DELEGATION AND ENDOGENOUS MERGERS IN OLIGOPOLY*

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A B S T R A C T

In this paper we consider the interactions between the use of strategic delegation and mergers in the context of a Cournot oligopoly with linear demand and cost functions. It is assumed that, after the merging process is completed, the owner of every independent firm decides its managerial incentive for his manager. In the context of endogenous mergers through acquisitions, we show that the incentive to merge, under delegation, is considerably increased with respect to the setting without delegation. In fact, we prove that the level of welfare in the setting with delegation is, in some cases, lower than the corresponding level under non delegation.

Keywords: Strategic Delegation; Endogenous Mergers; Cournot Oligopoly.

1. INTRODUCTION

In this paper, we deal with the interactions between the strategic use of delegation and the effects of mergers in oligopolistic markets. In the current literature about the profitability of mergers, it is usually assumed that the firms arising after the merging process compete directly in the product market. Under this assumption, Salant, Switzer and Reynolds (1983) have shown that merger is privately profitable only if a relatively high fraction of previously existing firms engage in the merging process. Specifically, they show that if demand and cost functions are linear, then an exogenous merger, followed by Cournot competition is only profitable if at least 80 percent of the firms engages in the merger. Nevertheless, some mergers have recently been observed in aeronautical and automobile industries which are not explained by this analysis[1]. González-Maestre et al. (1998) show that the required fraction of merging firms for a merger to be profitable, when delegation is considered, is substantially smaller than without delegation. This result, which agrees with the existence of mergers in industries with a small number of firms, suggests that delegation makes mergers more attractive for firms, relative to the no delegation case. This rises the issue of the competitive effects of endogenous mergers, since this incentive -might offset the more aggressive behavior of managers, associated to delegation and quantity competition. Regarding endogenous mergers, Kamien and Zang (1990) have analyzed a model without delegation where merger is carried out by means of acquisitions. Basically, they show that complete or partial monopolization, through acquisition, can only occur in industries with relatively few firms. In particular, with linear demand and cost functions, there is no merger in equilibrium, with more than two firms. This result does not agree with mergers observed in markets with a small number of firms.

The aim of this paper is to show that the implicit assumption of those authors that delegation by means of incentive contracts with managers is not feasible, is crucial in their results. We analyze a model with linear demand and cost functions where every firm, resulting from the previous merger process, delegates the output decision on its manager by means of a reward scheme which is a linear combination of revenues and profits. Thus, our model extends also the previous analysis by Fershtman and Judd (1987), Sklivas (1987) and Vickers (1985) to the case in which mergers are allowed. In a different approach to this issue, Faulí-Oller and Motta (1996), have analyzed a model with three initial firms, where only the owner of one firm can delegate both output and takeovers decisions on his manager, in

[1] In the aeronautical industry, McDonell-Douglas and Boeing have merged in a market with basically three firms (the other is Airbus). In the automobile indistry, Rolls Roice has been adquired by Wolksvagen, and Chirsler and Daimler-Benz have merged.

a linear Cournot framework. Their main conclusion is that, if the owner only knows the probability distribution of the production cost, then the equilibrium involves, in some cases, unprofitable acquisitions by the manager. In contrast, the basic consequence of our model is that delegation enhances the profitability of merger. In the setting of endogenous mergers through acquisitions, the incentive to merge is considerably greater in our model, compared with the model with no delegation. In fact, we show the following. First, complete monopolization arises as equilibrium with less than four firms, while in the model without delegation it only happens with two firms. Second, the duopoly is the only undominated equilibrium, with an initial number of firms greater than 3 and smaller than 7, while in the non-delegation game, merger is always impossible in equilibrium with more than two firms. Third, with 7 or more firms, no matter if delegation is considered or not, no merger will occur in equilibrium.

We also consider the welfare and competitive implications of our model. Relative to the model with no delegation, the equilibrium outcomes in our model are more competitive and involve more welfare if the initial number of firms is equal or greater than 7, since in both models the initial number of firms remains unchanged, but delegation makes firms more aggressive. With 2 initial firms, the equilibrium implies the same welfare in both models, since monopolization arises in the two cases. With 4 initial firms the welfare is also the same, since the higher equilibrium number of firms in the non-delegation game (4 instead of 2) is offset by the more aggressive managerial behavior of the resulting duopoly in the delegation game. However, with 3, 5 or 6 initial firms, the equilibrium outcome involves higher competition and welfare in the non-delegation model. The intuition for this result, relies on the fact that, although delegation makes managers more aggressive, it also increases the attractiveness of merger.

Regarding the dynamic implications of our model, it predicts that, with a sufficiently large number of initial firms, the market will remain unchanged over time in both models. However, in the delegation model the market structure moves quickly to a complete monopolization with less than 7 initial firms, while under no-delegation the number of firms is unchanged with more than two initial firms.

Our analysis is also extended to the case in which, after merger, the owner of each group of firms can operate independently more than one firm in the group (we will refer to this situation as the decentralized game). In this case, we show that in our delegation game each final owner has no incentive in operate more than one firm. However, in the non-delegation game, there is an incentive to operate more than one firm, as shown by Kamien and Zang (1990). We show that, with linear demand function, the non-delegation game gives a final monopoly with 4 of less initial firms, while for the rest of the cases, the initial number of active firms is reduced by one if the initial number of firms is even, and by two if this initial number is odd. Thus, the decentralized non-delegation game implies, in most cases, lower number of final active firms than our delegation game, contrary to what happens in the centralized setup.

The rest of the paper is organized as follows. In Section 2, we describe the model; Section 3 studies the game with endogenous mergers in the centralized case; Section 4 analyzes the decentralized game; and Section 5 gathers our conclusions.

2. THE MODEL

In this Section, we describe the assumptions and structure of our model. We assume that n initial owners of identical firms of a homogeneous good industry can engage in mergers through acquisitions. The inverse demand and cost functions of each firm are given , respectively, by $p(x) = a - x$, and $C(x_i) = cx_i$, where x is the total output, x_i is the output of firm i and $a > c > 0$.

The interactions among owners and managers is given by the following game:

Stage 1: Each owner $j \in N = \{1, 2, ..., n\}$ simultaneously announces a vector $B^j = (B_1^j, B_2^j ..., B_n^j) \in R^n$ of bids, where B_i^j is the owner j's bid for firm $i \neq j$, and B_j^j is the owner j's bid or asking price for his own firm. The allocation of firms to owners is assumed to be the following: a firm is owned by the owner offering the highest bid. In the case of tie among a buyer and a seller, then the transaction occurs. If the tie occurs among buyers, then the one with the lowest index takes the firm.

Let us define k_j and m, respectively, as the number of firms owned by owner j and the total number of active firms, after all mergers are completed.

Stage 2: The owner of every active firm, say i, resulting from the previous stage, decides, simultaneously, the incentive scheme with his unique manager. We assume that this incentive scheme is of the form $R_i = \pi_i + \lambda_i x_i$, where π_i is the profit of firm i and λ_i, the incentive variable decided by owner i. This contract can be shown to be equivalent to $R_i = (1 - \delta_i)\pi_i + \delta_i p x_i$, where δ_i satisfies $\lambda_i = c\delta_i$. Thus, our approach is equivalent to the assumption that the incentive scheme is a weighted average between profits and revenues (see Fershtman and Judd (1987), Vickers (1985) and Sklivas (1987)). We will keep our notation for simplicity.

Stage 3: Every manager decides, simultaneously, his output.

We assume, that every initial owner maximizes his payoff, given by his operating profit, less his payments for the firms he purchased, plus the payment he received for his firm if it was sold. Each manager is assumed to maximize his incentive scheme.

This game is an extension of the centralized version of the game considered by Kamien and Zang (1990), but with an intermediate stage where the managerial incentives are chosen, instead of assuming direct Cournot competition among the owners. In the rest of the paper, we will call our game the delegation game, while the model by Kamien and Zang will be referred as the non-delegation game.

Our delegation game is also an extension of the one considered by Fershtman and Judd (1987), Sklivas (1987) and Vickers (1985) but with the previous merger stage. The following auxiliary result characterizes the Subgame Perfect Equilibrium (SPE) of the subgame consisting in the two last stages of the previous delegation game.

Lemma 1. *In the SPE of the subgame given by Stages 2 and 3 of the delegation game, the total production, the operating profits obtained by each active firm and the delegation variable chosen by each firm are, respectively,* $x(m) = \frac{(a-c)m^2}{m^2+1}$, $\pi(m) = \frac{(a-c)^2 m}{(m^2+1)^2}$ *and* $\lambda(n) = \frac{(a-c)(n-1)}{n^2+1}$.

Proof. See Sklivas (1987). ■

According to Lemma 1, the use of managerial contracts makes the equilibrium more competitive, relative to the non-delegation game, if merger is ignored, since $\lambda(n) > 0$. However, we will show that if mergers are allowed, then this is not always true.

To save notation, in the rest of the paper we will assume, without loss of generality, $a - c = 1$.

Before analyzing the SPE of our delegation model, let us comment on the profitability of an exogenous merger that takes place before the Stage 2 of the game, instead of Stage 1. González-Maestre et al. (1998) analyze the profitability of an exogenous merger in a context whit delegation and compare their results with a similar approach, undertaken by Salant et al. (1983), in the context of linear Cournot oligopoly, but with no delegation. For an exogenous merger resulting in m final independent firms, let $\alpha = (n - m + 1)/n$ be the fraction of insiders, relative to the initial number of firms. The change in total profits by the insiders as a result of the merger is given by $f(n, \alpha) = \pi(n(1-\alpha)+1) - \alpha n \pi(n)$. González-Maestre et al. (1998) prove that , for any initial number $n \geq 3$ of firms, the merger is profitable ($f(n, \alpha) > 0$) if and only if $\alpha > \alpha(n)$, where $\alpha(\cdot)$ is a continuously differentiable function that strictly increases with n. The values summarized in the following table show that the required proportion for an exogenous merger to be profitable, is remarkably lower, compared with the non-delegation model of Salant et al. (1983).

n	2	3	4	5	6	7	8	9	10	50	100	500	1000
$\alpha(n)\%$	100	42	43	45	47	49	51	52	54	72	78	87	90

This suggest that delegation makes mergers more attractive for the firms, relative to the context where delegation is not considered and the issue of the competitive effects of endogenous mergers arises, since this incentive might offset the more aggressive behavior of managers, associated to delegation and quantity competition. The following section will confirm this conjecture, for some cases, by considering the endogenous incentive to merge by means of acquisitions.

3. THE ANALYSIS OF ENDOGENOUS MERGERS IN THE CENTRALIZED GAME

Let us define a merged subgame perfect equilibrium (SPE), as a SPE in which at least one owner owns more than one firm. We will consider, first, a necessary conditions for the existence of this merged SPE. If an owner has $k \geq 2$ firms in a SPE, then the following inequality must hold:

$$\pi(m) - (k-1)\pi(m+1) \geq \pi(m+k-1),$$

where m is the number of active firms in the merged equilibrium. That is, the payoff of the considered owner must be greater than what he can obtain if he decides to own just one firm, taking into account that $\pi(m+1)$ is the payment that must be made by this owner to any acquired firm. This condition is also discussed by Kamien and Zang (1990) in their analysis of the equilibrium in the non-delegation game. In our model, the above condition can be rewritten as

$$\begin{aligned} D(m,k) &\equiv \pi(m) - (k-1)\pi(m+1) - \pi(m+k-1) \\ &\equiv \frac{m}{(1+m^2)^2} - (k-1)\frac{m}{(1+m^2)^2} - \frac{m+k-1}{(1+(m+k)^2)^2} \geq 0, \;\; k \geq 2. \end{aligned}$$

from which we can get the following auxiliary result:

Lemma 2. *If $m \geq 2$, then $D(m,k)$ is strictly decreasing in k for any $k \geq 2$.*

Proof. See appendix A.1. ■

The following result characterizes the combination of values for m and n which are inconsistent with the necessary condition for SPE, given by $D(m,k) \geq 0$.

Proposition 1. *The following cases are inconsistent with a merged SPE in the delegation game:*
i) $m \geq 4$, $n \geq 1$,
ii) $m = 3$, $n \geq 7$,
iii) $m = 2$, $n \geq 7$,
iv) $m = 1$, $n \geq 4$.

Proof. See appendix A.2. ■

In other words, Proposition 1 says that a merged equilibrium with more than three final firms is always impossible (part (i)); a merged equilibrium yielding a duopoly or three firms is inconsistent with more than seven initial firms (parts (ii) and (iii)) and, finally, complete monopolization is impossible with four or more initial firms (part (iv)).

However, the following result shows that the rest of the logical configurations (a monopoly when the initial number of firms is 2 or 3, a duopoly when it is 3, 4, 5 or 6 and a three-firm oligopoly when it is 4, 5 or 6), actually can be supported as a SPE of our delegation game. In the proof we show that a given feasible configuration may be supported as SPE with the following set of bids. Each owner that finally possesses only one firm, demands a very high asking price and offers a very low bid for rival firms. Each owner that finally has two or more firms, demands a very high asking price, offers a bid equal to $\pi(m+1)$ for the firms that he will buy and offers a very low bid for the other firms. Each owner that finally possesses no firm, demands an asking price equal to $\pi(m+1)$ and offers a very low bid for rival firms.

Proposition 2. *The delegation game satisfies the following properties:*
i) Complete monopolization is a merged SPE only with 2 or 3 initial firms.
ii) The duopoly is a merged SPE if, and only if, $n \in \{3,4,5,6\}$.
iii) The three-firm oligopoly is a merged SPE only if $n \in \{4,5,6\}$.

Proof. See appendix A.3. ■

Note that, according to Proposition 2, there are multiple merged equilibria with 3 initial firms (in this case both, the duopoly and the monopoly are SPE), and with 4, 5 or 6 initial firms (in these cases the SPE might be either a duopoly or a market with three active firms). Moreover, it is easy to show that the absence of mergers is always a SPE in our model, which can be supported by a combination of bids in which the selling price by each owner is sufficiently high.

In order to be able to compare our results with the case in which delegation is not possible, let us assume that in our game, the players always choose a SPE which is undominated (i.e.: such that there is no other SPE with higher payoffs for all players). Let us denote by USPE those type of equilibria. Although there are some cases with many USPE, it is easy to show that all of them are equivalent in terms of the final structure of the market. The following result characterizes this USPE of the delegation game.

Proposition 3. *The delegation game satisfies the following properties:*

i) with 7 or more initial firms, the unique SPE implies no merger at all;

ii) if $n \in \{4, 5, 6\}$, each USPE implies a duopoly.

iii) with less than 4 firms, each USPE implies complete monopolization.

Proof. See appendix A.4. ■

Let us compare the SPE of our delegation game with the non-delegation game where firms compete directly a la Cournot. That is, the non-delegation game is the same as described in Section 2, but without the delegation stage. Standard calculations show the following

Remark 1. *With our linear demand and cost assumptions, the Cournot equilibrium with m firms and no delegation yields the following values for the total production and profits per firm, respectively:* $x_c(m) = \frac{m}{1+m}$, $\pi_c(m) = \frac{1}{(1+m)^2}$.

Hence, in the game where the owners compete directly a la Cournot, after the merger stage, the SPE of is characterized by the following

Lemma 3. *In the non-delegation game, the SPE implies the following:*

i) no merger occurs with more than two initial firms.

ii) with two firms, the complete monopolization is the unique USPE.

Proof. See appendix A.5. ■

Note that the above result characterizes the SPE of the game analyzed by Kamien and Zang (1991), but restricted to the linear demand case.

Let us denote by m_d and x_d, respectively, the number of active firms and the total production in the USPE of the delegation game. Similarly, we will denote by m_c and x_c, the number of active firms and total production in the USPE of the game where the active firms compete directly a la Cournot, with no delegation. The following result gives the comparisons between the number of firms and total production in both games.

Proposition 4. *The following properties hold, relative to the USPE of the delegation and non-delegation games:*

i) if $n \leq 2$, then $m_d = m_c = 1$ and $x_d = x_c$;

ii) if $n = 3$, then $m_d = 1$, $m_c = 3$ and $x_d < x_c$;

iii) if $n = 4$, then $m_d = 2$, $m_c = 4$ and $x_d = x_c$;

iv) if $n \in \{5, 6\}$, then $m_d = 2$, $m_c = n$ and $x_d < x_c$;

v) if $n \geq 7$, then $m_d = m_c = n$ and $x_d > x_c$.

Proof. See appendix A.6. ■

Given that we assume constant average costs and identical firms, it follows that the level of welfare, measured as the sum of profits plus consumer surplus, has the same properties as the level of output, relative to comparisons between the USPE of both games. Thus, according to Proposition 4, the level of competition and welfare in both games is the same if the initial number of firms is 2 or 4, but it is different in the rest of the cases. With 2 initial firms the explanation is obvious: monopolization is profitable for both firms in both models. With 4 firms, it happens that the higher aggressiveness induced by delegation is offset by the lower number of active firms (in the delegation game there are two mergers, yielding a final duopoly, while in the non-delegation game there is no incentive to merger). Obviously, with 7 or more initial firms, welfare and competition is higher under delegation, since in neither of both games there is a merged equilibrium but delegation makes the market more competitive. However, with 3, 5 or 6 initial owners, the result is reversed: competition and welfare is higher under non-delegation. The explanation is that delegation involves a higher incentive to merge, which results in complete monopolization with 3 initial firms, and duopoly with 5 or 6 firms, while in the case of non-delegation there is no merger at all.

Finally, let us investigate the dynamic implications of our model, by assuming that the delegation-game is repeated at each period $t = 1, 2, 3, ..., T$, with the number of firms n_{t-1} arising as USPE at period $t-1$. To simplify the analysis, we also assume that each owner only lives one period and has a unique heir who lives the next period. Let us define n_0 as the initial number of firms, before period 1. The following result characterizes the evolution of the number of firms in our delegation game:

Corollary 1. *Assume the delegation game is repeated several periods, and that each owner only lives one period. The following properties hold.*

i) If $n_0 \geq 7$, then $n_t = n_0$ for any $t \geq 1$.

ii) If $n_0 \in \{4, 5, 6\}$, then $n_1 = 2$ and $n_t = 1$ for any $t \geq 2$.

iii) If $n_0 \leq 3$, then $n_t = 1$ for any $t \geq 1$.

Proof. It follows directly from applying Proposition 3 at each period. ■

A remarkable conclusion from Corollary 1 is that with less than seven initial firms, the market always evolves to a monopoly in two or less periods. This is in contrast with the non-delegation game, where the incentive to merge only appears with two firms (see Lemma 3). Thus, the dynamic considerations in our model reinforce the anticompetitive effect of delegation, compared with the model of no delegation.

4. THE ANALYSIS OF ENDOGENOUS MERGERS IN THE DECENTRALIZED GAME

In this section, we extend the previous analysis to the case in which the owner of a group of firms can activate independently some or all the firms in his group. Specifically, let us assume the game described in the previous section, but with an intermediate stage (say Stage F), between Stage 1 and Stage 2, at which each owner decides the number of firms, in his group, to be activated independently, while at Stage 2 each owner decides, simultaneously, the incentive contract for the manager of each active firm.

The following result characterizes the SPE of this decentralized delegation game:

Lemma 4. *In the decentralized delegation game, the dominant strategy of each owner with one or more firms after the acquisition stage is to activate at most one firm.*

Proof. See appendix A.7. ■

In other words, the previous result shows that the decentralized delegation game is equivalent to the centralized delegation game considered in the previous Section, since each owner has no incentive to activate independently more than one firm. However, as shown by Kamien and Zang (1990), if delegation is not an available instrument, then there is an incentive to decentralize decisions by owners with several firms. In order to compare the results between our delegation game and the decentralized non-delegation, the following results characterize the properties of the decentralized non-delegation game with linear demand function. Specifically, we investigate the number of active firms in the merged SPE of the decentralized game where, at stage 1 owners engage in acquisitions, at stage 2 owners with one or more firms decide, simultaneously the number of active firms, and at stage 3 the active firms compete, independently a la Cournot. By a merged SPE we mean a SPE where the number of active firms is reduced, relative to the initial number of firms.

Lemma 5. *If there is a merged SPE to the decentralized non-delegation game, then the owner possessing the maximal number* k^**of firms, operates* $r(k^*) < k^*$*firms, and all other firms are operated by their owners. Moreover*[2]

i) When $n \neq 4$, *we have* $k^* = \left[\frac{n+1}{2}\right] + 1$, $r(k^*) = \left[\frac{n}{2}\right]$ *and the number of active firms is* $m_{cd} = 2\left[\frac{n}{2}\right] - 1$.

ii) When $n = 4$, *we have* $k^* = 3$, $r(k^*) = 2$, $m_{cd} = 3$ *or* $k^* = 4$, $r(k^*) = 1$, $m_{cd} = 1$.

Proof. See appendix A.8. ■

Lemma 6. *There exists merged SPE to the decentralized non-delegation game. Moreover, the following properties hold:*

i) If $n \neq 4$, *then in any merged SPE the number of active firms is reduced by one if* n *is even and by two if* n *is odd.*

ii) If $n = 4$, *then in any merged SPE the number of active firms is 3 or 1. The merged SPE, which implies complete monopolization, is the only USPE.*

Proof. See appendix A.9. ■

The comparisons between the number of active firms and production at the USPE of the delegation game and the decentralized non-delegation game are summarized in the following result, where subindex *cd* stands for the variables in the Cournot decentralized game, while *d* stands for the variables in the delegation game (centralized of decentralized, since both are equivalent).

Proposition 5. *The following properties hold, relative to the USPE of the delegation game and the decentralized non-delegation game:*

i) if $n \leq 3$, *then* $m_{cd} = m_d = 1$, *and* $x_{cd} = x_d$;

ii) if $n = 4$, *then* $m_{cd} = 1 < m_d = 2$, *and* $x_{cd} < x_d$;

iii) if $n = 5$,*then* $m_{cd} = 3 > m_d = 2$, *and* $x_{cd} = 3/4 < x_d = 4/5$;

iv) if $n = 6$, *then* $m_{cd} = 5 > m_d = 2$, *and* $x_{cd} = 5/6 > x_d = 4/5$;

v) if $n \geq 7$, *then* $m_{cd} = n - 1 < m_d = n$ *and* $x_{cd} < x_d$, *if* n *is even; while* $m_{cd} = n - 2 < m_d = n$ *and* $x_{cd} < x_d$, *if* n *is odd.*

Proof. See appendix A.10. ■

According to the previous result, contrary to what happened in the centralized context, if decentralization is a feasible commitment by each owner, then the active number of firms at the USPE in the non-delegation game is, in many cases, smaller than in the delegation game (see cases ii) and v) in Proposition 5). Moreover, only when there are six initial firms, the welfare and production with non-delegation is greater than with delegation (see part iv of the Proposition).

[2] $[x]$ denotes the highest integer lower than the real number x.

5. CONCLUSIONS

We have considered the effect of strategic delegation on the incentive to merger, in the context of a market where identical firms, with constant returns to scale, compete in the market of a homogeneous good. Our main conclusion is that, compared with the standard Cournot framework, where firms compete directly in quantities, the possibility of strategic delegation enhances substantially the incentive to merge. In particular, it is shown that with less than seven firms, there is always an incentive to merge. Moreover, in these cases the equilibrium evolves very quickly to complete monopolization, while in the standard Cournot model with no delegation, the incentive to merger only happens with two initial firms. As a consequence, the role for antitrust authorities is reinforced in the case of delegation, despite of the fact that, with quantity competition delegation enhances competition with a given number of firms. The basic explanation of this conclusion relies on the fact that delegation increases the incentive to merge.

Nevertheless, we have also shown that if the owners has the possibility of credibly decentralize output decisions after merger, then the above conclusions are substantially weakened in two main aspects: first, there are cases where the final number of active firms is greater under delegation, and, second, in most cases the welfare is greater under delegation.

A. APPENDIXES

A.1. Proof of Lemma 2.

It follows from $\partial_k D(m,2) = -\frac{m(m^2-2)}{(2+2m+m^2)^3} < 0 \ \forall \ m \geq 2$ and the fact that $\partial_{kk} D(m,k) = -\frac{12(k+m-2)(k+m-1)(k+m)}{(2-2k+k^2-2m+2km+m^2)^4} \leq 0 \ \forall \ m \geq 1, \ \forall \ k \geq 2$. ■

A.2. Proof of Proposition 1.

Part (i): follows from $D(m,2) = -\frac{2-2m-4m^2-4m^3-2m^4+m^5}{(1+m^2)^2(2+2m+m^2)^2} < 0 \ \forall m \geq 4$ and Lemma 2.

Part (ii): If $m = 3$ and $n \geq 7$ then there is an owner with $k \geq n/3 \geq 3$ firms, but $D(3,3) < 0$ and the result follows from Lemma 2.

Part (iii): If $m = 2$ and $n \geq 7$ then there is an owner with $k \geq n/2 \geq 4$ firms, but $D(2,4) < 0$ and the result follows from Lemma 2.

Part (iv): If $m = 1$, then there is an unique owner with $k = n$ firms, but, for all $k \geq 4$, we have $D(1,k) = -\frac{(k-1)(33-75k-9k^2-25k^3+8k^4)}{100(1+k^2)} < 0$, which completes the proof. ■

A.3. Proof of Proposition 2.

Consider a feasible configuration (m, n) in a merged SPE. Therefore, in that hypothetical merged SPE, there are $m < n$ owners possessing at least one firm. Without loss of generality, assume that these owners are $M = \{1, ..., m\}$. Let G_i be the group of firms owned by owner $i \in M$ after the merger process. Then, $\cup G_i = \eta$ and $k_1 + ... + k_m = n$ where $k_i = \#G_i$ for $i \in M$ and η is the set of all the initial firms. The necessary conditions for merged SPE reads $D(m, k_i) \geq 0$ if $k_i \geq 2$, $i \in M$. From Proposition 1, this condition implies that the feasible values $k_i \geq 2$ for a merged SPE are given in the following table:

$m \setminus n$	2	3	4	5	6
1	2	3	-	-	-
2	-	2	2 or 3	2 or 3	2 or 3
3	-	-	2	2	2

Besides, $D(1, 2) > 0$, $D(1, 3) > 0$, $D(2, 2) > 0$, $D(2, 3) > 0$ and $D(3, 2) > 0$. So, $D(m, k_i) > 0$ holds for the above cases. Note that, without loss of generality, we can suppose that each owner in M has not sold his proper firm and each owner in $N - M$ has bought no firm. Therefore, without loss of generality, we can divide de initial set of owners N into three disjoint parts $N_0 = N - M$, N_1, N_2, such that $N_1 \cup N_2 = M$ and $N_1 = \{i \in M \mid k_i = 1\}$, $N_2 = \{i \in M \mid k_i \geq 2\}$. Also, let us define η_0, η_1, η_2 and μ, respectively, as the set of firms initially owned by the owners in N_0, N_1, N_2 and M.

Consider now the following set of bids. For $i \in N_1$, $B_i^i = \alpha$ and $B_j^i = \beta$ if $j \in \eta - \{i\}$. For $i \in N_2$, $B_i^i = \alpha$, $B_j^i = \pi(m+1)$ if $j \in G_i$ and $B_j^i = \beta$ if $j \in \eta - G_i$. For $i \in N_0$, $B_i^i = \pi(m + 1)$ and $B_j^i = \beta$ if $j \in \eta - \{i\}$. The logic of these bids is the following. The owners in M, which has bought no firm, demands a very high asking price α and offers a very low bid $\beta \leq 0$ for rival firms. The owners in M, which has bought some firm, demands the asking price α, offers a bid equal to $\pi(m + 1)$ for the firms that they will buy and offers the bid β for the other firms. The owners in N_0 demands an asking price equal to $\pi(m+1)$ and offers the bid β for rival firms. We will show that these set of bids supports the above structure of acquisitions as SPE.

Obviously, in the subgame given by Stages 2 and 3, the operating profits for owners in M are equal to $\pi(m)$. The owners in N_0 does not produce. Consider a deviation in respect of the above structure of acquisitions. If an owner in N_0 deviates by buying some firm in μ, he will pay at least α; if he increases his asking price to result non purchased, he will get $\pi(m + 1)$; and if he buys some firm in η_0, he will obtain less than $\pi(m+1)$. Nevertheless, he obtain $u_0 = \pi(m+1)$ when his firm is purchased. If an owner in N_1 deviates by decreasing his asking price to result purchased, he will obtain β; if he buys some firm in μ, he will pay at least α; and if he buys some firm in η_0, he will get less than $\pi(m)$. Nevertheless, he obtains $u_1 = \pi(m)$ when he buys no firm and he is not purchased. Consider now an owner $i \in N_2$. If he decreases his asking price to be purchased, insisting on buying $k_i - 1$ firms, he will obtain less than or equal to $u_2 = \pi(m) - (k_i - 1)\pi(m+1)$, which is what he obtains with no deviation. If he buys $t_i - 1 < k_i - 1$ firms, with $1 \leq t_i < k_i$, and keeps his proper firm, he will obtain $\pi(m + k_i - t_i) - (t_i - 1)\pi(m+1)$, which is lower than u_2 for all the values for m, k_i, and t_i consistent with the above table (note that if $k_i = 2$, and/or $t_i = 1$ this property follows from $D(m, k_i) > 0$, while in the remaining two cases, in which $k_i = 3$ and $t_i = 2$ with $m = 2$ or $m = 1$, he will obtain 0 if he deviates). For any combination of previous deviations he will obtain less than u_2. This proves that, for a sufficiently high value of α and $\beta \leq 0$, the set of bids support the above structure of acquisitions as SPE, the equilibrium payoffs are u_0, u_1, and u_2 for owners in N_0, N_1 and N_2 respectively, and the possible equilibrium number of firms owned by owners with at least two firms are given in the above table. ■

A.4. Proof of Proposition 3.

Part (i) follows from Proposition 1. For the rest of cases, we will follow the notation in the proof of Proposition 2.

Case $n = 6$: Proposition 2 implies that the payoffs in a merged SPE with $m = 3$ are equal, w.l.o.g., to $v_1 = v_2 = v_3 = \pi(3) - \pi(4) = 0.0162$ and $v_4 = v_5 = v_6 = \pi(4) = 0.0138$. But, there is a merged SPE with $m = 2$ and payoffs $v_1' = v_2' = \pi(2) - 2\pi(3) = 0.02$, $v_3' = v_4' = v_5' = v_6' = \pi(3) = 0.03$, which dominates the former merged SPE.

Case $n = 5$: Proposition 2 implies that the payoffs in a merged SPE with $m = 3$ are equal, w.l.o.g., to $v_1 = v_2 = \pi(3) - \pi(4) = 0.0162$, $v_3 = \pi(3) = 0.03$ and $v_4 = v_5 = \pi(4) = 0.0138$. But, there is a merged SPE with $m = 2$ and payoffs $v_1' = \pi(2) - 2\pi(3) = 0.02$, $v_2' = \pi(2) - \pi(3) = 0.05$, $v_3' = v_4' = v_5' = \pi(3) = 0.03$, which dominates the former merged SPE.

Case $n = 4$: Proposition 2 implies that the payoffs in a merged SPE with $m = 3$ are equal, w.l.o.g., to $v_1 = \pi(3) - \pi(4) = 0.0162$, $v_2 = v_3 = \pi(3) = 0.03$ and $v_4 = \pi(4) = 0.0138$. But, there is a merged SPE with $m = 2$ and payoffs $v_1' = v_2' = \pi(2) - \pi(3) = 0.05$, $v_3' = v_4' = \pi(3) = 0.03$, which dominates the former merged SPE.

Case $n = 3$: Proposition 2 implies that the payoffs in a merged SPE with $m = 2$ are equal, w.l.o.g., to $v_1 = \pi(2) - \pi(3) = 0.05$, $v_2 = \pi(2) = 0.08$ and $v_3 = \pi(3) = 0.03$. But, there is a merged SPE with $m = 1$ and payoffs $v_1' = \pi(1) - 2\pi(2) = 0.09$, $v_2' = v_3' = \pi(2) = 0.08$, which dominates the former merged

SPE.

Case $n = 2$: The payoffs in the duopoly are $v_1 = v_2 = \pi(2) = 0.08$. But, from Proposition 2, there is a merged SPE with $m = 1$ and payoffs $v_1' = \pi(1) - \pi(2) = 0.17$, $v_2' = \pi(2) = 0.08$, which dominates the former merged SPE.

This proves Parts (ii) and (iii). ■

A.5. Proof of Lemma 3.

Without delegation, $D_c(m,k) = \pi_c(m) - (k-1)\pi_c(m+1) - \pi_c(m+k-1) \geq 0$ is a necessary condition for the existence of a merged SPE, where $m < n$ is the number of active firms and $k \geq 2$ is the number of firms owned by an owner that possesses at least two firm in a merged SPE for n initial firms (see the discussion at the beginning of this section). Since $\partial_{kk} D_c(m,k) = -6/(k+m)^4 < 0$ and $\partial_k D_c(m,2) = -m/(2+m)^3 < 0$ then, for all $m \geq 1$, $D_c(m,k)$ is strictly decreasing in k for $k \geq 2$. If $m \geq 2$ then $D_c(m,2) = -\frac{m^2-2}{(1+m)^2(2+m)^2} < 0$. So, $D_c(m,k) < 0$ for all $m \geq 2$ and all $k \geq 2$. This implies that only the complete monopolization may be a merged SPE. When there are $n \geq 3$ initial firms, a merged SPE will imply $m = 1$, but then $k = n \geq 3$ and $D_c(1,k) \leq D(1,3) < 0$ which is a contradiction. Therefore Part (i) holds. When there are $n = 2$ initial firms, the payoffs in the duopoly SPE are $v_1 = v_2 = \pi_c(2) = 0.1111$. Arguments like those employed in the proof of Proposition 2 imply that this equilibrium is dominated by the monopoly SPE with payoffs $v_1 = \pi_c(1) - \pi_c(2) = 0.1389$ and $v_2 = \pi_c(2)$. This proves Part (ii). ■

A.6. Proof of Proposition 4.

The properties about the equilibrium levels for the number of firms, come from Proposition 3 and Lemma 3. The properties about the level of production come from substitution of the relevant values for m_d and m_c, respectively, in $x_d = m_d^2/(1+m_d^2)$ and $x_c = m_c/(1+m_c)$ (see Lemma 1 and Remark 2). ■

A.7. Proof of Lemma 4.

Let us define k as the number of final owners with one or more firms, m_j as the number of active firms of owner j and $m \equiv \sum m_j$, $j = 1,..,k$. At Stage 3, the manager of each firm, say i, of owner j, maximizes

$$g_i = \pi_i + \lambda_j x_i = (a-c-x)x_i + \lambda_j x_i = (a-c)x_i - x_i^2 - x_i x_{-i} + \lambda_j x_i,$$

where λ_j is the common delegation variable for the managers of the same owner j, while x_{-i} stands for the output of other firms different from i. Without loss of generality, let us assume that $a - c = 1$. The first order conditions of Stage 3 give, after some standard calculations, $x = \frac{m+S}{m+1}$, $x_i = \frac{1-S+(m+1)\lambda_j}{m+1}$ and $p = \frac{1-S}{(m+1)} + c$, where $S \equiv \sum \lambda_j m_j$, $j = 1, 2, ..., k$. Thus, operative profits of owner j, at Stage 2 are given by $\pi_j = m_j \frac{1-S+(m+1)\lambda_j}{(m+1)^2}(1-S)$. The first order conditions of Stage 2 give

$$-m_j(1 - S + (m+1)\lambda_j) + (1 - S)(m - m_j + 1) = 0, \; j = 1, ..., k.$$

By adding from 1 to k in the above equations we obtain $S = \frac{(k-2)m+k}{(k-1)m+k+1}$, which substituted above gives the following expression for the total operative profits of owner j at Stage F of the decentralized game:

$$\pi_j(m_j, m_{-j}) = \frac{m_{-j} + 1}{\{(k-1)(m_j + m_{-j}) + k + 1\}^2},$$

where m_{-j} is the number of active firms of owners different from j. Thus, the Lemma holds, since the previous expression is decreasing in m_j. ■

A.8. Proof of Lemma 5.

For the decentralized non-delegation game, it can be shown [see Theorem 1 in Kamien and Zang (1990)] that if there is a merged SPE, then it has only one owner, say 1, with $k^* \geq \frac{n+1}{2}$ firms, which operates fewer firms than he owns, and all other firms are operated by their owners regardless of their ownership. For our model, with linear demand, we will obtain the above additional properties.

Let $T(r, t) = r\pi_c(r + t)$ be the gross profit of a given owner when he operates r firms and t is the number of firms operated by all other owners except the given owner. This function reaches an absolute maximum, with regard to r, at $r = t+1$. Therefore, owner 1 will operate $r(k^*) = n - k^* + 1$ firms and $k^* > (n+1)/2$ will hold. On the other hand, a necessary condition for a SPE [see Section VI in Kamien and Zang (1990)] is:

$$T(r(k^*), n - k^*) - (k^* - 1)T(1, r(k^* - 1) + n - k^*) - T(1, n - 1) \geq 0,$$

where $r(k^* - 1)$ is the number of firms that owner 1 will operate when an owner, which has sold his firm to owner 1, deviates by non selling it. This is equivalent to

$$\begin{aligned} D(n, k^*) &= \\ &= r(k^*)\pi_c(n - k^* + r(k^*)) - (k^* - 1)\pi_c(r(k^* - 1) + n - k^* + 1) - \pi_c(n) \geq 0. \end{aligned}$$

Since the absolute maximum of $T(r, n - (k^* - 1))$ is reached at $r = n - k^* + 2$ and, moreover, $n - k^* + 2 \leq k^* - 1$ holds if and only if $k^* \geq (n+3)/2$, we have

$r(k^*-1)=n-k^*+2$ if $k^* \geq (n+3)/2$ and $r(k^*-1)=k^*-1$ if $k^* < (n+3)/2$. In a SPE, owner 1's payoff has to be

$$u_1 = r(k^*)\pi_c(n-k^*+r(k^*)) - (k^*-1)\pi_c(r(k^*-1)+n-k^*+1).$$

Without loss of generality, we can assume that each owner, which has sold his firm to owner 1, has not bought any firm. Let N_1 be the set of these owners. So, the payoff of each owner $i \in N_1$ will be:

$$u_i = \pi_c(r(k^*-1)+n-k^*+1).$$

Let N_2 be the set of the rest of owners. Consider an owner in N_2 possessing k firms, with $1 \leq k \leq n-k^*$. Since the absolute maximum of $T(r, n-k^*-k+r(k^*))$ is reached at $r = n-k^*-k+r(k^*)+1 > k$, this owner will operate $r(k) = k$ firms, when owner 1 operates $r(k^*)$ firms and all other firms are operated by their owners. Moreover, it is easy to see that this owner of k firms is indifferent between possessing k firms and possessing only one. This implies that, in a merged SPE, the payoff of any owner $j \in N_2$ has to be

$$u_j = \pi_c(n-k^*+r(k^*)) = \pi_c(2n-2k^*+1).$$

For simplicity, the argument is divided in two cases.

Even case: $n = 2q$ with $q \geq 1$.

In this case, $k^* \geq q+1$ and we can write $k^* = q+d$, where $d = 1, \ldots, q$. In consequence, $r(k^*) = q-d+1$ and $r(k^*-1) = q$ if $d = 1$, and $r(k^*-1) = q-d+2$ if $d = 2, \ldots, n$. For the above function D, written with q and d, we have

$$D(q,1) = q\pi_c(2q-1) - (q+1)\pi_c(2q) = \frac{1}{4q(1+2q)^2} > 0,$$

for all $q \geq 1$. For $d = 2, \ldots, q$, we have:

$$\begin{aligned} D(q,d) &= \\ &= (q-d+1)\pi_c(2q-2d+1) - (q+d-1)\pi_c(2q-2d+3) - \pi_c(2q) = \\ &= -\frac{1}{(1+2q)^2} - \frac{d+q-1}{(2q-2d+4)^2} + \frac{1}{4q-4d+4}. \end{aligned}$$

Note that $D(2,2) = 9/400$. For $d = 2, \ldots, q$, with $q \geq 3$, we have

$$\frac{\partial D(q,d)}{\partial d} = \frac{1}{4}\left[\frac{1}{(q-d+1)^2} - \frac{3q+d}{(q-d+2)^3}\right].$$

Writing $q = d + h$, with $h \geq 0$, it follows

$$\begin{aligned}\frac{\partial D(d+h,d)}{\partial d} &= \frac{1}{4}\left[\frac{1}{(1+h)^2} - \frac{4d+3h}{(2+h)^3}\right] \leq \\ &\leq \frac{1}{4}\left[\frac{1}{(1+h)^2} - \frac{8+3h}{(2+h)^3}\right] = -\frac{h(7+8h+2h^2)}{4(1+h)^2(2+h)^3} \leq 0,\end{aligned}$$

because $d \geq 2$. This proves that $D(q,d)$ is decreasing in $d \in [2,q]$ for each $q > 2$. In consequence

$$D(q,d) \leq D(q,2) = \frac{1+4q+8q^2-4q^3}{4(q-1)q^2(1+2q)^2} < 0, \quad \forall q \geq 3,$$

for $d = 2,\ldots,q$. Thus, in the even case, for each $n = 2q$, with $n \neq 4$, a merged SPE must satisfy $k^* = q+1$ and $r(k^*) = q$, with $m_{cd} = 2q-1$ active firms. For $n = 4$, a merged SPE has to satisfy $k^* = 3$, $r(k^*) = 2$ and $m_{cd} = 3$, or $k^* = 4$, $r(k^*) = 1$ and $m_{cd} = 1$.

Odd case: $n = 2q+1$, with $q \geq 1$.

In this case $k^* \geq q+2$ and we can write $k^* = q+d+1$ for $d = 1,\ldots,q$. In consequence, $r(k^*) = q-d+1$ and $r(k^*-1) = q-d+2$ hold. In this case, the function D is

$$\begin{aligned}D(q,d) &= \\ &= (q-d+1)\pi_c(2q-2d+1) - (q+d)\pi_c(2q-2d+3) - \pi_c(2q+1) = \\ &= \frac{1}{4}\left[-\frac{1}{q-d+2} - \frac{1}{(1+q)^2} + \frac{1}{q-d+1} - \frac{2(d-1)}{(q-d+2)^2}\right].\end{aligned}$$

Then it follows

$$D(q,1) = \frac{1}{4q(1+q)^2} > 0, \quad \forall q \geq 1.$$

For $q \geq 2$ we have

$$\frac{\partial D(q,d)}{\partial d} = \frac{1}{4}\left[-\frac{4(d-1)}{(q-d+2)^3} + \frac{1}{(q-d+1)^2} - \frac{3}{(q-d+2)^2}\right],$$

and, therefore, for $h \geq 0$,

$$\begin{aligned}\frac{\partial D(d+h,d)}{\partial d} &= \frac{1}{4}\left[-\frac{4(d-1)}{(2+h)^3} + \frac{1}{(1+h)^2} - \frac{3}{(2+h)^2}\right] \leq \\ &\leq \frac{1}{4}\left[\frac{1}{(1+h)^2} - \frac{3}{(2+h)^2}\right].\end{aligned}$$

As the last function is negative if and only if $h \geq (\sqrt{3}-1)/2 = 0.366025$, it follows that $D(q,d)$ is strictly decreasing in $d \in [1, q-(\sqrt{3}-1)/2]$. Since

$$D(q,2) = \frac{2+3q+q^2-2q^3}{4(q-1)q^2(1+q)^2} < 0, \forall q \geq 2,$$

this implies that $D(q,d) < 0$ for $d = 2, \ldots, q-1$. Finally, as

$$D(q,q) = -\frac{q(q^2-3)}{8(1+q)^2} < 0, \forall q \geq 2,$$

in the odd case, for each $n = 2q+1$, a merged SPE must satisfy $k^* = q+2$, $r(k^*) = q$ and $m_{cd} = 2q-1$.

This proves the lemma. ■

A.9. Proof of Lemma 6.

From the previous lemma, consider first the necessary SPE-configuration given by $k^* = \left[\frac{n+1}{2}\right] + 1$, $r(k^*) = \left[\frac{n}{2}\right]$, $r(k^*-1) = \left[\frac{n+1}{2}\right]$, $m_{cd} = 2\left[\frac{n}{2}\right] - 1$. Suppose that owner 1 possesses k^* firms. Let N_1 be the set of owners which have sold their firms to owner 1 and let $N_2 = N - \{1\} \cup N_1$ the set of the rest of owners. We assume the configuration is such that owner 1 possesses his initial firm, each owner $i \in N_1$ sells his firm to owner 1 and owner i does not buy any firm and each owner in N_2 possesses only his proper firm. We will show that there is a structure of bids that supports the configuration as a SPE. The previous lemma implies that payoffs are

$$\begin{aligned} u_1 &= \left[\frac{n}{2}\right]\pi_c(m_{cd}) - \left[\frac{n+1}{2}\right]\pi_c(n), \\ u_i &= \pi_c(n), \forall i \in N_1, \\ u_j &= \pi_c(m_{cd}), \forall j \in N_2, \end{aligned}$$

where $m_{cd} = 2\left[\frac{n}{2}\right] - 1$. Consider the following set of bids. For owner 1, $B_1^1 = \alpha$, $B_i^1 = \pi(n)$ if $i \in N_1$and $B_j^1 = \beta$ if $j \in N_2$. For $i \in N_1$, $B_i^i = \pi(n)$ and $B_h^i = \beta$ if $h \neq i$. For $j \in N_2$, $B_j^j = \alpha$ and $B_h^j = \beta$ if $h \neq j$. Here, α is sufficiently high and β is sufficiently low such that the firms initially possessed by owners in $\{1\} \cup N_2$ are not purchased, and each owner does not sell his firm by lowering his asking price, in all potentially advantageous deviation. Therefore, there are three classes of relevant deviations.

(D1) Consider that owner 1 buys s firms, where $0 \leq s \leq \left[\frac{n+1}{2}\right] - 1$, from the set of firms initially possessed by the owners in N_1. In the new configuration, owner 1 will have $k'_1 = s+1$ firms and each one of the other owners will have, at most, one firm. Therefore, the last owners will activate all their firms and owner 1 will activate r firms, where r maximizes $T(r, n-s-1)$ for $0 \leq r \leq s+1$. The absolute maximum of this function is reached at $n-s \geq s+1$. In consequence, in the new configuration, owner 1 will operate $r'_1 = s+1$ firms and he will obtain $u'_1 = T(s+1, n-s-1) - s\pi_c(n) = \pi_c(n)$, which is strictly lower than u_1, from the necessary condition for SPE given in the proof of the previous lemma.

(D2) Consider that owner $i \in N_1$ deviates by buying some firms initially possessed by owners in N_1 and/or not selling his firm. In the new configuration, owner i will own $k'_i = s$ firms, where $1 \leq s \leq \left[\frac{n+1}{2}\right]$, with a net expenditure higher than $(s-1)\pi_c(n)$. Owner 1 will possess $k'_1 = \left[\frac{n+1}{2}\right] + 1 - s$ firms, and each one of the other owners will have, at most, one firm. Let r'_1 and r'_i be the number of firms that will respectively be activated by owners 1 and i. Since the absolute maximum, with respect to r, of $T(r, r'_i + n - \left[\frac{n+1}{2}\right] - 1)$ is reached at $r'_i + n - \left[\frac{n+1}{2}\right] \geq k'_1$, then owner 1 will operate $r'_1 = k'_1$ firms, in the new configuration. Since the absolute maximum, concerning r, of $T(r, r'_1 + n - \left[\frac{n+1}{2}\right] - 1) = T(r, n-s)$ is reached at $n-s+1 \geq s$, then owner i will activate $r'_i = k'_i$ firms. In consequence, in the new configuration, owner i will get $u'_i \leq T(s, n-s) - (s-1)\pi_c(n) = \pi_c(n) = u_i$.

(D3) Consider that owner $j \in N_2$ (then we will have $n - \left[\frac{n+1}{2}\right] \geq 2$) deviates by buying s firms, where $1 \leq s \leq \left[\frac{n+1}{2}\right]$, from the set of firms initially owned by the owners in N_1. He will pay at least $s\pi_c(n)$ and, in the new configuration, owner 1 will possess $k'_1 = \left[\frac{n+1}{2}\right] + 1 - s$ firms, owner j will have $k'_j = s+1$ firms, and there will be $n - \left[\frac{n+1}{2}\right] - 2$ owners possessing only one firm. These last owners will operate all their firms and the former owners will respectively activate r'_1 and r'_j verifying $r'_1 = \min(k'_1, r'_j + n - \left[\frac{n+1}{2}\right] - 1)$ and $r'_j = \min(k'_j, r'_1 + n - \left[\frac{n+1}{2}\right] - 1)$. Suppose that $k'_1 > r'_j + n - \left[\frac{n+1}{2}\right] - 1$. Then $2\left[\frac{n+1}{2}\right] - n + 2 - s > r'_j \geq 1$ holds and we must have $s = 1$ and $r'_j = 1$, or we will get a contradiction. Therefore, $r'_1 = \min(\left[\frac{n+1}{2}\right], n - \left[\frac{n+1}{2}\right]) = n - \left[\frac{n+1}{2}\right]$ and it follows $r'_1 + n - \left[\frac{n+1}{2}\right] - 1 > 2 = k'_j$ and $r'_j = 2$, which is a contradiction. This proves that we must have $k'_1 \leq r'_j + n - \left[\frac{n+1}{2}\right] - 1$ and, in consequence, $r'_1 = k'_1$. Now, we have two possibilities. If $1 \leq s \leq \left[\frac{n+1}{2}\right] - 1$, it follows $k'_j \leq r'_1 + n - \left[\frac{n+1}{2}\right] - 1$ and $r'_j = k'_j$, and owner j will get $u'_j \leq T(s+1, n-s-1) - s\pi_c(n) = \pi_c(n) < \pi_c(m_{cd}) = u_j$. If $s = \left[\frac{n+1}{2}\right]$, it follows $k'_j > r'_1 + n - \left[\frac{n+1}{2}\right] - 1$ and $r'_j = n - \left[\frac{n+1}{2}\right] = \left[\frac{n}{2}\right]$, and owner j will get $u'_j \leq \left[\frac{n}{2}\right]\pi_c(2\left[\frac{n}{2}\right] - 1) - \left[\frac{n+1}{2}\right]\pi_c(n)$. The inequality $\left[\frac{n}{2}\right]\pi(2\left[\frac{n}{2}\right] - 1) - \left[\frac{n+1}{2}\right]\pi(n) \leq u_j$ is equivalent to $\left(\left[\frac{n}{2}\right] - 1\right)\pi_c(2\left[\frac{n}{2}\right] - 1) - \left[\frac{n+1}{2}\right]\pi_c(n) \leq 0$. The last inequality holds because the left hand term is equal to $-\frac{1+3q}{4q^2(1+2q)^2} < 0$ if $n = 2q$, and equal to $-\frac{1}{4q^2(1+q)} < 0$ if $n = 2q + 1$. Therefore, in the new configuration, owner j will obtain $u'_j < u'_j$.

For $n = 4$, consider the necessary SPE-configuration given by $k^* = 4$, $r(k^*) = 1$, $m_{cd} = 1$. Owner 1 gets $u_1 = \pi(1) - 3\pi(3) = 1/16$ and each one of the others owners obtains $u_i = \pi(3) = 1/16$. Consider the following set of bids. For owner 1, $B_1^1 = \alpha$ and $B_i^1 = \pi(3)$ if $i \neq 1$. For owner $i \neq 1$, $B_i^i = \pi(3)$ and $B_h^i = \beta$ if $h \neq i$. Here α is sufficiently high and β is sufficiently low such that the firm initially possessed by owner 1 is not purchased, and each owner does not sell his firm by lowering his asking price, in all potentially advantageous deviation. Therefore, there are two classes of relevant deviations.

(D1) If owner 1 does not buy any firm, he will get $u'_1 = \pi(4) < u_1$, from the necessary condition in the proof of previous lemma. If he buys only one firm, he will activate 2 firms, and he will obtain $u'_1 = 2\pi(4) - \pi(3) = 7/400 < 1/16 = u_1$. If he buys 2 firms, he will activate 2 and he will get $u'_1 = 2\pi(3) - 2\pi(3) = 0 < u_1$.

(D2) If owner $i \neq 1$ deviates by purchasing some firms from the set of firms initially possessed by owners different to owner 1 and/or not selling his proper firm, he will get s firms, where $1 \leq s \leq 3$, with a net expenditure greater than $(s-1)\pi(3)$. In the new configuration, owners 1 and i will own, respectively, $k'_1 = 4 - s$ and $k'_i = s$ firms. If $s = 1$, they will activate, respectively, $r'_1 = 2$ and $r'_i = 1$ firms, and owner i will get $u'_i \leq T(1,2) = \pi(3) = u_i$. If $s = 2$, they will operate, respectively, $r'_1 = 2$ and $r'_i = 2$ firms, and he will obtain $u'_i \leq T(2,2) - \pi(3) = 2\pi(4) - \pi(3) = \frac{7}{400} < \frac{1}{16} = \pi(3) = u_i$. If $s = 3$, they will activate, respectively, $r'_1 = 1$ and $r'_i = 2$, and he will get $u'_i \leq T(2,1) - 2\pi(3) = 0 < u_i$.

These arguments prove that for any n there is a merged SPE. When $n \neq 4$, in any merged SPE, the number of active firms is equal to $m_{cd} = 2\left[\frac{n}{2}\right] - 1$. When $n = 4$, in any merged SPE, the number of active firms is 3 or 1. For a merged SPE with 3 active firms, the equilibrium payoffs are, w.l.o.g., equal to $u_1 = 2\pi(3) - 2\pi(4) = 9/200$, $u_2 = u_3 = \pi(4) = 1/25$ and $u_4 = \pi(3) = 1/16$. This merged SPE is dominated by the merged SPE with 1 active firm and payoffs $u_1 = \pi(1) - 3\pi(3) = \frac{1}{16}$, $u_i = \pi(3) = \frac{1}{16}$ for $i = 2, 3, 4$. ■

A.10. Proof of Proposition 5.

The properties about the equilibrium levels for the number of active firms, come from Proposition 3 and Lemma 5. The properties about the level of production come from substitution of the relevant values of m_d and m_{cd}, respectively, in the expressions for x_d and x_c obtained in Lemma 1 and Remark 1. ■

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