NON-ADMISSIBILITY AND THE SPECIFICATION OF UNOBSERVED COMPONENTS MODELS[*]

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A B S T R A C T

We deal with the problem of decomposing a time series into the sum of unobserved components as in detrending or seasonal adjustment. In particular, we analyze the situation in which the decomposition into orthogonal balanced components as performed by the ARIMA-Model-Based method is non-admissible. We show that considering top-heavy models for the components can help to solve the problem. The top-heavy decomposition is derived and the improvement achieved is illustrated by an application to a class of models often encountered in practice. Also, applying this procedure to a particular time series allows us to draw a comparison with the results yielded by the AMB decomposition of an approximated model, by using an ad-hoc filter such as X11-ARIMA, and by direct specification of the Structural Time Series models.

KEYWORDS: Signal Extraction; Unobserved Components ARIMA; Structural Time Series; Admissible Decompositions; Seasonal Adjustment; X11-ARIMA.

1 Introduction

Economists and short-term macroeconomic policy makers have shown some interest in focusing their analysis on some specific movements of time series. Typically, business cycle analysis concentrates on long term movements and cyclical fluctuations. Macroeconomic policy making and monitoring center on seasonally adjusted series, which come out of a decomposition into seasonal plus nonseasonal components.

A common practice is thus to estimate trends, seasonality and seasonally adjusted series. To do so, there are several procedures available. One set of methods involves the use of moving average filters, such as the Hodrick-Prescott (1980) filter for detrending time series, and the X11-ARIMA seasonal adjustment procedure; see Shiskin et al. (1967) and Dagum (1988). These methods are empirical and ad-hoc in the sense that they do not depend on the statistical properties of the series under analysis. In contrast, explicit modelling of the components characterize both the method of Structural Time Series (STS) models, developed by Engle (1978), Gersch and Kitagawa (1983), Harvey and Todd (1983), and Harvey (1989); and the ARIMA-model-based (AMB) approach put forward by Box, Hillmer and Tiao (1978), Burman (1980), Hillmer and Tiao (1982), and Bell and Hillmer (1984) (see also Maravall, 1996). At present, some units of the Statistical Office of the European Community (EUROSTAT), the Bank of Italy and the Bank of Spain are using the AMB approach for seasonally adjusting time series and for estimating trend components. See, for example, EUROSTAT (1996), Bank of Italy (1997) and Bank of Spain (1993).

The AMB approach starts from the (reduced-form) model for the observed series and derives the (structural) unobserved components models which are compatible with the "observed" one. There are time series models, however, for which the AMB decomposition cannot be drawn because the components spectra would take some negative values. These models, which yield the so-called "non-admissible decompositions", are typically met for series embodying a very unstable pattern. The existence of models which cannot be decomposed is an important drawback of the AMB approach since neither ad-hoc filtering nor structural modelling face that problem. In fact, the STS methodology follows the opposite route and proceeds by directly specifying the (structural) models for the components. Then, the parameters of these models are estimated through the information embedded in the observed series and the problem of negative values in spectral functions does not arise. Ad-hoc filtering consists of the direct application of pre-existing filters, so the component model is never explicitly stated and the decompositions are always admissible.

This paper discusses the issue of non-decomposable models and investigate whether a solution can be found in the AMB framework. Because it concerns time series embodying an unstable component, the problem is to find a modelling flexible enough to adequately represent unstable patterns. We shall focus on the possibility of extending the order of the components MA polynomial, i.e. considering top-heavy components models (in the terminology of Burman (1980), when the order of the autoregressive polynomial is greater (smaller) than the order of the moving average polynomial a model is said to be bottom-heavy (top-heavy); when the orders are equal the model is termed balanced.) The ability of top-heavy models to describe unstable patterns was pointed out in Findley (1996, p.391), but until now that modelling was only handled along the lines of the structural approach; see for example the data generation process of Ghysels et al. (1996). If top-heavy decompositions seem adequate for the treatment of series characterized by very unstable patterns, reduced-form decompositions design unobserved components models balanced in orders. In the present paper, we show how top-heavy decompositions can be obtained in the reduced-form approach, and we demonstrate that such decompositions can solve the problem of non-admissible decompositions encountered in the AMB methodology.

Sections 2 present the general model specification and section 3 discusses the link between non-admissibility and unobserved component model specification. General results about top-heavy decompositions are derived in section 4. The top-heavy modelling is then applied in section 5 to a time series characterized by a fast evolving seasonal pattern, so as to compare the approximate solution available in the AMB approach, the performance of ad-hoc filtering with X11-ARIMA, and the modelling flexibility of the STS framework. Finally, extensions and conclusions are presented in section 6, while proofs of theorems are gathered in the Appendix.

2 Model Specification

We consider the problem of decomposing an observed time series into orthogonal unobserved components according to

$$x_t = s_t + n_t + u_t \tag{2.1}$$

where s_t is a seasonal component, n_t a trend and u_t an irregular component catching the noise embodied in the series. The irregular component u_t is supposed to be a white noise with variance V_u. For the components s_t and n_t, it is assumed that they are realizations of stochastic linear processes as in

$$\begin{aligned} \phi_s(B)\, s_t &= \theta_s(B)\, a_{st}, \\ \phi_n(B)\, n_t &= \theta_n(B)\, a_{nt}, \end{aligned} \tag{2.2}$$

where $\phi_\bullet(B)$ and $\theta_\bullet(B)$ denote finite polynomials in the lag operator B, having all roots on or outside the unit circle. The variables a_{st} and a_{nt} are independent white noise with variances V_s and V_n, respectively. The polynomials $\phi_s(B)$ and $\phi_n(B)$ are prime, while the MA polynomials $\theta_s(B)$ and $\theta_n(B)$ share no unit roots in common.

Equations (2.1) and (2.2) imply that the observed series x_t follows an ARIMA model of the type

$$\phi(B)\, x_t = \theta(B)\, a_t, \tag{2.3}$$

where a_t is a white noise with variance V_a. Without loss of generality, we set $V_a = 1$. The respective orders of the polynomials $\phi(B)$, $\phi_s(B)$, $\phi_n(B)$, $\theta(B)$, $\theta_s(B)$, $\theta_n(B)$ will be denoted p, p_s, p_n, q, q_s and q_n. In order to ease the presentation, we concentrate on observed series models which are either bottom-heavy or balanced in orders ($q \leq p$). The case of top-heavy models for the observed series would only add undesired notational complications.

The polynomial $\phi(B)$ can be obtained as the product $\phi(B) = \phi_s(B)\phi_n(B)$. In the AMB approach, the roots of the polynomial $\phi(B)$ are allocated to the polynomials $\phi_s(B)$ and $\phi_n(B)$ according to the patterns that the components are expected to display. For instance, all positive roots of $\phi(B) = 0$ will be assigned to the trend component since they imply a spectral peak at the zero-frequency which reflects infinite period oscillations typical of a long term evolution. The MA part of (2.3) is related to the models for the components through

$$\theta(B)\, a_t = \phi_n(B)\theta_s(B)\, a_{st} + \phi_s(B)\theta_n(B)\, a_{nt} + \phi(B)\, u_t\,. \tag{2.4}$$

The derivation of the moving averages polynomials $\theta_s(B)$ and $\theta_n(B)$, and of the innovations variances V_s, V_n, V_u is not immediate. Use must be made of the constraint (2.4) which insures the coherence of the decomposition, but in general (2.4) does not have a unique solution and, unless some additional restrictions are imposed, the unobserved components models are not identified. Further, to be acceptable, any solution of (2.4) must imply nonnegative components spectra. We now discuss how the specifications of unobserved components models considered in practice face the constraint of nonnegative components spectra.

3 Identification and Admissibility

The problem of non-admissibility of a decomposition is readily understood when discussed in the frequency domain. Let $g_n(w)$ and $g_s(w)$, $w \in [0, \pi]$, denote the spectra of the trend and the seasonal components, defined as

$$g_i(w) = V_i \frac{\theta_i(e^{-iw})\theta_i(e^{iw})}{\phi_i(e^{-iw})\phi_i(e^{iw})}$$

where $i \equiv n, s$; while the spectrum of the process for the observed series $g_x(w)$ is given by

$$g_x(w) = V_a \frac{\theta(e^{-iw})\theta(e^{iw})}{\phi(e^{-iw})\phi(e^{iw})}$$

When the components are nonstationary $g_s(w)$, $g_n(w)$ and $g_x(w)$ are sometimes referred to as pseudo-spectra (see Harvey, 1989).

The orthogonality assumption implies that the components spectra must aggregate to the spectrum of the observed series. Hence,

$$g_x(w) = g_s(w) + g_n(w) + V_u$$

and identification of a particular decomposition consists of finding two polynomials $G_i(w) = G_{i0} + \sum_{j=1}^{q_i} G_{ij} \cos jw$, $i \equiv s, n$, and a constant k verifying

$$g_x(w) = \frac{G_s(w)}{\mid \phi_s(e^{-iw}) \mid^2} + \frac{G_n(w)}{\mid \phi_n(e^{-iw}) \mid^2} + k. \tag{3.1}$$

The derivation of the unknowns $k, G_{s0}, \cdots, G_{sq_s}, G_{n0}, \cdots, G_{nq_n}$ can be carried out by removing denominators in (3.1), developing the products of cos-terms using standard trigonometric formula, and equating the resulting terms in $\cos jw$ in the r.h.s. and the l.h.s. of (3.1). This yields a linear system of $1 + \max(q_n + p_s, q_s + p_n, p)$ equations where some restrictions must be imposed in order to get a unique solution for the $q_s + q_n + 3$ unknowns. Once an identifying assumption is made, the solution obtained is said to be admissible if k, $G_s(w)$ and $G_n(w)$ are nonnegative for all w. Only in that case the functions $G_i(w)$, $i \equiv s, n$, correspond to spectra of MA processes while k can be interpreted as an innovation variance. It is then possible to write

$$g_i(w) = \frac{G_i(w)}{\mid \phi_i(e^{-iw}) \mid^2} = V_i \frac{\mid \theta_i(e^{-iw}) \mid^2}{\mid \phi_i(e^{-iw}) \mid^2} \tag{3.2}$$

and $V_u = k$.

The nonnegativity constraint of the functions $G_i(w)$ and the constant k is closely related to the identifying assumption chosen. In unobserved components analysis, there are two main alternative (although related) ways of identifying a particular decomposition. A first one, which permits identification in Structural Time Series models, consists of imposing the following condition (see Hotta, 1989)

A1) Assumption 1: $q_s < p_s$ and $q_n < p_n$.

Assumption 1 implies bottom-heavy models for the components. However, given a model for the observed series, nothing guarantees that the bottom-heavy decomposition that Assumption 1 would isolate can be accepted. Consider for example an IMA$_2(1,1)$ model

$$\nabla_2 \, x_t = (1 + \theta B^2) a_t. \tag{3.3}$$

This model is considered by Hillmer and Tiao (1982), and, when θ is equal to zero, reduces to the prototypical seasonal adjustment model discussed in Maravall and Pierce (1987). A time series characterized in that way can be seen as made up of a trend n_t, a seasonal component s_t, and an irregular u_t. Factorizing the AR polynomial ∇_2 and imposing Assumption 1 leads to the specification given by

$$(1+B)\, s_t = a_{st},$$

$$\nabla\, n_t = a_{nt}. \tag{3.4}$$

Solving the system (3.1) yields the bottom-heavy spectra $g_s^{BH}(w)$ and $g_n^{BH}(w)$:

$$\begin{aligned} g_s^{BH}(w) &= \frac{(1+\theta)^2}{4}\frac{1}{2+2\cos w}, \\ g_n^{BH}(w) &= \frac{(1+\theta)^2}{4}\frac{1}{2-2\cos w}, \end{aligned} \tag{3.5}$$

with remaining term $k^{BH} = -\theta$. Thus, for positive θ this constant cannot represent the irregular component variance. The subset of the parameter space for which the $\text{IMA}_2(1,1)$ model admits a decomposition into bottom-heavy components is therefore $\theta < 0$, for which we have $V_u = -\theta$. Conversely, $\theta > 0$ gives the non-admissible region of the parameter space.

Notice that if the "structural" models are specified directly as in (3.4) the non-admissibility issue is no longer relevant. However, if the series follows a process like (3.3) with $\theta > 0$, the STS model specification is likely to produce bad fit, failing to catch a positive second order correlation in the stationary transformation of the series.

When a bottom-heavy decomposition cannot decompose a time series model, it might still be possible to solve the problem by redistributing some noise between the components. For instance, in expressions (3.5) it is readily seen that the minimum of the seasonal and trend spectra is $\epsilon = \min g_s^{BH}(w) = \min g_n^{BH}(w) = (1+\theta)^2/16$. Subtracting ϵ from both spectra, the remaining term becomes $k^{CB} = k^{BH} + 2\epsilon$, where CB stands for "canonical balanced" and its meaning will be made clear below. Since ϵ is positive, $k^{CB} > k^{BH}$, and as long as $2\epsilon > -k^{BH}$, the new decomposition will be admissible with irregular variance $V_u = k^{CB}$. Notice that, removing a constant from a bottom-heavy function yields a function with numerator and denominator balanced in orders. Hence, modifying the orthogonal noise allocation between components leads to the following assumption

A2) Assumption 2: $q_s = p_s$ and $q_n = p_n$.

Assumption 2, which is mostly used in the ARIMA-Model-Based methodology, does not guarantee identification of a unique decomposition. In the above $\text{IMA}_2(1,1)$ example, identification was achieved by removing all the orthogonal noise from the bottom-heavy spectra and, thus, letting the trend and the seasonal component as "clean" as possible of "superimposed" noise. The components specified in this way are termed *canonical* (see Box, Hillmer and Tiao, 1978, and Pierce, 1978), and it is a well known result that the canonical decomposition maximizes the irregular component variance. Thus, the inequality $k^{CB} \geq k^{BH}$ always holds, and it can also be that $k^{CB} \geq 0 > k^{BH}$. In the $\text{IMA}_2(1,1)$ example, values of θ within the interval $[0, .1716]$ implies that model (3.3) does admit a decomposition into balanced components but does not into bottom-heavy components (see Hillmer and Tiao, 1982, pp.66-67). In general, the property of canonical decompositions to maximize the irregular component variance implies that the region of the model parameters space where no decomposition is admissible will always be smaller when the components models are specified balanced rather than bottom-heavy.

When a canonical balanced decomposition is admissible the spectra of the components are readily obtained through

$$
\begin{aligned}
g_s^{CB}(w) &= g_s^{BH}(w) - \alpha_s \\
g_n^{CB}(w) &= g_n^{BH}(w) - \alpha_n
\end{aligned} \tag{3.6}
$$

with

$$\begin{aligned} \alpha_s &= \min_w[g_s^{BH}(w)] \\ \alpha_n &= \min_w[g_n^{BH}(w)] \end{aligned} \tag{3.7}$$

and $V_u = k^{CB} = k^{BH} + \alpha_s + \alpha_n$.

Some insights into the causes of the non-admissibility issue helps in understanding why it can happen that, for a given a model, a balanced decomposition can be acceptable while a bottom-heavy one is not. In the $\text{IMA}_2(1,1)$ model, the problem of non-admissibility occurs when θ takes large and positive values. These values are related to very unstable seasonal patterns: the larger θ, the more unstable the seasonality. All non-decomposable models that are encountered in practice embody a very unstable component. Balanced decompositions allow more flexibility in the components' structure and, hence, balanced models are more suitable to describe unstable patterns. Consequently, the region of the parameters space where a decomposition is non-admissible is smaller when balanced components are considered.

4 Top-heavy Models for Unobserved Components

In the previous section we argued that the problem of non-admissibility can be seen as a matter of finding a decomposition apt to represent particularly unstable patterns. Balanced model specifications may still be limited in their flexibility. In the example of (3.3), when $\theta > .1716$ the balanced decomposition is not admissible. For this reason, we investigate the viability of top-heavy models for the components.

The hypothesis of top-heavy components is stated as

A3) Assumption 3: $q_s = p_s + 1$ and $q_n = p_n + 1$.

Under A3, equation (3.1) yields a system of $p+2$ equations to identify $p+5$ unknowns.

There are thus 3 identification restrictions to define, and the point is to find a sensible procedure.

In the same way that a balanced decomposition is derived by adding a constant to the bottom-heavy spectra, a top-heavy decomposition can be obtained by adding a term of order 1 in $\cos w$ to the bottom-heavy spectra. Letting γ, α_s and α_n denote real scalars, we consider

$$\begin{aligned} g_s^{TH}(w) &= g_s^{BH}(w) - \alpha_s + \gamma \cos w \\ g_n^{TH}(w) &= g_n^{BH}(w) - \alpha_n - \gamma \cos w \end{aligned} \tag{4.1}$$

with

$$\begin{aligned} \alpha_s &= \alpha_s(\gamma) = \min_w [g_s^{BH}(w) + \gamma \cos w] \\ \alpha_n &= \alpha_n(\gamma) = \min_w [g_n^{BH}(w) - \gamma \cos w]. \end{aligned} \tag{4.2}$$

and $V_u = k^{TH} = k^{BH} + \alpha_s(\gamma) + \alpha_n(\gamma)$.

A balanced decomposition is achieved by removing the spectrum of a white noise from the bottom-heavy functions; similarly, a top-heavy decomposition can be obtained by subtracting the spectrum of a MA(1) process from the bottom-heavy functions. Importantly, the two MA(1) processes given in (4.1) have the property that they add up to a constant and this insures a white noise irregular component. The expressions in (4.2) are such that both $g_s^{TH}(w)$ and $g_n^{TH}(w)$ have a zero as minimum. Hence, $g_s^{TH}(w)$ and $g_n^{TH}(w)$ are well defined spectra, and the related components are specified as clean of noise as possible, in agreement with the canonical requirement. Notice that $\gamma = 0$ corresponds to the canonical decomposition into balanced components.

Contrary to balanced decompositions, the canonical property does not allow to isolate a unique top-heavy decomposition. The canonical property only defines two identification restrictions and therefore γ is still to be identified. For that concern, we focus our analysis on the remaining term given by $k^{TH}(\gamma) = k^{BH} + \alpha_s(\gamma) + \alpha_n(\gamma)$; if it is nonnegative, then $k^{TH}(\gamma)$ will give the variance of the irregular component and the top-heavy decomposition will be admissible.

Before discussing in details the top-heavy decomposition procedure, we set out the properties of the noise function $k^{TH}(\gamma)$:

Theorem 1: *The quantity of noise denoted $k^{TH}(\gamma)$ which can be isolated from the components in (4.1)-(4.2), is continuous and concave in γ. Also* $\lim_{\gamma\to\infty} k^{TH}(\gamma) = \lim_{\gamma\to-\infty} k^{TH}(\gamma) = -\infty$

Proof: see Appendix.

This theorem shows that the function $k^{TH}(\gamma)$ behaves regularly, and in particular each local maximum is a global one. We shall also state the following theorem:

Theorem 2: *Assume that the functions $g_i^{BH}(w)$, $i = n, s$, admit a single global minimum. If $g_s^{BH}(w)$ and $g_n^{BH}(w)$ do not have their minimum at the same frequency, then there exists some γ such that the top-heavy decomposition (4.1)-(4.2) yields an irregular component variance larger than the one obtained with a canonical decomposition in balanced components.*

Proof: see Appendix.

Theorem 2 states that, in general, the top-heavy model decompositions will increase the total amount of noise which can be extracted from the series. Hence, the property of canonical decompositions to maximize the irregular component variance only concerns balanced decompositions, and does not hold when top-heavy models are considered. Formally, theorem 2 says that it exists $\gamma \neq 0$ such that $k^{TH}(\gamma) > k^{TH}(0) = k^{CB}$ and, as a consequence, the subset of the parameter space where a decomposition is non-admissible is smaller when top-heavy components are considered. This results requires, as a regularity condition, that the bottom-heavy functions present a unique global minimum.

We now turn back to the identification issue and, for this purpose, a graphical example will simplify the discussion. For the case of model (3.3) with $\theta = 0.5$, figure 1 shows the function $k^{TH}(\gamma)$. Notice that, as expected, at $\gamma = 0$ the function takes a negative value ($k^{TH}(0) = -.219$) since, as we saw, for $\theta = .5$ the canonical balanced decomposition of model (3.3) is non-admissible. Any choice of γ in the interval [.125; .5] would isolate an admissible top-heavy canonical decomposition.

There are two obvious criteria to identify a top-heavy decomposition. The first one is to choose γ such that the irregular component variance is equal to zero. We suggest to use this first criterion if top-heavy decompositions are regarded as a simple extension to balanced ones when these last are non-admissible. Looking at figure 1, it is seen that it leads to take $\gamma = .125$. Yet, in agreement with Theorem 1, there is another value of γ for which $k^{TH}(\gamma) = 0$ (i.e. $\gamma = .5$ in the example). But given that the larger γ, the larger the departure from balanced decompositions and the more complex the structure of the resulting top-heavy components, we prefer the decomposition obtained with the smallest γ in absolute value. The second criterion is to take γ such that the irregular component variance is maximized (i.e. $\gamma = .281$ in figure 1, yielding an irregular component variance equal to .063). It would be a "reasonable" option if top-heavy models are considered as a general decomposition method for any observed series model. However, given the long tradition of the AMB canonical analysis we believe that the first criterion should be the preferred one. This view is also supported by the fact that the STS approach is likely to estimate the irregular variance as zero when a series with unstable patterns is dealt with (see the application of section 5).

What it is still to be verified is whether the top-heavy model specification always yields an admissible decomposition. If the observed series model is a simple $\text{IMA}_2(1,1)$ as defined in (3.3), it can be easily shown that $\max(k^{TH}(\gamma)) = \min(g_x(w)) \geq 0$, and therefore, in this case, top-heavy decompositions are always admissible. This need not to be true if we consider models that are of more practical interest. For this reason we have analyzed the so-called *airline* model. This model, popularized by Box and Jenkins (1970), has shown to provide good fit to a large number of seasonal time series. It is specified as follows

$$\nabla\nabla_m x_t = (1+\theta_1 B)(1+\theta_m B^m)a_t,$$

where m corresponds to the data-periodicity (e.g. $m = 4$ for quarterly series). A time series which can be represented in that way embodies a trend, a seasonal pattern and an irregular component, but not every value of θ_m in the interval $(-1, 1)$ allows a decomposition into balanced unobserved components (see for example Hillmer and Tiao, 1982). Problems arise for positive θ_m which here again characterize unstable patterns for the seasonal component.

For quarterly and monthly data, figure 2 display the admissible region of parameter space for balanced decompositions and for top-heavy decompositions. In both cases, the airline model always admit a canonical decomposition into balanced components below the solid line. The dotted line shows the frontier under which a top-heavy decomposition would be admissible. The region between the two curves shows the improvement obtained by increasing the order of the MA polynomial. Although the gain is much more pronounced for quarterly series, it is seen that the procedure we have built reduces the parameter space of models for observed series that yield non-admissible decompositions. In the final section, we shall give some insight into the possibility of increasing further the order of the components moving average part to be able to decompose any model.

It is interesting to notice that for both quarterly and monthly airline, there is one point of the parameter space for which no gain is obtained. These points, $(\theta_1, \theta_4) = (-.111, .374)$ and $(\theta_1, \theta_{12}) = (-.584, .337)$, are two points where $\gamma = 0$ correspond to a maximum of the function $\alpha_s(\gamma) + \alpha_n(\gamma)$. It can be verified that at these two points the regularity condition of theorem 2, concerning the existence of an unique global minimum in the component spectrum, is not fulfilled.

Finally, it is interesting to describe how the top-heavy models for the components are derived in practice. Given a model for the observed series, the bottom-heavy functions

$g_s^{BH}(w)$, $g_n^{BH}(w)$ plus the remaining term k^{BH} are first derived by partial fraction decomposition, as detailed in Burman (1980). Secondly, the modelling given in (4.1)-(4.2) is considered, and if it is checked that $g_s^{BH}(w)$ and $g_n^{BH}(w)$ have a unique global minimum at different frequencies, a grid on the parameter γ starts at $\gamma = 0$. The direction of the grid is indicated by the relative position of the spectra minima, as detailed in the proof of theorem 2. Continuity of $\alpha_s(\gamma) + \alpha_n(\gamma)$ ensures the convergence of the grid search. At each step, we compute the minima of $[g_s^{BH}(w) + \gamma \cos w]$ and $[g_n^{BH}(w) - \gamma \cos w]$ and the grid on γ is conducted until the sum of the minima plus k^{BH} is equal to zero. Finally, application of the formulae in (4.1) yields the spectra of the two components, both following a top-heavy model and presenting a zero in the spectrum, while the irregular is null.

In practice, a Newton-Raphson procedure can improve the numerical efficiency of the algorithm, and very few steps are needed. Usually, the number of iterations needed for convergence ranges from 2 to 5 depending on the amount of numerical accuracy required for the solution. In our calculations the algorithm was stopped when the sum of the minima plus k^{BH} was less then 10^{-12} in absolute value. Also notice that if one wishes to identify the top-heavy decomposition that maximizes the variance of the irregular component the procedure is analogous to the one described above except for running the grid on γ until the maximum of $k^{TH}(\gamma)$ is reached.

5 An Application to the EUBMP Series

Consider the European Basic Metals Production (EUBMP) series. It is a monthly series, whose sample covers 70 observations from January 1990 to October 1995. The series (in logs) is displayed in figure 3 and exercised eyes may see that it is characterized by a trend which becomes upward shaped at the end of 1993, and an unstable, rapidly evolving, seasonal component. In particular, the dips related to December seem to go vanishing. Estimating an airline model on the log transformed series, we obtain:

$$\nabla\nabla_{12}x_t = (1+\theta_1 B)(1+\theta_{12}B^{12})a_t \tag{5.1}$$

$$\theta_1 = -.412\,(.12)\,, \qquad \theta_{12} = .304\,(.15)\,, \qquad \text{and} \qquad \sigma_{(a_t)} = .040. \tag{5.2}$$

The relatively large and positive value of θ_{12} is in agreement with the unstable seasonal pattern shown by the series. The fit of the model is rather good since the statistics for residual randomness indicates no autocorrelations: the Ljung-Box statistics on the first 24 autocorrelations is not significant, $Q_{24}(a_t) = 25.61$. Further, the residuals seem normally distributed: for the skewness (S) and kurtosis (K) statistics, we have $S = -.11$ (.33) and $K = 2.34$ (.65).

The series is now decomposed into a trend n_t, a seasonal s_t plus a noise component u_t with models:

$$\begin{aligned} \nabla^2 \; n_t &= \theta_n(B)\; a_{nt}, \\ S(B)\; s_t &= \theta_s(B)\; a_{st}, \end{aligned} \tag{5.3}$$

where $S(B) = 1 + B + \cdots + B^{11}$. Specifying bottom-heavy model for n_t and s_t so that the MA polynomials $\theta_s(B)$ and $\theta_n(B)$ are of respective order 10 and 1, we get as remaining term: $k^{BH} = -.125$, which cannot be accepted. Allowing the models for the trend and the seasonal to be balanced and identifying them with the canonical requirement, then the remaining term becomes $k^{BH} + \alpha_s(0) + \alpha_n(0) = k^{CB} = -.089$, still not acceptable. We then move on to top-heavy models, and using the procedure described in section 4, we get:

$$\theta_n(B) = (1 - .831B - .839B^2 + .992B^3) \qquad V_n = .039$$

$$\begin{aligned} \theta_s(B) &= 1 + 1.104B + 1.258B^2 + 1.223B^3 + 1.014B^4 + .725B^5 + .453B^6 + \\ &+ .243B^7 + .061B^8 - .164B^9 - .433B^{10} - .521B^{11} - .081B^{12} \end{aligned}$$

$$
\begin{aligned}
V_s &= .485 \\
V_u &= 0.0
\end{aligned} \tag{5.4}
$$

The corresponding spectra are plotted in figure 4 together with the one of the observed series. As expected, the top-heavy model specification has produced a stable trend and very unstable seasonal component: the trend spectrum is very concentrated in the low frequencies region, while the seasonal component spectrum captures all the power of the series spectrum in the high frequencies region.

Having identified top-heavy models for the components which do not yield negative spectra, it is possible to estimate them. This is done by means of the Wiener-Kolmogorov filter (see Burman (1980)). The output can then be used to evaluate other possible answers to the problem of finding an admissible decomposition. We shall focus on the AMB decomposition of an approximated model, on the use of an ad-hoc filter, and on the direct specification of the components.

Given that model (5.1) is not decomposable into balanced components, a possible answer in the AMB framework consists of modifying slightly (5.1) so as to decompose an approximated model. This is the solution implemented in the seasonal adjustment software SEATS (see Gomez and Maravall, 1996), where in particular the default option is to set θ_{12} equal to zero. So the approximated model is:

$$
\nabla\nabla_{12}x_t = (1 - .412B)a_t \tag{5.5}
$$

This specification implies a loss in the goodness of fit: the Ljung-Box statistics gives $Q_{24}(a_t) = 35.71$, which indicates some correlations left in the residuals. Figure 5 shows the spectrum of the resulting adjusted series together with that of the top-heavy decomposition (5.3): it is clearly seen that the model approximation leads to a spectrum of the adjusted series above the spectrum of series. In particular, the spectrum of the adjusted series in the approximated model has an excess of power around the fundamental seasonal frequency. Estimating the components with SEATS, the seasonally adjusted series is reasonably close to the top-heavy one (see figure 6); yet, as a likely consequence of the power excess around the $2\pi/12$-frequency, it embodies some short-term movements of small amplitude that the top-heavy estimator does not show. To check whether this is related to the model approximation, we modify the specification and consider a value of θ_{12} on the borderline of the non-admissible region of the parameter space of the airline model:

$$\nabla\nabla_{12}x_t = (1 - .412B)(1 + .239B^{12})a_t.$$

The decomposition of this model into balanced components yields an irregular with null variance. The seasonally adjusted series in this "closer" model is plotted on figure 7 together with the top-heavy estimator: they are seen to be very close. This suggests that the trend smoothing performed by the top-heavy modelling was appropriate, and confirms that the movements in the seasonally adjusted series derived from (5.5) are related to the approximation.

With ad-hoc filters, the non-admissibility problem never arises since the components are estimated by empirical filtering of the series. The most known software for decomposing time series by ad-hoc filtering is incontestably X11-ARIMA and its variants, as developed and diffused by the Bureau of the Census (see Shiskin et al., 1967). We examine the answer provided by the X11-ARIMA procedure. Setting the length of the Henderson trend filter to 13 and choosing a 3×3 seasonal filter for unstable seasonality, the X11-ARIMA output file did not report any problem: the statistic $M7$ for fastly evolving seasonality took the value .174 while the summary statistic Q was such that $Q = .21$ (see Dagum, 1988). Yet the irregular to seasonal ratio suggested to use a 3×5 seasonal filter, but we discarded this possibility since the 3×3 filter has a larger band pass structure than the 3×5 filter and it is thus more able to catch an unstable seasonal pattern, while as expected the 3×5 seasonal filter could not improve the decomposition.

Figure 6 clearly shows that the X11 seasonally adjusted series presents a very erratic pattern. To understand the origin of this erratic behaviour, we derived the spectra of the adjusted series estimator obtained with X11, with the AMB decomposition of the approximated model (5.5), and with the top-heavy decomposition of the exact model. These spectra are related to historical estimators; for a simple way to derive X11 historical adjustment filters, see Bell and Monsell (1992). Figure 8 shows that the X11 filter was not able to remove all the seasonality from the series. Given that among the X11 filters, the 3×3 has the largest band-pass around the seasonal frequencies, there is little that can be done to improve the result. Figure 9, where the spectra of the seasonal component estimators are plotted, illustrates the incapacity of the X11 ad-hoc filters to deal with a time series characterized by a very unstable seasonality. Since ad-hoc filters do not adapt to the series under analysis, they can not deliver a satisfying result for time series lying outside of the range of patterns for which the filters are designed.

Figures 10 and 11 display the seasonal estimators yielded by the three methods for the months of August and December, the most important ones. It is seen that the top-heavy specification is the decomposition which allows the most flexibility in the seasonal component. The balanced decomposition of the approximated model yields a pattern which is reasonably close to the top-heavy one. The most stable seasonality is the one given by empirical filtering, but this stability is clearly not in agreement with the characteristics of the data.

We now examine the answer provided by Structural Time Series models. This framework guarantees that the problem of non-admissibility never occurs: the model for the components is directly specified and estimated using the information embedded in the series. However, the specification typically identifies bottom-heavy models for the components. It is then of interest to study the flexibility that structural models allow in practice. The basic STS modelling as discussed by Harvey (1989) offer two possibilities to model the seasonal component: using a pure AR representation or using a combination of trigonometric terms at the seasonal frequencies. We shall try the two alternatives. For the trend, we consider a second order random walk, and incorporate an irregular component as part of the series structure. So we have two models which correspond to two versions of the Basic Structural Model (see Harvey, 1989), that we shall denote BSM1 and BSM2, respectively.

The computations were executed with the software STAMP. The prediction error variances found in the two cases were close to σ_a^2(see 5.2); in order to draw comparisons, the variances associated to the unobservables have been rescaled in σ_a^2-units. For BSM1, the decomposition found verifies:

$$\nabla^2 \ n_t = (1 - .850B)\ a_{nt} \qquad V_n = .158,$$

$$S(B)\ s_t = a_{st} \qquad V_s = .182 \tag{5.6}$$

and $V_u = 0.0$. The model given for the trend is clearly the reduced form of the model fitted (see Maravall, 1985). As in the top-heavy decomposition, the irregular variance is set to zero so that trend and seasonally adjusted series are equivalent. The model for the trend implies that it embodies too much short-term fluctuations; see figure 12. This is mainly due to the pure AR model chosen for the seasonal: as can be seen on figures 13 and 14, this modelling does not allow for much flexibility in the seasonal pattern.

For BSM2 and using the reduced form representation of the trigonometric seasonality, the decomposition verifies:

$$\nabla^2 \ n_t = a_{nt} \qquad V_n = .009, \tag{5.7}$$

$$\begin{aligned} S(B)\ s_t &= (1 + .737B + .628B^2 + .430B^3 + .361B^4 + .220B^5 + \\ &+ .181B^6 + .089B^7 + .071B^8 + .020B^9 + .016B^{10})a_{st}, \\ V_s &= .471, \end{aligned} \tag{5.8}$$

and again $V_u = 0.0$: there is no orthogonal noise in the decomposition. Usually, second-order random walk specifications imply IMA(2,1) models for the trend, but in this case the value of the moving average parameter was practically 0 ($< 10^{-4}$). The trend is thus characterized by an I(2) model, with innovation variance noticeably reduced to .009. It can be seen on figure 12 that the resulting component is close to the top-heavy one. For the seasonality, the innovation variance is now increased to .471, quite close the one obtained in the top-heavy decomposition. Accordingly, figures 13 and 14 present the August and December swings and show that top-heavy seasonal and trigonometric seasonal are very close to each other. The trigonometric representation has brought enough flexibility in the description of the seasonal evolution to allow the trend to be sufficiently smooth, and the decomposition comes out as satisfactory. This is mainly due to the direct estimation of the unobserved components models, since it can be easily seen that (5.7) and (5.8) implies a reduced form for the observed series quite different from the airline model.

6 Conclusions

In this paper we analyze an important issue related to the specification of unobserved components in economic time series. We begin by discussing the problem of non-admissibility in the Arima-Model-Based approach and we show that it arises whenever one of the components (usually seasonality) displays a particularly unstable pattern. Although not directly addressable in terms of non-admissibility, this issue is also relevant when other signal extraction methods such as Structural Time Series and X11 are employed, as they may not be able to provide the flexibility required to deal with rapidly evolving components.

We then present a procedure to decompose time series into unobserved components when the AMB approach fail to yield nonnegative component spectra. This procedure is based on top-heavy models, and it can be seen as an extension of the standard canonical approach involving balanced components. Such an extension may be worth to consider since, under some conditions of regularity, it has been shown that an irregular component variance greater than the one yielded by a canonical decomposition into balanced components can be obtained.

Whether this improvement is enough to achieve a nonnegative irregular variance depends on the model for the observed series and cannot be determined a priori. However, we have computed the non-admissible region of the parameter space for two important cases, namely the monthly and quarterly airline models, and found that the improvement yielded by the top-heavy decompositions is significative. Still, a larger improvement would be possible by taking a further step in increasing the order of the MA polynomials of the components. Expressions (4.1)-(4.2) generalize readily since we can write:

$$\begin{aligned} g_s^{TH(R)}(w) &= g_s^{BH}(w) - \alpha_s + \gamma_1 \cos w + \cdots + \gamma_R \cos Rw \\ g_n^{TH(R)}(w) &= g_n^{BH}(w) - \alpha_n - \gamma_1 \cos w - \cdots - \gamma_R \cos Rw \end{aligned} \tag{6.1}$$

with α_s and α_n such that

$$\begin{aligned} \alpha_s = \alpha_s(\gamma_1, \ldots, \gamma_R) &= \min_w \left[g_s^{BH}(w) + \sum_{r=1}^{R} \gamma_r \cos rw \right] \\ \alpha_n = \alpha_n(\gamma_1, \ldots, \gamma_R) &= \min_w \left[g_n^{BH}(w) - \sum_{r=1}^{R} \gamma_r \cos rw \right] \end{aligned}$$

The remainder becomes $k^{TH(R)}(\gamma_1, \ldots, \gamma_R) = k^{BH} + \alpha_s + \alpha_n$, with the vector $\mathbf{\Gamma}= (\gamma_1, \ldots, \gamma_R)$ to identify. It can be shown that the total amount of orthogonal noise that can be extracted from the series with a top-heavy decomposition of order $R > 1$ is larger than the one obtained with the top-heavy decomposition of order 1. The main problem is to find out a unique value for $\mathbf{\Gamma}$. Selecting the decomposition for which the irregular variance is zero does not guarantee identification as there is not an unique value of $\mathbf{\Gamma}$ satisfying $k^{TH(R)}(\mathbf{\Gamma}) = 0$. A similar problem is met if we would focus on the decomposition maximizing the irregular component variance. A common sense solution suggests to select the "closest" decomposition to the balanced one by minimizing the norm of the vector $\mathbf{\Gamma}$. Notice that, although the dimension of $\mathbf{\Gamma}$ is expected to be low in practice, a solution can only be found by resorting to a very time consuming numerical approach.

Finally we apply the procedure developed to the series of European Basic Metals Production and the results are compared with those obtained with an STS decomposition, with X11 estimation and with the AMB decomposition of an approximated model. The example analyzed shows that the extreme characteristics of a time series which cannot be decomposed using a standard ARIMA-model-based approach, namely stable trend and very unstable seasonal, may make inadequate the use of an empirical filter. On the contrary, the top-heavy modelling has proved to be able to adequately represent the fastly evolving seasonal pattern of the series. Also, the canonical AMB decomposition of a closer approximated model produced good results while the Basic Structural Model with trigonometric seasonal component (BSM2) seems to catch unstable patterns in a very satisfactory way.

Furthermore, this paper has some bearing upon the importance of using a model-based approach (either ARIMA or STS based) for unobserved component estimators. Many practitioners and institutions seem to still prefer uniform filtering for estimating unobserved signals. In doing so, the problem of non-admissible decompositions is apparently avoided. The findings of this paper show that series so unstable so as to require a top-heavy decomposition presents patterns that the available ad-hoc filters cannot deal with. In any case, the debate is still open and the recent contribution of Ghysels (1997) makes a strong case in favour of ad-hoc procedures.

Appendix

Proof of Theorem 1.

By the "theorem of the maximum" (e.g. Varian (1984, p.326)), $\alpha_s(\gamma)$ and $\alpha_n(\gamma)$ are continuous in γ. Thus $k_{TH}(\gamma)$ is also continuous in γ. Furthermore, since $\min_w g_x(w) \geq k_{TH}(\gamma)$, $k_{TH}(\gamma)$ is bounded above. Now, consider any constants γ_1, γ_2 and $\lambda \in [0,1]$. Using the expressions (4.2), concavity of $\alpha_s(\gamma)$ follows from:

$$\begin{aligned}
\alpha_s(\lambda\gamma_1 + (1-\lambda)\gamma_2) &= \min_w [g_s^{BH}(w) + (\lambda\gamma_1 + (1-\lambda)\gamma_2)\cos w] = \\
&= \min_w [\lambda g_s^{BH}(w) + \lambda\gamma_1 \cos w + \\
&\quad +(1-\lambda)g_s^{BH}(w) + (1-\lambda)\gamma_2 \cos w] \\
&\geq \min_w [\lambda g_s^{BH}(w) + \lambda\gamma_1 \cos w] + \\
&\quad + \min[(1-\lambda)g_s(w) + (1-\lambda)\gamma_2 \cos w] = \\
&= \lambda\alpha_s(\gamma_1) + (1-\lambda)\alpha_s(\gamma_2).
\end{aligned}$$

It is straightforward to see that $\alpha_n(\gamma)$ shares this property, so $k_{TH}(\gamma) = k_{BH} + \alpha_s(\gamma) + \alpha_n(\gamma)$ is concave. ∎.

Proof of Theorem 2:

The canonical decomposition into balanced components defines $k_{BH} + \alpha_s(0) + \alpha_n(0)$ as a constant which would represent the irregular component variance if nonnegative. Hence the proof consists in obtaining the condition insuring that $\gamma = 0$ will not be maximum of $\alpha_s(\gamma) + \alpha_n(\gamma)$.

Let w_i^* denote the frequency where $g_i^{TH}(w)$, $i = n, s$, reaches its minimum. According to (4.1) w_i^* depends on γ, so we write $w_i^* \equiv w_i^*(\gamma)$. We shall prove that if $w_s^*(0) < w_n^*(0)$, then there exists $\eta > 0$, and an open interval $(0, \eta)$ in a neighborhood of 0, such that

$$\gamma \in (0, \eta) \quad \Rightarrow \quad k_{TH}(\gamma) > k_{TH}(0)$$

In contrast, if $w_s^*(0) > w_n^*(0)$, there exists $\eta > 0$, such that

$$\gamma \in (-\eta, 0) \quad \Rightarrow \quad k_{TH}(\gamma) > k_{TH}(0)$$

We first show that the derivatives $\frac{dw_i^*(\gamma)}{d\gamma}\mid_{\gamma=0}$, $i \equiv s, n$, exist and are continuous. The function $g_s^{BH}(w)$, $w \in [0, \pi]$, takes the form:

$$g_s^{BH}(w) = \frac{\sum_{k=0}^{p_s} a_k \cos kw}{\sum_{k=0}^{p_s} b_k \cos kw}$$

where the real numbers a_k, b_k, $k = 0, \cdots, p_s$ are obtained by partial fraction decomposition of $g_x(w)$ (see Burman, 1980). Since the ratio is bottom-heavy $a_{p_s} = 0$. The function $g_s^{BH}(w)$ is not strictly defined at the frequencies where $\phi_s(B)$ has unit roots, being asymptotically infinite. In the other points of $[0, \pi]$, the function $g_s^{BH}(w)$ is C^∞ since it is rational. In particular, it is C^∞ in some neighbourhood of its global minimum. That minimum is given by $w_s^*(0)$ and is unique by assumption. Now let us define W, U some neighbourhoods of $w_s^*(0)$ and of 0, respectively, and let $F(w, \gamma)$ be a function such that to $(w, \gamma) \in W \times U$ it associates:

$$F(w, \gamma) = \frac{dg_s^{BH}(w)}{dw} - \gamma \sin w$$

Since $w_s^*(0)$ is a minimum, it is easily seen that:

$$F(w_s^*(0), 0) = 0,$$

and that

$$\frac{dF(w_s^*(0), 0)}{dw} = \frac{d^2 g_s^{BH}(w_s^*(0))}{dw^2} > 0.$$

Given that $g_s^{BH}(w)$ is C^∞ in W, and that $F(w, \gamma)$ is linear in γ, standard results related to the implicit function theorem show that there exists a function $w(\gamma)$ which to $\gamma \in U$

associates $w(\gamma) \in W$ such that $F(w(\gamma), \gamma) = 0$ and which is moreover derivable in U. By construction that function is precisely $w_s^*(\gamma)$. The same reasoning shows the derivability of $w_n^*(\gamma)$ at $\gamma = 0$.

Omitting now the superscript BH in order to ease the presentation, from (4.2) the derivative of $\alpha_s(\gamma)$ with respect to γ is then given by:

$$\frac{d\alpha_s(\gamma)}{d\gamma} = g_s'[w_s^*(\gamma)] \frac{dw_s^*(\gamma)}{d\gamma} + \cos w_s^*(\gamma) + \gamma \frac{d \cos w_s^*(\gamma)}{d\gamma} \tag{6.1}$$

and, at $\gamma = 0$, we have

$$\frac{d\alpha_s(\gamma)}{d\gamma} |_{\gamma=0} = \cos w_s^*(0) . \tag{6.2}$$

To see this, notice that the last term of (6.1) is readily verified to be equal to 0 for $\gamma = 0$. The first term needs a more careful discussion. We distinguish two cases. If the minimum of $g_s(w)$ occurs at a corner solution, then for $w_s^*(0) = \pi$, the first term is such that

$$g_s'[w_s^*(\gamma)] |_{\gamma=0} \leq 0 \quad \text{and} \quad \frac{dw_s^*(\gamma)}{d\gamma} |_{\gamma=0} = 0 ,$$

while, for $w_s^*(0) = 0$, the derivative of $g_s(w_s^*(\gamma))$ at $\gamma = 0$ is positive.

If, instead, the minimum of $g_s(w)$ occurs in the interior of the $(0, \pi)$ interval, then, by the envelope theorem, it is

$$g_s'[w_s^*(\gamma)] |_{\gamma=0} = 0 .$$

In the some way, it is obtained that

$$\frac{d\alpha_n(\gamma)}{d\gamma} |_{\gamma=0} = - \cos w_n^*(0) . \tag{6.3}$$

Summing (6.1) and (6.2), we get:

$$\frac{d\alpha_s(\gamma)}{d\gamma} |_{\gamma=0} + \frac{d\alpha_n(\gamma)}{d\gamma} |_{\gamma=0} = \cos w_s^*(0) - \cos w_n^*(0), \tag{6.4}$$

which clearly shows that $w_n^*(0) \neq w_s^*(0)$ implies that $\gamma = 0$ is not a local maximum for $\alpha_s(\gamma) + \alpha_n(\gamma)$. Besides, $w_s^*(0) < w_n^*(0)$ implies that the sum $\alpha_s(\gamma) + \alpha_n(\gamma)$ will increase in γ around zero, and so it is for strictly positive values of γ in a neighborhood of 0 that we will obtain $\alpha_s(\gamma) + \alpha_n(\gamma) > \alpha_s(\gamma) + \alpha_n(\gamma)$. Otherwise, the result will hold for strictly negative values of γ. ∎

Figure 1: IMA$_2$(1,1); Irregular Variance in Top-Heavy Decompositions

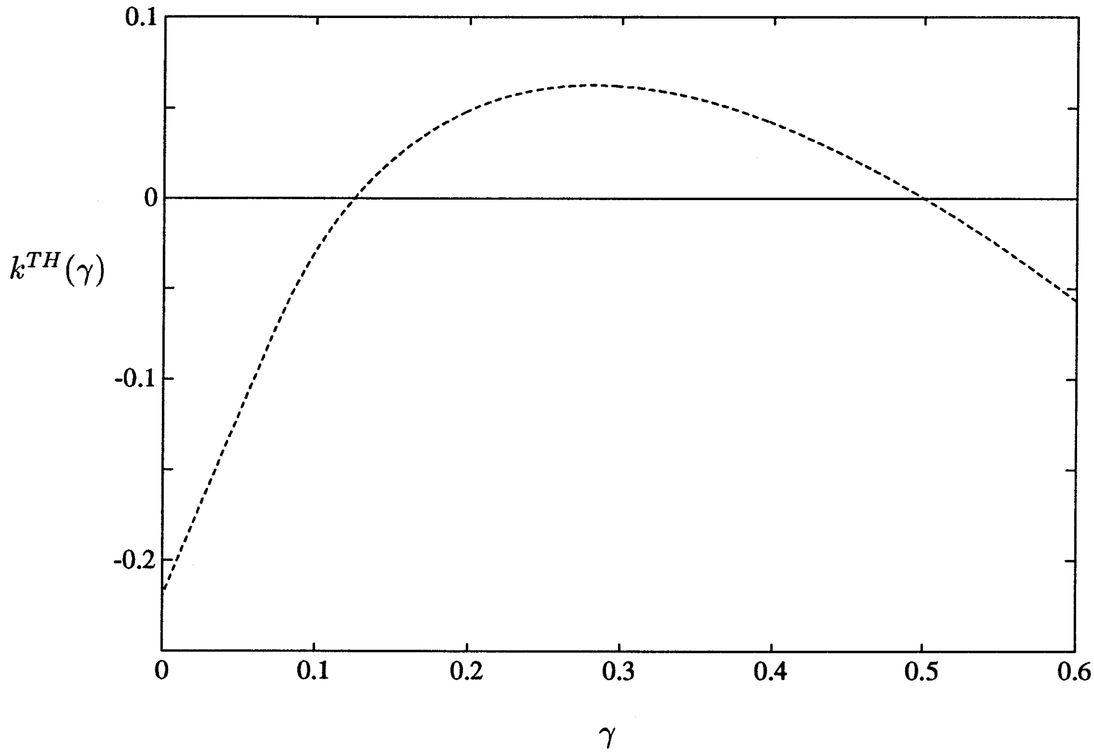
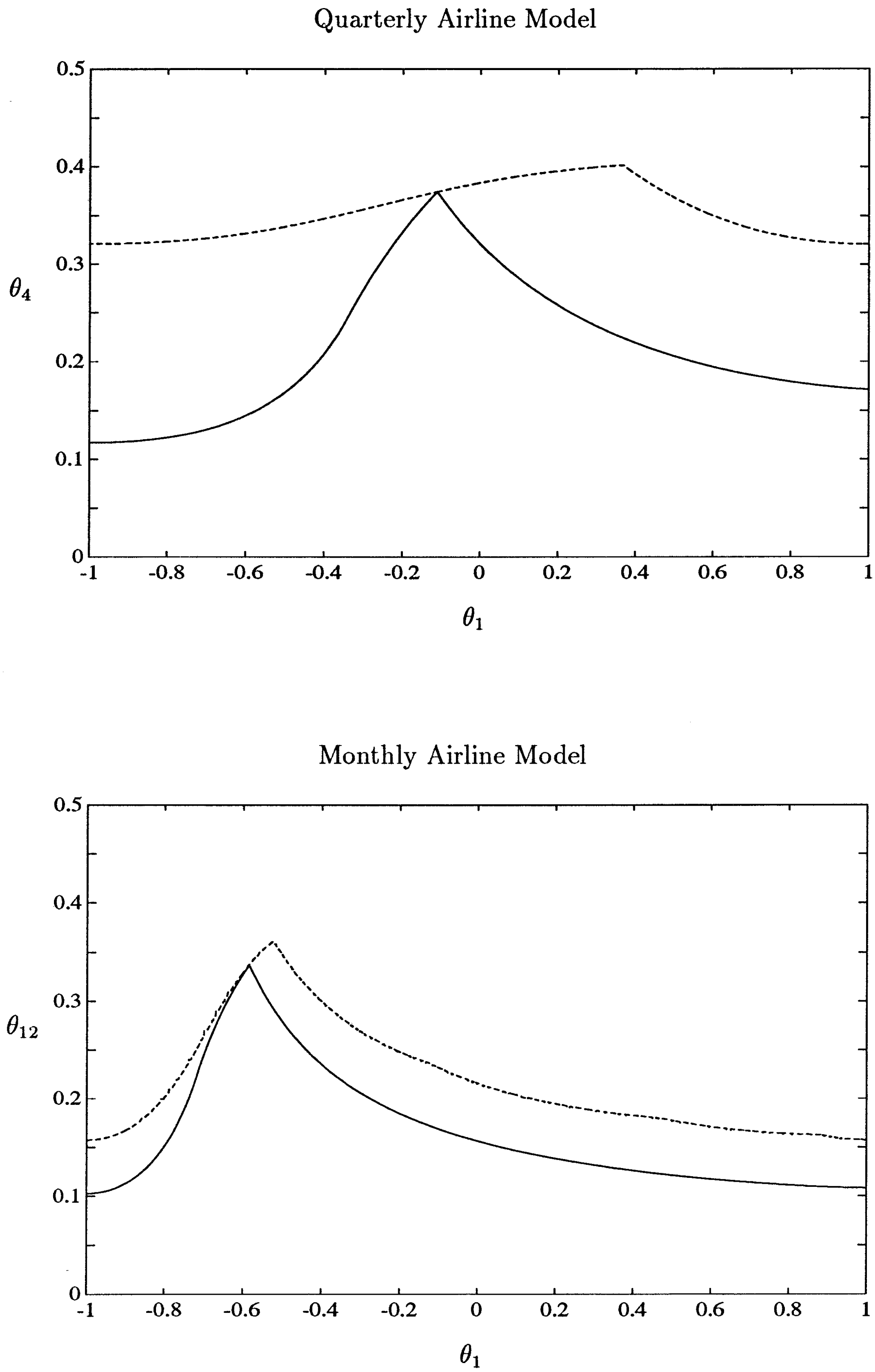


Figure 2: Non Admissible Region of Parameters Space



- - - Top-Heavy Models —— Balanced Models

Figure 3: Manufacture of Basic Metals in Europe (logs)

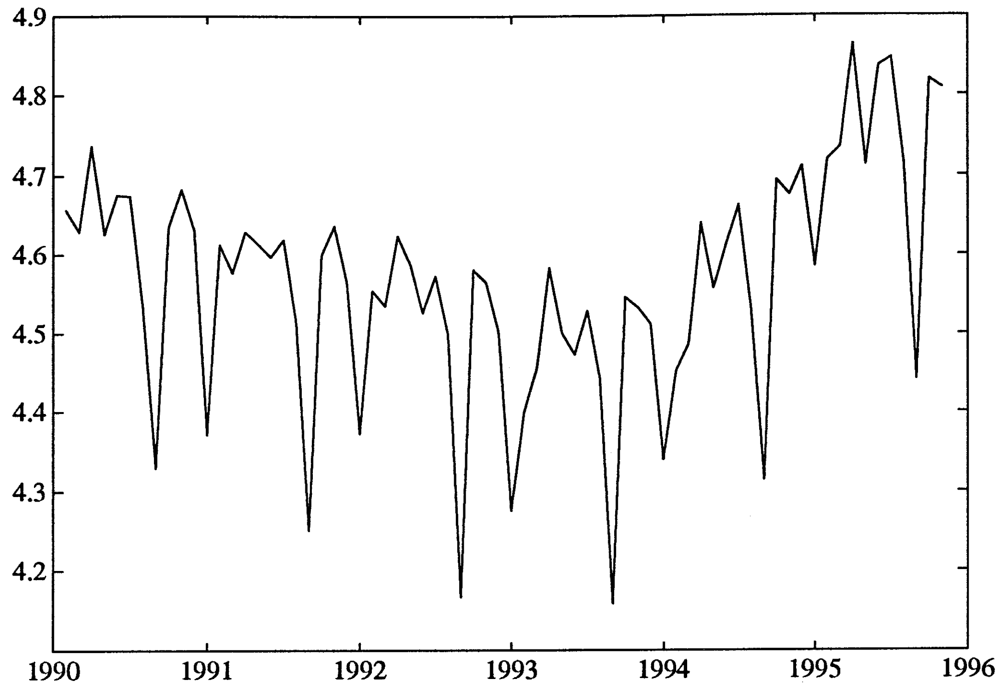


Figure 4: Series Spectrum and Top Heavy Component Spectra

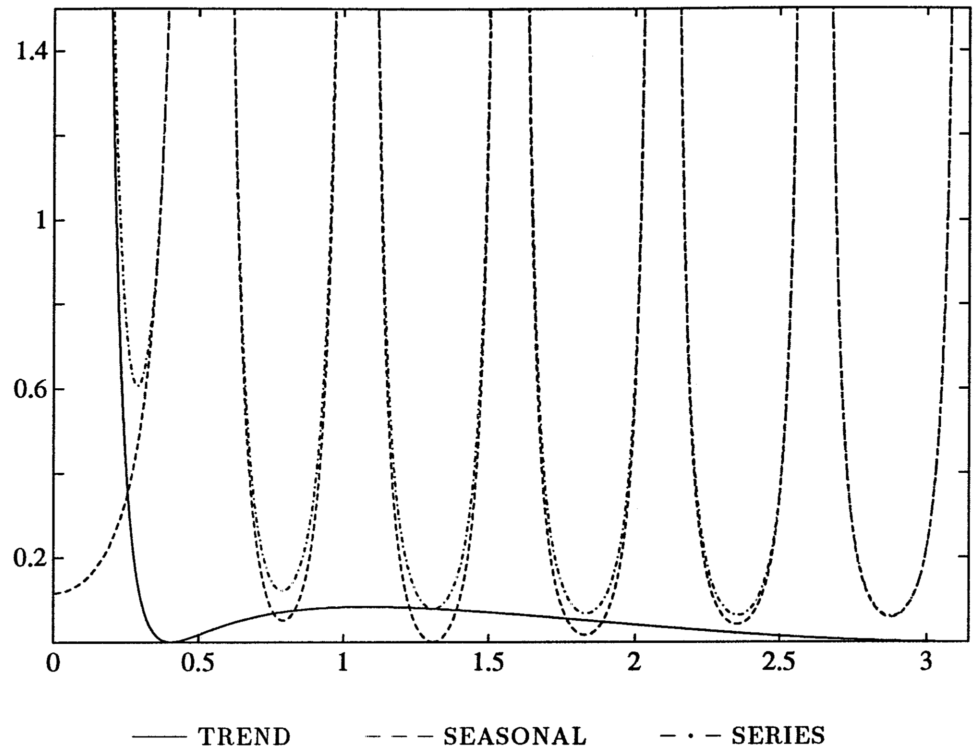


Figure 5: Series and Seasonally Adjusted Component Spectra

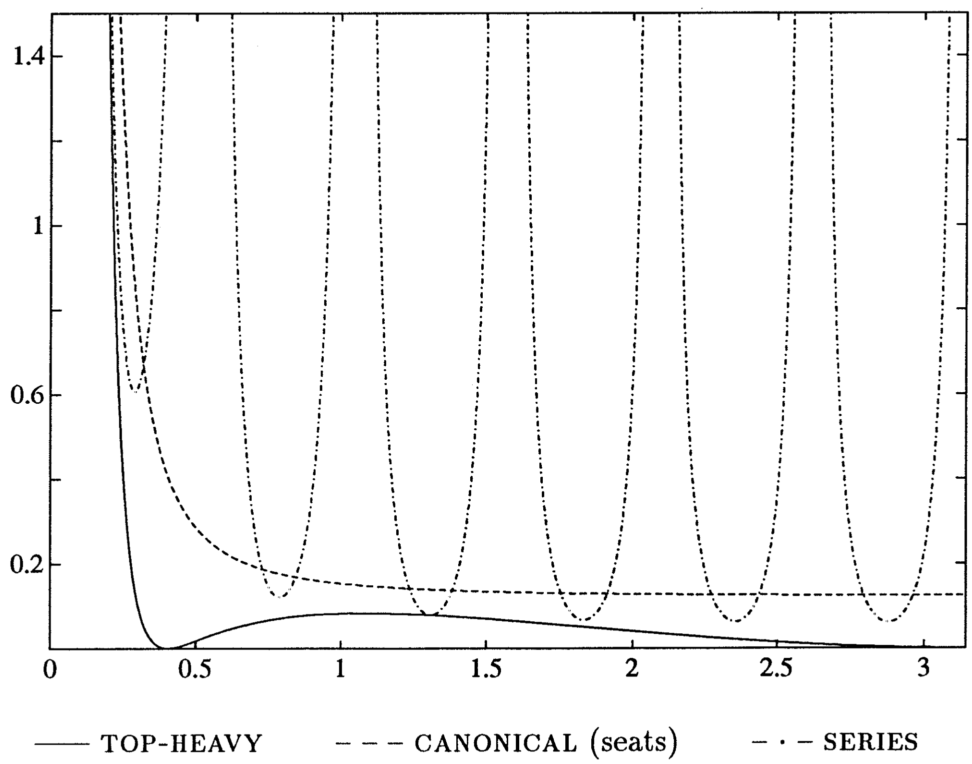


Figure 6: Seasonally Adjusted Estimators

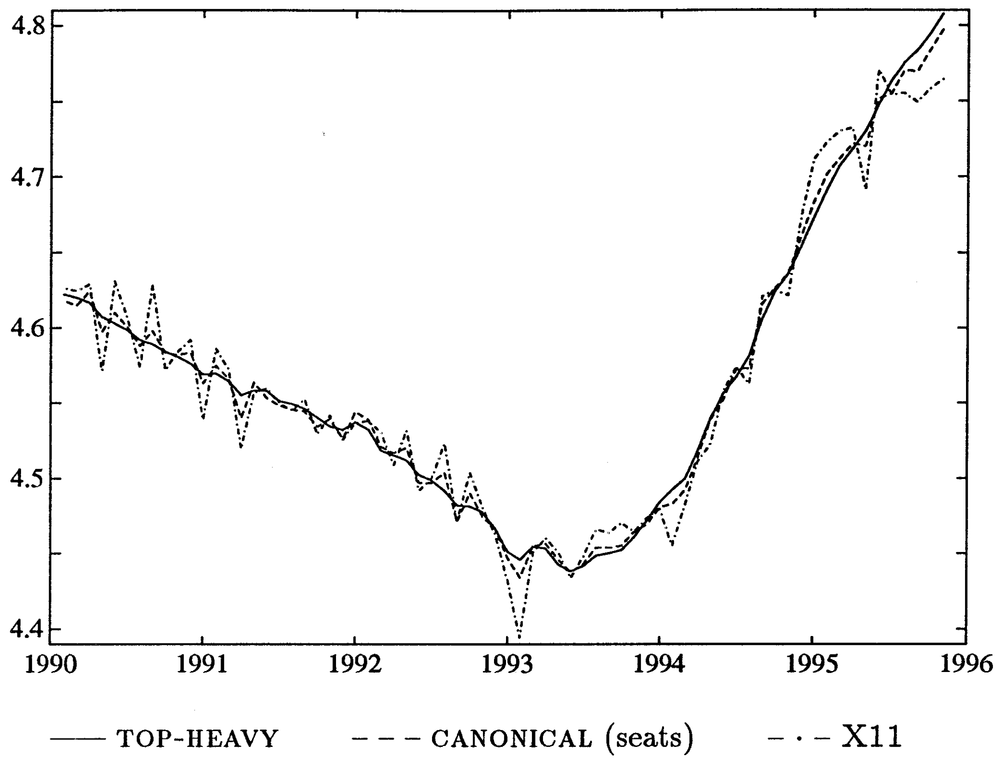


Figure 7: Seasonally Adjusted Estimators

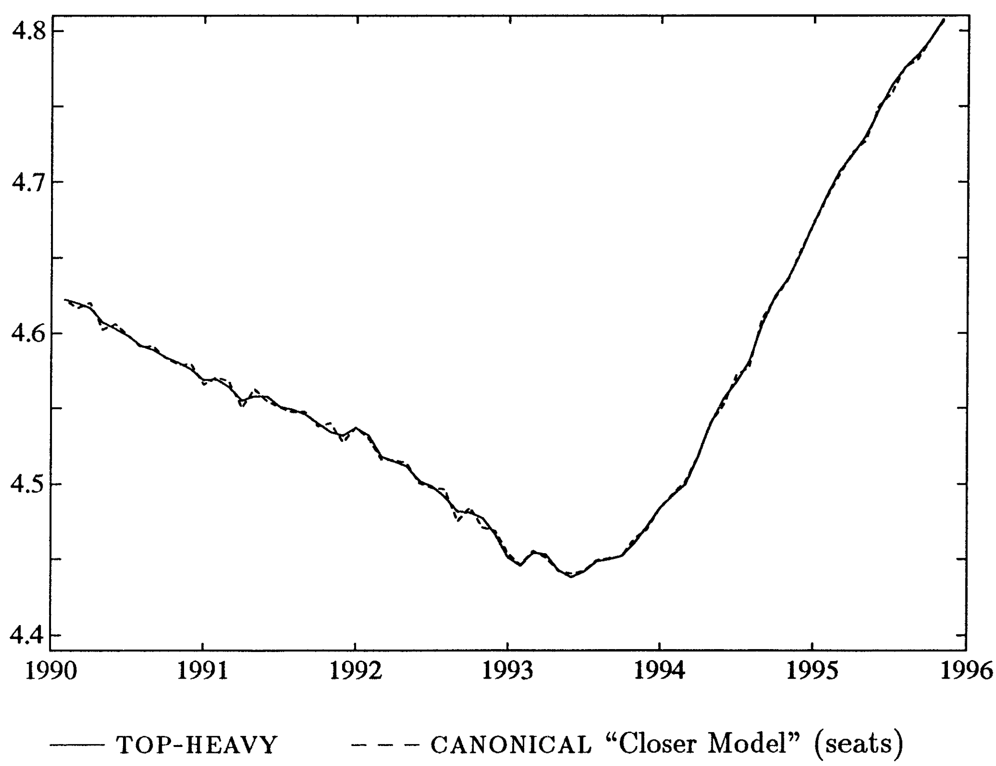


Figure 8: Spectra of Series and Adjusted Series Estimators

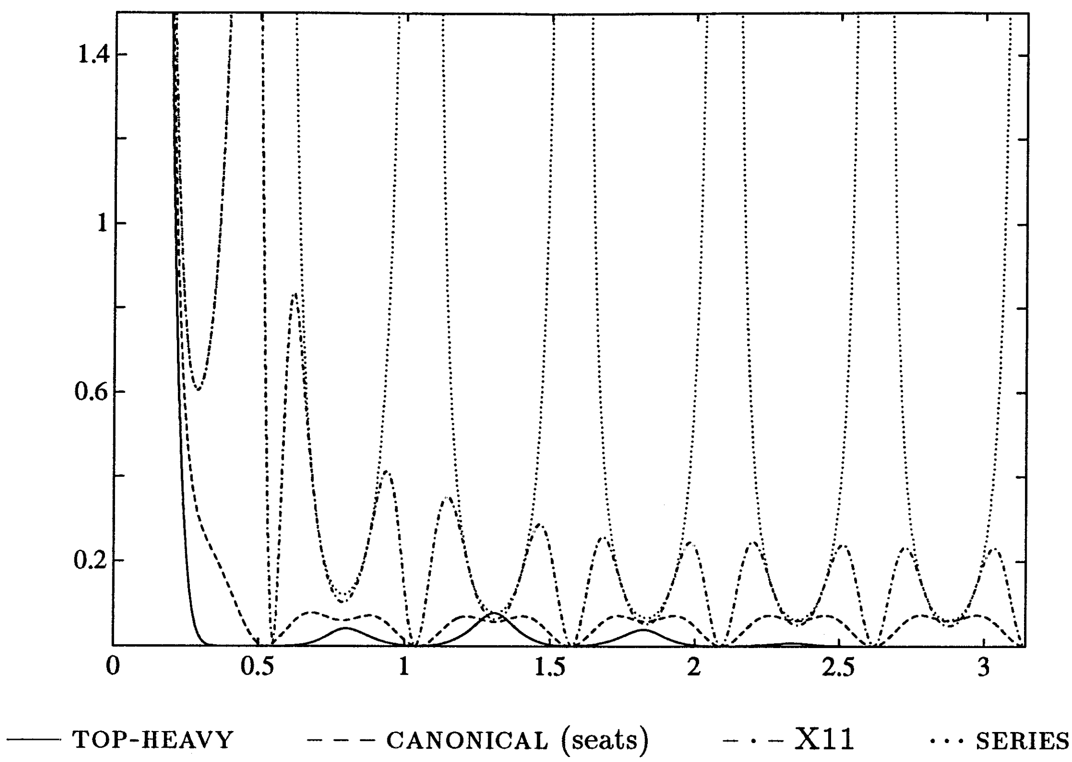


Figure 9: Spectra of Series and Seasonal Estimators

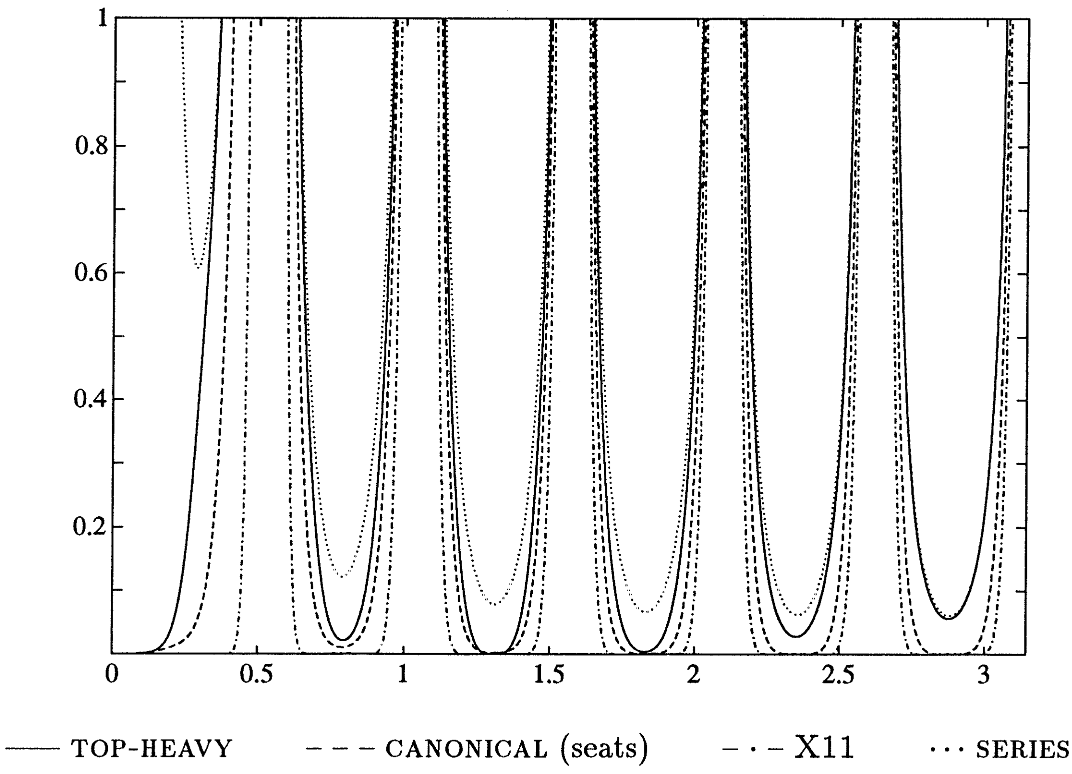


Figure 10: Seasonal Component Estimators - August

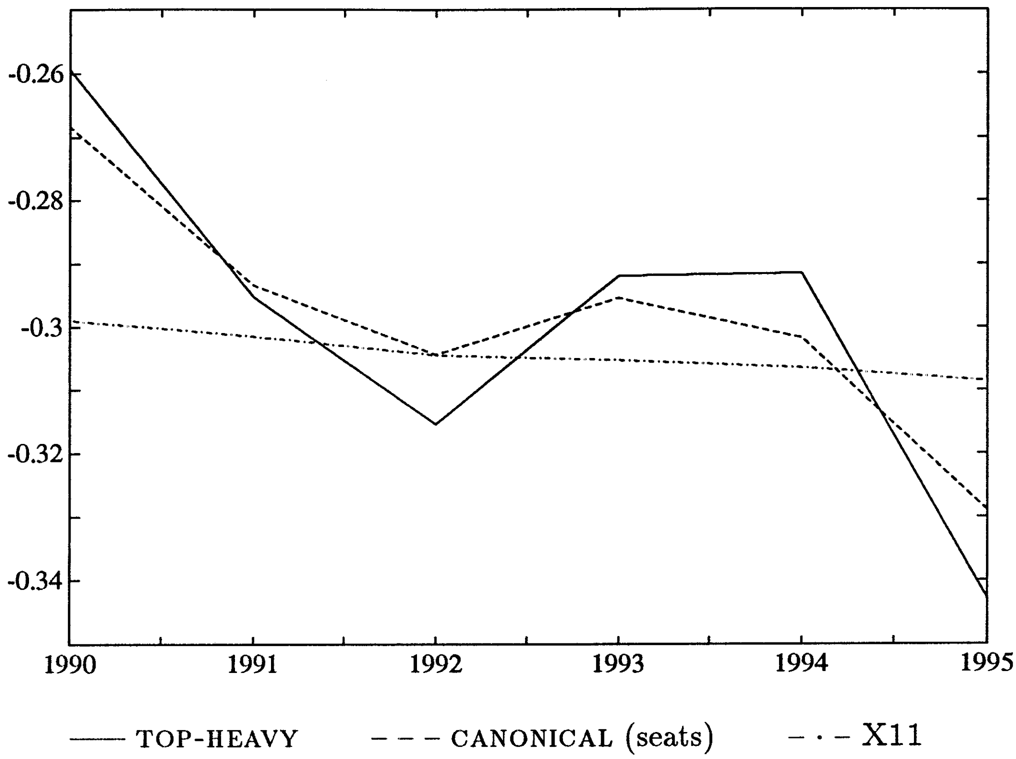


Figure 11: Seasonal Component Estimators - December

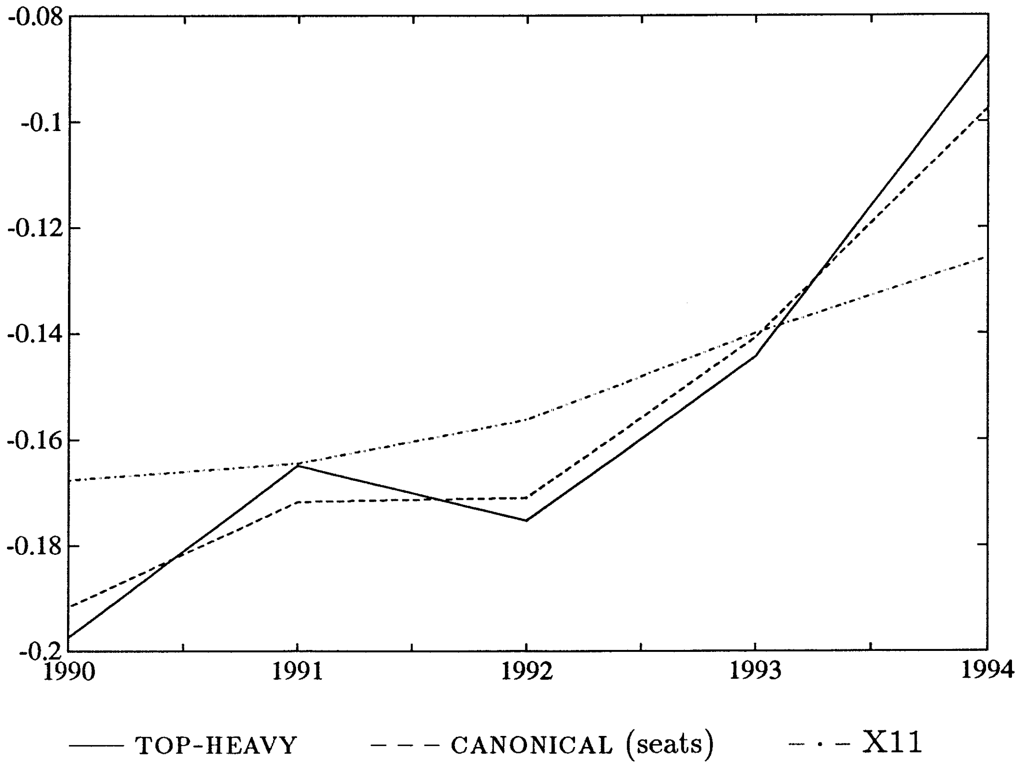


Figure 12: Seasonally Adjusted Estimators

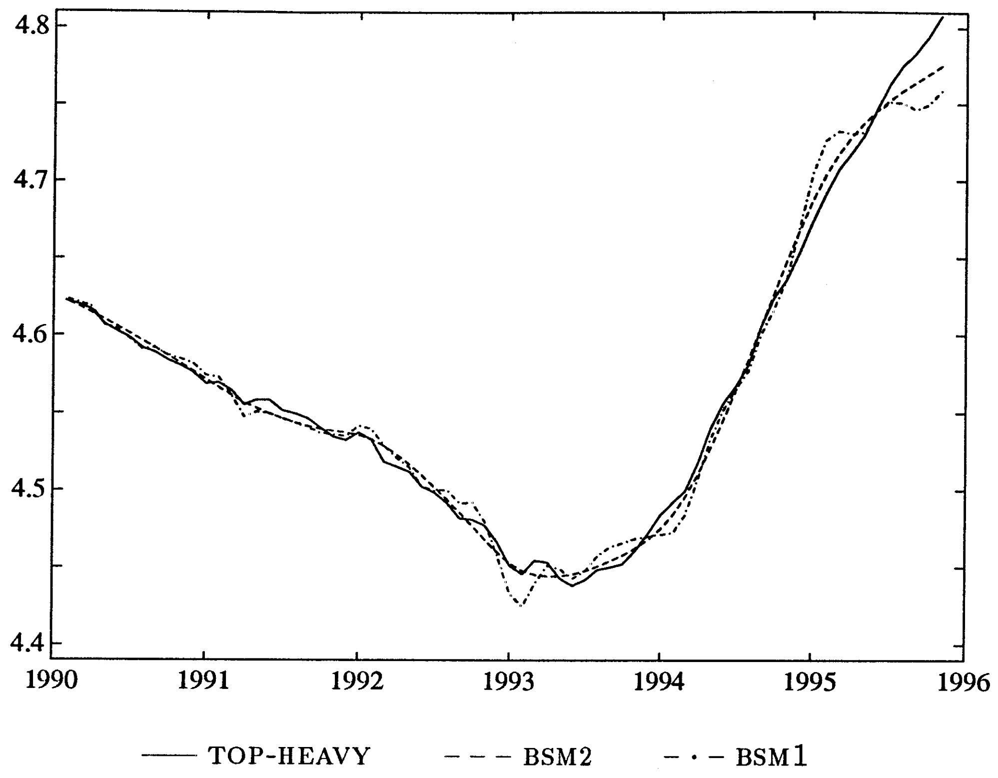


Figure 13: Seasonal Component Estimators - August

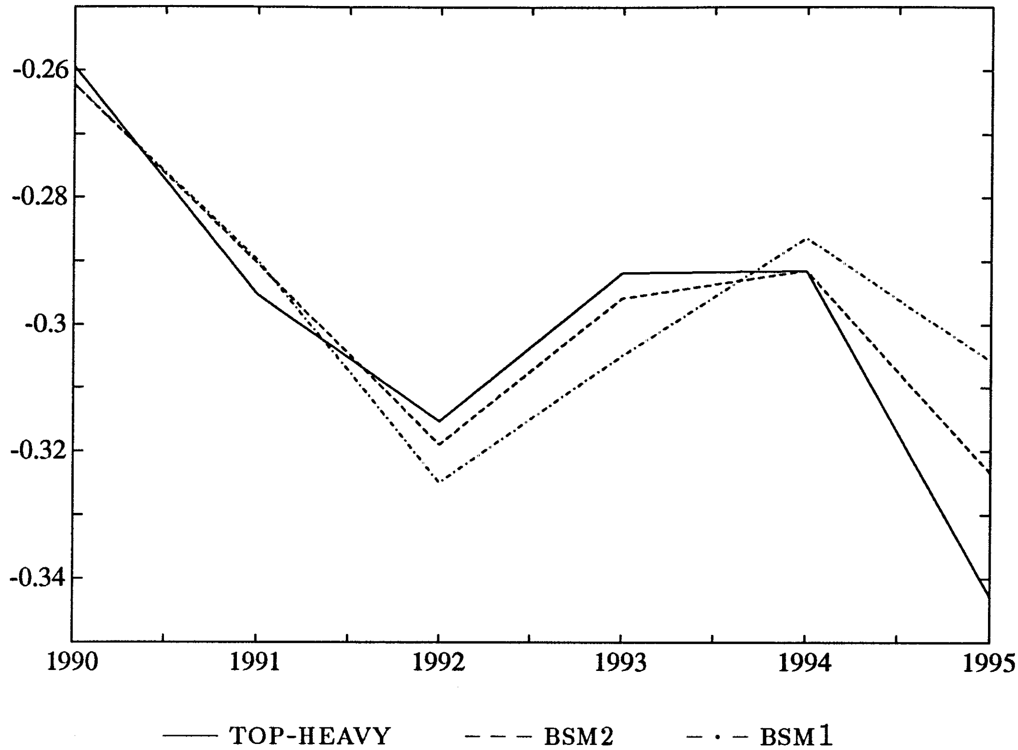
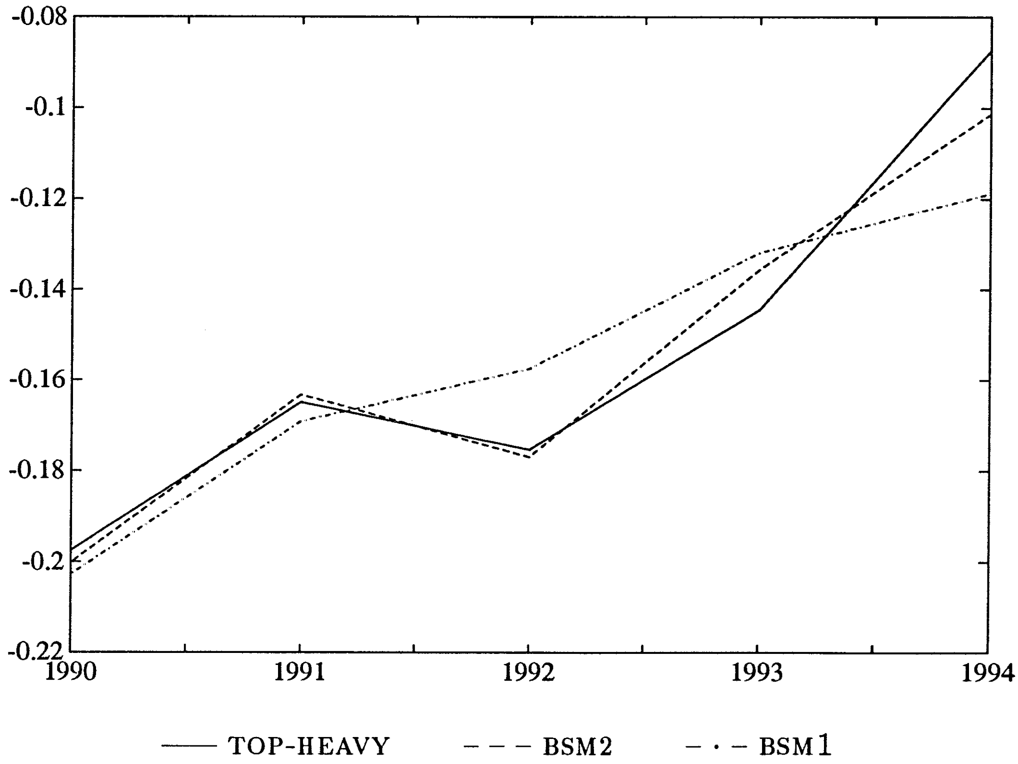


Figure 14: Seasonal Component Estimators - December



References

BANK OF ITALY (1997), *Economic Bulletin*, February.

BANK OF SPAIN (1993), *Boletin Económico*, February.

BELL, W.R. (1984), "Signal Extraction for Nonstationary Time Series", *The Annals of Statistics*, 12, 2, 646-664.

BELL W.R., AND HILLMER, S.C. (1984), "Issues Involved with the Seasonal Adjustment of Economic Time Series", *Journal of Business and Economic Statistics*, 2, 291-320.

BELL W.R., HILLMER S.C., AND TIAO G.C. (1981), "Modeling Considerations in the Seasonal Adjustment of Economic Time Series", *ASA-Census-NBER Conference on Applied Time Series Analysis of Economic Data.*

BELL, W.R. AND MONSELL, B.C. (1992), "X-11 Symmetric Linear Filters and their Transfer Functions", Bureau of the Census, Research Report n. RR 92/15, Washington.

BOX, G.E.P AND JENKINS, G.M (1970): *Time Series Analysis: Forecasting and Control*, San Francisco: Holden Day.

BOX, G.E.P, HILLMER S.C., AND TIAO G.C. (1978), "Analysis and Modeling of Seasonal Time Series", in *Seasonal Analysis of Time Series*, ed. A. Zellner, Washington, D.C: U.S. Department of Commerce, Bureau of the Census, 309-334.

BURMAN, J.P. (1980), "Seasonal Adjustment by Signal Extraction", *Journal of the Royal Statistical Society*, Ser. A, 143, 321-337.

CLEVELAND, W.P. AND TIAO, G.C. (1976) "Decomposition of Seasonal Time Series: a Model for the X-11 Program", *Journal of the American Statistical Association*, 71, 581-587.

DAGUM, E.B. (1988) "The X11-ARIMA/88 Seasonal Adjustment Method - Foundations and User's Manual", Time Series Research and Analysis Division, Statistics Canada, Ottawa.

ENGLE, R.F. (1978), "Estimating Structural Models of Seasonality", ed. A. Zellner, in *Seasonal Analysis of Economic Time Series*, Washington, D.C.: Dept. of Commerce, Bureau of the Census, 281-297.

EUROSTAT (1996), *Industrial Trends: Monthly Statistics*, Luxembourg, June.

FINDLEY, D.F. (1996), Comment to "Is Seasonal Adjustment Series a Linear or Nonlinear Data-Filtering Process?", by Ghysels et al., *Journal of Business and Economic Statistics*, 14, 389-393.

GERSCH, W. AND KITAGAWA, G. (1983), "The Prediction of Time Series with Trends and Seasonalities", *Journal of Business and Economic Statistics* 1, 253-264.

GHYSELS E. (1997), "Seasonal Adjustment and Other Data Transformations", *Journal of Business and Economic Statistics* 15, 410-418.

GHYSELS E., GRANGER C.W, AND SIKLOS, P.L. (1996), "Is Seasonal Adjustment Series a Linear or Nonlinear Data-Filtering Process?", with comments, *Journal of Business and Economic Statistics* 14, 374-397.

HARVEY, A.C. (1985), "Trends and Cycles in Macroeconomic Time Series", *Journal of Business and Economic Statistics*, 3, 3, 216-227.

HARVEY, A.C. (1989), *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge: Cambridge University Press.

HARVEY, A.C., AND TODD, P.H.J. (1983), "Forecasting Economic Time Series with Structural and Box-Jenkins Models : a Case Study", *Journal of Business and Economic Statistics*, 1, 299-307.

HAUSMAN, J.A., AND WATSON, M.W. (1985), "Errors-in-Variables and Seasonal Adjustment Procedures", *Journal of the American Statistical Association*, 80, 531-540.

HILLMER S.C. , AND TIAO G.C. (1982) "An ARIMA-Model-Based Approach to Seasonal Adjustment", *Journal of the American Statistical Association*, 77, 63-70.

HODRICK, R. AND PRESCOTT, E. (1980), "Post-War U.S. Business Cycles: An Empirical Investigation", Carnegie Mellon University Manuscript.

HOTTA, L.K. (1989), "Identification of Unobserved Components Models", *Journal of Time Series Analysis*, 10, 259-270.

MARAVALL, A. (1985), "On Structural Time series Models and the Characterization of Components", *Journal of Business and Economic Statistics*, 3, 4, 350-355.

MARAVALL, A. (1996), "Unobserved Components in Economic Time Series", *Handbook of Applied Econometrics*, Vol. 1.

MARAVALL, A. AND GÓMEZ, V. (1992), "Signal Extraction in Economic Time Series: Program SEATS", EUI Working Paper ECO No. 92/65, Department of Economics, European University Institute.

MARAVALL, A. AND PIERCE, D.A. (1987), "A prototypical Seasonal Adjustment Model", *Journal of Time Series Analysis*, 8, 177-193.

NELSON, C.R. AND PLOSSER, G.J. (1982), "Trends and Random Walks in Macroeconomic Time Series", *Journal of Monetary Economics*, 10, 139-162.

PIERCE, D.A. (1979), "Signal Extraction Error in Nonstationary Time Series", *Annals of Statistics*, 7, 1303-1320.

SHISKIN, J., YOUNG, A.H. AND MUSGRAVE, J.C. (1967), "The X-11 Variant of the Census Method II Seasonal Adjustment Program", Technical Paper 15, Washington, D.C.: Bureau of the Census.

VARIAN, H.R. (1984), *Microeconomic Analysis*, 2nd edition, W.W. Norton & Company, Inc. New York.