CONSUMER HETEROGENEITY

AND MARKET IMPERFECTIONS*

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ABSTRACT

With differentiated goods and heterogenous consumers, firms set prices above marginal costs when product choice is endogenous. When consumer tastes are identical and all consumers prefer one possible variant to all other possible variants at the marginal costs of production, then all firms provide the same variant in (subgame perfect) equilibrium and equilibrium prices in the subgame which follows, are equal to marginal costs. Intuition suggests that as the consumer density becomes more concentrated, firms will provide (weakly) closer substitutes in order to compete for the consumers in the high-density part of the distribution and prices are closer to marginal costs of production. Consequently, if the intuition is correct, price equal marginal costs can be seen as a good approximation when most consumers are rather the same. I give results specifying when prices do not approach marginal costs as consumers become more similar in taste. A small heterogeneity can give rise to large market imperfections.

1 Introduction

Strategic interaction among oligopolists is for more than 100 years in the mainstream of economic research. Imperfect competition can be seen as the starting point of industrial organization. One line of research has argued that 'perfect' competition can still serve as a reference point because it can be seen as some limit of imperfect competition. I consider comparative statics in the composition of demand.[1] As a reference point serves the well-known Bertrand model in which firms compete in prices and goods are homogeneous. Independent of the number of firms (≥ 2), equilibrium prices are equal to (constant) marginal costs of production and Bertrand competition mimics perfect competition.

My question is the following: Can competitive prices result as a limit when consumers have more and more similar tastes? And more general: Does more consumer heterogeneity necessarily lead to higher prices and more differentiated products? If the answer for a model is affirmative I say that the model satisfies intuitive comparative statics.

In order to address this question I explicitly take into account that firms not only affect prices but also product specification: firms first specify products and then compete in prices. Intuition suggests that as the consumer density becomes more concentrated,[2] firms will provide (weakly) closer substitutes in order to compete for the consumers

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[1]Other work focused on comparative statics in the number of firms. This increase of the number of firm can reduce the market power of the firms. The limit of an infinite number of firms has been analyzed intensively. In the Cournot model, prices turn to marginal costs as the number of firms goes to infinity (see e.g. Novshek, 1985). There exists also work on competitive equilibria as limits of equilibria in Bertrand-Edgeworth models. With exogenously differentiated products the competitive limit is no longer necessarily reached when the number of firms turns to infinity (Dixit and Stiglitz, 1977, Hart, 1985). The reason is that a large number of firms does not necessarily destroy the local market power each firm enjoys. Consequently, equilibrium prices do not turn to marginal costs as the number of firms becomes large. However, in address models of product differentiation additional firms lead to more intense price competition if firms are uniformly spread over a product space of finite dimension because demand reacts more sensitive to price changes.

[2]'Concentrated' means that the consumer mass is close to 1 in an appropriately defined neighborhood of a vector of the taste parameters.

in the high-density part of the distribution and prices are closer to marginal costs of production. Anderson, Goeree and Ramer (1997, p. 111) write that "the idea that price rises with heterogeneity is familiar in the theory of product differentiation". In their model they show that "tight density functions are a force of agglomeration".[3]

In the examples of my paper I show that the price of at least one firm approaches marginal costs for given product specifications (when the consumer heterogeneity is changed parametrically).[4] Endogenizing the product specification can reverse this result. For a particular model of an asymmetric duopoly and a class of density functions I will show that prices increase and products become more differentiated the more *homogeneous* the population; i.e. a small heterogeneity of consumer tastes (e.g. measured by the standard deviation of the distribution) can give rise to large market imperfections. Firms face a trade-off between market penetration and mark-up. As the consumer density becomes more concentrated it is too costly in terms of profits for a niche firm to recover lost market share by producing a closer substitute to the market leader. The firm rather separates itself further from the market leader and enjoys high mark-ups in its narrow market although it sells to fewer consumers.

The model provides a link between structure of the demand side and performance in an imperfectly competitive market. Under intuitive comparative statics a more homogeneous population is desirable for consumers with respect to prices. On the other hand, if there was a positive relationship between profits and consumer heterogeneity for some firms, at least one firm would have an incentive to affect consumer tastes in favor of more heterogeneity (e.g. by advertising). As pointed out, the intuitive comparative statics do not always hold. Thus one cannot base predictions on a general and

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[3]To take the Hotelling model with quadratic transportation costs as an example, when consumers are uniformly distributed on an interval then prices turn to marginal costs as the interval shrinks to a single point.

[4]Independently, Dierker and Dierker (1997) provide examples of *exogenous* product differentiation in duopoly in which there exist non-monotone relationships between the degree of heterogeneity and one or both prices. As they pointed out, the assumption of symmetric distributions such as in Perloff and Salop (1985) are not 'technical' assumptions but impose a particular structure upon the model which rule out counterintuitive comparative statics. They also show that, under some restrictive conditions without imposing symmetry, such counterintuitive comparative statics can be ruled out.

purely theoretical model. Put differently, easily observable variables such as prices and profits are rather imperfect indicators of the underlying consumer heterogeneity in a differentiated market.

This paper also provides an example to the rationale that some fringe or niche firms which operate on low volume cannot operate at higher profits in a more profitable location because this position is occupied by some other firms. When such a market niche becomes less favorable (because for example some consumers are replaced by others whose preferences favor some other products c.p.) increased product differentiation leads to nondecreasing prices of all firms which are operating in the market. This means that although consumer tastes are highly concentrated neither does one observe agglomeration of the firms in the product space nor prices close to marginal costs.

An important issue is whether the result will be robust to the introduction of asymmetric costs, noise (random utility model) or more elastic demand into the consumer choice. Although the discontinuity (of prices and product choices) in the limit does not hold in the latter two cases, I will argue that counterintuitive comparative statics are a robust result on some positive range in the sequence of densities.

2   Examples Confirming the Intuition

I present some standard examples which confirm the belief that making consumer tastes more homogeneous in a setup of discrete choice leads to more competitive prices. In this paper this means that in the subgame perfect equilibrium of the location-then-price game duopolists produce closer substitutes and prices are closer to (constant) marginal costs of production as consumer tastes are less dispersed.

As is known from the literature on product differentiation, firms set prices above marginal costs when goods are differentiated and consumers heterogenous. With endogenous product choice, it is determined in the model whether goods are good or bad substitutes in the aggregate. When consumer tastes are identical and all consumers prefer one possible variant to all other possible variants at the marginal costs of pro-

duction, then all firms provide the same variant in (subgame perfect) equilibrium and equilibrium prices in the subgame which follows, are equal to marginal costs; hence, the Bertrand model serves as a reference. The game is played under perfect information, consumer behavior is deterministic. In this and the following section I will look at different examples characterized by horizontal versus vertical product differentiation, unit versus unit-elastic demand, bounded versus unbounded support. Some of these examples with non-uniform consumer densities are presented in Goeree (1995), Anderson, Goeree, and Ramer (1997), and Peitz (1997b). The population of consumers is described by some distribution on the space of the underlying preference parameters (Caplin and Nalebuff, 1991, Dierker, 1991, Anderson, de Palma, and Thisse, 1992, Peitz, 1997a).

Example 1: The Hotelling Model with Quadratic Transportation Costs; bounded and unbounded product space.

To start with I consider the unavoidable Hotelling model with quadratic transportation costs (d'Aspremont, Gabszewicz, and Thisse, 1979). Consumers are distributed on a line segment. The location of the consumer $\theta \in \Re$ describes her ideal point in the product space. It is assumed that the consumer density is uniform on its support. Without loss of generality I consider densities $g : [-L/2, L/2] \rightarrow \Re_+$ when the line segment has length L. On this interval the density is $1/L$. Goods are positioned somewhere on the real line. I consider two cases: in the first case the product space is bounded and restricted to the support of the consumer density, in the second case it is unbounded. Consumers buy one unit of one of the two goods in the market. Their evaluation for good i takes the following value

$$v_i = r - p_i - \tau(\theta - l_i)^2$$

where r is the maximal willingness-to-pay, p_i the price of good i and $l_i \in \Re$ the location of good i in the product space. Consumers buy the good with the highest evaluation. Good 1 is located at a, good 2 at b, $a < b$. Firms maximize profits and have zero marginal costs of production. A consumer who is indifferent between the two goods is

located at

$$m = \frac{1}{2\tau}\frac{p_2 - p_1}{b - a} + \frac{1}{2}(a + b)$$

For given locations the unique price equilibrium is given by

$$
\begin{aligned}
p_1^*(a,b) &= \tau L(b - a) + \frac{1}{3}\tau(b^2 - a^2) \\
p_2^*(a,b) &= \tau L(b - a) - \frac{1}{3}\tau(b^2 - a^2)
\end{aligned}
$$

In the game with the bounded product space firms choose maximal differentiation in the unique subgame perfect equilibrium of the game. For these locations equilibrium prices are $p_i^* = \tau L^2$, $i = 1, 2$. Since the product space is restricted, firms have to produce closer and closer substitutes as L decreases. Hence, it is not surprising that prices converge to marginal costs as L turns to zero. When the product space is the real line firms choose $a^* = -(3/4)L$ and $b^* = (3/4)L$ and $p_i^* = (3/2)\tau L^2$ in the occurring subgame. The same qualitative results hold although firms are free not to choose close substitutes. Firms prefer to do so because they want to be sufficiently close to the area where consumers are concentrated. The comparative statics properties of this example are formally written as

$$\frac{\partial p_i}{\partial L} > 0, \quad \lim_{L \searrow 0} p_i = 0, \tag{1}$$

$$\frac{\partial \pi_i}{\partial L} > 0, \quad \lim_{L \searrow 0} \pi_i = 0, \tag{2}$$

$$\frac{\partial(b - a)}{\partial L} > 0, \quad \lim_{L \searrow 0}(b - a) = 0. \tag{3}$$

Note that firms tend to provide perfect substitutes as the consumer heterogeneity (measured by L) becomes small.

Example 2: The Hotelling Model with Quadratic Transportation Costs; more general consumer densities.

It might be argued that the previous results are due to the particular consumer density. Tails in the consumer density make it interesting for firms to locate further apart. I will show that for symmetric equilibria the comparative statics properties are the same when the consumer density is symmetric. This example has been worked out in detail

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by Anderson, Goeree, and Ramer (1997). Assume that G, $1 - G$ are log-concave. Then there exists a unique price equilibrium in the duopoly which is characterized by

$$
\begin{aligned}
p_1^*(a, b) &= 2\tau(b - a)\frac{G(m^*)}{g(m^*)} \\
p_2^*(a, b) &= 2\tau(b - a)\frac{1 - G(m^*)}{g(m^*)}
\end{aligned}
$$

where m^* solves $m = (a + b)/2 + (1 - 2G(m))/g(m)$. At the location stage firms maximize continuation profits with respect to their location.

$$
\tilde{\pi}_1(a, b) = p_1^*(a, b)G(m^*(a, b))
$$

and correspondingly for firm 2. Equilibrium candidates are characterized by (see Anderson, Goeree, and Ramer, 1997)

$$
\frac{g'(m^*)}{g^2(m^*)} = \frac{1}{G(m^*)} - \frac{1}{1 - G(m^*)}
$$

I assume that g is symmetric around 0. Then $m^* = 0$ solves the equation. When a symmetric subgame perfect equilibrium exists, its locations are given by

$$
\begin{aligned}
a &= -\frac{3}{4}\frac{1}{g(0)} \\
b &= \frac{3}{4}\frac{1}{g(0)}
\end{aligned}
$$

and in these locations equilibrium prices are $p_i^* = \frac{3\tau}{2g^2(0)}$. I consider a family of densities g^α, $\alpha > 0$ with $g^\alpha(\theta) \equiv \alpha g(\alpha\theta)$. As pointed out by Anderson, Goeree, and Ramer (1997) symmetric equilibrium locations are linear in $1/\alpha$ and prices are proportional to $1/\alpha^2$. Clearly, $\lim_{\alpha \to \infty} p_i^*(a^*(\alpha), b^*(\alpha)) = 0$ and prices converge to marginal costs. Anderson, Goeree, and Ramer (1997) provide a condition on the consumer density for subgame perfect equilibria to exist. A unique subgame perfect equilibrium exists if $G(\theta)(1 - G(\theta))/g(\theta)$ is strictly pseudo-concave which holds for example for the Normal distribution.

A Remark on How to Measure Heterogeneity. It has to be addressed which measure of heterogeneity is appropriate. In examples with uniform densities support,

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standard deviation, and median density provide the same ordering in terms of heterogeneity so that the question of the appropriate measure does not arise. Example 2 shows that global measures of heterogeneity (such as the standard deviation of a distribution) do not determine the endogenous variables of the model. The density of the median consumer (a local measure) determines equilibrium locations and prices in a symmetric model (in symmetric equilibrium). The median consumer does not matter when asymmetric densities are considered. In this case I will consider sequences of densities which are monotone in some sense and which converge to a homogeneous population (see discussion below). Along such a sequence a population is said to become less heterogeneous.

Note that, on a given support, the uniform density is the most heterogeneous among all logconcave densities according to its standard deviation and the value the density takes at the location of the median consumer. It is not the most heterogeneous in the class of density functions which satisfy that $G, 1 - G$ are logconcave (see e.g. Peitz, 1997a).

3 Asymmetric Markets: Dismantling the Intuition

Example 3: Vertical Product Differentiation with Unit Demand.
It has been noted by Neven (1986) and Cremer and Thisse (1991) that the Hotelling model has a counterpart as a model of vertical product differentiation with $v_i = r + \theta q_i - p_i$ where $q_i \in \Re_+$ stands for quality and firms have constant marginal cost of production cq_i^2, $c > 0$. With appropriate parameters one can interpret examples 1 and 2 as models of vertical product differentiation. Since quality is positively valued the consumer density has a support inside $\Re_+$.[5] Again one obtains the "intuitive" comparative statics result that products become more homogeneous($|q_2 - q_1| \to 0$) as the consumer density concentrates around a single point ($Var(\theta)$ small). A limit has

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[5]Instead of considering densities which are symmetric around 0 I consider densities which are symmetric around some $\tilde{\theta} > 0$. In the model of vertical product differentiation symmetric densities have necessarily bounded support. For the equivalence to hold, the density in the model of horizontal product differentiation must have bounded support.

to be taken such that a natural monopoly does not arise.

A first deviation from the intuitive comparative statics properties can be shown for a modification of this example. If the competitive pressure in case of more homogeneous goods is very strong, firms want to maximally differentiate their products as long as there is some heterogeneity of consumer tastes. Assume that firms have constant marginal costs of production cq_i, $c \in [\underline{\theta}, \overline{\theta}]$. This example is analyzed in detail in Peitz (1996). Note that for symmetric densities the market is symmetric if $c = (\overline{\theta} + \underline{\theta})/2$. Consider a population which is uniformly distributed on $[z - L/2, z + L/2]$, $l < 2z$, $l, z > 0$. The product space is assumed to be $[\underline{q}, \overline{q}]$ and r sufficiently high so that all consumers buy in the market. In the duopoly, firms choose $\underline{q}, \overline{q}$ in the unique subgame perfect equilibrium of the (restricted) quality then price game. This implies that $|q_2^* - q_1^*| = \overline{q} - \underline{q}$ as long as $l > 0$. Firms *do not* produce closer substitutes as the population becomes more homogeneous and property (3) is violated. When taking the limit $L \to 0$ one has to choose $c = (\overline{\theta} + \underline{\theta})/2$ in order to exclude natural monopolies. With maximal differentiation equilibrium prices are

$$
\begin{aligned}
p_1 &= c\underline{q} + \frac{1}{2}(\overline{\theta} - \underline{\theta})(\overline{q} - \underline{q}), \\
p_2 &= c\overline{q} + \frac{1}{2}(\overline{\theta} - \underline{\theta})(\overline{q} - \underline{q}).
\end{aligned}
$$

Although the product choice is unaffected by the dispersion of consumer tastes, prices converge to marginal costs. The result with a collapsing support of the consumer density supports the intuition to the extent that prices approach marginal costs. Two things should be investigated: the effect of an asymmetric density on prices given qualities and the effect of an asymmetric density on prices and product choices. In order to have possible effects on product choice, a model has to be constructed in which firms do not choose maximal differentiation.

Example 4. Vertical Product Differentiation with Unit Elastic Demand.

Here I use a modification of an example presented in Peitz (1997b). Two firms compete in a vertically differentiated market of fixed dollar size. In such a market high prices are less favorable than in a model with unit demand. Still, a monopolist would

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charge infinite prices. Firms have constant marginal costs of production c. The direct utility function is written as

$$u(x_0, x_1, x_2, q_1, q_2) = (\sum_i q_i^\theta x_i)^\alpha x_0^{1-\alpha}$$

Again q_i measures the quality of the good. A consumer with $\theta = 0$ does not put any value on quality and buys at the lowest price. A consumer of type $\theta \in [\underline{\theta}, \overline{\theta}] \subseteq \Re_+$ is indifferent between two variants if $p_2/p_1 = q_2/q_1$. Clearly, a good with 0 quality does not give any utility to consumers who value quality ($\theta > 0$).

In this setup each consumer spends αy in the differentiated market. When both firms are active, mean demand of the firms are

$$
\begin{aligned}
X_1(p_1, p_2, q_1, q_2) &= \int_{\underline{\theta}}^m \frac{\alpha y}{p_1} g^\beta(\theta) d\theta \\
X_2(p_1, p_2, q_1, q_2) &= \int_m^{\overline{\theta}} \frac{\alpha y}{p_2} g^\beta(\theta) d\theta
\end{aligned}
$$

where m is the marginal consumer defined by $(\log p_2 - \log p_1)/(\log q_2 - \log q_1)$ and g^β is a density function by which the population of consumers is described (see below). g^β represents the taste heterogeneity over the differentiated goods. It follows from Peitz (1997b) that there exists a unique price equilibrium for given qualities if g^β is log-concave in θ.[6]

For a given heterogeneity parameter $\beta \geq 0$, I assume that θ is distributed on $[0, 1]$ according to the density $g^\beta : \Re \to \Re_+$ with $g^\beta(\theta) = \theta^\beta(1+\beta)$ for $\theta \in [0, 1]$ and $g(\theta) = 0$, for $\theta \notin [0, 1]$. With this specification g is log-concave on its support and there exist a unique equilibrium in prices for differentiated products $q_1 \neq q_2$. For $\beta = 0$, the consumer density is uniform on $[0, 1]$. For $\beta \in (0, 1]$ the consumer density is increasing and concave. For $\beta = 1$ the density function is linear, for $\beta > 1$ the density function is increasing and strictly convex in the type θ. The higher β the more mass of the consumer distribution is concentrated near $\theta = 1$ and for $\beta \to \infty$, $G^\beta(\theta) \to 0$ for any $\theta \in [0, 1)$. For instance, at $\beta = 150$ more than 99.95 per cent of the consumer mass is located between 0.95 and 1 and more than 75 per cent between 0.99 and 1. β

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[6] Alternatively, one can allow for additional heterogenity in α and income y. Sufficient for existence and uniqueness is that the product of $\alpha(\theta)y(\theta)g(\theta)$ is log-concave in θ.

large implies that the distribution of consumer tastes has small variance. Comparing densities with $\beta_1 > \beta_2$, β_2 dominates β_1 in the sense that $G^{\beta_2}(\theta) \geq G^{\beta_1}(\theta)$ for all θ.

In the duopoly with $q_1 < q_2$ profit functions are

$$\pi_1(p_1, p_2, q_1, q_2) = \frac{p_1 - c}{p_1} \alpha y \frac{(\log p_2 - \log p_1)^{1+\beta}}{(\log q_2 - \log q_1)^{1+\beta}},$$

$$\pi_2(p_1, p_2, q_1, q_2) = \frac{p_2 - c}{p_2} \alpha y \left(1 - \frac{(\log p_2 - \log p_1)^{1+\beta}}{(\log q_2 - \log q_1)^{1+\beta}} \right).$$

For all $\beta \geq 0$ and for any parameter values $[\underline{q}, \overline{q}] \subseteq \Re_+$, a unique subgame perfect equilibrium of the location then price game exists. If firms are not restricted to a single quality one of the firms provides maximal quality and the other firm provides some lower quality.

I am now studying effects of the consumer heterogeneity on prices and qualities. Changes in the heterogeneity of the consumers amount to comparative statics in the parameter β. Consider for the moment a product space with $\underline{q} = 0$ so that maximal differentiation does not occur at any parameter value β. In equilibrium, the logarithmic price difference and the logarithmic quality difference are increasing in β, i.e. the fewer the consumers without much taste for high quality (low θ), the more differentiated are the goods (the computations are delegated to the appendix). The intuition is the following: for high β the low-quality producer moves far away from the high-quality producer giving his competitor a lot of monopoly power so that he can maximize profits. Moving closer would intensify competition and increased market share is overcompensated by deteriorated mark-ups. For high β almost the full mass of market expenditure belongs to consumers in the neighborhood of type $\theta = 1$ but there are no incentives for the firms to produce close substitute.

In the limit as $\beta \to \infty$ the whole expenditure mass is concentrated on $\theta = 1$. Consumers who buy in the market have the same taste parameter for quality. Hence, both firms set the highest quality in the unique subgame perfect equilibrium and prices are equal to marginal costs on the equilibrium path. Hence, one obtains a discontinuity in the limit . *A high concentration of consumer tastes can lead to large market imperfections.* The result is in contrast to a change of a uniform distribution such that the support shrinks to a single point. In that case (as in the examples of the previous sec-

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tion) one would obtain the result that firms play a Bertrand game with homogeneous products in the second stage and that prices approach marginal costs when the support turns to a single point. More general ($\underline{q} \geq 0$), the result is in contrast to the intuition that a more concentrated consumer distribution (in terms of expenditure) leads to less differentiated products (see below).

I state the comparative statics properties of this example as a proposition.

Proposition 1 *In example 4 with $\underline{q} = 0$ the following properties hold:*

$$\frac{\partial p_i}{\partial \beta} > 0, \qquad \lim_{\beta \to \infty} p_i > c, \tag{4}$$

$$\frac{\partial (q_2/q_1)}{\partial \beta} > 0, \qquad \lim_{\beta \to \infty} (q_2/q_1) = \infty. \tag{5}$$

The proposition is shown in the appendix. The result implies that the properties (1) and (3) of example 1 do not hold. In particular, (4) says that both prices are bounded away from marginal costs. Although firm 1 increases its price it does not grow without bound. One obtains $\lim_{\beta \to \infty} p_1(\beta) = 3c$ and $\lim_{\beta \to \infty} \pi_1(\beta) = 0$.

The results hold under the restriction that $\underline{q} = 0$ so that maximal differentiation never occurs. Putting a positive lower bound on the product space ($\underline{q} > 0$) destroys the limit results. The counterintuitive comparative statics hold on the set of parameter values β for which goods are not maximally differentiated in subgame perfect equilibrium. The set is nonempty for the maximal quality ratio $\overline{q}/\underline{q}$ sufficiently large.

The competitive forces at play are not due to the assumption of a duopoly. However, the complexity of the problem did not allow me to analyze a model with more than two firms. Below I discuss to which extent the results hold in variations of the model, one might want to consider.

A remark on asymmetric costs. Consider the case of a degenerate distribution. If firms have different marginal costs of production $c_1 > c_2$, the low-cost firm will serve the whole market. This firm is constrained in the price setting by the high-cost firm and, in subgame perfect equilibrium, sets its price $p_2 \leq c_1$. There exist many equilibria because the high-cost firm will be outside the market independent of its location under

a best reply of firm 2. After elimination of weakly dominated strategies both firms locate at the position of a protected single-product monopolist and $p_2 = c_1$. Clearly, the counterintuitive monotone comparative statics and discontinuity in the limit still hold in a specification which corresponds to example 4. Some heterogeneity makes firms produce products which are bad substitutes. p_1 and p_2 increase as the consumer density concentrates more mass in a neighborhood of a point. Under sufficient consumer heterogeneity and a sufficiently large cost difference the low-cost firm provides the high quality $\bar{q}$.

A remark on random decision making. If consumers make i.i.d. errors in their decision making, goods are not perfect substitutes in the aggregate even when physically identical. In a random utility model with physically identical goods prices are set above marginal costs because each firm enjoys some market power. As de Palma et al. (1985) have shown in a random utility model with horizontal product differentiation, firms may prefer not to physically differentiate their products in spite of consumer heterogeneity with respect to the product characteristics. Randomness, which corresponds to exogenous product differentiation in a second dimension, makes it unprofitable to use the dimension of physical product differentiation if the heterogeneity with respect to the evaluation of physical product characteristics is sufficiently small.

Adding noise to a model of product differentiation destroys the discontinuity in the limit one might otherwise obtain. This holds because the competitive pressure for goods at 'close' locations is reduced. For the heterogeneity of the population sufficiently small, firms choose identical products.

When the heterogeneity becomes large relative to the noise, firms do have an incentive to move apart in order to enjoy more market power. I conjecture that, for examples similar to the one above, one can show the counterintuitive monotonicity of prices with respect to a parameter which measures consumer heterogeneity.

A remark on elastic demand. With the Cobb-Douglas specification, expenditure in the market is fixed. When expenditure is decreasing in price (such as in a

CES-specification) this has a dampening effect on prices.[7] Since $\lim_{p_i \to \infty} \sum_i p_i X_i = 0$, firms' prices cannot explode. Introducing now noise into the utility function consequently destroys the limit result (4) because both firms produce the highest quality and enjoy profits in the setup of physically identical goods. For demand sufficiently inelastic, I expect the counterintuitive comparative statics to hold on a range of parameter values β.

4 Concluding Remark

This paper argued that a lower dispersion of taste parameters (more homogeneous population) does not necessarily lead to more competitive prices when firms choose product characteristics before prices. A firm might prefer to become a niche firm by differentiating its product from the market leader, i.e. it may rather want to exploit a small submarket than to compete head-on in the whole market. In a simple model I showed that this phenomenon leads to counterintuitive comparative statics. Such a phenomenon cannot be observed in a symmetric model. Finally, I discussed the robustness of the result to various changes in the model.

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[7]With CES-demand there exists a unique equilibrium in the price game. Since numerical computations for non-uniform densities are complicated, I only provide this remark.

Appendix: technical details of Example 4. First-order conditions of profit maximization reduce to

$$c(1 + \beta + \log p_2 - \log p_1) = (1 + \beta)p_1,$$

$$c\left(\frac{(\log q_2 - \log q_1)^{1+\beta}}{(\log p_2 - \log p_1)^\beta} + 1 + \beta - \log p_2 + \log p_1\right) = (1 + \beta)p_2.$$

Denote $\varphi = \log q_2 - \log q_1$ and $\phi = \log p_2 - \log p_1$. If both firms have a positive market share, $\phi > 0$. A solution to the first-order conditions satisfies

$$\frac{(\varphi^{1+\beta}/\phi^\beta) + 1 + \beta - \phi}{1 + \beta + \phi} = e^\phi. \tag{6}$$

With exogenous product differentiation φ is fixed. It is easy to show that $\varphi^{1+\beta} > 2\phi^{1+\beta}$. This implies that $\varphi > \phi$. For exogenous product differentiation, ϕ is consequently bounded from above. From the first order conditions it follows then that $\lim_{\beta\to\infty} p_1^*(\beta) = c$. Hence, $\lim_{\beta\to\infty} p_2^*(\beta) = c\frac{q_2}{q_1}$.

Consider now endogenous product choice. One obtains by implicit differentiation of (6) that

$$\frac{d\phi}{d\varphi} = \frac{(1+\beta)\left(\phi^\beta\left(e^\phi(1+\beta+\phi) - (1+\beta-\phi)\right)\right)^{\frac{\beta}{1+\beta}}}{\beta\phi^{\beta-1}\left(e^\phi(1+\beta+\phi) - (1+\beta-\phi)\right) + \phi^\beta\left(e^\phi(2+\beta+\phi) + 1\right)}.$$

This expression is positive for $\phi > 0$. Continuation profits can be expressed in terms of φ. It can be shown that continuation profits $\frac{d\tilde{\pi}_2}{d\varphi} > 0$ which implies that the high-quality firm prefers maximal differentiation. Since $p_1 = c(1 + \beta + \phi)/(1 + \beta)$ in price equilibrium, $(p_1 - c)/p_1 = \phi/(1 + \beta + \phi)$.

$$\tilde{\pi}_1(\varphi) = \alpha y \frac{(\phi(\varphi))^2}{e^{\phi(\varphi)}(1+\beta+\phi(\varphi))^2 - (1+\beta)^2 + (\phi(\varphi))^2}$$

In order to obtain prices and qualities in subgame perfect equilibrium of the two-stage game I look at the first-order condition of profit-maximization of firm 1.

$$\frac{d\tilde{\pi}_1}{d\varphi} = \frac{d\tilde{\pi}_1}{d\phi}\frac{d\phi}{d\varphi} = 0$$

$$\iff 2(1+\beta)^2 e^\phi + 2(1+\beta)\phi e^\phi - \phi^2 e^\phi - (1+\beta)^2 - \phi^2 = 0$$

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The function on the left-hand-side has a unique zero for $\phi > 0$. Any equilibrium with differentiated products has to satisfy $\phi > 0$. The solution to this equation is denoted by $\phi(\beta)$. Clearly, $d\phi(\beta)/d\beta > 0$. Note that $\lim_{\beta \to \infty}(2(1+\beta) - \phi(\beta)) = 0$. The distance approaches zero quite "fast": $\phi(0) \approx 1.74299$, $\phi(1) \approx 3.93588$, $\phi(2) \approx 5.98747$, $\phi(3) \approx 7.99776$. From $\phi(\beta)$ follow prices and qualities. From the first-order conditions of profit maximization it follows that

$$\lim_{\beta \to \infty} p_1(\beta) = \lim_{\beta \to \infty} \frac{1 + \beta + \phi(\beta)}{1 + \beta} c = \lim_{\beta \to \infty} \frac{1 + \beta + 2(1 + \beta)}{1 + \beta} c = 3c.$$

Since $\phi(\beta)/(1+\beta)$ is increasing, $dp_1(\beta)/d\beta > 0$ in subgame perfect equilibrium. Hence, the properties of (4) hold for firm 1 for all β.

Define the function $\tilde{\varphi} : \Re_+ \times \Re_+ \longrightarrow \Re_+$ which takes values

$$\tilde{\varphi}(\beta, \phi) = \left(\phi^\beta \left(e^\phi(1 + \beta + \phi) - (1 + \beta - \phi) \right) \right)^{\frac{1}{1+\beta}}. \tag{7}$$

Denote $\phi^{\lim}(\beta) = 2(1 + \beta)$. Replacing $\phi(\beta)$ by $\phi^{\lim}(\beta)$ in $\tilde{\varphi}(\beta, \phi(\beta))$ gives a solution $\varphi^{\lim}(\beta)$.

$$\varphi^{\lim}(\beta) = 2^{\frac{\beta}{1+\beta}} (3e^{2(1+\beta)} + 1)^{\frac{1}{1+\beta}} (1 + \beta).$$

Clearly, $\lim_{\beta \to \infty} \varphi(\beta) = \lim_{\beta \to \infty} \varphi^{\lim}(\beta) = \infty$ and $d\varphi(\beta)/d\beta \approx d\varphi^{\lim}(\beta)/d\beta$ for β "large". Note that φ increases approximately linearly in β for β "large". It is easily checked numerically that also for small β the inequality $d\varphi(\beta)/d\beta > 0$ holds. Hence, the properties of (5) hold for all β. Since $p_2 = \left((\varphi(\beta))^{1+\beta}/(\phi(\beta))^\beta + 1 + \beta - \phi(\beta) \right) /(1+\beta)$ property (5) implies that

$$\lim_{\beta \to \infty} p_2(\beta) = \lim_{\beta \to \infty} \frac{(\varphi^{\lim}(\beta))^{1+\beta}}{(\phi^{\lim}(\beta))^\beta (1 + \beta)} - 1 = \lim_{\beta \to \infty} \left(3e^{2(1+\beta)} \right) = \infty.$$

The formula shows that p_2 increases approximately exponentially in β for β "large". Since $\log p_2 = \log p_1 + \phi$ and $dp_1(\beta)/d\beta > 0$ and $d\phi(\beta)/d\beta > 0$, one has $dp_2(\beta)/d\beta > 0$.

For β "large", profits of firm 1 in subgame perfect equilibrium are approximately

$$\pi_1(\beta) \approx \alpha y \frac{4}{9e^{2(1+\beta)} + 3}$$

Since $\lim_{\beta \to \infty}(\phi^{\lim}(\beta) - \phi(\beta)) = 0$, one has $\lim_{\beta \to \infty} \pi_1(\beta) = 0$. Consequently, the market share of firm 2 approaches 1 and $\lim_{\beta \to \infty} \pi_2(\beta) = \alpha y$ (which is the monopoly profit) because p_2 turns to infinity.

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