1(\hat{\lambda})}{\partial \hat{\lambda}} \leq 0$. Consequently, there is a unique indifferent agent $\hat{\lambda}$. Finally, since

$$w_1(1-t_1) = 0 \leq w_2(1-t_2) = w_2, \tag{7.13}$$

INLP holds and by Proposition 2.2, $\{(1,0), P\}$ is an external equilibrium. And, given that by Lemma 5.1 we know that $\hat{\lambda} \in (0,1)$, it follows that $w_2(\hat{\lambda}) > 0$ which implies that inequality (7.13) is strict and so, in equilibrium the regions are different.

Second, suppose that E satisfies i") and ii). Then, in equilibrium $t_2 = 0$, and by i"), we have that $\frac{\partial \bar{w}_1(\hat{\lambda})}{\partial \hat{\lambda}} > 0$, so that t_1 may not be equal to 1. Let $t = (t_1, 0)$, then, by Lemma 5.1, there exists a partition of the population P that satisfies S and BI. In particular, there is an indifferent agent with skill $\hat{\lambda}$ where

$$\hat{\lambda} \in (0,1) \text{ satisfyies that } \phi\left(\hat{\lambda},t\right) \equiv Y_1\left(\hat{\lambda},t\right) - Y_2\left(\hat{\lambda},t\right) = 0 \tag{7.14}$$

However, $\hat{\lambda}$ may not be unique. Let us next show that it can be defined a function which selects for each tax vector the skill of the indifferent agent in a continuous way. Substituting the tax vector $t = (t_1, 0)$ in (7.14) we have that

$$t_1 = \frac{\hat{\lambda}\left(w_2(\hat{\lambda}) - w_1(\hat{\lambda})\right)}{\bar{w}_1(\hat{\lambda}) - \hat{\lambda} w_1(\hat{\lambda})} \tag{7.15}$$

where t_1 is obtained as a continuous and differentiable function[11] of $\hat{\lambda}$. And given that T_1 is twice continuously differentiable and f is continuous, (7.15) is continuously differentiable. However, it can not be guaranteed that for every t_1 there is a unique $\hat{\lambda}$. Let us next show that for all $t_1 \in [0,1]$, there is always $\hat{\lambda} \in (0,1)$ satisfying (7.15). Note that when $t_1 = 1$, by (7.15) we have that

$$\bar{w}_1(\hat{\lambda}) - \hat{\lambda} w_2(\hat{\lambda}) = 0. \tag{7.16}$$

[11] Note that $\bar{w}_1(\hat{\lambda}) - \hat{\lambda} w_1(\hat{\lambda}) = \left(\bar{\lambda} - \hat{\lambda}\right) w_1(\hat{\lambda})$ and since f is strictly positive, we have that for every $\hat{\lambda} \neq 0$, the term $\bar{\lambda} - \hat{\lambda} \neq 0$. Consequently, the derivative of (7.15) is always well defined.

And when $t_1 = 0$, by (7.15) we have that

$$w_1(\hat{\lambda}) - w_2(\hat{\lambda}) = 0. \tag{7.17}$$

And since $w_k = T_k'(L_k)$, by Assumption 1, we have that $\lim_{\hat{\lambda}\to 0} w_1(\hat{\lambda}) = \infty$. Consequently, as $\hat{\lambda}$ tends to 0, we have that $\bar{w}_1(\hat{\lambda}) - \hat{\lambda}w_2(\hat{\lambda}) > 0$ and $w_1(\hat{\lambda}) - w_2(\hat{\lambda}) > 0$, and as $\hat{\lambda}$ tends to 1, $\bar{w}_1(\hat{\lambda}) - \hat{\lambda}w_2(\hat{\lambda}) < 0$ and $w_1(\hat{\lambda}) - w_2(\hat{\lambda}) < 0$. Consequently, there is some $\hat{\lambda} \in (0,1)$ such that equation (7.16) satisfies, and also some $\hat{\lambda} \in (0,1)$ such that (7.17) satisfies. Thus, by continuity of function (7.15), we deduce that for each $t_1 \in [0,1]$, there is $\hat{\lambda} \in (0,1)$ satisfying (7.15).

In particular, for each $t_1 \in [0,1]$ we are going to select a unique value of $\hat{\lambda}$ in a continuous and differentiable path that we denote by Φ. Hence, we have that $\Phi(t_1) = \hat{\lambda}$ and since (7.15) is continuously differentiable, we have that Φ is also continuously differentiable. Let us next show in Step 1 that an internal equilibrium exists and in Steps 2 to 5, let us show that the obtained internal equilibrium is also an external equilibrium with different regions.

Step 1: Since the agents are sophisticated, for an agent living in region 1, his most preferred[12] tax rate is given by

$$t_1 \in \arg\max_{t_1 \in [0,1]} \quad w_1(\Phi(t))\lambda_i(1-t_1) + \bar{w}_1(\Phi(t))t_1. \tag{7.18}$$

The solution to the above problem is either extreme ($t_1 = 1$) or interior. In the case that the solution is interior, since we have assumed that $\frac{\partial^2 Y_1(\lambda_i,t)}{\partial^2 t_1} < 0$, second order condition hold. And since Φ is continuously differentiable and T_1 is twice continuously differentiable and f is continuous, then, as λ_i varies, the solution to problem (7.18) also varies in a continuous way. Hence, we can define a continuous function that we denote by M, that indicates for each $\lambda_i \in [0,1]$ the tax rate t_1 obtained as a solution to (7.18).

Now, for each $\hat{\lambda} \in [0,1]$ we are going to consider that the skill of the median agent in region 1 is given by $m(\hat{\lambda}) = \lambda_1^m$. And by continuity of f, we have that m is a continuous function of $\hat{\lambda}$. Thus, consider now the function $M : [0,1] \to [0,1]$ where $M\left(m\left(\hat{\lambda}\right)\right) = t_1$. Then, by majority voting, an internal equilibrium requires that

$$\Phi\left(M\left(m(\hat{\lambda})\right)\right) = \hat{\lambda} \tag{7.19}$$

[12] We assume that at this point the agents do not consider restrictions of the kind: $\lambda_i \leq \Phi(t)$.

By continuity of m, M and Φ, we have that Φ is a continuous function in the interval $[0,1]$. And since $\Phi : [0,1] \to [0,1]$, by Brower's fixed point theorem there is $\hat{\lambda} \in [0,1]$ such that (7.19) is satisfied. Now, we only have to show that this $\hat{\lambda}$ is unique (which is shown at the end of the proof).

By (7.19) the equilibrium policy in region 1 is given by $t_1 = M\left(m(\hat{\lambda})\right)$, and so $t = (t_1, 0)$ is an internal equilibrium.

Step 2: Let us show that S and BI hold. By (7.14), $\hat{\lambda}$ is the skill of the indifferent agent, and consequently it can be defined a partition $P = \{N_1, N_2\}$ where $N_1 = \left\{i : \lambda_i \leq \hat{\lambda}\right\}$, $N_2 = \left\{i : \lambda_i > \hat{\lambda}\right\}$ that satisfies S and BI.

Step 3: Let us show that $t_1 \neq 0$. The partial derivative of $Y_1(\lambda_1^m, t)$ respect to t_1 is given by

$$\frac{\partial Y_1(\lambda_1^m,t)}{\partial t_1} = \underset{-}{\frac{\partial \Phi(t)}{\partial t_1}} \left(\underset{-}{\frac{dw_1(\Phi(t))}{d\Phi}} (1-t_1)\lambda_1^m + \underset{+}{\frac{d\bar{w}_1(\Phi(t))}{d\Phi}} t_1 \right) - w_1\lambda_1^m + \bar{w}_1(\Phi(t))$$

where by ii), $(-w_1\lambda_1^m + \bar{w}_1(\Phi(t))) > 0$. In the case that $t_1 = 0$, we have that $\frac{\partial Y_1(\lambda_1^m,t)}{\partial t_1} > 0$, and so $t_1 = 0$ can not maximize $Y_1(\lambda_1^m, t)$.

Step 4: Let us next show that INLP holds. By BI and substituting that $t_2 = 0$, we have that

$$w_1(1-t_1)\hat{\lambda} + \bar{w}_1 t_1 = w_2\hat{\lambda} \tag{7.20}$$

Since by Step 3 we know that $t_1 \neq 0$ and by (7.14) we have that $\hat{\lambda} \in (0,1)$, in equilibrium $\bar{w}_1 t_1 \neq 0$, consequently, by (7.20) we find that $w_1(1-t_1) < w_2$.

Step 5: Let us show that the regions are different. Since $t_2 = 0$, we have that $\bar{w}_2 t_2 = 0$, and so, that $\bar{w}_1 t_1 \neq 0$ guarantees that the regions are different.

Finally, by steps 1 to 5 we have shown that for each $\hat{\lambda}$ satisfying (7.19), it exists a policy-based equilibrium with different regions and then, by remark 1, we know that the derivative $\frac{\partial \phi}{\partial \hat{\lambda}}$ is different from zero, and so, $\hat{\lambda}$ is unique, which implies that the equilibrium is unique. **Q.E.D.**

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